The behavior of basic fields

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Abstract

A basic field is defined in the realm of a mathematical modeling platform that is based on a collection of floating platforms and an embedding platform. Each floating platform is represented by a quaternionic separable Hilbert space. The embedding platform is a non-separable Hilbert space. A basic field is a continuum eigenspace of an operator that resides in the non-separable embedding Hilbert space. The continuum can be described by a quaternionic function, and its behavior is described by quaternionic differential calculus. The separable Hilbert spaces contain the point-like artifacts that trigger the basic field.

1 Modeling platform

The modeling platform is taken from the Hilbert Book Model. It is based on a series of quaternionic separable Hilbert spaces that each manage a private parameter space as eigenspace of a normal operator. The modeling platform applies versions of the quaternionic number system as parameter values, target values, and eigenvalues.

All Hilbert spaces use the same underlying vector space. A subspace of this vector space scans over the vector space as a function of a real progression parameter. The private parameter spaces of the separable Hilbert spaces float over a background parameter space, which is the private eigenspace of an infinite dimensional separable Hilbert space. This Hilbert space is the unique companion of a quaternionic non-separable Hilbert space, which has a continuum private parameter space that embeds the background parameter space and inhabits an operator that manages the continuum eigenspace, which represents the considered basic field. A dedicated quaternionic function describes this field. Quaternionic differential and integral calculus describe the behavior of the field.

The interaction of the field with excitations is defined by second order partial differential equations. This document treats the corresponding differential and integral calculus. The paper "Structure of physical reality" offers an overview of the highlights of the Wikiversity Hilbert Book Model Project. The document "The behavior of basic fields" treats the mathematics that is applied in the Hilbert Bool Model.

2 Quaternions

Hilbert spaces can only cope with number systems whose members form a divisions ring. Quaternionic number systems represent the most versatile division ring. Quaternionic number systems exist in many versions that differ in the way that coordinate systems can sequence them. Quaternions can store a combination of a scalar time-stamp and a three-dimensional spatial location. Thus, they are ideally suited as storage bins for dynamic geometric data.

In this paper, we represent quaternion q by a one-dimensional real part q_r and a three-dimensional imaginary part $\vec{q}$. The summation is commutative and associative

The following quaternionic multiplication rule describes most of the arithmetic properties of the quaternions.

$$c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \quad (2.1.1)$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system.

A quaternionic conjugation exists

$$q^* = (q_r + \vec{q})^* = q_r - \vec{q} \quad (2.1.2)$$

$$\left(ab\right)^* = b^* a^* \quad (2.1.3)$$

The norm $|q|$ equals

$$|q| = \sqrt{q_r^2 + \langle \vec{q}, \vec{q} \rangle} \quad (2.1.4)$$

$$q^{-1} = \frac{1}{q} = \frac{q}{|q|^2} \quad (2.1.5)$$

$$q = |q| \exp\left(q_\varphi \frac{\vec{q}}{|\vec{q}|} \right) \quad (2.1.6)$$

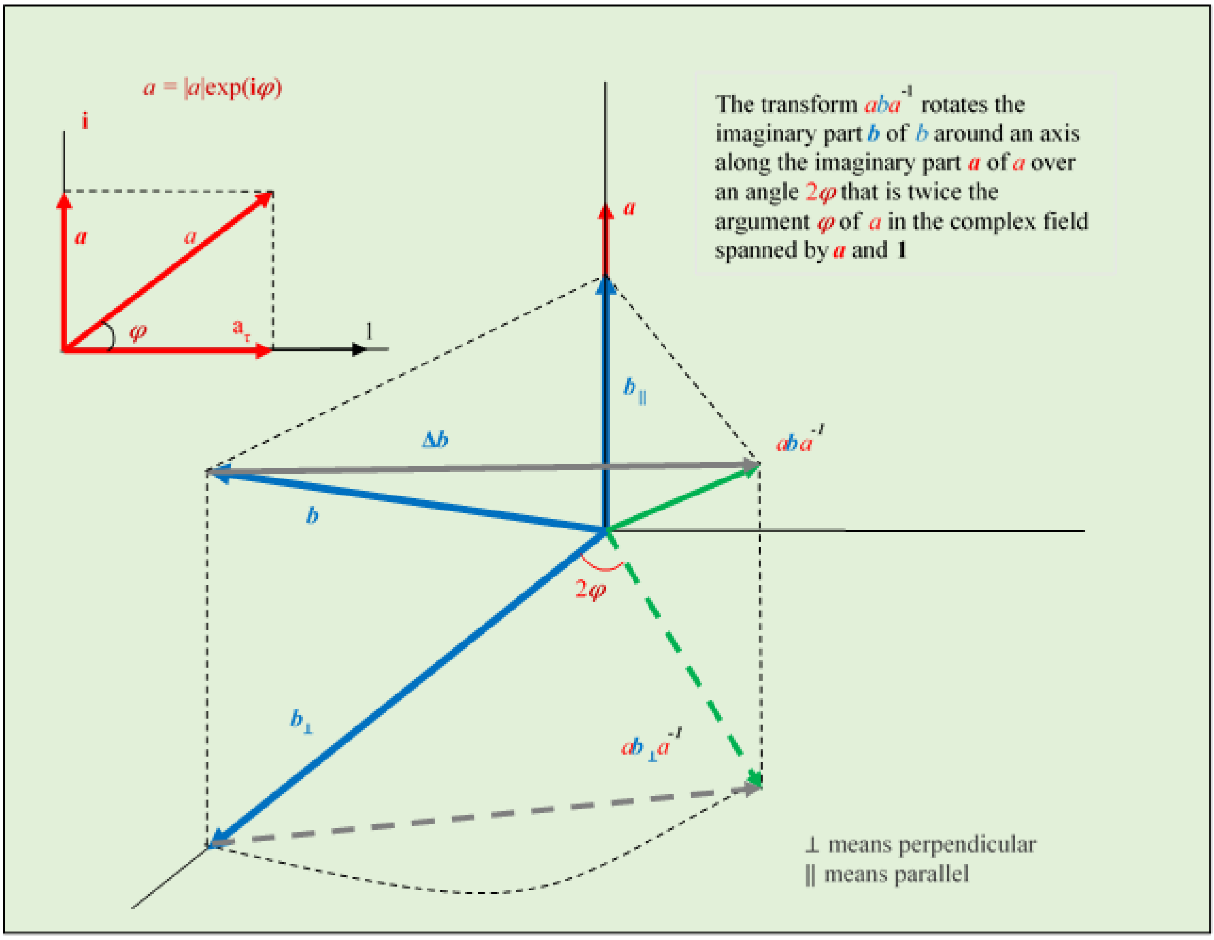
$\frac{\vec{q}}{|\vec{q}|}$ is the spatial direction of q .

