Once More About Quantum "Entanglement"[1)]

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Phenomenology

There are two aspects to the presentation of the problem – *conceptual* and *technological*. Here we will not touch upon the technological aspects of preparation and implementation of the technique of execution of experiments (for example, the A. Aspect's experiment), focusing on the conceptual principles of quantum theory.

In physics, topologically separable objects are initially distinguishable. Because of this, topological properties, we always have the opportunity to count objects to assign them a sequence number and be addressed to the objects by the assigned numbers.

1. Using this circumstance and considering photons as initially distinguishable particles (in the classical sense, although as we will see, it will lead us to a significant adjustment of this view), we will talk about two photons propagating in two opposite directions. In this case, the photon propagating to the left is assigned the number 1, and the photon propagating to the right is the number 2. The separability of photons is demonstrated by the fact that one photon can be registered on the left and the second on the right, thus illustrating the spatial distinctiveness and the possibility of their numbering. Instead of numbering the photons, it may be more convenient to use the arrow symbols "←" and "→", indicating the direction of movement. The separability of photons is demonstrated by the fact that one photon can be registered on the left and the second on the right, thus illustrating the spatial distinctiveness and the possibility of their numbering. Instead of numbering the photons, it may be more convenient to use the arrow symbols "←" and "→", indicating the direction of movement.

In the classical presentation, we will consider the Aspect's two-photon setting as a working model: two photons are born and propagated in opposite directions. We will consider another characteristic of the photon, except for the numbering of 1 and 2 – its polarization, which can take two values - "left" and "right", and in the spin notation – " ↑ " and " ↓ ".

So, photon 1 flies to the left (←), photon 2 – to the right (→). Symbolically, we can consider the following situations:

1) $|\uparrow,\leftarrow\rangle \equiv |\uparrow\rangle_1$ – the first photon is in the "spin up" state, without any information about the second photon;
2) $|\downarrow,\rightarrow\rangle \equiv |\downarrow\rangle_2$ – the second photon is in the "spin down" state, without any information about the first photon;
3) $|\uparrow,\leftarrow\rangle \otimes |\downarrow,\rightarrow\rangle \equiv |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$ – the joint state of two photons: the first photon is in the "spin up" state, the second photon is in the "spin down" state;
4) $|\downarrow,\leftarrow\rangle \otimes |\uparrow,\rightarrow\rangle \equiv |\downarrow\rangle_1 \otimes |\uparrow\rangle_2$ – the joint state of two photons: the first photon is in the "spin down" state, the second photon is in the "spin up" state.

Cases **1)** and **2)** are descriptions of single-particle systems, their formal solutions (without initial conditions) coincide, and there is no need for particle indexing.

Cases 3) and 4) are descriptions of two-particle systems, and indices specify the states of each particle.

In principle, for all these States, we can give the corresponding analogs of the description of one-and two-photon systems in classical physics. However, in quantum physics, there is a fundamentally new and important point, non-reproducible in classical physics. Explain this.

[1)] **I beg your pardon for my not very good English!** The original text in Russian**:** http://vixra.org/pdf/1804.0381v1.pdf

2. The wave equation of quantum mechanics can describe two-photon States as $|\uparrow\rangle_1 \otimes |\downarrow\rangle_2$ и $|\downarrow\rangle_1 \otimes |\uparrow\rangle_2$, so and superposition $|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + |\downarrow\rangle_1 \otimes |\uparrow\rangle_2$. Let's pay attention to the chain

$$\{|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + |\downarrow\rangle_1 \otimes |\uparrow\rangle_2\} \equiv \{|\uparrow, \leftarrow; \downarrow, \rightarrow\rangle\} \equiv \{|\uparrow, \downarrow\rangle\} \quad (1)$$

In the last equality $|\uparrow, \leftarrow; \downarrow, \rightarrow\rangle \equiv |\uparrow, \downarrow\rangle$ the connection between the spin projection and the direction of the particle motion or between the spin and the unique number of the particle is erased. This procedure can be called a procedure of symmetrization on the indices of the particles.

It can be seen that the result of following in the re-values along the given chain is a description of the non-index construction $\{|\uparrow, \downarrow\rangle\}$, which we can say that it represents the *path-tangled* or *"spin"-tangled* system of two photons. This construction is a kind of *integrity*, which is described by a single state vector or wave function, that is, is a single quantum object. Topological separability of photons disappears. And, if somehow we manage to assign numbers to the photons of the pair at their birth, we can not to know which of the photons (first or second) will be registered on the left or right. This is what was mentioned at the beginning of the article. Due to the integrity of a two-photon object, correlations can be observed between spatially separated points that are in the sphere of influence of this object. Indeed, when this integrity was observed from two spatially separated points in the experiment of the Aspect, the existence of a correlation between the ends of this integrity was confirmed.

3. It is further. The state of the quantum system can be expressed in a General form as $\psi = Ae^{i\varphi}$, which allows us to represent the left part (1) as

$$\{|\uparrow, \downarrow\rangle\} \equiv \psi = \mathrm{A}_1 e^{i\varphi_1} + A_2 e^{i\varphi_2} \quad (2)$$

Then

$$\psi = \mathrm{A}_1 e^{i\varphi_1} + A_2 e^{i\varphi_2} = \left\{\mathrm{A}_1 + A_2 e^{i\delta}\right\} e^{i\varphi_1} = \left\{\mathrm{A}_1 e^{-i\delta} + A_2\right\} e^{i\varphi_2}, \quad (3)$$

where A_1 и A_2 – real numbers and $\delta = \varphi_2 - \varphi_1$. Let us pay attention to the fragment of the equality (3):

$$\left\{\mathrm{A}_1 + A_2 e^{i\delta}\right\} e^{i\varphi_1} = \left\{\mathrm{A}_1 e^{-i\delta} + A_2\right\} e^{i\varphi_2}, \quad (4)$$

Here on the left is the phase φ_1 of the left part of the quantum object, on the right is the phase φ_2 of the right part, and in brackets – the phase δ, characterizing the integral construction of the quantum object. Measurements are made by left and right detectors of Aspect's device. Moreover, in the quantum description of the whole object exponential multipliers with phase φ_1 and φ_2 outside of the braces can be omitted, because quantum mechanics describes the states with accuracy up to an arbitrary multiplier modulo is equal to 1, i.e. $\left\{\mathrm{A}_1 + A_2 e^{i\delta}\right\}$ and $\left\{\mathrm{A}_1 e^{-i\delta} + A_2\right\}$ describe the same condition of two-photon quantum system.

And here – the most important thing: changing φ_2 while preserving δ we inevitably, according to (4), should get the change φ_1. From the point of view of the technology of the Aspect's experiment, it looks like this: by changing the phase φ_2 of a pair of photons (phase constancy δ) by the polarizer, we should observe the phase change φ_1. Since the phases φ_1, φ_2, δ do not contain dependences on the **z-axis** coordinate and time t, the spatiotemporal dependence φ_1 on φ_2 will not be deterministic, i.e. functional, and the observations can show only the statistical dependence $\varphi_1(\varphi_2)$ under an unknown mechanism of interaction between parts of coherent integrity[2)]. The state of the integral object while preserving δ is referred to as the *coherent* states of the "particles"[3)] of the pair.

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[2]) The determination of the dependence δ (z) can give an opportunity to describe the signal propagation through the modulation of this phase (for example, by the type of "pilot wave" de Broglie-Bohm). This is to the model (4) of the computational algorithm.

