Some topological paradoxes of relativity (EPR).

Einstein-Podolsky-Rosen Paradox[1)]

V.A. Kasimov (E-mail: *quadrica-m@mail.ru*)

In the wake of the article A. Aspect "BELL's THEOREM: the naive view of the experimenter" [2)]*.*
Since the formula (23) was found to be a mistake (or a typo), I took the trouble to check the calculations from 1 to 5 sections of the article. Some clarifying points important for understanding the essence are given.
Given an elementary derivation of the formulas (3), which is omitted in the article.

The limited applicability of the point description to real space-time relations has become apparent since the formulation of the uncertainty principle in 1927 by Heisenberg. It became clear that point topology does not work very well in quantum mechanics, but with the realization that the physical metric of space-time relations is lost in relativistic physics, these suspicions have become a problem today. Actually this is confirmed by the experiments of A. Aspect. As is known, velocity is the main topological parameters of conjugation of continuous space and time. Therefore, when interpreting the results of experiments A. Aspect and there are problems associated with the velocities of propagation of signals and cause preoccupation.

The connection of local theories with additional parameters and bell inequalities is a topic of the day in physics. In detail history of a question is covered in works [1, 2, 3, 4]. Results of A. Aspect and J. Bell belong to the physics of the microlevel description.

So what is the main thing in the Aspect's work?

1. The found theoretically correlations in the behavior of "*entangled*" States of the two photons can be explained by the introduction of *additional* parameters, "averaging" of which would theoretically allow to find agreement with the observed effects.

2. That Local Supplementary Parameters Theories (read-built on the basis of *PMC*-topology) are constrained by Bell's Inequalities.

3. That certain predictions of Quantum Mechanics violate Bell's Inequalities, and therefore that Quantum Mechanics is incompatible with Local Supplementary Parameters Theories.

4. Fundamental assumption for this conflict is the *Locality assumption*.

5. We will show that in a more sophisticated version of the E.P.R. thought experiment (« timing experiment »), the Locality Condition may be considered a consequence of Einstein's Causality, preventing faster-than-light interactions.

Concerning items 4 and 5 it is necessary to tell the following:

i4. The notion of locality of space-time relations in quantum mechanics lost its direct meaning after the appearance of Heisenberg uncertainty relations, retaining its indirect meaning only in connection with the classical ones macro interpretations of quantum mechanical measurements. The concept of a continuous trajectory that ensures the establishment of a genetic affinity of the positions of points of the same point body in a dynamic, as a causal sequence of events, has also disappeared.

[1)] In order not to disturb the coherence of the presentation, the author's words of the article A. Aspect will be highlighted in color and reproduced in the original.

Bell Theorem naive view 18 Alain Aspect.
Text was prepared for a talk at a conference in memory of John Bell, held in Vienna in December 2000. It has been published in "Quantum [Un]speakables – From Bell to Quantum information", edited by R. A. Bertlmann and A. Zeilinger, Springer (2002). https://www.dropbox.com/s/q8f5ehdudr2ixe6/0402001.pdf?dl=0

[2)]**I beg your pardon for my not very good English!** The original text in Russian**:** http://vixra.org/pdf/1804.0300v1.pdf i5. The Einstein's causality implies the need to include the notion of velocity as a "topological coupling" between a continuous time 1-manifold and a continuous 3-space manifold in *PMC*-topology. It is the speed that makes it possible to establish a genetic relationship between the positions of the trajectory points of a moving body that are or may be in the dynamic causal sequence of events. However, such concepts as speed, trajectory, genetic affinity, preserved in motion, are absent in quantum mechanics. Therefore to speak about violation of causality of big sense isn't present. Violation of the Einstein relativity principle for non-localized objects can also be seen in the macro-description of physical reality [6].

The positive *result* of experiments A. Aspect is concluded that PMC-topology in KM stops working properly.

The locality of classical physics, CTR and GTR, is most likely a consequence of their asymptotic approximations, when macrodistances and macrotime periods far exceed the micro scales.

The very same occurrence of locality is the subject of "condensation" of Hilbert's description (of arbitrary dimension, countable-separable spaces) into (3+1)-dimensional pseudo-euclidean (riemannian) diversity.

In physics, today, with the help of "entangled" (*mixed*) microstates, attempts are made to recreate macrorealities, the description of which cannot be reduced to the description in *PMC*- topology. For them, the concept of speed and genetic affinity through speed is alien. *Point* Lorentz transformations cannot be applied to them either.

1. Experimental scheme

Let us consider the optical variant of the Bohm's version[4] of the E.P.R. Gedankenexperiment (Fig. 1). A source S emits a pair of photons with different frequencies ν_1 and ν_2 , counterpropagating along Oz. Suppose that the polarization part of the state vector describing the pair is:

$$|\Psi(\nu_1,\nu_2)\rangle = \frac{1}{\sqrt{2}}\{|x,x\rangle + |y,y\rangle\} \qquad (1)$$

where $|x\rangle$ and $|y\rangle$ are linear polarizations states. This state is remarkable: it cannot be factorized into a product of two states associated to each photon, so we cannot ascribe any well defined state to each photon. In particular, we cannot assign any polarization to each photon. Such a state describing a system of several objects that can only be thought of globally, is an *entangled state*.

We perform linear polarization measurements on the two photons, with analysers *I* and *II*. The analyser *I*, in orientation **a**, is followed by two detectors, giving results + or −, corresponding to a linear polarization found parallel or perpendicular to **a**. The analyser *II*, in orientation **b**, acts similarly[‡].

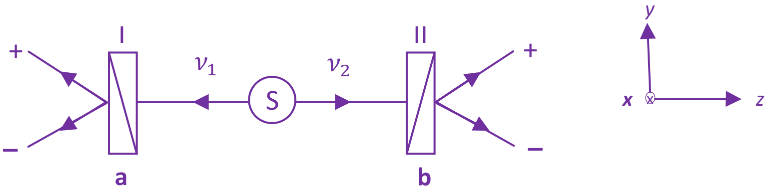

Fig. 1. Einstein-Podolsky-Rosen-Bohm Gedankenexperiment with photons*. The two photons ν_1 and ν_2 , emitted in the state $\Psi(\nu_1,\nu_2)\rangle$ of Equation (1), are analyzed by linear polarizers in orientations* **a** *and* **b**. *One can measure the probabilities of single or joint detections in the output channels of the polarizers.*

[‡] There is a one-to-one correspondance with the EPR Bohm Gedankenexperiment dealing with a pair of spin 1/2 particles, in a singlet state, analysed by two orientable Stern-Gerlach filters.

It is easy to derive the Quantum Mechanical predictions for these measurements of polarization, single or in coincidence. Consider first the singles probabilities $P_{\pm}(\mathbf{a})$ of getting the results $\pm$ for the photon v_1, and similarly, the singles probabilities $P_{\pm}(\mathbf{b})$ of obtaining the results $\pm$ on photon v_2. Quantum Mechanics predicts:

$$\begin{aligned} P_{+}(\mathbf{a}) &= P_{-}(\mathbf{a}) = \frac{1}{2} \\ P_{+}(\mathbf{b}) &= P_{-}(\mathbf{b}) = \frac{1}{2} \end{aligned} \qquad \text{Q.M.} \qquad (2)$$

These results are in agreement with the remark that we cannot assign any polarization to each photon, so that each individual polarization measurement gives a random result.

Let us now consider the probabilities $P_{\pm\pm}(\mathbf{a},\mathbf{b})$ of joint detections of v_1 and v_2 in the channels $+$ or $-$ of polarisers *I* or *II*, in orientations **a** and **b**. Quantum mechanics predicts [3]:

$$\begin{aligned} P_{++}(\mathbf{a},\mathbf{b}) &= P_{--}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cos^2(\widehat{\mathbf{a},\mathbf{b}}) \\ P_{+-}(\mathbf{a},\mathbf{b}) &= P_{-+}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\sin^2(\widehat{\mathbf{a},\mathbf{b}}) \end{aligned} \qquad \text{Q.M.} \qquad (3)$$

We are going to show that these quantum mechanical predictions have far reaching consequences.

