Exploring the Possibility of New Physics
Quantum Mechanics
Fundamental Particles & Gravity

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Abstract

In a completely different approach, this paper proposes fundamental particles formed from infinite superpositions, with mass borrowed from either a Higgs type scalar field, or the time component of zero point fields. Energy is also borrowed from zero point vector fields. Just as the Standard Model divides the fundamental particles into two types…those with mass and those without, with the Higgs mechanism providing the difference…infinite superpositions also seem to divide naturally into two sets: (a) those with "infinitesimal" mass, and (b) those with significant mass (from micro electron volts upwards). In the infinitesimal set (a), photons, gluons and gravitons (to fit with cosmology and the expansion of the cosmos) all have $\approx 10^{-33}$ eV mass, approximately the inverse of the causally connected horizon radius. These values are sufficiently close to zero, the symmetry breaking of the Standard Model remains essentially valid. With velocities this close to light, helicity is virtually fixed. The Higgs mechanism increases mass from infinitesimal type (a) to significant or measureable type (b) values. The energy in the zero point fields (borrowed to build fundamental particles) is limited. Cosmic wavelength gravitons (in contrast to high frequency borrowing from local invariant sources) borrow a redshifted supply of Planck scale zero point action modes from a holographic horizon, receding at virtually light velocity. Just as atomic wavefunctions require spherically symmetric scalar potentials, superposition wave functions require spherically symmetric squared vector potentials in the form of spin 1 quanta. In flat comoving coordinates the horizon is spherical but not at peculiar velocities. The borrowed spin 1 quanta, and the action from the horizon, transform together as a "Spherically Symmetric Four Volume Action Density" at cosmic wavelengths. It is invariant in all coordinates and metrics, and appears to relate with gravity. It only works in a flat on average continually expanding universe. This exponential expansion may relate with the present discrepancy in the different ways of measuring the Hubble parameter. There is an infinitesimal change to Einstein's stress tensor effective only at cosmic wavelengths. There is also a change in the metric tensor effective near black hole horizons that may increase apparent black hole masses slightly. This change has parallels to the Reissner-Nordstrom and Kerr-Newman metrics, but at solar system scale appears to be inside current experimental limits.

Foreword

As someone in their eighties, this paper represents the culmination of around sixty five years of thinking about physics, sometimes very intensely for long periods, and others less so; probably like many others of us interested in what makes the Universe tick. It combines two papers originally called Part I and Part II. That separation did not work so well because the ideas, though very simple are also very radical, and Part II frequently needed to refer to formulae and equations in Part I. These equations are now all hyperlinked for immediate reference, hopefully providing better continuity and consistency.

My love of Physics started in my teens and I apologize for the old fashioned methodology. I still tend to think on terms of rest mass for example where the modern generation don't. As will become apparent, perhaps slowly, the ideas presented here are basically very simple. To make them accessible to the widest possible audience, I have tried to express them as simply as I could, and much more detail is used than that required by experts in these fields. When looking at virtual photon and graviton probability densities for example, I did this in a simple way that works, and I think illustrates more effectively why gravitons around mass concentrations change the metric. I introduced a four vector type of graviton probability density, and what I called a "Spherically symmetric four volume action density from the receding horizon", invariant in all coordinates and metrics, only at a much later stage.

From early days I always puzzled over particles with zero radius having angular momentum, unless they have infinite mass; but like so many others, was always told it just has to be a mathematical fact. Newton would, I think, have turned in his grave. I always saw this as contradicting him much more so than Einstein's corrections to his absolute space and time, which I think, he would have eventually accepted. I am also almost certain he would have approved of spatially dependant wave functions with their orbital angular momentum. Richard Feynman was also my hero. I read and reread his wonderful lectures on physics; referring to them so many times they started to fall apart. Like Bibles to me, and treasured so much I had to get them rebound. They are a model of clarity and simplicity in the way he explains difficult topics. Some sections in my papers are almost verbatim from his books, but modified to illustrate my arguments. I hope he would not mind. When reading these books, I noticed that $l=3, m=\pm 2$ wavefunctions have an angular momentum vector at the same angle to the z axis as spin ½, $m=\pm\frac{1}{2}$ point particles. I started wondering whether it might be possible, to somehow build all the fundamental point particles out of superpositions of these spatially dependant $l=3$ wavefunctions; as $l=3, m=\pm 2$ states can emit both spins 1 & 2 bosons in $m=\pm 1$ & $m\pm 2$ states. Maybe someone has demonstrated how spin ½ particles can emit spin 2 gravitons, but I have never seen it, nor can I see how it is possible.

And so my long journey began. Back then there were various models using preon building blocks of real spin ½ particles. In contrast I used imaginary or virtual spin zero building blocks, I also called them preons. These virtual preons cannot be observed in the real world of experiments. There are only three; red, green and blue, all electrically charged with their anti-counterparts. Because they are spin zero they are not subject to the weak force. I found that groups of 8 coupling to the electromagnetic and 8 gluon colour fields can build all the fundamental point particles, providing they had non-zero mass. They spanned the frequency range from, a Planck Energy maximum to a low frequency cutoff $k_{\min}$; or maximum wavelength where the zero point energy/action density required, equalled that available. This cutoff $@\, k_{\min}$ is always approximately the inverse of the radius of the observable universe. They divided naturally into two sets, the normal mass set, and the infinitesimal mass set of $m \approx k_{\min}$ for photons, gluons and gravitons etc. This mass can be borrowed from either, a Higgs type scalar field, or the time component of a four potential zero point field; but they also borrow energy from the spatial component.

This mass and energy is borrowed for time $\Delta T \approx \hbar / 2\Delta E$, the time that each member of the superposition lasts before repeating this process. The density of zero point fields at the cosmic wavelength cutoff is infinitesimal. In the rest frame, in which the particle is built, these preons are born with zero momentum and infinite wavelength, allowing them to borrow Planck scale action quanta, redshifted from a distant receding horizon. Apart from the infinitesimal masses, it seems to mesh with the most basic version of the Standard Model. There are very approximately 10^{122} of these cosmic wavelength gravitons at the present time, exceeding by at least 10^{20} the number of all other particles. The action density they require controls the velocity of the horizon, which accelerates exponentially quite naturally with no need for Dark Energy. And it only works in a Euclidean or flat space on average universe.

In comoving coordinates, at any particular cosmic time, in a flat universe with homogeneous mass, there is a uniform (3 volume) density of cosmic wavelength gravitons, with a balancing uniform action density of cosmic wavelength quanta redshifted from the expanding spherical horizon. If mass concentrations gather locally, they are surrounded by concentrations of gravitons, and space around them expands to restore the required action density. This changing metric is equivalent to an “Invariant Spherically Symmetric Four Volume Action Density” for cosmic wavelength gravitons in all coordinates. It appears to be a different form of invariance, but must be related, to the action principle from which Einstein’s field equations can be derived. It does however require changes to Einstein’s energy momentum tensor. The effect locally is infinitesimal, but more significant near black hole horizons, and in a way that may relate with recent black hole merger observations.

It is even more significant at cosmic scale, such as requiring Euclidean on average space and exponential expansion. It adds an $\approx 1.4m^2/r^2$ dimensionless term to the metric which may be problematic. This is equivalent to adding ≈ 700 metres from the sun to all the planets in our solar system, with no change in the distance between them, or their orbital periods. This is probably within the current solar system constraints from experiments to test modified General Relativity, but not by much, and possibly detectable very soon. It would only affect the last few cycles of mergers and could increase the apparent mass of each member by up to $\approx$ 70%, but supercomputer simulation of this would be required. This m^2/r^2 term may also affect the inflow rate of matter into a black hole with possible implications for supermassive black hole build rates. It may also be a factor in the unexpected non alignment of black hole spins in some of the mergers observed so far.

When I was looking at the possible connections between superpositions and gravity, I watched the wonderful internet lectures by Leonard Susskind. Like Feynman's lectures, I watched and re watched the sessions on both General Relativity and Cosmology time and again. They helped me crystalize my ideas enormously. His books with Art Freidman have, like Feynman's, inspirationally simple explanations, but less voluminous of course. Even when you think you understand something, I have often found that someone who has a very deep grasp, explains things in such a way that frequently opens new pathways to understanding. It was only after reading these books that I started to see possible connections between gravity, and invariant four volume cosmic wavelength graviton action densities. I am deeply indebted to such people, and the countless others past and present; all treading similar paths, from whom we can all learn so much.

Repeating what has been said many times over the centuries, but in a slightly different way: Of all life forms, humanity has perhaps most refined the art "Of Standing on the shoulders of others, to enable us to try to see further" It may even turn out to be one of humanities defining features, and in reality is what all of us, trying to better understand the universe and world around us, each in our own different ways, are doing.

Contents

1 Introduction

Many Physicists today (probably a large majority) are; Supersymmetry supporters, String theory supporters, Inflation supporters, Metaverse supporters, etc. They will perhaps see the ideas presented in this paper as irrelevant. On the other hand there is a much smaller, but possibly growing band who are increasingly disillusioned at what seems to them to be a lack of real, concrete or testable progress over the last 30 to 40 years or so, since the development of the brilliantly successful and accurate Standard Model. This smaller band adheres to the tradition, started originally by the Greeks but more particularly over the last few centuries, of empirically testable science: Newton's theories, Maxwell's equations, General Relativity, Quantum mechanics and lastly the Standard Model representing the pinnacle of this testing by experiment era. All these great theories were developed from experiments. As instrumentation accuracy slowly improved, and experiments grew more refined, the above theories, each accurate in their day, slowly evolved from one to the next. The current situation in contrast, invites some important and relevant questions; for example:

1. Is Supersymmetry really the answer to the problems with the Standard Model?
2. Are the extra dimensions of String Theory really necessary?
3. Is "The Multiverse" the only explanation of accelerating cosmic expansion?
4. Is Inflation really necessary? And so on.

Approaching all this in a new direction, this paper explores possible solutions to these questions in a completely different way; but still using very simple basic principles of quantum mechanics and relativity. Apart from infinitesimal differences it is (almost) consistent with the Standard Model. It requires the universe to expand exponentially after the big bang in an accelerating manner that is testable. This is so regardless of the value of Ω, with no need for Dark Energy. It changes the metric around mass concentrations in accordance with an infinitesimally modified General Relativity. And it all only works if the Universe is flat on average, with no need for inflation.

1.1 Summary

Papers modifying the Standard Model are too numerous to list, however we briefly touch on a small number of some early versions of these in section 1.1.2. The approach in this paper is very different from that in most of these earlier papers. The main differences are summarized below.

1.1.1 General Relativity as our starting point

General Relativity tells us that all forms of mass, energy and pressure are sources of the gravitational field. Thus to create gravitational fields all spin ½ leptons & quarks, spin 1 gluons, photons, $W^{\pm}$ & Z^0 particles etc. emit virtual gravitons, except possibly gravitons themselves (section 6.2.6), as gravitational energy is not part of the Einstein tensor.

The starting point of this paper assumes there is a common thread uniting these fundamental particles making this possible. Equations are developed that unite the amplitudes of the colour and electromagnetic coupling constants with that of gravity. The precision required by quantum mechanics for half integral and integral angular momentum allows gravity to be included, despite the vast disparity in magnitude between gravity and the other two. This combination of colour, electromagnetic and gravitational amplitudes in the same equation is possible because of a radically different approach taken in this paper: An approach using infinite superpositions of positive and negative integral $\hbar$ angular momentum virtual wavefunctions for spin ½, spin 1 and spin 2 particles. The final result is almost identical to the Standard Model, with infinitesimal but important differences.

The total angular momentum can be summed over all wavenumbers k; from $k=0$ to some cutoff value k_{cutoff}. We will assume (as with many unification theories) that the cutoff for these infinite superpositions is somewhere near Planck scale. Firstly imagine a universe where the gravitational constant $G \to 0$. As $G \to 0$, the Planck length $L_P \to 0$, the Planck energy $E_P \to \infty$ and $k_{cutoff} \to \infty$ also. If we sum the angular momentum of these infinite superpositions when $G \to 0$ (i.e. from $k=0$ to $k_{cutoff} \to \infty$) we get precisely half integral or integral $\hbar$ for the fundamental spin ½, spin 1 & spin 2 particles in appropriate m states. If we now put $G > 0$ the infinitesimal effect of including gravity can be balanced by an equal but opposite effect due to the non-infinite cutoff value in k. A near Planck scale superposition cutoff requires gravity to be included to get precisely half integral or integral $\hbar$. (Section 4.2)

These infinite superpositions have another very relevant property relating to the fact that all experiments indicate that fundamental particles such as electrons behave as point particles. Each wavefunction with wavenumber k, which we label as ψ_k, has a maximum radial probability at $r \approx 1/k$ and they all look the same (Figure 1.1. 1.)

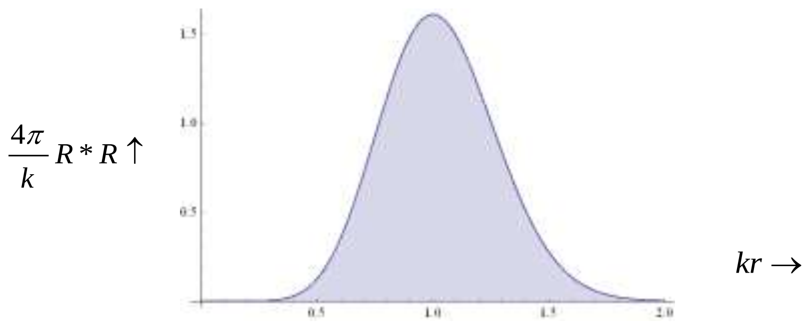


Figure 1.1. 1 The radial probability of the dominant $n=6$ for spin ½ wavefunction ψ_{6k}.

Every wavefunction ψ_k of these infinite superpositions, interacts only with virtual photons (for example) of the same k; if superpositions representing say an electron are probed with such photons (that interact only with wavefunction ψ_k) the resolution possible is of the same order as the dimensions of ψ_k, both have $r \approx 1/k$. The higher the energy of the probing particle the smaller the ψ_k it interacts with, the resolution of an observing photon can never be fine enough to see any ψ_k dimensions. Even if this energy approaches the Planck value, with a matching ψ_k radius near the Planck length it is still not possible to resolve it. This behaviour is consistent with the quantum mechanical properties of point particles.

1.1.2 **Primary interactions and Secondary interactions**

Supposing that superpositions can in fact build the fundamental spin ½, spin 1, and spin 2 particles, then what builds the superpositions? Before answering that question, this paper can only make sense if we divide the world of all interactions into two categories.

Secondary Interactions are those we are familiar with, and are covered by the Standard Model; but with the addition of gravity, which is not included in the Standard Model. They take place between the fundamental spin ½, spin 1 and spin 2 particles formed from infinite superpositions. They are the QED/QCD etc, interactions of all real world experiments.

Primary Interactions we conjecture on the other hand are those that build infinite superpositions. They are virtual, and completely hidden to the real world of experiments.

The majority of this paper is about these primary interactions, and the superpositions they build representing the fundamental spin ½, spin 1 and spin 2 particles. Primary interactions are between spin zero particles borrowed from a Higgs type scalar field and the zero point vector fields. In the 1970's models were proposed with preons as common building blocks of leptons and quarks [4] [5] [6] [7]. In contrast with the virtual particles in this paper some of these earlier models used real spin ½ building blocks. Real substructure has difficulties with large masses if compressed into the small volumes required to approach point particle behaviour. On the other hand with *virtual substructure borrowing energy from zero point fields the mass contribution at high k values can be cancelled* (section 3.2.1). As in earlier models this paper also calls the common building blocks preons, *but here the preons are both virtual and spin zero.* They also now build all spin ½ leptons and quarks, spin 1 gluons, photons, W & Z particles, plus spin 2 gravitons in contrast to only the leptons and quarks in the earlier models. (See Table 2.2. 1)

As these preons have zero spin they possess no weak charge, primary interactions (section 2.2.1) can take place only with the zero point colour, electromagnetic and gravitational fields. The three primary coupling constants for each of these three zero point fields are different from, (but related to) the secondary coupling constants. The behaviour of primary coupling is also entirely different from secondary coupling. Secondary coupling strengths vary (or run) with wavenumber k (the electromagnetic increasing with k and colour decreasing with k). In contrast, we conjecture primary coupling strengths (or constants) do not run. In this paper virtual preons are continually born with mass out of a Higgs type scalar field, existing only for time $\Delta t \approx \hbar / E$. At their birth, they interact while still bare with zero point vector fields at this instant of birth $t = 0$. The primary coupling constants consequently are fixed for all k: there is no time for charge canceling or reinforcing, which in secondary interactions forms around the bare charge progressively after its birth. The equations work only if this is true, and they also work only if the primary colour coupling constant is 1. This does not seem implausible as it simply means that primary colour coupling is certain (sections 2.2.2). The ratio between the primary and secondary colour coupling constants labelled χ_C is thus (*if primary colour coupling is 1*) the inverse of the secondary (or usual α_3^{-1} of QCD) colour coupling constant *at the superposition cutoff @ Planck Energy*. (Sections 3.3 & 4.2.2)

To enable the primary coupling to colour, electromagnetic and gravitational zero point fields, preons need colour, electric charge and mass. There are only three preons, red, green & blue with positive electric charge, and their anticouterparts. Their mass borrowed from some type of scalar Higg's field, or the time component of zero point fields must always be non-zero. This is discussed further in section 1.1.3. As there are 8 gluon fields, superpositions are built with 8 virtual preons for each virtual wavefunction ψ_k. The nett sum of these 8 electric charges is $0, \pm 2, \pm 4, \pm 6$, and never $> \pm 6$. This leads to the usual $0, \pm 1/3, \pm 2/3, \pm 1$ electric charge seen in the real world. Various combinations of these 8 preons in appropriate superpositions can build leptons and quarks, colour changing and neutral gluons, neutral photons, neutral massive Z^0 photons and the charged massive $W^{\pm}$ photons. (Table 2.2. 1)

1.1.3 Photons, gluons and gravitons with infinitesimal mass ($\approx 10^{-33} eV$).

For many decades after the discovery of the neutrino in the 1930s it was thought to be massless, and to travel at velocity c. Despite being in conflict with the Standard Model, towards the end of last century evidence slowly accumulated that this may not in fact be true, and that the family of 3 neutrinos have masses in the electron volt range. Due to this very low mass, and their normal emitted energies, they invariably travel at virtually the velocity of light c. Photons also have always been seen as massless traveling precisely at velocity c, except in the case of the massive $W^{\pm} \& Z^0$. Massless virtual photons have an infinite range, which has always been seen as an absolute requirement of the electromagnetic field. On the other hand, this paper requires some rest frame (even if this frame can move at virtually c) in which to build all the fundamental particles. Table 6.2 1 suggests photons, gluons and gravitons have $\approx 10^{-33} eV$ mass with *a range of approximately the inverse of the causally connected horizon radius*, and velocities sufficiently close to that of light their helicity remains essentially fixed. This allows some form of Higgs mechanism to increase this infinitesimal mass to the various values in the massive set. These infinitesimal masses are in line with some recent proposals [2] [3] where gravitons have a mass of $< 10^{-33} eV$ to explain accelerating expansion.

The virtual wavefunction we use is $\psi_{nk} = C_{nk} r^3 \exp(-n^2k^2r^2/18)Y(\theta,\varphi)$ an $l=3$ wavefunction. This virtual $l=3$ property is normally hidden. In the same way as scattering experiments on spin 0 pions show spin 0 properties, and not the properties of the two canceling spin ½ component particles, this $l=3$ property of the virtual components of superpositions is not visible in the real world. Scattering experiments can exhibit only the spin properties of the resulting particle. The individual angular momentum vectors $|\mathbf{L}| = 2\sqrt{3}\hbar$ of the infinite superposition all sum to a resulting: $|\mathbf{L}_{Total}| = (\sqrt{3}/2)\hbar$, $\sqrt{2}\hbar$ or $\sqrt{6}\hbar$ for spin ½ , spin 1 or spin 2 respectively, in a similar way to two spin ½ particles forming spin 0 or spin 1 states.

The wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2k^2r^2/18)Y(\theta,\varphi)$ has Eigenvalues $\mathbf{P}_{nk}{}^2 = n^2\hbar^2k^2$ with $|\mathbf{P}_{nk}| = n\hbar k$, suggesting it borrows n parallel $|\hbar\mathbf{k}|$ quanta from zero point vector fields provided n is integral. We can see this by letting $k \to \infty$ allowing energy $E \to n\hbar\omega$ by absorbing n quanta $\hbar\omega$ from the zero point vector fields (section 2.3.2). As spin 3 needs at least 3 spin 1 particles to create it, the lowest integral number n can be is 3. The virtual $l=3$ property can however be used to derive the magnetic moment of a charged spin ½, $m = \pm 1/2$ state as a function of n. Section 3.5 shows $g=2$ Dirac electrons need an average (over integral n states) of $\bar{n} \approx 6.0135$. Three member superpositions $\psi_k = \sum c_n \psi_{nk}$ with $n = 5, 6, \&7$ achieve this, creating Dirac spin ½ states. We also find that $n=6$ is the dominant member and each superposition ψ_k needs at least 3 members to make all the equations consistent for Dirac particles. Secondary interactions at any wavenumber k can occur with ψ_k if integers n change by ± 1, thus changing the Eigenvalues $|\mathbf{P}| = n\hbar k$ by $\pm\hbar k$ where this can be only a temporary rearrangement of the triplets of values of n. This is true, whether the interaction is with leptons, quarks, photons, gluons, W & Z particles, or gravitons. (Section 3.3)

1.1.4 Superposition wavefunctions require only squared vector potentials

The wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2k^2r^2/18)Y(\theta,\varphi)$ requires an invariant in all coordinates spherically symmetric squared vector potential to create it: $Q^2A^2 = n^4\hbar^2k^4r^2/81$. There are no linear potential terms in contrast with secondary interactions. The primary interaction operator is $\hat{P}^2 = -\hbar^2\nabla^2 + Q^2A^2$, with no linear potential terms included and Q simply represents a collective symbol for all the effective charges concerned. As an example, the dominant $n=6$ wavefunction of a spin ½ Dirac ψ_k requires a squared vector potential of $Q^2A^2 = n^4\hbar^2k^4r^2/81 = 16\hbar^2k^4r^2$ (section 2.3.1). Primary coupling between the 8 virtual preons and the colour, electromagnetic and gravitational zero point fields produces a vector potential squared value for all infinite superpositions which can be expressed as:

$$Q^2A^2 = \frac{\left[8 + 8\sqrt{\alpha_{EMP}} + im_0\sqrt{G_p/(2s\hbar c)}\right]^2 (\hbar^2k^4r^2)}{3\pi(sN)(1+\varepsilon)} \left[\frac{(sN)(1+\varepsilon)dk}{k}\right]$$

(Where the length of the complex vector is simply squared here.) The significance of the cancelling top and bottom factors (sN) is explained in section 2.1.2. Also the cancelling $(1+\varepsilon)$ factors are due to gravity and explained in section 4.2. The primary_colour coupling amplitude is conjectured to be 1 to each of the eight preons, and $\sqrt{\alpha_{EMP}}$ the primary electromagnetic coupling. This equation applies regardless of the individual preon colour or electric charge signs, whether positive or negative (section 2.2.3). The primary gravitational coupling is to the particle mass m_0. The primary gravitational constant is G_P divided by $\hbar c$ to put it in the same form as the other two coupling constants. The magnitude of the total angular momentum vector of the infinite superposition is $|\mathbf{L}_{Total}| = \sqrt{s(s+1)}$.) This Q^2A^2 without the gravity term generates superpositions with probability $(N \cdot s)dk/k$ where s is the superposition spin, $N=1$ for massive spin ½ fermion & massive boson superpositions but $N=2$ for infinitesimal mass boson superpositions (Table 4.3. 1, section 6 and its subsections cover this more fully). Section 4.2 includes gravity raising the superposition probability to $(1+\varepsilon)(N \cdot s)dk/k$ where the infinitesimal ε (not to be confused with infinitesimal mass) is $\varepsilon \approx 2m_0^2/Spin$ (in Planck units $\hbar = c = G = 1$) $\approx 7\times10^{-45}$ for electrons, and $\varepsilon \approx 10^{-34}$ for a Z^0. The ψ_k superpositions require at least three integral n members. The following three member superpositions fit the Standard Model best (see Table 4.3. 1)

Spin ½ massive $N=1$ fermion superpositions $\qquad \psi_k = \sum_{n=5,6,7} c_n \psi_{nk}$.

Spin 1 massive $N=1$ boson superpositions $\qquad \psi_k = \sum_{n=4,5,6} c_n \psi_{nk}$.

Spins 1 & 2 infinitesimal mass $N=2$ boson superpositions $\qquad \psi_k = \sum_{n=3,4,5} c_n \psi_{nk}$.

Below are infinite superpositions $|\psi_{\infty,s,m}\rangle$ for only spins ½ & 1. The symbol ∞ refers to the infinite sum, s the spin of the resulting real particle, m its angular momentum state, and ss a spherically symmetric state. Section 3.1.3 explains this format. Also square cutoffs in wavenumber k are used here for simplicity. Infinitesimal mass superpositions are introduced in section 6.2. (Complex number factors are not included here for clarity.)

$$\text{Massive} \qquad N=1 \text{ Spin } \tfrac{1}{2}, |\psi_{\infty,1/2,m}\rangle = \sum_{n=5,6,7} c_n \int_0^{k(cutoff)} \left[\frac{|\psi_{nk,ss}\rangle}{\gamma_{nk}} + \beta_{nk}|\psi_{nk,4m}\rangle) \right] \sqrt{\frac{1+\varepsilon}{2k}} dk \tag{1.1. 1}$$

$$\text{Infinitesimal mass } N=2 \text{ Spin } 1, |\psi_{\infty,1,m}\rangle = \sum_{n=3,4,5} c_n \int_0^{k(cutoff)} \left[\frac{|\psi_{nk,ss}\rangle}{\gamma_{nk}} + \beta_{nk}|\psi_{nk,2m}\rangle \right] \sqrt{\frac{2(1+\varepsilon)}{k}} dk$$

In these infinite superpositions the probability that the wavefunction is spherically symmetric is always $\gamma_{nk}^{-2} = 1-\beta_{nk}^2$ and the probability that it is an m state is β_{nk}^2 where β_{nk} is the magnitude of the velocity of the centre of momentum of the primary interactions that generate each ψ_{nk}. This is similar to the superposition of time and spatially polarized virtual photons in QED. For example spin ½ has probabilities of $\gamma_{nk}^{-2} = 1-\beta_{nk}^{2}$ spherically symmetric ψ_{nk} wavefunctions, and β_{nk}^{2} $\times(\psi_{nk}, m = \pm 2)$ wavefunctions. Each ψ_{k} is normalized to 1 but the infinite superpositions $\psi_{\infty,s,m}$ are not normalized, diverging logarithmically with k; the same logarithmic divergence that applies to virtual photon emission. (*Real wavefunctions have to be normalized to one as they refer to finding a real particle somewhere but this need not apply here.*) Each member of these spin ½ superpositions has probability $dk(1+\varepsilon)/2k$, and if electrically charged emits virtual photons with probability $4\alpha/\pi$. Ignoring the factor of $(1+\varepsilon) \approx 1 + \approx 10^{-44}$, the overall virtual scalar photon emission probability is the usual $(2\alpha/\pi)dk/k$. (Possible implications of the infinitessimal ε are discussed in section 10.1.8) Section 3.1 finds that $m = +2$ virtual wavefunctions have β_{nk}^{2} probability of leaving an $m = -2$ debt in the zero point fields. Integrating over all k produces a total angular momentum for a spin ½ state of $(\hbar/2)(1-\gamma_{Cutoff}^{-2})(1+\varepsilon) = \hbar/2$, (section 3.2.2). If $1/k_{Cutoff}$ is near the Planck length, $(1-\gamma_{Cutoff}^{-2}) = (1+\varepsilon)^{-1}$. A similar integration over all k for the rest energy of the infinite superposition also leads to $\pm m_0 c^2 (1-\gamma_{Cutoff}^{-2})(1+\varepsilon) = \pm m_0 c^2$, (section 3.2.1). The infinitesimal quantity ε vanishes in a zero gravity, zero Planck length universe where $k_{Cutoff}\ \&\ \gamma_{Cutoff} \to \infty$. In this paper each preon borrows virtual rest mass from a Higgs type scalar field. The superposition mass/energy is obtained by summing squared momenta over all k. The equations are based on probabilities of these in a similar manner to those for angular momentum. This suggests the superposition or equivalent particle mass is both energy borrowed from zero point *vector*, and mass borrowed from Higgs type *scalar* fields.

The second half of this paper argues that the limited zero point energies (required to generate virtual gravitons) available at causally connected cosmos wavelengths require it to expand exponentially in an accelerating manner (Figure 5.3. 4). Section 5.3 finds that the warping of spacetime around mass concentrations is consistent with local observers measuring a maximum wavelength virtual graviton probability density $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ where $K_{Gk\min}$ is invariant in all coordinates. The local measurement of $k_{\min} \approx R_{Horizon}^{-1}$ however depends on both the cosmic time *T* and the local metric clockrates $\sqrt{g_{00}}$. (Figure 5.3. 9.) This can only happen if at any radius *r* around a mass *m,* space expands proportionally to *m/r* in accordance with the Schwarzschild solution (Figure 5.2. 2). The first half of this paper is about the *primary interactions between spin zero preons and spin one quanta* that build the fundamental particles. The Standard Model is about the secondary interactions between them. (*The weak force is only between spin ½ particles and thus a secondary interaction. It can not be involved in primary interactions.*) Apart from infinitesimal effects, such as infinitesimal masses, the properties of fundamental particles covered in this paper seem consistent with their Standard Model counterparts. All $N = 1\ \&\ N = 2$ superpositions as in Table 4.3. 1 are conjectured to cutoff at the Planck energy E_P. If this is so both colour and electromagnetic *interaction energies* must cutoff at $E_P/\langle n\rangle \approx 2.03\times10^{18} GeV.$, or $\approx 1/6$ of the Planck energy. (The expectation value $\langle n \rangle$ is ≈ 6.0135 for spin ½ leptons and quarks Eq. (3.5. 16)). The electromagnetic and colour coupling constants at this cutoff are consistent with Standard Model predictions assuming three families of fermions and one Higgs field. (See Figure 4.1. 1 & Figure 4.1. 2). Only after attempting to show that infinite superpositions can be (almost) equivalent to the Standard Model fundamental particles do we try to connect them with General Relativity and the expansion of the cosmos.

2 Building Infinite Virtual Superpositions

2.1 The possibility of Infinite Superpositions

2.1.1 **Early ideas**

After World War II there was still much confusion about QED. In 1947 at the Long Island Conference the results of the Lamb shift experiment were announced [8]. Some of the first early explanations that gave approximately correct answers used simple semi classical thinking to get a better understanding of what seemed to be going on. These early ideas helped to eventually lead to the QED of today, perhaps in a similar manner to the way Bohr's original simple semi classical explanation of quantized atomic energy levels played such a large part in the eventual development of full three dimensional wavefunction solutions of atoms, and quantum mechanics. We start this paper with an example of a semi classical Lamb shift explanation that seems to lead into the possibility of fundamental particles and infinite virtual superpositions being one and the same.

The density of transverse modes of waves at frequency ω is $\omega^2 d\omega / \pi^2 c^3$ and the zero point energy for each of these modes is $\hbar\omega / 2$. The electrostatic and magnetic energy densities in electromagnetic waves are equal, thus for electromagnetic zero point fields:

$$\frac{\overline{\varepsilon_0 E^2}}{2} + \frac{\overline{\varepsilon_0 c^2 B^2}}{2} = \frac{\hbar\omega}{2}\left[\frac{\omega^2 d\omega}{\pi^2 c^3}\right] \quad \text{and} \quad \overline{\varepsilon_0 E^2} = \overline{\varepsilon_0 c^2 B^2} = \frac{\hbar\omega^4}{2\pi^2 c^3}\frac{d\omega}{\omega}.$$

For a fundamental charge e using $\alpha = e^2 / 4\pi\varepsilon_0 \hbar c,$ and provided $\beta << 1,$ this gives an

$$\text{Average force squared of} \quad \overline{F^2} = e^2 \overline{E^2} = \frac{2\alpha}{\pi}\frac{\hbar^2\omega^4}{c^2}\frac{d\omega}{\omega} \tag{2.1. 1}$$

Thinking semi classically, for an electron of rest mass m this can generate simple harmonic motion of amplitude r, where $F^2 = m^2\omega^4 r^2$ (if $\beta << 1$). Solving for r^2 (where r^2 is superimposed on the normal quantum mechanical electron orbit, $\lambda\!\!\!^{-}_C = \hbar / mc$ is the Compton wavelength, and $k = \omega / c$): $$r^2 = \frac{\hbar^2}{m^2 c^2} \frac{2\alpha}{\pi} \frac{d\omega}{\omega} = \left[\lambda\!\!\!^{-}{}_C^2\right] . \left[\frac{2\alpha}{\pi} \frac{dk}{k}\right]$$

Integrating r^2 (directions are random) : $r^2{}_{Total} = \lambda\!\!\!^{-}{}_C^2 \frac{2\alpha}{\pi} \int_{k\min}^{k\max} \frac{dk}{k} = \lambda\!\!\!^{-}{}_C^2 \frac{2\alpha}{\pi} \log(k_{\max} / k_{\min})$.

The minimum and maximum values for k are chosen to fit atomic orbits, and a root mean square value for r can be found. Combining this with the small probability that the electron will be found in the nucleus, this small root mean square deviation shifts the average potential by approximately the Lamb shift. This can also be thought of as simple harmonic motion of amplitude $\approx \lambda\!\!\!^{-}{}_C$, occurring with probability $(2\alpha / \pi) dk / k$. It can also be interpreted as the electron recoiling by $\approx \lambda\!\!\!^{-}{}_C$, (provided $\beta_{Recoil} << 1$) in random directions due to virtual photon emission with a probability of $(2\alpha / \pi) dk / k$.

2.1.2 Dividing probabilities into the product of two component parts

This probability $(2\alpha / \pi) dk / k$ can be thought of as the product of two terms A & B, where A includes the electromagnetic coupling constant α , B includes dk / k, and $AB = (2\alpha / \pi) dk / k$. This suggests that this same behaviour is possible if we have an appropriate superposition of virtual wavefunctions occurring with probability B, which emits virtual photons with probability A (by changing Eigenvalues $|\mathbf{p}_{nk}| = n\hbar k$ by $n = \pm 1$). For example, if a virtual superposition occurs with probability $B = (N \cdot s) dk / k$, and has a virtual photon emission probability for each member of these superpositions of $A = (N \cdot s)^{-1} (2\alpha / \pi)$, then the overall virtual photon emission probability remains as above at $AB = (2\alpha / \pi) dk / k$. This applies equally whether it is virtual gluon/photon/W&Z/graviton etc. emission. Provided A includes the appropriate coupling constant this same logic applies regardless of the type of boson emitted. As is usual to get integral or half integral total angular momentum $2s$ has to be integral and section 6.2 argues that N must also be integral. (This paragraph is simplified to illustrate the principle and will later be modified in section 3.3.)

In section 1.1.4 we said that these wavefunctions are built with squared vector potentials. If superpositions of them are to represent real particles they must be able to exist anywhere. This is possible only if they are generated by invariant fields. The only fields uniform in space-time are the zero point fields, and looking at the electromagnetic field first we can use section 2.1.1 above. Consider a vector $\mathbf{r}$ from some central origin O and a magnetic field vector $\mathbf{B}$ through origin O, then the vector potential at point $\mathbf{r}$ is $\mathbf{A} = (\mathbf{B} \times \mathbf{r}) / 2$ and the vector potential squared is $A^2 = (B^2 r^2 \sin^2 \theta) / 4$ where the angle between vectors $\mathbf{B}$ & $\mathbf{r}$ is θ.

As $\sin^2 \theta$ averages 2/3 over a sphere: $\overline{A^2} = B^2 r^2 / 6$ (2.1. 2)

This requires the source of these fields to be spherically symmetric, where B^2 here is the magnetic field squared at any point due to the invariant cubic intensity of zero point EM also as in section 2.1.1. (This is only true at higher frequencies, and we will find later that at cosmic wavelengths we need a similarly invariant spherically symmetric source from the receding horizon). Putting Eq's. (2.1. **1**) & (2.1. 2) together the vector potential squared is

$$\overline{e^2A^2} = \frac{e^2B^2r^2}{6} = \frac{\alpha}{3\pi}\frac{\hbar^2\omega^4r^2}{c^4}\frac{d\omega}{\omega} = \frac{\alpha}{3\pi}\hbar^2k^4r^2\frac{dk}{k} \tag{2.1. 3}$$

As in section 2.1.2 we can divide this into two parts, noting the inclusion of spin s and integer N in the numerator and denominator:

$$\overline{e^2A^2} = \left[\frac{\alpha}{3\pi sN}\hbar^2k^4r^2\right].\left[\frac{sN\cdot dk}{k}\right] \tag{2.1. 4}$$

But here a vector potential squared term $\left[\frac{\alpha}{3\pi sN}\hbar^2k^4r^2\right]$ *occurs with probability* $\left[\frac{sN\cdot dk}{k}\right]$.

Another way of looking at this is that a wavefunction ψ_k *that is generated by a vector potential squared term* $\left[\frac{\alpha}{3\pi sN}\hbar^2k^4r^2\right]$ *can occur with* $\left[\frac{sN\cdot dk}{k}\right]$ *probability.*

This is similar reasoning to that used in the semi classical Lamb shift explanation of section 2.1.1. In the first bracketed term of Eq. (2.1. 4), α is the electromagnetic coupling constant, but the same logic applies for the eight gluon and gravitational zero point vector fields where we will sum appropriate amplitudes of these and square this total as our effective coupling constant in Eq. (2.1. 4). But first we need to look at groups of spin zero preons that could build these wavefunctions. What mixtures of colours and electrical charges end up with the appropriate final colour and electrical charge for each of the fundamental particles or at least the ones we know of?

2.2 Spin Zero Virtual Preons from a Higgs type Scalar Field

2.2.1 Groups of eight preons that form superpositions

In this paper preons have zero spin and can have no weak charge. The only fields they can interact with (via Primary Interactions that build superpositions as in section 1.1.2) are colour, electromagnetic and gravity. In the simplest world there would be just one type of preon that comes in three colours, always positively charged say, with their three anti colours all negatively charged. We will inded find that this seems to work. Looking at Table 2.2. 1 we see that a minimum of 6 preons is required to get the correct charge ratios of 3:2:1 between electrons, and up and down quarks.

Table 2.2. 1 Groups of 8 virtual preons forming the fundamental particles. The electric charges we measure in the real world are one sixth of the Group electric charges in this table. The Higgs boson is discussed in section 10.1.7. If it is a superposition it would be in the neutral group at the top.

Fundamental Particles	Preon colour	Preon electric charge.	Group colour	Group electric charge.
Spin ½ Neutrino family. Spin 1 Photons, Z_0 & Neutral gluons. Spin 2 Gravitons.	Any colour + its Anticolour Red Antired Green Antigreen Blue Antiblue	1 -1 1 -1 1 -1 1 -1	Colourless	0
Spin ½ Electron family. Spin 1 W^-.	Any colour + its Anticolour Antired Antired Antigreen Antigreen Antiblue Antiblue	1 -1 -1 -1 -1 -1 -1 -1	Colourless	-6
Spin ½ Blue up quark family.	Red Antired Green Antigreen Green Blue Blue Red	1 -1 1 -1 1 1 1 1	Blue	+4
Spin ½ Red down quark family.	Green Antigreen Red Antired Green Antigreen Antiblue Antigreen	1 -1 1 -1 1 -1 -1 -1	Red	-2
Spin 1 Red to green Gluons.	Red Antigreen Red Antired Green Antigreen Blue Antiblue	1 -1 1 -1 1 -1 1 -1	Red plus antigreen	0

To get vector potential squared values that make all our equations work however, we need to couple to all 8 gluon fields requiring a total of 8 preons. Table 2.2. 1 has all the basic properties required to build infinite superpositions for the fundamental particles. We need to remember when looking at this table that from section 1.1.2 the *effective secondary charge is much less than the primary charge and we have no idea yet of just what effective value the primary preon electric charge is.*

Particles only are addressed in the groups of preons in Table 2.2. 1. To get anti particles it would seem that we can just change the signs of each preon in the groups of 8, excepting those that are already their own antiparticle. The first point to notice however is that both the electron and the W^- are predominantly anti preons, yet they are both defined as particles. Have we got something wrong? When we look at relativistic masses in section 3.2.1 we get the usual plus and minus solutions and Feynman showed us how to interpret the negative solutions as antiparticles. If this also applies in anti preons then because they are zero spin, and the weak force discriminates between particles and antiparticles by their helicity, this discrimination can apply only in secondary interactions. The preon anti preon content of the groups in Table 2.2. 1 does not necessarily tell us whether they produce particles or antiparticles. We will discuss this further in section 3.2.1, also as of now there is still no good understanding of the predominance of matter over antimatter in our universe. In Table 2.2. 1 only one example of colour is given for quarks and gluons. Different colours can be obtained by simply changing appropriate preon colours. Various combinations of 8 preons in this table are borrowed from a scalar field for time $\Delta T \approx \hbar / \Delta E$, this process continually repeating in time. Conservation of charge normally allows only opposite sign pairs of electric charges to appear out of the vacuum. Let us imagine that these virtual preons are building an electron for example whose electric charge exists continually unless it meets a positron and is annihilated. This charged electron is thus due to a continuous appearance out of and back into the vacuum of virtual charged preons in a steady state process existing for the life of the superposition, and not conflicting with conservation of charge. If the electron itself does not conflict then neither do the borrowed preons that build it.

