Revisiting the Derivation of Heisenberg's Uncertainty Principle: The Collapse of Uncertainty at the Planck Scale

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Abstract

In this paper, we will revisit the derivation of Heisenberg's uncertainty principle. We will see how the Heisenberg principle collapses at the Planck scale by introducing a minor modification. The beauty of our suggested modification is that it does not change the main equations in quantum mechanics; it only gives them a Planck scale limit where uncertainty collapses. We suspect that Einstein could have been right after all, when he stated, "God does not throw dice." His now-famous saying was an expression of his skepticism towards the concept that quantum randomness could be the ruling force, even at the deepest levels of reality. Here we will explore the quantum realm with a fresh perspective, by re-deriving the Heisenberg principle in relation to the Planck scale.

Our modified theory indicates that renormalization is no longer needed. Further, Bell's Inequality no longer holds, as the breakdown of Heisenberg's uncertainty principle at the Planck scale opens up the possibility for hidden variable theories. The theory also suggests that the superposition principle collapses at the Planck scale. Further, we show how this idea leads to an upper boundary on uncertainty, in addition to the lower boundary. These upper and lower boundaries are identical for the Planck mass particle; in fact, they are zero, and this highlights the truly unique nature of the Planck mass particle.

Key words: Heisenberg's uncertainty principle, certainty, wave function, Planck scale, Planck mass, Planck particle, Bell's inequality, superposition, entropy.

1 Introduction to the Momentum and Energy Operators

A commonly used wave function[1] in quantum mechanics is

$$\Psi(x,t) = e^{i(kx-\omega t)} \tag{1}$$

where $\omega = \frac{E}{\hbar}$, and

$$k = \frac{2\pi}{\lambda} \tag{2}$$

From the de Broglie matter wave, we know that

$$\lambda = \frac{h}{p} \tag{3}$$

This means we have

$$k = \frac{p}{\hbar} \tag{4}$$

and this means we can write the wave equation also as (well known)

$$\Psi = e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \tag{5}$$

Next we take the partial derivative with respect to x and get

$$\frac{\partial \Psi}{\partial x} = \frac{ip}{\hbar}\Psi \tag{6}$$

Multiplying each side by $\frac{\hbar}{i}$ we get

[1] The plane wave solution to the Klein–Gordon equation and the Schröedinger equation.

$$\begin{aligned} \frac{\hbar}{i}\frac{\partial \Psi}{\partial x} &= p\Psi \\ -i\hbar\frac{\partial \Psi}{\partial x} &= p\Psi \end{aligned} \tag{7}$$

and from this we have the well-known momentum operator

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \tag{8}$$

Further, the partial derivative of the wave equation with respect to time is

$$\begin{aligned} \frac{\hbar}{i}\frac{\partial \Psi}{\partial t} &= E\Psi \\ -i\hbar\frac{\partial \Psi}{\partial t} &= E\Psi \end{aligned} \tag{9}$$

which means the energy operator must be

$$\hat{E} = -i\hbar\frac{\partial}{\partial t} \tag{10}$$

Next, we will use this information to derive Heisenberg's uncertainty principle.

2 Introduction to Commutators, Operators, and Heisenberg's Uncertainty Principle

A standard commutator is given by

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \tag{11}$$

If $[\hat{A}, \hat{B}] \neq 0$, then $\hat{A}$ and $\hat{B}$ do not commute. If $[\hat{A}, \hat{B}] = 0$, then $\hat{A}$ and $\hat{B}$ do commute. Based on this, we have the following uncertainty

$$\sigma_A\sigma_B = \frac{1}{2}|\langle\hat{A}, \hat{B}\rangle| = \frac{1}{2}|\int \Psi^*[\hat{A}, \hat{B}]\Psi dt| \tag{12}$$

where Ψ^* is the complex conjugate of Ψ, and we see from the expression above that if $\hat{A}$ and $\hat{B}$ commute there is no uncertainty. The Heisenberg uncertainty principle [1, 2] can be derived from the following commutator

$$[\hat{p}, \hat{x}] = \hat{p}\hat{x} - \hat{x}\hat{p} \tag{13}$$

where the $\hat{p}$ is the momentum operator and $\hat{x}$ is the position operator. Again, the momentum operator is given by

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \tag{14}$$

and the position operator is given by

$$\hat{x} = x \tag{15}$$

From this we have

$$\begin{aligned} [\hat{p}, \hat{x}]\Psi &= [\hat{p}\hat{x} - \hat{x}\hat{p}]\Psi \\ &= \left(-i\hbar\frac{\partial}{\partial x}\right)(x)\Psi - (x)\left(-i\hbar\frac{\partial}{\partial x}\right)\Psi \\ &= -i\hbar\left(\Psi + x\frac{\partial \Psi}{\partial (x)}\right) + i\hbar x\frac{\partial \Psi}{\partial (x)} \\ &= -i\hbar\left(\Psi + x\frac{\partial \Psi}{\partial (x)} - \frac{\partial \Psi}{\partial (x)}\right) \\ &= -i\hbar\Psi \end{aligned} \tag{16}$$

And we have the following uncertainty

$$\begin{aligned}
\sigma_p\sigma_x &\geq \frac{1}{2}|\int \Psi^*[\hat{p},\hat{x}]\Psi dt| \\
&\geq \frac{1}{2}|\int \Psi^*(-i\hbar)\Psi dt| \\
&\geq \frac{1}{2}|-i\hbar\int \Psi^*\Psi dt|
\end{aligned} \tag{17}$$

and since $\int \Psi^*\Psi dt$ must sum to 1 (there must be 100% probability for the particle to be somewhere), we are left with

$$\begin{aligned}
\sigma_p\sigma_x &\geq \frac{1}{2}|-i\hbar| \\
\sigma_p\sigma_x &\geq \frac{\hbar}{2}
\end{aligned} \tag{18}$$

that is as expected, we arrive at the Kennard version of Heisenberg's uncertainty principle. The Heisenberg uncertainty principle is the foundation of many of the results and interpretations of quantum mechanics. If we derive the uncertainty principle from the energy and time operators instead, we get

$$\sigma_E\sigma_t \geq \frac{\hbar}{2} \tag{19}$$

This is shown in detail in the Appendix.