A quaternion and its inverse can rotate a part of a third quaternion. The imaginary part of the rotated quaternion that is perpendicular to the imaginary part of the first quaternion is rotated over an angle that is twice the angle of the argument φ between the real part and the imaginary part of the first quaternion. This makes it possible to shift the imaginary part of the third quaternion to a different dimension. For that reason, must $\varphi = \pi / 4$.

Each quaternion c can be written as a product of two complex numbers a and b of which the imaginary base vectors are perpendicular

$$\begin{aligned} c &= \left(a_r + a_1 \vec{i}\right)\left(b_r + b_2 \vec{j}\right) \\ &= a_r b_r + \left(a_1 + b_r\right)\vec{i} + \left(a_r + b_2\right)\vec{j} + a_1 b_2 \vec{k} = c_r + c_1 \vec{i} + c_2 \vec{j} + c_3 \vec{k} \end{aligned} \quad (2.1.7)$$

Where $\vec{k} = \vec{i} \times \vec{j}$



3 Quaternionic Hilbert spaces

Around the turn of the nineteenth century into the twentieth century David Hilbert and others developed the type of vector space that later got Hilbert's name.

The Hilbert space is a particular vector space because it defines an inner product for every pair of its member vectors.

That inner product can take values of a number system for which every non-zero member owns a unique inverse. This requirement brands the number system as a division ring.

Only three suitable division rings exist:

- The real numbers
- The complex numbers
- The quaternions

Hilbert spaces cannot cope with bi-quaternions or octonions

3.1 Bra's and ket's

Paul Dirac introduced a handy formulation for the inner product that applies a bra and a ket.

The bra $\langle f|$ is a covariant vector, and the ket $|g\rangle$ is a contravariant vector. The inner product $\langle f|g\rangle$ acts as a metric.

For bra vectors hold

$$\langle f| + \langle g| = \langle g| + \langle f| = \langle f+g| \tag{3.1.1}$$

$$(\langle f+g|) + \langle h| = \langle f| + (\langle g+h|) = \langle f+g+h| \tag{3.1.2}$$

For ket vectors hold

$$|f\rangle + |g\rangle = |g\rangle + |f\rangle = |f+g\rangle \tag{3.1.3}$$

$$(|f+g\rangle) + |h\rangle = |f\rangle + (|g+h\rangle) = |f+g+h\rangle \tag{3.1.4}$$

For the inner product holds

$$\langle f|g\rangle = \langle g|f\rangle^* \tag{3.1.5}$$

For quaternionic numbers α and β hold

$$\langle \alpha f|g\rangle = \langle g|\alpha f\rangle^* = (\langle g|f\rangle \alpha) = \alpha^* \langle f|g\rangle \tag{3.1.6}$$

$$\langle f|\beta g\rangle = \langle f|g\rangle \beta \tag{3.1.7}$$

$$\langle (\alpha+\beta) f|g\rangle = \alpha^* \langle f|g\rangle + \beta^* \langle f|g\rangle = (\alpha+\beta)^* \langle f|g\rangle \tag{3.1.8}$$

Thus

$$\alpha |f\rangle \tag{3.1.9}$$

$$\langle \alpha f| = \alpha^* \langle f| \tag{3.1.10}$$

$$|\alpha g\rangle = |g\rangle \alpha \tag{3.1.11}$$

We made a choice. Another possibility would be $\langle \alpha f| = \alpha \langle f|$ and $|\alpha g\rangle = \alpha^* |g\rangle$

In mathematics a topological space is called separable if it contains a countable dense subset; that is, there exists a sequence $\{|f_i\rangle\}_{i=\infty}^{i=0}$ of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence.

Its values on this countable dense subset determine every continuous function on the separable space $\mathfrak{H}$.

The Hilbert space $\mathfrak{H}$ is separable. That means that a countable row of elements $\{|f_n\rangle\}$ exists that spans the whole space.

If $\langle f_m|f_n\rangle = \delta(m,n)$ [1 if n=m; otherwise 0], then $\{|f_n\rangle\}$ is an orthonormal base of Hilbert space $\mathfrak{H}$.