2.2.2 Primary coupling constants behave differently and actually are constant

Q.E.D. tells us that the bare (electric) charge of an electron for example increases logarithmically inversely with radius from its centre. Polarizations of the vacuum (of virtual charged pairs) progressively shield the bare charge from a radius of approximately one Compton radius $\lambda\!\!\!^{-}_C$ inwards towards the centre. When an electron (for example) is created in some interaction the full bare charge is exposed for an infinitesimal time. Instantaneously after its creation, shielding due to polarization of the vacuum builds progressively outward from the centre of its creation at the velocity of light.

For radii $\geq \lambda\!\!\!^{-}_C$ we measure the usual fundamental charge e. There are similar but more complicated processes that occur to the colour charge. Camouflage is the dominant one where the colour charge grows with radius as the emitted gluons themselves have color charge. At the instant of their birth the preons are bare and at this time $t = 0$ say, all the zero point vector fields can act on these bare colour and electric charges as there is simply no time for shielding and other effects to build. The primary coupling constants that we use must consequently be the same for all values of k in complete contrast to those for secondary interactions. We don't know what this *primary electromagnetic coupling constant* is so we will just call it α_{EMP}. Also we will find that to get any sense out of our equations the *primary colour coupling* has to be very close to 1. A coupling of 1 is a natural number and simply reflects certainty of coupling. Provided the *secondary colour coupling can be in line with the Standard Model* and there does not seem to be any other good reason to pick a number less than 1, we will make the (apparently arbitrary) *assumption that the bare primary colour coupling is exactly 1*. (In section 4.1.1 we will find that this seems to be consistent with the Standard Model.)

2.2.3 Primary interactions also behave differently

Let us define a frame in which the central origin of the wavefunctions ψ_k of our infinite superposition is at rest: The laboratory or rest frame we will refer to as the LF. The preons that build each ψ_k are born from a Higg's type scalar field with zero momentum in this frame. This has very relevant consequences as their *wavelength is infinite* in this rest frame at time $t = 0$, and after they become wavefunction ψ_k their wavelength is of the order $1/k$ for times $0 < t < \hbar / 2E$. This implies that there could possibly be significant differences in the way amplitudes are handled between primary and secondary interactions.

Let us consider secondary interactions first with an electron and positron for example located approximately distance r apart. For photon wavelengths $<< r$ both the electron and the positron each emit virtual photons with probabilities proportional to α, but for wavelengths $>> r$ their amplitudes cancel. Returning to primary interactions, zero momentum preons must always have an infinite wavelength which is greater than the wavelengths (or $1/k$ values) of the zero point quanta they interact with, for all $k \neq 0$. This implies that we cannot simply add or subtract amplitudes algebraically as the charged preons can be always further apart than the wavelength of the interacting quanta (except when $k = 0$, but we will see there is always a minimum k value, ie $k_{\min} > 0$ in sections 5 & 6). In fact if algebraic addition of amplitudes did apply in primary interactions, infinite superpositions for colourless and electrically neutral neutrinos would be impossible. So how can infinitely far apart preons of differing charge generate wavefunctions of all dimensions down to Planck scale? This can happen only if the amplitudes of all 8 preons are somehow linked over infinite space, all at the same time $t = 0$ contributing to generating the wavefunction ψ_k. This non-local behaviour is not new.

Recent experiments have confirmed that what Einstein struggled to come to terms with is in fact true; he called it "spooky action at a distance". While these experiments are so far limited in the distance over which they demonstrate entanglement, there is now wide acceptance that it can reach across the Universe. In the same manner wavefunctions covering all space can instantly collapse. We want to suggest that this same non-locality applies in primary interactions: our 8 virtual preons all unite instantaneously at time $t = 0$ across infinite space in generating each ψ_k. Also the vector potential squared equations that they generate must always be the same for all the preon combinations in Table 2.2. 1. This can happen only if the amplitudes of all 8 are added regardless of charge sign for primary interactions. This applies to both colour and electric charge.

The opposite is true for the secondary interactions. At time $t = 0$ all 8 preons instantaneously collapse into some sort of virtual composite particle that for times $0 < t < \hbar / 2E$ obeys wavefunction ψ_k. The dimensions of ψ_k are of the same order as the wavelength of the interacting quanta, and the usual algebraic total electric charge and nett colour charge must now apply as in the group charges in Table 2.2. 1. All of this may seem contrary to current thinking which has gradually been built up over several centuries of secondary interaction experiments; however it may not be so out of place when viewed in the context of the counter intuitive results of entanglement experiments. *The key point to bear in mind is that the predictions of this paper must agree or at least be able to fit the Standard Model, or secondary interaction experiments; as we may never be able to look into virtual primary interactions, but only observe their effects.*

Amplitudes to interact are complex numbers which we can draw as a vector. This applies to both colour and electric coupling, where these two vectors can be at the same complex angle or at different angles. The simplest case is if they are in line and we will assume this is true for both colour and electromagnetic primary interactions which are both spin 1. This seems to work and when we later include gravity, a spin 2 interaction, we find that the spin 2 vector only works if it is at right angles to the two in line spin 1 vectors. *Let us start in a zero gravity world* by simply adding the 8 preon colour vectors of amplitude 1 and the eight primary electromagnetic vectors of amplitude $\sqrt{\alpha_{EMP}}$ together, as all this only works if they are all in line.

The total colour plus electromagnetic primary amplitude is $8 + 8\sqrt{\alpha_{EMP}}$ (2.2. 1)

This equation is always true regardless of signs as in section 2.2.3

The colour plus electromagnetic primary coupling constant is $\left(8 + 8\sqrt{\alpha_{EMP}}\right)^2$ (2.2. 2)

Inserting this into Eq. (2.1. 4) we get

$$Q^2A^2 = \left[\frac{\left[8+8\sqrt{\alpha_{EMP}}\right]^2}{3\pi sN} \hbar^2 k^4 r^2 \right] \cdot \left[\frac{sN \cdot dk}{k} \right] \tag{2.2. 3}$$

Again we interpret this just as we did in section 2.1.2 and Eq. (2.1. 4) as a vector potential squared term

$$Q^2A^2 = \frac{\left[8+8\sqrt{\alpha_{EMP}}\right]^2}{3\pi sN} \hbar^2 k^4 r^2 \text{ occurring with probability } = \frac{sN \cdot dk}{k} \tag{2.2. 4}$$

Where Q is a symbol representing the entire 8 colour and 8 electric amplitudes combined, with s the spin and $N=1$ for massive superpositions, but $N=2$ for infinitesimal mass superpositions. (Table 4.3. 1, section 6 and its subsections cover this more fully.)

2.3 Virtual Wavefunctions that form Infinite Superpositions

2.3.1 **Infinite families of similar virtual wavefunctions**

Consider the family of wave functions where ignoring time:

$$\begin{aligned} \psi_{nk} &= U(nrk)Y(\theta\varphi) \\ U(nrk) &= C_{nk} r^l \exp(-n^2k^2r^2/18) \end{aligned} \tag{2.3. 1}$$

$U(nrk)$ is the radial part of ψ_{nk} ,$Y(\theta\phi)$ the angular part, C_{nk} a normalizing constant, and we will find that l is the usual angular momentum quantum number. There is an infinite family of ψ_{nk} , one for each value k where $0<k<\infty$ *in a zero gravity world*.

$$\text{Now put } R(nrk) = rU(nrk) = C_{nk} r^{l+1} \exp(-n^2k^2r^2/18) \tag{2.3. 2}$$

As we are dealing with zero spin preons we use Klein-Gordon equations [9]. The Klein-Gordon equation is based on the relativistic equation $\mathbf{p}^2 = E^2/c^2 - m_0^2c^2$ and in a spherically symmetric squared vector potential the Time Independent Klein Gordon Equation is

$$\hat{P}^2\psi = -\hbar^2\nabla^2\psi + Q^2A^2\psi = \left[\frac{E^2}{c^2} - m_0^2c^2\right]\psi \tag{2.3. 3}$$

Using $\dfrac{\nabla^2\psi}{\psi} = \dfrac{1}{R}\dfrac{\partial^2 R}{\partial r^2} - \dfrac{l(l+1)}{r^2}$ we get the Time Independent

Radial Klein Gordon Equation $\dfrac{\hbar^2}{R}\dfrac{\partial^2 R}{\partial r^2} = \dfrac{l(l+1)\hbar^2}{r^2} + Q^2A^2 - \left[\dfrac{E^2}{c^2} - m_0^2c^2\right]$ (2.3. 4)

For each ψ_{nk} the energy is E_{nk} a function of n & k, and we will label the rest mass as m_{0snk} a function of spin s, n & k, but also a function of the particle rest mass m_0 and this becomes

$$\frac{\hbar^2}{R}\frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + Q^2A^2 - \left[\frac{E_{nk}^2}{c^2} - m_{0snk}^2c^4\right] \qquad (2.3.\ 5)$$

Differentiating $R(nrk) = rU(nrk) = C_{nk}r^{l+1}\exp(\dfrac{-n^2k^2r^2}{18})$ twice with respect to r, multiplying by $\hbar^2$ and dividing by R

$$\frac{\hbar^2}{R}\frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + \frac{n^4\hbar^2k^4r^2}{81} - \frac{(2l+3)n^2\hbar^2k^2}{9} \qquad (2.3.\ 6)$$

Comparing Eq's. (2.3. 5) & (2.3. 6) we see that l is the usual angular momentum quantum number and the vector potential squared required to generate these wavefunctions is

$$Q^2A^2 = \frac{n^4\hbar^2k^4r^2}{81} = \left[\frac{n}{3}\right]^4\hbar^2k^4r^2 \qquad (2.3.\ 7)$$

The momentum squared is $\mathbf{p}_{nk}^2 = \dfrac{E_{nk}^2}{c^2} - m_{0snk}^2c^2 = \dfrac{(2l+3)n^2\hbar^2k^2}{9}$ (2.3. 8)

For $l = 3$ wavefunctions this becomes $\mathbf{p}_{nk}^2 = n^2\hbar^2k^2$ & $|\mathbf{p}_{nk}| = n\hbar k$ (2.3. 9)

2.3.2 Eigenvalues of these virtual wavefunctions and parallel momentum vectors

From Eq.'s (2.3. 8) & (2.3. 9) as $k \to \infty$, the energy squared $E_{nk}^2 \to \mathbf{p}_{nk}^2c^2 = n^2\hbar^2\omega^2$ and thus

If $l = 3$ when $k \to \infty$ energy $E_{nk} \to n\hbar\omega$ (considering only the positive solution). (2.3. 10)

This suggests that n must be integral. If it is integral when $k \to \infty$, we will conjecture that it must be integral for all values of k. This is a virtual or "off shell" process, where energy can depart from $E^2 = m_0^2 c^4 + \mathbf{p}^2 c^2$ for time $\Delta T \approx \hbar / 2\Delta E$. We can also perhaps think of Eq.(2.3. 9) as integral n parallel momentum vector $|\mathbf{p}| = \hbar k$ quanta, transferring total momentum $|\mathbf{p}_{nk}| = n\hbar k$ and energy $E \le n\hbar\omega$ from the zero point fields to generate the virtual wavefunction ψ_{nk}. Thus provided $Q^2 A^2 = (n/3)^4 \hbar^2 k^4 r^2$ as in Eq. (2.3. 7) the operator $\hat{P}^2 = (-\hbar^2 \nabla^2 + Q^2 A^2)$ applied to the virtual wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta\varphi)$ produces $\hat{P}^2 |\psi_{nk}\rangle = (-\hbar^2 \nabla^2 + Q^2 A^2)|\psi_{nk}\rangle = n^2 \hbar^2 k^2 |\psi_{nk}\rangle$,
where n is integral, but k is continuous as for free particles. Thus we conjecture that:

$\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta\varphi)$ are Eigenfunctions with (2.3. 11)

Eigenvalues $\mathbf{p}_{nk}^2 = n^2 \hbar^2 k^2$ with continuous k but integral n.

Also there are no scalar potentials involved, only squared vector potentials, so this is a magnetic or vector type interaction. Particles in classical magnetic fields have a constant magnitude of linear momentum which is consistent with the squared momentum Eigenvalues of Eq. (2.3. 11). *This also implies that each ψ_{nk} is formed from quanta of wave number k only and that secondary interactions with ψ_{nk} emit or absorb $|\hbar k|$ virtual quanta if n changes by* ± 1. The wavefunction ψ_{nk} is virtual and in this sense both the energy E_{nk} and rest mass m_{0snk} in Eq. (2.3. 8) are also virtual quantities borrowed from zero point vector fields and a scalar Higgs type field. We use these virtual quantities to calculate the amplitude that ψ_{nk} is in an m state of angular momentum in section 3.1, and in section 3.2 to calculate the total angular momentum and rest mass. As in section 2.3.2 above, we can think of $|\mathbf{p}_{nk}| = n\hbar k$ as n parallel momentum vectors $|\mathbf{p}| = \hbar k$. As spin 3 (or $l = 3$) needs at least 3 spin 1 quanta to build it, n must be at least 3. When $n = 3$ we can think of this as 3 of the 8 preons each absorbing quanta $|\hbar k|$ at time $t = 0$. We will find that a spin ½ state has a dominant $n = 6$ Eigenfunction where 6 of the 8 preons each absorb quanta $|\hbar k|$. It needs at least two smaller side Eigenfunctions $n = 5$ & $n = 7$ with either 5 or 7 respectively, of the 8 preons each absorbing quanta $|\hbar k|$ respectively at $t = 0$. (Figure 3.1. 4 illustrates the three n modes of a positron superposition.)

From Eq. (2.3. 7) $Q^2 A^2 = \left[\frac{n}{3}\right]^4 \hbar^2 k^4 r^2 = 16\hbar^2 k^4 r^2$ for this dominant $n = 6$ mode.

Thus using Eq. (2.2. 4) $Q^2 A^2 = \frac{\left[8 + 8\sqrt{\alpha_{EMP}}\right]^2}{3\pi sN} \hbar^2 k^4 r^2 = 16\hbar^2 k^4 r^2$ for an $n = 6$ mode.

Now $s = 1/2$ & $N = 1$ for spin ½ fermions and $\frac{2\left[8 + 8\sqrt{\alpha_{EMP}}\right]^2}{3\pi} = 16$ if we have only an $n = 6$ mode.

Thus $8+8\sqrt{\alpha_{EMP}}=\sqrt{24\pi}$ and $\alpha_{EMP}^{-1}\approx 137.1$, but this is true for an $n=6$ Eigenfunction only, and we have a superposition where the amplitudes of the smaller side Eigenfunctions $n=5$ & $n=7$ determine the ratio between the primary to secondary (colour and electromagnetic) coupling amplitudes or the value of $\alpha_3^{-1} @ k_{cutoff}$ (Section 3.3). The Q^2A^2 required to produce this superposition with amplitudes c_n is, using Eq. (2.3. 7)

$$Q^2A^2=\sum_{n=5,6,7} c_n * c_n \frac{n^4\hbar^2k^4r^2}{81} \tag{2.3. 12}$$

Repeating the same procedure as above for three member superpositions using Eq. (2.3. 12) we find the strength of α_{EMP} required increases considerably (see section 4.1 & Table 4.1. 1.) As the secondary electromagnetic coupling $\alpha_{EMS}^{-1} @ k_{cutoff}$ must be constant for all spin ½ leptons and quarks the amplitudes of the smaller side Eigenfunctions $n=5$ & $n=7$ that determine this must also be constant for all the fermions, implying that Eq. (2.3. 12) must be the same for all fermions. The same arguments apply to the other groups of fundamental particles but we return to this in sections 3.3 where we see that the same also applies with graviton emission.

3 Properties of Infinite Superpositions

3.1 What is the Amplitude that ψ_{nk} is in an *m* state?

3.1.1 Four vector transformations

The rules of quantum mechanics tell us that if we carry out any measurement on a real spherically symmetric $l=3$ wavefunction it will immediately fall into one of the seven possible states $l=3, m=0,\pm1,\pm2,\pm3$ [10]. But ψ_{nk} is a virtual $l=3$ wave function so we cannot measure its angular momentum. During its brief existence it must always remain in some virtual superposition of the above seven possible states and we can describe only the amplitudes of these. So is there any way to calculate these amplitudes as they must relate to the amplitudes of the angular momentum states of the spin 1 quanta it absorbs from the zero point vector fields? First consider the 4 vector wavefunction of a spin 1 particle and start with a time polarized state which has equal probability of polarization directions. It is thus spherically symmetric, which we will label as ss. Using 4 vector (t, *x*, *y*, *z*) notation:

In frame A, a time polarized or ss spin 1 state is (1,0,0,0).

Let frame B move along the z axis at velocity $\beta = v/c$ in the z direction.

In frame B the polarization state transforms to $(\gamma, 0, 0, \gamma\beta)$.

But this is γ^2 time polarized $|ss\rangle$ states minus $\gamma^2\beta^2 \times z$ polarized or $|m=0\rangle$ states

In frame B the probabilities are $\gamma^2 |ss\rangle - \gamma^2\beta^2 |m=0\rangle$ states.

Now $\gamma^2 - \gamma^2\beta^2 = \gamma^2(1-\beta^2) = 1$ is an invariant probability in all frames and in removing $\gamma^2\beta^2 \times m = 0$ states from $\gamma^2 \times ss$ states, the new ratio of spherical symmetry is $(\gamma^2 - \gamma^2\beta^2)/\gamma^2 = 1-\beta^2$. Thus a spherically symmetric state is transformed from probability 1 in frame A, to $1-\beta^2$ in frame B. Also removing $m=0$ states from spherically symmetric states leaves a surplus of $m=\pm 1$ states, as spherically symmetric states are equal superpositions of $|m=-1\rangle$, $|m=0\rangle$, & $|m=+1\rangle$ states.

Thus in Frame B the probabilities are $(1-\beta^2)|ss\rangle + \beta^2 |m=\pm 1\rangle$ states. (3.1. 1)

We can describe this as a virtual superposition of $\frac{1}{\gamma}|ss\rangle + \beta |m=\pm 1\rangle$ states. (3.1. 2)

As $\beta^2 \to 1$ we have transverse polarized states, the same as real photons. Now transverse polarized spin 1 states can be either left $(m=-1)$, or right $(m=+1)$ circular polarization, or equal superpositions of $(1/\sqrt{2})|L\rangle + (1/\sqrt{2})|R\rangle$ as in $x \,\&\, y$ polarization. If we think of individual spin zero preons absorbing these spin 1 quanta at $t=0$ they must also have this same β^2 probability of transversely polarized spin 1 states. If they then merge into some composite $l=3$ particle (as in Figure 3.1. 4) for time $0 < t < \hbar/2E$, the probability of it being in some particular state $(l=3, m=0)$, $(l=3, m=\pm 1)$, $(l=3, m=\pm 2)$ or $(l=3, m=\pm 3)$, must be the same β^2. If we look at Eq.'s (1.1. 1) we can see what is behind them. We initially write the amplitudes in these three equations in terms of β_{nk} & γ_{nk} as this is the most convenient way to express them. Velocity operators are momentum operators over relativistic masses. Our Eigenvalues are $\mathbf{p}_{nk}^2 = n^2\hbar^2k^2$ for each $n \,\&\, k$, and this allows the velocity operators to give constant β_{nk}^2. Later in Eq's. (3.1. 11) & (3.1. 12) we write β_{nk} & γ_{nk} in momentum terms. Even though the mass in these operators is virtual, we can still use it to calculate $|\beta_{nk}|$. For each k and integral n there will be a constant $|\beta_{nk}|$ and $\gamma_{nk} = (1-\beta_{nk}^2)^{-1/2}$. As we will see, β_{nk} can be thought of as the magnitude of the velocity of an imaginary centre of momentum frame in which these interactions take place. We will also draw our Feynman diagrams of these interactions in terms of β_{nk} & γ_{nk} for convenience, even though this is unconventional. To proceed from here we define two frames as follows:

1) The Laboratory Frame (LF) or Fixed Frame as in section 2.2.3

The infinite superposition has rest mass m_0 and zero nett momentum in this frame. Each ψ_{nk} is centered here with magnitude of momentum $|\mathbf{p}_{nk}| = n\hbar k$. Even though we have no idea of the direction of this momentum vector we will define it as the z direction. The eight preons are born in this frame with zero momentum, and can thus be considered here as being at rest or with zero velocity and *infinite wavelength at their birth*. The Feynman diagram of the interaction in this frame that builds ψ_{nk} is illustrated in Figure 3.1. 3.

2) The Center of Momentum Frame (CMF)

This (imaginary) frame is the center of momentum of the interaction that builds ψ_{nk}. The CMF moves at velocity β_{nk} relative to the laboratory frame in the z direction or parallel to the unknown momentum vector direction $\mathbf{p}_{nk}$. In this CMF the momenta and velocities of the preons at birth and after the interaction are equal and opposite. This is illustrated in Figure 3.1. 2 again in terms of $m_0, \beta_{nk}, \& \gamma_{nk}$. In the LF the velocity of the preons at birth is zero, in the CMF this is $-\beta_{nk}$ and after the interaction $+\beta_{nk}$, where both $-\beta_{nk}$ and $+\beta_{nk}$ are in the unknown z direction. In the LF the particle velocity $\beta_{nk}(particle) = \beta_{nkp}$ is the simple relativistic addition of the two equal velocities β_{nk} as in Figure 3.1. 1.

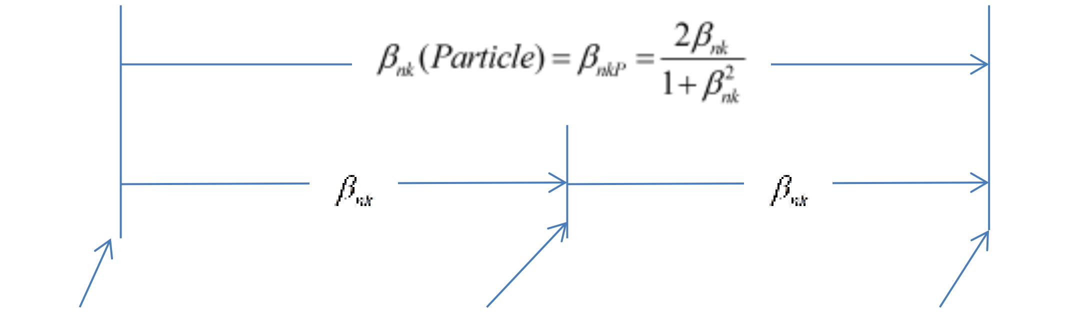


Figure 3.1. 1

3.1.2 Feynman diagrams of primary interactions

Let us start with

$$\beta_{nk}(Particle) = \beta_{nkP} = \frac{2\beta_{nk}}{1+\beta_{nk}^2} \text{ and } \gamma_{nkP} = (1-\beta_{nkp}^2)^{-1/2} = \gamma_{nk}^2(1+\beta_{nk}^2) \qquad (3.1.\ 3)$$

If the particle rest mass is m_0 let each preon have a virtual rest mass $m_0 / (8\gamma_{nk}\sqrt{2s})$.

The eight preons are effectively a virtual particle of rest mass $m_{0snk} = \dfrac{m_0}{\gamma_{nk}\sqrt{2s}}$ (3.1. 4)

The particle momentum in the LF is zero at birth. After the interaction using these equations

$$\left|\mathbf{p}_{nk}\right| = n\hbar k \;\; = m_{0snk}\beta_{nkP}\gamma_{nkP}c = \left[\frac{m_0}{\gamma_{nk}\sqrt{2s}}\right]\left[\frac{2\beta_{nk}}{1+\beta_{nk}^2}\right]\left[\gamma_{nk}^2(1+\beta_{nk}^2)\right]c$$

The particle momentum after the interaction in the LF $\left|\mathbf{p}_{nk}\right| = n\hbar k = \dfrac{2m_0\beta_{nk}\gamma_{nk}c}{\sqrt{2s}}$ (3.1. 5)

Using Eq. (3.1. 4), in the LF the particle energy at birth is

$$m_{0snk}c^2 = \frac{m_0c^2}{\gamma_{nk}\sqrt{2s}} \tag{3.1. 6}$$

In the LF the particle energy after the interaction is using Eq's. (3.1. 3)

$$m_{0snk}\gamma_{pnk}c^2 = \frac{m_0}{\gamma_{nk}\sqrt{2s}}\gamma_{nk}^2(1+\beta_{nk}^2)c^2 = \frac{m_0\gamma_{nk}}{\sqrt{2s}}(1+\beta_{nk}^2)c^2 \tag{3.1. 7}$$

In the CMF the momentum at birth is using Eq. (3.1. 4)

$$-m_{0snk}\gamma_{nk}\beta_{nk} = \frac{-m_0\beta_{nk}}{\sqrt{2s}} \tag{3.1. 8}$$

In the CMF the momentum after the interaction is equal but in the opposite direction

$$= \frac{+m_0\beta_{nk}}{\sqrt{2s}} \tag{3.1. 9}$$

In the CMF the energy at birth, and after the interaction is

$$m_{0snk}\gamma_{nk}c^2 = \frac{m_0c^2}{\sqrt{2s}} \tag{3.1. 10}$$

These values are all summarized in Figure 3.1. 2 and Figure 3.1. 3 but with $c = 1$.

From Eq. (3.1. 5) $\left|\mathbf{p}_{nk}\right| = n\hbar k = \dfrac{2m_0\beta_{nk}\gamma_{nk}c}{\sqrt{2s}}$ and $\beta_{nk}\gamma_{nk} = \dfrac{n\hbar k\sqrt{2s}}{2m_0c} = \dfrac{\lambda\!\!\!^{-}_C nk\sqrt{2s}}{2}$

(where $\lambda\!\!\!^{-}_C$ is the Compton wavelength). We can now express β_{nk} & γ_{nk} in momentum terms:

$$\text{Let } K_{nk} = \beta_{nk}\gamma_{nk} = \frac{n\hbar k\sqrt{2s}}{2m_0c} = \frac{\lambda\!\!\!^{-}_C nk\sqrt{2s}}{2} \tag{3.1. 11}$$

$$\text{In terms of } K_{nk}\text{:} \quad \beta_{nk}^2 = \frac{K_{nk}^2}{1+K_{nk}^2} \text{ and } \gamma_{nk}^2 = 1+K_{nk}^2 \tag{3.1. 12}$$

Each infinite superposition has fixed $\lambda\!\!\!^{-}_C$. Each wavefunction ψ_{nk} of this infinite superposition has fixed n & s, thus $K_{nk} \propto k$.

$$\text{For example we can put}\quad \frac{dK_{nk}}{K_{nk}} = \frac{dk}{k} \tag{3.1. 13}$$

These simple expressions and what follows are not possible if $m_{0snk} \neq m_0 / \gamma_{nk}\sqrt{2s}$, and *when we include gravity* we find $m_{0snk} = m_0 / (\gamma_{nk}\sqrt{2s})$ is essential (section 4.2).

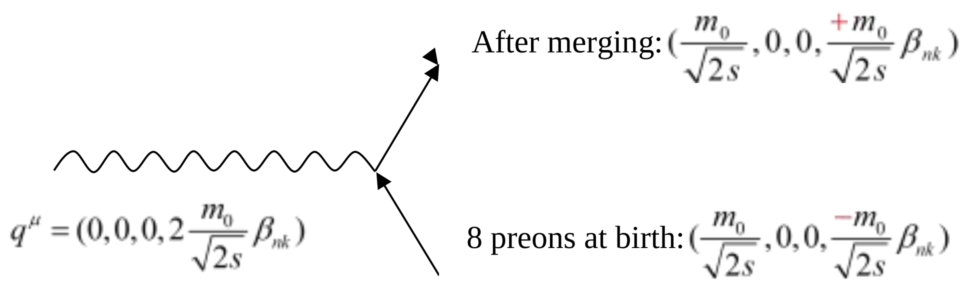


Figure 3.1. 2 Feynman diagram in an imaginary centre of momentum frame.

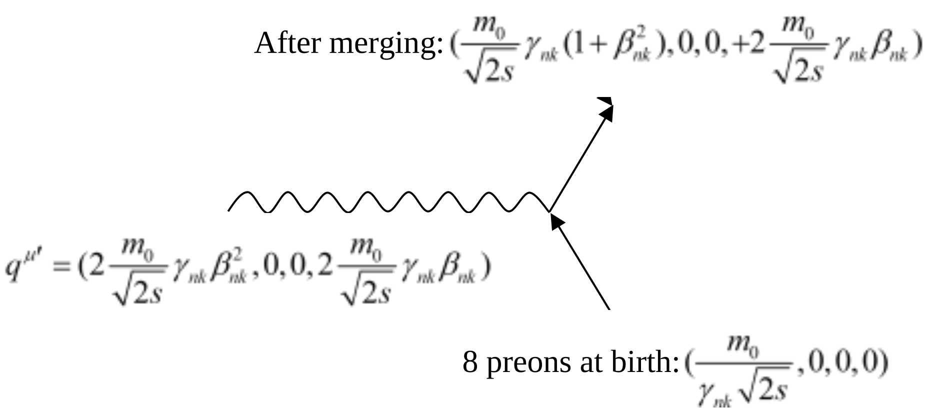


Figure 3.1. 3 Feynman diagram in the laboratory frame.

The interaction in the Feynman diagrams above is with spin 1 quanta. The Feynman transition amplitude of this interaction tells us that the polarization states of these exchanged quanta is determined by the sum of the components of the initial, plus final 4 momentum $(p_i + p_f)^{\mu}$. Ignoring all other common factors this tells us that the space polarized component is the sum of the momentum terms $(\mathbf{p}_i + \mathbf{p}_f)$ and the time polarized component is the sum of the energy terms $(p_i + p_f)^0$. We have defined our momentum as in an unknown z direction:

The ratio of z polarization to time polarization amplitudes is $\frac{(p_i + p_f)^z}{(p_i + p_f)^0}$ (3.1. 14)

In the CMF $(p_i + p_f)^z = 0$, thus an interaction in the CMF exchanges only time polarized, or spherically symmetric $l = 1$ states. In the LF the ratio of z (or $m = 0$) polarization, to time polarization in the LF is β_{nk}^2,

$$\text{where} \quad \frac{(p_i + p_f)^z}{(p_i + p_f)^0} = \frac{2m_0\gamma_{nk}\beta_{nk}}{2m_0\gamma_{nk}} = \beta_{nk} \tag{3.1. 15}$$

From section 3.1.1 these are probabilities of $\gamma_{nk}^2 \left|ss\right\rangle - \gamma_{nk}^2\beta_{nk}^2 \left|m = 0\right\rangle$ states, or as $l = 1$ here $(1-\beta_{nk}^2)\left|ss\right\rangle + \beta_{nk}^2 \left|m = \pm 1\right\rangle$ states.

In the LF this is a virtual superposition of $(\frac{1}{\gamma_{nk}}\left|ss\right\rangle + \beta_{nk}\left|m = \pm 1\right\rangle)$ states. (3.1. 16)

From section 3.1.1 as these quanta from the scalar and vector zero point fields build each ψ_{nk} this implies that:

In the LF ψ_{nk} has virtual superposition amplitudes $\frac{1}{\gamma_{nk}}\left|ss\right\rangle + \beta_{nk}\left|m\right\rangle$ states. (3.1. 17)

From section 3.1.1 appropriate $l = 1, m = \pm 1$ superpositions can build any $l = 3,\ m$ state. Figure 3.1. 4 is an example of such a ψ_{nk} for $n = 5, 6, \&7$ $\left|l = 3, m = +2\right\rangle$ states.

3.1.3 Different ways to express superpositions

We have expressed all superpositions here in terms of spherically symmetric and m states for convenience and simplicity. We could have expressed them in the form:

$$\frac{1}{\gamma_{nk}\sqrt{7}}\left[\left|m=-3\right\rangle + \left|m=-2\right\rangle + \left|m=-1\right\rangle + \left|m=0\right\rangle + \left|m=+1\right\rangle + \left|m=+2\right\rangle + \left|m=+3\right\rangle\right] + \beta_{nk}\left|m=+2\right\rangle$$

Which is equivalent to (as above ignoring complex number amplitude factors for clarity)

$$\psi_{nk} = \frac{1}{\gamma_{nk}}\left|ss\right\rangle + \beta_{nk}\left|m = +2\right\rangle \text{ where we have put m = +2 in this example.}$$

Because all these wavefunctions are virtual they cannot be measured in the normal way that collapses them into any of these Eigenstates, it is more convenient to use the method adopted here which is similar to QED virtual photon superpositions.

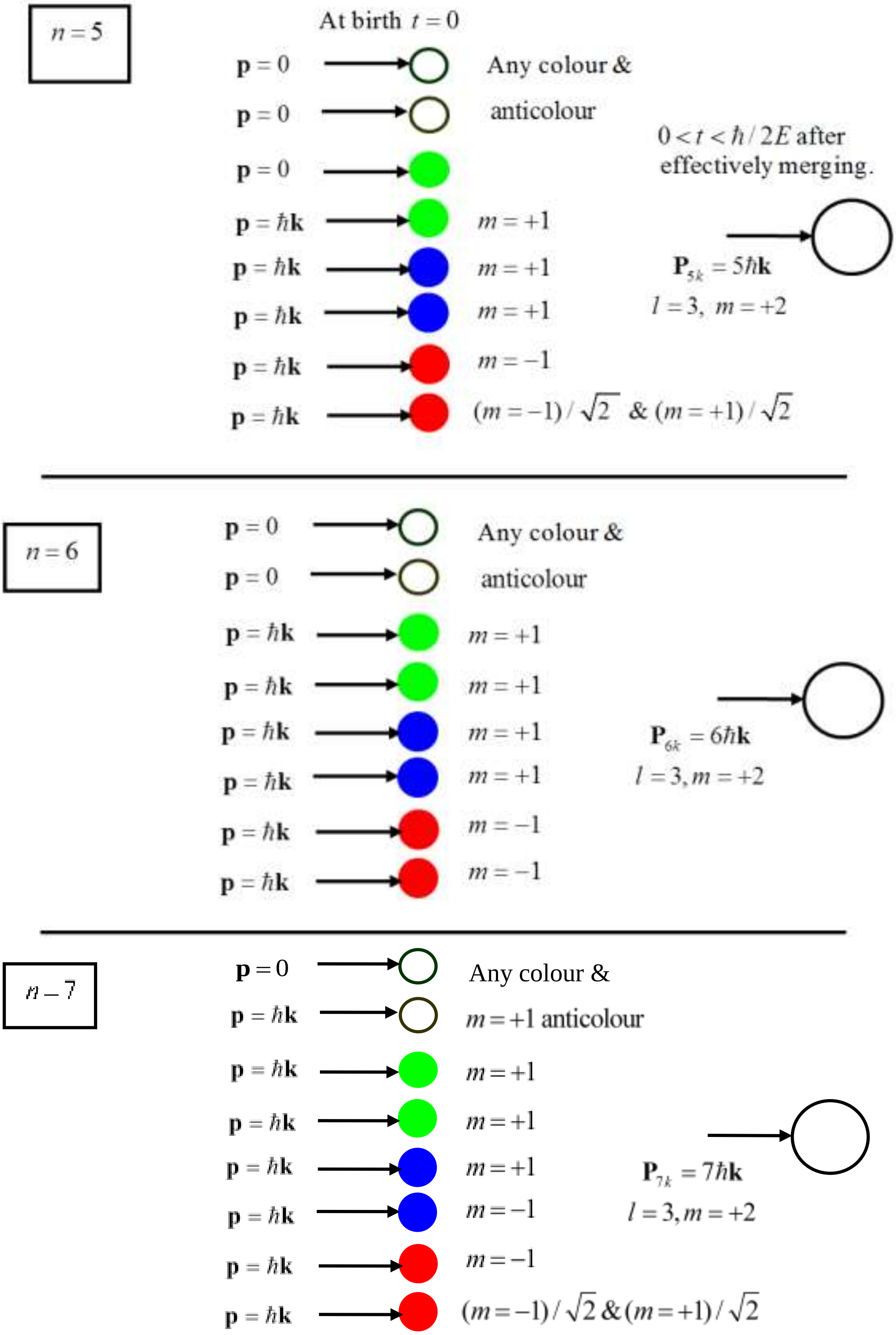


Figure 3.1. 4 Eight preons forming $m = +2$ states as part of a positron superposition.

There is no significance in which preons absorb quanta in the above.

3.2 Mass and Total Angular Momentum of Infinite Superpositions

3.2.1 **Total mass of massive infinite superpositions**

We will consider first the total mass of an infinite superposition, and to help illustrate, consider only one integral n Eigenfunction ψ_{nk} at a time; temporarily assuming that the amplitude c_n of each ψ_{nk} has magnitude $|c_n| = 1$. Each time ψ_{nk} is born it borrows virtual mass from a scalar Higgs field and virtual energy from vector zero point fields. Each time ψ_{nk} is born the virtual mass that it borrows is exactly cancelled by an equal debt in the Higgs scalar field so this should sum to zero for all k. But what about the momenta borrowed from the zero point fields, do these momenta also leave momentum debts in the vacuum? From section 2.3.2 as $k \to \infty$, $E_{nk}^2 \to \mathbf{p}_{nk}^2 c^2 = n^2\hbar^2\omega^2$ or $E_{nk} \to n\hbar\omega$ and n quanta of energy $\hbar\omega$ and momentum $|\hbar k|$ are absorbed. We know that in some unknown direction $\mathbf{p}_{nk} = n\hbar\mathbf{k}$, which implies these n absorbed quanta must leave a cancelling debt in the opposite direction of $\mathbf{p}_{nk}(debt) = -n\hbar\mathbf{k}$ in the vacuum. But this is true only as $k \to \infty$ & $\beta_{nk}^2 \to 1$ and the virtual quanta energy transferred $E_X \to \hbar\omega$. So what happens when $\beta_{nk}^2 << 1$? Our wavefunctions ψ_{nk} are generated from a vector potential squared term A^2 derived in section 2.1.2 which in turn came from a B^2 type term as in section 2.1.1. As discussed in section 2.3.2 the Eigenvalues $\mathbf{p}_{nk}^2 = n^2\hbar^2k^2$ confirm the constant momentum squared feature of magnetic type interactions. Also in section 2.1.1 the scalar virtual photon emission probability is directly related to the force squared term $F^2 = \varepsilon^2E^2$. Magnetic type coupling probabilities are related to a magnetic type force squared term $F^2 = \beta^2\varepsilon^2B^2/c^2 = \beta^2\varepsilon^2E^2$, where from section 3.1.2 and Eq's. (3.1. 14) & (3.1. 15) the ratio of this scalar to magnetic coupling is β_{nk}^2. Thus when $k < \infty$ and the exchanged energy $E_X \neq \hbar\omega$, $\beta_{nk}^2 n$ quanta $|\hbar k|$ are absorbed from the vacuum and:

$$\text{We can expect a momentum debt of } \mathbf{p}_{nk}(debt) = -\beta_{nk}^2 n\hbar\mathbf{k} \tag{3.2. 1}$$

We could sum $\sum \mathbf{p}_{nk}^2$ & $\sum \mathbf{p}_{nk}^2(debt)$ but both vectors $\mathbf{p}_{nk}$ and $\mathbf{p}_{nk}(debt)$ are antiparallel in the same unknown direction. We can pair them together giving a nett momentum per pair of:

$$\mathbf{p}_{nk}(nett) = \mathbf{p}_{nk} + \mathbf{p}_{nk}(debt) = (1-\beta_{nk}^2)n\hbar\mathbf{k} = \frac{n\hbar\mathbf{k}}{\gamma_{nk}^2} = \frac{\mathbf{p}_{nk}}{\gamma_{nk}^2} \text{ at wavenumber } k. \tag{3.2. 2}$$

We have said above that the mass of each virtual particle is cancelled by an equal and opposite debt in the Higgs scalar field so we can now use the relativistic energy expression

$$E_n^2 = \sum_{k=0}^{k=\infty} \mathbf{p}_{nk}(nett)^2 c^2 \text{ times the probability of each pair at each wavenumber } k.$$

We will initially look at only $N = 1$ massive infinite superpositions in Eq. (2.2. 4).