3 The Planck Scale and Haug's Maximum Velocity for Matter

In 1899, Max Planck [3, 4] introduced what he called the 'natural units': the Planck mass, the Planck length, the Planck time, and the Planck energy. He derived these units using dimensional analysis, assuming that the Newton gravitational constant, the Planck constant and the speed of light were the most important universal constants. Lloyd Motz, while working at the Rutherford Laboratory in 1962, [5, 6, 7] suggested that there was probably a very fundamental particle with a mass equal to the Planck mass that he called the "Uniton." Motz acknowledged that his Unitons (Planck mass particles) had far too much mass compared to known subatomic masses. He tried to address this issue by claiming that the Unitons had radiated most of their energy away:

> *According to this point of view, electrons and nucleons are the lowest bound states of two or more Unitons that have collapsed down to the appropriate dimensions gravitationally and radiated away most of their energy in the process.* – Lloyd Motz

Others have suggested that there were plenty of Planck mass particles around just after the Big Bang; see [8], but that most of the mass of these super-heavy particles has radiated away. Modern physics has also explored the concept of a hypothetical Planck particle that has $\sqrt{\pi}$ more mass than the Uniton originally suggested by Motz. Some physicists, including Motz and Hawking, have suggested such particles could be micro-black holes [9, 10, 11]. Planck mass particles have even been proposed as candidates for cosmological dark matter, [12, 13]. We will suggest that the Planck mass particle only lasts for one Planck second and that its mass should be seen as approximately 1.17×10^{-51} kg compared to other particles. The Planck mass particle is, in our view, the mass-gap. It is a observational time-window dependent mass. We suspect that all other masses are time-dependent as well, but this will first be noticeable when one is trying to measure their mass in a observational time window below their reduced Compton time, something we are not capable of doing at the moment. The electron's mass can experimentally be found from the electron's reduced Compton length, [14]. To measure the reduced Compton wavelength of the electron we would, at a minimum, need an observational time interval of $\frac{\bar{\lambda}_e}{c}$. This is due to the maximum velocity of a signal being the speed of light. This indicates that our idea that the elementary particle mass could be dependent on the observed time interval, if that interval is below the reduced Compton time, $t_c = \frac{\bar{\lambda}}{c}$, of the particle in question

In a series of recent publications, Haug [15, 16, 17, 18] has suggested that there is a maximum velocity for anything with rest-mass given by

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{20}$$

where l_p is the Planck length. Further, $\bar{\lambda}$ is the reduced Compton wavelength of the elementary particle in question. For any observed particle, the maximum velocity will be very close to that of the speed of light, but considerably above the speed achieved in the Large Hadron Collider. An electron has a reduced Compton wavelength of $\bar{\lambda}_e \approx 3.86159 \times 10^{-13}$ m and here we suggest that it can never be accelerated to a velocity faster than

$$v = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}_e^2}} = c \times 0.99999999999999999999999999999999999999999999912416 \tag{21}$$

In the above calculation, we have assumed a Planck length of 1.616199×10^{-35}. As there is considerable uncertainty about the exact value for the Planck length, there is also some uncertainty about the theoretical value for the maximum speed limit of the electron. In our framework, the Planck length and the Planck mass can be measured independent of any prior knowledge of Newtonian gravity or the gravitational constant, as recently shown by [18, 19]. For additional context, Haug's maximum velocity was first derived in [15], and has later been derived from Heisenberg's uncertainty principle when assuming the minimum uncertainty in position is $\sigma_x \geq l_p$, see [20]. Here we will show that the Heisenberg uncertainty principle breaks down at the Planck scale if the maximum velocity for matter follows this expression, which can be derived from special relativity equations by assuming that the reduced Compton wavelength can never undergo more length contraction than the Planck length. Alternatively, we can assume that the maximum frequency is the Planck frequency, or that the maximum relativistic mass of an elementary particle is the Planck mass.

This also means there is a maximum limit on the relativistic momentum of

$$P_{max} = \frac{mv_{max}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} = \frac{mc\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\sqrt{1 - \frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} = m_p c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{22}$$

From the formula we see that the maximum momentum for most particles is very close to the Planck mass momentum $m_p c$. However the Planck mass particle has zero momentum as $\bar{\lambda} = l_p$. We think just as there is rest-mass energy, there is also what we can call "potential momentum," which is mc. The Planck mass particle is always at rest, so it has potential momentum of $m_p c$, but zero momentum in its current state. No particle with rest-mass can move at the speed of light, and that the Planck mass momentum is assumed to be $m_p c$ indicates it is not a normal momentum, but rather a potential momentum.

However, in the wave equation it is momentum and kinetic energy that are relevant. Further, the maximum kinetic energy is given by

$$\begin{aligned} E_k &= \frac{mc^2}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} - mc^2 \\ &= \frac{mc^2}{\sqrt{1 - \frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} - mc^2 \\ &= \frac{mc^2}{\sqrt{1 - 1 + \frac{l_p^2}{\bar{\lambda}^2}}} - mc^2 \\ &= m_p c^2 - mc^2 \\ &= \frac{\hbar}{l_p}\frac{1}{c}c^2 - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 \\ &= \hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right) \end{aligned} \tag{23}$$