A ket base $\{|k\rangle\}$ of $\mathfrak{H}$ is a minimal set of ket vectors $|k\rangle$ that span the full Hilbert space $\mathfrak{H}$.

Any ket vector $|f\rangle$ in $\mathfrak{H}$ can be written as a linear combination of elements of $\{|k\rangle\}$.

$$|f\rangle = \sum_k |k\rangle\langle k|f\rangle \tag{3.1.12}$$

A bra base $\{\langle b|\}$ of $\mathfrak{H}^\dagger$ is a minimal set of bra vectors $\langle b|$ that span the full Hilbert space $\mathfrak{H}^\dagger$.

Any bra vector $\langle f|$ in $\mathfrak{H}^\dagger$ can be written as a linear combination of elements of $\{\langle b|\}$.

$$\langle f| = \sum_b \langle f|b\rangle\langle b| \tag{3.1.13}$$

Usually, a base selects vectors such that their norm equals 1. Such a base is called an orthonormal base

3.2 Operators

Operators act on a subset of the elements of the Hilbert space.

An operator L is linear when for all vectors $|f\rangle$ and $|g\rangle$ for which L is defined and for all quaternionic numbers α and β

$$L|\alpha f\rangle + L|\beta g\rangle = L|f\rangle\alpha + L|g\rangle\beta = L(|f\rangle\alpha + |g\rangle\beta) = L(|\alpha f\rangle + |\beta g\rangle) \tag{3.2.1}$$

The operator B is **colinear** when for all vectors $|f\rangle$ for which B is defined and for all quaternionic numbers α there exists a quaternionic number γ such that

$$\alpha B|f\rangle = B|f\rangle\gamma\alpha\gamma^{-1} \equiv B|\gamma\alpha\gamma^{-1}f\rangle \tag{3.2.2}$$

If $|a\rangle$ is an eigenvector of the operator A with quaternionic eigenvalue α ,

$$A|a\rangle = |a\rangle\alpha \tag{3.2.3}$$

then $|\beta a\rangle$ is an eigenvector of A with quaternionic eigenvalue $\beta^{-1}\alpha\beta$.

$$A|\beta a\rangle = A|a\rangle\beta = |a\rangle\alpha\beta = |\beta a\rangle\beta^{-1}\alpha\beta \tag{3.2.4}$$

$A^\dagger$ is the **adjoint** of the **normal** operator A

$$\langle f|Ag\rangle = \langle fA^\dagger|g\rangle = \langle g|A^\dagger f\rangle^* \tag{3.2.5}$$

$$A^{\dagger\dagger} = A \tag{3.2.6}$$

$$(A+B)^\dagger = A^\dagger + B^\dagger \tag{3.2.7}$$

$$(AB)^\dagger = B^\dagger A^\dagger \tag{3.2.8}$$

If $A = A^\dagger$ then A is a **self-adjoint** operator.

A linear operator L is normal if $LL^\dagger$ exists and $LL^\dagger = L^\dagger L$

For the normal operator N holds

$$\langle Nf|Ng\rangle = \langle NN^\dagger f|g\rangle = \langle f|NN^\dagger g\rangle \tag{3.2.9}$$

Thus

$$N = N_r + \vec{N} \tag{3.2.10}$$

$$N^\dagger = N_r - \vec{N} \tag{3.2.11}$$

$$N_r = \frac{N + N^\dagger}{2} \tag{3.2.12}$$

$$\vec{N} = \frac{N - N^\dagger}{2} \tag{3.2.13}$$

$$NN^\dagger = N^\dagger N = N_r N_r + \langle \vec{N}, \vec{N} \rangle = |N|^2 \tag{3.2.14}$$

N_r is the Hermitian part of N.

$\vec{N}$ is the anti-Hermitian part of N.

For two normal operators A and B holds

$$AB = A_r B_r - \langle \vec{A}, \vec{B} \rangle + A_r \vec{B} + \vec{A} B_r \pm \vec{A} \times \vec{B} \tag{3.2.15}$$

For a unitary transformation U holds

$$\langle Uf \mid Ug \rangle = \langle f \mid g \rangle \tag{3.2.16}$$

The closure of separable Hilbert space $\mathfrak{H}$ means that converging rows of vectors of $\mathfrak{H}$ converge to a vector in $\mathfrak{H}$.

3.2.1 Operator construction

$| f \rangle \langle g |$ is a constructed operator.

$$|g\rangle\langle f| = (|f\rangle\langle g|)^\dagger \tag{3.2.17}$$

For the orthonormal base $\{|q_i\rangle\}$ consisting of eigenvectors of the reference operator holds

$$\langle q_n \mid q_m \rangle = \delta_{nm} \tag{3.2.18}$$

The reverse bra-ket method enables the definition of new operators that are defined by quaternionic functions.

$$\langle g \mid F \mid h \rangle = \sum_{i=1}^{N} \{ \langle g | q_i \rangle F(q_i) \langle q_i | h \rangle \} \tag{3.2.19}$$

The symbol F is used both for the operator F and the quaternionic function $F(q)$. This enables the shorthand

$$F \equiv |q_i\rangle F(q_i) \langle q_i| \tag{3.2.20}$$

It is evident that

$$F^{\dagger} \equiv |q_i\rangle F^{*}(q_i)\langle q_i| \tag{3.2.21}$$

For reference operator $\mathfrak{R}$ holds

$$\mathfrak{R} = |q_i\rangle q_i \langle q_i| \tag{3.2.22}$$

If $\{|q_i\rangle\}$ consists of all rational values of the version of the quaternionic number system that $\mathfrak{H}$ applies then the eigenspace of $\mathfrak{R}$ represents the private parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines operator F in the formula (3.2.20).

