Structure formation in the Early Universe as a result of non Linear Electrodynamics influencing scale factor size with attendant changes in gravitational potential and its relationship to the 3 body problem.

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Abstract. We find that having the scale factor close to zero due to a given magnetic field value in, an early universe magnetic field affects how we would interpret Mukhanov's chapter on 'self reproduction of the universe' in in his reference "Physical foundations of cosmology" terms of production of inhomogeneity during inflation and its aftermath. The stronger an early universe magnetic field is, the greater the likelihood of production of about 20 new domains of size 1/ *H*, with *H* early universe Hubble's constant, per Planck time interval in evolution. One final caveat to consider. What may happen is that the Camara (2004) density and Quintessential density (Corda et al.) are both simultaneously satisfied, which would put additional restrictions on the magnetic field which in turn affects structure formation. In time, once Eq.(16) of this paper is refined further, the author hopes that some of the issues raised by Kobayashi and Seto as to allowed inflation models may be addressed, once further refinement of these preliminary results commences . We close as to how fluctuations in the Hubble expansion parameter, H, as given below may affect structure as given in reference [10] below. We close with statements as to the value of α in a gravitational potential proportional to $r^{-\alpha}$ and how this adjustment affects the 3 body problem.

1. Introduction

Part 1 : We first of all will look at how a scale factor is affected by the NLED paradigm which in fact also is linked to the idea of 'self reproduction' as given in [1], which is a different way as to outline how this affects the evolution of density in the early universe.

Part 2 : Next , after having done this, we give a description of an equation for setting the value of α in a gravitational potential proportional to $r^{-\alpha}$. This α has real and complex values, unlike the Newtonian real value

Part 3: We then summarize what this has to do with possible revisions of the three body problem [2] with a particular mention as to how this would affect material as of page 141 and beyond in [2] with implications as to astrophysics.

2. Part 1, We first of all will look at how a scale factor is affected by the NLED paradigm which in fact also is linked to the idea of 'self reproduction'

This part of the paper has several routes as to identifying NLED phenomenon pertinent to cosmology structure formation. First we look at what Mukhanov [1] writes as far as structure formation. Mainly that there is a formulation of what is called self reproduction of inhomogeneity in terms of early universe condition [1]. In this, the starting point is if one used the meme of chaotic inflation, i.e. inflation generated by a potential of the form as given by Guth [3] as well as Mukhanov [1]

$$V\left(potential\right) \sim \phi^2 \quad (1)$$

In this, Mukhanov [1] write that one can look at a scalar field at the end of (chaotic) inflation, with an amplitude given by, with ϕ_i for the initial value of the inflaton such that (where m will be determined by NLED inputs to be brought up later.)

$$\delta_{\phi}^{Max} \sim m \cdot \phi_i^2 \quad (2)$$

In terms of the initial inflaton, inhomogenities do not form if the initial inflaton is bounded [1] as given by

$$m^{-1} > \phi_i > m^{-1/2} \quad (3)$$

This leads to (low?) inhomogeneity in the space-time generated by inflation. Inflation is eternal [1,3] if. there is only the inequality

$$\phi_i > m^{-1/2} \quad (4)$$

2a. NLED applied to Eq. (4) plus details of structure formation added

What we will do is to look at the following treatment of mass, and this will be our starting point. i.e. we will be looking at , if l_P is Planck length, and α >0, then

$$m \sim 10^{\alpha} \cdot l_p^3 \cdot \rho\left(density\right) \quad (5)$$

Then we can consider the following formulation of density given below.