2. Correlations

Consider first the particular situation $(\widehat{\mathbf{a},\mathbf{b}}) = 0$, where polarisers are parallel. The Quantum Mechanical predictions for the joint detection probabilities (equations 3) are :

$$\begin{aligned} P_{++}(\mathbf{a},\mathbf{a}) &= P_{--}(\mathbf{a},\mathbf{a}) = \frac{1}{2} \\ P_{+-}(\mathbf{a},\mathbf{a}) &= P_{-+}(\mathbf{a},\mathbf{a}) = 0 \end{aligned} \qquad (4)$$

According to this result, and taking into account (2), we conclude that when the photon v_1 is found in the $+$ channel of polarizer *I*, v_2 is found *with certainty* in the $+$ channel of *II* (and similarly for the $-$ channels). For parallel polarizers, there is thus a *total correlation*[4] between the individually random results of measurements of polarization on the two photons v_1 and v_2.

A convenient way to measure the amount of correlations between random quantities, is to calculate the correlation coefficient. For the polarization measurements considered above, it is equal to

$$E(\mathbf{a},\mathbf{b}) = P_{++}(\mathbf{a},\mathbf{b}) + P_{--}(\mathbf{a},\mathbf{b}) - P_{+-}(\mathbf{a},\mathbf{b}) - P_{-+}(\mathbf{a},\mathbf{b}) \qquad (5)$$

Note 1

To verify that (5) adequately reflects the degree of correlation between the orientations of vectors **a** and **b**, two cases can be considered: complete independence (no correlation) and functional uniquely dependent between their values (full correlation).

1) When the vectors a and b are completely independent and (2) is satisfied, we have:

$$\begin{aligned} &P_{++}(\mathbf{a},\mathbf{b}) = P_{+}(\mathbf{a})\cdot P_{+}(\mathbf{b}) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}; \quad P_{--}(\mathbf{a},\mathbf{b}) = P_{-}(\mathbf{a})\cdot P_{-}(\mathbf{b}) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}; \\ &P_{+-}(\mathbf{a},\mathbf{b}) = P_{+}(\mathbf{a})\cdot P_{-}(\mathbf{b}) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}; \quad P_{-+}(\mathbf{a},\mathbf{b}) = P_{-}(\mathbf{a})\cdot P_{+}(\mathbf{b}) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}. \end{aligned} \qquad (a)$$

Then from (5) we obtain:

$$E(\mathbf{a},\mathbf{b}) = 0, \qquad (b)$$

that means that there is no correlation between the accepted values of **a** and **b**.

[3] The output of the formulas (3) is presented in Appendix.

[4] The correlation dependence, in contrast to the functional, determines the effect of the value of one quantity on the distribution of another. The correlation coefficient determines the tightness (strength) of this dependence.

2) Now let each of the vectors **a** and **b** can take one of two mutually exclusive values: " $+$ " or " $-$ ". Consider two options for functional dependence:
   a) if the projection of vector **a** is positive $(+)$, the projection of vector **b** also takes a uniquely positive value $(+)$;
   b) if the projection of vector **a** is positive $(+)$, the projection of vector **b** takes a uniquely negative value $(-)$.

Denote by $P(B|A)$ - probability of occurrence of the event B ($+$ or $-$ the projection of the vector **b**), provided that has occurred event is A ($+$ or $-$ the projection of the vector **a**). Considering vector **a** as a cause, we obtain:

$$P_{++}(\mathbf{a},\mathbf{b}) = P(\mathbf{a}_+)\cdot P(\mathbf{b}_+|\mathbf{a}_+) \qquad P_{--}(\mathbf{a},\mathbf{b}) = P(\mathbf{a}_-)\cdot P(\mathbf{b}_-|\mathbf{a}_-)$$
$$P_{+-}(\mathbf{a},\mathbf{b}) = P(\mathbf{a}_+)\cdot P(\mathbf{b}_-|\mathbf{a}_+) \qquad P_{-+}(\mathbf{a},\mathbf{b}) = P(\mathbf{a}_-)\cdot P(\mathbf{b}_+|\mathbf{a}_-) \tag{c}$$

Then
for the case of a):

$$P(\mathbf{b}_+|\mathbf{a}_+) = P(\mathbf{b}_-|\mathbf{a}_-) = 1\,, \qquad P(\mathbf{b}_-|\mathbf{a}_+) = P(\mathbf{b}_+|\mathbf{a}_-) = 0\,; \tag{d}$$

for case b):

$$P(\mathbf{b}_+|\mathbf{a}_+) = P(\mathbf{b}_-|\mathbf{a}_-) = 0\,, \qquad P(\mathbf{b}_-|\mathbf{a}_+) = P(\mathbf{b}_+|\mathbf{a}_-) = 1\,. \tag{e}$$

Substituting the found values (d) and (e) in (c), in both cases, taking into account (2), from (5) we obtain:

$$|E(\mathbf{a},\mathbf{b})| = 1, \tag{f}$$

that characterizes the complete correlation between the accepted values of the projections of vectors **a** and **b**.
It is easy to obtain similar results when considering the value of the projection of the vector **b** as the initial event.

Using the prediction (3) of Quantum Mechanics, we find a correlation coefficient E_{QM}

$$E_{QM}(\mathbf{a},\mathbf{b}) = \cos 2\cdot(\widehat{\mathbf{a},\mathbf{b}}) \tag{6}$$

In the particular case of parallel polarizers ($(\widehat{\boldsymbol{a},\boldsymbol{b}}) = 0$), we find $E_{QM}(0) = 1$: this confirms that the correlation is total.

In conclusion, the quantum mechanical calculations suggest that although each individual measurement gives random results, these random results are correlated, as expressed by equation (6). For parallel (or perpendicular) orientations of the polarizers, the correlation is total ($|E_{QM}| = 1$).

3. Difficulty of an image derived from the formalism of Quantum Mechanics

As a naive physicist, I like to raise the question of finding a simple image to understand these strong correlations. The most natural way to find an image may seem to follow the quantum mechanical calculations leading to (3). In fact, there are several ways to do this calculation. A very direct one is to project the state vector (1) onto the eigenvector corresponding to the relevant result. This gives immediately the joint probabilities (3). However, since this calculation bears on state vectors describing globally the two photons, I do not know how to build a picture in our ordinary space.

The situation with the experiment on a pair of "entangled" photons(see **Note 1**) is similar to the variant 2 of the functional dependence of the States of projections of vectors **a** and **b** at $|E(\mathbf{a},\mathbf{b})| = 1$ described by the formulas (c), (d), (e), (f). Indeed, when measuring the projection of the first photon, the state of which is unknown, the value of the projection of the second photon automatically becomes known, no matter how remote it is.

In order to overcome this problem, and to identify separately the two measurements happening on both ends of the experiment, we can split the joint measurement in two steps. Suppose for instance that the measurement on photon ν_1 takes place first, and gives the result $+$, with the polarizer I in orientation **a**. The $+$ result (associated with the polarization state $|\mathbf{a}\rangle$) has a probability of $1/2$. To proceed with the calculation, we must then use the postulate of reduction of the state vector, which states that after this measurement, the new state vector $|\Psi'(\nu_1,\nu_2)\rangle$ describing the pair is obtained by projection of the initial state vector $|\Psi\ (\nu_1, \nu_2)\rangle$ (equation 1) onto the eigenspace associated to the result $+$: this two dimensional eigenspace has a basis $\{|\mathbf{a},\mathbf{x}\rangle, |\mathbf{a},\mathbf{y}\rangle\}$. Using the corresponding projector, we find after a little algebra

$$\Psi'(\nu_1, \nu_2) = |\mathbf{a},\mathbf{a}\rangle \tag{7}$$

This means that immediately after the first measurement, photon ν_2 takes the polarization $|\mathbf{a}\rangle$: this is obvious because it has been measured with a polarizer oriented along **a**, and the result $+$ has been found. More surprisingly, the distant photon ν_2 , which has not yet interacted with any polarizer, has also been projected into the state **a** with a well defined polarization, parallel to the one found for photon ν_1 . This surprising conclusion however leads to the correct final result (3), since a straightforward application of Malus law[5)] shows that a subsequent measurement performed along **b** on photon ν_2 will lead to

$$P_{++}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cos^2(\widehat{\boldsymbol{a},\boldsymbol{b}}) \tag{8}$$

The calculation in two steps therefore gives the same result as the direct calculation. But in addition it suggests a picture for the two steps measurement:

i. Photon ν_1 , which had not a well defined polarization before its measurement, takes the polarization associated to the obtained result, at the moment of its measurement: this is not surprising.

ii. When the measurement on ν_1 is done, photon ν_2 , which had not a well defined polarization before this measurement, is projected into a state of polarization parallel to the result of the measurement on ν_1. This is very surprising, because this change in the description of ν_2 happens instantaneously, whatever the distance between ν_1 and ν_2 at the moment of the first measurement.