3.3 Non-separable Hilbert space

Every infinite dimensional separable Hilbert space $\mathfrak{H}$ owns a unique non-separable companion Hilbert space $\mathcal{H}$. This is achieved by the closure of the eigenspaces of the reference operator and the defined operators. In this procedure, on many occasions, the notion of the dimension of subspaces loses its sense.

Gelfand triple and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces.

In the non-separable Hilbert space, for operators with continuum eigenspaces, the reverse bra-ket method turns from a summation into an integration.

$$\langle g|F|h\rangle \equiv \int\iiint\{\langle g|q\rangle F(q)\langle q|h\rangle\} dV d\tau \tag{3.3.1}$$

Here we omitted the enumerating subscripts that were used in the countable base of the separable Hilbert space.

The shorthand for the operator F is now

$$F \equiv |q\rangle F(q)\langle q| \tag{3.3.2}$$

For eigenvectors $|q\rangle$ the function $F(q)$ defines as

$$F(q) = \langle q|Fq\rangle = \int\iiint\{\langle q|q'\rangle F(q')\langle q'|q\rangle\} dV' d\tau' \tag{3.3.3}$$

The reference operator $\mathcal{R}$ that provides the continuum background parameter space as its eigenspace follows from

$$\langle g|\mathcal{R}h\rangle \equiv \int\iiint\{\langle g|q\rangle q\langle q|h\rangle\} dV d\tau \tag{3.3.4}$$

The corresponding shorthand is

$$\mathcal{R} \equiv |q\rangle q\langle q| \tag{3.3.5}$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, it becomes clear that every infinite dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that can be considered to embed its separable companion.

The reverse bracket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology.

4 Quaternionic differential calculus

4.1 Field equations

Maxwell equations apply the three-dimensional nabla operator in combination with a time derivative that applies coordinate time. The Maxwell equations derive from results of experiments. For that reason, those equations contain physical units.

In this treatment, the quaternionic partial differential equations apply the quaternionic nabla. The equations do not derive from the results of experiments. Instead, the formulas apply the fact that the quaternionic nabla behaves as a quaternionic multiplying operator. The corresponding formulas do not contain physical units. This approach generates essential differences between Maxwell field equations and quaternionic partial differential equations.

The quaternionic partial differential equations form a complete and self-consistent set. They use the properties of the three-dimensional spatial nabla.

The corresponding formulas are taken from Bo Thidé's EMTF book., section Appendix F4.

Another online resource is Vector calculus identities.

The quaternionic differential equations play in a Euclidean setting that is formed by a continuum quaternionic parameter space and a quaternionic target space. The parameter space is the eigenspace of the reference operator of a quaternionic non-separable Hilbert space. The target space is eigenspace of a defined operator that resides in that same Hilbert space. The defined operator is specified by a quaternionic function that completely defines the field. Each basic field owns a private defining quaternionic function. All basic fields that are treated in this chapter are defined in this way.

Physical field theories tend to use a non-Euclidean setting, which is known as spacetime setting. This is because observers can only perceive in spacetime format. Thus, Maxwell equations use coordinate time, where the quaternionic differential equations use proper time. In both settings, the observed event is presented in Euclidean format. The hyperbolic Lorentz transform converts the Euclidean format to the perceived spacetime format. Chapter 8 treats the Lorentz transform. The Lorentz transform introduces time dilation and length contraction. Quaternionic differential calculus describes the interaction between discrete objects and the continuum at the location where events occur. Converting the results of this calculus by the Lorentz transform will describe the information that the observers perceive. Observers perceive in spacetime format. This format features a Minkowski signature. The Lorentz transform converts from the Euclidean storage format at the situation of the observed event to the perceived spacetime format. Apart from this coordinate transformation, the perceived scene is influenced by the fact that the retrieved information travels through a field that can be deformed and acts as the living space for both the observed event and the observer. Consequently, the information path deforms with its carrier field and this affects the transferred information. In this chapter, we only treat what happens at the observed event. So, we ignore the Lorentz coordinate transform, and we are not affected by deformations of the information path.

The Hilbert Book Model archives all dynamic geometric data of all discrete creatures that exist in the model in eigenspaces of separable Hilbert spaces whose private parameter spaces float over the background parameter space, which is the private parameter space of the non-separable Hilbert space. For example, elementary particles reside on a private floating platform that is implemented by a private separable Hilbert space.

Quantum physicists use Hilbert spaces for the modeling of their theory. However, most quantum physicists apply complex-number based Hilbert spaces. Quaternionic quantum mechanics appears to represent a natural choice. Quaternionic Hilbert spaces store the dynamic geometric data in the Euclidean format in quaternionic eigenvalues that consists of a real scalar valued time-stamp and a spatial, three-dimensional location.

