Draft version May 26, 2018
Typeset using LaTeX **preprint** style in AASTeX62

The 'constant Lagrangian' fit of galaxy rotation curves as caused by cosmic space expansion under energy conservation conditions.

E.P.J. de Haas[1]

[1] *Nijmegen, The Netherlands*

(Dated: May 25, 2018)

ABSTRACT

In my opinion, the problem of the galaxy rotation curves can be solved on the basis of the combined and competing principles of space expansion and energy conservation on the one hand and gravitational contraction and the virial theorem on the other hand. Thus far it has been assumed that the existence of galaxies is proof of the dominance of gravitational contraction in galaxies. In this paper I present arguments in favor of my conviction that this assumption is wrong and that space expansion is one of the two dominating and competing principles active in galaxies. Compared to the hypothesis of a revolutionary and mysterious Dark Matter particle, WIMP or otherwise, my approach is rather conservative. All I propose to reset, rethink and rescale is the presupposed boarder between Newtonian gravitational contraction and Hubble space expansion.

Keywords: Galaxies: kinematics and dynamics, Galactic rotation curves, Dark Matter, MOND, Schwarzschild

1. INTRODUCTION

In a recent draft I introduced a 'constant Lagrangian' model for galactic dynamics (de Haas 2018d). In a few sequential drafts I went from a qualitative attempt at fitting real rotational velocity curves using the proposed model, see (de Haas 2018f,e), towards a quantitative analysis by including the error bars of the measured velocity, (de Haas 2018g,c,b). In that last preprint I presented the analysis of the full set of 175 galaxies at the SPARC database, as provided by (Lelli et al. 2016) in the file Rotmod-LTG.zip. That rotation curve fitting result was presented in a non-categorized order, it just followed the order of the alphabetic-numerical list. I subsequently categorizes the fitting curves according to the fitting result. After having fit and categorized those 175 rotation curves, I realized that it allowed me to go from a rather weak deductive to a more robust inductive justification of the 'constant Lagrangian' model(de Haas 2018a).

In that analysis, I split the 175 galaxies in several categories. The most significant group was the single fit category, galaxies that directly fit to the model. This was also the least interesting group, in the sense that it didn't add additional dynamics to the analysis of galaxies. Then there were four dual fit categories, categories that turned out to provide essential galactic dynamics due to the situation

haas2u@gmail.com

of having two competing fitting curves. Two with a crossover transition dual fit and two with a parallel transition dual fit. The galaxies with three or more fits could be dynamically analyzed using the four dual categories. At the end there was a rest category of galaxies that defy simple fits and subsequent categorization, 27 percent in total. That meant that 73 percent of the galaxies allowed for perfect to moderate analysis and explanation on the basis of the constant Lagrangian model (de Haas 2018a).

In this paper I will briefly give a possible reason, or explanation, for the appearance of the constant Lagrangian galactic rotation curves.

The explanation of the galactic rotation curves can be presented in terms of the cosmic competition between space expansion on the one hand and gravitational contraction on the other hand. Gravitational contraction follows the virial curve, space expansion the Lagrangian curve. Where the two meet face to face with equal strengths, complex galactic rotation curves result.

2. A 'CONSTANT LAGRANGIAN' MODEL GALAXY WITH FLATTENING ROTATION CURVE

In (de Haas 2018d) I presented a model galaxy consisting of a model bulge and a model outside of the bulge. I presented this in the context of both a geodetic Lagrangian approach and a GNSS atomic clock satellite syntonization application as a completely syntonized model galaxy. Fundamental my approach of galactic dynamics was to analyze gravity using relative frequency shifts, and thus $\frac{d\tau}{dt}$, as one of the basic experimental inputs. The key to my approach was to extend this clock frequency perspective towards gravity from geodesy to galaxies. When I connected the GNSS result for clock-rate syntonization

$$\frac{d\tau^2}{dt^2} = 1 + \frac{2\Phi}{c^2} - \frac{v_{orbit}^2}{c^2} = 1 - \frac{2L}{U_0} \tag{1}$$

to the problem of the galactic rotation curve, I realized that the flat rotation curve implies atomic clock syntonization in those areas. In order to study the relativistic clock-rate behavior in the inner regions of galaxies, I had construct a model galaxy that was completely syntonized of the entire rotation curve. This model galaxy was build of a model bulge with mass M and radius R and a Schwardschild metric emptiness around it. The model bulge was a quasi solid sphere.