This picture seems in contradiction with relativity. According to Einstein, what happens in a given region of space-time cannot be influenced by an event happening in a region of space-time that is separated by a space like interval. It therefore not unreasonable to try to find more acceptable pictures for «understanding » the EPR correlations. It is such a picture that we consider now.

4. Supplementary parameters

Correlations between distant measurements on two separated systems that had previously interacted are common in the classical world. For instance, if a mechanical object with a null linear (or angular) momentum is split in two parts by some internal repulsion, the linear (or angular) momenta of the two separated parts remain equal and opposite in the case of a free evolution. In the general case where each fragment is submitted to some interaction, the two momenta remain correlated since they are at each moment determined by their initial values, which had a perfectly defined sum.

It is tempting to use such a classical picture to render an account of the EPR correlations, in term of common properties of the two systems. Let us consider again the perfect correlation of polarization measurements in the case of parallel polarisers $(\widehat{\mathbf{a},\mathbf{b}}) = 0$. When we find $+$ for ν_1, we are sure to find also $+$ for ν_2. We are thus led to admit that there is some property (Einstein said « an element of physical reality ») pertaining to this particular pair, and determining the result $++$. For another pair, when the results is $--$, we can similarly invoke a common property, determining the result $--$. It is then sufficient to admit that half the pairs are emitted with the property $++$, and half with the property $--$, to reproduce all the results of measurement in this configuration. Note however that such properties, differing from one pair to another one, are not taken into account by the Quantum Mechanical state vector $|\Psi(\nu_1, \nu_2)\rangle$ which is the same for all pairs. This is why we can conclude with Einstein that *Quantum*

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5) The Malus law is a physical law that expresses the dependence of the intensity of linearly polarized light after its passage through the polarizer from the angle φ between the polarization planes of the incident light and the polarizer $I = k_a I_0 \cos^2\varphi$, where I_0 is the intensity of the light incident on the polarizer, I is the intensity of the light exiting the polarizer, k_a is the transmittance of the polarizer.

Mechanics is not complete. And this is why such additional properties are referred to as « *supplementary parameters* », or «*hiddenvariables*»[6)].

As a conclusion, it seems possible to « understand » the EPR correlations by such a classical-looking picture, involving supplementary parameters differing from one pair to another one. It can be hoped to recover the statistical Quantum Mechanical predictions when averaging over the supplementary parameters. It seems that so was Einstein's position[5,6,7]. Note that at this stage of the reasoning, a commitment to this position does not contradict quantum mechanics: there is no logical problem to fully accept the predictions of quantum mechanics, *and* to invoke supplementary parameters giving an acceptable picture of the EPR correlations. It amounts to considering Quantum Mechanics as the Statistical Mechanics description of a deeper level.

Three decades after the publication of the EPR paper [4], Bell expressed in mathematical form the consequences arising from the previous discussion and explicitly introduced additional parameters[2]. The details of the events related to the subsequent statements about the impossibility to introduce additional parameters in quantum mechanics without contradictions AND the results of real experiments confirming the correlation connections of type (5) are presented in the work cited here [1]. It is our task here was to once again address the essence of space-time contradictions in the interpretation of experiments such as EPR with the refinement of additional calculations.

5.Bell' inequalitoes

5.1. Formalism

Three decades after the EPR paper, Bell translated into mathematics the consequences of the preceding discussion, and he explicitly introduced supplementary parameters, denoted l. Their distribution on an ensemble of emitted pairs is specified by a probability distribution $\rho(\lambda)$, such that

$$\rho(\lambda) \geq 0$$
$$\int d\lambda \rho(\lambda) = 1 \tag{9}$$

For a given pair, characterized by a given supplementary parameter λ, the results of measurements are given by the bivalued functions

$$\begin{aligned} &A(\lambda, \mathbf{a}) = \pm 1 \text{ at analyzer I (in orientation } \mathbf{a}) \\ &B(\lambda, \mathbf{b}) = \pm 1 \text{ at analyzer II(in orientation } \mathbf{b}) \end{aligned} \tag{10}$$

A particular Supplementary Parameter Theory is completely defined by the explicit form of the function $\rho(\lambda)$, $A(\lambda, \mathbf{a})$ and $B(\lambda, \mathbf{b})$. It is then easy to express the probabilities of the various results of measurements. For instance, noting that the function ½$[A(\lambda, \boldsymbol{a}) + 1]$ assumes the value $+1$ for the $+$ result, and 0 otherwise (and similarly ½$[1 - B(\lambda, \boldsymbol{b})]$) assumes the value $+1$ for the $-$ result, and 0 otherwise, we can write

$$\begin{aligned} P_{+}(\mathbf{a}) &= \int d\lambda \rho(\lambda) \frac{[A(\lambda, \mathbf{a}) + 1]}{2} \\ P_{+-}(\mathbf{a}, \mathbf{b}) &= \int d\lambda \rho(\lambda) \frac{[A(\lambda, \mathbf{a}) + 1]}{2} \frac{[1 - B(\lambda, \mathbf{b})]}{2} \end{aligned} \tag{11}$$

Similarly, the correlation function assumes the simple form

$$E(\mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) A(\lambda, \mathbf{a}) B(\lambda, \mathbf{b}) \tag{12}$$

[6]) Einstein actually did not speak of « hidden variables » or « supplementary parameters », but rather of « elements of the physical reality ». Accordingly, many authors refer to « realistic theories » rather than to « hidden variable theories », or to « supplementary variable theories ».

Note 2.

On the comparison of formulas for correlations (5) and (12). Along with the General definitions of correlation, as bearing information about the tightness of the influence of the value of one random variable on the distribution of another, the definition (5) is the most adequate. Its value of 1 indicates a direct functional dependence of the directions of vectors a and b (see Note 1). To determine the correlation in the General model for random variables X and Y, we have chosen for them a uniform distribution, which does not carry any information about the probabilities of their values, except for certain ones. Its value of 1 indicates a direct functional dependence of the directions of vectors a and b (see Note 1). To determine the correlation in the General model for random variables X and Y, we have chosen for them a uniform distribution, which does not carry any information about the probabilities of their values, except for certain ones.

In table. A data and results for the calculation of the standard correlation coefficient $r(X,Y)$) of random variables X and Y, coinciding covariance $COV(X,Y)$ [7]:

$$COV(X,Y)/(\sigma_x\sigma_v) = \left(M[X,Y] - \mu_x\mu_v\right)/\sigma_x\sigma_v = r(X,Y) \tag{g}$$

Table A

x_i	$+1$	-1
x_i^2	$+1$	$+1$
p_i	$1/2$	$1/2$
$\boldsymbol{\mu_x = M[X]} = (x_1 + x_2)/2$	**0**	
$M[X^2] = (x_1^2 + x_2^2)/2$	1	
$D[X] = M[X^2] - (M[X])^2$	**1**	
$\boldsymbol{\sigma_x}$	**1**	

The additional parameter λ introduced in (12) provides the generality of the bell model.

5.2. (Naive) example of a theory with additional parameters

As an example of a theory with an additional parameter, we consider a model in which each photon propagating along the Oz axis is assumed to have a well-defined linear polarization given by its angle $(\lambda_I и\ \lambda_{II})$ with the x axis. To account for the tight correlation, we assume that two photons of the same pair are emitted with the same linear polarization, defined by the General angle λ (Fig. 2).

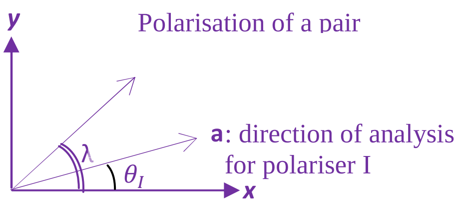

Fig. 2 - The naive example. *Each pair of photons has a « direction of polarisation », defined by λ, which is the supplementary parameter of the model. Polariser I makes a polarisation measurement along* **a**, *at an angle θ_I from the x axis..*

The polarisation of the various pairs is randomly distributed, according to the probability distribution $\rho(\lambda)$, that we take rotationally invariant:

$$\rho(\lambda) = \frac{1}{2\pi} \tag{13}$$

To complete our model, we must give an explicit form for the functions $A(\lambda,\boldsymbol{a})$ and $B(\lambda,\boldsymbol{b})$. We take the following form

$$A(\lambda,\mathbf{a}) = \mathrm{sign}\{\cos 2\,(\theta_I - \lambda)\},$$
$$B(\lambda,\mathbf{b}) = \mathrm{sign}\{\cos 2\,(\theta_{II} - \lambda)\}, \tag{14}$$

where the angles θ_I and θ_{II} indicate the orientations of the polarisers. Note that these forms are very reasonable: $A(\lambda,\mathbf{a})$ assumes the value $+1$ when the polarisation of photon v_1 makes an angle less than π/4 with the direction of analysis **a**, and -1 for the complementary case (polarisation closer to the perpendicular to **a**).