This means the wave function at the suggested maximum velocity for the anything with rest-mass is given by

$$\begin{aligned} \Psi &= e^{i\left(\frac{p_{max}}{\hbar}x - \frac{E_{max}}{\hbar}t\right)} \\ &= e^{i\left(\frac{m_p c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{\hbar}x - \frac{\hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right)}{\hbar}t\right)} \end{aligned} \tag{24}$$

from this we have

$$\begin{aligned}
\frac{\partial \Psi}{\partial x} &= \frac{i m_p c \sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{\hbar}\Psi \\
\frac{\hbar}{i}\frac{\partial \Psi}{\partial x} &= m_p c \sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\Psi \\
-i\hbar\frac{\partial \Psi}{\partial x} &= p\Psi
\end{aligned} \tag{25}$$

so the momentum operator is

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \tag{26}$$

and the energy operator must be

$$\hat{E} = -i\hbar\frac{\partial}{\partial t} \tag{27}$$

That is to say, the same momentum and energy operators are just as before, so this will not change Heisenberg's uncertainty principle. However, there is one exception to the rule, namely for a Planck mass particle where the reduced Compton wavelength is $\bar{\lambda} = l_p$. Inserted into the wave equation, we get

$$\begin{aligned}
\Psi &= e^{i\left(\frac{m_p c\sqrt{1-\frac{l_p^2}{l_p^2}}}{\hbar}x - \frac{\hbar c\left(\frac{1}{l_p}-\frac{1}{l_p}\right)}{\hbar}t\right)} \\
&= e^{i\left(\frac{m_p c\sqrt{1-1}}{\hbar}x - \frac{\hbar c(1-1)}{\hbar}t\right)} \\
&= e^{i\left(\frac{m_p c\times 0}{\hbar}x - \frac{\hbar c\times 0}{\hbar}t\right)} = 1
\end{aligned} \tag{28}$$

This means we have

$$\frac{\partial \Psi}{\partial x} = 0 \tag{29}$$

and

$$\frac{\partial \Psi}{\partial t} = 0 \tag{30}$$

Thus, the momentum operator and the energy operator must be zero for the Planck mass particle. This means we must have

$$\begin{aligned}
[\hat{p}, \hat{x}]\Psi &= [\hat{p}\hat{x} - \hat{x}\hat{p}]\Psi \\
&= \left(-0\times\frac{\partial}{\partial x}\right)(x)\Psi - (x)\left(-0\times\frac{\partial}{\partial x}\right)\Psi \\
&= 0
\end{aligned} \tag{31}$$

That is, $\hat{p}$ and $\hat{x}$ commute for the Planck particle, but do not commute for any other particle. For formality's sake, the uncertainty in the special case of the Planck particle must be

$$\begin{aligned}
\sigma_p\sigma_x &\geq \frac{1}{2}|\int \Psi^*[\hat{p}, \hat{x}]\Psi dt| \\
&\geq \frac{1}{2}|\int \Psi^*(0)\Psi dt| \\
&\geq \frac{1}{2}| -0\times\int \Psi^*\Psi dt| = 0
\end{aligned} \tag{32}$$

and also

$$\begin{aligned} \sigma_E\sigma_t &\geq \frac{1}{2}|\int \Psi^*[\hat{E},\hat{t}]\Psi dt| \\ &\geq \frac{1}{2}|\int \Psi^*(0)\Psi dt| \\ &\geq \frac{1}{2}|-0\times\int \Psi^*\Psi dt| = 0 \end{aligned} \tag{33}$$

In the special case of the Planck mass particle, the uncertainty principle collapses to zero. In more technical terms this implies that the quantum state of a Planck mass particle can simultaneously be a position and a momentum eigenstate. That is, for the special case of the Planck mass particle we have certainty. In addition, the probability amplitude of the Planck mass particle will always be one $\Psi_p = e^0 = 1$, which likely should be interpreted to mean that we can know the momentum and position of the Planck mass particle at the same time; this is unlike any other particles that follow unmodified quantum mechanics.

We will claim the Planck mass particle always has potential momentum equal to m_pc, and if we have detected a Planck mass particle, we know its position and its momentum. However, within one Planck second the Planck mass particle dissolves into energy. We also get a hint about the lifetime of a Planck particle from the Planck acceleration, $a_p = \frac{c^2}{l_p} \approx 5.56092\times10^{51}\ m/s^2$. The Planck acceleration is assumed to be the maximum possible acceleration by several physicists; see [21, 22], for example. The velocity of a particle that undergoes Planck acceleration will actually reach the speed of light within one Planck second: $a_pt_p = \frac{c^2}{l_p}\frac{l_p}{c} = c$. However, we know that nothing with rest-mass can travel at the speed of light, so no "normal" particle can undergo Planck acceleration if the shortest possible acceleration time interval is the Planck second. The solution is simple. The Planck acceleration is an internal acceleration inside the Planck particle that within one Planck second turns the Planck mass particle into pure energy. This also explains why the Planck momentum is so special, namely always m_pc, unlike for any other particles, which can take a wide range of velocities and therefore a wide range of momentums.

4 The Wave Function for the Planck Mass Particle

The wave function

$$\Psi(x,t) = e^{i(kx-\omega t)} \tag{34}$$

can be written as

$$\Psi(x,t) = \cos(kx-\omega t) + i\sin(kx-\omega t) \tag{35}$$

where $\cos(kx-\omega t)$ is the real part of the wave function and $i\sin(kx-\omega t)$ is the imaginary part. Complex numbers in the wave function and thereby the imaginary part are, to our knowledge, necessary precisely because we cannot know the momentum and the position of the particle at the same time. However, we have shown that this is not the case for the Planck mass particle. We should, therefore, also be able to write the wave function for the Planck mass particle simply, without the imaginary part, as

$$\Psi_P = \cos(kx-\omega t) = \cos\left(\frac{P}{\hbar}x - \frac{E}{\hbar}t\right) \tag{36}$$

As the momentum and kinetic energy of the Planck mass particle are zero, the wave function will just be one for the Planck mass particle. Squaring this and we get one. That means that the probability related to the Planck mass particle is always one. For other particles, we are back to the standard wave function.