In the Hilbert Book Model, the instant of storage of the event data is irrelevant if it coincides with or precedes the stored time stamp. Thus, the model can store all data at an instant, which precedes all stored timestamp values. This impersonates the Hilbert Book Model as a creator of the universe in which the observable events and the observers exist. On the other hand, it is possible to place the instant of archival of the event at the instant of the event itself. It will then coincide with the archived time-stamp. In both interpretations, after sequencing the time-stamps, the repository tells the life story of the discrete objects that are archived in the model. This story describes the ongoing embedding of the separable Hilbert spaces into the non-separable Hilbert space. For each floating separable Hilbert space this embedding occurs step by step and is controlled by a private stochastic process, which owns a characteristic function. The result is a stochastic hopping path that walks through the private parameter space of the platform. A coherent recurrently regenerated hop landing location swarm characterizes the corresponding elementary object.

Elementary particles are elementary modules. Together they constitute all other modules that occur in the model. Some modules constitute modular systems. A dedicated stochastic process controls the binding of the components of the module. This process owns a characteristic function that equals a dynamic superposition of the characteristic functions of the stochastic processes that control the components. Thus, superposition occurs in Fourier space. The superposition coefficients act as gauge factors that implement displacement generators, which control the internal locations of the components. In other words, the superposition coefficients may install internal oscillations of the components. These oscillations are described by differential equations.

4.2 Fields

In the Hilbert Book Model fields are eigenspaces of operators that reside in the non-separable Hilbert space. Continuous or mostly continuous functions define these operators, and apart from some discrepant regions, their eigenspaces are continuums. These regions might reduce to single discrepant point-like artifacts. The parameter space of these functions is constituted by a version of the quaternionic number system. Consequently, the real number valued coefficients of these parameters are mutually independent, and the differential change can be expressed in terms of a linear combination of partial differentials. Now the total differential change df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x}\vec{i}dx + \frac{\partial f}{\partial y}\vec{j}dy + \frac{\partial f}{\partial z}\vec{k}dz \tag{4.2.1}$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ are quaternions.

The quaternionic nabla ∇ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system. Thus

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \tag{4.2.2}$$

The Hilbert Book Model assumes that the quaternionic fields are moderately changing, such that only first and second order partial differential equations describe the model. These equations can describe fields of which the continuity gets disrupted by point-like artifacts. Warps, clamps and Green's functions describe the reaction of the field on such disruptions.

4.3 Field equations

Generalized field equations hold for all basic fields. Generalized field equations fit best in a quaternionic setting.

Quaternions consist of a real number valued scalar part and a three-dimensional spatial vector that represents the imaginary part.

The multiplication rule of quaternions indicates that several independent parts constitute the product.

$$c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \tag{4.3.1}$$

The $\pm$ indicates that quaternions exist in right-handed and left-handed versions.

The formula can be used to check the completeness of a set of equations that follow from the application of the product rule.

We define the quaternionic nabla as

$$\nabla \equiv \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{4.3.2}$$

$$\vec{\nabla} \equiv \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \tag{4.3.3}$$

$$\nabla_r \equiv \frac{\partial}{\partial \tau} \tag{4.3.4}$$

$$\phi = \phi_r + \vec{\phi} = \nabla \psi = \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right)\left(\psi_r + \vec{\psi}\right) = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{4.3.5}$$

$$\phi_r = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \tag{4.3.6}$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{4.3.7}$$

Further,

$\vec{\nabla} \psi_r$ is the gradient of ψ_r

$\left\langle \vec{\nabla}, \vec{\psi} \right\rangle$ is the divergence of $\vec{\psi}$

$\vec{\nabla} \times \vec{\psi}$ is the curl of $\vec{\psi}$

The change $\nabla \psi$ divides into five terms that each has a separate meaning. That is why these terms in Maxwell equations get different names and symbols. Every basic field offers these terms!

$$\vec{E} = -\nabla_r \vec{\psi} - \vec{\nabla} \psi_r \tag{4.3.8}$$

$$\vec{B} = \vec{\nabla} \times \psi \tag{4.3.9}$$

It is also possible to construct higher order equations. For example

$$\vec{J} = \vec{\nabla} \times \vec{B} - \nabla_r \vec{E} \qquad (4.3.10)$$

The equation (4.3.6) has no equivalent in Maxwell's equations. Instead, its right part is used as a gauge.

Two special second-order partial differential equations use the terms $\frac{\partial^2 \psi}{\partial \tau^2}$ and $\langle \vec{\nabla}, \vec{\nabla} \rangle \psi$

$$\phi = \left\{ \frac{\partial^2}{\partial \tau^2} - \langle \vec{\nabla}, \vec{\nabla} \rangle \right\} \psi \qquad (4.3.11)$$

$$\rho = \left\{ \frac{\partial^2}{\partial \tau^2} + \langle \vec{\nabla}, \vec{\nabla} \rangle \right\} \psi \qquad (4.3.12)$$

The equation (4.3.11) is the quaternionic equivalent of the wave equation.

The equation (4.3.12) can be divided into two first order partial differential equations.

$$\chi = \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = (\nabla_r + \vec{\nabla})(\nabla_r - \vec{\nabla})(\psi_r + \vec{\psi}) = (\nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle)\psi \qquad (4.3.13)$$

This composes from $\chi = \nabla^* \varphi$ and $\varphi = \nabla \psi$

$\frac{\partial^2}{\partial \tau^2} - \langle \vec{\nabla}, \vec{\nabla} \rangle$ is the quaternionic equivalent of d'Alembert's operator $\Box$.

The operator $\frac{\partial^2}{\partial \tau^2} + \langle \vec{\nabla}, \vec{\nabla} \rangle$ does not yet have an accepted name.