With this explicit model, we can use equations (11) to calculate the probabilities of the various measurements. We find for instance single probabilities

$$P_+(\mathbf{a}) = P_-(\mathbf{a}) = P_+(\mathbf{b}) = P_-(\mathbf{b}) = \frac{1}{2} \tag{15}$$

identical to the Quantum Mechanical results. The model also allows us to calculate the joint probabilities, or equivalently the correlation function, and we find, using (12) :

$$E(\mathbf{a},\mathbf{b}) = 1 - 4\frac{|\theta_I - \theta_{II}|}{\pi} = 1 - 4\frac{|(\mathbf{a},\mathbf{b})|}{\pi} = \quad \text{for } -\frac{\pi}{2} \leq \theta_I - \theta_{II} \leq \frac{\pi}{2} \tag{16}$$

Note 3

Let us present the conclusions of the formula (16).

To facilitate the understanding of the relationship between the angles arising in the example, let us present two analyzers with the same orientation $\theta = \pi/4$with respect to the axis **X.** By rotating the first analyzer counterclockwise by an angle Δ_1 Bt orientating it along the direction **a,** the vector acquires the orientation $\theta_I = \theta + \Delta_1$. By rotating the second analyzer clockwise at an angle Δ_2, orienting it in the direction **b**, we obtain an angle $\theta_{II} = \theta - \Delta_2$. Together with the analyzers, the areas of positive values for the coefficients $A(\lambda,\mathbf{a})$ and $B(\lambda,\mathbf{b})$ − blue and green , respectively, rotate

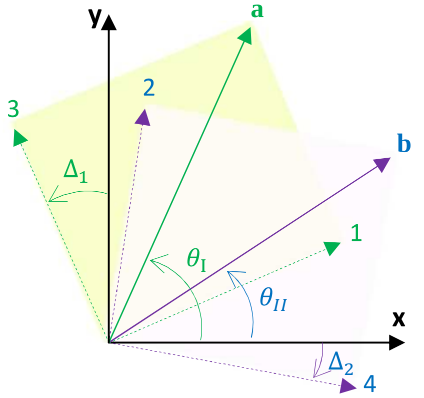

***Fig. A**. Directions **a** and **b** with their "neighborhoods" of positive values of $A(\lambda, \mathbf{a})$ and $B(\lambda, \mathbf{b})$". Maximum "raster" between the directions of polarization − θ_I and θ_{II}, is π⁄2, that is: $-\pi/2 \leq \theta_I - \theta_{II} \leq \pi/2$.*

As a result, we obtain the following regions with the distribution of signs for $A(\lambda, \mathbf{a})$ and $B(\lambda, \mathbf{b})$.

Table B

Domain	The range of angles	The absolute value of the angle	A	B
1.	$\Delta_1 \geq \lambda \geq -\Delta_2$	$\Delta_1 + \Delta_2$	−	+
2.	$(\pi/2) - \Delta_2 \geq \lambda \geq \Delta_1$	$(\pi/2) - (\Delta_1 + \Delta_2)$	+	+
3.	$(\pi/2)+\Delta_1 \geq \lambda \geq (\pi/2) - \Delta_2$	$\Delta_1 + \Delta_2$	+	−
4.	$2\pi - \Delta_2 \geq \lambda \geq (\pi/2) + \Delta_1$	$3\pi/4 - (\Delta_1 + \Delta_2)$	−	−

The relationship between $\theta_i$and Δ_i is expressed by the pair of formulas:

$$\theta_I = \Delta_1 + \pi/4; \quad \theta_{II} = -\Delta_2 + \pi/4;$$
$$\Delta_1 = \theta_I - \pi/4; \quad \Delta_2 = -\theta_{II} + \pi/4; \tag{h}$$
$$\theta_I - \theta_{II} = \Delta_1 + \Delta_2.$$

The integral (12) in a closed loop counter-clockwise starting from the direction 4 given the data table B leads to the results (16)

This is a remarkable result. Note first that $E(\widehat{\boldsymbol{a},\boldsymbol{b}})$ depends only on the relative angle $(\widehat{\boldsymbol{a},\boldsymbol{b}})$, as the Quantum Mechanical prediction (6). Moreover, as shown on Fig 3, the difference between the predictions of the simple supplementary parameters model and the quantum mechanical predictions is always small, and the agreement is exact for the angles $0, \pm\pi/4$ and $\pm\pi/2$ *i.e.* cases of total correlation. This result, obtained with an extremely simple supplementary parameters model, is very encouraging, and it might be hoped that a more sophisticated model could be able to reproduce exactly the Quantum Mechanical predictions. *Bell's discovery is the fact that the search for such models is hopeless*, as we are going to show now.

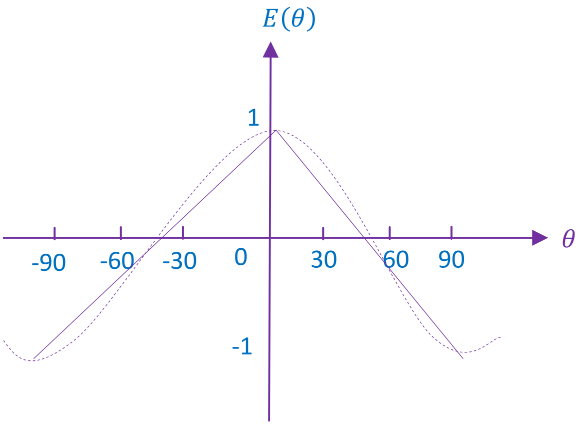

Fig. 3 *- Polarisation correlation coefficient, as a function of the relative orientation of the polarisers: (i) Dotted line : Quantum Mechanical prediction ; (ii) solid line : the naive model.*

5.3. Bell's Inequalities

There are many different forms, and demonstrations of Bell'inequalities. We give here a very simple demonstration leading to a form directly applicable to the experiments[7)].

Let us consider the quantity

$$s = A(\lambda,\mathbf{a})B(\lambda,\mathbf{b}) - A(\lambda,\mathbf{a})B(\lambda,\mathbf{b}') + A(\lambda,\mathbf{a}')B(\lambda,\mathbf{b}) + A(\lambda,\mathbf{a}')B(\lambda,\mathbf{b}') =$$
$$= A(\lambda,\mathbf{a})[B(\lambda,\mathbf{b}) - B(\lambda,\mathbf{b}')] + A(\lambda,\mathbf{a}')[B(\lambda,\mathbf{b}) + B(\lambda,\mathbf{b}')] \tag{17}$$

Remembering that the four numbers A and B take only the values ± 1, a simple inspection of the second line of (17) shows that

$$s(\lambda,\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') = \pm 2 \tag{18}$$

The average of s over l is therefore comprised between $+\ 2$ and $-\ 2$:

$$-2 \le \int d\,\lambda \rho(\lambda) s(\lambda,\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') \le 2 \tag{19}$$

According to (12), we can rewrite these inequalities

$$-2 \le S(\lambda,\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') \le 2 \tag{20}$$

with

$$S(\lambda,\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') = E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}') \tag{21}$$

These are B.C.H.S.H. inequalities, *i.e.* Bell's inequalitites as generalized by Clauser, Horne, Shimony, Holt [8]. They bear upon the combination S of the four polarization correlation coefficients, associated to two directions of analysis for each polarizer (**a** and **a**′ for polarizer I, **b** and **b**′ for polarizer II). Note that they apply to any Supplementary Parameter Theory of the very general form defined in section 5.1 (equations 9, 10, and 12), of which our naive model is only an example.