We can now also re-derive the uncertainty principle from our new wave function in the Planck mass particle. If we derive the Planck mass particle wave function with respect to the position, we get

$$\frac{\partial\Psi_P}{\partial x} = 0 \tag{37}$$

Again, this means the momentum operator must be zero in the special case of the Planck mass particle. We will have momentum and position operators that commute, as well as energy and time operators that commute. In other words, this is fully consistent with our analysis above: in the special case for the Planck mass particle, we can know its position and momentum at the same time.

5 Maximum Uncertainty in Addition to Minimum Uncertainty

Next let us look at the maximum kinetic energy multiplied by the relativistic reduced Compton time of the particle in question

$$\begin{aligned}
E_k t &= \left(\frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - mc^2\right)\frac{\bar{\lambda}}{c}\sqrt{1-\frac{v^2}{c^2}} \\
&= \left(\frac{mc^2}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} - mc^2\right)\frac{\bar{\lambda}}{c}\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}} \\
&= \left(\frac{mc^2}{\sqrt{1-1+\frac{l_p^2}{\bar{\lambda}^2}}} - mc^2\right)\frac{\bar{\lambda}}{c}\sqrt{1-1+\frac{l_p^2}{\bar{\lambda}^2}} \\
&= (m_p c^2 - mc^2)\frac{l_p}{c} \\
&= \left(\frac{\hbar}{l_p}\frac{1}{c}c^2 - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2\right)\frac{l_p}{c} \\
&= \hbar - \hbar\frac{l_p}{\bar{\lambda}} \\
&= \hbar\left(1-\frac{l_p}{\bar{\lambda}}\right)
\end{aligned} \tag{38}$$

we will suggest that this is the maximum uncertainty for an elementary particle, so we must have

$$\frac{\hbar}{2} \le \sigma_E \sigma_t \le \hbar\left(1-\frac{l_p}{\bar{\lambda}}\right) \tag{39}$$

This means we have an extended uncertainty principle with lower boundary, similar to that of Heisenberg, as well as an additional new upper boundary. However, in the special case of a Planck mass particle the lower and upper boundaries on uncertainty are zero. The correct interpretation here is that for the Planck mass particle we have a certainty principle. The energy times time for a Planck mass particle is always

$$E_p t_p = m_p c^2 \frac{l_p}{c} = \hbar \tag{40}$$

Basically, this means if we detect a Planck mass particle we know it is at rest and it has a reduced Compton wavelength of l_p that cannot undergo any length contraction, which is why it is at rest. The Planck mass particle can only have a rest-mass of $m_p c^2$ and a rest-mass momentum of $m_p c$, and must have zero momentum and zero kinetic energy. It stands absolutely still, but only for one Planck second before it dissolves into pure energy. This is also why its relativistic reduced Compton wavelength is certain, because its velocity is always zero (when it exists, but it only exists for one Planck second). Other particles have a velocity that can vary from zero to almost c; this means great uncertainty in their position, their relativistic reduced Compton wavelength, their relativistic mass, their relativistic momentum, and their relativistic kinetic energy. This interpretation is not the standard one, but we find it to be more logical.

The Planck mass particle, in our view, is also linked to photon-photon collisions. The velocity of a light particle at the precise moment when it collides with another light particle is the "meeting point" of light and matter; see also [23].

Our analysis is fully consistent with our maximum velocity and the relativistic energy momentum relation

$$
\begin{aligned}
E &= \sqrt{p^2c^2 + (mc^2)^2} \\
E &= \sqrt{\left(\frac{mv_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}}\right)^2 c^2 + (mc^2)^2} \\
E &= \sqrt{\frac{m^2v_{max}^2c^2}{1-\frac{v_{max}^2}{c^2}} + m^2c^4} \\
E &= \sqrt{\frac{m^2\frac{v_{max}^2}{c^2}c^4}{1-\frac{v_{max}^2}{c^2}} + m^2c^4} \\
E &= \sqrt{\frac{m^2c^4\left(\frac{v_{max}^2}{c^2}-1\right)}{1-\frac{v_{max}^2}{c^2}} + \frac{m^2c^4}{1-\frac{v_{max}^2}{c^2}} + m^2c^4} \\
E &= \sqrt{-m^2c^4 + \frac{m^2c^4}{1-\frac{v_{max}^2}{c^2}} + m^2c^4} \\
E &= \sqrt{\frac{m^2c^4}{1-\frac{v_{max}^2}{c^2}}} \\
E &= \frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}}
\end{aligned}
\tag{41}
$$

That in the special case for a Planck mass particle is

$$
\begin{aligned}
E &= \sqrt{p^2c^2 + (m_pc^2)^2} \\
E &= \sqrt{\left(\frac{m_p \times 0}{\sqrt{1-\frac{0^2}{c^2}}}\right)^2 c^2 + (m_pc^2)^2} \\
E &= m_pc^2
\end{aligned}
\tag{42}
$$

This confirms that the Planck mass particle is unique and can only consist of rest-mass, and no kinetic energy or momentum. We will claim the Planck mass particle, and thereby the Planck length and Planck time are invariant across all reference frames. That is we predict Lorentz symmetry is broken at the Planck scale, something that several quantum gravity theories also predict, see [24], for example.

The uncertainty principle is, in this new perspective, actually an uncertainty about the velocity of the particle in question, that is linked to the uncertainty in the relativistic reduced Compton wavelength of the particle. The uncertainty in the reduced Compton wavelength of a particle with momentum or kinetic energy different from zero must be

$$
\begin{aligned}
l_p \geq \quad & \bar{\lambda}\sqrt{1-\frac{(\Delta v)^2}{c^2}} \quad \leq \bar{\lambda} \\
l_p \geq \quad & \Delta x \quad \leq \bar{\lambda}
\end{aligned}
\tag{43}
$$

while for the Planck mass particle we have $\Delta\lambda = 0$ because for the Planck mass particle it is always $\bar{\lambda} = l_p$, which must mean that the Planck mass particle cannot move; it is at absolute rest for one Planck second.