The Poisson equation equals

$$\rho = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi \qquad (4.3.14)$$

A very special solution of this equation is the Green's function $\frac{1}{\vec{q} - \vec{q'}}$ of the affected field

$$\nabla \frac{1}{\vec{q} - \vec{q'}} = -\frac{(\vec{q} - \vec{q'})}{|\vec{q} - \vec{q'}|^3} \qquad (4.3.15)$$

$$\langle \vec{\nabla}, \vec{\nabla} \rangle \frac{1}{|\vec{q} - \vec{q'}|} \equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{|\vec{q} - \vec{q'}|} \right\rangle = -\left\langle \vec{\nabla}, \vec{\nabla} \frac{(\vec{q} - \vec{q'})}{|\vec{q} - \vec{q'}|^3} \right\rangle = 4\pi\delta(\vec{q} - \vec{q'}) \qquad (4.3.16)$$

The spatial integral over Green's function is a volume.

(4.3.11) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| - c\left(\tau-\tau'\right)\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{4.3.17}$$

A one-dimensional type of shock front solution is

$$\psi = \vec{f}\left(\left|\vec{q}-\vec{q}'\right| - c\left(\tau-\tau'\right)\right) \tag{4.3.18}$$

The equation (4.3.11) is famous for its wave type solutions

$$\nabla_r \nabla_r \psi = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = \omega\psi = \exp\left(2\pi\vec{x}\tau\right) \tag{4.3.19}$$

Periodic harmonic actuators cause the appearance of waves,

The Helmholtz equation considers the quaternionic function that defines the field separable.

$$\psi\left(q_r, \vec{q}\right) = A\left(\vec{q}\right) T\left(q_r\right) \tag{4.3.20}$$

$$\frac{\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A}{A} = \frac{\nabla_r \nabla_r T}{T} = -k^2 \tag{4.3.21}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -k^2 A \tag{4.3.22}$$

$$\nabla_r \nabla_r T = -k^2 T \tag{4.3.23}$$

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A\left(r,\theta,\varphi\right) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left\{ \left(a_{lm} j_l\left(kr\right)\right) + b_{lm} Y_l^m\left(\theta,\varphi\right) \right\} \tag{4.3.24}$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics. These solutions play a role in the spectra of atomic modules.

A more general solution is a superposition of these basic types.

(4.3.12) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi = \frac{f\left(\vec{q}-\vec{q}' + c\left(\tau-\tau'\right)\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{4.3.25}$$

A one-dimensional type of shock front solution is

$$\psi = \vec{f}\left(\vec{q}-\vec{q}' + c\left(\tau-\tau'\right)\right) \tag{4.3.26}$$

Equation (4.3.12) offers no waves as part of its solutions.

During travel, the amplitude and the lateral direction $\frac{\vec{f}}{|\vec{f}|}$ of the one-dimensional shock fronts are fixed. The longitudinal direction is along $\frac{\vec{q}-\vec{q}'}{|\vec{q}-\vec{q}'|}$.

The shock fronts that are triggered by point-like actuators are the tiniest field excitations that exist. The actuator must fulfill significant restricting requirements. For example, a perfectly isotropic actuator must trigger the spherical shock front. The actuator can be a quaternion that belongs to another version of the quaternionic number system than the version, which the background platform applies. The symmetry break must be isotropic. Electrons fulfill this requirement. Neutrinos do not break the symmetry but have other reasons why they cause a valid trigger. Quarks break symmetry, but not in an isotropic way.

5 Line, surface and volume integrals

5.1 Line integrals

The curl can be presented as a line integral

$$\langle \vec{\nabla}\times\vec{\psi},\vec{n}\rangle \equiv \lim_{A\to 0}\left(\frac{1}{A}\oint_C \langle \vec{\psi}, d\vec{r}\rangle\right) \tag{5.1.1}$$

5.2 Surface integrals

With respect to a local part of a closed boundary that is oriented perpendicular to vector $\vec{n}$ the partial differentials relate as

$$\vec{\nabla}\psi = -\langle \vec{\nabla},\vec{\psi}\rangle + \vec{\nabla}\psi_r \pm \vec{\nabla}\times\vec{\psi} \Leftrightarrow \vec{n}\psi = -\langle \vec{n},\vec{\psi}\rangle + \vec{n}\psi_r \pm \vec{n}\times\vec{\psi} \tag{5.2.1}$$

This is exploited in the surface-volume integral equations that are known as Stokes and Gauss theorems.

$$\iiint \vec{\nabla}\psi dV = \oiint \vec{n}\psi dS \tag{5.2.2}$$

$$\iiint \langle \vec{\nabla},\vec{\psi}\rangle dV = \oiint \langle \vec{n},\vec{\psi}\rangle dS \tag{5.2.3}$$

$$\iiint \vec{\nabla}\times\vec{\psi} dV = \oiint \vec{n}\times\vec{\psi} dS \tag{5.2.4}$$

$$\iiint \vec{\nabla}\psi_r dV = \oiint \vec{n}\psi_r dS \tag{5.2.5}$$

This result turns terms in the differential continuity equation into a set of corresponding integral balance equations.