---

[7]) It is important to distinguish between inequalities which show a mathematical contradiction with quantum mechanics, but without the possibility of an experimental test with (necessarily) imperfect apparatus, and inequalities allowing an experimental test provided that the experimental imperfections remain in certain limits.

6. Conflict with quantum mechanics

6.1. Evidence

We can use the predictions (6) of Quantum Mechanics for EPR pairs, to evaluate the quantity $S(\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}')$ defined by equation (21). For the particular set of orientations shown on Fig. 4.a, the result is

$$S_{QM} = 2.83 > 2 \tag{22}$$

This quantum mechanical prediction definitely conflicts with the Bell's inequality (20) which is valid for any Supplementary Parameter Theory of the general form defined in §5.1.

We have thus found a situation where the quantum mechanical predictions cannot be reproduced (mimicked) by Supplementary Parameters Theories. This is the essence of Bell's theorem: it is impossible to find a Supplementary Parameter Theory, of the general form defined in § 3.1, that reproduces **all** the predictions of quantum mechanics. This statement is the generalisation of what appears on Fig. 3, for the particular supplementary parameter model considered in § 5.2: the model exactly reproduces the predictions of quantum mechanics for some particular angles $(0, \pi/4, \pi/2)$, but it somewhat deviates at other angles. The importance of Bell's theorem is that it is not restricted to a particular supplementary parameters model, but it is general.

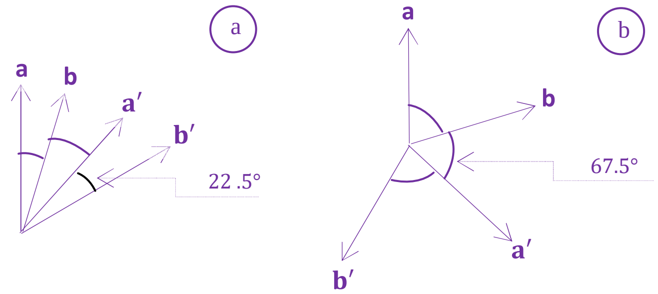

***Fig. 4** - Orientations yielding the largest conflict between Bell's inequalities and Quantum Mechanics..*

6.2. *Maximum conflict*

It is interesting to look for the maximum violation of Bell's inequalities by the quantum mechanical predictions. Let us take the quantum mechanical value of S_{QM}:

$$S_{QM}(\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') = \cos 2\cdot(\widehat{\mathbf{a},\mathbf{b}}) - \cos 2\cdot(\widehat{\mathbf{a},\mathbf{b}'}) + \cos 2\cdot(\widehat{\mathbf{a}',\mathbf{b}}) + \cos 2\cdot(\widehat{\mathbf{a}',\mathbf{b}'}) \tag{23}$$

It [8]) is a function of three independent variables $(\widehat{\mathbf{a},\mathbf{b}}), (\widehat{\mathbf{b},\mathbf{a}'})$ и $(\widehat{\mathbf{a}',\mathbf{b}'})$. Note that

$$(\widehat{\mathbf{a},\mathbf{b}'}) = (\widehat{\mathbf{a},\mathbf{b}}) + (\widehat{\mathbf{b},\mathbf{a}'}) + (\widehat{\mathbf{a}',\mathbf{b}'})$$

Note 4

Introduce the notation: $(\widehat{\mathbf{a},\mathbf{b}}) = \alpha$, $(\widehat{\mathbf{b},\mathbf{a}'}) = \beta$, $(\widehat{\mathbf{a}',\mathbf{b}'}) = \gamma$, $(\widehat{\mathbf{a},\mathbf{b}'}) = \delta$ with

$$\alpha + \beta + \gamma = \delta. \tag{i}$$

The Lagrangian takes the form: $L(\alpha,\beta,\gamma,\delta) = \cos 2\alpha - \cos 2\delta + \cos 2\beta + \cos 2\gamma + \mu(\alpha+\beta+\gamma-\delta)$, where μ is the Lagrange multiplier.

Differentiating $L(\alpha,\beta,\gamma,\delta)$ by independent variables α,β,γ, taking into account the relationship between the angles $\alpha,\beta,\gamma,\delta$ we obtain a system of equations:

$$\begin{cases} \partial L/\partial\alpha = 0 \\ \partial L/\partial\beta = 0 \\ \partial L/\partial\gamma = 0, \end{cases} \quad \Rightarrow \quad \begin{cases} \sin 2\alpha = \mu/2 \\ \sin 2\beta = \mu/2 \\ \sin 2\gamma = \mu/2, \end{cases} \tag{j}$$
$$\alpha+\beta+\gamma=\delta, \qquad \alpha+\beta+\gamma=\delta,$$

which has the solution

$$\alpha = \beta = \gamma = \theta, \tag{k}$$

[8]) The original text of the article contains the formula: $S_{QM}(\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') = \cos(\widehat{\mathbf{a},\mathbf{b}}) - \cos(\widehat{\mathbf{a},\mathbf{b}'}) + \cos(\widehat{\mathbf{a}',\mathbf{b}}) + \cos(\widehat{\mathbf{a}',\mathbf{b}'})$

and by virtue of the last equality

$$\delta = 3\theta. \tag{l}$$

Equation

$$\partial L/\partial\delta = 2\sin 2\delta = \mu \text{ или } \sin 2\delta = \mu/2 = \sin 2\theta \tag{m}$$

we rewrite in the form

$$\sin 2\delta - \sin 2\theta = 2\cos(\delta+\theta)\sin(\delta-\theta) = 2\cos 4\theta \sin 2\theta = 0\,. \tag{n}$$

The equation (n) has solutions for: $4\theta = \frac{\pi}{2} + 2\pi k;\, 2\theta = \pi k,$ from where $\theta = \frac{\pi}{8} + \frac{\pi}{4}k;\ \theta = \frac{\pi}{2}k$. At $k = 0$ we have: $\theta = \frac{\pi}{8}$ и 0; при $k = 1$: $\theta = \frac{3}{8}\pi$ и $\frac{\pi}{2}$.

For a rigid construction defined by equations (i) and (k), the ratio is true

$$S_{QM}(\theta) = 3\cos 2\theta - \cos 6\theta \tag{p}$$

The table C shows values of $S_{QM}(\theta)$ for configurations with deviations from predictions of quantum mechanics are given:

Таблица C

θ	$S_{QM}(\theta)$
$0 = 0°$	2
$\pi/8 = 22{,}5°$	2.83
$3\pi/8 = 67{,}5°$	-2.83
$\pi/2 = 90°$	2

These values are the results of substitution of the values θ in (p). The orientation sets for для $\theta = \pi/8$и $\theta = 3\pi/8$ are shown in Fig. B. they give the greatest violations of Bell's inequalities.

A more General consideration shows that there is a large range of orientations that involve conflict with bell's inequalities (see Fig. 5); it is also clear, however, that there are many sets of orientations for which there is no such conflict.

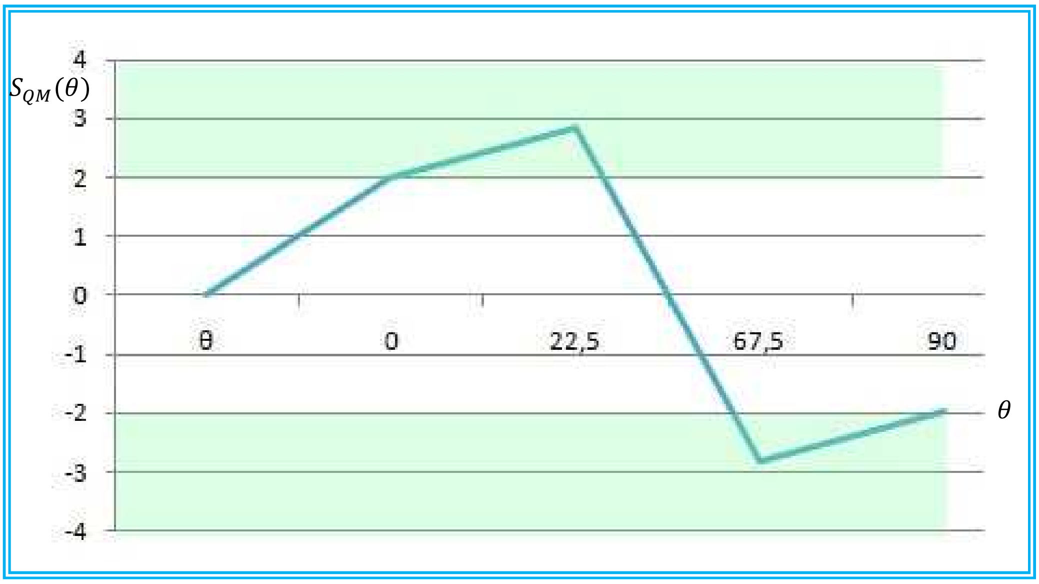

***Fig. B**: linear approximation based on the dependence $S_{QM}(\theta)$ from θ*

***Fi.e 5**: $S_{QM}(\theta)$, predicted by quantum mechanics for EPR-pairs. The conflict with Bell's inequalities occurs at $|S| > 2$ and it reaches a maximum for the set of orientations shown in Figure 4.*

Appendix. *A simple derivation of the basic formulas for a pair of "tangled" photons*

1. Light polarization

1. Polarization and helicity are examples of physical quantities taking only two quantized values.

Description of a photon as an object with dichotomous properties is possible using the property of helicity: the helicity of a photon in any direction can take only two values - positive and negative. The situation is similar to the state of an electron with spin 1⁄2 in the singlet state.