If we do not wish to consider a rotating universe, then Camara et al, [4] has an expression as to density, with a B field contribution to density, and we also can used the Weinberg result [5]of scaling density with one over the fourth power of a scale factor, which we will remark upon in the general section, as well the Corda and Questa result of [6] for density of (note reference [6] is for a star, whereas [4] is for a universe)

In addition, Corda, and others in [6] use quintessential density to falsify the null energy condition of a Penrose theorem cited in [7], Further details of what Penrose was trying to do as to this issue of GR, can be seen in [8], and to answer how to violate the null energy condition, one should go to [6] for quintessential density defined, with the constant in Eq.(4) greater than zero. Then in both the massive star and the early universe, the density result below is applicable.

$$\rho_{\gamma} = \frac{16}{3} \cdot c_1 \cdot B^4 \quad (6)$$

Keeping in mind what was said as to choices of what to do about density, and its relationship to Eq.(5) above, we then can reference what Mukhanov[1] says about structure formation as follows, namely look at how a Hubble parameter changes with respect to cosmic evolution. It changes with respect to H_{today} being the Hubble parameter in the recent era, and the scale factor a , with this scale factor being directly responsive to changes in density according to [5], i.e.

$$\rho \sim a^{-4} \tag{7}$$

In the next section, we will examine how [4] suggests how to vary the scale factor cited in Eq. (7), and we will in this section take note of what the scale factor cited in [4] does to the Hubble parameter given in Eq.(8) below, and then in the section afterwards review a possible reconciliation of what Eq.(6) and Eq.(7) say about defining early universe parameters. But to know why we are doing it, we should take into consideration what happens to the Hubble parameter, as given below

$$H \sim H_{today} / a^{3/2} \tag{8}$$

According to [1], if Eq.(4) holds, then inhomogeneous patches of space time appear in a causal region of space time for which

$$Causal-domain \sim H^{-1} \sim 1/\left(H_{today} / a^{3/2}\right) \tag{9}$$

Furthermore, [1] states that about 20 such domains are created in a Hubble time interval $\Delta t_H \propto H^{-1}$ i.e. As a function of say 10^{α} times Planck time, for a domain size given by Eq.(9) above and that this requires then a clear statement as to how the scale factor changes, due to considerations given by [4] and reconciling the density expression given in Eq.(6) and Eq.(7) above**.**

2b. Showing a non zero initial radius of the universe due to non linear space-time E&M

What we are asserting is. in [1] there exists a scaled parameter λ, and a parameter a_0 which is paired with α_0. For the sake of argument, we will set the $a_0 \propto \sqrt{t_{Planck}}$, with t_{Planck} ~ 10^ - 44 seconds. Also, Λ is a cosmological 'constant' parameter which is described later, as in quintessence , via reference [9], and is in [4] via:

$$\alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}} B_0 \tag{10}$$

$$\lambda = \Lambda c^2 / 3 \tag{11}$$

Then if , initially, Eq. (11) is large, due to a very large Λ the time, given in Eq.(53) of [4] is such that we can write , most likely, that even though there is an expanding and contracting universe, that the key time parameter may be set , due to very large Λ as

$$t_{\min} \approx t_0 \equiv t_{Planck} \sim 10^{-44} s \tag{12}$$

Whenever one sees the coefficient like the magnetic field, with the small 0 coefficient, for large values of Λ, this should be the initial coefficient at the beginning of space-time which helps us make sense of the non zero but tiny minimum scale factor[1]

$$a_{\min} = a_0 \cdot \left[\frac{\alpha_0}{2\lambda} \left(\sqrt{\alpha_0^2 + 32\lambda\mu_0\omega B_0^2} - \alpha_0 \right) \right]^{1/4} \tag{13}$$

The minimum time, as referenced in Eq.(12) most likely means, due to large Λ that Eq. (13) is of the order of about 10^{-55}, i.e. 33 orders of magnitude smaller than the square root of Planck time, in magnitude. We next will be justifying the relative size of the Λ

2c. Showing How to obtain a varying Λ with a large initial value and its relationship to obtaining a scale factor value for the early universe via NLED methods