The gravitational potential in such a case is well known. Inside the sphere the potential is

$$\Phi = -\frac{GM}{2R}\left(3 - \frac{r^2}{R^2}\right), \tag{2}$$

and outside the sphere the potential is

$$\Phi = -\frac{GM}{r}. \tag{3}$$

Inside this bulge the classical virial theorem would hold side by side of the constant Lagrangian condition. So $2K = -V$, with on the boundary where $r = R$ we would have $\frac{K}{m} = \frac{GM}{2R}$ and $\frac{L}{m} = \frac{K-V}{m} = \frac{3GM}{2R}$. At the center of the rotating sphere, $K = 0$ and we also have $\frac{L}{m} = \frac{3GM}{2R}$. One can further calculate that $L = K - V$ is a constant in between $r = 0$ and $r = R$, so everywhere' inside this quasi-solid sphere. We can write for this region:

$$\frac{L}{m} = \frac{v_{orbit}^2}{2} + \frac{GM}{r} = \frac{3GM}{2R} = constant. \tag{4}$$

In the model galaxy that I thus constructed, $L = constant$ inside the model bulge. I then added the condition that $L = constant$ too in the whole of space outside the bulge. In that region however, the virial theorem was assumed invalid, without changing the Newtonian potential.

This leads to $K = L + V$ and $L = V(r = 0)$, so for the region $0 \leq r \leq R$ we get

$$v_{orbit}^2 = \frac{GM}{R} \cdot \frac{r^2}{R^2} \tag{5}$$

and outside the model bulge, where $R \leq r \leq \infty$, we have

$$v_{orbit}^2 = \frac{3GM}{2R} - \frac{GM}{r}. \tag{6}$$

In Fig.(1) I sketched the result, with $-V = +K_{escape}$.