6 Implications

Our maximum velocity of matter, which is directly linked to the Planck scale, has a series of important implications for quantum mechanics.

Renormalization

Renormalization should no longer be needed. Even though renormalization has become an accepted method over time, this was not the case originally. One prominent critic of renormalization was Richard Feynman [25]. Clearly, he had a central role in the development of quantum electrodynamics, and yet he claimed

> *The shell game that we play ... is technically called 'renormalization'. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate.* – Richard Feynman, 1985

In 1987, Feynman [33] again commented on renormalization

> *Some twenty years ago one problem we theoretical physicists had was that if we combined the principles of quantum mechanics and those of relativity plus certain tacit assumptions, we seemed only able to produce theories (the quantum field theories), which gave infinity for the answer to certain questions. These infinities are kept in abeyance (and now possibly eliminated altogether) by the awkward process of renormalization.* – Richard Feynman, 1987

Our maximum velocity limit provides a clear cut-off point on energy limits in elementary particles and renormalization should no longer be needed.

Bell's Theorem

Several researchers have pointed out that by implicitly assuming all possible Bell measurements occur simultaneously, then all proofs of Bell's Theorem [26] violate Heisenberg's uncertainty principle [27]. We wonder what it could mean for the interpretation of Bell's Theorem if Heisenberg's uncertainty principle breaks down at the Planck scale and we then go from uncertainty to certainty (determinism). Interestingly, Clover states [28]

> *By implicitly assuming that all measurements occur simultaneously, Bell's Theorem only applied to local theories that violated Heisenberg's uncertainty principle.*

If Heisenberg's uncertainty principle breaks down at the Planck scale, this should open up the possibility of hidden variables, as suggested by Einstein, Podolsky, and Rosen in 1935; see [29]. We have shown that, under our theory, Planck mass particles can commute. Further, we claim that the Planck mass particle may be the building block of all other particles. Our theory opens up the way for hidden variable theories and in this framework, Bell's Theorem likely is invalid.

Negative Probabilities and Negative Energy: A New Logical Interpretation

In addition to a minimum uncertainty of $\sigma_p \sigma_x \geq \frac{\hbar}{2}$, there is a maximum uncertainty of

$$\sigma_E \sigma_t \leq \hbar c \left(1 - \frac{l_p}{\bar{\lambda}}\right) \tag{44}$$

Assume that we now multiply both sides with minus one and we get

$$-\sigma_E \sigma_t \geq -\hbar c \left(1 - \frac{l_p}{\bar{\lambda}}\right) \tag{45}$$

In other words, we are basically flipping the sign of the energy operator (and the momentum operator). We speculate that the theoretical negative energy one can mathematically obtain from the relativistic energy momentum relationship when used in connection to the Klein–Gordon equation, for example, should be interpreted to mean that there is an upper limit on the relativistic energy level of elementary particles, still this view will need further investigation.

Negative quasi probabilities are typically related to negative energies, as first pointed by Dirac [30] (see also [31, 32, 33])

> *Thus the two undesirable things, negative energy and negative probability, always occur together.* – Paul Dirac, 1942

Negative probabilities could be linked to negative uncertainty. Both negative probabilities and negative uncertainty cannot exist in the real world, which naturally is impossible, but mathematically it could simply mean we have flipped the sign of the inequality and that there is a maximum limit on uncertainty, in addition to a lower bound? Negative probabilities have been used in recent times in an attempt to explain the spooky action at distance in Bell's Theorem, [34]. We do not think that is the ultimate solution to the challange, but it does give a hint about what might have been missing in quantum mechanics, namely an exact upper limit on energy for elementary particles, and thereby (in our theory) also a minimum distance (in terms of minimum reduced Compton wavelength for a particle) where uncertainty collapses. As we have discussed, in the special case of the Planck mass, the upper and lower bound are zero, and thus there is no uncertainty in that case. There are no negative probabilities per-se, they are just an indication of an upper boundary condition on the maximum velocity for anything with rest-mass.

Entropy

It has been shown that Heisenberg's uncertainty principle and wave mechanics are closely linked to entropy [35]. In 1957, Hirschman [36] showed that the Heisenberg principle could be expressed as a lower bound on the sum of entropies; see also [37]. This likely indicated that a minimum entropy is a function of Heisenberg's uncertainty principle. Here we have shown that for the Planck mass particle there is no uncertainty. This could mean that entropy collapses at the Planck scale (for the Planck mass particle). This collapse of entropy would possibly only last for one Planck second.

When we are working with non-Planck mass particles, we have suggested there must be an upper limit on uncertainty equal to $\sigma_E\sigma_t \leq \hbar\left(1-\frac{l_p}{\bar{\lambda}}\right)$ in addition to the Heisenberg lower limit. If this is true, then it would likely mean an upper bound on entropy. This would change our entire view on entropy and include the concept that entropy collapses at the Planck scale. At this time, this is purely speculative and we will leave it for another time, and deeper examination to see how this could be linked to entropy, in both a mathematical and a logical manner.