The concept of polarization, inherited from the classical properties of light, refers to the description of the properties of transverse electromagnetic waves. Polarization is described by two vectors, orthogonal to the direction of motion. The quantum dichotomy of the classical polarization parameter is manifested here. These two vectors are also orthogonal to each other, and if you know the direction of one vector, the orthogonal direction of the other vector can be represented by two directions - left or right, which distinguishes right or left polarized waves.

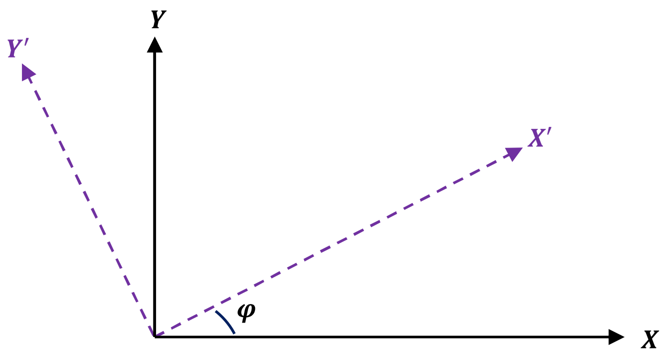

Fig. 1

2. The state of a photon with respect to its polarization can be described by two basic vectors of a 2-linear space $|x\rangle, |y\rangle$:

$|x\rangle -$ vector describing polarization along the axis X;
$|y\rangle -$ vector describing polarization along the axis X; (1)

Vectors $|x\rangle$ and $|y\rangle$ represent the complete orthonormal system, so

$$\langle x|x\rangle = \langle y|y\rangle = 1, \langle x|y\rangle = \langle y|x\rangle^* = 0 \quad (2)$$

Any state of the photon $|\Psi\rangle$ can be represented as a linear combination of these two basic states

$|\Psi\rangle = A|x\rangle + B|y\rangle$, where A and B are in generally $-$ complex numbers (3)

Let's

$$|x'\rangle = A|x\rangle + B|y\rangle \quad (4)$$

describes the new polarization state (Fig. 1). From the obvious equalities $\langle x|x'\rangle = \cos\varphi\,, \langle y|x'\rangle = \sin\varphi$ should be $A = \cos\varphi\,, B = \sin\varphi$, that is

$$|x'\rangle = \cos\varphi\,|x\rangle + \sin\varphi\,|y\rangle \quad (5)$$

a (5) will represent a photon polarized along the new axis X'.

Let's find the second basis vector $|y'\rangle$ and write down

$$|y'\rangle = C|x\rangle + D|y\rangle. \quad (6)$$

From orthogonality of vectors $|x^{'}\rangle$ and $|y'\rangle$ we have:

$$\langle y'|x'\rangle = C^*\cos\varphi + D^*\sin\varphi = 0, \quad (7)$$

and from the normalization of the vector $|y'\rangle$:

$$\langle y'|y'\rangle = C^*C + D^*D = 1 \quad (8)$$

The joint solution (6), (7) and (8) in real numbers gives

$$C = \mp\sin\varphi$$
$$D = \pm\cos\varphi \quad (9)$$

So for $|y'\rangle$ we have two solutions:

$$|y'\rangle = -\sin\varphi\,|x\rangle + \cos\varphi\,|y\rangle \tag{10}$$

$$|y'\rangle = \sin\varphi|x\rangle - \cos\varphi\,|y\rangle \tag{11}$$

corresponding to the right and left polarizations of the photon.

2. Experiment

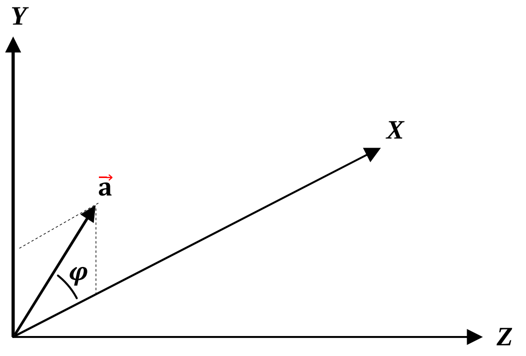

Fig. 2

Let's consider a two-photon system. Since photons do not interact with each other, the equation of state for this system allows for separation of variables and representation of the solution as a product of the state vectors of individual photons (1st and 2nd):

$$|\Psi\rangle_{1,2} = |\Psi\rangle_1 * |\Psi\rangle_2 \tag{12}$$

To represent the entangled state " of a pair of photons moving in opposite directions, needs choose states with opposite polarizations. Then, given (5), (10) and (11) when $\varphi = 0$, the expression for the first $|\Psi\rangle_1$ and the second $|\Psi\rangle_2$ vectors of states take the form:

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2}}\{|x\rangle_1 + |y\rangle_1\} \tag{13}$$

$$|\Psi\rangle_2 = \frac{1}{\sqrt{2}}\{|x\rangle_2 - |y\rangle_2\} \tag{14}$$

According to (12), we have

$$\begin{aligned} |\Psi\rangle_{1,2} &= \frac{1}{2}(|x\rangle_1 + |y\rangle_1) * (|x\rangle_2 - |y\rangle_2) = \\ &= \frac{1}{2}\{|x\rangle_1 * |x\rangle_2 - |x\rangle_1 * |y\rangle_2 + |y\rangle_1 * |x\rangle_2 - |y\rangle_1 * |y\rangle_2\} \end{aligned} \tag{15}$$

To satisfy the requirement of the particle identity principle, this state vector must be symmetric. This procedure will describe the real entanglement of photons, after which it is impossible to distinguish the first and second photons. Therefore :

$$\begin{aligned} |\Psi\rangle^{initial} &= \frac{1}{\sqrt{2}}\{|\Psi\rangle_{1,2} + |\Psi\rangle_{2,1}\} = \\ &= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \cdot \{|x\rangle_1 * |x\rangle_2 - |x\rangle_1 * |y\rangle_2 + |y\rangle_1 * |x\rangle_2 - |y\rangle_1 * |y\rangle_2 \\ &+ |x\rangle_2 * |x\rangle_1 - |x\rangle_2 * |y\rangle_1 + |y\rangle_2 * |x\rangle_1 - |y\rangle_2 * |y\rangle_1\} = \\ &= \frac{1}{\sqrt{2}}\{|x\rangle_1 * |x\rangle_2 - |y\rangle_1 * |y\rangle_2\} \end{aligned}$$

Finally, the vector of the singlet initial state of a pair of photons will take the form:

$$|\Psi\rangle^{initial} = \frac{1}{\sqrt{2}}\{|x\rangle_1 * |x\rangle_2 - |y\rangle_1 * |y\rangle_2\} \tag{16}$$

1. Let us consider the result of the measurement of pair, when the first $|\chi\rangle_1$ and the second $|\chi\rangle_2$ photons are recorded with the same polarization described by the vector (5) $|x'\rangle = \cos\varphi\,|x\rangle + \sin\varphi\,|y\rangle$:

$$|\chi\rangle_1 = \{\cos\varphi_1|x\rangle_1 + \sin\varphi_1|y\rangle_1\} \tag{17}$$

$$|\chi\rangle_2 = \{\cos\varphi_2|x\rangle_2 + \sin\varphi_2|y\rangle_2\} \tag{18}$$