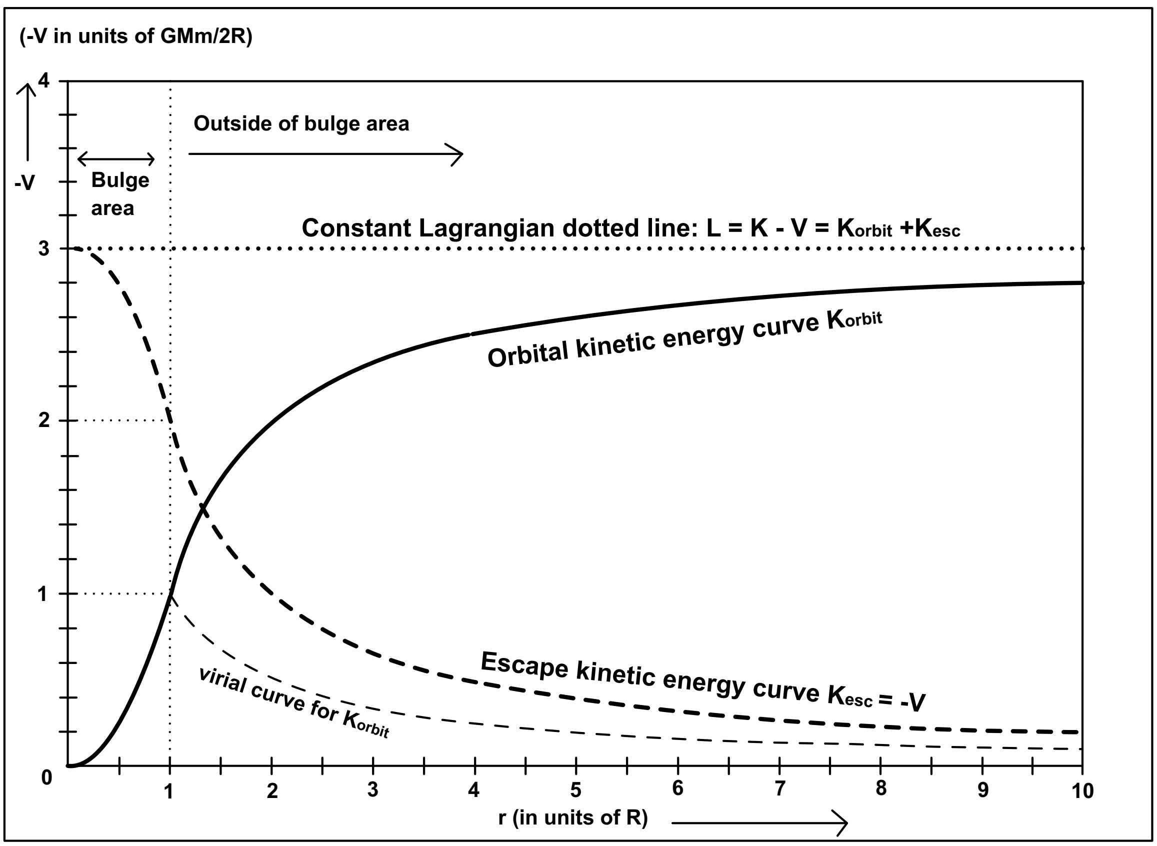


Figure 1. The square of the orbital velocity profile in the model galaxy with $L = constant$.

From the perspective of a free fall Einstein elevator observer, the free fall on a radial geodetic from infinity towards the center of the bulge, the other free fall tangential geodetics seem to abide the law of conservation of energy, because the escape kinetic energy plus the orbital kinetic energy is a constant on my model galaxy with galactic constant L. An Einstein elevator system with test mass m that would be put in an orbital collapse situation, magically descending from orbit to orbit in a process in thermodynamic equilibrium, would have constant total kinetic energy, from the radial free fall perspective. This can be expressed as $L = K_{orbit} - V = K_{orbit} + K_{escape} = K_{final}$.

3. SPACE EXPANSION VERSUS GRAVITATIONAL CONTRACTION

During the analysis of real galactic rotation curves, see (de Haas 2018a), I began to realize that the constant Lagrangian rotation curve had to be understood from the inside out, not from the outside in as with virial rotation curves. The latter one's are under the Newtonian regime of gravitational contraction. From an energy perspective it makes sense to analyze Newtonian gravitation as a system that is in a natural state of contraction. Contracting mass looses gravitational energy, of which, according to the virial theorem, one half is converted into orbital energy and one half has to be dissipated into any other form of energy. Dissipation examples are heat, internal rotation and emission of radiation.

For the constant Lagrangian curves, it makes more sense to look at them as situations where orbital kinetic energy is being transformed into gravitational energy under extremely stable conditions of total Lagrangian energy conservation. This is a process that implies some form of escape from the central mass. On a galactic scale, only cosmic expansion of space seems to provide such an escape mechanism. So my hypothesis is that the constant Lagrangian curve is a product of the combination of cosmic space expansion and energy conservation.

In the model galactic bulge, the virial theorem and the constant Lagrangian are both valid, implying that gravitational contraction and space expansion balance each other out with a perfectly stable bulge as a result. But outside the bulge, the extreme low density presumed in the model makes that gravitational contraction is no match for space expansion. If space expansion was to happen without affecting the orbital velocity and kinetic energy, then gravitational energy would increase automatically due to space expansion. Then the cosmic expansion of space would create energy out of its own process, so out of space expansion. The appearance of the constant Lagrangian rotation curve of galaxies show that conservation of energy is maintained in the process of space expansion: orbital kinetic energy is the source of the increasing gravitational energy.

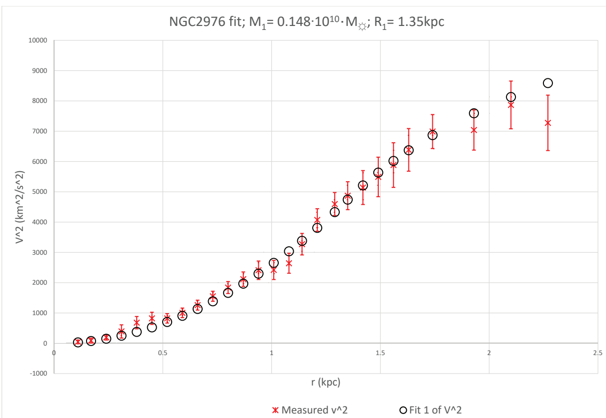
Perceived from the inside out, the mass realizing the flattening rotation curve in the outer regions of galaxies started out as mass being close to the bulge. This mass then slowly expanded away from the center, not on its own initiative but by being dragged along the expansion of space itself. But during the process this mass had to maintain its own energy balance. The only available free source for balancing the increasing gravitational energy was its orbital energy. The result is a 'constant Lagrangian' rotation curve for my model galaxy.

But in regions of galaxies where the matter density is high, gravitational contraction might can win the battle with space expansion and the virial theorem partly reasserts itself. In regions where space expansion is negligible, the virial gravitational contraction rules alone.