Thus using probability $sN \cdot dk / k = s \cdot dk / k$, also Eq's. (3.1. 11), (3.1. 12),(3.1. 13),&(3.2. 2).

$$E_n^2 = c^2 \int_{k=0}^{k=\infty} \mathbf{p}_{nk}(nett)^2 \frac{s \cdot dk}{k} = c^2 \int_0^\infty \frac{n^2\hbar^2 k^2}{\gamma_{nk}^4} \frac{s \cdot dk}{k} = 4m_0^2 c^4 \int_0^\infty \frac{K_{nk}^2}{(1+K_{nk}^2)^2} \frac{dK_{nk}}{2K_{nk}}$$

$$E_n^2 = m_0^2 c^4 \left[\frac{-1}{1+K_{nk}^2} \right]_0^\infty = m_0^2 c^4 \text{ or } E_n = \pm m_0 c^2 \tag{3.2. 3}$$

This energy is due to summing momenta squared and it must be real, with a mass $\pm m_0$ for infinite superpositions of Eigenfunctions ψ_{nk}. These superpositions can form all the non infinitesimal mass fundamental particles. The equations do not work if the mass m_0 is zero. (We will look at infinitesimal masses in section 6.2.) Negative mass solutions in Eq. (3.2. 3) must be handled in the usual Feynman manner, and treated as antiparticles with positive energy going backwards in time. If they are spin ½ this also determines how they interact with the weak force.

3.2.2 Angular momentum of massive infinite superpositions

We will use the same procedure for the total angular momentum of $N=1$ type infinite superpositions with non infinitesimal mass in Eq. (2.2. 4).
Wavefunctions $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta, \varphi)$ have angular momentum squared Eigenvalues $\mathbf{L}^2 = 12\hbar^2$ and the various m states have angular momentum Eigenvalues $\mathbf{L}_z = m\hbar$. We will treat both angular momentum and angular momentum debts as real just as we did for linear momentum. Even though m state wavefunctions are part of superpositions they still have probabilities just as the linear momenta squared above and it seemed to work. Using exactly the same arguments as in section 3.2.1 , *if ψ_{nk} is in a state of angular momentum $\mathbf{L}_{zk} = m\hbar$*, then it must leave an angular momentum debt in the vacuum of $\mathbf{L}_{zk}(debt) = -\beta_{nk}^2 m\hbar$ (or as in section 3.2.1) $\mathbf{L}_{zk}(nett) = \mathbf{L}_{zk} - \mathbf{L}_{zk}(debt)$.

$$\mathbf{L}_{zk}(nett) = (1-\beta_{nk}^2) m\hbar = (1-\beta_{nk}^2)\mathbf{L}_{zk} = \frac{\mathbf{L}_{zk}}{\gamma_{nk}^2} \quad (\text{if } \mathbf{L}_{zk} \text{ is in state } m\hbar) \tag{3.2. 4}$$

But from Eq. (3.1. 17) the probability that $\mathbf{L}_{zk}$ is in an m state is also β_{nk}^2 so that

$$\text{Including this extra } \beta_{nk}^2 \text{probability term: } \mathbf{L}_{zk}(nett) = m\hbar \frac{\beta_{nk}^2}{\gamma_{nk}^2} \text{ at wavenumber } k. \tag{3.2. 5}$$

For an $N=1$ type infinite superposition $\mathbf{L}_z(Total) = \int_{k=0}^{k=\infty} \mathbf{L}_{zk}(nett) \frac{s \cdot dk}{k}. = sm\hbar \int_0^\infty \frac{\beta_{nk}^2}{\gamma_{nk}^2} \frac{dk}{2k}$

Using Eq's. (3.1. 11) to (3.1. 13) $\mathbf{L}_z(Total) = sm\hbar \int_0^\infty \frac{K_{nk}^2}{(1+K_{nk}^2)^2} \frac{dK_{nk}}{K_{nk}} = \frac{sm\hbar}{2} \left[\frac{-1}{1+K_{nk}^2} \right]_0^\infty$

$$\mathbf{L}_z(Total) = m'\hbar = \frac{sm\hbar}{2} \qquad \text{or} \qquad m' = \frac{s}{2}m \tag{3.2. 6}$$

Where m' *is the angular momentum state of the infinite superposition and* m *the state of* ψ_{nk}. Thus for spin ½ particles with $s=$ ½ in Eq.(3.2. 6) $m' = m/4$ but m' can be only ± ½, implying the m state of ψ_{nk} that generates spin ½ must be $m = \pm 2$. An $N=1$ massive spin 1 particle has $s=1$ with $m' = m/2$. ($N=2$ is covered in section 6.2.) This is summarized in the following three member infinite superpositions *ignoring complex number factors*.

$$\text{Massive } (N=1) \text{ Spin } \tfrac{1}{2},\ \left|\psi_{\infty,1/2,\pm 1/2}\right\rangle = \sum_{n=5,6,7} c_n \int_{k=0}^{k=\infty} \left[\frac{\left|\psi_{nk,ss}\right\rangle}{\gamma_{nk}} + \beta_{nk}\left|\psi_{nk,\pm 2}\right\rangle\right]\sqrt{\frac{1}{2k}}dk \tag{3.2. 7}$$

$$\text{Massive } (N=1) \text{ Spin 1},\ \left|\psi_{\infty,1,m}\right\rangle = \sum_{n=4,5,6} c_n \int_{k=0}^{k=\infty} \left[\frac{\left|\psi_{nk,ss}\right\rangle}{\gamma_{nk}} + \beta_{nk}\left|\psi_{nk,2m}\right\rangle\right]\sqrt{\frac{1}{k}}dk \tag{3.2. 8}$$

The spin vectors of each ψ_{nk} with $|\mathbf{L}| = 2\sqrt{3}\hbar$, and their spin vector debts in the zero point vector fields, have to be aligned such that the sum in each case is the correct value: $|\mathbf{L}| = \sqrt{3}\hbar/2$, $|\mathbf{L}| = \sqrt{2}\hbar$ or $|\mathbf{L}| = \sqrt{6}\hbar$ for spins ½ , 1 & 2 respectively. Gravity (the ε term) is included in Eq. (1.1. 1) in our summary also spin 1 in Eq. (3.2. 8) is for $N=1$.

Spherically symmetric massive $N=1$ spin 1 states are a superposition of three states $\frac{1}{\sqrt{3}}\left[\left|m'=-1\right\rangle + \left|m'=0\right\rangle + \left|m'=+1\right\rangle\right]$, and using Eq. (3.2. 8) can be formed as follows

$$\text{Massive spin 1} \begin{bmatrix} \frac{1}{\sqrt{3}}\left|\psi_{\infty,1,m'=-1}\right\rangle = \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=\infty} \left[\frac{\left|\psi_{nk,ss}\right\rangle}{\gamma_{nk}} + \beta_{nk}\left|\psi_{nk,m=-2}\right\rangle\right]\sqrt{\frac{1}{k}}dk \\ +\frac{1}{\sqrt{3}}\left|\psi_{\infty,1,m'=0}\right\rangle = \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=\infty} \left[\frac{\left|\psi_{nk,ss}\right\rangle}{\gamma_{nk}} + \beta_{nk}\left|\psi_{nk,m=0}\right\rangle\right]\sqrt{\frac{1}{k}}dk \\ +\frac{1}{\sqrt{3}}\left|\psi_{\infty,1,m'=+1}\right\rangle = \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=\infty} \left[\frac{\left|\psi_{nk,ss}\right\rangle}{\gamma_{nk}} + \beta_{nk}\left|\psi_{nk,m=+2}\right\rangle\right]\sqrt{\frac{1}{k}}dk \end{bmatrix} \tag{3.2. 9}$$

3.2.3 Mass and angular momentum of multiple integer *n* superpositions

In sections 3.2.1 & 3.2.2 for simplicity we looked at single integer *n* superpositions ψ_{nk}. For superpositions $\psi_k = \sum_n c_n \psi_{nk}$, we replace K_{nk}^2 with $\left\langle K_k\right\rangle^2$. Equation (2.3. 9) appears to suggest $|\mathbf{p}_k|^2 = \sum_n c_n{}^* c_n n^2\hbar^2k^2 = \left\langle n^2\right\rangle \hbar^2 k^2$ and $\left\langle|\mathbf{p}_k|\right\rangle = \hbar k\sqrt{\left\langle n^2\right\rangle}$. In section (3.5.1) we discuss why $\left\langle|\mathbf{p}_k|\right\rangle \neq \hbar k\sqrt{\left\langle n^2\right\rangle}$ but $\left\langle|\mathbf{p}_k|\right\rangle = \hbar k\sum_n c_n{}^* c_n \cdot n = \hbar k\left\langle n\right\rangle$. Thus using Eq. (3.1. 11)

$$\left\langle K_k\right\rangle = \frac{\lambda_C k\sqrt{2s}}{2}\left\langle n\right\rangle \ \ \& \ \left\langle K_k\right\rangle^2 = \frac{\lambda_C^2 k^2 s}{2}\left\langle n\right\rangle^2 \ \text{ but } \left\langle K_k\right\rangle^2 \neq \frac{\lambda_C^2 k^2 s}{2}\left\langle n^2\right\rangle \tag{3.2. 10}$$

Replacing K_{nk}^2 with $\langle K_k \rangle^2 = \bar{\lambda}_C^2 k^2 s \langle n \rangle^2 / 2$ in the key equations (3.2. 3) & (3.2. 6) does not change the final results. The laws of quantum mechanics tell us the total angular momentum is precisely integral $\hbar$ or half integral $\hbar / 2$. Looking at the above integrals used to derive total angular momentum we see that *N* must be 1 (we discuss *N*=2 in section 6.2) also s must be exactly ½ or 1 for spin ½ & spin 1 massive particles respectively, in Eq. (2.2. 4) our probability formula. Also these integrals are infinite sums of positive and negative integral $\hbar$ that are virtual and cannot be observed. If an infinite superposition for an electron is in a spin up state and flips to spin down in a magnetic field, a real $m = \pm 1$ photon is emitted carrying away the change in angular momentum. This is the only real effect observed from this infinity of $(l = 3, m = +2)$ virtual wavefunctions all flipping to $(l = 3, m = -2)$ states, plus an infinite flipping of the virtual zero point vector debts. Also Eq's. (3.2. 3) & (3.2. 6) are true *only if our high energy cutoff is at infinity and the low frequency cutoff is at zero*. We look at high energy Planck scale cutoffs in section 4.2 and in section 6.1 low energy cutoffs near the radius of the causally connected horizon.

3.3 Ratios between Primary and Secondary Coupling

3.3.1 **Initial simplifying assumptions**

This section is based on a special case thought experiment that tries to illustrate, hopefully in a simple way, how superpositions interact with one another; in the same way as virtual photons interact with electrons for example. It is unfortunately long and not very rigorous, but it illustrates how, in all interactions between fundamental particles represented as infinite superpositions, the actual interaction is between *only the same* k *single wavenumber* superpositions of each particle. We will later conjecture that the interacting virtual particle is a single wavenumber k superposition only and not a full infinite superposition. Only real particles whose properties we can measure are full infinite superpositions. The full properties do not exist until measurement, just as in so many other examples in quantum mechanics. This will be clearer as we proceed. *It is also important to remember here that because primary coupling constants are to bare charges (section 2.2.2), and thus fixed for all k, while secondary coupling constants run with k, that the coupling ratios can be defined only at the cutoff value of k applying to the bare charge* (sections 4.1.1 & 4.2.2). From Table 2.2. 1 there are 6 fundamental primary charges for electrons and positrons. But electrons and positrons are defined as fundamental charges. In other words what we define as a fundamental electric charge is in reality 6 primary charges. Of course we can never in reality measure 6 as their effect is reduced by the ratio between primary and secondary coupling. Because electromagnetic and colour coupling are both via spin one bosons their *coupling ratios* are fundamentally the same but because of the above they are related simply as $6^2 = 36:1$.

$$\frac{1}{\chi_{Colour}} = \frac{36}{\chi_{EM}} \tag{3.3. 1}$$

We define the colour and electromagnetic ratios as follows (leaving gravity till section 6.2.6)

$$\frac{1}{\chi_{Colour}} = \frac{\alpha_{Colour\,(\mathrm{Secondary})}}{\alpha_{Colour\,(\mathrm{Primary})}} = \frac{\alpha_{3S}}{\alpha_{3P}} \qquad \text{and} \qquad \frac{1}{\chi_{EM}} = \frac{\alpha_{EM\,(\mathrm{Secondary})}}{\alpha_{EM\,(\mathrm{Primary})}} = \frac{\alpha_{EMS}}{\alpha_{EMP}} \tag{3.3. 2}$$

The secondary coupling constants α_{EMS} & α_{3S} *are the bare charge values, both at the fermion interaction cutoff near the Planck length* Eq. (4.2. 11). Also we assumed in section 2.2.2 that $\alpha_{3P} = 1$; thus from Eq.(3.3. 2)

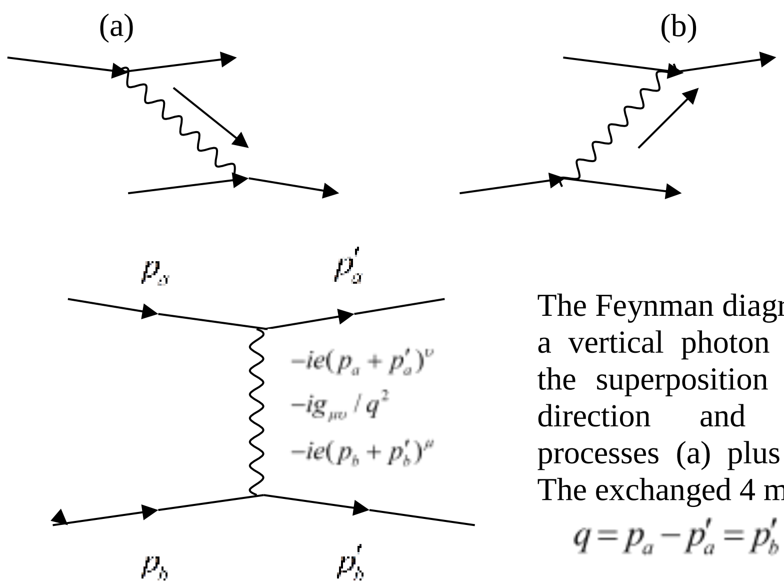
$$\chi_C = \alpha_{3S}^{-1} = \alpha_3^{-1} \,@\, k_{cutoff} \approx 2.029 \times 10^{18}\, GeV \tag{3.3. 3}$$

In other words provided $\alpha_{3P} = 1$, the ratio χ_C (or χ_{Colour}) is also the inverse of the colour coupling constant α_3 at *the high energy interaction cutoff* near the Planck length. In this respect χ_C *or* χ_{Colour} *is the fundamental ratio we will use mainly from here on*. From the above paragraphs to find the coupling ratios we need secondary interactions that are between bare charges. But this implies extremely close spacing where the effects of spin dominate. If the spacing is sufficiently large the effects of spin can be ignored but then we are not looking at bare charges. However *we can ignore the effects of shielding due to virtual charged pairs by imagining as a simple thought experiment, an interaction between bare charges even at such large spacing.* We can also simplify things further by considering only scalar or coulomb type *elastic* interactions at this large spacing. We are also going to temporarily ignore Eq. (3.3. 2) and imagine that we have only one primary electric and or one colour charge. Consider two superpositions and (*due to the above simplifying assumptions*) imagine them as spin zero charges. QED considers the interaction between them as a single covariant combination of two separate and opposite direction non-covariant interactions (a) plus (b) as in the Feynman diagram of Figure 3.3. 1 below. The Feynman transition amplitude is invariant in all frames [9]. So let us consider a special simple case in a CM frame where we have identical particles on a head on (elastic) collision path with spatial momenta:

$$\mathbf{p}_a = -\mathbf{p}'_a = -\mathbf{p}_b = +\mathbf{p}'_b \tag{3.3. 4}$$

From Eq. (3.3. 4) the initial and final spatial momenta are reversed with mirror images of each other at each vertex. Also in this simple special scalar case the transferred four momentum squared is simply the transferred three momentum squared, where ignoring the minus sign for q^2 in what we are doing here for simplicity:

$$q^2 = (p_a - p_a{}')^2 = (p_b - p_b{}')^2 = 4\mathbf{p}_a^2 = 4\mathbf{p}_b^2. \tag{3.3. 5}$$



The Feynman diagram is drawn with a vertical photon line representing the superposition of two opposite direction and non covariant processes (a) plus (b).
The exchanged 4 momentum is:

$$q = p_a - p'_a = p'_b - p_b.$$

Figure 3.3. 1 Feynman diagram of virtual photon exchange between two spin zero particles of charge e.

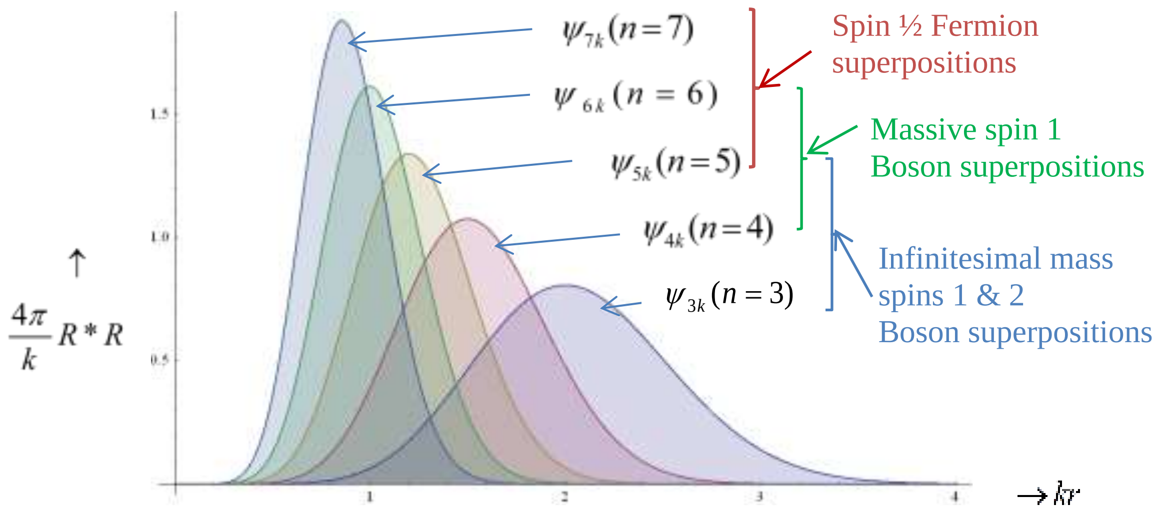


Figure 3.3. 2 All Eigenfunctions ψ_{nk} in the groups of three overlap at a fixed wavenumber *k.*

If we look at Figure 3.3. 2 we see that at any fixed value of *k*, all modes ψ_{nk} in the groups of three overlapping superpositions for the various spins ½, 1 & 2 occupy very similar regions of space (provided they are all on the same centre.) The directions of their linear momenta are unknown but let us imagine some particular vector $\hbar\mathbf{k}$ that is parallel to the above vectors $\mathbf{p}_a = \mathbf{p}_b$. As we are considering only scalar interactions, all these modes must be spherically symmetric (as in section 3.2.2 for spins 1 & 2, and for spin ½ provided k or in turn β_{nk} is small enough the probability that it is not spherically symmetric can be extremely low) and at a fixed value of k they have momenta $\pm n\hbar\mathbf{k}$. Also as they overlap each other we can imagine units of $\pm\hbar\mathbf{k}$ quanta somehow transferring between these superpositions so that the values of n in each mode can change temporarily by ± 1 for times $\Delta T \approx \hbar / \Delta E$. The directions of these momentum transfers causing either repulsion or attraction depending on the charge signs of the superpositions at each vertex, whether the same or opposite.

3.3.2 Restrictions on possible Eigenvalue changes

Before we look at changing these Eigenvalues by $n = \pm 1$ we need to consider what restrictions there are on these changes.

From Eq. (2.3. 12) superposition ψ_k requires $Q^2A^2 = \sum_n c_n * c_n \frac{n^4\hbar^2k^4r^2}{81}$ and Eq. (2.2. 4) tells us the available $Q^2A^2 = \frac{\left[8+8\sqrt{\alpha_{EMP}}\right]^2}{3\pi sN}\hbar^2k^4r^2$ occurs with probability $= \frac{sN \cdot dk}{k}$.

For very brief periods the required value of Q^2A^2 can fluctuate, such as during these changes of momentum, but if its average value changes over the entire process then Eq. (2.2. 4) tells us that probability $sN \cdot dk / k$ changes also, and we have shown in section 3.2.1 that this is not allowed. For example in a spin ½ superposition $\psi_{5k}, \psi_{6k}, \psi_{7k}$, the average values of $|c_5|, |c_6|$ & $|c_7|$ must each remain constant. This can happen only if n remains within its pre-existing boundaries of $(5 \le n \le 7)$. For example if ψ_7 adds $+\hbar\mathbf{k}$, it can create ψ_8, but $|c_8|$ must average zero, which it can do only if it fluctuates either side of zero, and $|c_n|$ cannot be negative. Similarly $|c_4|$ must average zero, thus ψ_4 & ψ_8 are forbidden states. Keeping the average values of $|c_n|$ constant is also equivalent to a constant internal average particle energy (we have shown in section 3.2.1 that rest mass is a function of $\sum c_n * c_n \cdot \mathbf{p}_{nk}^2$). By changing these Eigenvalues by $n = \pm 1$ there are only four possibilities; ψ_6 & ψ_7 can both reduce by $-\hbar\mathbf{k}$ quanta, ψ_6 & ψ_5 can both increase by $+\hbar\mathbf{k}$ quanta. If ψ_6 becomes ψ_7, $|c_7|$ also increases and $|c_6|$ decreases, but then ψ_7 has to drop back becoming ψ_6, with $|c_7|$ decreasing back down and $|c_6|$ increasing back up in exact balance. If we view this as one overall process the average values of both $|c_6|$ and $|c_7|$ remain constant but fluctuate continuously. We can use exactly the same argument if ψ_5 increases which has to be followed by ψ_6 dropping, where if we view this as one process again, the average values of both $|c_5|$ and $|c_6|$ remain constant. This is similar to a particle not being able to absorb a photon in a covariant manner, it has to reemit within time $\Delta T \approx \hbar / E$. With *spherical symmetry* the momentum $\mathbf{p} = \pm n\hbar\mathbf{k}$. If we change n by ± 1 the sign of $\mathbf{p} = \pm n\hbar\mathbf{k}$ determines the *direction of the momentum transfer* $\Delta\mathbf{p}$. In the above if $\psi_{5k} \to \psi_{6k}$ then returns $\psi_{6k} \to \psi_{5k}$, and $\mathbf{p} = \pm n\hbar\mathbf{k}$ *keeps the same sign during this process*, there is *no nett momentum transfer* and there is a probability of this, but it is not the probability we need right now. (We discuss this possibility in section 5.3.8 when looking at spin 2 graviton exchanges which don't appear to exchange 3 momentum). However consider the process as in Figure 3.3. 3. *To get a net momentum transfer the momenta have to be in opposite directions for each half of this process*. (Conservation of momentum allows this only if there is an equal and opposite transfer of momentum at the other vertex of the interaction.) The problem with this is that a total transfer of $\Delta\mathbf{p} = -2\hbar\mathbf{k}$ implies superpositions ψ_k interact with virtual $2k$ photons. Section 3.5 shows that interactions only with virtual k photons give the correct Dirac spin ½ magnetic energy. However just as transversely polarized photons are equal left and right polarization superpositions $|L\rangle/\sqrt{2}+|R\rangle/\sqrt{2}$, we can perhaps regard the Figure 3.3. 3 process as a similar equal superposition $|a\rangle/\sqrt{2}+|b\rangle\sqrt{2}$.

$\begin{matrix} 6\hbar\mathbf{k}\rightarrow \\ \downarrow \\ 5\hbar\mathbf{k}\rightarrow \\ |a\rangle \end{matrix}$ produces $\Delta\mathbf{p} = -\hbar\mathbf{k}$, but there is another $\Delta\mathbf{p} = -\hbar\mathbf{k}$ if returning via $\begin{matrix} -5\hbar\mathbf{k}\leftarrow \\ \downarrow \\ -6\hbar\mathbf{k}\leftarrow \\ |b\rangle \end{matrix}$

Figure 3.3. 3

The figure 3.3.3 process becomes the superposition $\frac{|a\rangle}{\sqrt{2}}+\frac{|b\rangle}{\sqrt{2}}=\frac{|-\hbar\mathbf{k}\rangle}{\sqrt{2}}+\frac{|-\hbar\mathbf{k}\rangle}{\sqrt{2}}$ (3.3. 6)

We have two equal 50% probabilities of states $|a\rangle$ & $|b\rangle$ producing the required total $\Delta\mathbf{p} = -\hbar\mathbf{k}$. Also as from the above paragraphs the average values of $|c_5|$ and $|c_6|$ remain constant:

The probability of transitions $\psi_5 \rightleftarrows \psi_6$ must be the same in either direction. (3.3. 7)

As spherically symmetric states have momentum $\mathbf{p} = \pm n\hbar\mathbf{k}$:

We can also think of $\mathbf{p} = \pm n\hbar\mathbf{k}$ as a superposition $\mathbf{p} = |+n\hbar\mathbf{k}\rangle/\sqrt{2}+|-n\hbar\mathbf{k}\rangle/\sqrt{2}$. (3.3. 8)

3.3.3 Looking at just one vertex of the interaction first

In Table 4.3. 1 and section 6.2 we introduce infinitesimal rest mass photons and gluons as superpositions of $\psi_{3k},\psi_{4k},\psi_{5k}$ where $N=2$ in Eq. (2.2. 4). Consider just one vertex of an infinitesimal rest mass spin 1 photon superposition $\psi_{3k},\psi_{4k},\psi_{5k}$ interacting with a spin ½ superposition $\psi_{5k},\psi_{6k,}\psi_{7k}$ at the same k. Looking at one possibility first, ψ_{4k} & ψ_{5k} for spin 1 and ψ_{6k} & ψ_{7k} for spin ½, we can apply the Figure 3.3. 3 process to get a nett momentum transfer. For this combination of Eigenfunctions there are four possible ways of getting the momentum transfer as in Figure 3.3. 4. In each of these 4 cases the amplitude for this to happen includes the factors $c_4 \cdot c_5 \cdot c_6 \cdot c_7$. Let us temporarily imagine $|c_4 \cdot c_5 \cdot c_6 \cdot c_7.| = 1$. Then $\mathbf{p} = +n\hbar\mathbf{k}$ as in $|a\rangle$ of Figure 3.3. 3 with an amplitude of $1/\sqrt{2}$ from Eq. (3.3. 8) transfers $\Delta\mathbf{p} = -\hbar\mathbf{k}$ also with an amplitude of $1/\sqrt{2}$, which is the required first half of our superposition Eq.(3.3. 6) $|a\rangle/\sqrt{2}+|b\rangle/\sqrt{2}$. Similarly $\mathbf{p} = -n\hbar\mathbf{k}$ as in $|b\rangle$ of Figure 3.3. 3

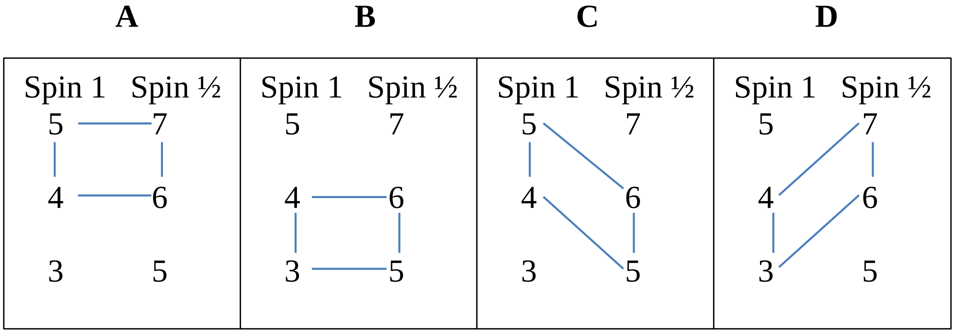
gives the second half. It would thus seem that our amplitude is simply $c_5 \cdot c_6 \cdot c_6 \cdot c_7$. However from Eq. (3.3. 7) there is a 50% probability of the transitions $\psi_5 \rightleftarrows \psi_6$ in either direction, or an extra $1/\sqrt{2}$ amplitude factor for $\psi_5 \rightleftarrows \psi_6$ in either direction, similarly an extra $1/\sqrt{2}$ amplitude factor for $\psi_6 \rightleftarrows \psi_7$. These two extra $1/\sqrt{2}$ factors reduce the amplitude $c_4 \cdot c_5 \cdot c_6 \cdot c_7$ to $c_4 \cdot c_5 \cdot c_6 \cdot c_7 / (\sqrt{2} \times \sqrt{2}) = c_4 \cdot c_5 \cdot c_6 \cdot c_7 / 2$. Thus adding the four cases in Figure 3.3. 4 together and *treating all other factors as 1*:

$$\text{Figure 3.3. 4 process amplitude factor is } 4 \times (c_4 \cdot c_5 \cdot c_6 \cdot c_7)/2 = 2c_4 \cdot c_5 \cdot c_6 \cdot c_7 \qquad (3.3.\ 9)$$

Spin 1 4 goes to 5 with **p** $\leftarrow$ 5 returns to 4 with **p** $\rightarrow$	Spin 1/2 7 goes to 6 with **p** $\leftarrow$ 6 returns to 7 with **p** $\rightarrow$	Spin 1 4 goes to 5 with **p** $\leftarrow$ 5 returns to 4 with **p** $\rightarrow$	Spin 1/2 6 goes to 7 with **p** $\rightarrow$ 7 returns to 6 with **p** $\leftarrow$
Spin 1 5 goes to 4 with **p** $\rightarrow$ 4 returns to 5 with **p** $\leftarrow$	Spin 1/2 7 goes to 6 with **p** $\leftarrow$ 6 returns to 7 with **p** $\rightarrow$	Spin 1 5 goes to 4 with **p** $\rightarrow$ 4 returns to 5 with **p** $\leftarrow$	Spin 1/2 6 goes to 7 with **p** $\rightarrow$ 7 returns to 6 with **p** $\leftarrow$

Figure 3.3. 4

The four possibilities in Figure 3.3. 4 are all between the same sets of Eigenfunctions ψ_{4k} & ψ_{5k} for spin 1, ψ_{6k} & ψ_{7k} for spin ½. But there are also four different sets of these A, B, C & D, between groups of four Eigenfunctions as in Figure 3.3. 5; with their amplitudes from Eq. (3.3. 9) below each relevant box, which we also label as *A*, *B*, *C* & *D*. (Subscripts *a* refer to spin ½ and *b* to spin 1.)



Amplitudes: $A = 2c_{4b}c_{5b}c_{6a}c_{7a}$, $B = 2c_{3b}c_{4b}c_{6a}c_{5a}$, $C = 2c_{4b}c_{5b}c_{6a}c_{5a}$, $D = 2c_{3b}c_{4b}c_{6a}c_{7a}$.

Figure 3.3. 5

3.3.4 Assumptions when looking at both vertexes of the interaction

Because we are looking at an interaction between identical spin ½ fermions each vertex has the same groups of Eigenfunctions A,B,C&D as in Figure 3.3. 5. From section 2.2.2 and Figure 3.1. 4 the three Eigenfunctions forming each of the interacting particles are born simultaneously. It would thus seem reasonable to assume that *the amplitudes of each group of three Eigenfunctions have the same complex phase angle.* The two fermions and one boson can be at different complex phase angles to each other but each one individually is a superposition of three Eigenfunctions at the same complex phase angle. Thus the four amplitudes *A,B,C&D* from Figure 3.3. 5 (*A,B,C &D* each comprising two fermion amplitudes and two boson amplitudes) must all have the same complex phase angle. Similarly the four amplitudes $A', B', C' \& D'$ of vertex 2 in Figure 3.3. 6 also have a common phase angle.

Eigenfunction Groups	A	B	C	D
Vertex 1	Amplitude A	Amplitude B	Amplitude C	Amplitude D
Vertex 2	Amplitude A'	Amplitude B'	Amplitude C'	Amplitude D'

Figure 3.3. 6

We are also going to assume that Eigenfunctions A of vertex 1 interact only with Eigenfunctions A of vertex 2 and Eigenfunctions B of vertex 1 interact only with Eigenfunctions B of vertex 2 etc. Eigenfunctions A of vertex 1 do not interact with Eigenfunctions B of vertex 2 etc. Thus *if all other amplitude factors are 1*:

$$\text{The total interaction amplitude } = AA' + BB' + CC' + DD' \qquad (3.3.\ 10)$$

Apart from a different complex phase angle this is equivalent to: ($A \& A', B \& B'$ etc. all differ by *the same complex phase angle*.)

$$\text{Total interaction amplitude} = A^2 + B^2 + C^2 + D^2 \qquad (3.3.\ 11)$$

$$\text{Interaction probability} = (A^2 + B^2 + C^2 + D^2)*(A^2 + B^2 + C^2 + D^2) \qquad (3.3.\ 12)$$

Using $(A^2 * A^2) = (A*A)(A*A)$ etc. this is equivalent to

$$\text{Interaction probability} = (A*A + B*B + C*C + D*D)^2 \qquad (3.3.\ 13)$$

From Figure 3.3. 5 $A = 2c_{4b}c_{5b}c_{6a}c_{7a}$, $B = 2c_{3b}c_{4b}c_{6a}c_{5a}$, $C = 2c_{4b}c_{5b}c_{6a}c_{5a}$, $D = 2c_{3b}c_{4b}c_{6a}c_{7a}$.

Putting $P_{5a} = c_{5a} * c_{5a}, P_{4b} = c_{4b} * c_{4b}$ etc. & $A * A = 4P_{4b}P_{5b}P_{6a}P_{7a}$ etc. this is equivalent to

$$(A*A + B*B + C*C + D*D)^2 = 16\left[P_{4b}P_{5b}P_{6a}P_{7a} + P_{3b}P_{4b}P_{6a}P_{5a} + P_{4b}P_{5b}P_{6a}P_{5a} + P_{3b}P_{4b}P_{6a}P_{7a}\right]^2$$

$$= 16\left[P_{4b}(P_{3b} + P_{5b})\right]^2\left[P_{6a}(P_{5a} + P_{7a})\right]^2$$

Then using $c_{3b} * c_{3b} + c_{4b} * c_{4b} + c_{5b} * c_{5b} = c_{5a} * c_{5a} + c_{6a} * c_{6a} + c_{6a} * c_{6a} = 1$ the interaction probability is

$$(A*A + B*B + C*C + D*D)^2 = 2^4\left[c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})\right]^2\left[c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})\right]^2 \qquad (3.3.\ 14)$$

We have assumed to here that *all other amplitude factors are 1*. However at each vertex there are both fermion and boson superposition probabilities from Eq. (2.2. 4). Writing the superposition probability at each vertex $sN \cdot dk / k$ as $s_{1/2}N_1 dk / k$, $s_1 N_2 dk / k$ for clarity where spin $1 = s_1$, $N = 1$ is N_1 etc. Including these factors (*if all other factors are one*) in Eq. (3.3. 14) our overall probability at wavenumber k is

$$\left[\frac{2s_{1/2}N_1 c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})}{k}\right]^2\left[\frac{2s_1 N_2 c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})}{k}\right]^2$$

$$= \frac{\left[2s_{1/2}N_1 c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})\right]^2\left[2s_1 N_2 c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})\right]^2}{(k)^4}.$$

The momentum per transfer is a total of $\pm\hbar\mathbf{k}$ and using Eq's. (3.3. 5), (3.3. 6) & Figure 3.3. 3 we have $(\pm\hbar\mathbf{k})^4 = q^4$ (then putting $\hbar = 1$) the interaction probability:

$$= \frac{\left[2s_{1/2}N_1 c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})\right]^2\left[2s_1 N_2 c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})\right]^2}{q^4} \qquad (3.3.\ 15)$$

This is the *scalar interaction probability between two spin ½ fermions exchanging infinitesimal rest mass spin 1 bosons at very large spacings, where the fermions are effectively spin zero, imagining them as bare charges and all other factors being one*. Going through exactly the same procedure but similarly exchanging spin 2 infinitesimal rest mass scalar gravitons (with $N = 2 = N_2$ for clarity) the *gravitational interaction probability* between fermions becomes (using *subscript c* for spin 2) *if all other amplitude factors are 1*:

$$= \frac{\left[2s_{1/2}N_1 c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})\right]^2 \left[2s_2 N_2 c_{4c} * c_{4c}(1 - c_{4c} * c_{4c})\right]^2}{q^4} \text{ for fermions.} \tag{3.3. 16}$$

And if for example two spin 1 photons exchange spin 2 gravitons (all infinitesimal rest mass with $N = 2 = N_2$) the *interaction probability* becomes *if all other amplitude factors are 1*:

$$= \frac{\left[2s_1 N_2 c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})\right]^2 \left[2s_2 N_2 c_{4c} * c_{4c}(1 - c_{4c} * c_{4c})\right]^2}{q^4} \text{ for } N = 2 \text{ photons.} \tag{3.3. 17}$$

If two massive $N = 1$ photons (as in Figure 3.3. 2) exchange spin 2 gravitons the *interaction probability* becomes *if all other factors are 1:*

$$= \frac{\left[2s_1 N_1 c_{5b} * c_{5b}(1 - c_{5b} * c_{5b})\right]^2 \left[2s_2 N_2 c_{4c} * c_{4c}(1 - c_{4c} * c_{4c})\right]^2}{q^4} \text{ for } N = 1 \text{ photons.} \tag{3.3. 18}$$

General Relativity (section 1.1.1) tells us the emission of gravitons is identical for both mass and energy. Keeping all other factors (such as mass/energy) in Eq's. (3.3. 16), (3.3. 17) & (3.3. 18) constant, the exchange probabilities must be the same in each. We can thus put them equal to each other and cancel out the red terms:

$$2s_1 N_2 c_{4b} * c_{4b}(1 - c_{4b} * c_{4b}) = 2s_1 N_1 c_{5b} * c_{5b}(1 - c_{5b} * c_{5b}) = 2s_{1/2} N_1 c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})$$

or

$$\underset{N = 2 \text{ Spin } 1}{4c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})} = \underset{N = 1 \text{ Spin } 1}{2c_{5b} * c_{5b}(1 - c_{5b} * c_{5b})} = \underset{N = 1 \text{ Spin } 1/2}{c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})} \tag{3.3. 19}$$

Now assume that all other factors (other than coupling constants) are 1, and remember that we are simplifying with a thought experiment by looking at spin ½ superpositions sufficiently far apart so we can treat them *as approximately spherically symmetric or effectively spin zero even if they are supposed to be bare charges with spin*. Under these same scalar exchange conditions QED tells us that with electrons for example:

The probability of scalar or coulomb exchange in Eq (3.3. 15). $= \dfrac{4\alpha^2}{q^4}.$ (3.3. 20)

Let us temporarily ignore the fact that gluons have limited range, and imagine our thought experiment applying to colour charges exchanging gluons. The α of Eq. (3.3. 20) becomes the usual colour coupling α_3. To get the *fundamental coupling ratio* labelled as $\chi_C = \alpha_3^{-1}$ $@\, k_{cutoff}$ we substitute the α of Eq. (3.3. **20**) with $\alpha = \chi_C^{-1}$ as we have assumed $\alpha_3(\text{Primary}) = 1$. Also substitute $2s_{1/2} = 1,\ 2s_1 = 2,\ N_1 = 1\ \&\ N_2 = 2$ and equate Eq's. (3.3. 15) & (3.3. 20)

$$\frac{\left[c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})\right]^2 \left[4c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})\right]^2}{q^4} = \frac{4(\chi_C^{-1})^2}{q^4}$$
$$\text{or}\quad \left[c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})\right]\left[4c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})\right] = 2\chi_C^{-1} \qquad (3.3.\ 21)$$

But from Eq. (3.3. 19) the blue and green terms are equal (also the magenta terms) and we can solve for *the fundamental coupling ratio* by combining Eq's. (3.3. 19) & (3.3. 21).