According to (16), (17), (18), get for the final state

$$|\chi\rangle_{1,2}^{final} = |\chi\rangle_1 * |\chi\rangle_2 = \{\cos\varphi_1|x\rangle_1 + \sin\varphi_1|y\rangle_1\} * \{\cos\varphi_2|x\rangle_2 + \sin\varphi_2|y\rangle_2\} =$$
$$= \{\cos\varphi_1\,\cos\varphi_2|x\rangle_1|x\rangle_2 + \cos\varphi_1\,\sin\varphi_2|x\rangle_1|y\rangle_2 + \sin\varphi_1\,\cos\varphi_2|y\rangle_1|x\rangle_2 + \sin\varphi_1\,\sin\varphi_2|y\rangle_1|y\rangle_2\} \quad (19)$$

$${}_{2,1}^{final}\langle\chi| =$$
$$= \{\cos\varphi_1\,\cos\varphi_2\,{}_2\langle x|\,{}_1\langle x| + \cos\varphi_1\,\sin\varphi_2\,{}_2\langle y|\,{}_1\langle x| + \sin\varphi_1\,\cos\varphi_2\,{}_2\langle x|\,{}_1\langle y| + \sin\varphi_1\,\sin\varphi_2\,{}_2\langle y|\,{}_1\langle y|\} \quad (19')$$

For the amplitude of transition from the initial state $|\Psi\rangle^{initial}$ to final state $|\chi\rangle_{1,2}^{final}$ are:

$${}_{2,1}^{final}\langle\chi|\Psi\rangle^{initial} =$$
$$= \{\cos\varphi_1\,\cos\varphi_2\,{}_2\langle x|\,{}_1\langle x| + \cos\varphi_1\,\sin\varphi_2\,{}_2\langle y|\,{}_1\langle x| + \sin\varphi_1\,\cos\varphi_2\,{}_2\langle x|\,{}_1\langle y| + \sin\varphi_1\,\sin\varphi_2\,{}_2\langle y|\,{}_1\langle y|\} * \quad (20)$$
$$* \frac{1}{\sqrt{2}}\{|x\rangle_1 * |x\rangle_2 - |y\rangle_1 * |y\rangle_2\} = \frac{1}{\sqrt{2}}\{\cos\varphi_1\,\cos\varphi_2 - \sin\varphi_1\,\sin\varphi_2\} = \frac{1}{\sqrt{2}}\cos(\varphi_1+\varphi_2)$$

2. Now let us consider the result of the measurement of the pair, when the states of the first $|\chi\rangle_1$ and the second $|\chi\rangle_2$ photons are fixed with the same polarization, but described by the vector (10) $|y'\rangle = -\sin\varphi\,|x\rangle + \cos\varphi\,|y\rangle$:

$$|\chi\rangle_1 = \{-\sin\varphi_1|x\rangle_1 + \cos\varphi_1|y\rangle_1\} \quad (21)$$
$$|\chi\rangle_2 = \{-\sin\varphi_2|x\rangle_2 + \cos\varphi_2|y\rangle_2\} \quad (22)$$

According to (16), (21), (22), get for the final state

$$|\chi\rangle_{1,2}^{final} = |\chi\rangle_1 * |\chi\rangle_2 = \{-\sin\varphi_1|x\rangle_1 + \cos\varphi_1|y\rangle_1\} * \{-\sin\varphi_2|x\rangle_2 + \cos\varphi_2|y\rangle_2\} =$$
$$= \{\sin\varphi_1\,\sin\varphi_2|x\rangle_1|x\rangle_2 - \sin\varphi_1\,\cos\varphi_2|x\rangle_1|y\rangle_2 - \cos\varphi_1\,\sin\varphi_2|y\rangle_1|x\rangle_2 + \cos\varphi_1\,\cos\varphi_2|y\rangle_1|y\rangle_2\} \quad (23)$$

$${}_{2,1}^{final}\langle\chi| =$$
$$= \{\sin\varphi_1\,\sin\varphi_2\,{}_2\langle x|\,{}_1\langle x| - \sin\varphi_1\cos\varphi_2\,\,{}_2\langle y|\,{}_1\langle x| - \cos\varphi_1\,\sin\varphi_2\,{}_2\langle x|\,{}_1\langle y| + \cos\varphi_1\,\cos\varphi_2\,{}_2\langle y|\,{}_1\langle y|\} \quad (23')$$

For the amplitude of transition from the initial state $|\Psi\rangle^{initial}$ to final state $|\chi\rangle_{1,2}^{final}$ are:

$${}_{2,1}^{final}\langle\chi|\Psi\rangle^{initial} =$$
$$= \{\sin\varphi_1\,\sin\varphi_2\,{}_2\langle x|\,{}_1\langle x| - \sin\varphi_1\cos\varphi_2\,\,{}_2\langle y|\,{}_1\langle x| - \cos\varphi_1\,\sin\varphi_2\,{}_2\langle x|\,{}_1\langle y| + \cos\varphi_1\,\cos\varphi_2\,{}_2\langle y|\,{}_1\langle y|\} * \quad (24)$$
$$* \frac{1}{\sqrt{2}}\{|x\rangle_1 * |x\rangle_2 - |y\rangle_1 * |y\rangle_2\} = \sin\varphi_1\,\sin\varphi_2 - \cos\varphi_1\,\cos\varphi_2 = -\frac{1}{\sqrt{2}}\cos(\varphi_1+\varphi_2)$$

3. Let us consider the result of the measurement of pair when the states of the first $|\chi\rangle_1$ and the second $|\chi\rangle_2$ photons are fixed with different polarizations described by vectors (5) and (10): $|x'\rangle = \cos\varphi\,|x\rangle + \sin\varphi\,|y\rangle$, $|y'\rangle = -\sin\varphi\,|x\rangle + \cos\varphi\,|y\rangle$:

$$|\chi\rangle_1 = \cos\varphi\,|x\rangle + \sin\varphi\,|y\rangle \quad (25)$$
$$|\chi\rangle_2 = \{-\sin\varphi_2|x\rangle_2 + \cos\varphi_2|y\rangle_2\} \quad (26)$$

According to (16), (25), (26), get for the final state

$$|\chi\rangle_{1,2}^{final} = |\chi\rangle_1 * |\chi\rangle_2 = \{\cos\varphi_1|x\rangle_1 + \sin\varphi_1|y\rangle_1\} * \{-\sin\varphi_2|x\rangle_2 + \cos\varphi_2|y\rangle_2\} =$$
$$= \{-\cos\varphi_1\,\sin\varphi_2|x\rangle_1|x\rangle_2 + \cos\varphi_1\,\cos\varphi_2|x\rangle_1|y\rangle_2 - \sin\varphi_1\,\sin\varphi_2|y\rangle_1|x\rangle_2 + \sin\varphi_1\,\cos\varphi_2|y\rangle_1|y\rangle_2\} \quad (27)$$

$${}_{2,1}^{final}\langle\chi| =$$
$$= \{-\cos\varphi_1\,\sin\varphi_2\,{}_2\langle x|\,{}_1\langle x| - co\varphi_1\cos\varphi_2\,\,{}_2\langle y|\,{}_1\langle x| - \sin\varphi_1\,\sin\varphi_2\,{}_2\langle x|\,{}_1\langle y| + \sin\varphi_1\,\cos\varphi_2\,{}_2\langle y|\,{}_1\langle y|\} \quad (27')$$

For the amplitude of transition from the initial state $|\Psi\rangle^{initial}$ to final state $|\chi\rangle_{1,2}^{final}$ are:

$${}_{2,1}^{final}\langle\chi|\Psi\rangle^{initial} =$$
$$= \{-\cos\varphi_1\,\sin\varphi_2\,{}_2\langle x|\,{}_1\langle x| - co\varphi_1\cos\varphi_2\,\,{}_2\langle y|\,{}_1\langle x| - \sin\varphi_1\,\sin\varphi_2\,{}_2\langle x|\,{}_1\langle y| + \sin\varphi_1\,\cos\varphi_2\,{}_2\langle y|\,{}_1\langle y|\} \quad (28)$$
$$* \frac{1}{\sqrt{2}}\{|x\rangle_1 * |x\rangle_2 - |y\rangle_1 * |y\rangle_2\} = -\cos\varphi_1\,\sin\varphi_2 - \sin\varphi_1\,\cos\varphi_2 = -\frac{1}{\sqrt{2}}\sin(\varphi_1 + \varphi_2)$$

In formulas (20), (24), (28) the angles $\varphi_1$and φ_2 are represented in different coordinate systems − right and left. To bring their values to one coordinate system, it is necessary to replace $\varphi_2 \;\Rightarrow\; -\varphi_2$. Fig. 3 explains the need for this replacement.