The method also applies to other partial differential equations. For example

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{\psi}) = \vec{\nabla}\langle\vec{\nabla},\vec{\psi}\rangle - \langle\vec{\nabla},\vec{\nabla}\rangle\vec{\psi} \Leftrightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{\psi}) = \vec{n}\langle\vec{n},\vec{\psi}\rangle - \langle\vec{n},\vec{n}\rangle\vec{\psi} \tag{5.2.6}$$

$$\iiint_V \{\vec{\nabla}\times(\vec{\nabla}\times\vec{\psi})\} dV = \oiint_S \{\vec{\nabla}\langle\vec{\nabla},\vec{\psi}\rangle\} dS - \oiint_S \{\langle\vec{\nabla},\vec{\nabla}\rangle\vec{\psi}\} dS \tag{5.2.7}$$

One dimension less, a similar relation exists.

$$\iint_S \left(\left\langle \vec{\nabla} \times \vec{a}, \vec{n} \right\rangle\right) dS = \oint_C \left\langle \vec{a}, d\vec{l} \right\rangle \tag{5.2.8}$$

5.3 Derivation of physical laws

The quaternionic equivalents of Ampère's law are

$$\vec{J} \equiv \vec{\nabla} \times \vec{B} = \nabla_r \vec{E} \Leftrightarrow \vec{J} \equiv \vec{n} \times \vec{B} = \nabla_r \vec{E} \tag{5.3.1}$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{B}, \vec{n} \right\rangle dS = \oint_C \left\langle \vec{B}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{J} + \nabla_r \vec{E}, \vec{n} \right\rangle dS \tag{5.3.2}$$

The quaternionic equivalents of Faraday's law are:

$$\nabla_r \vec{B} = \vec{\nabla} \times \left(\nabla_r \vec{\psi}\right) = -\vec{\nabla} \times \vec{E} \Leftrightarrow \nabla_r \vec{B} = \vec{n} \times \left(\nabla_r \vec{\psi}\right) = -\vec{\nabla} \times \vec{E} \tag{5.3.3}$$

$$\oint_c \left\langle \vec{E}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{\nabla} \times \vec{E}, \vec{n} \right\rangle dS = -\iint_S \left\langle \nabla_r \vec{B}, \vec{n} \right\rangle dS \tag{5.3.4}$$

$$\vec{J} = \vec{\nabla} \times \left(\vec{B} - \vec{E}\right) = \vec{\nabla} \times \vec{\varphi} - \nabla_r \vec{\varphi} = \vec{v} \rho \tag{5.3.5}$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{\varphi}, \vec{n} \right\rangle dS = \oint_C \left(\left\langle \vec{\varphi}, d\vec{l} \right\rangle\right) = \iint_S \left\langle \vec{v} \rho + \nabla_r \vec{\varphi}, \vec{n} \right\rangle dS \tag{5.3.6}$$

The equations (5.3.4) and (5.3.6) enable the derivation of the ***Lorentz force***.

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{B} \tag{5.3.7}$$

$$\frac{d}{d\tau} \iint_S \left\langle \vec{B}, \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \dot{\vec{B}}(\tau_0), \vec{n} \right\rangle ds + \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle ds \tag{5.3.8}$$

The Leibniz integral equation states

$$\begin{aligned} &\frac{d}{dt} \iint_{S(\tau)} \left\langle \vec{X}(\tau_0), \vec{n} \right\rangle dS \\ &= \iint_{S(\tau_0)} \left\langle \dot{\vec{X}}(\tau_0) + \left\langle \vec{\nabla}, \vec{X}(\tau_0) \right\rangle \vec{v}(\tau_0), \vec{n} \right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{X}(\tau_0), d\vec{l} \right\rangle \end{aligned} \tag{5.3.9}$$

With $\vec{X} = \vec{B}$ and $\left\langle \vec{\nabla}, \vec{B} \right\rangle = 0$ follows

$$\begin{aligned} \frac{d\Phi_B}{d\tau} &= \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \dot{\vec{B}}(\tau), \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \\ &= -\oint_{C(\tau_0)} \left\langle E(\tau_0), d\vec{l} \right\rangle - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \end{aligned} \tag{5.3.10}$$

The electromotive force (EMF) ε equals

$$\varepsilon = \oint_{C(\tau_0)} \left\langle \frac{\vec{F}(\tau_0)}{q}, d\vec{l} \right\rangle = -\frac{d\Phi_B}{d\tau}\Big|_{\tau=\tau_0}$$
$$= \oint_{C(\tau_0)} \left\langle \vec{E}(\tau_0), d\vec{l} \right\rangle + \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \tag{5.3.11}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{5.3.12}$$

6 Polar coordinates

In polar coordinates, the nabla delivers different formulas.

In pure spherical conditions, the Laplacian reduces to:

$$\langle \vec{\nabla}, \vec{\nabla} \rangle \psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) \tag{6.1.1}$$

The Green’s function blurs the location density distribution of the hop landing location swarm of an elementary particle. If the location density distribution has the form of a Gaussian distribution, then the blurred function is the convolution of this location density distribution and the Green’s function. The Gaussian distribution is

$$\rho(r) = \frac{1}{(\sigma\sqrt{2\pi})^3} \exp\left(-\frac{r^2}{2\sigma^2} \right) \tag{6.1.2}$$

The shape of the deformation of the field for this ***example*** is given by:

$$\mathfrak{T}(r) = \frac{ERF\left(-\frac{r}{\sigma\sqrt{2}} \right)}{4\pi r} \tag{6.1.3}$$

In this function, every trace of the singularity of the Green’s function has disappeared. It is due to the distribution and the huge number of participating hop locations. This shape is just an example. Such extra potentials add a local contribution to the field that acts as the living space of modules and modular systems. The shown extra contribution is due to the local elementary module that the swarm represents. Together, a myriad of such bumps constitutes the living space.

