question 462: Integrals

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Introduction

The Number Pi

The number pi is defined by:

$$\pi = \int_0^1 \frac{1}{x^{3/4} + x^{5/4}}\, dx = 3.14159265\ldots \tag{1}$$

In this note we presents some integrals involving pi.

Integrals

$$\pi = \int_0^{\infty} \frac{1}{\sqrt{x}\,\cosh\left(\sqrt{x}\right)}\, dx \tag{2}$$

Let $u = \dfrac{\sqrt{3}}{\ln 3}$, then

$$\pi = \frac{3}{4}\sqrt{3}\ln 3 + 3\int_u^{\infty} \left\{\frac{1}{x}\operatorname{sech}\left(\frac{1}{x}\operatorname{sech}\left(\frac{1}{x}\operatorname{sech}\left(\frac{1}{x}\ldots\right)\right)\right)\right\}^2 dx \tag{3}$$

Let $u = \dfrac{\sqrt{6}+\sqrt{2}}{4\ln\left(\sqrt{6}-\sqrt{3}-\sqrt{2}+2\right)}$, then

$$\pi = \frac{3\left(\sqrt{6}+\sqrt{2}\right)\ln\left(\sqrt{6}-\sqrt{3}-\sqrt{2}+2\right)}{2} + 6\int_u^{\infty} \left\{\frac{1}{x}\operatorname{sech}\left(\frac{1}{x}\operatorname{sech}\left(\frac{1}{x}\operatorname{sech}\left(\frac{1}{x}\ldots\right)\right)\right)\right\}^2 dx \tag{4}$$

References

1. Boros, G. and Moll, V.H.: Irresistible Integrals , Cambridge, University Press, 2004.