Non withstanding the temperature variation in reference [9] for the cosmological Hubble parameter, we also can reference what is done in reference [4] namely

$$\Lambda(t) \sim (H_{\text{inflation}})^2 \tag{14}$$

1. In short, what we obtain, via looking at due to [9], that Eq.(14) is also equivalent to

$$\Lambda_{Max} \sim c_2 \cdot T_{temperature}{}^{\tilde{\beta}} \tag{15}$$

Comparing Eq.(6) and Eq.(7) above, leads to the following constraints, i.e.

$$\left(\rho \sim a^{-4}\right)^{-1} \sim a^4 \sim \frac{16}{3} \cdot c_1^{-1} \cdot B^{-4} \sim a_0^{-4} \Big/ \left[\frac{\alpha_0}{2\lambda} \left(\sqrt{\alpha_0^2 + 32\lambda\mu_0\omega B_0^2} - \alpha_0 \right) \right] \tag{16}$$

The above relationship will argue in favor of a large value for Eq.(15) and Eq.(16) B field and also the cosmological 'constant' parameterized in Eq. (14) and Eq.(15), i.e. once fully worked out, the allowed values of B, for initial conditions will be large but tightly constrained, and this in turn will allow for Eq. (9) having initially extremely small inhomogeneity behavior, in line with being proportional to the inverse of an allowed Hubble parameter based upon Eq. (8). Note that from [11] we have

$$\frac{\Delta H}{H} \sim \Omega_m^2 h^2 \Delta_{\Re} \sim 10^{-5} \tag{17}$$

Here, we have that if there is a flat universe, that according to Guth [12] and taking note of

$$H^2 = \frac{8\pi}{3} \cdot \rho \tag{18}$$

Roughly put, what we are predicting is, that if we use what Lloyd wrote, namely [13] as well as use the magnetic field relations to density brought up in reference [6], then

$$\begin{aligned}
&\#operations \sim \rho_{crit} \times t^4 \sim (t/t_p)^2 \\
&\Leftrightarrow \rho_{crit} \sim 1/t^2 \propto \rho_\gamma = \frac{16}{3} \cdot c_1 \cdot B^4 \\
&\Leftrightarrow \#operations \sim (t/t_p)^2 / B^8 \\
&\sim 1/(t_p B^4) \sim 1/B^4
\end{aligned} \tag{19}$$

If we have such a treatment of information as given by Lloyd [13], plus the above, we can estimate that there is a fluctuation due to early universe cosmology along the lines of, if we have a base line number for initial (expansion) value of the Hubble parameter, we call $H_{base-line}$ .as a starting point for an expanding universe, and with $\#operations$, as given by Lloyd [13] as a function of entropy, initially. So then, in terms of what may be generated and show up in the CMBR we may see

$$\Delta H(thermal) \sim H_{base-line} \cdot (\#operations)^{1/4} \cdot 10^{-5} \cdot \sqrt{t / t_{Planck}} \tag{20}$$

Eq. (16) to Eq.(18) , if we write a change in H, as given by Eq.(17) and that along the lines of figure 2 of reference [11], we have , perhaps, the beginning of how NLED may impact fluctuations in H, which in turn may lead to the issue we started our discussion over. Eq (20) may give some insight as to the fluctuations which show up in figure 2, of [11]

2d. Conclusion for Part 1 . Tightly constrained but very large magnetic fields allow for inhomogeneous patches due to NLED showing up in CMBR: Relevance to Bicep 2 dispute?

Note that Eq. (11) to Eq.(13) are arguing in favor of a very small scale factor, implying a large initial density while Eq. (16) appears to give credence to a large Hubble parameter. Further work will come up with a set of constraints as to admissible early universe quintessence, ie. Answering the question if the cosmological constant is significantly larger, and if it plays a role in structure formation is important, especially in lieu of the Bicep 2 results which purport to favor large field inflation [10]. While not wishing to get immersed in the details of the data controversy surrounding Bicep 2 at this immediate time , further refinements of NLED , and the relationship in Eq.(16) , as to structure formation may give credence, or help falsify the conclusions of reference [10], with great refinements and equalities needed in defining more precisely the suggested relationships implied in Eq.(16)

Eq. 16 to Eq. (18) will in tandem lead to some of the variation of structure given in Figure 2 of [10} which we argue should be seen as a compliment to the work given by [11] . In addition, the interplay between Eq. (20) and Eq. (9) may be ripe for computer simulation work.