7 A New Type of Quantum Probabilities

We will here suggest a new way to look at particles that is somewhat related to Schrödinger's [38] hypothesis in 1930 of a ("trembling motion" in German) in the electron. It is also linked to our Heisenberg derivation that shows a breakdown of uncertainty for Planck masses. Schrödinger indicated that the electron was in a sort of trembling motion $\frac{2mc^2}{\hbar} \approx 1.55269 \times 10^{21}$ per second. We will suggest that the electron is in a Planck mass state $\frac{c}{\bar{\lambda}_e} \approx 7.76344 \times 10^{20}$ per second (exactly half of that of Schrödinger's "Zitterbewegung" frequency. However, each Planck mass state only lasts for one Planck second and we therefore get the normal electron mass from

$$\frac{c}{\bar{\lambda}_e} m_p \frac{l_p}{c} = \frac{\hbar}{\bar{\lambda}_e}\frac{1}{c} \approx 9.10938 \times 10^{-31}\ \text{kg} \tag{46}$$

This model leads to a new probabilistic quantum probability theory that seems to be consistent with our uncertainty relationship described above. We will claim that every elementary particle has a quantum probability of

$$P = \frac{l_p}{\bar{\lambda}} \tag{47}$$

which, in this model, is shown when we find an elementary particle in its Planck mass state in a one Planck second observational time window. As only the Planck mass particle has reduced Compton wavelength equal to the Planck length, only the mass state of the Planck mass is certain if one finds a Planck mass particle. So, we clearly see the probability for elementary particles will vary between zero and one. For zero probability, we need an infinite reduced Compton wavelength. An electron, for example, will have a probability of $P \approx 4.185 \times 10^{-23}$ to be in a Planck mass state for any hypothetical observational time window of one Planck second. This means that every elementary particle can be expressed as

$$m = m_p P = m_p \frac{l_p}{\bar{\lambda}} \tag{48}$$

The formula that shows how an elementary particle can be expressed as a Planck mass multiplied by $\frac{l_p}{\bar{\lambda}}$ is not new, it was possibly first suggested by Hoyle, Burbidge, and Narlikar in 1994; see [39]. What is new here is that we are interpreting $\frac{l_p}{\bar{\lambda}}$ as a quantum probability for the particle to be in a Planck mass state, and that this probability is directly linked to the Planck length, in addition to the reduced Compton length of the particle in question.

Further, we will assume the reduced Compton wavelength must undergo standard length contraction as measured with Einstein-Poincaré synchronized clocks when the particle is moving relative to the observer. This will also affect the quantum probability for the particle to be in a Planck mass state. Every elementary particle must then have a relativistic quantum probability that is

$$P = \frac{\frac{l_p}{\bar{\lambda}}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{l_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}} \tag{49}$$

Many will likely protest here, because if we only rely on combining this with Einstein's special relativity theory [40] it means we can get relativistic probabilities above unity and even close to infinite probabilities. This would be absurd and would not lead to a good theory. However, Haug's maximum velocity of matter comes into play, for simplicity we repeat it:

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{50}$$

More important is that at this maximum velocity for each particle, the quantum relativistic probability can take on a maximum value of

$$\begin{aligned}
P &= \frac{l_p}{\bar{\lambda}\sqrt{1 - \frac{v_{max}^2}{c^2}}} \\
P &= \frac{l_p}{\bar{\lambda}\sqrt{1 - \frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} \\
P &= \frac{l_p}{\bar{\lambda}\sqrt{1 - \frac{c^2\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{c^2}}} \\
P &= \frac{l_p}{\bar{\lambda}\sqrt{1 - 1 + \frac{l_p^2}{\bar{\lambda}^2}}} \\
P &= \frac{l_p}{\bar{\lambda}}\frac{\bar{\lambda}}{l_p} = 1
\end{aligned} \tag{51}$$

Thus, Haug's maximum velocity very elegantly leads to a maximum quantum probability of one. This means we get a boundary condition on the quantum probability for each elementary particle for each Planck second of

$$\begin{aligned}
\frac{l_p}{\bar{\lambda}} &\leq P \leq \frac{l_p}{\bar{\lambda}\sqrt{1 - \frac{v_{max}^2}{c^2}}} \\
\frac{l_p}{\bar{\lambda}} &\leq P \leq 1
\end{aligned} \tag{52}$$

Still, the relativistic quantum probability range will be different for each elementary particle. This means Einstein's relativistic mass formula for elementary particles can be seen as

$$\frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\hbar}{\bar{\lambda}\sqrt{1 - \frac{v^2}{c^2}}}\frac{1}{c} \tag{53}$$

The maximum relativistic mass for any particle is the Planck mass multiplied by the maximum relativistic probability, which is one, and not surprisingly we get

$$\text{Expected relativistic maximum mass electron} = m_p \frac{l_p}{\bar{\lambda}_e\sqrt{1 - \frac{v_{max}^2}{c^2}}} = m_p \times 1 = m_p \tag{54}$$

How should we interpret this? It means at its maximum velocity any subatomic particle becomes a Planck mass, when relying on our new type of quantum probabilities. This also means that the original Heisenberg uncertainty principle collapses and becomes the certainty principle at the Planck scale. In addition, the Lorentz symmetry is broken at the Planck scale, but not before that.

The Planck mass particle is a particularly interesting case; its reduced Compton wavelength is $\bar{\lambda} = l_p$, which gives a probability range for the Planck mass particle of

$$\begin{aligned}
\frac{l_p}{l_p} &\leq P_p \leq 1 \\
1 &\leq P_p \leq 1
\end{aligned} \tag{55}$$

This can only be true if the Planck particle quantum probability is always $P_p = 1$. This naturally means there is no uncertainty for the Planck mass particle, as we have shown when re-deriving the Heisenberg uncertainty principle. Our interpretation is that the Planck mass particle is the very collision point of the light particles making up each elementary particle.

Table 1 shows the standard relativistic mass as well as the probabilistic approach; they are consistent. Be aware that there must be a maximum velocity limit on anything with mass; this will be equal to Haug's maximum velocity.

	Standard appproach	**Probabilistic approach observation in one Planck second**
Electron mass	$m = \frac{m_e}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{\hbar}{\bar{\lambda}_e}\frac{1}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$	$E[m] = m_p P_e = m_p \frac{l_p}{\bar{\lambda}_e\sqrt{1-\frac{v^2}{c^2}}} \geq m_e$
Proton mass	$m = \frac{m_{\mathbf{P}}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{\hbar}{\bar{\lambda}_P}\frac{1}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$	$E[m] = m_p P_P = m_p \frac{l_p}{\bar{\lambda}_P\sqrt{1-\frac{v^2}{c^2}}} \geq m_{\mathbf{P}}$
Planck mass particle	$m_p = \frac{m_p}{\sqrt{1-\frac{0^2}{c^2}}} = m_p$	$E[m] = m_p P_p = m_p \frac{l_p}{l_p\sqrt{1-\frac{0^2}{c^2}}} = m_p$

Table 1: This table shows the standard relativistic mass as well as the probabilistic approach. Be aware of the notation difference between the Planck mass m_p and the proton rest-mass $m_{\mathbf{P}}$, and $E[m]$ stands for the expected mass.