[3]) Realizing that the concept of *particle* in quantum mechanics is rather conventional, we will use this term for brevity in quotes. However, in the process of decoherence (see below) the notion of a particle acquires classical certainty

Formalism

When understanding the results of the Aspect's experiment, it is necessary to speak the language of quantum mechanics, not the language of some Argo insights. The subject of one of these representations is the concept of "quantum entanglement". Meanwhile, the language of quantum mechanics makes it possible to clearly and unambiguously fill the issues raised in this regard with concrete content. For the analysis the elementary model used in [1, 2] is offered.

We consider two particles and mark them with index α, $(\alpha = 1, 2)$. Each of them can be in one i of two states $(i = 1,2)$, often indicated by arrows (in the spin notation): ↑ for $(i = 1)$ and ↓ for $(i = 2)$.

We introduce a two-index designation x_i^α, where the upper index α denotes a particle, the lower index $i-$ its state $(\alpha, i = 1, 2)$. Then the state of each of the two particles can be represented by the vector $|x_i^\alpha\rangle$ of a single-particle space. For example, the vector $|x_1^1\rangle$ represents the first particle in the state ↑; $|x_2^2\rangle$ is the second particle in state ↓, and so on. These vectors are obtained as a solution of the one-particle equation of quantum mechanics.

We also introduce four-index expression x_{ik}^{12}, which will determine the vector $|x_{ik}^{12}\rangle$ is already two-particle space using a direct product of one-particle spaces $|x_i^1\rangle$ and $|x_k^2\rangle$. If the Hamiltonian of a two-particle system assumes the possibility of separating the variables of the wave equation, then at least the order of the solution of the wave equation is known and allows the representation of the solution $|x_{ik}^{12}\rangle$ in the form:

$$|x_{ik}^{12}\rangle = |x_i^1\rangle \otimes |x_k^2\rangle, \tag{5}$$

that is, the vector $|x_{ik}^{12}\rangle$ is obtained as a composition of solutions of single-particle wave equations.

Integrating upon continuous variables and summing upon all quantum numbers of one particle in (5), of course, as a convolution with complex conjugate parameters, we will obtain a wave vector (function) for another particle. It should be noted that the *possibility of performing the convolution operation will allow us to talk about the particles as some separable entities* (in this case, as two particles).

In the case of particles of the same type[4)], we are interested in two possible solutions for a pair of particles according to the particle identity principle: symmetric (for bosons) and antisymmetric (for fermions). According to the superposition principle, these solutions can be represented as:

$$|\hat{x}_{ik}\rangle = |x_{ik}^{12}\rangle \pm |x_{ik}^{21}\rangle, \tag{6}$$

where the upper sign corresponds to the case of Bose-particles, the lower sign corresponds to Fermi-particles.

Let select the phases φ_1 and φ_2 in $|x_{ik}^{12}\rangle$ and $|x_{ik}^{21}\rangle$:

$$|x_{ik}^{12}\rangle = \left||x_{ik}^{12}|\right\rangle e^{i\varphi_1}, |x_{ik}^{21}\rangle = \left||x_{ik}^{21}|\right\rangle e^{i\varphi_2}, \tag{7}$$

and (6) we will rewrite in the form

$$|\hat{x}_{ik}\rangle = ||x_{ik}^{12}|\rangle e^{i\varphi_1} \pm ||x_{ik}^{21}|\rangle e^{i\varphi_2} = \left\{||x_{ik}^{12}|\rangle \pm ||x_{ik}^{21}|\rangle e^{i\delta}\right\} e^{i\varphi_1}, \tag{8}$$

where $\delta = \varphi_2 - \varphi_1$.

[4)] The case of the same type particles is considered due to the fact that the pair of photons (bosons) or a pair of fermions (in the EPR version) is used in the interpretation of the Aspect's experiments. The conclusions of the preliminary results become more transparent. This particular can be overcome by considering the separation and symmetrization of variables of not all particle parameters, but a group of identical properties. Then "entanglement" as an argo-concept gets its certainty for different particles of micro- and mesoscales, although there is no reason for refusal of and macroscales.

Note the arbitrariness of the phases φ_1 and φ_2 is a property solutions $|x_i^1\rangle$, $|x_k^2\rangle$ and $|x_{ik}^{12}\rangle$, defined accurate to an arbitrary phases. These solutions represent *pure* quantum mechanical states as described by the state vectors: $|x_i^1\rangle$ and $|x_k^2\rangle$ in the single-particle space and $|x_{ik}^{12}\rangle$ – in the two-particle space.

Locking the phase difference $\delta = \varphi_2 - \varphi_1$, we transform the two-particle system $|\hat{x}_{ik}\rangle$ in the holistic object of the coherent coexistence of quantum objects $|x_{ik}^{12}\rangle$ and $|x_{ik}^{21}\rangle$. Actually this is the essence of manifestation of the phenomenon of *entanglement of states of two quantum objects.* By freeing the phases φ_1 and φ_2, we give quantum objects the possibility of free and independent coexistence for each of them.

In this regard, *the principle of identity of quantum particles* can be considered as a static aspect of entanglement and the correlation in the Aspect's experimental results as a dynamic aspect of exchange interaction.

It should be noted that the procedure of projection (associated with the reduction) of the symmetrized vector of a two-particle space $|\hat{x}_{ik}\rangle$ onto a one-particle subspace will no longer be represented by a unitary transformation, which is necessary to describe evolution according to the Schrödinger's equation, and is an important feature of the *decoherence* phenomenon. This projection will not represent a *pure* one-particle state because the corresponding particle will already be in the environment of the other. Because of this, its state can not be described by the state vector, it must be described by the *density matrix*. Decoherence as a phenomenon allows describing the transition from quantum mechanics to classical mechanics (see, for example, [4]).

Appendix 1

Single-particle density matrix in 2-space

In the basis of the one-particle state space

$$|\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix};\ |\downarrow\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad (\text{п}1.1)$$

the state $|x_i^\alpha\rangle = \begin{pmatrix}x_1^\alpha\\x_2^\alpha\end{pmatrix}$ can be represented as a superposition

$$|x_i^\alpha\rangle = x_1^\alpha \begin{pmatrix}1\\0\end{pmatrix} + x_2^\alpha \begin{pmatrix}0\\1\end{pmatrix} = x_1^\alpha|\uparrow\rangle + x_2^\alpha|\downarrow\rangle, \qquad (\text{п}1.2)$$

where x_1^α and x_2^α are probability amplitudes for the states $|\uparrow\rangle$ and $|\downarrow\rangle$ of the first $(\alpha = 1)$ or second $(\alpha = 2)$ particles.

From the condition of normalization of the vector (п1.2) follows

$$|x_1^\alpha|^2 + |x_2^\alpha|^2 = 1. \qquad (\text{п}1.3)$$

Matrix elements $\bar{M}_{ik}^\alpha$ of the density matrix $\bar{M}^\alpha$ take the form[5]:

$$\bar{M}_{ik}^\alpha = |x_i^\alpha\rangle\langle x_k^\alpha| = \begin{pmatrix}|x_1^\alpha|^2 & x_1^\alpha * \tilde{x}_2^\alpha\\ \tilde{x}_1^\alpha * x_2^\alpha & |x_2^\alpha|^2\end{pmatrix}. \qquad (\text{п}1.4)$$

[5]) The tilde sign "~" here denotes the complex conjugate quantities.