$N = 2$ Spin 1	$N = 1$ Spin 1	$N = 1$ Spin 1/2	(3.3. 22)
Photons or Gluons	Massive Photons	Fermions	

$$4c_{4b} * c_{4b}(1 - c_{4b} * c_{4b}) = 2c_{5b} * c_{5b}(1 - c_{5b} * c_{5b}) = c_{6a} * c_{6a}(1 - c_{6a} * c_{6a}) = \sqrt{2}\chi_C^{-1}$$

The coupling ratio is fundamentally the same for colour and electromagnetism apart from the six primary electric charges of Eq. (3.3. 1) because of the way electric charge is defined. Equations (3.3. 19), (3.3. 21) & (3.3. 22) tell us that for any interactions between two superpositions, *the inverse coupling ratio always involves the product of the central superposition member probability by the probability of the other two members combined* $\times N \times spin$ of the first superposition, times *the equivalent product for the other superposition.* In section 4 we introduce gravity and solve these ratios. Despite all the simplifications and lack of rigorousness, the above equations are surprisingly consistent with the Standard Model, provided there are only three families of fermions. Even though we used gravity to derive Eq.(3.3. 19) we leave discussing the gravity coupling ratio till section 6.2.6.

3.4 Electrostatic Energy between two Infinite Superpositions

3.4.1 Using a simple quantum mechanics early QED approach

In section 3.3 we have shown that fermion superpositions can exchange boson superpositions in the same way as electrons can exchange virtual photons for example. Providing the superposition amplitudes are appropriate, the coupling constants can be just as in QED, though we will look further at this in section 4.1.1. So it might seem that evaluating electrostatic energy between superpositions is unnecessary. However when we look at gravity we find that the spacetime warping around mass concentrations appears to be strongly related to cosmic wavelength virtual graviton probability densities. Virtual particle exchange probabilities, in QED/QCD etc, use perturbation theory to calculate particle scattering crossections, and electron g factor corrections with incredible precision; but as we later focus on virtual graviton probability densities, it is more appropriate here to use a simple, but only approximate, quantum mechanical method based on virtual photon probability densities to find the scalar potentials between two charges (or infinite superpositions). This same method also allows a simple solution to the magnetic energy between superpositions in Section 3.5, where we modify relevant equations in a simple manner. We later use some of these same equations when looking at why borrowing energy and mass from zero point fields, requires the universe to expand after the Big Bang, and distort spacetime around mass concentrations.

We assume spherically symmetric $l=3$ superpositions emit virtual scalar photons in this section and $l=3, m=\pm 2$ superpositions emit virtual $m=\pm 1$ photons in section 3.5. As section 3.3 has shown that we can achieve the same electromagnetic coupling constant α we can use the scalar photon emission probability $(2\alpha/\pi)(dk/k)$ covered in section 2.1.1. From section 3.3 we can also see that the effective average emission point has to be the center of superpositions. For a virtual photon $\Delta E \cdot \Delta T \approx \hbar/2$, and the range over which it can be found is roughly $r \approx \Delta T \approx 1/2\Delta E \approx 1/2k$ when $\hbar = c = 1$. The radial probability of finding the centre of the spin 1 superposition representing the interacting virtual photon decays exponentially with radius as e^{-2kr}. The normalized wavefunction ψ for such a virtual scalar photon of wave number k emitted at $r=0$ is:

$$\psi = \sqrt{\frac{2k}{4\pi}}\,\frac{e^{-kr}e^{+i(kr-\omega t)}}{r} = \sqrt{\frac{2k}{4\pi}}\,\frac{e^{-kr}e^{+ikr}}{r} \quad @ \text{ time } t=0.$$

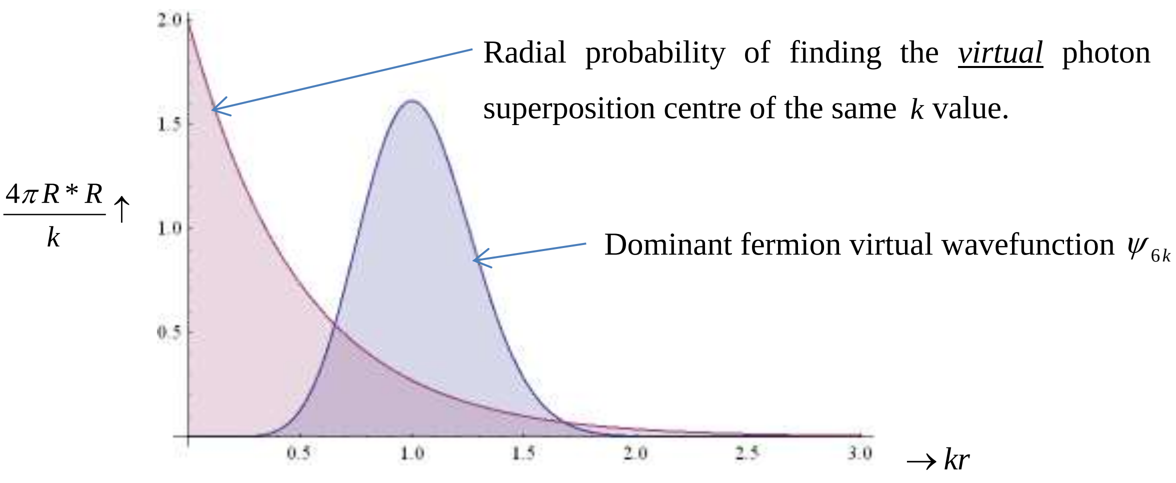


Figure 3.4. 1 Radial probabilities of ψ_{6k} and the exponential decay with radius of a virtual photon of the same k value $R^*R \propto 2ke^{-2kr}$. These curves look the same for all k, applying equally to virtual photons, gravitons and to large k value gluons etc.

Wavefunction ψ is spherically symmetric as scalar photons are time polarized. Figure 3.4. 1 plots the radial probabilities of the exponentially decaying with radius virtual photon and the dominant $n=6$ mode of its relating superposition ψ_k. The effective range of a wavenumber k virtual photon is of a similar order to the radial probability dimensions of ψ_{6k}. For simplicity in what follows we locate two superpositions (which we refer to as sources) in cavities that are small in relation to the distance between them. The accuracy of our results depends on how far apart they are in relation to the cavity size. Consider two spherically symmetric sources distance $2C$ apart emitting virtual scalar photons as in Figure 3.4. 2 where point P is r_1 from source 1, & r_2 from source 2. Let ψ_1 be the amplitude from source 1, and ψ_2 be the amplitude from source 2 and for simplicity and clarity let $t=0$.

$$\text{Thus} \qquad \psi_1 = \sqrt{\frac{2k}{4\pi}}\frac{e^{-kr_1+ikr_1}}{r_1} \quad \& \quad \psi_2 = \sqrt{\frac{2k}{4\pi}}\frac{e^{-kr_2+ikr_2}}{r_2} \tag{3.4. 1}$$

Consider $\quad (\psi_1+\psi_2)^*(\psi_1+\psi_2) \quad = \psi_1{}^*\psi_1 + \psi_1{}^*\psi_2 + \psi_2{}^*\psi_1 + \psi_2{}^*\psi_2$

Now $\psi_1{}^*\psi_1$ & $\psi_2{}^*\psi_2$ are just the normal probability densities around sources 1 & 2 as though they are infinitely far apart but the work done per pair of superpositions k on bringing 2 sources closer together is in the interaction term: $\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1$.

$$\psi_1{}^*\psi_2 = \frac{2k}{4\pi r_1 r_2} e^{-kr_1}e^{-kr_2}e^{-ikr_1}e^{+ikr_2} = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)}e^{-ik(r_1-r_2)}$$

$$\psi_2{}^*\psi_1 = \frac{2k}{4\pi r_1 r_2} e^{-kr_2}e^{-kr_1}e^{-ikr_2}e^{+ikr_1} = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)}e^{+ik(r_1-r_2)}$$

$$\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1 = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)}\left[e^{+ik(r_1-r_2)} + e^{-ik(r_1-r_2)}\right]$$

$$= \frac{4k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} \cos k(r_1-r_2)$$

$$\text{Now put } (A=r_1+r_2, B=r_1-r_2) \quad \& \quad \psi_1{}^*\psi_2 + \psi_2{}^*\psi_1 = \frac{4k}{4\pi r_1 r_2} e^{-Ak}\cos(kB) \tag{3.4. 2}$$

Real work is done when bringing superpositions together and we can treat these interacting virtual photons as having real energy $\hbar\omega = \hbar kc$. Using virtual photon emission probability $(2\alpha/\pi)(dk/k)$ from section 2.1.1

$$\text{Energy per virtual photon} \times \text{Probability} = \hbar kc \times \left[\text{Probability } \frac{2\alpha}{\pi}\frac{dk}{k}\right] = \frac{2\alpha\hbar c}{\pi}dk \tag{3.4. 3}$$

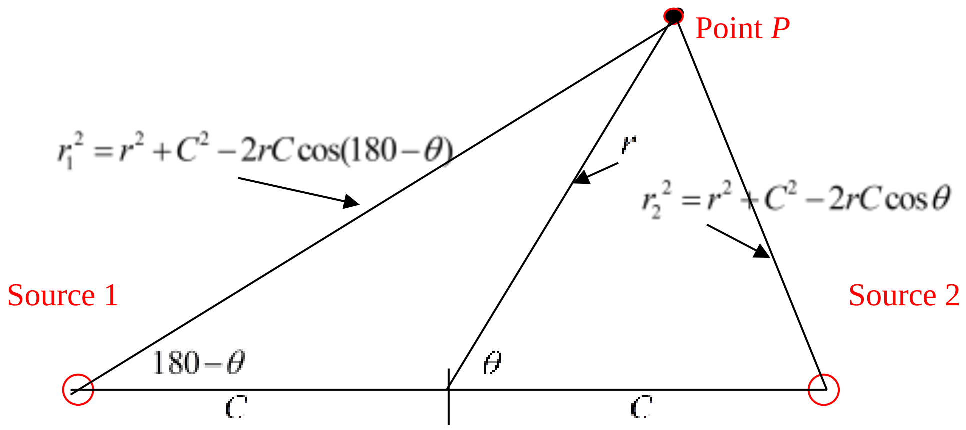


Figure 3.4. 2

Including Eq.(3.4. 3) the interaction energy @ k is thus $(\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)\left[\frac{2\alpha\hbar c}{\pi}dk\right]$ and using Eq. (3.4. 2) the interaction energy @ k is $\left[\frac{2\alpha\hbar c}{\pi}dk\right]\frac{4k}{4\pi r_1 r_2}e^{-Ak}\cos(kB)$.

The total interaction energy density due to $\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1$ for all k is

$$\frac{2\alpha\hbar c}{\pi}\frac{4}{4\pi r_1 r_2}\int_0^\infty ke^{-Ak}\cos(Bk)dk \tag{3.4. 4}$$

$$\int_0^\infty ke^{-Ak}\cos(Bk)dk = \frac{A^2 - B^2}{(A^2+B^2)^2} \tag{3.4. 5}$$

Where $A^2 = (r_1+r_2)^2 = r_1{}^2 + 2r_1r_2 + r_2{}^2$ & $B^2 = (r_1-r_2)^2 = r_1{}^2 - 2r_1r_2 + r_2{}^2$

Thus $A^2 = (r_1+r_2)^2 = r_1{}^2 + 2r_1r_2 + r_2{}^2$ & $A^2 + B^2 = 2(r_1{}^2 + r_2{}^2)$ (3.4. 6)

$= 2(r^2 + C^2)$ as $\cos(180-\theta) = -\cos\theta$

and $A^2 + B^2 = 4(r^2 + C^2)$ (3.4. 7)

Putting Eq's. (3.4. 4), (3.4. 5), (3.4. **6**) & (3.4. 7**)** together $\frac{A^2-B^2}{(A^2+B^2)^2} = \frac{4r_1r_2}{16(r^2+C^2)^2}$

$$\int_0^\infty ke^{-Ak}\cos(Bk)dk = \frac{r_1r_2}{4(r^2+C^2)^2}$$

$$\frac{2\alpha\hbar c}{\pi}\frac{4}{4\pi r_1 r_2}\int_0^\infty ke^{-Ak}\cos(Bk)dk = \frac{2\alpha\hbar c}{\pi}\frac{4}{4\pi r_1 r_2}\frac{r_1 r_2}{4(r^2+C^2)^2}$$

$$\frac{2\alpha\hbar c}{\pi}\frac{4}{4\pi r_1 r_2}\int_0^\infty ke^{-Ak}\cos(Bk)dk = \frac{2\alpha\hbar c}{\pi}\frac{1}{4\pi}\frac{1}{(r^2+C^2)^2} \tag{3.4. 8}$$

This is the total interaction energy density of time polarized virtual photons at point P due to $\psi_1{}^*\psi_2+\psi_2{}^*\psi_1$ for all k and there are no directional vectors to take into account. We will use similar equations for the vector potential ($m=\pm1$) photons for magnetic energies but will then need directional vectors. Equation (3.4. **8**) is the energy due to the interaction of amplitudes at any radius r from the centre of the pair. It is independent of θ, and to get the total energy of interaction we multiply by $4\pi r^2 dr$ for layer dr and integrate from $r=0\to\infty$.

The total interaction energy is $\quad \dfrac{2\alpha\hbar c}{\pi}\displaystyle\int_0^\infty\int_0^\infty(\psi_1{}^*\psi_2+\psi_2{}^*\psi_1)dk\;4\pi r^2 dr$

Using Eq. (3.4. **8**) $\quad = \dfrac{2\alpha\hbar c}{\pi}\dfrac{1}{4\pi}\displaystyle\int_0^\infty\frac{4\pi r^2 dr}{(r^2+C^2)^2}$

Thus $\quad \dfrac{2\alpha\hbar c}{\pi}\displaystyle\int_0^\infty\int_0^\infty(\psi_1{}^*\psi_2+\psi_2{}^*\psi_1)dkdv = \frac{2\alpha\hbar c}{\pi}\int_0^\infty\frac{r^2 dr}{(r^2+C^2)^2}$

$$\int_0^\infty\frac{r^2 dr}{(r^2+C^2)^2} = \frac{1}{2C}\frac{\pi}{2}$$

The interaction or potential energy is $\dfrac{\alpha\hbar c}{2C}=\dfrac{\alpha\hbar c}{R}$ (3.4. 9)

If $R=2C$ is the distance between the centres of our assemblies, this is the classical potential. The procedure used here with small changes, simplifies the derivation of the magnetic moment; we reuse some equations, but in a slightly modified form taking polarization vectors into account. We also reuse some of these simple but approximate derivations when looking at gravity in Section 5.

3.5 Magnetic Energy between two spin aligned Infinite Superpositions

In this section we are going to consider two infinite superpositions that form Dirac spin ½ states. We will look at the magnetic energy between them when they are both in a spin up state say along some z axis as in Figure 3.5. 1. *We are not looking at the magnetic energy here when they are both coupled in a spin 0 or spin 1 state*. That is, both Dirac spin ½ states have their $\sqrt{3}\hbar/2$ spin vectors randomly oriented around the z axis with $\hbar/2$ components aligned along this z axis. Also in this section we will be dealing with transversely polarized virtual photons and must take account of polarization vectors. In section 3.2.2 and Eq. (3.2. 7) spin ½ states are generated only from $l=3, m=2$ states and as transversely polarized photons are superpositions of $m=\pm 1$ photons they can only be emitted from these $l=3, m=2$ states, the remaining states are spherically symmetric and cannot emit transversely polarized photons. We don't yet know the value of amplitudes $|c_n|$ so we will derive the magnetic energy in terms of these. We will then equate this energy to the Dirac values assuming a g value of 2 before QED corrections; this allows us to evaluate in section 4.3 the amplitudes $|c_n|$ in terms of the ratio χ_{EM} between primary and secondary electromagnetic coupling. We can then evaluate in section 4.1 the primary electromagnetic coupling constant α_{EMP} in terms of the ratio χ_{EM}. (Section 3.5 uses the same format as Chapter 18, "The Feynman Lectures on Physics" Volume 3, Quantum Mechanics [11].)

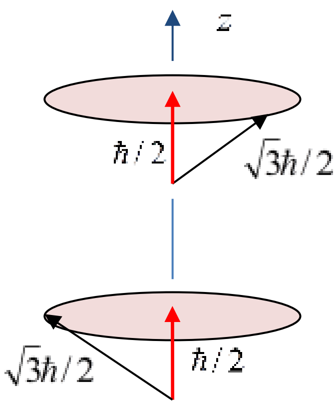


Figure 3.5. 1

An $l=3, m=2$ state can emit a right hand circularly (R.H.C.) polarized $(m=+1)$ photon in the $+z$ direction. Let the amplitude for this be temporarily $|R\rangle$.

An $l=3, m=-2$ state can emit a left hand circularly (L.H.C.) polarized $(m=-1)$ photon in the $+z$ direction. Let the amplitude for this also be temporarily $|L\rangle$.

First rotate the z axis about the y axis by angle θ (call this operation $S|R\rangle$) then use

$\langle x'| = (1/\sqrt{2})[\langle R'| + \langle L'|]$ and multiply on the right by operation $S|R\rangle$.

The amplitude to emit a transversely polarized photon in the x' direction is thus

$$\langle x'|S|R\rangle = \frac{1}{\sqrt{2}}\left[\langle R'|S|R\rangle + \langle L'|S|R\rangle\right]$$

Where $\langle R'|S|R\rangle = \langle 3,+2'|S|3,+2\rangle = (1/4)\left[2+2\cos\theta - 4\sin^2\theta + 3\sin^2\theta\cos\theta\right]$ is the amplitude an $l=3, m=2$ state remains in an $l=3, m=2$ state after rotation by angle θ.

Also $\langle L'|S|R\rangle = -\langle 3,-2'|S|3,+2\rangle = (1/4)\left[2-2\cos\theta - 4\sin^2\theta - 3\sin^2\theta\cos\theta\right]$ is minus the amplitude that an $l=3, m=2$ state is in an $l=3, m=-2$ state after rotation by θ.

Putting this together $$\langle x'|S|R\rangle = \frac{1-2\sin^2\theta}{\sqrt{2}} = \frac{\cos 2\theta}{\sqrt{2}} \tag{3.5. 1}$$

An $l=3, m=2$ state can also emit an $(m=+1)$ photon in the $-z$ direction but it will now be left hand circularly polarized. Let this amplitude be temporarily: $|L\rangle$.

Similarly an $l=3, m=-2$ state can emit an $(m=-1)$ photon in the $-z$ direction which is right hand circularly polarized. Let this amplitude be temporarily: $|R\rangle$.

We can go through the same procedure as above to get $$\langle x'|S|L\rangle = \frac{\cos 2\theta}{\sqrt{2}} \tag{3.5. 2}$$

This amplitude Eq. (3.5. 2) is for a photon emitted in the opposite direction to amplitude Eq. (3.5. 1) but $\cos 2\theta = \cos 2(180+\theta)$ and we can simply add these two amplitudes. Let us assume however that an $l=3, m=2$ state has equal amplitudes to emit in the $+z$ & $-z$ directions of $|R\rangle/\sqrt{2}$ and $|L\rangle/\sqrt{2}$.

With these amplitudes; $$\frac{1}{\sqrt{2}}\left[\langle x'|S|R\rangle + \langle x'|S|L\rangle\right] = \frac{\cos 2\theta}{2} + \frac{\cos 2\theta}{2} = \cos 2\theta \tag{3.5. 3}$$

Eqation (3.5. 3) is the angular component of the amplitude for a transverse x' polarization in the new z' direction where $x \to x'$ & $z \to z' = \theta$. When $\theta = 0$ or 180 the on axis amplitude for transverse polarization is one as expected ignoring other factors. Using the same normalization factors (we check the validity of this in section 3.5.2 we can still use the amplitudes and phasing of our original time mode photons Eq's. (3.4. 1) but instead of including polarization vectors we will for simplicity just use the cosine of the angle $(\gamma-\delta)$ between them (as in Figure 3.5. 2) as a multiplying factor. Including the angular factor Eq. (3.5. 3) in our earlier scalar amplitudes Eq's. (3.4. 1) we have for our new wavefunctions:

$$\psi_1 = \cos 2\delta \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr_1+ikr_1}}{r_1} \quad \& \quad \psi_2 = \cos 2\gamma \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr_2+ikr_2}}{r_2} \tag{3.5. 4}$$

The transverse polarized photons from sources (1) & (2) have polarization vectors $|x_1\rangle$ and $|x_2\rangle$ at angle to each other $(\gamma-\delta)$, (Figure 3.5. 2) and the complex product becomes:

$$(\psi_1 + \psi_2)^*(\psi_1 + \psi_2) \quad = \psi_1{}^*\psi_1 + (\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)(\cos(\gamma-\delta) + \psi_2{}^*\psi_2$$

Where the interaction term is now: $(\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)\cos(\gamma-\delta)$ and as in the scalar case (section 3.4.1) but now using Eq's. (3.5. 4)

$$\psi_1{}^*\psi_2 \cos(\gamma-\delta) = \cos 2\delta \cos 2\gamma \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{-ik(r_1-r_2)} \cos(\gamma-\delta)$$

$$\psi_2{}^*\psi_1 \cos(\gamma-\delta) = \cos 2\delta \cos 2\gamma \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{+ik(r_1-r_2)} \cos(\gamma-\delta)$$

$$(\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)\cos(\gamma-\delta) = \cos 2\delta \cos 2\gamma \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos(kB) \cos(\gamma-\delta) \tag{3.5. 5}$$

(Where as in section 3.4.1, Eq. (3.4. 2) $A = r_1 + r_2$ & $B = r_1 - r_2$.)

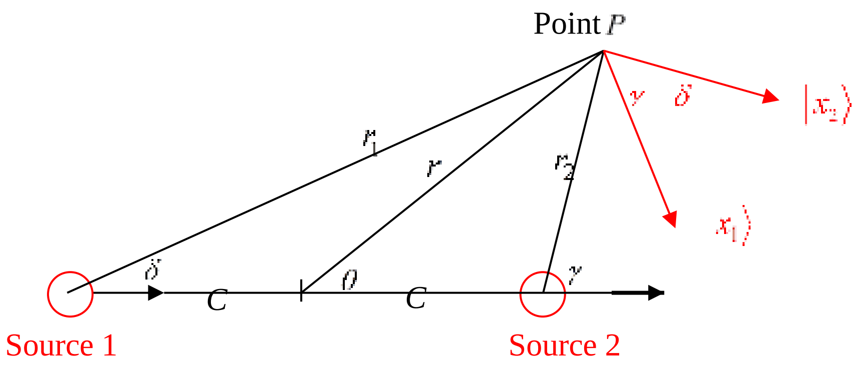


Figure 3.5. 2 Two sources $2C$ apart, both with $\beta_{nk}^2 \times (m=+2)$ states along the joining line, δ & γ are the respective angles to P , r_1 & r_2 are the respective distances to point P.

3.5.1 Amplitudes of transversely polarized virtual emmited photons

In the laboratory frame ψ_{nk} *has amplitude* β_{nk} *to be in an* $m=+2$ *state* (section 3.1). For a multiple integer n superposition $\psi_k = \sum_n c_n \psi_{nk}$. At each fixed wavenumber k we cannot distinguish which integer n a virtual photon comes from, so we must add amplitudes from each individual integer n superposition. To keep integrals simple we will assume that $\beta_{nk} <<<< 1$ or that spacing $2C$ is very large, and our interacting k values are very small. (We can make a comparison with the Dirac values at any large spacing so accuracy need not be affected.) Thus if $\beta_{nk} <<< 1$ & $\gamma_{nk} \approx 1$, we can approximate Eq. (3.1. 11) as

$$K_{nk} = \beta_{nk}\gamma_{nk} \approx \beta_{nk} \approx \frac{n\hbar k\sqrt{2s}}{2m_0 c} \approx \frac{|\mathbf{p}_{nk}|\sqrt{2s}}{2m_0 c} \approx \frac{\lambda\!\!\!^{-}_c nk\sqrt{2s}}{2} \approx \frac{\lambda\!\!\!^{-}_c nk}{2} \text{ for spin ½ fermions.}$$

$$\text{Adding amplitudes for multiple integer } n \text{ superpositions} \langle \beta_k \rangle \approx \frac{\lambda\!\!\!^{-}_c \langle n \rangle k}{2} \qquad (3.5.\ 6)$$

(When deriving Eq. (3.2. 10) we said $\langle |\mathbf{p}_k| \rangle = \hbar k \langle n \rangle$ *and not* $\langle |\mathbf{p}_k| \rangle = \hbar k \sqrt{\langle n^2 \rangle}$. How do we justify this? When $\beta_{nk} << 1$ as above $\beta_{nk} \propto n\hbar k = |\mathbf{p}_{nk}|$ So adding amplitudes β_{nk} to get $\langle \beta_k \rangle$ is equivalent to adding $\mathbf{p}_{nk}$ to get $\langle \mathbf{p}_k \rangle$ *and not adding* $\mathbf{p}_{nk}^2 = n^2\hbar^2 k^2$ *to get* $\langle |\mathbf{p}_k| \rangle = \hbar k \sqrt{\langle n^2 \rangle}$. If this is true when $\beta_{nk} << 1$ it must be true for $0 \le \beta_{nk} \le 1$.)

3.5.2 Checking our normalization factors

Let us pause and check the reasonableness of all this and our normalization factors. From Eq's. (3.4. 1) for scalar photons $\left[\psi^* \psi = \frac{2k}{4\pi} \frac{e^{-2kr}}{r^2} \right]$ × (emission probability $\frac{2\alpha}{\pi} \frac{dk}{k}$) gives a

$$\text{Scalar } \psi_k \text{ emission probability density } \psi^* \psi \left[\frac{2\alpha}{\pi} \frac{dk}{k} \right] = \frac{2k}{4\pi} \frac{e^{-2kr}}{r^2} \left[\frac{2\alpha}{\pi} \frac{dk}{k} \right].$$

The transversely polarized probability density, using Eq's. (3.5. 4) & (3.5. 7) plus $\langle \beta_k \rangle^2$ is

$$\text{Transverse emission probability density } \langle \beta_{nk} \rangle^2 \psi'^* \psi' \frac{2\alpha}{\pi} \frac{dk}{k} = \langle \beta_{nk} \rangle^2 \left[\cos^2 2\delta \frac{2k}{4\pi} \frac{e^{-2kr}}{r^2} \right] \frac{2\alpha}{\pi} \frac{dk}{k}$$

(Where $2\delta = 2\gamma$ & $r_1 = r_2$.) If we now consider the on axis $\delta = 0$ case the transverse polarized on axis emission probability density at k is:

$$\langle \beta_k \rangle^2 \left[\frac{2k}{4\pi} \frac{e^{-2kr}}{r^2} \right] \frac{2\alpha}{\pi} \frac{dk}{k} = \langle \beta_k \rangle^2 \quad \psi^* \psi \frac{2\alpha}{\pi} \frac{dk}{k}$$

Just as in QED the factor $\langle \beta_k \rangle^2$ is the factor we need for this on axis emission probability density *ratio between transverse and scalar polarization*. This justifies using the same normalization constant $\sqrt{2k/4\pi}$ for both the scalar and magnetic wavefunctions. We seem to be on the right track and using the same virtual photon emission probability and energy $\hbar kc$ as in Eq. (3.4. 3) for both the scalar and transverse polarization cases ie

$$\text{Energy per transverse photon} \times \text{Probability} = \hbar kc \times \left[\text{Probability } \frac{2\alpha}{\pi}\frac{dk}{k} \right] = \frac{2\alpha\hbar c}{\pi}dk \qquad (3.5.\ 7)$$

Multiplying Eq. (3.5. 5) by Eq. (3.5. 6) squared, and Eq. **(3.5. 7)** we get the transverse interaction energy @ wavenumber k :

$$\langle\beta_k\rangle^2 (\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)\cos(\gamma-\delta)\left[\frac{2\alpha\hbar c}{\pi}dk\right]$$

$$= \left[\frac{\lambda_C^2 \langle n\rangle^2 k^2}{4}\right]\cos 2\delta \cos 2\gamma \frac{4k}{4\pi r_1 r_2} e^{-Ak}\cos(kB)\ \cos(\gamma-\delta)\left[\frac{2\alpha\hbar c}{\pi}dk\right]$$

Rearranging this: $\langle\beta_k\rangle^2 (\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)\cos(\gamma-\delta)\left[\frac{2\alpha\hbar c}{\pi}dk\right]$

$$= \frac{2\langle n\rangle^2 \lambda_C^2 \alpha\hbar c}{\pi}\ \frac{\cos 2\delta \cos 2\gamma \cos(\gamma-\delta)}{4\pi r_1 r_2}\left[k^3 e^{-Ak}\cos(kB)dk\right] \qquad (3.5.\ 8)$$

As in the scalar case we integrate over k first but now with a k^3 term due to the inclusion of the $\langle\beta_k\rangle^2$ factor which is approximately proportional to k^2 from Eq. (3.5. 6).

Using $A = r_1 + r_2$ & $B = r_1 - r_2$ and Eq's. (3.4. 6) & (3.5. 6)

$$\int_0^\infty \left[k^3 e^{-Ak}\cos(kB)dk\right] = \frac{3}{8}\left[\frac{2r_1{}^2 r_2{}^2 - (r^2+C^2)^2}{(r^2+C^2)^4}\right]$$

And thus: $\int_0^\infty \langle\beta_k\rangle^2 (\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)\cos(\gamma-\delta)\ \frac{2\alpha\hbar c}{\pi}dk$

$$= \frac{2\langle n\rangle^2 \lambda_C^2 \alpha\hbar c}{\pi}\ \frac{\cos 2\delta \cos 2\gamma \cos(\gamma-\delta)}{4\pi r_1 r_2} \times \frac{3}{8}\left[\frac{2r_1{}^2 r_2{}^2 - (r^2+C^2)^2}{(r^2+C^2)^4}\right] \qquad (3.5.\ 9)$$

Equation (3.5. 9) is the magnetic interaction energy density at point P for all wave numbers k. Figure 3.5. 2 is a plane of symmetry that can be rotated through angle 2π around the axis of symmetry (the joining line along the axis of the 2 spin aligned sources). To evaluate the total magnetic energy density over all space we just multiply by $4\pi r^2 \sin\theta d\theta dr$.

We thus integrate Eq. (3.5. 9) × $4\pi r^2 \sin\theta d\theta dr$. =

$$\frac{3\langle n\rangle^2 \lambda_C^2 \alpha\hbar c}{4\pi}\int_0^{\infty}\int_0^{\pi/2}\frac{\cos 2\delta\cos 2\gamma\cos(\gamma-\delta)}{r_1 r_2}\left[\frac{2r_1^2 r_2^2-(r^2+C^2)^2}{(r^2+C^2)^4}\right]r^2\sin\theta d\theta dr \qquad (3.5.\ 10)$$

Now $\int_0^{\infty}\int_0^{\frac{\pi}{2}}\frac{\cos 2\delta\cos 2\gamma\cos(\gamma-\delta)}{r_1 r_2}\left[\frac{2r_1^2 r_2^2-(r^2+C^2)^2}{(r^2+C^2)^4}\right]r^2\sin\theta d\theta dr$ can be reduced to the single integral: $\frac{1}{8C^3}\int_0^1\sqrt{1-x^2}\left[\frac{(7-5x^2)}{x^3}\ln\frac{1+x}{1-x}-\frac{14}{x^2}+\frac{16}{3}\right]dx$ which can be also expressed as an infinite series in p (to not confuse with superposition value n):

$$\frac{1}{8C^3}\sum_{p=1}^{p=\infty}\left[\frac{14}{2p+3}-\frac{10}{2p+1}\right]\frac{(2p-1)!}{(p-1)!(p+1)!4^p}.\frac{\pi}{2} \quad =\frac{1}{8C^3}\frac{(160-51\pi)}{6}.\frac{\pi}{2}$$

$$\text{(Putting } R=2C\text{)} \quad =\frac{1}{R^3}\frac{(160-51\pi)}{6}.\frac{\pi}{2} \qquad (3.5.\ 11)$$

$$\text{This infinite series is approximately } \approx -\frac{1}{R^3}\frac{\pi}{54(1.0045062....)} \qquad (3.5.\ 12)$$

Putting Eq.(3.5. 12) into Eq.(3.5. 9) the total magnetic interaction energy over all frequencies and all space for 2 spin aligned infinite superpositions is:

$$U'\approx\frac{3\langle n\rangle^2 \lambda_C^2\alpha\hbar c}{4\pi}\left[-\frac{1}{R^3}\frac{\pi}{54(1.0045062....)}\right]$$

$$\text{We will call this } \quad U\left(\text{superpositions}\right)\approx-\left[\frac{\langle n\rangle^2 \lambda_C^2\alpha\hbar c}{72R^3(1.0045062....)}\right] \qquad (3.5.\ 13)$$

We can equate this magnetic energy to the classical value assuming the Dirac value of $g=2$ for spin ½ (No QED corrections have been applied so it must be $g=2$). For the arrangement of spins as in Figure 3.5. 1 the Dirac magnetic energy between two spin ½ states is

$$U\left(\text{Dirac}\right)=-\left[\frac{2\mu^2}{4\pi\varepsilon_o c^2 R^3}\right] \qquad (3.5.\ 14)$$

Using the Dirac magnetic moment $\mu=\frac{e\hbar}{2m_0}=\frac{e\hbar c}{2m_0 c}=\frac{ec\lambda_C}{2}$ the Dirac magnetic energy is

$$U(\text{Dirac}) = -\left[\frac{\lambda_C^2 \alpha \hbar c}{2R^3}\right]$$

The approximation used in deriving Eq. (3.5. 6) $\gamma^2\beta^2 \approx \beta^2$ for $\beta^2 <<< 1$ is true only when $R >>> \lambda_C$. This error in β^2 is of the order of λ_C^2 / R^2 and rapidly tends to zero with increasing R. There is no upper limit on the value of distance R we can choose. Thus comparing our estimate of the magnetic energy with Dirac's value when $R >>> \lambda_C$.

$$U(\text{Dirac}) = U(\text{Superpositions}) \text{ or } -\left[\frac{\lambda_C^2 \alpha \hbar c}{2R^3}\right] \approx -\left[\frac{\langle n\rangle^2 \lambda_C^2 \alpha \hbar c}{72R^3(1.0045062....)}\right] \tag{3.5. 15}$$

All symbols cancel except $\langle n\rangle$ leaving: $\langle n\rangle^2 \approx 36(1.0045062.....)$

The expectation value $\langle n\rangle$ in our superposition is slightly more than $n=6$ our dominant mode. This is why we have used a three member superposition centred on this dominant $n=6$ mode. The two side modes $n=5$ & $n=7$ are smaller so that:

$$\langle n\rangle = \sum_{n=5,6,7.} (c_n * c_n)n \approx \sqrt{36(1.0045062...)} \approx 6.01350345 \tag{3.5. 16}$$

This is for Dirac spin ½ particles. This mean value of *n* creates a $g=2$ fermion which *QED corrections* (*which are secondary interactions*) *increase slightly to the experimental value.* In section 4.1 we solve the primary electromagnetic coupling constant in terms of ratio χ_{EM} using Eq. (3.5. 16). It is important to remember this magnetic energy derivation applies to two infinite assemblies (or particles) localized in small cavities in relation to their distance R apart. They must be both on the z axis with spins aligned (or anti aligned) along this z axis as in Figure 3.5. 1 & Figure 3.5. 2. Also the agreement with Dirac and in what follows is possible *if superposition* ψ_k *interacts only with virtual photons of the same wavenumber* k.

4 High Energy Superposition Cutoffs

4.1 Electromagnetic Coupling to Spin ½ Infinite Superpositions

Equation (3.5. 16) is the key requirement for spin ½ superpositions to behave as Dirac fermions, allowing us to solve α_{EMP}^{-1} as a function of coupling ratio χ using Eq. (3.5. 16).

$$\langle n\rangle = \sum_{n=5,6,7.} (c_n * c_n)n \approx \sqrt{36(1.0045062...)} \approx 6.01350345$$

Thus $5c_5*c_5+6c_6*c_6+7c_7*c_7=6.01350345$ but $6c_5*c_5+6c_6*c_6+6c_7*c_7=6$

and $$c_7*c_7-c_5*c_5=0.01350345$$

As $c_7*c_7+c_5*c_5=1-c_6*c_6$ we can now solve for c_7*c_7 & c_5*c_5 in terms of c_6*c_6

$$c_7*c_7\approx 0.50675172-\frac{c_6*c_6}{2} \quad \& \quad c_5*c_5\approx 0.49324827-\frac{c_6*c_6}{2} \tag{4.1. 1}$$

From Eq. (2.3. 12) the Q^2A^2 required to produce this superposition with amplitudes c_n is

$$Q^2A^2=\sum_{n=5,6,7}c_n*c_n\frac{n^4\hbar^2k^4r^2}{81} \text{ and using Eq. (4.1. 1)}$$

$$\sum_{n=5,6,7}c_n*c_n\ n^4=625c_5*c_5+1296c_6*c_6+2401c_7*c_7\approx 1524.991-217c_6*c_6$$

Thus $Q^2A^2=\sum_{n=5,6,7}c_n*c_n\frac{n^4\hbar^2k^4r^2}{81}\approx[18.82705-2.67901c_6*c_6]\hbar^2k^4r^2$ is the *required* vector potential squared to produce this spin ½ superposition. From Eq. (2.2. 4) with s = ½ & $N=1$ for massive fermions $Q^2A^2=\frac{2\left[8+8\sqrt{\alpha_{EMP}}\right)\Big]^2}{3\pi}\hbar^2k^4r^2$ is the *available* Q^2A^2.

Equating *required and available*: $2\left[8+8\sqrt{\alpha_{EMP}}\right)\Big]^2\approx 3\pi[18.82705-2.67901c_6*c_6]$

$$\left[1+\sqrt{\alpha_{EMP}}\right)\Big]^2\approx[1.386256-0.197258c_6*c_6]$$

$$\alpha_{EMP}\approx\left[\sqrt{1.386256-0.197258c_6*c_6}-1\right]^2 \tag{4.1. 2}$$

From Eq's. (3.3. 1) & (3.3. 22), $c_6*c_6(1-c_6*c_6)=\sqrt{2/\chi_C}=6\sqrt{2/\chi_{EM}}$ and we can solve for α_{EMP} as a function of either χ_{EM} or χ_C. We then use Eq. (3.3. 22) again to get $\alpha_{EMS}^{=1}@k_{cutoff}$. Now both χ_{EM} and χ_C are fundamentally the same ratio differing only by 36:1, because electron superpositions have six primary charges whereas we define them as one fundamental charge (section 3.3.1) and quarks have only one colour charge (Table 2.2. 1). Because $\chi_C=\alpha_3^{-1}$ at the cutoff near L_P it is more convenient to work with. From Eq. (3.3. 22)

$$c_6*c_6=\frac{1}{2}\pm\frac{1}{2}\sqrt{1-4\sqrt{\frac{2}{\chi_C}}} \quad \text{and there are two solutions for each } \chi_C.$$

One has c_6*c_6 *dominant* with two smaller c_5*c_5 & c_7*c_7 side modes, the other is *the reverse* with c_6*c_6 *the minor player* and two larger c_5*c_5 & c_7*c_7 side modes. As the values for α_{EMP} with c_6*c_6 dominant fit the Standard Model very closely, we include only these. (This only applies to spin ½ fermions and in Table 4.3. 1 spins 1 & 2 boson superpositions have minor centre modes.) Table 4.1. 1 shows these dominant $c_6 * c_6$ mode results for $\chi_C = \alpha_3^{-1}$ at various possible cutoffs in the range $\chi_C = 50 \to 51$, as this range fits the Standard Model. Of course there can be *only one solution for this cutoff*.

Coupling Ratio χ_C	$c_6 * c_6$	$\alpha^{-1}_{\text{EMPrimary}}$	$\alpha^{-1}_{EM\text{Secondary}} @ k_{cutoff}$
50.00	≈ 0.723607	≈ 75.4414	≈ 104.7798
50.20	≈ 0.724497	≈ 75.5447	≈ 105.3429
50.40	≈ 0.725378	≈ 75.6472	≈ 105.9060
50.4053	≈ 0.725401	≈ 75.6499	≈ 105.9210
50.60	≈ 0.726250	≈ 75.7488	≈ 106.4692
50.80	≈ 0.727115	≈ 75.8497	≈ 107.0324
51.00	≈ 0.727970	≈ 75.9499	≈ 107.5956

Table 4.1. 1 Possible coupling ratios χ_C versus $\alpha^{-1}_{EMSecondary}$ in the range $\chi_C = 50 \to 51$. The yellow row corresponds to the interaction cutoff energy in Figure 4.1. 2 & Eq. (4.2. 11).