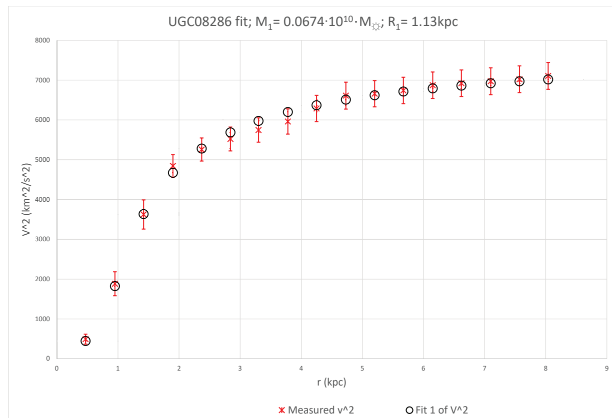
4. IDENTIFYING SPACE EXPANSION ZONES VERSUS GRAVITATIONAL CONTRACTION ZONES IN REAL GALAXY ROTATION CURVES

In (de Haas 2018b,a) I showed that 25 percent of the 175 LTG galaxies of the SPARC database could be fitted on a single 'constant Lagrangian' curve. The real velocity curves including the error margins are from (Lelli et al. 2016). This database functioned as a random set relative to my model. I analyzed, fitted, the full set of 175 galaxies at the SPARC database, as provided by (Lelli et al. 2016) in the file Rotmod-LTG.zip. The SPARC website also provided a luminosity and mass distribution analysis of those 175 galaxies. The fits of this database shows that for galaxies that didn't had a single fit, at least huge stretches of those galaxy rotation curves could be plotted on a constant Lagrangian curve.

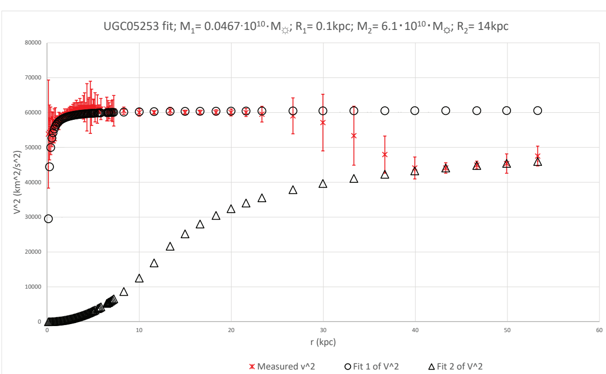
In the results of Fig.(2, UGC01281) and Fig.(2, NGC2976) the three aspects of the model curve are clearly present. First the model bulge patter is clearly present in the ascending parabolic part of the curve. This part of the model is classical because it combines the virial theorem and the constant Lagrangian. Then secondly the shift from bulge to free space as a continuous increasing function instead of the abrupt decrease as would be expected classically with the virial theorem. Thirdly is the type of ascending towards a maximum. This part of the graph is more clearly visible in Fig.(2, UGC08286). Using the analysis of this paper I can conclude that these two galaxy rotation curves are the product of space expansion under energy conservation as the dominating process outside their bulge.



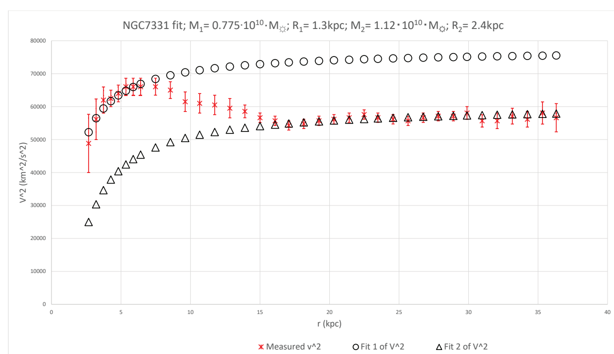
NGC2976



UGC08286



UGC05253



NGC7331

Figure 2. Two single fit constant Lagrangian galaxies and two dual fit constant Lagrangian galaxies. Where the actual measurements descend from the first fit to the second, gravitational contraction dominated over space expansion.

The other two galaxies could only be fitted on two curves, with an intermediate zone of rotation curve descent. In the analysis of this paper, in these last zones gravitational contraction has managed to assert itself relative to space expansion. But the result is a mixture of both because the descent is not by far on a pure virial curve. The pure virial curve would have resulted in a much steeper descent.

These zones should be recognizable as having a relatively high mass density, allowing gravity and gravitational contraction to partially reassert itself.