Using (п1.4) taking into account (п1.3), we obtain:

$$\sum_n \bar{M}^{\alpha}_{in}\bar{M}^{\alpha}_{nk} = \begin{pmatrix} |x_1^\alpha|^2 & x_1^\alpha * \tilde{x}_2^\alpha \\ \tilde{x}_1^\alpha * x_2^\alpha & |x_2^\alpha|^2 \end{pmatrix} * \begin{pmatrix} |x_1^\alpha|^2 & x_1^\alpha * \tilde{x}_2^\alpha \\ \tilde{x}_1^\alpha * x_2^\alpha & |x_2^\alpha|^2 \end{pmatrix} = \tag{п1.5}$$

$$= \begin{pmatrix} |x_1^\alpha|^2 * |x_1^\alpha|^2 + x_1^\alpha * \tilde{x}_2^\alpha * \tilde{x}_1^\alpha * x_2^\alpha & |x_1^\alpha|^2 * x_1^\alpha * \tilde{x}_2^\alpha + x_1^\alpha * \tilde{x}_2^\alpha * |x_2^\alpha|^2 \\ \tilde{x}_1^\alpha * x_2^\alpha * |x_1^\alpha|^2 + |x_2^\alpha|^2 * \tilde{x}_1^\alpha * x_2^\alpha & \tilde{x}_1^\alpha * x_2^\alpha * x_1^\alpha * \tilde{x}_2^\alpha + |x_2^\alpha|^2 * |x_2^\alpha|^2 \end{pmatrix} = \bar{M}^{\alpha}_{ik},$$

Notation 1

$|x_1^\alpha|^2 * |x_1^\alpha|^2 + x_1^\alpha * \tilde{x}_2^\alpha * \tilde{x}_1^\alpha * x_2^\alpha = |x_1^\alpha|^2 * |x_1^\alpha|^2 + |x_1^\alpha|^2 * |x_2^\alpha|^2 = |x_1^\alpha|^2 * (|x_1^\alpha|^2 + |x_2^\alpha|^2) = \boldsymbol{|x_1^\alpha|^2}$

$|x_1^\alpha|^2 * x_1^\alpha * \tilde{x}_2^\alpha + x_1^\alpha * \tilde{x}_2^\alpha * |x_2^\alpha|^2 = x_1^\alpha * \tilde{x}_2^\alpha * (|x_1^\alpha|^2 + |x_2^\alpha|^2) = \boldsymbol{x_1^\alpha * \tilde{x}_2^\alpha}$

$\tilde{x}_1^\alpha * x_2^\alpha * |x_1^\alpha|^2 + |x_2^\alpha|^2 * \tilde{x}_1^\alpha * x_2^\alpha = \tilde{x}_1^\alpha * x_2^\alpha * (|x_1^\alpha|^2 + |x_2^\alpha|^2) = \boldsymbol{\tilde{x}_1^\alpha * x_2^\alpha}$

$\tilde{x}_1^\alpha * x_2^\alpha * x_1^\alpha * \tilde{x}_2^\alpha + |x_2^\alpha|^2 * |x_2^\alpha|^2 = |x_2^\alpha|^2 * (|x_2^\alpha|^2 + |x_2^\alpha|^2) = \boldsymbol{|x_2^\alpha|^2}$

Thus,

$$(\bar{M}^{\alpha})^2 = \bar{M}^{\alpha}, \tag{п1.6}$$

a density matrix $\bar{M}^{\alpha}$, determined by (п1.4), describes the *pure* state as well as the vector (п1.2) [6].

Two-particle density matrix in 4-space

In the basis of the two-particle states space

$$|\uparrow\uparrow\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad |\uparrow\downarrow\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad |\downarrow\uparrow\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix};\ |\downarrow\downarrow\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{п1.7}$$

the state $|x_{ik}^{\alpha\beta}\rangle = \begin{pmatrix} x_{11}^{12} \\ x_{12}^{12} \\ x_{21}^{12} \\ x_{22}^{12} \end{pmatrix}$ can be represented as a superposition

$$|x_{ik}^{\alpha\beta}\rangle = x_{11}^{12}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + x_{12}^{12}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + x_{21}^{12}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + x_{22}^{12}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = x_{11}^{12}|\uparrow\uparrow\rangle + x_{12}^{12}|\uparrow\downarrow\rangle + x_{21}^{12}|\downarrow\uparrow\rangle + x_{22}^{12}|\downarrow\downarrow\rangle\ , \tag{п1.8}$$

where $x_{11}^{12},\ x_{12}^{12}, x_{21}^{12}, x_{22}^{12}$ the probability amplitudes for the states of a pair of particles $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ the first and the second, respectively.

Matrix elements $\bar{\bar{M}}_{ik}$ of the density matrix $\bar{\bar{M}}$ take the form [7] :

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[6] Operators that have the idempotence property $M^2 = M$, are called projection operators. Thus, the pure state density matrices are represented by projection operators.

[7] see Appendix 2

$$\bar{\bar{M}}_{ik} = \begin{Vmatrix} |x_{11}^{12}|^2 & x_{11}^{12} * \tilde{x}_{12}^{12} & x_{11}^{12} * \tilde{x}_{21}^{12} & x_{11}^{12} * \tilde{x}_{22}^{12} \\ x_{12}^{12} * \tilde{x}_{11}^{12} & |x_{12}^{12}|^2 & x_{12}^{12} * \tilde{x}_{21}^{12} & x_{12}^{12} * \tilde{x}_{22}^{12} \\ x_{21}^{12} * \tilde{x}_{11}^{12} & x_{21}^{12}\tilde{x}_{12}^{12} & |x_{21}^{12}|^2 & x_{21}^{12}\tilde{x}_{22}^{12} \\ x_{22}^{12} * \tilde{x}_{11}^{12} & x_{22}^{12} * \tilde{x}_{12}^{12} & x_{22}^{12} * \tilde{x}_{21}^{12} & |x_{22}^{12}|^2 \end{Vmatrix} \quad (\text{п1.9})$$

Using (п1.9) taking into account the normalization of the vector (п1.8), we obtain:

$$\sum_n \bar{\bar{M}}_{in}\bar{\bar{M}}_{nk} = \quad (\text{п1.10})$$

$$= \begin{Vmatrix} |x_{11}^{12}|^2 & x_{11}^{12} * \tilde{x}_{12}^{12} & x_{11}^{12} * \tilde{x}_{21}^{12} & x_{11}^{12} * \tilde{x}_{22}^{12} \\ x_{12}^{12} * \tilde{x}_{11}^{12} & |x_{12}^{12}|^2 & x_{12}^{12} * \tilde{x}_{21}^{12} & x_{12}^{12} * \tilde{x}_{22}^{12} \\ x_{21}^{12} * \tilde{x}_{11}^{12} & x_{21}^{12}\tilde{x}_{12}^{12} & |x_{21}^{12}|^2 & x_{21}^{12}\tilde{x}_{22}^{12} \\ x_{22}^{12} * \tilde{x}_{11}^{12} & x_{22}^{12} * \tilde{x}_{12}^{12} & x_{22}^{12} * \tilde{x}_{21}^{12} & |x_{22}^{12}|^2 \end{Vmatrix} * \begin{Vmatrix} |x_{11}^{12}|^2 & x_{11}^{12} * \tilde{x}_{12}^{12} & x_{11}^{12} * \tilde{x}_{21}^{12} & x_{11}^{12} * \tilde{x}_{22}^{12} \\ x_{12}^{12} * \tilde{x}_{11}^{12} & |x_{12}^{12}|^2 & x_{12}^{12} * \tilde{x}_{21}^{12} & x_{12}^{12} * \tilde{x}_{22}^{12} \\ x_{21}^{12} * \tilde{x}_{11}^{12} & x_{21}^{12}\tilde{x}_{12}^{12} & |x_{21}^{12}|^2 & x_{21}^{12}\tilde{x}_{22}^{12} \\ x_{22}^{12} * \tilde{x}_{11}^{12} & x_{22}^{12} * \tilde{x}_{12}^{12} & x_{22}^{12} * \tilde{x}_{21}^{12} & |x_{22}^{12}|^2 \end{Vmatrix} =$$