4.1.1 Comparing this with the Standard Model

In the real world of Standard Model secondary interactions the electromagnetic force splits into two components $\alpha_1 \,\&\, \alpha_2$ at energies greater than the mass/energy of the Z_0 boson or $\approx 91.1876\ GeV$. **[12]**. However we want to compare these Standard Model couplings with the values derived in Table 4.1. 1 at the $\approx 2.0288 \times 10^{18} GeV$. cutoff of Eq. (4.2. 11). Assuming three families of fermions and one Higgs field the SM [13] predicts

$$\begin{aligned} \alpha_1^{-1} &\approx 58.98 \pm 0.08 - \frac{4.1}{2\pi}\log_e \frac{Q}{91.1876} \\ \alpha_2^{-1} &\approx 29.60 \pm 0.04 + \frac{19}{6\times 2\pi}\log_e \frac{Q}{91.1876} \\ \alpha_3^{-1} &\approx 8.47 \pm 0.22 + \frac{7}{2\pi}\log_e \frac{Q}{91.1876} \end{aligned} \tag{4.1. 3}$$

The weak force split obeys $\alpha_{EM}^{-1} = \frac{5}{3}\alpha_1^{-1} + \alpha_2^{-1}$

(4.1. 4)

Also $\alpha_1^{-1} = \frac{3}{5}\alpha_{EM}^{-1} Cos^2\theta_W$ & $\alpha_2^{-1} = \alpha_{EM}^{-1} Sin^2\theta_W$ where θ_W is the Weinberg angle.

Combining Eq's. (4.1. 3) & (4.1. 4)

$$\alpha_{EM}^{-1} = \frac{5}{3}\alpha_1^{-1} + \alpha_2^{-1} \approx 127.90 \pm 0.173 - \frac{11}{3\times 2\pi}\log_e \frac{Q}{91.1876} \tag{4.1. 5}$$

Figure 4.1. 1 plots these four inverse coupling constants. Figure 4.1. 2 plots the intersection of $\alpha^{-1}_{EMSecondary}$ predicted in Table 4.1. 1 and the Standard Model prediction for α_{EM}^{-1} in Eq. (4.1. 5). It would initially seem in Figure 4.1. 2 that there is an unusually large error band in

the predicted results. However $\Delta\alpha^{-1}_{EMSecondary} / \Delta\chi^{-1} \approx 2.8$ is approximately constant in this table and the error band in the Standard Model colour coupling α_3^{-1} of ± 0.22 in Eq's (4.1. 3) translates into the larger error band for $\alpha^{-1}_{EMSecondary}$ of $\pm 0.22 \times 2.8 \approx \pm 0.62$ in Figure 4.1. 2.

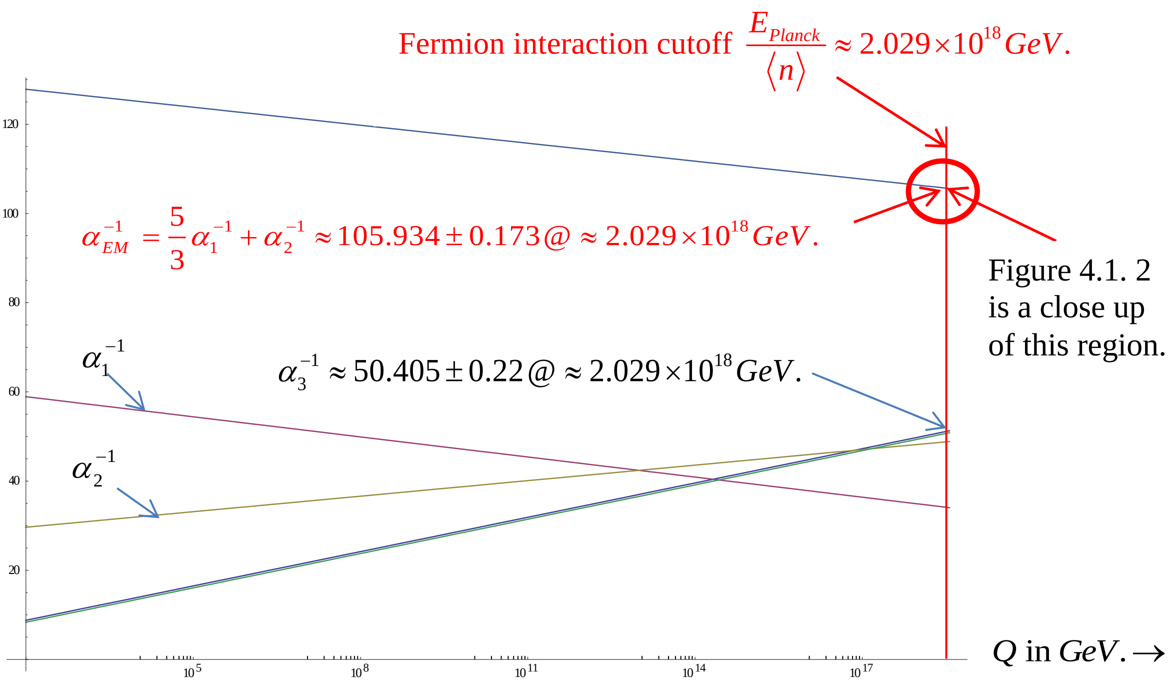


Figure 4.1. 1 Standard Model based on three families of fermions and one Higgs field.

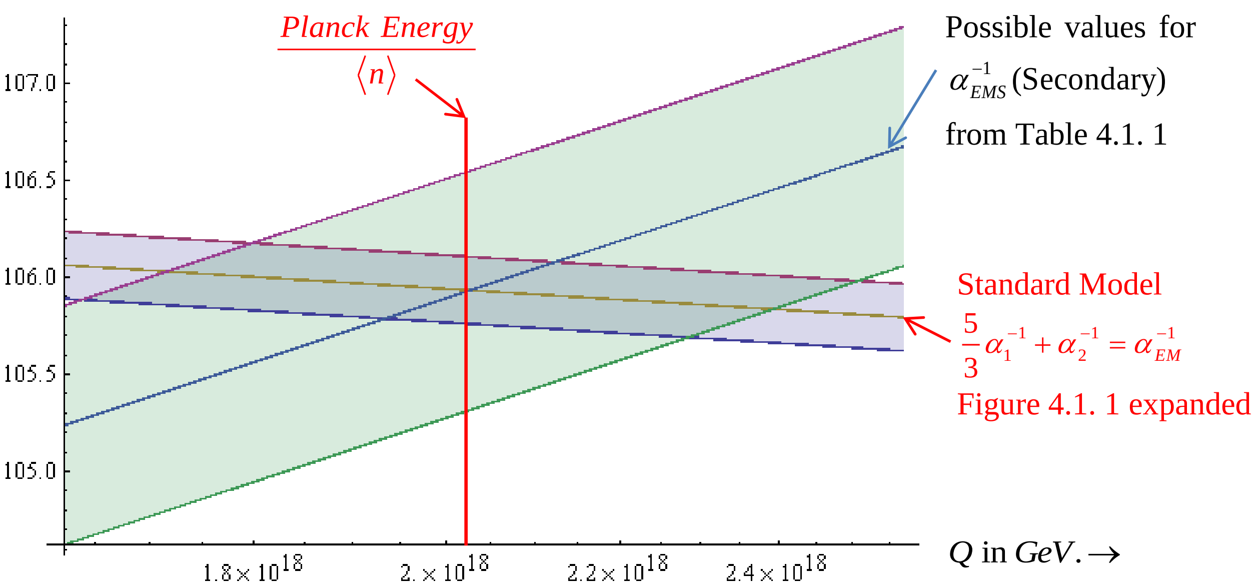


Figure 4.1. 2 A close up of the intersecting region of the Standard Model Eq. (4.1. 5) and Table 4.1. 1 predictions. This fermion interaction cutoff is perhaps more consistent with the Standard Model than we might expect; as we have assumed, for simplicity, a square superposition cutoff at k_{Cutoff}. An exponential cutoff of some type is much more likely.

4.2 Introducing Gravity into our Equations

4.2.1 Simple square superposition cutoffs

In section 3.2 we looked at single integer n superpositions of ψ_{nk} initially for clarity, and later found multiple integer n superpositions gave the same results; we will do the same here. We also found in Eq's. (3.2. 3) & (3.2. 6) that the integrals for both angular momentum and rest masses are of similar form. They both ended up including the term

$$\left[\frac{-1}{1+K_{nk}^2}\right]_0^{\infty} \text{ which if } K_{nk}cutoff < \infty \text{ becomes } \left[\frac{-1}{1+K_{nk}^2}\right]_0^{K_{nk}cutoff} \text{ and this is equal to}$$

$$1-\frac{1}{1+K_{nk}^2 cutoff}=\frac{K_{nk}^2 cutoff}{1+K_{nk}^2 cutoff}=\frac{1}{1+1/K_{nk}^2(cutoff)}=\frac{1}{1+\varepsilon} \tag{4.2. 1}$$

where using Eq. (3.1. 11) the infinitesimal $\varepsilon=\dfrac{1}{K_{nk}^2 cutoff}=\dfrac{2m_0^2c^2}{n^2\hbar^2(k_{cutoff})^2 s}$ (4.2. 2)

For integral or half integral $\hbar$ angular momentum precision is required but Eq. (3.2. 6) now gives us $\mathbf{L}_z(Total)=\dfrac{sm\hbar}{2}\left[\dfrac{-1}{1+K_{nk}^2}\right]_0^{K_{nk}cutoff}=\dfrac{sm\hbar}{2}\dfrac{1}{1+\varepsilon}$. So can the effect of gravity increase our probabilities from $sN\cdot\dfrac{dk}{k}$ to $sN\cdot(1+\varepsilon)\dfrac{dk}{k}$? We will initially address only *massive infinite superpositions* where $N=1$ in Eq. (2.2. 4).

The first question we need to address is what is the effective preon mass to be used when coupling to gravity? In Eq. (3.1. 4) we said the preon rest mass is $m_0/(8\gamma_{nk}\sqrt{2s})$ for each of the 8 preons that build a spin ½ particle of rest mass m_0. Now gravity couples to the total mass including the kinetic energy. It also couples to other terms in Einstein's energy-momentum tensor, but *we conjecture that in primary interactions such as this (section 1.1.2), gravitons only couple to the mass/energy*, and the equations are consistent only if this is so. (Sections 6.2.1 & 6.2.3 also discuss this further.)

At the start of the interaction each preon mass is $m_0/(8\gamma_{nk}\sqrt{2s})$ and after the interaction (Figure 3.1. 3) it is $m_0\gamma_{nk}(1+\beta_{nk}^2)/(8\sqrt{2s})$. Let us think semi classically again and see where it leads us. We have been using magnitudes of velocities as they are the most convenient way to express our equations even if not the conventional language of quantum mechanics. The interaction with the zero point fields takes the momentum of each preon from zero to $2m_0\gamma_{nk}\beta_{nk}c/(8\sqrt{2s})$ (Figure 3.1. 3). While this happens as a quantum step change let us imagine it as a virtually infinite acceleration from zero velocity to $2\beta_{nk}/(1+\beta_{nk}^2)$, which is the relativistic velocity addition (see Figure 3.1. 1) of 2 equal steps of β_{nk}. At the half way point after one step the velocity is β_{nk} (the velocity of the CMF, the preon mass has increased to $m_0/(8\sqrt{2s})$. We can imagine this as being like the central point of a quantum interaction.

We will conjecture this midway point preon mass $m_0/(8\sqrt{2s})$ is the mass value that gravity acts on and we will see that it is indeed the only value that fits all equations. Also it does not make sense to choose either of the end point masses. We can also get reassurance from the properties of the Feynman transition amplitude which tells us in Eq. (3.1. 15)

$\frac{(p_i+p_f)^z}{(p_i+p_f)^0}=\frac{2m_0\gamma_{nk}\beta_{nk}}{2m_0\gamma_{nk}}=\beta_{nk}$ and the ratio of space to time polarization in the LF is β_{nk}^2.

This centre of momentum velocity tells us the key properties of the interaction. We will thus assume we have 8 preons in each ψ_{nk} of effective gravitational mass $m_0/(8\sqrt{2s})$ with effective *total* gravitational mass $m_0/\sqrt{2s}$. To put the gravitational constant in the same form as the other coupling constants we need to divide it by $\hbar c$. The gravitational coupling amplitude is thus $m_0\sqrt{G_P/(2s\hbar c)}$ to the gravitational zero point field, where $\sqrt{G_P}$ *is the primary amplitude for a Planck mass to emit or absorb a graviton*. Now this gravitational amplitude can be regarded as a complex vector just as colour and electromagnetism. We assumed for simplicity, as they are both spin 1 field particles, that colour and electromagnetism are parallel. Spin 2 gravity could be at a different complex angle to the other two. In fact the equations only have the correct properties if gravity is at right angles to colour and electromagnetism. Putting $G_{Primary}=\chi'_G\cdot G_{Secondary}$ we conjecture that:

$$\text{The gravitational coupling amplitude is } im_0\sqrt{G_P/(2s\hbar c)}=im_0\sqrt{\chi'_G\cdot G_S/(2s\hbar c)} = im_0\sqrt{\chi'_G\cdot G/(2s\hbar c)} \qquad (4.2.\ 3)$$

Where we have put the *secondary gravitational coupling constant to a bare Planck mass* G_s *in Eq. (4.2. 3) equal to the measured gravitational constant G and temporarily labelled the ratio between the primary the primary and secondary gravitational constants as* χ'_G and return to this in section 6.2.6. So modifying Eq's. (2.2. 1) to (2.2. 3) by adding Eq. (4.2. 3)

$$Q^2A^2=\left[\frac{\left|8+8\sqrt{\alpha_{EMP}}+im_0\sqrt{\chi'_G\cdot G/(2s\hbar c)}\right|^2}{3\pi sN}\hbar^2k^4r^2\right]\cdot\left[\frac{sN\cdot dk}{k}\right]$$

$$Q^2A^2 = \left[\frac{\left[8+8\sqrt{\alpha_{EMP}}\right)\right]^2}{3\pi sN}\hbar^2k^4r^2\right].\left[\frac{sN(1+\varepsilon')dk}{k}\right] \text{ where } \varepsilon' = \frac{m_0^2\chi_G'\cdot G}{2s\hbar c(8+8\sqrt{\alpha_{EMP}})^2}$$

Our previous wavefunctions ψ_k required $Q^2A^2 = \frac{\left[8+8\sqrt{\alpha_{EMP}}\right)\right]^2}{3\pi sN}\hbar^2k^4r^2$ from Eq. (2.2. 4).

Thus primary graviton interaction can increase the probability of our previous wavefunctions ψ_k by $1+\varepsilon'$ as required to obtain precision in our integrals for $\hbar/2\,\&\,\hbar$ if $K_{nk}cutoff < \infty$.

Using Eq.(4.2. 2) now put $\varepsilon' = \frac{m_0^2\chi_G'\cdot G}{2s\hbar c(8+8\sqrt{\alpha_{EMP}})^2} = \varepsilon = \frac{1}{K_{nk}^2 cutoff} = \frac{2m_0^2c^2}{sn^2\hbar^2(k_{cutoff})^2}$ (4.2. 4)

Thus

$$\frac{\chi_G'\cdot G}{4\hbar c(8+8\sqrt{\alpha_{EMP}})^2} \approx \frac{c^2}{n^2\hbar^2(k_{cutoff})^2}$$

$$\frac{\chi_G'}{256(1+\sqrt{\alpha_{EMP}})^2}\frac{G\hbar}{c^3} \approx \frac{1}{n^2(k_{cutoff})^2}$$

But $L_P{}^2 = \frac{G\hbar}{c^3}$ and $\frac{\chi_G'\cdot L_P{}^2}{256(1+\sqrt{\alpha_{EMP}})^2} \approx \frac{1}{n^2(k_{cutoff})^2}$

For $N=1$ single integer n superpositions $\chi_G' \approx \frac{256(1+\sqrt{\alpha_{EMP}})^2}{n^2(k_{cutoff}L_P)^2}$ (4.2. 5)

For $N=1$ superpositions $\psi_k = \sum_n c_n\psi_{nk}$, we can use the logic of section 3.5.1; replacing K_{nk}^2 with $\langle K_k\rangle^2$, and n^2 with $\langle n\rangle^2$ in Eq. (4.2. 4), so that Eq. (4.2. **5**) becomes

For $N=1$ multiple integer n superpositions $\chi_G' \approx \frac{256(1+\sqrt{\alpha_{EMP}})^2}{\langle n\rangle^2(k_{cutoff}L_P)^2}$ (4.2. 6)

If we now go back to Eq's. (2.3. 9) & (2.3. 10) as $k\to\infty$ the energy squared $E_{nk}^2 \to \mathbf{p}_{nk}^2c^2$ $= n^2\hbar^2\omega^2$. Again using the logic of section 3.5.1 for multiple integer n superpositions the expectation value for energy squared as $k\to\infty$ is $\langle E_k\rangle^2 \to \langle|\mathbf{p}_k|\rangle^2c^2 = \langle n\rangle^2\hbar^2k^2c^2$ thus

For multiple integer n superpositions as $k\to\infty$, $\langle E_k\rangle \to \langle|\mathbf{p}_k|\rangle c = \langle n\rangle\hbar kc$ (4.2. 7)

4.2.2 All *N* = 1 superpositions cutoff at Planck Energy but interactions at less

It is reasonable to assume that the cutoff superposition energy cannot exceed the Planck energy E_{Planck} (*at least for square cutoffs*) and that this is true for all $N=1$ superpositions. (Section 6.2.1 discusses $N = 2$ superposition E_{Planck} cutoffs.) So for *simple square cutoffs*:

$$N=1 \text{ multiple integer } n \text{ superpositions cutoff energy } \left\langle E_{k(cutoff)}\right\rangle = \langle n\rangle \hbar k_{cutoff} c = E_{Planck} \tag{4.2. 8}$$

$$\text{This can be written as} \qquad \langle n\rangle k_{cutoff} \hbar c = E_{Planck} = \frac{\hbar c}{L_{Planck}}$$

$$\text{For } N=1 \text{ multiple integer } n \text{ superpositions } \langle n\rangle k_{cutoff} = \frac{1}{L_{Planck}} \quad \& \quad \langle n\rangle k_{cutoff} L_P = 1 \tag{4.2. 9}$$

$$N=1 \text{ multiple integer } n \text{ superposition interaction cutoff energy } \hbar c k_{cutoff} = \frac{E_{Planck}}{\langle n\rangle} \tag{4.2. 10}$$

Using Eq. (4.2. 10) with Planck energy $1.22\times10^{19}\,GeV.$ and $\langle n\rangle \approx 6.0135$ from Eq.(3.5. 16) for *simple square cutoffs* (also see Figure 4.1. 2).

$$\text{Interactions between } N=1 \text{ fermions cutoff @} \approx 2.0288\times10^{18}\,GeV. \tag{4.2. 11}$$

From Table 4.3. 1 we see that all other particles such as photons, gluons and gravitons etc. have $\langle n\rangle < 6$ and thus higher interaction cutoff energies than fermions ie. $> 2.03\times10^{18}\,GeV.$, but < E_P. Putting $2.0288\times10^{18}\,GeV.$ in the Standard Model equations (4.1. 3) & (4.1. 4).

$$\begin{aligned}
&\alpha_1^{-1} \text{ @ } 2.0288\times10^{18}\,GeV. \approx 34.4179 \pm 0.08 \text{ @ } k(cutoff)\\
&\alpha_2^{-1} \text{ ....} \approx 48.5707 \pm 0.04 \text{...................}\\
&\alpha_3^{-1} \text{ ....} \approx 50.4053 \pm 0.22 \text{....................}\\
&\alpha_{EM}^{-1} = \frac{5}{3}\alpha_1^{-1} + \alpha_2^{-1} \text{........} \approx 105.934 \pm 0.173 \text{..................}
\end{aligned} \tag{4.2. 12}$$

Real world high energy secondary interactions only involve α_1, α_2 & α_3, but spin zero primary interactions do not involve the weak force. Table 4.1. 1 can thus only predict $\alpha_{EM}^{-1} \approx 105.921$ at the cutoff compared to the Standard Model combination of $(5/3)\alpha_1^{-1} + \alpha_2^{-1} = \alpha_{EM}^{-1}$ $\approx 105.934 \pm 0.173$ of Eq. (4.2. 12). (See Figure 4.1. 1 & Figure 4.1. 2). Also using Eq's. (3.3. 3) & (4.2. 12) we get the primary to secondary fundamental coupling ratio χ_C.

$$\text{Coupling Ratio } \chi_C = \alpha_3^{-1} \text{ @ } k_{cutoff} \approx 50.405 \pm 0.22 \;\; (\text{ie.@ } 2.0288\times10^{18}\,GeV.) \tag{4.2. 13}$$

The secondary coupling constants in Eq. (4.2. 12) can be thought of as those to the bare colour and electromagnetic charges.

If we now put Eq. (4.2. 9) into Eq. (4.2. 6) we get $\chi'_G \approx \dfrac{256(1+\sqrt{\alpha_{EMP}})^2}{\langle n \rangle^2 (k_{cutoff} L_P)^2} = 256(1+\sqrt{\alpha_{EMP}})^2$

From Eq's.(4.1. 2) and Table 4.3. 1 we find $(1+\sqrt{\alpha_{EMP}}) \approx 1.115$ and Eq.(4.2. 6) becomes

$$\chi'_G \approx 256(1.115)^2 \approx 318.3 \tag{4.2. 14}$$

Using Eq. (4.2. 3) $\chi'_G \approx 318.3$ is the ratio between the primary graviton coupling to bare preons, and the *normal measured gravitational constant (Big* G). In other words the primary graviton coupling to preons is $\alpha_G(\text{Primary}) \approx (318.3)G$. (Section 5.1.2, Eq. (5.1. 7)) defines the secondary graviton coupling between Planck masses α_G and Eq. **(5.3. 14**) finds that $\alpha_G \approx 1$ so as in Eq.(6.2. 8) *the primary to secondary graviton coupling ratio is* $\chi_G \approx 1$ and $\chi'_G \approx 318.3$.) When $\chi'_G \approx 318.3$ in Eq.(4.2. 4) the contribution from gravity (the ε' in Eq.(4.2. 4)) cancels any deficit in primary interactions (the ε in Eq.(4.2. 4)) if these superpositions cutoff at Planck energy, which we argue is true for all $N=1$ superpositions. (Sections 6.2 & 6.2.1 discuss $N=2$ superposition E_P cutoffs.) To enable high energy interactions $N=2$ (infinitesimal mass) bosons must also cutoff at Planck energy just as $N=1$ superpositions do, or as in Eq. (4.2. 10). Figure 4.2. 1 plots radial probabilities for all $n=3,4,5,6\,\&\,7$ Planck Energy cutoff modes. They are identical as the radial probability $P_R \propto r^8 \times Exp(n^2k^2r^2/9)$, but from Eq. (4.2. 7) $nk=1$ in each Planck energy mode, so they all have radial probability $P_R \approx 8.74\times10^{-6} \times r^8 Exp(r^2/9)$.

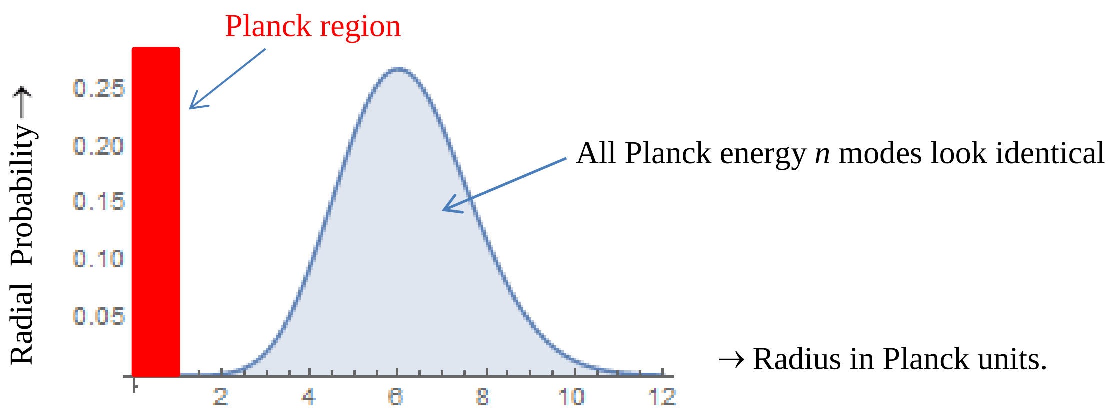


Figure 4.2. 1

Despite each $n=3,4,5,6\,\&\,7$ mode having Planck energy the probability in every case of being inside the Planck region is virtually zero at $\approx 8.9\times10^{-7}$.

4.3 Solving for spin ½, spin 1 and spin 2 superpositions

Superpositions with $N=2$ are covered in section 6.2 but Eq.(4.2. 13) and Eq. (3.3. 22) extended by keeping $N \cdot s$ constant as in Eq. (4.4. 1) allow us to solve various combinations of spins ½, 1 or 2 and $N=1$ or $N=2$.

$$\begin{array}{ccccc} (N=2)\times(\text{Spin } 1) & & (N=1)\times(\text{Spin } 1) & & (N=1)\times(\text{Spin } \tfrac{1}{2}) \\ \text{or } (N=1)\times(\text{Spin } 2) & & \text{or } (N=2)\times(\text{Spin } \tfrac{1}{2}) & & \\ 4c_{4b}*c_{4b}(1-c_{4b}*c_{4b}) & = & 2c_{5b}*c_{5b}(1-c_{5b}*c_{5b}) & = & c_{6a}*c_{6a}(1-c_{6a}*c_{6a}) \end{array}$$
$$=\sqrt{2/\chi_C} \approx \sqrt{2/50.4053} \approx 0.199194 \tag{4.4. 1}$$

Starting with spin ½ we can solve this to get $c_6 * c_6 \approx 0.7254$ as the dominant value. Putting $c_6 * c_6 \approx 0.7254$ into Eq.(4.1. 2) or alternatively using Table 4.1. 1

$$\alpha_{EMP} \approx \left[\sqrt{1.386256-0.197258c_6 * c_6} - 1\right]^2 \approx 75.6499^{-1} \tag{4.4. 2}$$

From Eq. (2.2. 4) the available $Q^2A^2 = \dfrac{\left[8+8\sqrt{\alpha_{EMP}}\,)\right]^2}{3\pi sN}\hbar^2k^4r^2$ with probability $\dfrac{sN \cdot dk}{k}$ where we ignore the infinitesimal factor of $(1+\varepsilon)$ due to gravitons. And from Eq. (2.3. 12)

$$Q^2A^2 = \sum_n c_n * c_n \frac{n^4\hbar^2k^4r^2}{81} = \frac{\left[8+8\sqrt{\alpha_{EMP}}\,)\right]^2}{3\pi sN}\hbar^2k^4r^2$$
$$\begin{aligned} \sum c_n * c_n \cdot n^4 &\approx 1367.58 \text{ for (spin } 1/2\times N=1) \\ &\approx 683.79 \text{ for (spin } 1\times N=1) \text{ or (spin } 1/2\times N=2) \\ &\approx 341.9 \text{ for (spin } 1\times N=2) \text{ or (spin } 2\times N=1) \\ &\approx 170.95 \text{ for (spin 2, } N=2) \text{ by extension.} \end{aligned} \tag{4.4. 3}$$

The same primary electromagnetic coupling α_{EMP} builds all fundamental particles, allowing Eq.(4.4. 3) to be true. Using Eq's (4.4. 1),(4.4. 3) & $\sum_n c_n * c_n = 1$ we get Table 4.3. 1. We define the coupling ratio for gravitons $\chi_G \approx 23,200$ in Eq.(6.2. 8) section 6.2.6, where we also solve infinitesimal mass graviton superpositions. In Table 4.3. 1 three member superpositions fit the Standard Model best. In section 4.1 we solved spin ½ superpositions with a dominant centre mode $c_6 * c_6$ that fitted the Standard Model. However when solving for spins 1 & 2 we must initially comply with Eq. (4.4. 1) which defines interaction probabilities (see Eq. (3.3. 22) and final paragraph section 3.3.4). We must also comply with Eq.(4.4. 3) which determines centre or side mode dominance. In this table we have also included *a massive* $N=1$ *spin 2 graviton type Dark Matter possibility interacting only with* $N=2$ *spin 2 gravitons.* There are other possibilities which we have not included. To this point this paper has attempted to demonstrate that infinite superpositions can behave as the Standard Model fundamental particles.

The methods used may seem unconventional, but it is important to remember that primary interactions are very different to secondary interactions (see sections 2.2.2 & 2.2.3).These methods are however based on simple basic quantum mechanics and special relativity. There is also surprising consistency with the Standard Model. If the principles behind the outcomes of these derivations are at least on the right track, and fundamental particles can be built by borrowing energy and mass from zero point fields then, as we will see in what follows, this may possibly have some significant and profound consequences.

Mass Type	Spin	N	$c_3 * c_3$	$c_4 * c_4$	$c_5 * c_5$	$c_6 * c_6$	$c_7 * c_7$
Infinitesimal mass gravitons	2	2	0.8317	0.0039	0.1644		
Infinitesimal mass bosons	1	2	0.4847	0.0526	0.4627		
Massive (dark matter?) gravitons	2	1	0.4847	0.0526	0.4627		
Massive bosons	1	1		0.0134	0.8878	0.0988	
Massive fermions	½	1			0.1305	0.7254	0.1441

Table 4.3. 1 Approximate probabilities for various possible superpositions.

5 The Expanding Universe and General Relativity

5.1 Zero point energy densities are limited

If the fundamental particles can be built from energy borrowed from the spatial component of zero point fields and this energy source is limited, (particularly at cosmic wavelengths) there must be implications for the maximum possible densities of these particles. In section 2.2.3 we discussed how the preons that build fundamental particles are born from a Higg's type scalar field with zero momentum in the laboratory rest frame. Infinitesimal mass particles such as gravitons borrow their mass from the time component of the same zero point fields.In this frame they have infinite wavelength and can borrow from anywhere in the universe. This suggests there should be little effect on localized densities, but possibly on overall average densities in any universe. So which fundamental particle is there likely to be most of? Working in Planck, or natural units with $G = 1$ and a graviton coupling constant between Planck masses of one, there are approximately $M \approx 10^{61}$ Planck masses within the causally connected observable universe. Their average distance apart is approximately the radius R_{OH} of this region. Thus there should be approximately $M^2 \approx 10^{122}$ virtual gravitons with wavelengths of the order of radius R_{OH} within this same volume. No other fundamental particle is likely to approach these values, for example the number of virtual photons *of this extreme wavelength* is much smaller. (Virtual particles emerging from the vacuum are covered in section 6.2.3.) If this density of virtual gravitons needs to borrow more energy from the zero point fields than what is available at these extreme wavelengths does this somehow control the maximum possible density of a causally connected universe?

5.1.1 Virtual Particles and Infinite Superpositions

Looking carefully at Section 3.3 we showed there that, for all interactions between fundamental particles represented as infinite superpositions, the actual interaction is only between a single wavenumber k superposition of each particle. *We are going to conjecture that a virtual particle of wavenumber k for example is just such a single wavenumber k member*. Only if we actually measure the properties of real particles do we observe the properties of the full infinite superposition. The full properties do not exist until measurement, just as in so many other examples in quantum mechanics. We will use this conjectured virtual property below when looking at the probability density of virtual gravitons of the maximum cutoff wavelength. These virtual gravitons would be a superposition of the three modes $n=3,4,5$ as in Table 4.3. 1, but of a single wavenumber k only. Time polarized, or spherically symmetric, versions we conjecture (See section 7.1.1) are a further equal $(1/\sqrt{3})$ superposition of $m=-2,0,+2$ states of the above $n=3,4,5$ mode superpositions. A spin 2 virtual graviton in an $m=+2$ state is simply a superposition of the three modes $n=3,4,5$ as above but all in an $m=+2$ state.

5.1.2 Virtual graviton density at wavenumber k in a causally connected Universe

From here on we will use Planck units $\hbar=c=G=1$. When we looked at scalar potentials between electric charges in section 3.4 we used time polarized virtual photons in a simple example that works for both photons and gravitons, using field energy densities rather than exchanged 4 momentum. However in quantum field theory, scalar or coulomb forces are due to exchanged 3 momentum with time polarized photons (or gravitons), but gravitational forces appear to be due to changes in the metric, and not exchanged 3 momentum. However, in the meantime we will continue to use time polarized graviton densities as earlier with photons, discussing exchanged momentum in section 5.3.8 . Also if observers, at the centre of their universe, are moving at a peculiar velocity β_P relative to comoving coordinates, the average velocity of all mass in the universe is moving with the opposite velocity $-\beta_P$. Over all thin spherical shells of matter at the same radius, they can choose pairs of small areas in opposite directions. The spatially polarized vectors due to their velocity exactly cancel for all pairs. Central observers see only time polarized gravitons regardless of peculiar velocity. This is the same as magnetic vectors cancelling at the centre of long current conductors. Spin 2 gravitons couple to the stress tensor in contrast to 4 currents for spin 1. Because of the above the only important term is the mass/energy density T_{00} (or simply ρ) or its transformed value in any other coordinates, as flow of momentum density terms cancel out. We can thus use the same wavefunctions for time polarized graviton densities as we did for photons. Using $(\psi_1+\psi_2)^*(\psi_1+\psi_2)=\psi_1{}^*\psi_1+\psi_1{}^*\psi_2+\psi_2{}^*\psi_1+\psi_2{}^*\psi_2$ we showed in section 3.4.1 (see Figure 3.4. 2) that the interaction term for virtual photons but now for gravitons is

$$\psi_1{}^*\psi_2+\psi_2{}^*\psi_1=\frac{4k}{4\pi r_1r_2}e^{-k(r_1+r_2)}\cos k(r_1-r_2) \tag{5.1. 1}$$

This equation is strictly true only in flat space but it is still approximately true if the curvature is small or when $2m/r <<<< 1$, *which we will assume applies almost everywhere throughout the universe except in the infinitesimal fraction of space close to black holes*. In both sections 3.4 & 3.5, for simplicity and clarity, we delayed using coupling constants and emission probabilities in the wavefunctions until necessary. We do the same here. There will also be some minimum wavenumber k which *we call* $k_{\min}$ where for all $k < k_{\min}$ there will be insufficient zero point energy available, and Eq. (5.1. 1) cuts off exponentially. We will find that this maximum wavelength is where $k_{\min} \approx 1/R_{OU} (=1/R_{ObsevableUniverse})$. In Section 6 we find gravitons have an infinitesimal rest mass m_0 of the same order as this minimum wavenumber $k_{\min}$. At these extreme k values this rest mass must be included in the wavefunction exponential term. It is normally irrelevant for infinitesimal masses. Section 6.2 looks at $N=2$ infinitesimal rest masses finding $\langle K_{k\min}\rangle^2 \approx 1$. Using Eq.(3.1. 11) with $\hbar=c=1$

$$\langle K_{k\min}\rangle^2=\frac{s\langle n\rangle^2 k_{\min}^2}{2m_0{}^2}\approx 1 \text{ and for spin 2 gravitons } \frac{\langle n\rangle^2 k_{\min}^2}{m_0{}^2}\approx 1 \text{ or } m_0\approx\langle n\rangle k_{\min} \tag{5.1. 2}$$

From Table 4.3. 1 we find

$$\text{For } N=2 \text{ spin 2 gravitons } \langle n\rangle \approx 3.33 \quad \text{so that} \quad m_0 \approx 3.33k_{\min} \tag{5.1. 3}$$

This virtual mass m_0 increases the ΔE term in $\Delta E\cdot\Delta T \approx \hbar/2$ for a virtual graviton from $\Delta E = k$ to $\Delta E=\sqrt{k^2+m_0^2}$ when $\hbar=c=1$. This reduces the range $r \approx \Delta T \approx \Delta E^{-1}$ over which it can be found, which is controlled by the exponential decay term e^{-kr} in its wavefunction. This term becomes $e^{-\sqrt{k^2+m_0^2}}$ as we approach $k_{\min}$. So we can define a k' using Eq. (5.1. 3)

$$k'=\sqrt{k^2+m_0^2}\approx\sqrt{k^2+11.09k_{\min}^2} \quad \text{and} \quad k'_{\min}\approx\sqrt{k_{\min}^2+11.09k_{\min}^2}\approx 3.477k_{\min} \tag{5.1. 4}$$

The normalized virtual graviton wavefunction in Eq. (3.4. 1)

$$\text{A massless } \psi=\sqrt{\frac{2k}{4\pi}}\frac{e^{-kr+ikr}}{r} \text{ becomes with infinitesimal mass } \sqrt{\frac{2k'}{4\pi}}\frac{e^{-k'r+ikr}}{r} \tag{5.1. 5}$$

Thus the massless interaction term in Eq. (5.1. 1) becomes with this infinitesimal mass m_0

$$\psi_1{}^*\psi_2+\psi_2{}^*\psi_1=\frac{4k'}{4\pi r_1r_2}e^{-k'(r_1+r_2)}\cos k(r_1-r_2) \tag{5.1. 6}$$

Let point P in Figure 5.1. 1 be anywhere in the interior region of a typical universe. . Let the average density (or its equivalent transformed value) be ρ_U (subscript U for universe) Planck masses/energy density per unit volume. Consider two spherical shells initially in comoving coordinates around the central point P of radii r_1 & r_2 and thicknesses dr_1 & dr_2 with masses $dm_1 = \rho_U dv_1 = 4\pi r_1^2 dr_1 \rho_U$ & $dm_2 = \rho_U dv_2 = 4\pi r_2^2 dr_2 \rho_U$

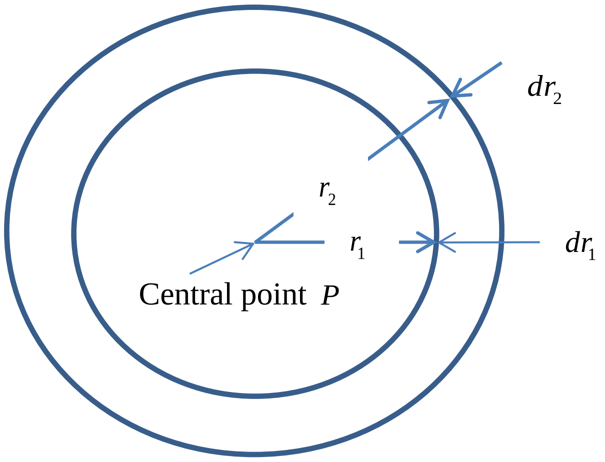


Figure 5.1. 1

Now we expect the graviton coupling constant α_G to be $=1$ between Planck masses, but we will assume we don't know this and solve it in section 0 to find it appears to be true.

$$\text{The Secondary graviton coupling constant between Planck masses } = \alpha_G \qquad (5.1.\ 7)$$

Section 3.4.1 in Eq. (3.4. 3) used a scalar emission probability $(2\alpha/\pi)(dk/k)$ which becomes $(2\alpha_G/\pi)(dk/k)$ between Planck masses. But we must include $(1-Exp[-0.61k^2/k_{\min}^2])$ as an exponential cutoff @ $k_{\min}$ as in Eq. **(6.2. 7**). Now distant galaxies recede at light like and greater velocities, but quantum interactions are instantaneous over all space. Thus as we integrate over radii r_1 & $r_2 = 0 \to \infty$ we can still use the same equations as if the distant galaxies are not moving. (The vast majority of mass is moving relatively slowly in these comoving coordinate systems and we return to this important comoving coordinate property in section 5.3.1). Using this new coupling probability between Planck masses $(2\alpha_G/\pi)(dk/k)$ we can now integrate over both radii r_1 & r_2; but to avoid counting all pairs of masses dm_1 & dm_2 twice, we must divide the integral by two. The total probability density of virtual gravitons at any point P in the universe at wavenumber k is using Eq.(5.1. 6)

$$\rho_{Gk} = \frac{\rho_U^2}{2}\alpha_G(1-e^{-0.61k^2/k_{\min}^2})\frac{2}{\pi}\frac{dk}{k}\iint\limits_{0\to\infty} 4\pi r_1^2 dr_1 \cdot 4\pi r_2^2 dr_2 \cdot \frac{4k'}{4\pi r_1 r_2} e^{-k'(r_1+r_2)} \cos k(r_1-r_2)$$

$$= 16\alpha_G(1-e^{-0.61k^2/k_{\min}^2})\rho_U^2 \frac{k'}{k} dk \iint\limits_{0\to\infty} r_1 r_2 e^{-k'(r_1+r_2)} \cos k(r_1-r_2)\cdot dr_1 \cdot dr_2$$

Expanding $\cos k(r_1 - r_2) = \cos kr_1 \cos kr_2 + \sin kr_1 \sin kr_2$, then using:

$$\int_{r=0}^{r=\infty} rExp(-k'r)\cos kr = \frac{k'^2 - k^2}{(k'^2 + k^2)^2} \quad \text{and} \quad \int_{r=0}^{r=\infty} rExp(-k'r)\sin kr = \frac{2k'k}{(k'^2 + k^2)^2}$$

$$\text{yields } \rho_{Gk} = 16\alpha_G (1 - e^{-0.61k^2/k_{\min}^2})\, \rho_U^2 \frac{k'}{k} dk \frac{(k'^2 + k^2)^2}{(k'^2 + k^2)^4} \tag{5.1. 8}$$

$$= 16\alpha_G (1 - e^{-0.61k^2/k_{\min}^2})\rho_U^2 \frac{k'}{k} dk \frac{1}{(k'^2 + k^2)^2}$$

From Eq.(5.1. 4) $k' = \sqrt{k^2 + m_0^2} \approx \sqrt{k^2 + 11.09k_{\min}^2}$ and we can write Eq.(5.1. 8) as

$$\rho_{Gk} = 16\alpha_G \rho_U^2 (1 - e^{-0.61k^2/k_{\min}^2}) \frac{\sqrt{k^2 + 11.09k_{\min}^2}}{\left(2k^2 + 11.09k_{\min}^2\right)^2} \frac{dk}{k}$$

$$= 16\alpha_G \frac{\rho_U^2}{k_{\min}^4} dk \frac{(1 - e^{-0.61x^2})\sqrt{x^2 + 11.09}}{x\left(2x^2 + 11.09\right)^2} \quad \text{where } x = \frac{k}{k_{\min}}$$

$$\text{Wavelength } k \text{ Probability Density } \rho_{Gk} \approx \frac{0.149\alpha_G \rho_U^2}{k_{\min}^4} dk \left[\frac{108}{x} \frac{(1 - e^{-0.61k^2/k_{\min}^2})\sqrt{x^2 + 11.09}}{\left(2x^2 + 11.09\right)^2} \right]$$

Where the blue square bracket is 1 when $k / k_{\min} = x = 1$

$$\text{Cutoff wavelength Probability Density } \rho_{Gk\min} \approx \frac{0.149\alpha_G \rho_U^2}{k_{\min}^4} dk_{\min} \quad \text{when } \frac{k}{k_{\min}} = x = 1 \tag{5.1. 9}$$

As we think $K_{G\min}$ will prove to a spacetime invariant we will write this as follows.

$$\text{Cutoff wavelength Probability Density } \rho_{Gk\min} = K_{Gk\min} dk_{\min} \text{ where } K_{Gk\min} \approx \frac{0.149\alpha_G \rho_U^2}{k_{\min}^4} \tag{5.1. 10}$$

5.2 Can we relate all this to General Relativity?

The above assumes a homogeneous universe that is essentially flat on average. At any cosmic time T it also assumes there is always some value $k_{\min}$ where the borrowed energy density $E_{Gk\min} = E_{ZP\min}$ the available zero point energy density @ $k_{\min}$. We have initially assumed comoving coordinates, but at peculiar velocities our spherical shells become ellipses and our equation $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ should remain true at any peculiar velocity, also in all coordinates as we hope to show later. So what happens if we put a small mass concentration $+m_1$ at some point? The gravitons it emits must surely increase the local density of $k_{\min}$ gravitons upsetting the balance between borrowed energy and that available. However General Relativity tells us that near mass concentrations the metric changes, radial rulers shrink and local observers measure larger radial lengths. This expands locally measured volumes lowering their measurement of the background $\rho_{Gk\min}$. But clocks slowdown also, increasing the locally measured value of $k_{\min}$. Let us look at whether we can relate these changes in rulers and clocks with the $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ of Eq. (5.1. 10).