But as we go further out, mass densities inevitably reduces to very low values and space expansion, which should be a function of the amount of space available, inevitably has to come out as the dominating factor in the far out regions of all galaxies. This also leads to the conclusion that only mass that originates from a galaxy can be part of its constant Lagrangian rotation curve. Really, really far out, there will be mass that didn't originate from the inside the galaxy concerned and that mass should have a fixed constant Lagrangian energy relation to that galaxy. Mass that has been gravitationally captured by a galaxy from the outside can originally have any rotation curve relative to the host galaxy. Only as far as and for the time it has participated in the space expansion process of the galaxy that has trapped it can it be on a constant Lagrangian curve of its own.

5. MOND, VERLINDE AND THE CONNECTION BETWEEN BTF AND HUBBLE EXPANSION

One of the few non-particle approaches to the problem of Dark Matter is MOND or MOdified Newtonian Dynamics. MOND started in 1983 with two seminal paper of Milgrom. Instead of assuming the Newtonian theory to remain valid in and around galaxies, Milgrom modified Newtons second law by making inertia a function of acceleration (Milgrom 1983b). Milgrom replaced $m_g\mathbf{a} = \mathbf{F}$ by

$$m_g\mu\left(\frac{a}{a_0}\right)\mathbf{a} = \mathbf{F}. \tag{7}$$

With such a deviation only reveals itself for accelerations with $a \approx a_0$. When $a \gg a_0$, $\mu \approx 1$ and the Newtonian regime reasserts itself. This resulted in the capacity to reasonably fit most of the galaxy rotation curves and it lead to an intrinsic connection to the baryonic Tully-Fisher relation as $V_\infty^4 = a_0GM$ (Milgrom 1983a).

The original Tully-Fisher relation is a relation between the luminosity of a spiral galaxy and its, maximum, rotation velocity (Tully & Fisher 1977). The physical basis of the Tully-Fisher relation is the relation between a galaxy's total baryonic mass and the velocity at the flat end of the rotation curve, the final velocity. According to McGaugh both stellar and gas mass of galaxies have to be taken into account in the relation that is referred to as the Baryonic Tully-Fisher (BTF) relation. In 2005 McGaugh determined the baryonic version of the LT relation as $M_d = 50v_f^4$, see (McGaugh 2005) and Fig(3). In this form, M_d is expressed in solar mass $M_\odot = 1,99 \cdot 10^{30}\ kg$ units and the final velocity of the galactic rotation velocity curve v_f is expressed in km/s. If we express the galactic mass in kg and the velocity in m/s we get the total baryonic mass, final velocity relations in SI unit values as $M_b = 1,0 \cdot 10^{20}v_f^4$.

In 1983, Milgrom interpreted the BTF relation as indicative of his proposed deviation from Newtonian gravity, justifying his modification of Newtonian dynamics or MOND (Milgrom 1983b). Using McGaug's 2005 values in SI units, Milgrom's presentation of the BTF relation can be cast in the form $v_f^4 = 1,0 \cdot 10^{-20}M_b = Ga_0M_b$, resulting in an acceleration $a_0 = 1,5 \cdot 10^{-10}\ m/s^2$ in McGaug's values. Milgrom hypothesized that this relation should hold exactly, thus interpreting it as an inductively found law of nature, instead of looking at it as just a coincidental empirical relation (Milgrom 1983a). The resulting acceleration can be written as $5 \cdot a_0 \approx cH_0$, with the velocity of light c and the Hubble constant H_0. According to Milgrom, the deeper significance of this relation between this special galactic acceleration and the Hubble acceleration should be revealed by future cosmological insights

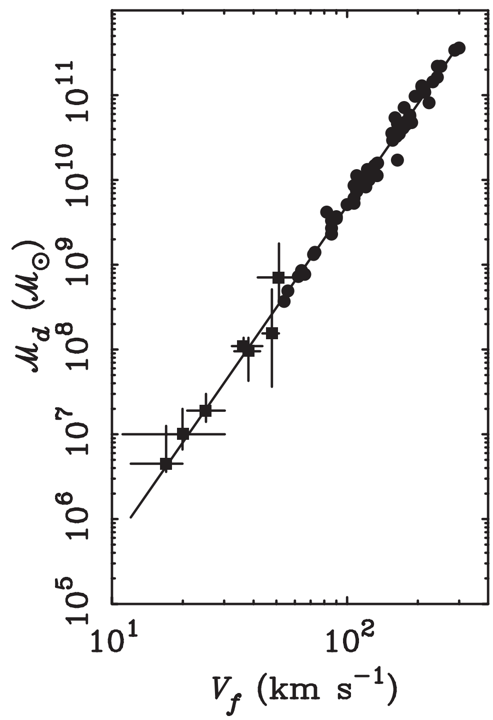


Figure 3. The Baryonic Tully-Fisher relation. Reprint from McGaug 2005 (McGaugh 2005).