$$= \bar{\bar{M}}_{ik}$$

The first line of the multiplication result (п1.10) of the density matrix itself is presented in Notation 2. Similarly, the results can be obtained for the following lines. Thus, as in the case of a single-particle vector space, in the two-particle space, hold true the equality:

$$\bar{\bar{M}}^2 = \bar{\bar{M}}, \quad (\text{п1.11})$$

and a density matrix $\bar{\bar{M}}$ defined by (п1.9) describes the pure state as and the vector (п1.8).

Notation 2

$$|x_{11}^{12}|^2 * |x_{11}^{12}|^2 + x_{11}^{12} * \tilde{x}_{12}^{12} * x_{12}^{12} * \tilde{x}_{11}^{12} + x_{11}^{12} * \tilde{x}_{21}^{12} * x_{21}^{12} * \tilde{x}_{11}^{12} + x_{11}^{12} * \tilde{x}_{22}^{12} * x_{22}^{12} * \tilde{x}_{11}^{12} =$$

$$|x_{11}^{12}|^2(|x_{11}^{12}|^2 + \tilde{x}_{12}^{12} * x_{12}^{12} + \tilde{x}_{21}^{12} * x_{21}^{12} + \tilde{x}_{22}^{12} * x_{22}^{12}) = |x_{11}^{12}|^2(|x_{11}^{12}|^2 + |x_{12}^{12}|^2 + |x_{21}^{12}|^2 + |x_{22}^{12}|^2) = \boldsymbol{|x_{11}^{12}|^2}$$

$$|x_{11}^{12}|^2 * x_{11}^{12} * \tilde{x}_{12}^{12} + x_{11}^{12} * \tilde{x}_{11}^{12} * |x_{12}^{12}|^2 + x_{11}^{12} * \tilde{x}_{21}^{12} * x_{21}^{12} * \tilde{x}_{12}^{12} + x_{11}^{12} * \tilde{x}_{22}^{12} * x_{22}^{12} * \tilde{x}_{12}^{12} =$$
$$= x_{11}^{12} * \tilde{x}_{12}^{12} * (|x_{11}^{12}|^2 + |x_{12}^{12}|^2 + |x_{21}^{12}| + |x_{22}^{12}|) = \boldsymbol{x_{11}^{12} * \tilde{x}_{12}^{12}}$$

$$|x_{11}^{12}|^2 * x_{11}^{12} * \tilde{x}_{21}^{12} + x_{11}^{12} * \tilde{x}_{12}^{12} * x_{12}^{12} * \tilde{x}_{21}^{12} + x_{11}^{12} * \tilde{x}_{21}^{12} * |x_{21}^{12}|^2 + x_{11}^{12} * \tilde{x}_{22}^{12} * x_{22}^{12} * \tilde{x}_{21}^{12} =$$
$$x_{11}^{12} * \tilde{x}_{21}^{12} * (|x_{11}^{12}|^2 + |x_{12}^{12}|^2 + |x_{21}^{12}|^2 + |x_{22}^{12}|^2) = \boldsymbol{x_{11}^{12} * \tilde{x}_{21}^{12}}$$

(*important*) **Note 1**

Consideration of wave equations for a two-particle system in 4-space of state in representation (п1.7) with corresponding Hamiltonians allows for chains of solutions of the type

$$\begin{cases} \xi_{11}^{12}|\uparrow\uparrow\rangle \Rightarrow ([x_{11}^{12}|\uparrow\uparrow\rangle + x_{12}^{12}|\uparrow\downarrow\rangle + x_{21}^{12}|\downarrow\uparrow\rangle + x_{22}^{12}|\downarrow\downarrow\rangle), \\ (x_{11}^{12}|\uparrow\uparrow\rangle + x_{12}^{12}|\uparrow\downarrow\rangle + x_{21}^{12}|\downarrow\uparrow\rangle + x_{22}^{12}|\downarrow\downarrow\rangle) \Rightarrow \chi_{22}^{12}|\downarrow\downarrow\rangle, \end{cases}$$

that will allow to obtain equivalent to the evolutionary (using Schrödinger equations) description of the change of states and "transfer" of the first particle to the opposite state):

$$\{\xi_{11}^{12}|\uparrow\uparrow\rangle \Leftrightarrow \chi_{21}^{12}|\downarrow\uparrow\rangle \quad (\text{п1.12})$$

The main property of this method of describing the evolution of a quantum mechanical two-particle system is the representation of the solution as pure states *with full preservation of information about the individual behavior of each of the particles* in the system.

Reduction: 4⇒2

Let us now consider the possibility of describing a particle of a two-particle system in a single-particle space.

The first and most important question: *can we describe the behavior of one particle in the basis (n1.1)?*

Using wave functions, the equality (5) can be represented as:

$$\psi_{ik}(X,Y) = \varphi_i(X)\chi_k(Y), \qquad (\text{п}1.13)$$

where φ and χ are functions obtained by separating variables in the solution of the wave equation. In general case, they are represented by functions of different types. We will identify the particles by the parameters: $(i,X)-$ for the first particle, $(k,Y)-$ for the second particle, and the previous relation is presented as[8)]:

$$\psi_{ik}^{12}(X,Y) = \varphi_i^1(X)\chi_k^2(Y), \qquad (\text{п}1.13')$$

By folding this equality by variables (X,i) or (Y,k) of one particle (upper indices), we can obtain the wave function of another. This operation allows you to get rid of information about one particle, providing "clean" information about another. The information about the particle itself is contained in the properties described by variables (X,i) or (Y,k).

Given that the permutation of particles, each particle changes only its interface with "internal content", but not the environment of their stay, we have:

$$\psi_{ik}^{12}(X,Y) \to \psi_{ik}^{21}(X,Y) = \psi_{ki}^{12}(Y,K), \qquad (\text{п}1.14)$$

and the condition of symmetry takes the form:

$$\psi_{ik}(\mathrm{X},\mathrm{Y}) = \psi_{ki}(\mathrm{Y},\mathrm{X}). \qquad (\text{п}1.15)$$

Given that, we have for the symmetrized function $\hat{\psi}_{ik}(X,Y)$:

$$\hat{\psi}_{ik}(X,Y) = \varphi_i(X)\chi_k(Y) \pm \varphi_k(Y)\chi_i(X). \qquad (\text{п}1.16)$$

The symmetrization procedure (п1.16) records the fact of identity of particles by the selected group of properties.

It follows from the relation (п1.16) that convolution operation on the state variables of one particle was possible, for example, in the cases (5) and (п1.13), 5, but cannot be performed for the expression (п1.16), since here as a result of symmetrization, the entanglement between the particles parameters occurred[9)]. The latter means that in this case, the description of a particle in a one-particle state space in the medium of another particle as a certain separable entity is unrealizable, and to present its state as a superposition of base states (п1.1) impossible.