5.2.1 Approximations with possibly important consequences

Let us refer back to Eq. (3.4. 2) and the steps we took in section 3.4.1 to derive it; but now including $k' = \sqrt{k + m_0} \approx \sqrt{k^2 + 11.09 k_{\min}^2}$ as in Eq.(5.1. 4)

$$\psi_1 {}^* \psi_2 + \psi_2 {}^* \psi_1 = \frac{4k'}{4\pi r_1 r_2} e^{-k'(r_1 + r_2)} \cos[k(r_1 - r_2)] \tag{5.2. 1}$$

Assume that *space has to be approximately flat* with errors $\propto 1 - (1 - 2m/r)^{1/2} \approx m/r$. If we now focus on Figure 3.4. 2 , equation (5.2. 1) is the probability that a virtual graviton of wavenumber k is at the point P *if all other factors are one*. Let us now put a mass of m_1 Planck masses at the Source 1 point in Figure 3.4. 2 or as in Figure 5.2. 1.

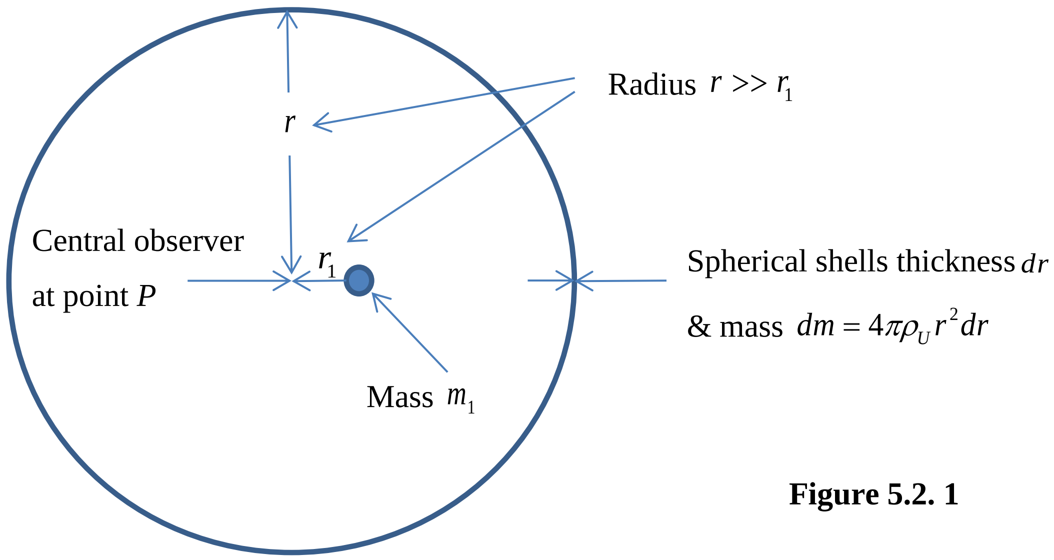


Figure 5.2. 1

Also assume that the *point P is reasonably close to mass* m_1 *(in relation to the horizon radius)* at distance r_1 as in Figure 5.2. 1 and *the vast majority of the rest of the mass inside the causally connected or observable horizon* R_{OH} is at various radii r, equal to r_2 of Eq.(5.2. 1) where $r_2 = r >> r_1$ and thus $\cos[k(r_1 - r)] \approx \cos(-kr)$ & $e^{-k'(r_1+r_2)} \approx e^{-k'r}$. This is equivalent to localizing General Relativity to much smaller than horizon radii, but still to vast cosmic radii. *Only under these conditions can we approximate Eq.* (5.2. 1) *as*

$$\psi_1 {}^* \psi_2 + \psi_2 {}^* \psi_1 \approx \frac{4k'}{4\pi r_1 r} e^{-k'r} \cos(-kr) \tag{5.2. 2}$$

The background gravitons are time polarized and we are effectively looking at the scalar potential of this central mass relative to the rest of the universe, so this is a time polarized or scalar interaction with no directional effects due to spatial polarization. We can consider simple spherical shells (again initially in comoving coordinates) of thickness dr and radius r around a central observer at the point P which have mass $dm = \rho_U 4\pi r^2 dr$. At each radius r the coupling factor including an exponential cutoff is $(1-e^{-0.61k^2/k_{\min}^2})(2\alpha_G/\pi)(dk/k)$ between Planck masses. Again assuming instantaneous quantum coupling as if space is not expanding:

$$\text{Coupling factor } = (1-e^{-0.61k^2/k_{\min}^2})\frac{2\alpha_G m_1}{\pi}dm\frac{dk}{k} = (1-e^{-0.61k^2/k_{\min}^2})\frac{2\alpha_G m_1}{\pi}\rho_U 4\pi r^2 dr\frac{dk}{k} \tag{5.2. 3}$$

Including this coupling factor in Eq. (5.2. 2)

$$\begin{aligned}\text{Including this coupling factor } & (1-e^{-0.61k^2/k_{\min}^2})\left(\frac{2\alpha_G m_1}{\pi}\frac{dk}{k}\rho_U 4\pi r^2 dr\right)(\psi_1{}^*\psi_2+\psi_2{}^*\psi_1) \\ \approx\; & (1-e^{-0.61k^2/k_{\min}^2})\left(\frac{2\alpha_G m_1}{\pi}\frac{dk}{k}\rho_U 4\pi r^2 dr\right)\left(\frac{4k'}{4\pi r_1 r}e^{-k'r}\cos(-kr)\right) \\ \approx\; & (1-e^{-0.61k^2/k_{\min}^2})\alpha_G\frac{m_1}{r_1}\frac{8\rho_U}{\pi}\frac{k'dk}{k}re^{-k'r}\cos(-kr)dr\end{aligned} \tag{5.2. 4}$$

This is virtual graviton density at point P due to each spherical shell. (Ignoring the relatively *small number of particularly* $k_{\min}$ gravitons emitted by mass m_1 itself $\psi_{m_1}{}^*\psi_{m_1}$, (see section ????). Integrating over radius $r = 0 \to \infty$ the virtual graviton density at wavenumber k using Eq's.(5.1. 4) & (5.2. 4)

$$\begin{aligned}\Delta\rho_G &= (1-e^{-0.61k^2/k_{\min}^2})\alpha_G\frac{m_1}{r_1}\frac{8\rho_U}{\pi}\frac{k'dk}{k}\int_0^\infty re^{-k'r}\cos(-kr)dr \\ &= (1-e^{-0.61k^2/k_{\min}^2})\alpha_G\frac{m_1}{r_1}\frac{8\rho_U}{\pi}\frac{k'dk}{k}\left[\frac{(k'^2-k^2)}{(k'^2+k^2)^2}\right]\end{aligned} \tag{5.2. 5}$$

Now $k'^2 = k^2 + m_0^2 \approx k^2 + 11.09k_{\min}^2$ and if $k = k_{\min}$ then $k_{\min}'^2 \approx 12.09k_{\min}^2$ and so when $k = k_{\min}$:

$$\begin{aligned}\Delta\rho_{Gk\min} &\approx (1-e^{-0.61k^2/k_{\min}^2})\alpha_G\frac{m_1}{r_1}\frac{8\rho_U}{\pi}\frac{\sqrt{12.09k_{\min}^2}\,dk_{\min}}{k_{\min}}\left[\frac{(12.09k_{\min}^2-k_{\min}^2)}{(12.09k_{\min}^2+k_{\min}^2)^2}\right] \\ \Delta\rho_{Gk\min} &= (\psi_{\text{Universe}}{}^*\psi_{m_1}+\psi_{m_1}{}^*\psi_{\text{Universe}}) \approx (1-e^{-0.61k^2/k_{\min}^2})\cdot\alpha_G\frac{m_1}{r_1}0.573\frac{\rho_U}{k_{\min}^2}dk_{\min}\end{aligned} \tag{5.2. 6}$$

Equation (5.1. 10) hypothesizes $\rho_{Gk\min} = K_{Gk\min}dk_{\min}$. In a metric far from masses where $g_{\mu\nu} = \eta_{\mu\nu}$, $k_{\min}$ has its lowest value. As we approach any mass $k_{\min}$ increases to $k''_{\min}$ where we use green double primes when $g_{\mu\nu} \neq \eta_{\mu\nu}$ to avoid confusion with the k' & $k'_{\min}$ of Eq.(5.1. 4). At a radius r from mass m the Schwarzchild metric is $(1-2m/r)^{\pm 1/2}$ for the time and radial terms. Radial rulers shrink and clocks slow, measured volumes and frequencies both increase locally as $\approx 1+\dfrac{m}{r}$.

$$\text{Thus both } \frac{V+\Delta V}{V} \approx 1+\frac{m}{r} \text{ and also } \frac{k''_{\min}}{k_{\min}} \approx 1+\frac{m}{r} \text{ if } r >>> m$$

$$\text{Then using } \rho_{Gk\min} = K_{Gk\min} dk_{\min} \;\; \& \;\; \rho''_{Gk\min} = K_{Gk\min} dk''_{\min}$$

$$1+\frac{m}{r} \approx \frac{V+\Delta V}{V} = 1+\frac{\Delta V}{V} \approx \frac{k''_{\min}}{k_{\min}} = \frac{dk''_{\min}}{dk_{\min}} = \frac{\rho''_{k\min}}{\rho_{k\min}} \tag{5.2. 7}$$

So in this metric the total number of $k_{\min}$ gravitons is the original $(g_{\mu\nu} = \eta_{\mu\nu})$ $\rho_{Gk\min}$ of Eq. (5.1. 10) plus the extra due to a local mass of Eq. (5.2. 6), but we have to divide this number by the increased volume to get the new density $\rho''_{Gk\min} \approx (1+\dfrac{m}{r})\rho_{Gk\min}$. Thus using Eq. (5.2. 7)

$$\text{The new } \rho''_{Gk\min} \approx \frac{\rho_{Gk\min}+\Delta\rho_{Gk\min}}{1+\Delta V/V} \approx \frac{\rho_{Gk\min}+\Delta\rho_{Gk\min}}{(1+m/r)} \approx (1+m/r)\rho_{Gk\min}$$

$$\rho_{Gk\min}+\Delta\rho_{Gk\min} = (1+m/r)^2\rho_{Gk\min} \approx (1+2m/r)\rho_{Gk\min} \qquad (\text{if } r >>> m)$$

$$\frac{\rho_{Gk\min}+\Delta\rho_{Gk\min}}{\rho_{Gk\min}} \approx 1+\frac{2m}{r} \qquad \text{or} \qquad \frac{\Delta\rho_{Gk\min}}{\rho_{Gk\min}} \approx 2\frac{m}{r} \tag{5.2. 8}$$

We can now put Eq's.

(5.1. **9**) & (5.2. 6) into (5.2. 8) and dropping the now unnecessary subscripts, the graviton coupling constant α_G and exponential cutoff $(1-e^{-0.61k^2/k^2_{\min}})$ cancel:

$$\frac{\Delta\rho_{Gk\min}}{\rho_{Gk\min}} \approx \frac{(1-e^{-0.61k^2/k^2_{\min}})\alpha_G \left[\dfrac{m}{r}\right] 0.573\dfrac{\rho_U}{k^2_{\min}} dk_{\min}}{(1-e^{-0.61k^2/k^2_{\min}})\alpha_G\, 0.3247\dfrac{\rho_U^2}{k^4_{\min}} dk_{\min}} \approx \left[\frac{m}{r}\right]\left[\frac{1.765k^2_{\min}}{\rho_U}\right] \approx 2\frac{m}{r} \tag{5.2. 9}$$

(Strictly speaking we should be using $dk''_{k\min}$ in the top line of this equation but the error is second order as we are approximating with $r >>> m$. We will do this more accurately below for large masses.) In any metric both ρ_U & $k_{\min}$ transform their values but $k^2_{\min}/\rho_U$ is invariant.

For the above to be consistent with General Relativity this suggests that:

"At all points inside the horizon, and at any cosmic time *T*, the red highlighted part is ≈ 2 in Planck units. This is simply equivalent to putting $G/c^2 = 1 = G = c$".

Thus we can say

$$\text{The average density of the universe } \rho_U \approx (0.8823)k^2_{\min} \approx 0.8823\frac{\Upsilon^2}{R^2_{OU}} \tag{5.2. 10}$$

Where the parameter $\Upsilon = k_{\min}R_{OH}$ is in radians, and Υ is close to 1.

Putting Eq. (5.2. 10) the average density ρ_U into Eq. (5.1. 10) gives $\rho_{Gk\min}$ & $K_{Gk\min}$.

$$\text{Cutoff Wavelength Graviton Probability Density } \rho_{Gk\min} \approx \frac{0.149\alpha_G \rho_U^2}{k_{\min}^4} dk_{\min}$$

$$\rho_{Gk\min} \approx \frac{0.149\alpha_G (0.8823 k_{\min}^2)^2}{k_{\min}^4} dk \approx 0.115\alpha_G dk_{\min} = K_{Gk\min} dk_{\min} \tag{5.2. 11}$$

Where we label $K_{Gk\min} \approx 0.115\alpha_G$ as "The $k_{\min}$ Graviton Invariant".

If our conjectures are true, this is the number density of maximum wavelength gravitons excluding possible effects of virtual particles emerging from the vacuum. In section 6.2.3 we argue these do not change the $K_{Gk\min}$ of Eq. (5.2. 11). However $K_{Gk\min}$ does depend on the graviton coupling constant α_G between Planck masses, but α_G cancels out in Eq. (5.2. 9).
It does not affect the allowed universe average density ρ_U in Eq. (5.2. 10).

5.2.2 The Schwarzchild metric near large masses

At a radius r from a mass m (dropping the now unnecessary suffixes) the Schwarzchild metric is $(1-2m/r)^{\pm 1/2}$ for the time and radial terms which can be written as

$$\sqrt{g_{rr}} = \frac{1}{\sqrt{1-2m/r}} = \frac{1}{\sqrt{g_{00}}} = \frac{1}{\sqrt{1-\beta_M^2}} = \gamma_M \tag{5.2. 12}$$

Velocity β_M ($c=1$) is that reached by a small mass falling from infinity and $\gamma_M^{\pm 1}$ is the metric change in clocks and rulers due to mass m. We are using green symbols with the subscript M for metrics $g_{\mu\nu} \neq \eta_{\mu\nu}$ as we did for $k''_{\min}$ above. The symbols $\gamma_M^{\pm 1}$ help clarity in what follows.

$$\beta_M^2 = \frac{2m}{r}$$

$$\gamma_M^2 = \frac{1}{1-2m/r} = g_{rr} = \frac{1}{g_{00}}$$

$$\text{Using these symbols } k''_{\min} = \gamma_M k_{\min} \quad \& \quad dk''_{\min} = \gamma_M dk_{\min} \quad \& \quad \rho''_{Gk\min} = \gamma_M \rho_{Gk\min} \tag{5.2. 13}$$

In sections 5.1.2 & 5.2.1 we approximated in flat space. The wavelength of $k_{\min}$ gravitons span approximately to the horizon. They fill all of space. We can think of the non flat space around even a large black hole as an infinitesimal bubble on the scale of the observable universe. The normalizing constant of a $k_{\min}$ wavefunction emitted from a localized mass is only altered very close to this mass. Over the vast majority of space it is unaltered. Only close to this mass will local observers measure $k''_{\min} = \gamma_M k_{\min}$ due to the change in clocks. There is also a local dilution of the normalizing constant due to the change in radial rulers. We will consider both these changes in two steps to help illustrate our argument. Now repeat the derivation of $\Delta\rho_{Gk\min}$ as in section 5.2.1 but with a large central mass as in Figure 5.2. 1.

At the point P consider Eq. (5.2. 2) $\psi_1 {}^* \psi_2 + \psi_2 {}^* \psi_1 \approx \frac{4k'}{4\pi r_1 r} e^{-k'r} \cos(-kr)$.

The red part is the normalizing factor discussed above where we will *initially ignore the dilution* due to the local increase in volume. In deriving Eq**. (5.2. 2**) we ignored the exponential decay term and phase angle term from the local mass as (even in the space around large black holes) $e^{-k'_{\min} r_1}$ & $\cos(-k_{\min} r_1) \approx 1$. The green $k'r$ & kr terms are phase angles that are virtually fixed by the time they approach even large black holes, as they apply to the vastly distant masses, and only increase infinitesimally in any local metric. So treating them as fixed and *ignoring the dilution factor* this equation is unaltered. As the exponential cutoff is unchanged we are left with the coupling factor

$\frac{2\alpha_G}{\pi} \frac{dk_{\min}}{k_{\min}}$ which is the same as $\frac{2\alpha_G}{\pi} \frac{dk''_{\min}}{k''_{\min}} = \frac{2\alpha_G}{\pi} \frac{\gamma_M dk_{\min}}{\gamma_M k_{\min}}$ in the changed metric.

Dropping the now uneccesary subscripts and temporarily ignoring dilution factors and clock changes we can rederive Eq's **(5.2. 4)**, **(5.2. 5)** & **(5.2. 6)** to get with large masses:

$$\Delta\rho_{Gk\min} \approx (1 - e^{-0.61k^2/k^2_{\min}})\alpha_G \frac{m}{r}(0.573)\frac{\rho_U}{k^2_{\min}} dk_{\min} \qquad \approx \frac{m}{r} 0.261\alpha_G \frac{\rho_U}{k^2_{\min}} dk_{\min} \ @\ k_{\min}$$

But $\frac{\rho_U}{k^2_{\min}} \approx 0.8823$ from Eq.(5.2. 10) so $\Delta\rho_{Gk\min} \approx \frac{2m}{r} 0.115\alpha_G dk_{\min}$

Using Eq, (5.2. 11) $K_{Gk\min} \approx 0.115\alpha_G$ $\qquad \Delta\rho_{Gk\min} \approx \frac{2m}{r} K_{Gk\min} dk_{\min} \ @\ k_{\min}$

Using $\beta_M^2 = \frac{2m}{r}$ and to help illustrate metric changes we temporarily include the factor γ_M^2

$$\text{Before metric changes} \quad \Delta\rho_{Gk\min} \approx \beta_M^2 \gamma_M^2 K_{Gk\min} dk_{\min} \tag{5.2. 14}$$

The total $k_{\min}$ graviton density before metric changes is the original $\rho_{Gk\min} \approx K_{Gk\min} dk_{\min}$ plus the extra $\Delta\rho_{Gk\min} \approx \beta_M^2 \gamma_M^2 K_{Gk\min} dk_{\min}$. So before metric changes

$$\rho_{Gk\min}(\text{Total}) = K_{Gk\min} dk_{\min} + \beta_M^2 \gamma_M^2 K_{Gk\min} dk_{\min} \qquad = (1 + \beta_M^2 \gamma_M^2) K_{Gk\min} dk_{\min}$$

$$\text{But} \quad (1 + \beta_M^2 \gamma_M^2) = 1 + \frac{\beta_M^2}{1 - \beta_M^2} = \gamma_M^2 \tag{5.2. 15}$$

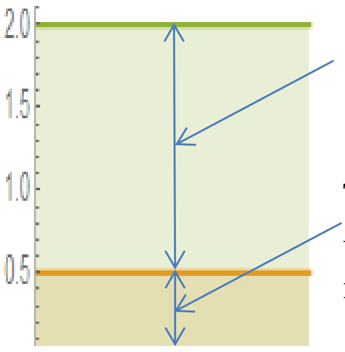
Before metric changes $\rho_{Gk\min}(\text{Total}) = K_{Gk\min} dk_{\min} + \beta_M^2 \gamma_M^2 K_{Gk\min} dk_{\min} = \gamma_M^2 K_{Gk\min} dk_{\min}$

If we now increase the volume to that in the new metric, the new volume is $\sqrt{g_{rr}} = \gamma_M$ times the original volume. So in the new metric we must divide this value by γ_M to get

Diluted $\rho_{Gk\min}(\text{Total}) = \dfrac{K_{Gk\min} dk_{\min}}{\gamma_M} + \beta_M^2 \gamma_M K_{Gk\min} dk_{\min} = \gamma_M K_{Gk\min} dk_{\min}$ but in the new metric, time changes make $k''_{\min} = \gamma_M k_{\min}$ and $dk''_{\min} = \gamma_M dk_{\min}$ or $dk_{\min} = dk''_{\min} / \gamma_M$

$$\text{In the new metric } \rho_{3Gk\min}(\text{Total}) = K_{Gk\min}\frac{dk''_{\min}}{\gamma_M^2} + \beta_M^2 K_{Gk\min} dk''_{\min} = K_{Gk\min} dk''_{\min} \qquad (5.2.\ 16)$$

If for example $\gamma_M = 2$, frequencies are doubled so $k''_{\min} = 2k_{\min}$, the number density of gravitons ($\rho''_{Gk\min} = 2\rho_{Gk\min}$) is doubled, but so is the measurement of a local small volume element, which is now $V = 2$. The above equations tell us that the original $\rho_{Gk\min}$ background gravitons which occupied one unit of volume is now compressed into 1/2 a unit of volume and the remaining 3/2 units of volume is taken up by the gravitons due to the central mass. Figure 5.2. 2 illustrates this. The metric adjusts itself so that $K_{Gk\min}$ *(the cutoff wavelength graviton probability constant)* is an invariant number, and this should be true in all metrics at any peculiar velocity (See Figure 5.3. 9 also.) What we have done in this section is only true if the increase in measured volume is equal to the increase in measured frequency. In the Schwarzchild metric this is equivalent to saying that $g_{rr} \cdot g_{tt} = 1$ or $|g| = 1$. We discuss angular momentum in section 7.



Measured local volumes double, & 3/2 units of volume × the increased number density equals the extra maximum wavelength gravitons at that point due to a central mass.

The background $k_{\min}$ gravitons that originally occupied one unit of volume are compressed into 1/2 a unit of volume as number densities are doubled in this new metric.

Figure 5.2. 2 An infinitesimal local volume in a metric where $\sqrt{g_{rr}} = \gamma_M = 2$.

5.3 The Expanding Universe

Section 5.1.1 describes virtual gravitons as superpositions of the three modes $n = 3,4,5$ at a single wavenumber k as in Table 4.3. 1 which also tells us $\langle n \rangle = 3.33$. Equation (3.2. 1) tells us $\langle \mathbf{p}_k(debt) \rangle = -\langle \beta_k \rangle^2 \langle n \rangle \hbar \mathbf{k}$ is the debt of $\hbar\mathbf{k}$ spatially polarized quanta they borrow. Equation (5.1. 2) tells us $m_0 = \langle n \rangle k_{\min}$ & Eq. (3.1. 4) says they borrow from time polarized quanta a mass $m_0 / (\gamma\sqrt{2s}) = m_0 / (2\gamma) = \langle n \rangle k_{\min} / (2\gamma)$ for spin 2 gravitons. Equation (6.2. 2) tells us that @ $k_{\min}$ for gravitons $\gamma_{\min}^2 = 2$ & $\beta_{\min}^2 = 1/2$. From this we can see that @ $k_{\min}$, the spatially polarized debt is $\sqrt{2}$ larger that the time polarized debt.

So we only need to consider the spatially polarized debt when equating quanta available to quanta required, as when these are equal there is a small surplus of time polarized quanta. However to plot these curves near $k = k_{\min}$ we need the number density of gravitons at any wavenumber k, so rewiting Eq. **(5.1. 9**) using Eq.(5.2. 10) for $\rho_U^2 / k_{\min}^4$ & Eq.(5.2. 11

$$\rho_{Gk} \approx \frac{0.149\alpha_G \rho_U^2}{k_{\min}^4} dk \left[\frac{108}{x} \frac{(1\text{-e}^{-0.61x^2})\sqrt{x^2+11.09}}{\left(2x^2+11.09\right)^2} \right] \approx 0.115\alpha_G dk \left[\frac{108}{x} \frac{(1\text{-e}^{-0.61x^2})\sqrt{x^2+11.09}}{\left(2x^2+11.09\right)^2} \right] \quad (5.3.\ 1)$$

Both blue boxes are one when $k / k_{\min} = x = 1$. Using Eq's. (3.1. 11), (3.1. **12**) & (3.2. 10) $\langle \beta_k \rangle^2 = \frac{\langle K_k \rangle^2}{1 + \langle K_k \rangle^2}$. For $N = 2$ spin 2 $\langle K_k \rangle = \frac{\langle n \rangle k}{m_0}$ and using $m_0 \approx 3.33 k_{\min}$ we can show that $\langle \beta_k \rangle^2 = \frac{k^2}{k^2 + k_{\min}{}^2} = \frac{x^2}{x^2+1}$ where $x = \frac{k}{k_{\min}}$ so wavefunction ψ_k borrows $\langle \beta_k \rangle^2 \langle n \rangle \approx (3.33) \frac{x^2}{x^2+1}$ wavenumber k quanta. But wavenumber k virtual quanta last for time $\Delta T \approx \hbar / 2k$, whereas the time the superposition lasts is $\Delta T' \approx \hbar / 2E$. We are borrowing $\text{Energy} \times \text{Time}$ or Action, and the superposition energy $E = k' = \sqrt{k^2 + 11.09 k_{\min}^2}$ as in Eq. (5.1. 4) lasts for a shorter time when k is near $k_{\min}$. So the Action Quanta of $\text{Energy} \times \text{Time}$ required, reduces as $\frac{k}{k'} = \frac{k}{\sqrt{k^2 + 11.09 k_{\min}^2}} = \frac{x}{\sqrt{x^2+11.09}}$ and the quanta density required @k by gravitons is thus

$$\rho_{Quanta@k} \approx \frac{0.115\alpha_G}{2.09} dk \left[2.09 \cdot \frac{3.33 \times x^2}{x^2+1} \cdot \frac{x}{\sqrt{x^2+11.09}} \right] \cdot \left[\frac{108}{x} \cdot \frac{(1\text{-e}^{-0.61x^2})\sqrt{x^2+11.09}}{\left(2x^2+11.09\right)^2} \right]$$

$$\rho_{Quanta@k} \approx 0.0555\alpha_G dk \left[\frac{751x^2}{(x^2+1)} \frac{(1\text{-e}^{-0.61x^2})}{\left(2x^2+11.09\right)^2} \right]$$ All blue boxes are one when $\frac{k}{k_{\min}} = x = 1$ & $\rho_{Quanta@k\min} = \rho_{Qk\min} \approx 0.0555\alpha_G dk_{\min}$ @ $k_{\min}$ (5.3. 2)

But the density of zero point modes available @ $k_{\min}$ is $k_{\min}^2 dk / \pi^2$. Even if $\alpha_G << 1$ this is too small by about $k_{\min}^2 \approx 1 / R_{OH}^2$. However the area of the causally connected horizon $4\pi R_{OH}^2$ suggests possible connections with Holographic horizons and the AdS/CFT correspondence [14], but in a very different way.

5.3.1 Holographic horizons and red shifted Planck scale zero point modes

Malcadena proposed AntiDesitter or Hyperbolic spacetime where Planck modes on a 2D horizon are infinitely (almost) redshifted at the origin by an (almost) infinite change in the metric. In contrast we have assumed flat space on average to the horizon. In section 2.2.3 we defined a rest frame in which preons with zero momentum and infinite wavelength build superpositions. If we also have a spherical horizon with Planck scale modes, but receding locally at the velocity of light, these Planck modes can be absorbed by infinite wavelength preons (from that receding horizon) and red shifted in a radially focussed manner inwards. We will argue in what follows, that at the centre where the infinite superpositions are built, approximately 1/6 of these Planck modes can be absorbed from that horizon with wavelengths of the order of the horizon radius. *This potential possibility only exists because zero momentum preons have an infinite wavelength.* If any source of radiation recedes at velocity $\beta = v/c$ the frequency/wavenumber reduces as $k_{observer} = k_{source}\left[\gamma(1-\beta)\right]$ where $\gamma = (1-\beta^2)^{-1/2}$. In the extreme relativistic limit $\beta \to 1$ & we can put $1-\beta = \Delta\beta = \varepsilon$.

$$\text{Putting } 1-\beta = \Delta\beta = \varepsilon \text{ implies } \beta = 1-\varepsilon \text{ and } \beta^2 \approx 1-2\varepsilon$$
$$1-\beta^2 = \gamma^{-2} \approx 2\varepsilon \text{ and } \gamma \approx 1/\sqrt{2\varepsilon} \tag{5.3. 3}$$
$$\text{Thus } \frac{k_{Observer}}{k_{Source}} = \left[\gamma(1-\beta)\right] \approx \frac{\varepsilon}{\sqrt{2\varepsilon}} \approx \sqrt{\frac{\varepsilon}{2}} \approx \frac{1}{2\gamma}$$

There is always some rest frame travelling at nearly light velocity that can redshift Planck energy modes into a $k_{\min} \approx 1/R_{OH}$ mode and also many other frames travelling at various lower velocities that can redshift Planck energy modes into any $k > k_{\min}$ mode . This is special relativity applying locally. But in sections 5.1.2 & 5.2.1 we used the fact that clocks in comoving coordinates tick at the same rate. So how does Eq.(5.3. 3) help? Space between comoving galaxies expands with cosmic or proper time t and is called the scale factor $a(t)$. It is normally expressed as $a(t) \propto t^p$ and we will start at time $t = T_0$ with time T now.

$$\text{Thus } \dot{a}(t) \propto pt^{p-1} \text{ and the Hubble parameter } H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{p}{t} \tag{5.3. 4}$$

We have been assuming to here that space is flat on average and will use the properties that in flat space at the current time the coordinate, proper and comoving distances are all equal. Writing the present scale factor normalized to one so that $a(T) = 1$ implies $a(t) = t^p/T^P$, we can get the causally connected horizon radius and the horizon velocity *V*. Using Eq.(5.3. 4)

$$\text{The horizon radius } R_{OH} = \int_{T_0}^{T} \frac{dt}{a(t)} = T^p \int_{T_0}^{T} \frac{dt}{t^p} = \frac{T-T_0}{1-p} \approx \frac{T}{1-p} \text{ if } T >> T_0 \text{ \& } p \text{ constant.} \tag{5.3. 5}$$

$$\text{In flat space horizon velocity } V = \frac{dR_{OH}}{dT} = \frac{d}{dT}\left[T^p \int_{T_0}^{T} \frac{dt}{t^p}\right] \text{ then using } d(u\cdot v) = u\cdot dv + v\cdot du \text{ for } (T >> T_0):$$

$$\frac{dR_{OH}}{dT}=\frac{T^{p}}{T^{p}}+\frac{R_{OH}}{T^{p}}(pT^{p-1})=1+\frac{p}{T}R_{OH}.\ \text{But } \frac{p}{T} \text{ is the Hubble constant at time } T. \tag{5.3. 6}$$

In flat space the horizon velocity $V=1+H(T)R_{OH}$ regardless of how p behaves.

The *Hubble flow velocity of a comoving galaxy on the horizon* is $V'=H(T)R_{OH}$ and thus from Eq.(5.3. 6) the horizon velocity is always $V=1+V'$. In other words the horizon is moving at light velocity relative to comoving coordinates instantaneously on the horizon as measured by a central observer. Now clocks tick at the same rate in all comoving galaxies but clocks moving at almost the horizon light velocity (relative to comoving coordinates instantaneously on the horizon) will tick extremely slowly or as $1/\gamma$ from Eq.(5.3. 3) as special relativity applies locally in this case. Thus Planck modes on the receding horizon will obey Eq's.(5.3. 3) as seen in all comoving coordinates. Let us now imagine an infinity of frames all travelling at various relativistic velocities relative to comoving coordinates instantaneously on the horizon and radially as seen by central observers. We can think of these as spherical shells on the horizon *all of one Planck length thickness as measured by observers moving radially with them.* Transverse dimensions do not change for all radially moving observers and the effective surface area of all these shells is $4\pi R_{OH}^2$. The internal volume of all these shells as measured in rest frames by observers moving radially with them as each of these observers measures their thickness ΔR as one Planck length is

$$\text{Rest frame internal shell volume } V=4\pi R_{OH}^2\Delta R=4\pi R_{OH}^2 \tag{5.3. 7}$$

We want the zero point quanta available where *these quanta have Planck energy* ΔE *lasting for Planck time* ΔT *such that* $\Delta E\times\Delta T\approx\hbar/2$. Before redshifting a single zero point quanta thus has Planck energy (temporarily using a single primed k' that is not the k' of Eq. (5.1. 4)) where $k'=1$ before redshifting and k after redshifting. The density of modes in this shell is k'^2dk'/π^2 (where each quanta & the superposition it builds both last for time $\Delta T\approx\hbar/(2\Delta E)$)

$$\frac{k'^2dk'}{\pi^2} \text{ quanta, which we will write as zero point quanta density } \frac{k'^3}{\pi^2}\frac{dk'}{k'}. \tag{5.3. 8}$$

At Planck scale $k'=1$ and redshifing to k then using Eq's.(5.3. 3) $k=k'\sqrt{\varepsilon/2}$ & $dk=dk'\sqrt{\varepsilon/2}$. Thus $dk'/k'=dk/k$. As $k=1$ Eq. (5.3. 8) becomes

$$\text{Planck Scale Quanta Density before redshifting}=\frac{1^3}{\pi^2}\frac{dk'}{k'}=\frac{1}{\pi^2}\frac{dk}{k} \tag{5.3. 9}$$

Multiplying density by volume ie. Eq's. (5.3. 7) & (5.3. 9) gived the total Planck scale quanta inside the rest frame shell as $4\pi R_{OH}^2 \cdot \frac{1}{\pi^2}\frac{dk}{k}$. Two thirds of these quanta are transverse and one third radial, so only $1/6$ of these quanta are available for redshifting radially inwards.