7 Material penetrating field

7.1 Field equations

Basic fields can penetrate homogeneous regions of the material. Within these regions, the fields get crumpled. Consequently, the average speed of warps, clamps, and waves diminish or these vibrations just get dampened away. The basic field that we consider here is a smoothed version $\vec{\psi}$ of the original field ψ that penetrates the material.

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{7.1.1}$$

$$\vec{\varphi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{\mathfrak{C}} \pm \vec{\mathfrak{B}} \tag{7.1.2}$$

The first order partial differential equation does not change much. The separate terms in the first order differential equations must be corrected by a material-dependent factor and extra material dependent terms appear.

These extra terms correspond to polarization $\vec{P}$ and magnetization $\vec{M}$ of the material, and the factors concern the permittivity ε and the permeability μ of the material. This results in corrections in the $\vec{\mathfrak{E}}$ and the $\vec{\mathfrak{B}}$ field and the average speed of warps and waves reduces from 1 to $\frac{1}{\sqrt{\varepsilon\mu}}$.

$$D = \varepsilon\vec{\mathfrak{E}} + \vec{P} \tag{7.1.3}$$

$$\vec{H} = \frac{1}{\mu}\vec{\mathfrak{B}} - \vec{M} \tag{7.1.4}$$

$$\rho_b = -\left\langle \vec{\nabla}, \vec{P} \right\rangle \tag{7.1.5}$$

$$\rho_f = -\left\langle \vec{\nabla}, \vec{D} \right\rangle \tag{7.1.6}$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M} + \nabla_r \vec{P} \tag{7.1.7}$$

$$\vec{J}_f = \vec{\nabla} \times \vec{H} - \nabla_r \vec{D} \tag{7.1.8}$$

$$\rho = \frac{1}{\varepsilon}\left\langle \vec{\nabla}, \vec{\mathfrak{E}} \right\rangle = \rho_b + \rho_f \tag{7.1.9}$$

$$\vec{J} = \frac{1}{\mu}\vec{\nabla} \times \vec{\mathfrak{B}} - \frac{\varepsilon}{\mu}\nabla_r \vec{\mathfrak{E}} = \vec{J}_b + \vec{J}_f \tag{7.1.10}$$

$$\vec{\phi} = \vec{\mathfrak{E}} - \vec{\mathfrak{B}} = \frac{1}{\varepsilon}\left(\vec{D} - \vec{P}\right) - \mu\left(\vec{H} + \vec{M}\right) \tag{7.1.11}$$

The subscript $_b$ signifies bounded. The subscript $_f$ signifies free.

The homogeneous second order partial differential equations hold for the smoothed field ψ.

$$\left\{\nabla_r \nabla_r \pm v^2 \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right\}\psi = 0 \tag{7.1.12}$$

7.2 Pointing vector

The **Poynting vector** represents the directional energy flux density (the rate of energy transfer per unit area) of a basic field. The quaternionic equivalent of the Poynting vector is defined as:

$$\vec{S} = \vec{E} \times \vec{H} \tag{7.2.1}$$

u is the electromagnetic energy density for linear, nondispersive materials, given by

$$u = \frac{\left\langle \vec{E}, \vec{B} \right\rangle + \left\langle \vec{B}, \vec{H} \right\rangle}{2} \tag{7.2.2}$$

$$\frac{\partial u}{\partial \tau} = -\langle \vec{\nabla}, \vec{S} \rangle - \langle \vec{J}_f, \vec{E} \rangle \tag{7.2.3}$$

8 Lorentz transform

The shock fronts move with speed c . In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2 \tau^2 \tag{8.1.1}$$

Swarms of clamp triggers move with lower speed v .

For the geometric centers of these swarms still holds:

$$x^2 + y^2 + z^2 - c^2\tau^2 = x'^2 + y'^2 + z'^2 - c^2\tau'^2 \tag{8.1.2}$$

If the locations $\{x, y, z\}$ and $\{x', y', z'\}$ move with uniform relative speed v , then

$$ct' = ct\cosh(\omega) - x\sinh(\omega) \tag{8.1.3}$$

$$x' = x\cosh(\omega) - ct\sinh(\omega) \tag{8.1.4}$$

$$\cosh(\omega) = \frac{\exp(\omega) + \exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \tag{8.1.5}$$

$$\sinh(\omega) = \frac{\exp(\omega) - \exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \tag{8.1.6}$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \tag{8.1.7}$$

This is a hyperbolic transformation that relates two coordinate systems.

This transformation can concern two platforms P and P' on which swarms reside and that move with uniform relative speed .

However, it can also concern the storage location P that contains a timestamp τ and spatial location $\{x, y, z\}$ and platform P' that has coordinate time t and location $\{x', y', z'\}$.

In this way, the hyperbolic transform relates two individual platforms on which the private swarms of individual elementary particles reside.

It also relates the stored data of an elementary particle and the observed format of these data for the elementary particle that moves with speed relative to the background parameter space.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x, y, z\}$ and proper time stamps τ into the perceived coordinate system that consist of the spacetime coordinates $\{x', y', z', ct'\}$ in which t' plays the role of proper time. The uniform velocity v causes time dilation $\Delta t' = \frac{\Delta\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$ and length contraction $\Delta L' = \Delta L\sqrt{1 - \frac{v^2}{c^2}}$