[8)] We will assume the possibility of some freedom in establishing the correspondence allowed by the procedure of solving the wave equation by the method of separating variables: : $1 \leftrightarrow (i,X), 2 \leftrightarrow (k,Y)$, which will be used in the procedure of symmetrization.

[9)] It should be noted that it is possible to talk about the entanglement of not all the properties of particles, that is, about the complete entanglement, and only about their some common properties.

Since it is impossible to describe a particle of a two-particle system in a vector 2-space as a superposition of basic States (п1.1), a different method is used, known in quantum mechanics as a method of describing using density matrices.

The standard approach here is to solve the evolutionary equation for the density matrix of the quantum system described by the Hamiltonian [3]. However, if the previous approach in the solution of the Schrödinger equation with subsequent symmetry, leading to entanglement of particles and their States, here the main problem is the fragmentation of the system into component parts and factorization of the description of a single particle.

In the end, the second question arises: *how to describe the behavior of one particle in the environment of another using a single-particle density matrix (n1.4)?*

In such a situation, when measuring the characteristics of one of the two particles, it is necessary to provide first the possibility of choosing both the first and the second particles.

In this case, to describe a single particle, it is first necessary to establish the possibility of its choice in a two-particle system characterized by a combination of pure states $M^{\alpha}(\alpha = 1,2)$. We define this capability by the corresponding probabilities p_{α}. Then the expression for the density matrix M of the particle will take the form:

$$M = \sum_{\alpha} p_{\alpha} \bar{M}^{\alpha} , \qquad (п1.17)$$

where $\sum_{\alpha} p_{\alpha} = 1$, $p_{\alpha} \geq 0, \alpha = 1,2$. The equality (п1.17) will be to determine the *mixture* of pure states of two particles, each of which being described in the one-particle space of states. The density matrix M itself is also an element of the one-particle space of states and, therefore, allows a one-particle description of a particle in the medium of the another. This is how one can describe entanglement as a physical phenomenon.

It is not difficult to show that for a matrix of the form (п1.17) in general, the equality

$$M^2 = M, \qquad (п1.18)$$

is not fulfilled, which is the criterion that the particle is in a pure state. However, this equality is possible if the condition of the probability unit p_{α} for one particle and zero for another is met.

Notation 3

Let us consider a system consisting of two particles, each of which ∀α (α=1,2) being in the pure state of 2-space, and is described by its density matrix $\bar{M}_{\alpha}$, satisfying the condition

$$(\bar{M}_{\alpha})^2 = \bar{M}_{\alpha}$$

The *holistic* two-particle system of the same 2- space is described by the density matrix

$$M = p_1 \bar{M}_1 + p_2 \bar{M}_2 \qquad (п1.19)$$
$$(0 < p_1 < 1), (0 < p_2 < 1), (p_1 + p_1 = 1).$$

The square of this matrix

$$M^2 = p_1^2 \bar{M}_1^2 + p_2^2 M^{\bar{}}{}_2^2 + p_1 p_2 (\bar{M}_1 \bar{M}_2 + \bar{M}_2 \bar{M}_1)$$

by virtue of identities

$$\bar{M}_1 \bar{M}_2 + \bar{M}_2 \bar{M}_1 = \bar{M}_1^2 + \bar{M}_2^2 - (\bar{M}_1 - \bar{M}_2)^2$$

and

$$p_1^2 + p_1 p_2 = p_1(p_1 + p_2) = p_1, \qquad p_2^2 + p_1 p_2 = p_2$$

can be written as

$$M^2 = p_1 \bar{M}_1^2 + p_2 \bar{M}_2^2 - p_1 p_2 (\bar{M}_1 - \bar{M}_2)^2$$

Since the matrices M_1 and M_2, each separately, satisfy the condition $M^2 = M$, we find

$$M - M^2 = p_1 p_2 (\bar{M}_1 - \bar{M}_2)^2 \qquad (\text{п}1.20)$$

The right part represents positive matrix; hence M is equal M^2 only if $(\bar{M}_1 - \bar{M}_2)^2 = 0$. The square of the hermitian matrix vanishes only when all elements are zero. One of the necessary conditions for the implementation of the equality $M^2 = M$ is the equality $\bar{M}_1 = \bar{M}_2$. The same result is achieved when one of p_1 and p_2 is equal to zero.

It follows from the relation (п1.20) that a two-particle system with particles in different states at $p_1 > 0$ and $p_2 > 0$ behaves as a mixture of particles (п1.19), each particle being described in an *isolated* form by its density matrix $\bar{M}_1$ or $\bar{M}_2$.Место для формулы.

When $p_1 = 0$ or $p_2 = 0$, the particle α with $p_\alpha = 1$ will be described as being in the pure state. However, at $p_1 > 0$ and $p_2 > 0$, the state of any of the particles cannot be considered pure, which means that it will be described by density matrices for which the equality $(M)^2 = M$ is not satisfied , which is a criterion of the pure state.

The determinant of the density matrix constructed from the pure state vector as it is easy to verify from (п1.2) and (п1.4) is always zero. It follows that the mixture of states (п1.17) in general cannot be described by the state vector.

Notation 4

Using (п1.17) in the form

$$M = p_1 * M_1 + p_2 * M_2,$$

for the determinant of the matrix M we obtain:

$$\{(p_1 * |x_1|^2 + p_2 * |y_1|^2) * (p_1 * |x_2|^2 + p_2 * |y_2|^2)\} - \{(p_1 \tilde{x}_1 * x_2 + p_2 \tilde{y}_1 * y_2) * (p_1 * x_1 * \tilde{x}_2 + p_2 * y_1 * \tilde{y}_2)\} =$$
$$\{p_1 * |x_1|^2 * p_1 * |x_2|^2 + p_1 * |x_1|^2 * p_2 * |y_2|^2 + p_2 * |y_1|^2 * p_1 * |x_2|^2 + p_2 * |y_1|^2 * p_2 * |y_2|^2\} -$$
$$-\{p_1 \tilde{x}_1 * x_2 * p_1 * x_1 * \tilde{x}_2 + p_1 \tilde{x}_1 * x_2 * p_2 * y_1 * \tilde{y}_2 + p_2 \tilde{y}_1 * y_2 * p_1 * x_1 * \tilde{x}_2 + p_2 \tilde{y}_1 * y_2 * p_2 * y_1 * \tilde{y}_2\} =$$
$$= \{p_1^2 * |x_1|^2 * |x_2|^2 + p_1 * p_2 * |x_1|^2 * |y_2|^2 + p_1 * p_2 * |y_1|^2 * |x_2|^2 + p_2^2 * |y_1|^2 * |y_2|^2\} -$$
$$-\{p_1^2 * |x_1|^2 * |x_2|^2 + p_1 * p_2 * \tilde{x}_1 * x_2 * y_1 * \tilde{y}_2 + p_1 * p_2 * \tilde{y}_1 * y_2 * x_1 * \tilde{x}_2 + p_2^2 * |y_1|^2 * |y_2|^2\} =$$
$$= p_1 * p_2 \{|x_1|^2 * |y_2|^2 + |x_2|^2 * |y_1|^2 + x_1 * \tilde{x}_2 \;\; \tilde{y}_1 * y_2 + \;\; \tilde{x}_1 * x_2 * y_1 * \tilde{y}_2\}$$

Since the determinant of the two-row pure state density matrix is zero, its rank is one. This means that the matrix itself can be represented in a one-dimensional linear subspace, and the state by a vector in two-dimensional space. This vector may be is decomposed into basis vectors (п1.1), that is, it can be represented by a superposition, and thus represent a pure state.