Table 2 shows the relativistic mass when a particle is traveling at its maximum velocity. This will always correspond to a relativistic mass equal to the Planck mass, and a quantum probability of one. Be aware that the particle when reaching this velocity, which is above what can be achieved at LHC, likely will burst into energy within one Planck second. So, the certainty we predict can only last for one Planck second when we are dealing with single particles.

	"Standard" appproach	**Probabilistic approach observation in one Planck second**
Electron mass	$m = \frac{m_e}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = m_p$	$E[m] = m_p P_e = m_p \frac{l_p}{\bar{\lambda}_e\sqrt{1-\frac{v_{max}^2}{c^2}}} = m_p \times 1 = m_p$
Proton mass	$m = \frac{m_{\mathbf{P}}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = m_p$	$E[m] = m_p P_P = m_p \frac{l_p}{\bar{\lambda}_P\sqrt{1-\frac{v_{max}^2}{c^2}}} = m_p \times 1 = m_p$
Planck mass particle	$m_p = \frac{m_p}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = m_p$	$E[m] = m_p P_p = m_p \frac{l_p}{l_p\sqrt{1-\frac{v_{max}^2}{c^2}}} = m_p \times 1 = m_p$

Table 2: This table shows the standard relativistic mass as well as the probabilistic approach at the maximum velocity only. Be aware of the notation difference between the Planck mass m_p and the proton rest-mass $m_{\mathbf{P}}$, and $E[m]$ stands for the expected mass.

8 Conclusion

Based on Haug's recently suggested maximum velocity for matter, we have shown that the momentum and position operators, as well as the energy and time operators, commute at the Planck scale, but not before that. This means that Einstein may have been right, as it opens up the possibility for hidden variable techniques, and also means that Bell's Inequality does not necessarily hold. Further, this means that we get a relativistic quantum mechanics where there should no longer be a need for renormalization, as we get an exact upper limit on energies linked to the Planck scale. Our new theory seems to be consistent in all aspects. It means Lorentz symmetry is broken at the Planck scale, but not before that, something that a series of quantum gravity theories predict could be the case. We think that the so-called negative energies that come out from the relativistic energy momentum relationship and therefore are embedded in the Klein–Gordon equation could be reinterpreted, as there also is an upper energy limit.

It is important to note that our modified quantum theory does not conflict with any common experiments in quantum mechanics. Our theory simply gives a new and we would say more logical interpretation. The question partly boils down to whether we want to have a theory that needs renormalization and that predicts spooky instantaneous action at distance (entanglement), or a theory that opens up the possibility for hidden variables and does not need renormalization, and where negative energies and negative probabilities can simply be interpreted as an upper boundary on uncertainty. Hopefully future research efforts will come up with experiments that can distinguish between this theory and other theories concerning quantum probabilities and their interpretation.

Appendix

Here we will derive the Kennard version of Heisenberg's uncertainty principle relation from the energy and time operators (instead of the momentum and position operators)

$$[\hat{E}, \hat{t}] = \hat{E}\hat{t} - \hat{t}\hat{E} \tag{56}$$

where $\hat{E}$ is the energy operator and $\hat{t}$ is the time operator. The energy operator is given by

$$\hat{E} = -i\hbar\frac{\partial}{\partial t} \tag{57}$$

and the time operator is given by

$$\hat{t} = t \tag{58}$$

From this we have

$$\begin{aligned}
[\hat{E}, \hat{t}]\Psi &= [\hat{E}\hat{t} - \hat{t}\hat{E}]\Psi \\
&= \left(-i\hbar\frac{\partial}{\partial t}\right)(t)\Psi - (t)\left(-i\hbar\frac{\partial}{\partial t}\right)\Psi \\
&= -i\hbar\left(\Psi + t\frac{\partial\Psi}{\partial(t)}\right) + i\hbar t\frac{\partial\Psi}{\partial(t)} \\
&= -i\hbar\left(\Psi + t\frac{\partial\Psi}{\partial(t)} - \frac{\partial\Psi}{\partial(t)}\right) \\
&= -i\hbar\Psi
\end{aligned} \tag{59}$$

And we have the following uncertainty

$$\begin{aligned}
\sigma_E\sigma_t &\geq \frac{1}{2}|\int \Psi^*[\hat{E}, \hat{t}]\Psi dt| \\
&\geq \frac{1}{2}|\int \Psi^*(-i\hbar)\Psi dt| \\
&\geq \frac{1}{2}| - i\hbar\int \Psi^*\Psi dt|
\end{aligned} \tag{60}$$

and since $\int \Psi^*\Psi dt$ must sum to 1 (there must be 100% probability for the particle to be somewhere), we are left with

$$\begin{aligned}
\sigma_E\sigma_t &\geq \frac{1}{2}| - i\hbar| \\
\sigma_E\sigma_t &\geq \frac{\hbar}{2}
\end{aligned} \tag{61}$$

that is, we get the same uncertainty relation as derived from the momentum and position operators. However, for the Planck mass particle there is no uncertainty, as the energy operator for a Planck mass particle must be zero.

References

[1] W. Heisenberg. Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik. *Zeitschrift für Physik*, (43):172–198, 1927.

[2] E. H. Kennard. Zur quantenmechanik einfacher bewegungstypen. *Zeitschrift für Physik*, (44):326–352, 1927.