We shall next investigate how part 1 and its results affect gravitational potential behaviour, in the Pre Planckian to Planckian universe evolution, next.

3. Part 2, the problem of the α in a gravitational potential proportional to $r^{-\alpha}$

In order to review this, we need to look at [14] where we can use the following treatment of the Klein Gordon equation which we write as

$$\begin{aligned}
&\ddot{\varphi}_k + 3H\dot{\varphi}_k + \frac{k^2}{a^2}\varphi_k = 0 \\
&\& \\
&\varphi_k \approx \frac{H\tau}{\sqrt{2k}} \cdot \left(1 + \left(ik\tau\right)^{-1}\right) \cdot \exp\left[-ik\tau\right] \\
&\& \\
&\tau = -H^{-1} \cdot \exp\left[-H \cdot t\right]
\end{aligned} \tag{21}$$

Here, k is the value of wave number, and H is assumed, in the early universe to be a constant. The net result is that $k = 2\pi / \lambda$, with λ proportional to the 'width' of a would be pre universe 'bubble' as seen in [15] place of a singularity, and also that one would have, for a constant H, during this time as seen by [16, 17]

$$
\begin{aligned}
&H = \sqrt{\frac{8\pi G}{3} \cdot \rho - \frac{\kappa}{a^2}} \\
&\rho = '\mathit{energy} - \mathit{density}' \\
&\kappa = '\mathit{curvature}'
\end{aligned}
\tag{22}
$$

Further use of [17] will lead to the situation that

$$
\begin{aligned}
H &= \sqrt{\frac{8\pi G}{3} \cdot \rho - \frac{\kappa}{a^2}} \\
&= \sqrt{\frac{8\pi G}{3}} \cdot \sqrt{\rho - \frac{3}{8\pi G} \cdot \frac{\kappa}{a^2}} \\
&\approx \sqrt{\frac{8\pi G}{3}} \cdot \sqrt{V(\varphi) - \frac{\dot{\varphi}^2}{2}} \\
&\Leftrightarrow \frac{\dot{\varphi}^2}{2} = \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + V(\varphi) \right) - \rho
\end{aligned}
\tag{23}
$$

Chaotic inflation would be using the approximation that

$$
V(\varphi) \approx \frac{k^2}{a^2} \cdot \varphi^2 \tag{24}
$$

Use the approximation that the time derivative is $d / d\tau$, and $\varphi \equiv \varphi_k$, and if so, then

$$
\begin{aligned}
&\frac{\dot{\varphi}_k^{\ 2}}{2} = \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^{\ 2} - \rho \right) \\
&\& \\
&\frac{\dot{\varphi}_k^{\ 2}}{2} = \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^{\ 2} - \frac{16}{3} \cdot c_1 \cdot B^4 \right)
\end{aligned}
\tag{25}
$$

The last line of Eq. (25) states that , if we apply it to the Pre Planckian to Planckian regime, that there will be a change in the energy, which we will call

$$
\Delta E \approx \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^{\ 2} - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \tag{26}
$$

We then will call this shift in energy, as equivalent to a change in KINETIC energy, and then reference the Virial theorem which in a general form, will be interpreted as

$$\langle \psi \mid Kinetic - Energy \mid \psi \rangle = \langle \psi \mid r \cdot \nabla V \left(Potential - energy \right) \mid \psi \rangle \quad (27)$$

Leading to

$$\left\langle \psi \middle| \left[Kinetic - Energy \approx \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k{}^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \right] \middle| \psi \right\rangle \approx \left\langle \psi \middle| \left(r \cdot \nabla \left[V \left(Potential - energy \right) \approx c_2 / r^{\alpha} \right] \right) \middle| \psi \right\rangle \quad (28)$$