In general, if the rank of a two-row matrix is not equal to one, then the matrix itself cannot be represented in a one-dimensional subspace, since it is actually a two-dimensional object. That is, the matrix itself is representable only a mixture (п1.17) of two 1-objects. These objects will be interacting particles, that is, one particle in the environment of another. These objects will represent interacting particles, that is, one particle in the environment of another. There is no reason to demand the uniqueness of such a representation.

(*important*) **Note 2**

The reduction procedure 4⇒2 describes the" dissolution " of the individuality of one particle in the environment of another particle, accompanied by the *loss of information about the individuality of their behavior*, while maintaining, however, the holistic. The subject of the reduction procedure description is the *decoherence theory*, which aims to formalize the transition from quantum mechanics to classical mechanics. Important feature: *this procedure is not described by unitary transformations, that is – solutions of Schrödinger equations.*

Appendix 2

Let consider the experimental installation of Aspect. We introduce the most transparent symbols (as far as possible) and use the results of Appendix 1.

To identify individual configurations, 4-index symbols $x_{ik}^{\alpha\beta}$, will be used, where the subscript refers to the state of the particles, and the upper one – to the ends of the installation of Aspect: left and right. The correspondence between the observer and the state of its particle is established "vertically" in the expression $x_{ik}^{\alpha\beta}$. Here the state i corresponds to the observer α, the state k corresponds to the observer β.

As it is often done, we place on the left side of the installation of Aspect – Alice (A), on the right side – Bob (B). Since their presence is not necessary, we will denote the left and right ends of the installation by upper indices: 1 - (A) and 2 - (B). Since the position of observers is fixed by our condition, the designation of the upper indices not changing and will correspond to the representation of "12" or "AB", that is always $\alpha = 1,\ \beta = 2$.

For clarity, the possible States of the Alice and Bob particle vectors are also denoted by the spin symbolism – ↑ and ↓, and for compactness – by the lower indices. With the index designation, the state $|\uparrow\rangle$ will receive the value of the lower index 1, and the state $|\downarrow\rangle$ – the value of the lower index 2. In the representation of the states of the two-particle system with arrows, in the first place (left) will be represented by the state of the particle of the left end of the installation (A), in the second place (right) - the state of the particle of the right end of the installation (B). Then, for example, the state $|\uparrow\uparrow\rangle$ will correspond to such a configuration, in which the particles of Alice and Bob are in a state with spins "up", this situation is associated with the parameter x_{11}^{12}; the state $|\downarrow\uparrow\rangle$ will correspond to the configuration, when the particle of Alice has a spin "down", and Bob – spin "up", this situation corresponds to the parameter x_{21}^{12}, etc.

System of vectors

$$\vec{e}_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad \vec{e}_2 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad \vec{e}_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix};\quad \vec{e}_4 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

it is an orthonormal basis in a linear 4-space. We associate the possible configurations of a two-particle system with this basis as follows:

$$|\uparrow\uparrow\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad |\uparrow\downarrow\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad |\downarrow\uparrow\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix};\quad |\downarrow\downarrow\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}. \qquad (\text{п}2.1)$$

Then an arbitrary state vector $|x\rangle$ of a two-particle system of 4-space state can be expressed by a superposition:

$$|x\rangle = x_{11}^{12}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + x_{12}^{12}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + x_{21}^{12}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + x_{22}^{12}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = x_{11}^{12}|\uparrow\uparrow\rangle + x_{12}^{12}|\uparrow\downarrow\rangle + x_{21}^{12}|\downarrow\uparrow\rangle + x_{22}^{12}|\downarrow\downarrow\rangle, \qquad (п2.2)$$

where $x_{ik}^{\alpha\beta}$, will correspond to the components of vectors in the representation of the two-particle state in the basis $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$, which are the amplitudes of the probabilities of finding the system in one of the basis states.

To illustrate the definition of the elements of the density matrix, we introduce new notation of the "coordinates" of the 4-vector and present the 4-vector (п2.2) in the form

$$x_{11}^{12}|\uparrow\uparrow\rangle + x_{12}^{12}|\uparrow\downarrow\rangle + x_{21}^{12}|\downarrow\uparrow\rangle + x_{22}^{12}|\downarrow\downarrow\rangle = \xi_1|\uparrow\uparrow\rangle + \xi_2|\uparrow\downarrow\rangle + \xi_3|\downarrow\uparrow\rangle + \xi_4|\downarrow\downarrow\rangle. \qquad (п2.3)$$

Then the elements of the density matrix can be expressed by the formula

$$\bar{\bar{M}}_{ik} = \xi_i\tilde{\xi}_k, \qquad (п2.4)$$

and the matrix itself takes the form

$$\bar{\bar{M}}_{ik} = \begin{Vmatrix} |\xi_1|^2 & \xi_1 * \tilde{\xi}_2 & \xi_1 * \tilde{\xi}_3 & \xi_1 * \tilde{\xi}_4 \\ \xi_2 * \tilde{\xi}_1 & |\xi_2|^2 & \xi_2 * \tilde{\xi}_3 & \xi_2 * \tilde{\xi}_4 \\ \xi_3 * \tilde{\xi}_1 & \xi_3 * \tilde{\xi}_2 & |\xi_3|^2 & \xi_3 * \tilde{\xi}_4 \\ \xi_4 * \tilde{\xi}_1 & \xi_4 * \tilde{\xi}_2 & \xi_4 * \tilde{\xi}_3 & |\xi_4|^2 \end{Vmatrix} \qquad (п2.5)$$

Returning to the variables $x_{ik}^{\alpha\beta}$ we get finally:

$$\bar{\bar{M}}_{ik} = \begin{Vmatrix} \left|x_{11}^{12}\right|^2 & x_{11}^{12} * \tilde{x}_{12}^{12} & x_{11}^{12} * \tilde{x}_{21}^{12} & x_{11}^{12} * \tilde{x}_{22}^{12} \\ x_{12}^{12} * \tilde{x}_{11}^{12} & \left|x_{12}^{12}\right|^2 & x_{12}^{12} * \tilde{x}_{21}^{12} & x_{12}^{12} * \tilde{x}_{22}^{12} \\ x_{21}^{12} * \tilde{x}_{11}^{12} & x_{21}^{12}\tilde{x}_{12}^{12} & \left|x_{21}^{12}\right|^2 & x_{21}^{12}\tilde{x}_{22}^{12} \\ x_{22}^{12} * \tilde{x}_{11}^{12} & x_{22}^{12} * \tilde{x}_{12}^{12} & x_{22}^{12} * \tilde{x}_{21}^{12} & \left|x_{22}^{12}\right|^2 \end{Vmatrix} \qquad (п2.6)$$

From the representation (п2.6) it is possible to see the full configuration of the probability amplitudes of the pair of particles at both ends of the experimental setup, emitted by the source. Here, as already noted, the upper indices refer to the observers: at the left (Alice) and right (Bob) ends of the installation, and the lower indices refer to the states of the particles. The values x_{11}^{12}, $x_{12}^{12}, x_{21}^{12}, x_{22}^{12}$ represent the probability amplitudes for the States of the pair of particles $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ – the first and the second, respectively. It is in this configuration of entanglement represented by the coherent superposition of states (п2.2) or the density matrix (п2.6) of 2-system of for subsequent measurements appear