(Milgrom 1983b). At the moment, (Verlinde 2016, p. 38) calls it the Hubble acceleration scale and assumes $6 \cdot a_0 \simeq cH_0$. Then we get the approximate BTF-MOND-Hubble relation

$$v_f^4 \simeq \frac{GcH_0M_b}{6}. \tag{8}$$

Thus, in galactic research, the connection between galaxy rotation curves and the Hubble expansion parameter is a recognized empirical peculiarity.

The expansion of the cosmos is determined by the Hubble constant. It is a universal constant, because the same in all directions where we can observe receding galaxies. The deviations from the virial rotation curve always appears below a certain acceleration, when

$$\frac{GM}{r^2} < \frac{cH_0}{2}, \tag{9}$$

see (Verlinde 2016) and Emergent Gravity and the Dark Universe, lecture by Prof. Erik Verlinde at IPS 2017. In my perspective, this clearly indicates that Newtonian gravitational contraction is limited in its capacity to counterbalance Hubble space expansion. The MOND acceleration a_0 or Verlinde's acceleration scale a_M present the practical limit in this sense. In today's Standard Cosmology Dark Matter effects kick in beyond this limit. In my perspective, beyond this limit Hubble space expansion starts to measurably dominate over Newtonian gravitational contraction. This Hubble space expansion is a process within the principle of conservation of energy and the mass dragged along with this expanding space therefore follows the constant Lagrangian rotation curves, implicating that orbital kinetic energy is being transformed into gravitational energy during this expansion.

The Hubble acceleration defined as $g_H \simeq cH_0$ indicates the order of magnitude below which value expansion starts to dominate contraction. The notion that this already happens deep inside galaxies and not only well outside galaxies and even outside galaxy clusters is the 'revolutionary' new of my 'constant Lagrangian' approach towards the Dark Matter issue. Compared to the hypothesis of a revolutionary and mysterious Dark Matter particle, WIMP or otherwise, my approach is rather conservative. All I propose to reset, rethink and rescale is the presupposed boarder between Newtonian contraction and Hubble expansion.

6. CONCLUSION

In my opinion, the problem of the galaxy rotation curves can be solved on the basis of the combined and competing principles of space expansion and energy conservation on the one hand and gravitational contraction and the virial theorem on the other hand.

Thus far it has been assumed that the existence of galaxies is proof of the dominance of gravitational contraction in galaxies. It is my conviction that this assumption is wrong and that space expansion is one of the two dominating principles active in galaxies. The other being gravitational contraction.

Rotation curves of Galaxy clusters should allow analogous analysis, with perhaps a somewhat different balance between space expansion and gravitational contraction.

REFERENCES

de Haas, E. P. J. 2018a, Preprint on viXra.org:Astrophysics, viXra:1805.0402

—. 2018b, Preprint on viXra.org:Astrophysics, viXra:1805.0342

—. 2018c, Preprint on viXra.org:Astrophysics, vixra:1805.0168

—. 2018d, Preprint on viXra.org:Astrophysics, vixra:1804.0298

—. 2018e, Preprint on viXra.org:Astrophysics, vixra:1804.0386

—. 2018f, Preprint on viXra.org:Astrophysics, vixra:1804.0328

—. 2018g, Preprint on viXra.org:Astrophysics, vixra:1805.0047

Lelli, F., McGaugh, S. S., & Schombert, J. M. 2016, The Astronomical Journal, 152, 157

McGaugh, S. S. 2005, The Astrophysical Journal, 632, 859, arXiv:astro-ph/0506750v2

Milgrom, M. 1983a, The Astrophysical Journal, 270, 371, Astronomy Abstract Service pdf

—. 1983b, The Astrophysical Journal, 270, 365, Astronomy Abstract Service

Tully, R. B., & Fisher, J. R. 1977, Astronomy and Astrophysics, 54, 661

Verlinde, E. 2016, SciPost Phys., 2, arXiv:1611.02269v2 [hep-th]