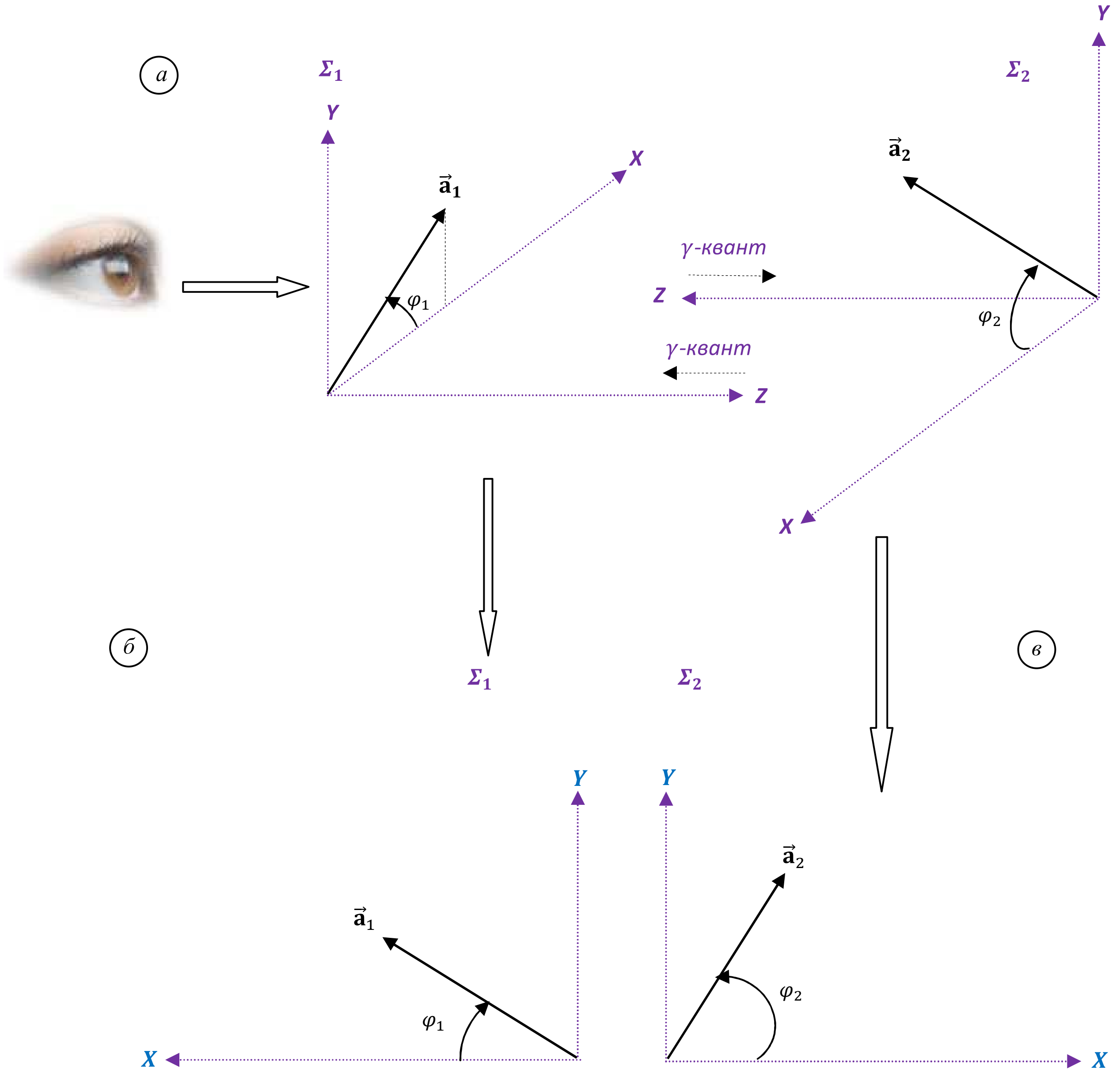

Fig. 3

When replacing $\varphi_2 \;\Rightarrow\; -\varphi_2$ then the results (20), (24), (28) rewrite the form:

$$ {}^{final}_{2,1}\langle\chi|\Psi\rangle^{initial} = \frac{1}{\sqrt{2}}\cos(\varphi_1-\varphi_2) \tag{20'}$$

$$ {}^{final}_{2,1}\langle\chi|\Psi\rangle^{initial} = -\frac{1}{\sqrt{2}}\cos(\varphi_1-\varphi_2) \tag{24'}$$

$$ {}^{final}_{2,1}\langle\chi|\Psi\rangle^{initial} = -\frac{1}{\sqrt{2}}\sin(\varphi_1-\varphi_2) \tag{28'}$$

We introduce explicitly the angle difference $\varphi_1 - \varphi_2$ as the angle between vectors $\mathbf{a}_1, \mathbf{a}_2$. Then the probability of transition, for example, (20') will take the form:

$$\left| {}_{2,1}^{final}\langle\chi|\Psi\rangle_{1,2}^{initial} \right|^2 = \frac{1}{2}\cos^2(\widehat{\mathbf{a}_1, \mathbf{a}_2})$$

Following the Aspect[1], we introduce notation:

$P_{++}(\mathbf{a_1}, \mathbf{a_2})$ –	the probability to detect the polarization of the first photon along the vector $\mathbf{a}_1$, and the second photon along the vector $\mathbf{a}_2$
$P_{--}(\mathbf{a_1}, \mathbf{a_2})$ –	the probability to detect the polarization of the first photon perpendicular to the vector $\mathbf{a}_1$, and the second photon perpendicular to the vector $\mathbf{a}_2$
$P_{+-}(\mathbf{a_1}, \mathbf{a_2})$ –	the probability to detect the polarization of the first photon along the vector $\mathbf{a}_1$, and the second photo on the perpendicular vector $\mathbf{a}_2$
$P_{-+}(\mathbf{a_1}, \mathbf{a_2})$ –	the probability to detect the polarization of the first photon perpendicular to the vector $\mathbf{a}_1$, and the second photon along the vector $\mathbf{a}_2$

From (20'), (24'), (28') should

$$P_{++}(\mathbf{a}_1, \mathbf{a}_2) = \frac{1}{2}\cos^2(\widehat{\mathbf{a}_1, \mathbf{a}_2}) \tag{29}$$

$$P_{--}(\mathbf{a}_1, \mathbf{a}_2) = \frac{1}{2}\cos^2(\widehat{\mathbf{a}_1, \mathbf{a}_2}) \tag{30}$$

$$P_{+-}(\mathbf{a}_1, \mathbf{a}_2) = P_{-+}(\mathbf{a}, \mathbf{b}) = \frac{1}{2}\sin^2(\widehat{\mathbf{a}_1, \mathbf{a}_2}) \tag{31}$$

Links

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V.A.Kasimov. *Some topological paradoxes of relativity (EPR).* (English version)/

Abstract

In the footsteps of the article by A. Aspect "BELL'S THEOREM: the naive view of an experimentalist". As in equation (23) has detected an error (or typo), I took the trouble to verify calculations from 1 to 5 sections of the article. Are some clarifying points that are important for understanding the essence. Given an elementary conclusion of formulas (3), which is omitted in the article.

The Bell's inequality, derived on the basis of the general model for a dichotomous variable, disturbed the quantum mechanical model for a pair of "entangled" photons. In Bell's article it is clearly (though not very detailed) shown.

No " artificial gadgets" is not able to resolve this contradiction. The only thing that causes confusion is the procedure of creating a mixed state of two photons and the essence conceptual view of mathematics experiment. Theoretically, this procedure can be represented as a symmetrization of the wave function of the pair. However, how does the transfer of this idea to the technical essence of the experiment is unclear.

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For communicating:
quadrica-m@mail.ru

Author's seminar:
http://my.mail.ru/community/physiks.princips/?ref=cat

http://quadrica.ucoz.net/
https://independent.academia.edu/KasimovVladimir
https://vk.com/public128913510
https://www.facebook.com/quadrica.m
http://orcid.org/0000-0002-1435-9220

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