Let the state of the particle with its spin up, i.e. $-\uparrow$, be recorded as a result of measurement at the right end of the installation (B). This means that the system is in a state:

$$x_{11}^{12}|\uparrow\uparrow\rangle + x_{12}^{12}|\uparrow\downarrow\rangle + x_{21}^{12}|\downarrow\uparrow\rangle + x_{22}^{12}|\downarrow\downarrow\rangle = x_{11}^{12}|\uparrow\uparrow\rangle + 0 * |\uparrow\downarrow\rangle + x_{21}^{12}|\downarrow\uparrow\rangle + 0 * |\downarrow\downarrow\rangle, \qquad (п2.7)$$

with amplitudes $x_{12}^{12} = 0$ $x_{22}^{12} = 0$ and the density matrix took the form:

$$\begin{Vmatrix} |x_{11}^{12}|^2 & 0 & x_{11}^{12} * \tilde{x}_{21}^{12} & 0 \\ 0 & 0 & 0 & 0 \\ x_{21}^{12} * \tilde{x}_{11}^{12} & 0 & |x_{21}^{12}|^2 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix} \quad (\text{п}2.8)$$

Further. There is a correlation between the particles-the connection established by the Aspect. Therefore, it is reasonable to expect a change in the state of the particle at the left end of the installation, after changing the state of the particle at its right end. The relationship between the states of the particles is not functional, but correlation, which means that the parameters of one particle can influence only the parameters of the probability distribution for another particle. In a single measurement, this relationship may not"work". In this case, we cannot judge the speed of the impact of one particle on another. This speed can be judged only statistically and on average, taking into account the arising noise suppressing the real value of the speed of influence.

The further development of events after the particle measurement at the right end of the installation should be considered as the process of evolutionary relaxation of the system with a real Hamiltonian, described by the wave equation for the density matrix (п2.8). Since this relaxation mechanism is unknown today, the quantum mechanical prediction (probabilistic) of matrix elements (п2.8) it is not possible. However, A. Aspects of his experimental results proved the reality of the existence of such a mechanism.

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Summary

1. There are two ways to describe quantum objects − using state vectors and density matrices. Both methods give equivalent descriptions of non-interacting with the external environment, i.e. closed quantum systems. The states described in these cases are called *pure* states.

2. The quantum mechanical description of a closed two-particle system requires consideration of a solution in the 4-space of quantum States. It can be realized both with the help of 4-vectors and 4-matrix density .

3. When describing a pair of particles, one particle is always in the medium of another. Although each isolated particle of the pair can be described as a pure state, however, in the two-particle description, one of them will always be in the environment of the other, and the system after the symmetry procedure of the equation solution will be a mixture of two particles. In this case, each of them can be described separately by its density matrix, through which the mixture of states is determined. A mixture of state turns into a pure state in the "disappearance" of a single particle, that is, when $p_1 = 0$ or $p_2 = 0$. This circumstance is fully consistent with the fact that a quantum system located in the external environment, can't be described with vector of states, as it cannot be in pure state.

4. The reduction procedure: $4 \Rightarrow 2$,allows you to switch from two-particle 4-dimensional description to a one-particle 2-dimensional description, as if sticking together the history of the evolutions of two parts: the unitary and the post unitary. The result of non-unitary reduction is the boundary of these stories.

5. It is the procedure of symmetrization in the description of quantum mechanical systems that causes the phenomenon of entanglement of particle States (see section 1).

6. The concept of entanglement can arise only when several quantum objects (particles) are considered. In this case, we can talk about the entanglement of particle states. The entanglement of the states of one quantum object is nothing but a superposition of the states of a single object. When the characteristics of several objects are entangled, a new integrity can arise, that is, a new quantum object.

The mixture of states of different objects cannot lead to the emergence of a new object, because mixing as an operation is not identical to the superposition of states. However, a mixture of states admits a factorization of the description and simulation of the fragmentation of the system into two interacting subsystems. It should be emphasized that the considered model of states (п1.17), which is a mixture of states, becomes fictitious if the experiment in principle cannot obtain optimal information about the subsystems (particles) of the entire object.

Q & A

According to results of previous on-line dialogues on the author's seminar a number of questions which require clarification

The description of the phenomenon of entanglement for identical particles by means of the operation of symmetrization in general convinces. The concept of coherence of states also arises here. More or less, it's understandable. But how to imagine the mechanism of decoherence is not strict, but clear?

The coherency is strictly formalized concept in the framework of the phase dependences of the wave functions. Information on these issues can be found on the example of a two-particle quantum system in the present article [see (2,3,4)].

Violation of phase dependencies between the elements of the whole is decoherence. Decoherence as a phenomenon is described on the basis of density matrices.

The emergence of many coherences of the object with the environment, that is, "dissolution" of its own phase on the elements of the environment - this can also be considered as decoherence, but already known to be different... and this is a common problem solved by the "theory of decoherence" as a discipline.

As for the experimental and technological mechanisms of the realization of coherence and decoherence at the present time – they are at the stage of experimental work with different results."

I would like to hear briefly about Neumann's "Process 1" and his non-unitarily.

General

There are two possibilities for the construction of quantum mechanics: the use of the *state vectors* apparatus (Schrödinger wave functions) and the *density matrix* apparatus (hermitian operators), [Landau, 1927].

Representations using state vectors and density matrices are formally equivalent when considering *closed* quantum systems. These systems are exhaustively described by so-called *pure* states.

The evolution of closed quantum systems or their temporal dynamics for both descriptions is represented by unitary transformations with Hamiltonian as a generator of infinitesimal shifts in time (solutions of the Schrödinger-type equation with initial data). Density matrices of such solutions, as well as any operators of the observed ones, are represented by *hermitian* operators. Hermitian matrices, in turn, can be reduced to a diagonal form by unitary transformations by choosing the corresponding representation basis. Unitary transformations leave the information status of the quantum system unchanged. It is the finding of unitary transformations (solutions of Schrödinger equations) in both cases that make up the essence of solutions to traditional problems of quantum mechanics.

However, quantum subsystems immersed in the external environment cannot be considered closed. Such systems are represented by so-called *mixtures* of states. The fact is that mixtures of States cannot be described by vectors of pure States and their superpositions. Of the two possibilities, the description of the mixture of states can be realized only with the help of density matrices. This circumstance was used by von Neumann to clarify the Bohr's interpretation of quantum mechanics in terms of the interpretation of measurement procedures, that is, in terms of the interaction of the measured system with the measuring device or, in general, with the external environment. Consideration of transformations of quantum systems that *violate unitary evolution* leads to the necessity of consideration of issues related to changes in the information status of the resulting states. The same approach is used in the description of the transition from quantum systems to classical systems in the theory of decoherence, since the transition from the quantum description to the classical one is associated with the inevitable loss of phase information about quantum states.

In particular

1. Consider a specific example of a two-particle quantum system, for example, in an Aspect's experiment. This system must be described in Hilbert's 4-space. Description of one of the particles is possible in a single-particle 2-space. However, in our case - in the case when one particle is in the environment of another – it is impossible to describe this particle by means of the apparatus of the pure states. This is where the need arises to consider the transformation

associated with the transition from Hilbert's 4-space to 2-space. Such *reduction* cannot be realized by means of unitary (evolutionary) transformation, because it is associated with the loss of information about the state of the system, while all unitary transformations leave the system in pure States with entropy equal to zero, that is, with the maximum information status. The loss of information and the increase in entropy during this transition is obvious. This is the first aspect of non-unitary reduction.