After redshifting to wavenumber k, in flat on average space in a thought experiment, we can imagine them forming spherical standing waves, with a central spherical first node at radius $R = \lambda/4 = \pi/2k$, where λ is the De Broglie wavelength of momentum k particles or waves. The polarization directions are spherically symmetric (as required to build infinite superpositions in their rest frame, see section 5.3.7), forming virtual spin 1 quanta with a radial probability of $\psi * \psi \propto 2k\cos^2 kr$. Inside this sphere the expectation value of the radius that a quantum is at is $\langle r \rangle = \pi/4k$ as $\langle \cos^2 kr \rangle = 1/2$, so the expectation value of the probability density is

$$\frac{2k\langle \cos^2 kr \rangle}{4\pi \langle r \rangle^2} = \frac{2k\cos^2(\pi/4)}{4\pi(\pi/4k)^2} = \frac{k}{4\pi}\frac{16k^2}{\pi^2} = \frac{1.62k^3}{4\pi} = \frac{1.62\Upsilon^3}{4\pi R_{OH}^3}\left[\frac{k}{k_{\min}}\right]^3 \text{ where we have used } \Upsilon = k_{\min} R_{OH}$$

This is the average probability density of a single quantum. So the total density is this single quantum probability density, times the number from the horizon; but we also need to divide by 2 as we are only considering the spatially polarized or vector half. Again using $\Upsilon = k_{\min} R_{OH}$ the total quanta density becomes, after dividing by the two factors of 2 & 6

$$\rho_{Quanta@k} \approx \frac{1}{2}\cdot\frac{1}{6}\cdot\left[4\pi R_{OH}^2 \frac{1}{\pi^2}\frac{dk}{k}\right]\frac{1.62\Upsilon^3}{4\pi R_{OH}^3}\left[\frac{k}{k_{\min}}\right]^3 \approx \frac{\Upsilon^2}{7.4\pi^2}dk\left[\frac{k}{k_{\min}}\right]^2 \approx \frac{\Upsilon^2}{7.4\pi^2}dk\cdot x^2 \text{ where } x = \frac{k}{k_{\min}}$$

Density of vector quanta available after redshifting $\rho_{Quanta@k} \approx \frac{\Upsilon^2}{7.4\pi^2}x^2 dk$ (5.3. 10)

Now an observer at the centre of all this sees space being added inside the horizon at the rate of the horizon velocity $V = 1 + H(T)R_{OH}$ as in Eq.**(5.3. 6**). *We will conjecture that the space added in one unit of Planck time inside the expanding horizon also creates the source of these zero point quanta that we can borrow.* Thus Eq. (5.3. 10) becomes

$$\text{Density of quanta available } \rho_{Quanta@k} \approx \frac{\Upsilon^2 V}{7.4\pi^2}\left[\frac{k}{k_{\min}}\right]^2 dk \approx \frac{\Upsilon^2 V}{7.4\pi^2}x^2 dk \qquad (5.3.\ 11)$$

$$\approx \frac{\Upsilon^2(1 + H\cdot R_{OH})}{7.4\pi^2}x^2 dk \quad \text{where } x = \frac{k}{k_{\min}}$$

5.3.2 Plotting available quanta densities and required quanta densities

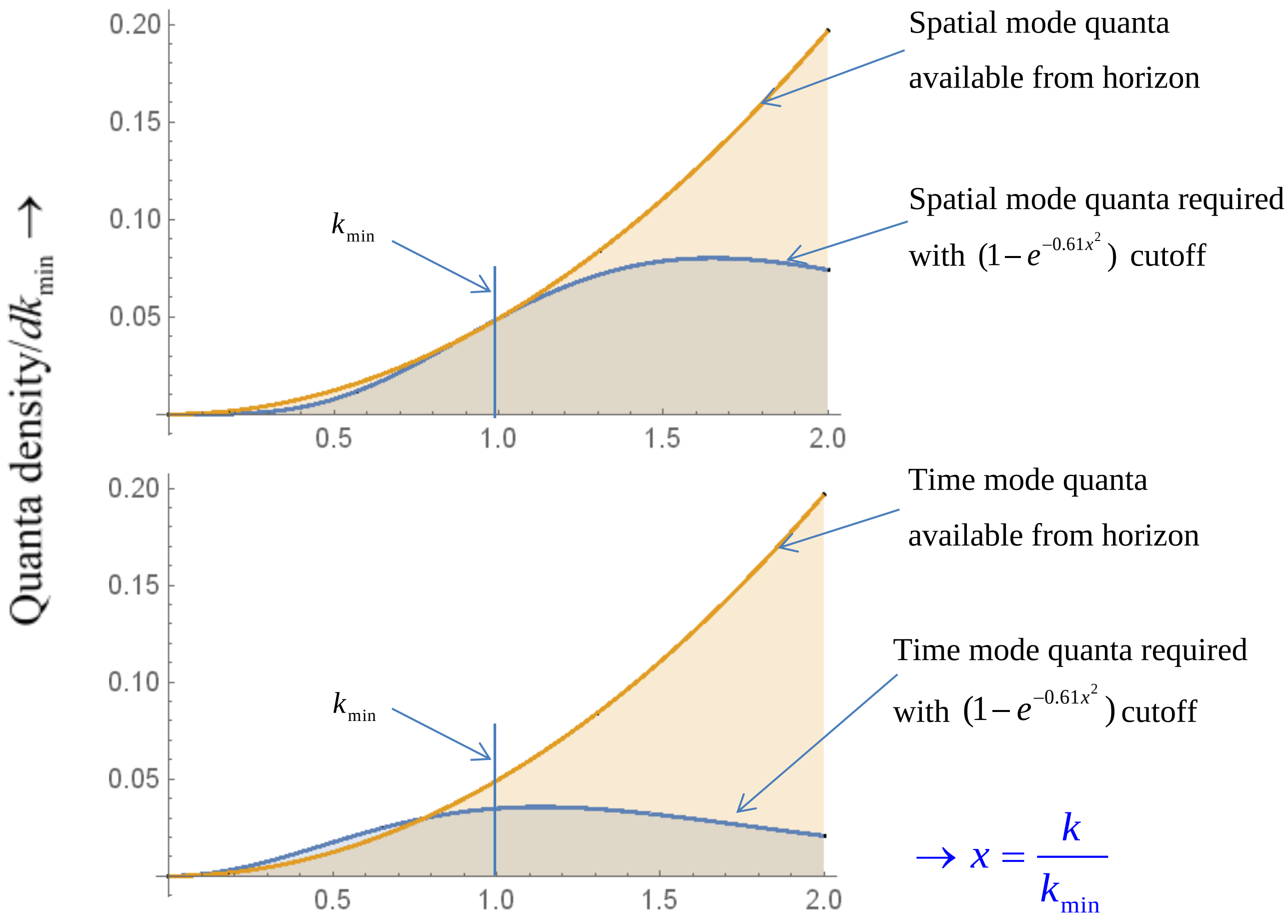


Figure 5.3. 1 plots Eq's.(5.3. 2) & (5.3. 11)as a function of $x = k / k_{\min}$ Going through similar procedures for the time mode quanta as for space modes, we have plotted both time and space. An exponential $e^{-0.61x^2}$ cutoff fits available and required reasonably for $k < k_{\min}$ in both cases; also showing there should be an adequate supply of time mode quanta from the horizon for all infinitesimal mass particles. The spatial mode crosses @ $k = k_{\min}$. Both plots always look the same at all cosmic times *T*. And, in any metric only the value of $k_{\min}$ changes. It only works in an expanding flat universe. Equating required and available spatial modes @ $k = k_{\min}$

$$\text{Quanta available} \approx \frac{\Upsilon^2 V}{7.4\pi^2} dk_{\min} = \text{Quanta required} \approx 0.055\alpha_G dk_{\min} = K_{Qk\min} dk_{\min}$$
$$\text{Where } K_{Qk\min} = 0.055\alpha_G \text{is the "Quanta required @ } k_{\min}\text{Constant" \& } \alpha_G \approx \frac{\Upsilon^2 V}{4} \qquad (5.3.\ 12)$$

Equation (5.2. 10) the average density of the universe $\rho_U \approx 0.8823\dfrac{\Upsilon^2}{R_{OH}^2}$ allows us to solve the present value of $\Upsilon = k_{\min} R_{OH}$. Using WMAP data for Baryonic and Dark Matter density $\rho_U \approx 5.6\times10^{-124}$ in Planck units & the radius $R_{OH} \approx 2.7\times10^{61}$ Planck lengths ($\approx 46.5\times10^9$ light years) puts $\rho_U \times R_{OH}^2 \approx 0.42$ in Planck units so $\rho_U \times R_{OH}^2 \approx 0.8823\Upsilon^2 \approx 0.42$ now.

$$\text{Using } \Lambda\text{-CDM model \& WMAP data} \quad \Upsilon = k_{\min} R_{OH} \approx 0.69 \text{ now} \tag{5.3. 13}$$

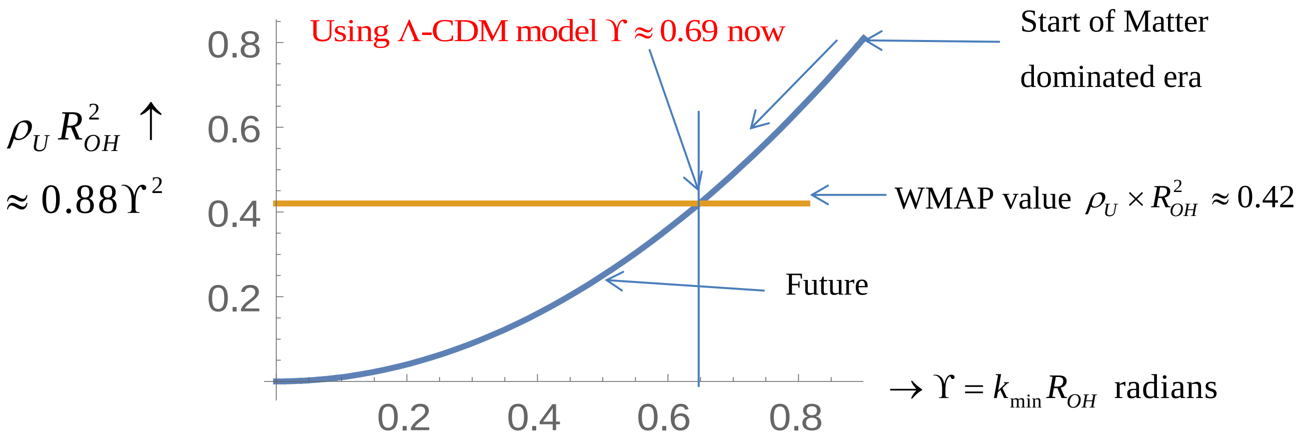


Figure 5.3. 2 Plots $\rho_U \times R_{OH}^2 \approx 0.8823\Upsilon^2$

The $\Lambda-\text{CDM}$ model &WMAP data Horizon velocity $V = 1 + H(T)R_{OH} \approx 1 + 3.37 \approx 4.37$ if $H(T) \approx 1/T$ now, and putting this and $\Upsilon \approx 0.69$ into Eq. (5.3. 12) we can get an approximate value for the graviton coupling constant α_G .

$$\text{Using } \Lambda\text{-CDM model \& WMAP data } \alpha_G \approx \frac{\Upsilon^2 \text{V}}{4} \approx \frac{(0.69)^2 \times 4.37}{4} \approx \frac{2.08}{4} \approx 0.52 \tag{5.3. 14}$$

$$\text{Or, as is more probable } \alpha_G = 1\text{, and we should expect } \Upsilon^2 \text{V} \approx 4$$

Using the current $\Lambda-\text{CDM}$ model and WMAP data with $\Upsilon^2 V \approx 2.08$, puts α_G in the right ball park, suggesting that our approximate analyses is not too far off the mark, and that we can perhaps turn it around to show we should expect $\Upsilon^2\text{V} \approx 4$. The $\Lambda-\text{CDM}$ model has to be fine tuned for flatness requiring a critical density. It also has to have a fixed ratio of Dark Energy to total matter to get the observed accelerated expansion. The current figures are $\approx 5\%$ Baryonic $\approx 23\%$ Dark matter and the rest Dark energy. This puts the ratio of Dark Matter to Baryonic as $\approx$ 4.5 to 1 whereas it can be as much as 9 or even 10 to 1 in some galaxies. If for example it was approaching 10 to 1 then $\Upsilon^2\text{V} \approx 4$ at the current horizon velocity and horizon radius we used above. However the most important part of the above is that $\Upsilon^2 V$ has to be constant, and as we will see this naturally leads to exponential expansion. In the next section we will find slightly different values for both the horizon radius and velocity, which combined with a smaller increase in Dark matter of about $6.5 \text{ to } 1$, can give $\Upsilon^2\text{V} \approx 4$. And it all only works in flat on average space.

5.3.3 A possible exponential expansion solution in flat space

In flat on average space we can simplify things greatly. We have equal coordinate, proper and comoving distances at the current time. Let the scale factor be a then density $\rho \propto \frac{1}{a^3}$ and Eq. (5.2. 10) average universe density $\rho_U \approx 0.8823\frac{\Upsilon^2}{R_{OH}^2}$ or $\rho_U = K\frac{\Upsilon^2}{R_{OH}^2} = \frac{1}{a^3}$ where $K = 0.8823$

$$\text{Thus} \quad a^3 = KR^2\Upsilon^{-2} \to a = K'R^{2/3}\Upsilon^{-2/3} \quad \text{where } R = R_{OH} \tag{5.3. 15}$$

The Hubble parameter H is

$$H = \frac{\dot{a}}{a} = \frac{(2/3)K'R^{-1/3}\Upsilon^{-2/3}\, dR/dt}{K'R^{2/3}\Upsilon^{-2/3}} - \frac{(2/3)K'R^{2/3}\Upsilon^{-5/3}\, d\Upsilon/dt}{K'R^{2/3}\Upsilon^{-2/3}} = \frac{2}{3}\left[\frac{1}{R}\frac{dR}{dt} - \frac{1}{\Upsilon}\frac{d\Upsilon}{dt}\right]$$

$$\text{Thus the Hubble Horizon velocity @ } R_{OH} \text{ is } V' = H \cdot R = \frac{2}{3}\left[\frac{dR}{dt} - \frac{R}{\Upsilon}\frac{d\Upsilon}{dt}\right] \tag{5.3. 16}$$

We can also write Eq. **(5.3. 14)** $\Upsilon^2 V \approx 164\alpha_G$ = a constant, hence $\Upsilon^2 dV + 2\Upsilon d\Upsilon V = 0$.

Thus $\frac{1}{2V}\frac{dV}{dT} = -\frac{1}{\Upsilon}\frac{d\Upsilon}{dT}$ and Eq.**(5.3. 6)** tells us that the Horizon velocity $V = \frac{dR_{OH}}{dt} = \frac{dR}{dt}$.

Equation **(5.3. 6)** also tells us that $V' = H \cdot R = V - 1$ so we can write Eq. (5.3. 16) as

$$\left[3(V-1) - 2V = -\frac{2R}{\Upsilon}\frac{d\Upsilon}{dt} = \frac{2R}{2V}\frac{dV}{dt}\right] \to V - 3 = \frac{R}{V}\frac{dV}{dt} \quad \to \quad \frac{dV}{dt} = \frac{V}{R}(V-3) \tag{5.3. 17}$$

We will look for an exponential increase of the horizon velocity so $dV/dt > 0$ and $3 \le V \le \infty$.

Let us try first a simple $V = 3Exp(bt)$ with $V > 3$ for all values of $b \,\&\, t > 0$.

Also simply put $\quad R = \int_0^t Vdt = \int_0^t 3Exp(bt)dt \quad$ thus $\quad R = \frac{3[Exp(bt) - 1]}{b}$.

Putting this value for R plus $V = 3Exp(bt)$ & $V - 3 = 3[Exp(bt) - 1]$ into Eq. (5.3. 17)

$$\frac{dV}{dt} = \frac{V}{R}(V-3) = 3Exp(bt)\cdot\frac{b}{3[Exp(bt)-1]}\cdot 3[Exp(bt)-1] = 3bExp(bt).$$

But $V = 3Exp(bt)$ and again $\frac{dV}{dt} = \frac{d}{dt}[3Exp(bt)] = 3bExp(bt)$. Thus Eq's. (5.2. 10) & (5.3. **14**) are consistent with $V = 3Exp(bt)$ for positive b .

$$\text{A possible expansion solution is } V = 3Exp(bt) \;\&\; R = \frac{3[Exp(bt)-1]}{b},\; b > 0. \tag{5.3. 18}$$

In flat space this should be consistent with the local special relativity requirement for R_{OH} but does R @ time $T = a(T)\int_0^T \frac{dt}{a(t)} = \frac{3[Exp(bT)-1]}{b}$? Equation (5.3. 15) $a^3 = KR^2\Upsilon^{-2}$ gives the scale factor $a = K'R^{2/3}\Upsilon^{-2/3}$ & Eq, **(5.3. 14)** says $\Upsilon^2 \propto 1/V$ so the scale factor $a \propto V^{1/3}R^{2/3}$.
From Eq.**(5.3. 18)**, ignoring the constant factors 3 & b, $V \propto Exp(bt)$ & $R \propto [Exp(bt)-1]$

$$\text{The scale factor } a(t) \propto Exp(bt)^{1/3}[Exp(bt)-1]^{2/3} \text{ and } R = a(T)\int_0^T \frac{dt}{a(t)}$$

$$\begin{aligned} &= Exp(bT)^{1/3}[Exp(bT)-1]^{2/3}\int_0^T \frac{dt}{Exp(bt)^{1/3}[Exp(bt)-1]^{2/3}} \\ &= \frac{3[Exp(bT)-1]}{b} \end{aligned} \qquad (5.3.\ 19)$$

And Eq.**(5.3. 18)** appears to be a consistent exponential expansion for both V and R.
From Eq. **(5.3. 14)** we showed $\frac{1}{2V}\frac{dV}{dT} = -\frac{1}{\Upsilon}\frac{d\Upsilon}{dT}$. Using Eq.**(5.3. 18)** $V = 3Exp(bt)$ & $\frac{dV}{dt} = 3bExp(bt)$ implies $\Upsilon = K \cdot Exp(-bt/2)$. The current $\Lambda-$CDM /WMAP value $\Upsilon \approx 0.69$ from Eq.(5.3. 13) and our best guess of $b \approx 0.48$ from Figure 5.3. 3 yields

$$\Upsilon = k_{\min}R_{OH} \approx 0.88Exp(-0.24t) \text{ in radians} \qquad (5.3.\ 20)$$

This simple exponential expansion starting at the Big Bang is very different to current Λ-CDM models keeping the Hubble parameter $H = \dot{a}/a \approx 2/3t$ constant (if $\Omega = 1$) until Dark Energy starts to take effect. Current Λ-CDM models put the Hubble parameter as $H = \dot{a}/a \approx 1/T$ at present (based on $T \approx 13.8\times10^9$ years). In the plots below we put $T \approx 13.8\times10^9$ years $=1$, with R_{OH} or radius R becoming multiples of $T = 1$. Using Eq.**(5.3. 6)** $V = 1 + H(T)R$, Figure 5.3. 3 plots the Hubble parameter by time $(T = 1)$ now as a function of the exponential time coefficient b showing if $b = 0$ that H always $= 2/(3t)$ as in current cosmology at critical density with no dark energy. Also if $H \approx 1/T$ now the best guess is $b \approx 0.48$. This yields $R \approx 3.85T$ or $\approx 15\%$ greater than current cosmology. Figure 5.3. 4 plots horizon velocity which @ $V \approx 4.85$ now is also $\approx 11\%$ greater. The current $\Lambda-$CDM model puts Baryonic matter at $\approx 5\%$ and Dark matter at $\approx 23\%$ but if we make this ratio say about 6.4 to 1, the total matter density of the universe increases from $\approx 28\%$ to $\approx 37\%$, and ρ_U increases as $37/28 \approx 1.32$. Now $\rho_U \times R_{OH}^2 \approx 0.8823\Upsilon^2$ and if R_{OH} is 15% greater, then $\Upsilon'^2 \approx 1.32\times1.15^2\times\Upsilon^2 \approx 1.75\times0.47 \approx 0.825$ as $\Upsilon^2 \approx 0.47$ in current $\Lambda-$CDM models. If $V' \approx 4.85$ then $\Upsilon'^2V' \approx 4$ which fits our model. Figure 5.3. 5 plots scale factor based on $b \approx 0.48$, but of course the actual value of b or rate of change with time must be in agreement with the redshifts currently observed when looking back towards the big bang. These could well change b and radius R. Figure 5.3. 6 plots the transition to positive acceleration of the scale factor showing the effect of changing the value of b.

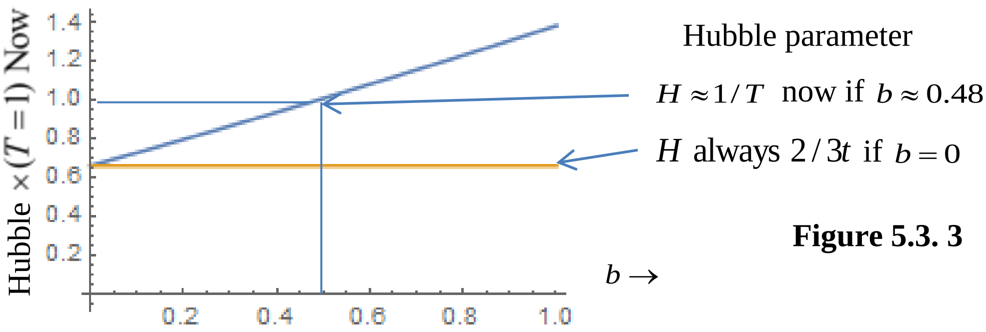


Figure 5.3. 3

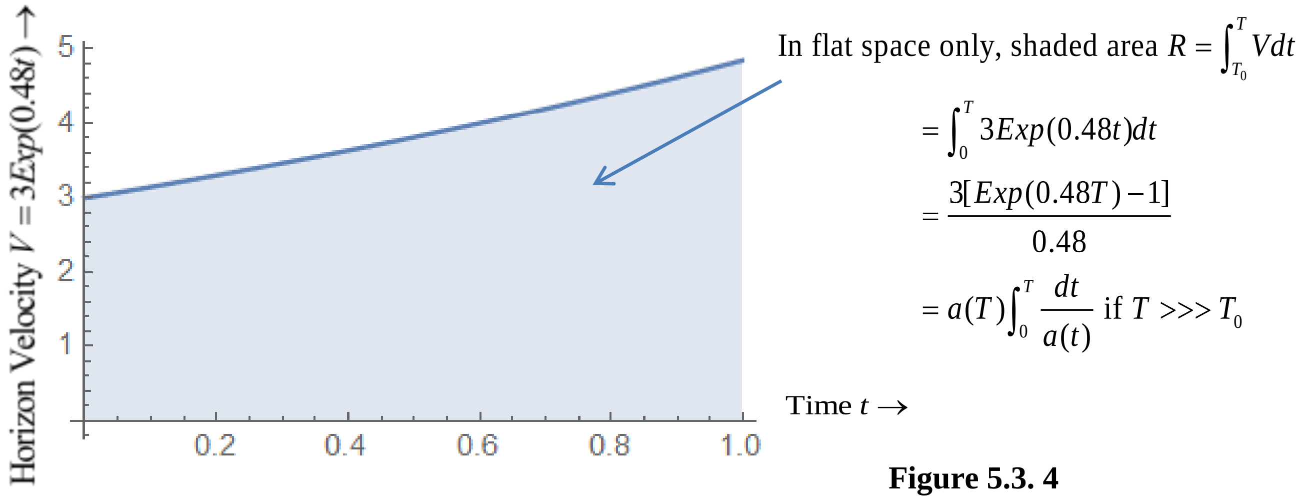


Figure 5.3. 4

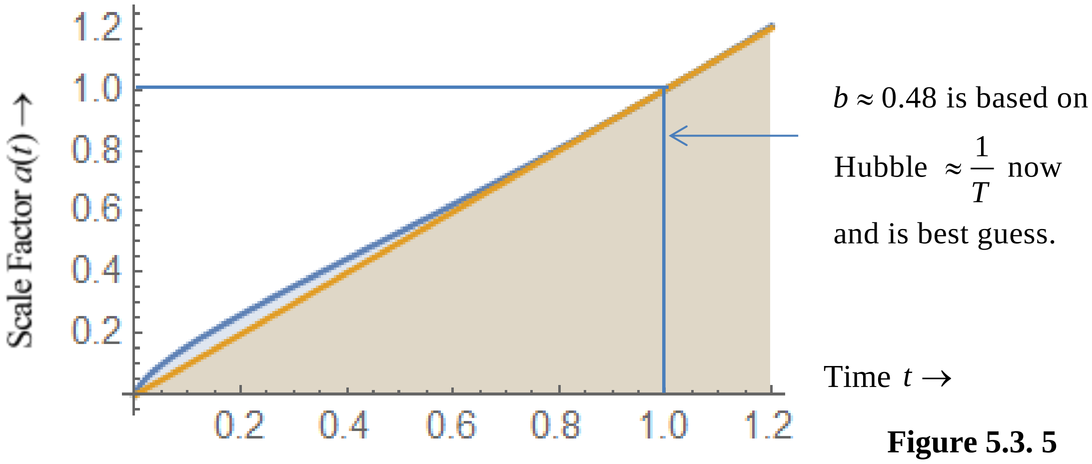


Figure 5.3. 5

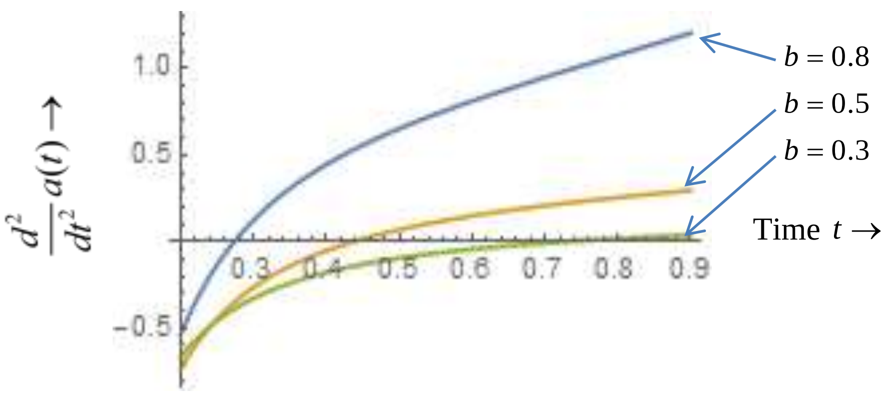


Figure 5.3. 6

These plots show an $\approx 15\%$ increase in Horizon Radius, an $\approx 11\%$ increase in Horizon Velocity, which if combined with an increased Dark Matter to Baryonic Matter ratio of ≈ 6.5 (versus the Λ-CDM ratio ≈ 4.5) gives the required $\Upsilon^2 V \approx (0.91)^2 \times 4.85 \approx 4$

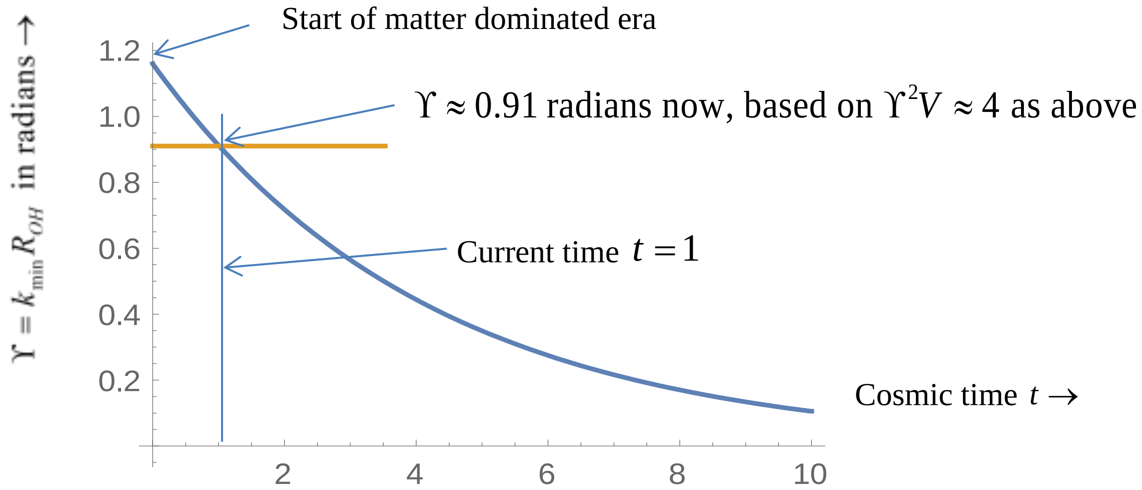


Figure 5.3. 7 Plots $\Upsilon = k_{\min} R_{OH} \approx 1.155 Exp(-0.24t)$ to 10 times the age of the universe.

5.3.4 The radiation dominated era up to the transition

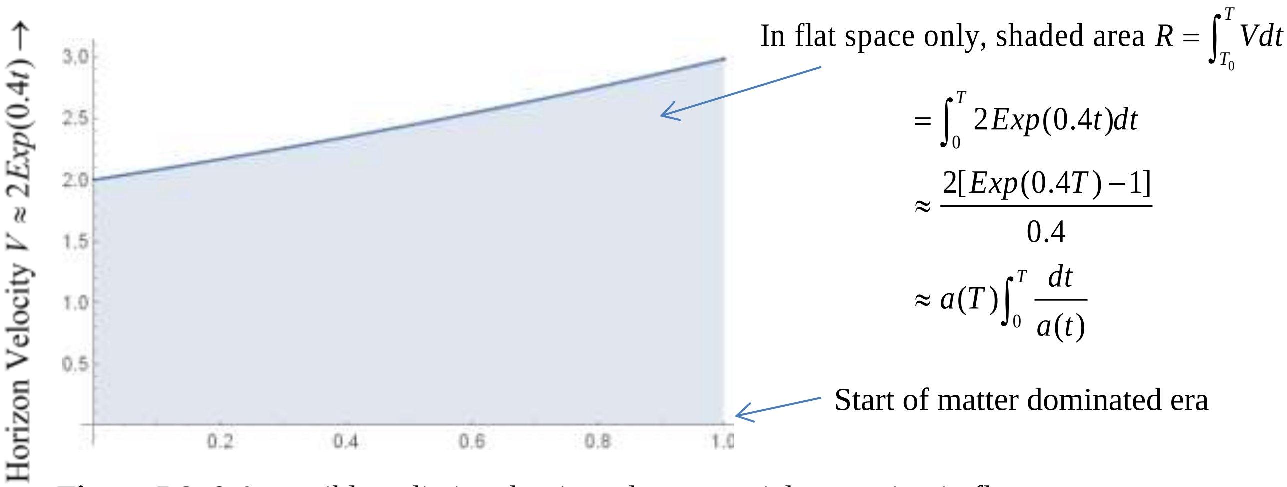


Figure 5.3. 8 A possible radiation dominated exponential expansion in flat space.

In the mass dominated era the density $\rho \propto 1/a^3$, and in the preceding radiation dominant era $\rho \propto 1/a^4$. We can repeat section 5.3.3 with horizon velocity $V = 2Exp(ct)$, & horizon radius $R = \int_0^t V dt = \int_0^t 2Exp(ct)dt \; = 2[Exp(ct) - 1]/c$. The horizon velocity starts $@V = 2$ and horizon radius $@R = 2t$ with a scale factor $a \propto t^{1/2}$ and is the value used in current models of this era before transition, which predict results in close agreement with the current measurements of normal matter in the universe. At the start of the matter dominated era, $V = 3$, and one possibility is $V \approx 2Exp(0.4t)$, where time is normalized to one at the end of this era. This exponential expansion can, with some smooth transition, continue on into the $V = 3Exp(0.48t)$ of the matter era, with the normalization of time then changed to the current cosmos age. If $\alpha_G = 1$ as in Eq, **(5.3. 14)** with $V = 2$, Υ has to be ≈ 1.4 initially to get $\Upsilon^2 V \approx 4$. By the end of this era when the horizon velocity has increased to $V \approx 3$, Υ has exponentially reduced to $\Upsilon \approx 1.155$ to keep $\Upsilon^2 V \approx 4$. Because the transition time is so small in relation to the $\approx 10^{10}$ year age of the universe, this era has insignificant affect on all our above graphs.

5.3.5 Peculiar velocities and non-comoving coordinates

We have up to here been focussing more on comoving coordinates for simplicity. Velocities relative to comoving coordinates are usually referred to as peculiar velocities, so, does all our previous work still apply in non-comoving coordinates? The average momentum of the universe is zero in comoving coordinates and all background gravitons are time polarized. As explained in section 5.1.2, if we move at a peculiar velocity, equal and opposite gravitomagnetic vectors all sum to zero, resulting in zero spatially polarized gravitons, just as the magnetic field is zero at the centre of long circular conductors with uniform currents. Thus the background, at the centre of any observer's universe, in non-rotating spacetime, always contains only time polarized or spherically symmetric $k_{\min}$ gravitons, regardless of peculiar velocities. We can think of a box of these $k_{\min}$ gravitons fixed in comoving coordinates. It will have a 3 volume density $\rho^{3V}_{Gk\min} = K_{Gk\min} dk_{\min}$ as we have previously calculated. (The superscripts here are for convenience only, and nothing to do with tensors.) If we now move relative to it at peculiar velocity β_P it will shrink in size as $\gamma_P^{-1} = \sqrt{1-\beta_P^2}$ so that it's new 3 volume density $\rho''^{3V}_{Gk\min} = K_{Gk\min} dk''_{\min}$, where $dk''_{\min} / dk_{\min} = \gamma_P$ is the local increase in wavenumber $k_{\min}$. If we repeat our derivations of the background 3 volume density, and the extra emitted by local mass concentrations, we find they also both increase by $dk''_{\min} / dk_{\min} = \gamma_P$ with no change in the ratio $\Delta\rho / \rho$, so all our logic is unchanged at any peculiar velocity. But all this, is the same as saying that at any peculiar velocity, and in any metric, the 4 volume density of $k_{\min}$ gravitons is Invariant at any cosmic time *T*.

5.3.6 Invariant 4 volume cosmic wavelength graviton densities

Define $\rho^{3V}_{Gk\min} = \dfrac{k_{\min}\text{Gravitons}}{3\text{Volume}} = \dfrac{k_{\min}\text{Gravitons}}{\Delta x \Delta y \Delta z}$ and as 4 volume $\Delta x \Delta y \Delta z \Delta t = \Delta x' \Delta y' \Delta z' \Delta t'$

$$\rho^{4V}_{Gk\min} = \frac{k_{\min}\text{Gravitons}}{4\text{Volume}} = \frac{k_{\min}\text{Gravitons}}{\Delta x \Delta y \Delta z \Delta t} = \frac{k_{\min}\text{Gravitons}}{\Delta x' \Delta y' \Delta z' \Delta t'} \quad \text{is an invariant.}$$

We will define 4 volume $k_{\min}$ graviton density at any point in space-time as

$$4 \text{ Volume Density } \rho^{4V}_{Gk\min} \quad = \quad 3 \text{ Volume Density } \rho^{3V}_{Gk\min} \tag{5.3. 21}$$

but only in commoving flat space coordinates, however

$\rho^{4V}_{Gk\min}$ is invariant in all coordinates and in any metric.

This is equivalent to dividing $k''_{\min}$, in any metric, at any peculiar velocity, by $\gamma_P\gamma_M$ thus returning it to flat space comoving value $k_{\min}$ at any cosmic time T. Similarly Eq. (5.2. 16)

$$\rho^{3V}_{Gk\min}(\text{Total}) = K_{Gk\min}\frac{dk''_{\min}}{\gamma_M^2} + \beta_M^2 K_{Gk\min}dk''_{\min} = K_{Gk\min}dk''_{\min} \text{ in comoving coordinates,}$$

which can be written as

$$\rho^{3V}_{Gk\min}(\text{Total}) = K_{Gk\min}\frac{dk''_{\min}}{\gamma_M^2} + \Delta\rho^{3V}_{Gk\min}\text{, becomes using invariant 4 volume notation}$$

$$\rho^{4V}_{Gk\min}(\text{Total}) = K_{Gk\min}\frac{dk_{\min}}{\gamma_M^2} + \beta_M^2 K_{Gk\min}dk_{\min} = K_{Gk\min}dk_{\min} \text{ which can be written as}$$

$$\rho^{4V}_{Gk\min}(\text{Total}) = K_{Gk\min}\frac{dk_{\min}}{\gamma_M^2} + \Delta\rho^{4V}_{Gk\min}$$

Where $\Delta\rho^{4V}_{Gk\min} = \beta_M^2 K_{Gk\min}dk_{\min}$; $\quad \Delta\rho^{3V}_{Gk\min} = \beta_M^2 K_{Gk\min}dk''_{\min}$; $\quad \dfrac{dk''_{\min}}{dk_{\min}} = \dfrac{\Delta x\Delta y\Delta z}{\Delta x'\Delta y'\Delta z'} = \dfrac{\Delta t'}{\Delta t}$

As both $\rho^{4V}_{Gk\min}$ & $\Delta\rho^{4V}_{Gk\min}$ are invariant, their ratio is also invariant in any coordinates, and at any peculiar velocity at any particular cosmic time. But the flat space comoving value of $k_{\min}$ decreases with cosmic time. We also know that $\beta_M^2 = 2m/r$ is always true.

5.3.7 Cosmic wavelength graviton action densities and spherical symmetry

In deriving Eq.(5.3. 2) we said that each $k_{\min}$ graviton always borrows a fixed amount of action, where $Action = \Delta E \cdot \Delta T$ per graviton is constant but $\Delta E \propto k_{\min}$. So if four volume density $\rho 4V_{Gk\min}$ is invariant the four volume action density required by $k_{\min}$ gravitons must also be invariant. In Eq (5.3. 11) we calculated the $k_{\min}$ action or quanta density available from the horizon in comoving coordinates. But if we move at a peculiar velocity β_P, both energy ΔE and time ΔT increase as γ_P, so the four volume action density available from this source should increase by γ_P^2 and not be invariant, appearing to destroy our logic. (If there is more action available than what is required by $k_{\min}$ gravitons there is nothing to keep their density controlled.) If we go back to the building of superpositions, in their rest frame, they have a wave equation generated from a vector potential squared, or A^2 term Eq. (2.3. 7). This can only come from a spherically symmetric source of spatially polarized quanta. At high frequencies this spherically symmetric source is the invariant vector potential squared portion of the local zero point fields. At low frequencies this source is from the receding horizon, but it must also be spherically symmetric (see section 5.3.1). We derived Eq. (5.3. 11) in comoving coordinates where the spatially polarized source from the horizon is spherically symmetric as required to build superpositions in their rest frame. But it is not spherically symmetric at peculiar velocities. Equation **(3.1. 1)** shows that if we move at a peculiar velocity β_P relative to a spherically symmetric spin one source, its probability of being spherically symmetric reduces as $1/\gamma_P^2$. But the 4 volume action density increases as γ_P^2 so these two factors cancel. Exactly the same happens in any metric. The 4 volume $k_{\min}$ spin one quanta action density from the horizon increases as γ_M^2 but the spherical symmetry probability drops as $1/\gamma_M^2$, and both these effects cancel again.

Spherically symmetric 4 volume $k_{\min}$ action density available from the receding horizon is always invariant at any particular cosmic time in a region of space. (At any cosmic time it depends on the value of $k_{\min}$ in comoving flat space but decreases with cosmic time.)

So our hypothesis is that at any point in spacetime: Gravity is consistent with the spherically symmetric 4 volume action density available from a receding horizon always being equal to the spherically symmetric 4 volume $k_{\min}$ action density required by gravitons; with both remaining invariant in any coordinates.

We will use the superscript ss for spherically symmetric invariant 4 volume densities.

Define Invariant Spherically Symmetric 4 Volume Action Density as $\rho_{Qk\min}^{4VSS}$

Where $\rho_{Qk\min}^{4VSS}$ action available $\approx 0.48(\rho_{Gk\min}^{4VSS})$ required by $k_{\min}$ gravitons. (5.3. 22)

This equation is true in any coordinates, and at any point in spacetime.

5.3.8 If Gravity is due to metric changes then what about exchanged momentum?

Let us now consider exchanging $k_{\min}$ time polarized gravitons between Planck masses (or in fact between any masses) instead of simply considering $k_{\min}$ graviton densities. As we have noted many times, by far the vast majority of gravitons in the universe are near $k_{\min}$, so we will only look at the effect of $k_{\min}$ exchanges. If the 4 volume densities of both spherically symmetric $k_{\min}$ action available, and $k_{\min}$ gravitons are invariant everywhere, each mass is interacting with the rest of the universe in a spherically symmetric manner. Quantum field theory tells us that coulomb or scalar forces are due to the exchange of virtual photon 3 momentum. Assuming virtual gravitons are no different, and this exchange is happening in a spherically symmetric manner, there can be no nett force in any direction due to these exchanges, but only momentum squared terms. If two masses are orbiting each other they will also be exchanging higher frequency gravitons, and this would seem to be not in a spherically symmetric manner. If there is a nett force this would cause a deviation from their geodesic paths which has never been observed. Section 3.3.2 mentions a possible such interaction between superpositions with no 3 momentum exchanged, and maybe this is what happens with spin 2 interactions, but only time will tell. Einstein always thought gravitational forces were fictitious, that gravity was due to metric changes only, and not exchanged gravitons. If in time, our hypothesis proves to be true, he may well prove to have been correct.

5.3.9 An infinitesimal change to General Relativity at cosmic scale

We started everything here in flat space with no mass concentrations. So, uniform densities don't curve space. We introduced mass concentrations and space has to curve around them so as to keep our spherically symmetric 4 volume $k_{\min}$ action densities, required and available, invariant. If we think of the mass in the universe as a dust of density ρ_U essentially at rest in comoving coordinates we can define a tensor $T_{\mu\upsilon}(\text{Background})$. In comoving coordinates $T_{\mu\upsilon}(\text{Background})$ has only one non zero term $T_{00}(\text{Background}) = \rho_U$. In any other coordinates this same $T_{\mu\upsilon}(\text{Background})$ tensor is transformed by the usual tensor transformations that apply in GR. If these coordinates move at peculiar velocity β_P then $T_{00}'''(\text{Background}) = \gamma_P^2\rho_U$ $= \gamma_P^2 T_{00}(\text{Background})$. The same transformation happens in any metric but with $\gamma_M^2\rho_U$. We argue that Eq. (5.3. **22**) is consistent with the infinitesimally modified Einstein field equations

$$G_{\mu\upsilon} \equiv R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = \frac{8\pi G}{c^4}\left[T_{\mu\upsilon} - T_{\mu\upsilon}(\text{Background})\right] \tag{5.3. 23}$$

This infinitesimal modification is only relevant in the extreme case as $T_{\mu\upsilon}$ approaches $T_{\mu\upsilon}(\text{Background})$. Far from mass concentrations $T_{\mu\upsilon} \le T_{\mu\upsilon}(\text{Background})$. Space curvature, in these remote voids, is in general somewhere between slightly negative and zero; but the causally connected universe is always flat on average regardless of the value of Ω. Equation (5.3. 23) is also consistent with Eq. (5.2. 11) $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ at all cosmic times. If there is no inflation, in flat comoving coordinates, at the Big Bang or slightly after, (using $\Upsilon = k_{\min} R_{OH}$) $k_{\min}$ starts at just under one and is always close to the inverse of the causally connected horizon radius. It is also close to the inverse of cosmic time T. It is always at its minimum far from mass concentrations, but increases with the slower clock rates in the local metric around mass concentrations as in Figure 5.3. 9.