[3] L. Planck. *Naturlische Masseinheiten.* Der Königlich Preussischen Akademie Der Wissenschaften, p . 479, 1899.

[4] L. Planck. *Vorlesungen über die Theorie der Wärmestrahlung.* Leipzig: J.A. Barth, p. 163; see also the English translation "The Theory of Radiation" (1959) Dover, 1906.

[5] L. Motz. Gauge invariance and the structure of charged particles. *Il Nuovo Cimento*, 26(4), 1962.

[6] L. Motz. A gravitational theory of the mu meson and leptons in general. *Rutherford Observatory, Columbia University*, 1966.

[7] L. Motz. The quantization of mass. *Rutherford Observatory, Columbia University*, 1971.

[8] S. S. De. Quantum creation of highly massive particles in the very early universe. *International Journal of Theoretical Physics*, 2001.

[9] S. Hawking. Gravitationally collapsed objects of very low mass. *Monthly Notices of the Royal Astronomical Society*, 152, 1971.

[10] L. Motz and J. Epstein. The gravitational charge $1/2\sqrt{\hbar c}$ as a unifying principle in physics. *Il Nuovo Cimento*, 51(1), 1979.

[11] G. M. Obermair. Primordial Planck mass black holes (ppmbhs) as candidates for dark matter? *Journal of Physics, Conference Series*, 442, 2013.

[12] J. H. Macgibbon. Can Planck-mass relics of evaporating black holes close the universe? *Nature*, 329, 1987.

[13] V. I. Dokuchaev and Y. N. Eroshenko. Black hole atom as a dark matter particle candidate. *https://arxiv.org/abs/1403.1375*, 2014.

[14] S. Prasannakumar, S. Krishnaveni, and T. K. Umesh. Determination of rest-mass energy of the electron by a Compton scattering experiment. *European Journal of Physics*, 33(1), 2012.

[15] E. G. Haug. *Unified Revolution: New Fundamental Physics*. Oslo, E.G.H. Publishing, 2014.

[16] E. G. Haug. The gravitational constant and the Planck units. A simplification of the quantum realm. *Physics Essays Vol 29, No 4*, 2016.

[17] E. G. Haug. The ultimate limits of the relativistic rocket equation. The Planck photon rocket. *Acta Astronautica*, 136, 2017.

[18] E. G. Haug. Can the Planck length be found independent of big G ? *Applied Physics Research*, 9(6), 2017.

[19] E. G. Haug. Planck mass measured totally independent of big G utilising McCulloch-Heisenberg Newtonian equivalent gravity. *Working Paper: Norwegian University of Life Sciences*, 2018.

[20] E. G. Haug. Does Heisenberg's uncertainty principle predict a maximum velocity for anything with rest-mass below the speed of light? *orking Paper: Norwegian University of Life Sciences, under review*, 2018.

[21] G. Scarpetta. *Letter Nuovo Cimento,*, 51, 1984.

[22] D. F. Falla and P. T. Landsberg. Black holes and limits on some physical quantities. *European Journal of Physics*, 15, 1994.

[23] O. J. Pike, F. Mackenroth, E. G. Hill, and S. J. Rose. A photon–photon collider in a vacuum hohlraum. *Nature Photonics*, 8, 2014.

[24] F. Kislat and H. Krawczynski. Planck-scale constraints on anisotropic Lorentz and CPT invariance violations from optical polarization measurements. *Physical Review D*, 95, 2017.

[25] R. P. Feynman. *QED: The Strange Theory of Light and Matter*. Penguin, p. 128, 1985.

[26] J. S. Bell. On the Einstein Podolsky Rosen paradox. *Physics*, 1, 1964.

[27] M. Clover. Bell's Theorem: A critique. *arXiv:quant-ph/0502016*, 2005.

[28] M. Clover. Bell's Theorem: A new derivation that preserves Heisenberg and locality. *arXiv:quant-ph/0409058*, 2005.

[29] A. Einstein, B. Podolsky, and N. Rosen. Can quantum mechanical description of physical reality be considered complete? *Phys. Rev.*, (47):777–780, 1935.

[30] P. Dirac. The physical interpretation of quantum mechanics. *Proc. Roy. Soc. London*, (A 180):1–39, 1942.

[31] E. P. Wigner. On the quantum correction for thermodynamic equilibrium. *Phys. Rev.*, 40, 1932.

[32] W. Pauli. Remarks on problem connected with the renormalization of quantized field. *Il Nuovo Cimento*, 4, 1956.

[33] R. P. Feynman. Negative probability. *First published in the book Quantum Implications: Essays in Honour of David Bohm, by F. David Peat (Editor), Basil Hiley (Editor) Routledge & Kegan Paul Ltd, London & New York*, pp. 235–248, http://kh.bu.edu/qcl/pdf/feynmanr19850a6e6862.pdf, 1987.

[34] A. Khrennikov. Bell's Inequality: Physics meets probability. *arXiv:0709.3909*, 2007.

[35] I. Bialynicki-Birula and J. Mycielski. Uncertainty relations for information entropy in wave mechanics. *Communications in Mathematical Physics*, 44(2), 1975.

[36] R. Hirschman. A note on entropy. *American Journal of Mathematics*, 79, 1957.

[37] R. Leipnik. Entropy and the uncertainty principle. *Information and Control*, 2, 1959.

[38] E. Schrödinger. Über die kräftefreie bewegung in der relativistischen quantenmechanik. *Sitzungsberichte der Preuischen Akademie der Wissenschaften. Physikalisch-mathematische Klasse*, 1930.

[39] F. Hoyle, G. Burbidge, and J.V. Narlikar. The basic theory underlying the quasi-steady state cosmology. *Working paper, Inter-University Center for Astronomy and Astrophysics*, 1994.

[40] A. Einstein. On the electrodynamics of moving bodies. *Annalen der Physik, English translation by George Barker Jeffery 1923*, (17), 1905.