2. The second aspect of reduction is related to the loss of information on phase coherence, which is absent in classical manifestations.

The essence of the matter is as follows. Information about phase dependencies and hence quantum correlations is contained in nondiagonal elements of the density matrix. As already mentioned, with the help of unitary transformations, it is possible to bring the density matrix to the diagonal form, which would allow to give a probabilistic interpretation of diagonal elements without phase correlations. With these transformations, the information status of the quantum system will not change. This means that information about the correlations do not disappear which means it will be obliged anyway to show itself that is contradict experience macroexperiments. Thus, unitary transformations cannot realize the transition from quantum description to classical one. Non-unitary operation of the density matrix diagonalization (reduction) is the process that led von Neumann as a necessary element for matching quantum mechanics (with quantum correlations) with classical (without quantum correlations).

The loss of information and the increase of entropy in the reduction in both cases is quite obvious, and these processes cannot be described in the framework of unitary evolution.

Von Neumann introduced *Process 1* to ensure the integrity of the consideration of the transition from closed quantum systems to systems interacting with the environment.

The first aspect is presented in detail in this paper; the second – in the article:
https://www.dropbox.com/s/x3dtk8w4vv9kv3n/ZurekEn.pdf?dl=0 (*original*),
https://www.dropbox.com/s/w6ewl825x8m01nh/Zurek.pdf?dl=0 (*translation into Russian*).

Does this mean that the collapse of wave functions should be described by non-unitary transformations, that is, outside the framework of traditional quantum mechanics (solutions of Shrödinger equations)?

That's right. Moreover, there is a radical view that in this area (the famous "*FWT*-theorem" Conway and Cohen, which is called "Free Will Theorem") do not work cause-effect relationships and it is confirmed experimentally. An elementary demonstration of the absence of a functional connection between the perturbation and the response in the spin system is the Cohen-Specker paradox described in the work of the same authors. This paradox has stirred the physical world.

Unlike von Neumann, Zurek diagonalized the density matrix using a unitary transformation and" closed " the dynamics of the system under consideration to its Hamiltonian. But you needed a non-unitary transformation in an article about entanglement!

Zurek

It is known from quantum mechanics that any "quantum history" develops due to two types of independent processes - free evolution and measurement procedures that define a new branch of evolution. Both processes are not reducible to one another. Bohr proposed such an interpretation of the integrity of "quantum history" (Copenhagen interpretation): to consider the evolution of quantum systems using Schrödinger equations, and measurements over quantum systems using classical devices. Despite the striking effectiveness of such a construction, there is a natural question of reducibility of one (quantum) description language with another (classical) and this question, first of all, concerns the coherence of States at the quantum level and decoherence, that is, the loss of this coherence at the macrolevel. It is the decision of this problem article is dedicated by Zurek.

Zurek's article considers the interaction of the 1/2-spin $\mathcal{S}$ subsystem with the $\mathcal{D}$detector. The detector can only respond to the following states: ↑ and ↓. As closed, this composite system is described by a pure state, which can be represented by both a vector and a density matrix in a Hilbert's 4-space. Actually, Zurek uses the density matrix, but in contrast to von Neumann, considering the unitary transformation, leading to the matrix diagonal form, but in the selected basis.

In our article on quantum entanglement, we consider the description of an holistic integral two - particle system in 4-space, those wholeness that is due to the coherence of the States of the components. The transition from a two-particle description in 4-space to a one-particle description in 2-space, that is, the *destruction* of the wholeness of the system during observation, it is the subject of our article.

One thing in common is a discussion of the non-unitarity of the transformation called in one word reduction. Zurek managed to link the integrity of the joint system without the non-unitary transformation of von Neumann. In our case, it is impossible. Just the tasks we consider are different.

Read more about the Zurek's reasons "scattering of coherence" in the environment.

The density matrix in general contains nondiagonal elements responsible for the correlation interactions between the subsystems that make up the integrity of the original system. This form of the matrix clearly did not fit into the picture of classical measurements. To bring it to the uncorrelated form required for the classical interpretation of the measurement procedure, von Neumann proposed to introduce a non-unitary matrix diagonalization procedure (that is, beyond the limits of quantum mechanics), whose diagonal elements could then be interpreted as classical probabilities (without quantum correlations). Why not unitary?

Just because the " *Unitary evolution condemns every closed quantum system to 'purity.'* " (p. 9)[10)], preserving the quantum correlations, which cannot be deduced in the classical manner. In addition, unitary transformations are equivalent in terms of the evolution of the system with respect to the coherence of phase relations according to the Schrödinger equations. This was the reason for the introduction of non-unitary transformation. Von Neumann developed his idea by moving from (6) to (7) [10)] and called this procedure a *reduction* of the density matrix with a special emphasis on its non-unitary character, which corresponds to the transition to the classical measurement procedure.

Zurek, in contrast to von Neumann, introduced the external environment $\mathcal{E}$ as the third unclosed component of the general system, allowing to implement a complete description of the measurement procedure, while remaining within the framework of quantum mechanics. "*The final state of the combined $\mathcal{SDE}$ "von Neumann chain" of correlated systems extends the correlation beyond the $\mathcal{SD}$ pair*" [10)]. Здесь: $\mathcal{S}$ – система, $\mathcal{D}$ – детектор. Thus, it represents an element of the mechanism of coherence scattering in the environment.

If the states of the environment corresponding to the detector States are orthogonal, the density matrix folded over uncontrolled and unknown degrees of freedom takes the form (14) [10)], which was proposed by von Neumann. In this case, if the observed Λ of the subsystem $\mathcal{S}$ commutes with the Hamiltonian of the H_{int} detector-environment interaction, its dynamics invariantly closes on the diagonal of the reduced density matrix, that is, when the system is in its eigenstate Λ, the interaction with the environment will leave it unperturbed, and the observed Λ will be the motion integral.

The last remark allows us to divide the system parameters into controlled, preserving coherence (entanglement), and uncontrolled (unknown, destroying entanglement). This fact makes it possible to "raise" the entanglement on a small group of controlled parameters to meso- and macro- levels and, at the same time, to explain the disappearance of coherence on a large group of uncontrolled parameters.

Llinks

1. Касимов В.А. *Некоторые топологические парадоксы СТО*. Новосибирск. 2014 г.
https://www.academia.edu/32427340/
2. Касимов В.А. *О парадоксе ЭПР. Особенности разрешения*. Новосибирск. 2015 г.
https://www.academia.edu/32427334/
3. Касимов ВА. *Квантовая механика (принципы)*.Новосибирск. "Сибпринт", 2013 г.
https://www.academia.edu/32434510/
4. E. Joos, C. Kiefer, J. ... *Декогеренция и возникновение классического мира в квантовой теории*. Springer. DecoherenceBook.djvu

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10) https://www.dropbox.com/s/x3dtk8w4vv9kv3n/ZurekEn.pdf?dl=0

V.A.Kasimov. Once more about quantum "entanglement" (English version)

Abstract
During the conceptual design of the experimental results of Aspect one must speak the language of quantum mechanics, not the language Argo of the private **insights.** One of these insights is the concept of "*entanglement*" (of particles or states is unclear!) The language of quantum mechanics allows for a clear and unambiguous manner to give concrete content to the questions on this occasion. For the analysis of the proposed elementary model used in [1, 2].