In the Pre Planckian to Planckian space time, we will approximate, in the instant before time is initialized, formally, the mean value theorem as to the computed values of both the Left and right hand sides of Eq. (27) and Eq. (28) with the results that we obtain

$$\begin{gathered} \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k{}^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \approx -\alpha / r^{\alpha} \equiv -\alpha / (Planck - length)^{\alpha} \\ \Leftrightarrow \left[\alpha / (Planck - length)^{\alpha} \right] \approx \frac{16}{3} \cdot c_1 \cdot B^4 - \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k{}^2 \right) \end{gathered} \quad (29)$$

Here, the magnetic field would be determined in part by the value of B, as given in [19], and the scale factor a, is given by Eq. (13), and φ_k is given by Eq. (21).

This shows in part that α is no longer strictly real valued but is strongly influence by the input from φ_k, i.e. which has real and imaginary components

We will next, in Part 3 of this document conclude with a specific statement as to how the 3 body problem, and in fact other experimental science could be impacted by Eq. (29) and a careful reading of [2], page 134, has a very carefully done section on the so called Lagrangian equilateral triangle and possible orbits 3 body problem, and [20] has its section on the KAM theory, page 15, which are subsequently modified, and which may yield more rigorous simulations, in the computer numerical sense which in turn could give us more useful input into experiments.

4. How to reconcile our developments with ascertaining limitations and also improvements on the 3 body Problem of Classical physics (plus its quantum analogues)

What we are doing is to consider both [2] and [20] in terms of KAM theory , [20], as of page 15, has in its classical mode a highly restricted set of equations, as given by having the following quote

From [20],

Quote

Now, the KAM theorem tells us that when the system is slightly perturbed most of the invariant tori are not destroyed but only slightly shifted in the phase space. This has important implications on stability of orbits in the general and restricted three body problem. The proof of the KAM theorem by Moser [1962] and Arnold [1963] also demonstrated that convergent power series solutions exist for the three-body (as well as for the n-body) problem. The KAM theorem seems to be very useful for *studying the global stability in the three-body problem [Robutel, 1993a, Montgomery, 2001, Sim´o, 2002]; however, some of its applications are limited only to small masses of the third body*

The limitation as stated as of KAM theory is that MOST of the time, we have that the KAM results require a third body to be low mass.

This is a classical dynamical systems result. What we should endeavour through judicious application of Eq. (29) is to remove dependence upon the smallness of the third mass, and to examine if this can still , with a non trivial third mass recover still much of the stability analysis

Later, at an appropriate time this question in terms of a serious application of the value of Eq. (29) will be pursued

Secondly, as of [2] , the section give on page 154, entitled "6.4 Orbital changes in encounters with planets", which is a restricted 3 body problem , frequently is used as to the interaction of say comets (comparatively small mass) with a planet, circulating the Sun, where we have 2 'massive' masses, and the third body , in this case a comet, which gives usually parameters of how a hyperbolic orbit for a comet, i.e. one which enters in a planet- Sun system are impacted via how the parameters of say a hyperbolic trajectory of a minor mass object (comet) are impacted in a simple solar system model.

We hope that by judicious investigation of the arguments given as of to Eq. (29) that the restrictions as to the smallness of the 'third' mass may be partly ameliorated.

If this is done, and it will require through investigations, then a template as to how to reliably simulate N bodies interacting, may be a doable problem, but first of all, to do this, it should be seen if the classical KAM problem may be generalized beyond its present strictures

And, now for an overall conclusion

We conclude that a worthy application of our techniques is in investigating if restrictions as to the KAM results, and restricted small mass of a third body in terms of interaction with two other bodies is doable and made more likely via Eq. (29)

We argue that Eq. (29) in its limiting characterization may allow certain classical 3 body simulations more flexibility than what exist presently.

We also leave the door open as to other applications of Eq.(29), especially in early universe conditions.

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