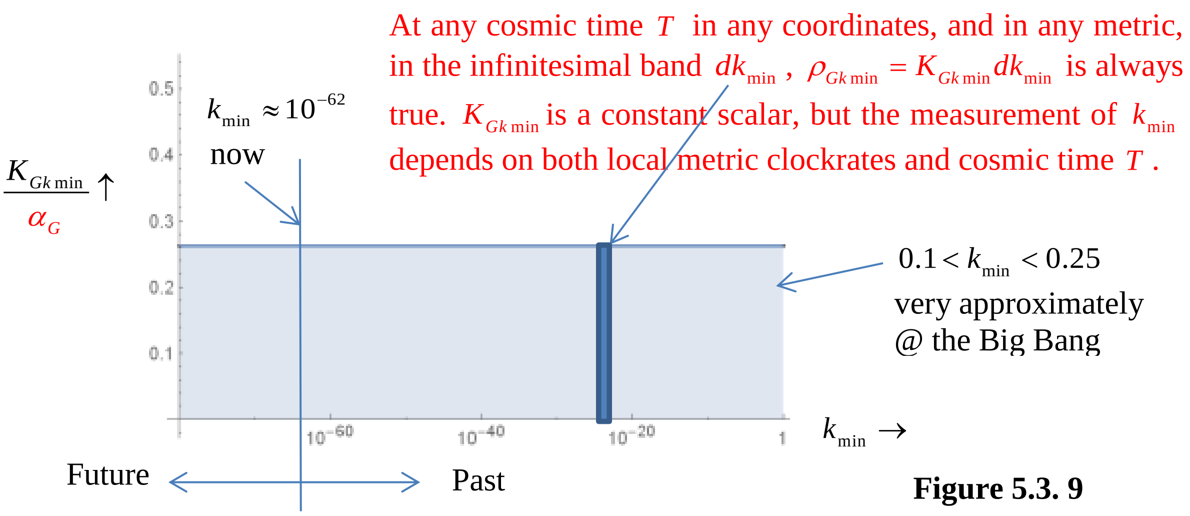


Figure 5.3. 9

5.3.10 **Is Inflation really necessary in this proposed scenario?**

There are two main reasons, usually given, for why inflation is necessary:

(a) The average flatness of space.
(b) The almost uniform temperature of the cosmic microwave background from regions that were initially out of causal contact.

If we put $T_{\mu\upsilon}(\text{Local}) = T_{\mu\upsilon}(\text{Background})$ in Eq.(5.3. 23) the right hand side is identically zero, and $G_{\mu\upsilon} \equiv R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R \equiv 0$ on average throughout all space. The average curvature of all space must be zero and space is compelled to be flat on average.

In section 5.3.3 we found that space has to expand exponentially as in Eq. (5.3. 18) and plotted in Figure 5.3. 4. The actual value of the constant b in $V = 3Exp(bt)$ has to fit experimental observations. But if it is some fundamental constant, which does not seem unreasonable, it must be the same for all comoving observers. If this is so the physics is identical for all such observers, regardless of whether they are in causal contact. Provided we can assume identical starting points everywhere, of say the Planck temperature at cosmic time $T = 0$, then apart from quantum fluctuations, the average background temperature should be some function of cosmic time T for all comoving observers, or at least up to the time the universe became transparent. The physics controlling this should be identical in each comoving frame. Causal contact should not be essential for this. Inflation only guarantees that the starting temperature is uniform everywhere when it stops at approximately $T = 0$. It also has to assume identical physics everywhere from $T \approx 0$ for about the first 375,000 years, or until the universe is transparent. This is virtually identical to what we are proposing in the scenario in this paper.

6 Further Consequences of Infinite Superpositions

6.1 Cosmic Wavelength Superposition Cutoffs

In section 4.2 when we introduced gravity, for the lower limit in our integrals we assumed $k_{\min} = 0$, and then in section 5 showed that there is a lower limit $k_{\min} > 0$. It turns out that for massive $N = 1$ superpositions the effect of this is negligible in comparison to the high frequency cutoff $k_{cutoff} < \infty$, which we showed gravity can address in section 4.2. For infinitesimal rest mass $N = 2$ superpositions we cannot however ignore the effect of $k_{\min} > 0$.

6.1.1 Quantifying the approximate effect of $k_{\min} > 0$ on infinite superpositions

If we look again at section 4.2.1 we can repeat what we did there as follows. Initially to illustrate these effects we will consider only $N=1$ superpositions where we can say that

When $K_{nkCutoff} \to \infty$ & (for $N=1$ only) $K_{nk\min} \to 0$ and thus

$$\left[\frac{-1}{1+K_{nk}^2}\right]_{K_{nk\min}}^{K_{nk}cutoff} = \frac{1}{1+K_{nk\min}^2} - \frac{1}{1+K_{nkCutoff}^2} \approx 1-\left[\frac{1}{K_{nkCutoff}^2}+K_{nk\min}^2\right] \approx 1-\varepsilon'' \approx \frac{1}{1+\varepsilon''} \quad (6.1.\ 1)$$

Our earlier infinitesimal $\varepsilon \to \varepsilon'' \approx \dfrac{1}{K_{nkCutoff}^2}+K_{nk\min}^2$ and from Eq. (3.1. 11) $K_{nk}^2 = \dfrac{n^2 s}{2}\lambda_C^2 k^2$.

For spin ½ fermions for example $\langle n^2 s/2\rangle \approx 9$. Also $k_{Cutoff}^2 \approx 1/L_P^2$ and $k_{\min}^2 \approx 1/R_{OH}^2$ so that

Putting ε as the original $1/K_{nk}^2 cutoff$ at the E_{Planck} of Eq. (4.2. 2) and $\Delta\varepsilon$ as due to $K_{nk\min}$

$$\varepsilon'' \approx \frac{1}{K_{nkCutoff}^2}+K_{nk\min}^2 = \varepsilon+\Delta\varepsilon \approx \frac{L_P^2}{9\lambda_C^2}+\frac{9\lambda_C^2}{R_{OH}^2} \approx \frac{(L_P R_{OH})^2+(9\lambda_C^2)^2}{9\lambda_C^2 R_{OH}^2}$$

The ratio of the extra contribution $\Delta\varepsilon$ to ε (where $\varepsilon''=\varepsilon+\Delta\varepsilon$) is $\dfrac{\Delta\varepsilon}{\varepsilon} \approx \left[\dfrac{9\lambda_C^2}{L_P R_{OH}}\right]^2$ (6.1. 2)

Eq. (6.1. 2) is for spin ½, but the numerical factor 9 only changes slightly for spins 1 & 2. In Planck units $L_P R_{OH} \approx 10^{61}$, but for electrons say $\lambda_C^2 \approx 6\times 10^{44}$, so the effect is of order $\Delta\varepsilon/\varepsilon \approx 10^{46}/10^{61} \approx 10^{-25}$ which we have been ignoring. We cannot ignore this however in the case of infinitesimal rest masses as we will see.

6.2 Infinitesimal Masses and *N* = 2 Superpositions

Looking again at angular momentum and rest masses in section 3.2 the key factor in our final integrals is in Eq. (6.1. 1). Using Eq. (3.1. 12) we can rewrite Eq. (6.1. 1) as

$$\left[\frac{-1}{1+K_{nk}^2}\right]_{K_{nk\min}}^{K_{nk}cutoff} = \frac{1}{\gamma_{nk\min}^2} - \frac{1}{\gamma_{nkCutoff}^2} \quad (6.2.\ 1)$$

With massive $N=1$ superpositions as above the difference between $\gamma_{nk\min}^2$ & 1 is vanishingly small, i.e. $(\gamma_{nk\min}^2-1) \to 1/\infty$ and as in section 6.1.1 this first term is of much less significance than the $\gamma_{nkCutoff}^2$ term. Now define an approximate equality between N & $\langle\gamma_{k\min}\rangle^2$ using Eq. (3.1. 12) as follows

$$N \approx \left[\langle\gamma_{k\min}\rangle^2 = 1+\langle K_{k\min}\rangle^2\right] \quad (6.2.\ 2)$$

In section 3.2 we derived angular momentum and rest masses for only massive or what we called $N = 1$ particles. To get integral angular momentum we had to assume in deriving Eq. (3.2. 6) that the minimum value of K_{nk} or $K_{nk\min} = 0$. For massive $N = 1$ particles such as the fermions the error in this assumption (as in section 6.1.1) is $\approx 10^{-25}$ times smaller than ε, which for an electron is already $\varepsilon \approx 10^{-45}$ due to the high frequency cutoff @ $\approx 10^{18.31} GeV$. (We allowed for this $\varepsilon \approx 10^{-45}$ when we included gravity in section 4.2.) From section 6.1.1 above we approximated $K^2_{nk\min}$ as $\approx 9\bar{\lambda}^2_C / R^2_{OH}$ for a spin ½ fermion. So we can express Eq. (6.2. 2) in terms of this approximation for *fermions with non infinitesimal mass*

$$N \approx \left[\langle \gamma_{k\min} \rangle^2 = 1 + \frac{9\bar{\lambda}^2_C}{R^2_{OH}} \approx 1 \right] \quad \text{as} \quad \frac{9\bar{\lambda}^2_C}{R^2_{OH}} \to 0 \tag{6.2. 3}$$

For example an electron has $\dfrac{9\bar{\lambda}^2_C}{R^2_{OH}} \approx 10^{-77}$

For the massive particles it appears we can safely say that $N = 1$. Even if neutrino masses were as low as $10^{-4} eV$ then $\langle \gamma_{k\min} \rangle^2 - 1 \approx 10^{-59}$. If the mass is too small however Eq. (6.2. 1) tells us we cannot get the correct angular momentum unless something else changes. *Infinitesimal increases above 1 of the order of* $\approx 10^{-50}$ *or so can be handled perhaps by a small change in the actual high frequency cutoff details, but this probably does not allow massive particles to be much less than sub micro electron volts.* So if massive particles are a group with $N = 1$, then it would not seem unreasonable to imagine there could possibly be another group with $N = 2 = 1 + \langle K_{k\min} \rangle^2$ implying that $\langle K_{k\min} \rangle^2 = 1$. Repeating the derivation of Eq. (3.2. 6) but with $N = 2 = 1 + \langle K_{k\min} \rangle^2$ and for clarity and simplicity let $K_{nk\text{cutoff}} \to \infty$.

$$\mathbf{L}_z(Total) = s \cdot (N = 2) m\hbar \int_{K_{nk\min}}^{\infty} \frac{K^2_{nk}}{(1 + K^2_{nk})^2} \frac{dK_{nk}}{K_{nk}} = sm\hbar \left[\frac{-1}{1 + K^2_{nk}} \right]_{K_{nk\min}}^{\infty} \tag{6.2. 4}$$

$$\mathbf{L}_z(Total) = sm\hbar \left[\frac{1}{1 + K^2_{nk\min}} \right] = sm\hbar \left[\frac{1}{(N = 2)} \right] = \frac{sm\hbar}{2} \quad \text{as previously.}$$

Provided we have doubled the probability of superpositions as in Eq. (2.1. 4) from $s \cdot (N = 1) dk / k$ to $s \cdot (N = 2) dk / k$, the final angular momentum results in Eq's. (3.2. 6) & (6.2. 4) are identical. The same is true for rest mass calculations. For multiple integer n infinite superpositions if $N = 2$ then the expectation value $\langle K_{k\min} \rangle^2 = 1$.

We thus conjecture that all $N = 2$ infinite superpositions have $\langle K_{k\min} \rangle^2 = 1$.

From Table 4.3. 1

$N = 2$ infinitesimal rest mass spin 1 superpositions have $\langle n \rangle \approx 3.98$

$N = 2$ infinitesimal rest mass spin 2 superpositions have $\langle n \rangle \approx 3.33$

Using Eq's. (3.1. 11) and Eq.**(5.2. 10)**.

$$\langle K_{k\min}\rangle^2 = \frac{\langle n\rangle^2 s}{2}\lambda_C^2 k_{\min}^2 \approx \frac{15.82}{2}\lambda_C^2 k_{\min}^2 = 1 \text{ or } \lambda_C \approx 0.355\frac{R_{OH}}{\Upsilon} \text{ for Spin 1}$$
$$\approx \frac{11.09\times 2}{2}\lambda_C^2 k_{\min}^2 = 1 \text{ or } \lambda_C \approx 0.300\frac{R_{OH}}{\Upsilon} \text{ for Spin 2} \qquad (6.2.\ 5)$$

Using the value for $\Upsilon \approx 0.65$ from Eq. (5.3. 13) based on WMAP data which also puts the horizon radius at $\approx 46\times 10^9$ light years $R_{OH} \approx 2.7\times 10^{61}$ Planck lengths.

Spin	$\langle n\rangle$	Compton Wavelength λ_c	Infinitesimal Rest Mass
1	3.98	$\approx 0.55 R_{OH}$	$\approx 8.3\times 10^{-34} eV.$
2	3.33	$\approx 0.46 R_{OH}$	$\approx 9.8\times 10^{-34} eV.$

Table 6.2 1 Infinitesimal rest masses of $N=2$ photons, gluons & gravitons.

These Compton wavelengths and rest masses are the present values, reducing slowly but exponentially with cosmic time *T*. They are based on WMAP data where $\Omega = 1$ and could be slightly different if $\Omega \neq 1$ and $\Upsilon^2 V \approx 4$ as in Eq. **(5.3. 14)**. They also depend on the actual value of *b* in the exponential expansion $V = 3Exp(bt)$. These infinitesimal rest masses limit the range of virtual photons and gravitons to approximately the horizon. The graviton rest masses above are also close to recent proposals for the accelerating expansion of the cosmos [2] [3].

6.2.1 Cutoff behaviours for *N* = 1 & *N* = 2 superpositions

Equation (6.2. 1) can be written for both $N=1$ & $N=2$ superpositions using the results of sections 4.2 & 6.2 as follows

$$\left[\frac{-1}{1+K_{nk}^2}\right]_{K_{nk\min}}^{K_{nk}cutoff} = \frac{1}{\left[\gamma_{nk\min}^2 = 2\right]} - \frac{1}{\gamma_{nkCutoff}^2} = \frac{1}{2(1+\varepsilon'')} \quad \text{when } N=2 \qquad (6.2.\ 6)$$
$$\left[\frac{-1}{1+K_{nk}^2}\right]_{K_{nk\min}}^{K_{nk}cutoff} = \frac{1}{\left[\gamma_{nk\min}^2 \approx 1\right]} - \frac{1}{\gamma_{nkCutoff}^2} = \frac{1}{1+\varepsilon''} \quad \text{when } N=1$$

(We should be using expectation values, but for clarity we simply imply them.) We have shown in section 6.2 that $\langle 1/\gamma_{k\min}{}^2\rangle = 1/2$ when $N=2$, but in reality it is Eq. (6.2. 6) that must be true. In section 4.2 we showed that for $N=1$ superpositions the primary coupling of gravity to preons infinitesimally increased the interaction probability by ε' to $(1+\varepsilon')$ where

from Eq. (4.2. 4) $$\varepsilon' = \frac{m_0^2\chi_G'\cdot G}{2s\hbar c(8+8\sqrt{\alpha_{EMP}})^2} = \varepsilon = \frac{1}{K_{nk}^2 cutoff} = \frac{2m_0^2c^2}{sn^2\hbar^2(k_{cutoff})^2}.$$

In the $N=1$ case this meant that any deficits due to a non infinite cutoff were exactly balanced by the contribution from gravity, but in the $N=2$ case this infinitesimal correction is out by a factor of two. However Eq. (6.2. 6) tells us that exactness can be maintained in the $N=2$ case by an infinitesimal change from $\langle 1/\gamma_{k\min}^2 \rangle = 1/2$ to $\langle 1/\gamma_{k\min}^2 \rangle \approx 1/2$. Thus both $N=1$ & $N=2$ superpositions can cut off at Planck energy as in section 4.2.2. The low frequency cutoff for all superpositions must be at $k_{\min} \approx \Upsilon / R_{OH}$ if they are to affect gravity.

6.2.2 An exponential cutoff at cosmic wavelengths

We used a square cutoff above for $k_{\min}$ but an exponential cutoff is most probable. As superposition wavefunctions are squared exponentials we will use a similar type cutoff finding that it fits quite well (see Figure 5.3. 1). Going over what we did in Eq.**(6.2. 4**) and

Putting $x = \dfrac{k}{k_{\min}} = \dfrac{K_{nk}}{K_{nk\min}}$ then using $\displaystyle\int_{x=1}^{x=\infty} \frac{xdx}{(1+x^2)^2} = 0.25 \approx \int_{x=0}^{x=\infty} \frac{(1-e^{-0.61x^2})xdx}{(1+x^2)^2} \approx 0.249991$

An exponential cutoff of $(1-e^{-0.61x^2})$ @ $k_{\min}$ is $\approx$ the same as a square cutoff @ $k_{\min}$ (6.2. 7)

6.2.3 Virtual particle pairs emerging from the vacuum and space curvature

For almost a century it has been a puzzle why spacetime is not massively curved by Planck scale zero point energy densities. However space appears to be flat on average regardless of this massive Planck scale zero point energy density so *something must be different and what is it?* In section 5.1.1 we conjectured that virtual particles are just single wavenumber k superposition members, whereas real particles are full infinite superpositions of all wavenumbers k from $k_{\min}$ to k_{Planck}. We assumed this was true in all of section 5. We are going to raise this claim to be the actual difference between virtual and real particles. Only full infinite superpositions have real properties that can be measured (such as measured mass/energy) rather than implied. Because $k_{\min}$ virtual gravitons are such single members they couple to $k_{\min}$ members of full infinite superpositions. On the other hand virtual particles out of the vacuum, are mainly short lived high k single value members that will not couple to $k_{\min}$, if our claim above is true. The density of $k_{\min}$ virtual pairs from the vacuum is virtually zero as it is based on the Lorentz invariant supply of local zero point fields, not from the receding horizon (see sections 6.2.4 & 6.2.5 below). But this is not the full story. The virtual particles that dress electrons and quarks for example add mass to the real particles. In fact the majority of the proton and neutron mass is due to the virtual gluons interacting between quarks. If short lived virtual particles somehow contribute to the mass of full infinite superpositions, then these virtual particles indirectly contribute to the $k_{\min}$ virtual graviton coupling, which is based on the actual mass of the infinite superposition as in Eq. (3.2. 3). The conservation of energy or in reality 4 momentum says that what we call "real matter or energy" can last for close to the age of the universe. It will have mass and by definition it can be weighed. It can move around, even close to the speed of light, but it is conserved.

Gravitons that last this long we have called $k_{\min}$ gravitons and they can only couple to real, or long lasting energy/matter that can be weighed in whatever manner. The rotating dark matter in galaxies we cannot weigh directly, but it contributes to the theoretical weight of a galaxy. We have to allow for this mass when studying galaxy dynamics. The particle beams in accelerators have real energy which can be temporarily converted into virtual particles. The total energy or 4 momentum is always conserved, but can fluctuate for time $\Delta T \approx 1/\Delta E$. The long term average is what counts. In this sense the mass of short lived virtual particles can contribute to $k_{\min}$ virtual graviton coupling, just as it does in the virtual particle dressing of real charged particles as above.

6.2.4 Redshifted zero point energy from the horizon behaves differently to local

As we said above local zero point energies are Lorentz invariant. At high frequencies there is no shortage locally to build the high frequency components of full infinite superpositions. But as we have shown this is not so as we approach cosmic wavelengths. If there were no redshifted supply from the horizon there would be only a few modes of the local supply of $k_{\min} \approx 1/R_{OU}$ quanta inside the horizon. Because preons are born with zero momentum and infinite wavelength they can however absorb a different (but also spherically symmetric supply as required to build superpositions) of redshifted $k_{\min} \approx 1/R_{OU}$ quanta from the receding horizon as we have discussed. This $k_{\min}$ quanta redshifted supply behaves differently to normal Lorentz invariant zero point local fields. It behaves as

$$K_{Qk\min} = 0.055\alpha_G \text{"The Quanta required @ } k_{\min}\text{Constant"} \text{ of Eq. } (\mathbf{5.3.\ 12}).$$
$$\text{Where } K_{Qk\min} \approx 0.48 \times K_{Gk\min} \text{"The } k_{\min}\text{Graviton Constant" of Eq. (5.2. 11).}$$

This redshifted supply is only available to zero spin preons that are born with zero momentum, or infinite wavelength, in the rest frame in which infinite superpositions are built.

6.2.5 Revisiting the building of infinite superpositions

In section 2 we developed equations to determine the probability of each mode of a superposition using local zero point fields. In section 5, when we found the cosmic wavelength supply inadequate, we switched to a different redshifted supply for long range quanta. So how do we justify our use of the local zero point fields to determine mode probabilities and behaviours? As we said above there is a plentifull supply of high frequency local zero point fields. This local supply is adequate for high densities of superpositions for all modes from the Planck energy $k=1$ high energy mode cutoffs to somewhere around $k \approx 10^{-20}$ or near nuclear wavelengths. The coupling to local zero point fields in this high frequency region determines the behaviour of all the standard model particles. There is however a gradual transition to absorbing quanta from the redshifted horizon supply as the wavelength increases. Because the redshited supply of $k_{\min}$ quanta behaves as the invariants $K_{Qk\min}$ or $K_{Gk\min}$ above and entirely differently to Lorentz invariant local zero point fields, spacetime has to warp around mass concentrations and the universe has to expand.

6.2.6 The primary to secondary graviton coupling ratio χ_G

In Eq. (4.2. 14) we found $\chi'_G \approx 318.3$ as the ratio between the primary graviton coupling to a bare Planck mass and the *normal measured gravitational constant G*. Equation (5.1. 7) defined graviton coupling between Planck masses α_G. If $\alpha_G = 1$ as we had expected, the ratio between primary and secondary graviton coupling (as defined for colour and electromagnetism in Eq. (3.3. 2) would be $\chi_G = \alpha_G^{-1}\chi'_G = \chi'_G \approx 318.3$.

$$\text{The primary to secondary graviton coupling ratio } \chi_G = \alpha_G^{-1}\chi'_G \approx 318 \qquad (6.2.\ 8)$$

To solve graviton superpositions we can use Eq. (3.3. 16) which is the *gravitational interaction probability* between fermions and we can now put on the RHS the coupling ratio $\chi_G \approx 23,200$ in the same way as we did for Eq.(3.3. 21). (This $c_{4c} * c_{4c}(1-c_{4c} * c_{4c})$ we are going to calculate here is for spin 2 & $N=2$. It is different to the double combination of $(N=2)\times(\text{Spin 1})$ or $(N=1)\times(\text{Spin 2})$ for $4c_{4b} * c_{4b}(1-c_{4b} * c_{4b})$ we derived in Eq. (4.4. 1)).

$$\frac{\left[2s_{1/2}N_1 c_{6a} * c_{6a}(1-c_{6a} * c_{6a})\right]^2 \left[2s_2 N_2 c_{4c} * c_{4c}(1-c_{4c} * c_{4c})\right]^2}{q^4} = \frac{4(\chi_G^{-1})^2}{q^4}$$

$$2s_{1/2} = 1, N_1 = 1, \& 2s_2 = 4,\ N_2 = 2 \quad \text{so } \left[c_{6a} * c_{6a}(1-c_{6a} * c_{6a})\right]\left[8c_{4c} * c_{4c}(1-c_{4c} * c_{4c})\right] = 2\chi_G^{-1} = 2/318$$

$$\text{or } c_{4c} * c_{4c}(1-c_{4c} * c_{4c}) \approx \left[\frac{1}{318}\right]\frac{1}{8\left[c_{6a} * c_{6a}(1-c_{6a} * c_{6a})\right]}$$

But from Eq. (4.4. 1) $c_{6a} * c_{6a}(1-c_{6a} * c_{6a}) = \sqrt{2/\chi_C} \approx \sqrt{2/50.4053} \approx 0.199194$

$$\text{So} \quad c_{4c} * c_{4c}(1-c_{4c} * c_{4c}) \approx \frac{1}{4 \times 318 \times 0.199194} \approx 3.9 \times 10^{-3} \approx \frac{1}{254}.$$

Using Eq.(4.4. 3), $\sum c_n * c_n \cdot n^4 \approx 170.95$ for spin 2, $N=2$ we get the infinitesimal mass graviton superposition values in Table 4.3. 1.

6.2.7 Massive Bosons and the Higg's mechanism

In the Standard Model the Higg's mechanism adds mass to zero mass photons but here we say it adds mass to infinitesimal mass photons. But not only does it do that, it also converts them from from $N=2$ to $N=1$, and also from $n=3,4,5$ to $n=4,5,6$ superpositions.

7 Angular Momentum and the Kerr Metric

In the next two sections we revert to simple 3 volume densities for $k_{\min}$ gravitons.

In the Schwarzchild metric the increase in volume is the same as the frequency increase as $g_{rr} \cdot g_{tt} = 1$ and $g_{\theta\theta} \cdot g_{\phi\phi} = r^4 \sin^2\theta$ is invariant if there is no angular momentum. With angular momentum both $g_{\theta\theta}$ & $g_{\phi\phi}$ change. The volume ratio of $g_{\mu\nu} \neq \eta_{\mu\nu}$ space, to $g_{\mu\nu} \neq \eta_{\mu\nu}$ space in

any metric at fixed $r\,\&\,\theta$ is $\dfrac{V'}{V} = \sqrt{\dfrac{(g'_{rr} \cdot g'_{\theta\theta} \cdot g'_{\phi\phi})(g_{\mu\nu} \neq \eta_{\mu\nu})}{(g_{rr} \cdot g_{\theta\theta} \cdot g_{\phi\phi})(g_{\mu\nu} = \eta_{\mu\nu})}} = \sqrt{\dfrac{(g'_{rr} \cdot g'_{\theta\theta} \cdot g'_{\phi\phi})(g_{\mu\nu} \neq \eta_{\mu\nu})}{r^4 \sin^2\theta}}$ (7.1. 1)

The Kerr metric can be written in Boyer-Lindquist coordinates as

$$\left[\begin{array}{l} g_{\theta\theta} = r^2 + \alpha^2 \cos^2\theta \\ g_{\varphi\varphi} = (r^2 + \alpha^2 + \dfrac{r_S r}{\rho^2}\alpha^2 \sin^2\theta)\sin^2\theta \\ g_{t\varphi} = \dfrac{r_S r}{g_{\theta\theta}}\alpha \sin^2\theta \\ g_{rr} = \dfrac{g_{\theta\theta}}{\Delta} \qquad \& \qquad g_{tt} = 1 - \dfrac{r_S r}{g_{\theta\theta}} \end{array}\right.$$

Where $\Delta = r^2 + r_S r + \alpha^2$ and $\alpha = \dfrac{J}{mc}$ and $r_S = \dfrac{2Gm}{c} = 2m$ is the Schwarzchild radius in Planck units where $G = c = 1$. Everything is in units of length or (length)^2, except g_{rr} & g_{tt} which are dimensionless. Because we want volume ratios as in Eq. (7.1. 1) we can write the above version of the Kerr metric in a dimensionless form, leaving the length squared, and length terms $r^2, r^2\sin^2\theta\,\&\,r\sin\theta$ in $r^2 d\theta^2$, $r^2\sin^2\theta d\varphi^2$ & $r\sin\theta d\varphi$ etc outside the metric tensor. This effectively gives us the denominator $r^4\sin^2\theta$ we want in Eq. (7.1. 1) as we will see. Also, the angular momentum parameter α is a length dimension.

Writing the above in dimensionless form as follows, using $-+++$ for the line element ds^2:

A Dimensionless form of the Kerr Metric where $\Delta = 1 + \dfrac{\alpha^2}{r^2} - A$ and $A = \dfrac{2m}{r}$ but we will add an also dimensionless $\dfrac{m^2}{r^2}$ later. See section 8

(We assume silent $G = c^2 = 1$ Planck value constants in $A = \dfrac{2m}{r} +$ a dimensionless term)

$$\left[\begin{array}{l} g_{\varphi\varphi} = 1 + \dfrac{\alpha^2}{r^2} + \dfrac{A}{g_{\theta\theta}}\dfrac{\alpha^2}{r^2}\sin^2\theta \\ g_{\theta\theta} = 1 + \dfrac{\alpha^2}{r^2}\cos^2\theta \\ g_{rr} = \dfrac{g_{\theta\theta}}{\Delta} \\ g_{t\varphi} = \dfrac{A}{g_{\theta\theta}}\dfrac{\alpha}{r}\sin\theta \\ g_{tt} = 1 - \dfrac{A}{g_{\theta\theta}} \end{array}\right. \qquad (7.1.\ 2)$$

The space surrounding a rotating mass corotates with it. If we move in this corotating reference frame there is a new metric time component, which after some rearranging of plus and minus signs just for convenience, we can write as: $g'_{tt} = g_{tt} + \dfrac{g_{t\phi}^2}{g_{\phi\phi}}$.

Thus using Eq. (7.1. 2) $\quad g'_{tt} = g_{tt} + \dfrac{g_{t\varphi}^2}{g_{\varphi\varphi}} = (1 - \dfrac{A}{g_{\theta\theta}}) + \dfrac{\dfrac{A^2}{g_{\theta\theta}^2}\dfrac{\alpha^2}{r^2}\sin^2\theta}{\left[1 + \dfrac{\alpha^2}{r^2} + \dfrac{A}{g_{\theta\theta}}\dfrac{\alpha^2}{r^2}\sin^2\theta\right]}$

$$= (1 - \frac{A}{g_{\theta\theta}}) + \frac{\dfrac{A^2}{g_{\theta\theta}}\dfrac{\alpha^2}{r^2}\sin^2\theta}{g_{\theta\theta}\left[1 + \dfrac{\alpha^2}{r^2} + \dfrac{A}{g_{\theta\theta}}\dfrac{\alpha^2}{r^2}\sin^2\theta\right]}$$

$$= \frac{g_{\theta\theta}(1 + \dfrac{\alpha^2}{r^2} + \dfrac{A}{g_{\theta\theta}}\dfrac{\alpha^2}{r^2}\sin^2\theta) - A(1 + \dfrac{\alpha^2}{r^2}) - \dfrac{A^2}{g_{\theta\theta}^2}\dfrac{\alpha^2}{r^2}\sin^2\theta + \dfrac{A^2}{g_{\theta\theta}^2}\dfrac{\alpha^2}{r^2}\sin^2\theta}{g_{\theta\theta}(1 + \dfrac{\alpha^2}{r^2} + \dfrac{A}{g_{\theta\theta}}\dfrac{\alpha^2}{r^2}\sin^2\theta)}$$

$$= \frac{-A(1 + \dfrac{\alpha^2}{r^2} - \dfrac{\alpha^2}{r^2}\sin^2\theta) + g_{\theta\theta}(1 + \dfrac{\alpha^2}{r^2})}{g_{\theta\theta}g_{\phi\phi}}$$

$$= \frac{-A(1 + \dfrac{\alpha^2}{r^2}\cos^2\theta) + g_{\theta\theta}(1 + \dfrac{\alpha^2}{r^2})}{g_{\theta\theta}g_{\phi\phi}}$$

$$g'_{tt} = g_{\theta\theta}\frac{-Ag_{\theta\theta} + g_{\theta\theta}(1 + \dfrac{\alpha^2}{r^2})}{g_{\phi\phi}} = \frac{g_{\theta\theta}(1 + \dfrac{\alpha^2}{r^2} - A)}{g_{\theta\theta}g_{\phi\phi}} = \frac{\Delta}{g_{\phi\phi}} \qquad (7.1.\ 3)$$

We have explicitly gone through this to show that if the parameter $A = 2m/r$ is dimensionless, there is potentially freedom to change it without changing this equation.
(See section 8.1.6 as this is similar to what happens in the Kerr-Newman metric, where instead of a dimensionless m^2/r^2 term, a dimensionless r_Q^2/r^2 or equivalently a dimensionless Q^2/r^2 is included in term A . See for example Table 8.1. 2 &Table 8.1. 3.

We will work in corotating frames. Space is swirling around the black hole effectively at rest in these frames, simplifying our calculations and equations. (Section 9 puts this into a four vector form, invariant in all frames.) If a small mass, at rest at infinity in the same rest frame as the rotating black hole, falls inwards, it will have the same circumferential velocity as the corotating rest frames at all radii. It will be falling radially through these corotating frames. As in section 5.2.2 we call this radial velocity β_M where as in the non-rotating case

$$\frac{1}{\sqrt{1-\beta_M^2}} = \gamma_M \quad \text{but now} \quad \frac{1}{\sqrt{1-\beta_M^2}} = \gamma_M = \frac{1}{\sqrt{g'_{tt}}} \quad \text{the new inverse rate of clocks.}$$

In corotating frames
$$\frac{1}{\gamma_M^2} = g'_{tt} = \frac{\Delta}{g_{\phi\phi}} \qquad (7.1.\ 4)$$
$$\gamma_M^2 = \frac{g_{\phi\phi}}{\Delta}$$

Frequencies measured in corotating frames increase as γ_M . Similarly using Eq's. (7.1. 1) & (7.1. 4) we can get the (three) volume element ratio in this corotating refence frame.

The 3 volume element ratio $V = \sqrt{(g_{rr} \cdot g_{\theta\theta} \cdot g_{\phi\phi})} = \sqrt{\frac{g_{\theta\theta}}{\Delta} g_{\theta\theta} g_{\phi\phi}} = g_{\theta\theta}\sqrt{\frac{g_{\phi\phi}}{\Delta}} = g_{\theta\theta}\gamma_M$ (7.1. 5)

With angular momentum we no longer have the same increase in frequency as volume as in the Schwarzchild case. With no angular momentum we found that the probability density of time polarized $k_{\min}$ gravitons Eq. (5.2. 14) $\Delta\rho_{Gk\min} \approx \gamma_M^2 \beta_M^2 K_{Gk\min} dk_{\min} = \gamma_M^2 \frac{2m}{r} K_{Gk\min} dk_{\min}$. (Again temporarily adding γ_M^2). With rotation we will find a circularly polarized $\cos^2\theta$ type distribution of gravitons around the axis. These add to the time polarized dimensionless number $\frac{2m}{r}$ to get an as yet unknown number we simply label as X where $X > \frac{2m}{r}$

(7.1. 6)

Let us rewrite Eq.(5.2. 14) as $\Delta\rho_{Gk\min} \approx \gamma_M^2 X K_{Gk\min} dk_{\min}$ with rotation

Where the factor γ_M^2 is for clarity only. Repeating the derivation of Eq. (5.2. 15)

$$\rho_{Gk\min}(\text{Undiluted Total}) = K_{Gk\min} dk_{\min} + \gamma_M^2 X K_{Gk\min} dk_{\min} = (1+\gamma_M^2 X) K_{Gk\min} dk_{\min}$$

As in the derivation of Eq. (5.2. 16) but in a slightly different order, we divide this undiluted total by the new volume $V = g_{\theta\theta}\gamma_M$ in Eq. (7.1. 5) to get the new $k_{\min}$ graviton density $\rho''_{Gk\min}$. If our conjectures are correct $\rho''_{Gk\min} = K_{Gk\min} dk''_{\min}$ is always true, and as our measurement of $k_{\min}$ increases to $k''_{\min} = \gamma_M k_{\min}$ in the new metric, $\rho''_{Gk\min} = K_{Gk\min}\gamma_M dk_{\min}$; rewriting as follows

$$\rho''_{Gk\min} = \frac{(1+\gamma_M^2 X) K_{Gk\min} dk_{\min}}{V} = \frac{(1+\gamma_M^2 X) K_{Gk\min} dk_{\min}}{g_{\theta\theta}\gamma_M} = \gamma_M K_{Gk\min} dk_{\min} = K_{Gk\min} dk''_{\min}$$
$$(1+\gamma_M^2 X) K_{Gk\min} dk_{\min} = g_{\theta\theta}\gamma_M^2 K_{Gk\min} dk_{\min}$$

$$1+\gamma_M^2 X = g_{\theta\theta}\gamma_M^2$$

$$X = g_{\theta\theta} - \frac{1}{\gamma_M^2} = (1 + \frac{\alpha^2}{r^2}\cos^2\theta) - \frac{1}{\gamma_M^2}$$

$$X = (1 + \frac{\alpha^2}{r^2}\cos^2\theta) - \frac{\Delta}{g_{\varphi\varphi}} \quad \text{using Eq. (7.1. 4)}$$

$$X = 1 + \frac{\alpha^2}{r^2}\cos^2\theta - \frac{1 + \frac{\alpha^2}{r^2} - A}{1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}}\frac{\alpha^2}{r^2}\sin^2\theta} \quad \text{using Eq's.(7.1. 2)}$$

We can write this as $$X = \frac{\alpha^2}{r^2}\cos^2\theta + \frac{\left[1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}}\frac{\alpha^2}{r^2}\sin^2\theta\right] - \left[1 + \frac{\alpha^2}{r^2} - A\right]}{1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}}\frac{\alpha^2}{r^2}\sin^2\theta}$$

$$X = \frac{\alpha^2}{r^2}\cos^2\theta + \frac{A\left[1 + \frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\theta\theta}}\right]}{1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}}\frac{\alpha^2}{r^2}\sin^2\theta}$$

$$X = \frac{\alpha^2}{r^2}\cos^2\theta + \frac{A\left[1 + \frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\theta\theta}}\right]}{g_{\phi\phi}} \quad \text{using Eq's.(7.1. 2)}$$

$$X = \frac{\alpha^2}{r^2}\cos^2\theta \quad + \quad \frac{A}{g_{\varphi\varphi}} \quad + \quad A\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\phi\varphi}g_{\theta\theta}}$$

Section 7.1.3 discusses why there is no separate term in $A\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\phi\varphi}g_{\theta\theta}}$ so we will write this as

$$X = \frac{\alpha^2}{r^2}\cos^2\theta \quad + \quad \frac{Ag_{\theta\theta}}{g_{\varphi\varphi}g_{\theta\theta}} \quad + \quad A\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\phi\varphi}g_{\theta\theta}}$$

$$X = \frac{\alpha^2}{r^2}\cos^2\theta \quad + \quad \frac{A(1 + \frac{\alpha^2}{r^2}\cos^2\theta)}{g_{\varphi\varphi}g_{\theta\theta}} \quad + A\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\phi\varphi}g_{\theta\theta}}$$

Which we finally write as $$X = \frac{\alpha^2}{r^2}\cos^2\theta + \frac{A(1 + \frac{\alpha^2}{r^2})}{g_{\varphi\varphi}g_{\theta\theta}} \qquad (7.1.\ 7)$$

Putting $A = \frac{2m}{r}$, the *extra* $k_{\min}$ virtual gravitons $\gamma_M^2 X$ (due to a mass m rotating with angular parameter α that has dimensions of length) are the following two polarization groups (The background $k_{\min}$ virtual gravitons have been normalized to one when $\gamma_M = 1$)

Circularly polarized spin2: $\left[\frac{\alpha^2}{r^2}\cos^2\theta\right]\times(m=\pm 2)$ & Time polarized spin 2: $\left[\frac{\frac{2m}{r}(1+\frac{\alpha^2}{r^2})}{g_{\varphi\varphi}g_{\theta\theta}}\right]$

We can rewrite Eq. (5.2. 16) using $1+\gamma_M^2 X = g_{\theta\theta}\gamma_M^2$ or $\frac{1}{\gamma_M^2}+X = g_{\theta\theta} = 1+\frac{\alpha^2}{r^2}\cos^2\theta$

$$\left[\frac{1}{\gamma_M^2}+\frac{\alpha^2}{r^2}\cos^2\theta+\frac{A(1+\frac{\alpha^2}{r^2})}{g_{\varphi\varphi}g_{\theta\theta}} = 1+\frac{\alpha^2}{r^2}\cos^2\theta\right] \text{ or } \left[\frac{1}{\gamma_M^2}+\frac{A(1+\frac{\alpha^2}{r^2})}{g_{\varphi\varphi}g_{\theta\theta}} = 1\right] \text{ where } \left[\frac{A(1+\frac{\alpha^2}{r^2})}{g_{\varphi\varphi}g_{\theta\theta}} = \beta_M^2\right]$$

We have a 4 vector equation again as circular polarization cancels on both sides. The main thing to notice here is that the circularly polarized $k_{\min}$ gravitons are independent of the central mass, suggesting they are due to the effect of the rotation of space, or frame dragging, on the $k_{\min}$ graviton background. We will discuss this in section 7.1.2. The extra $k_{\min}$ gravitons due to the central mass have a $(1+\alpha^2/r^2)/(g_{\varphi\varphi}g_{\theta\theta})$ factor, distorting them from spherical symmetry. Figure 7.1. 1 & Figure 7.1. 2 compare the above with spinning charged spheres in electromagnetism. The electrostatic energy density surrounding a charged sphere however, reduces with radius as r^{-4}, and magnetic energy as r^{-6}, or two more powers of radius. With gravity however we have been looking at the probability density of minimum wavenumber $k_{\min}$ gravitons surrounding a mass. With no angular momentum there are only time polarized $k_{\min}$ gravitons, and their extra probability density drops as r^{-1}, as so far we have only focussed on those $k_{\min}$ gravitons (the vast majority), that interact with the rest of the mass in the universe. If a charged sphere rotates, there is a radial magnetic field of circularly polarized $m=\pm 1$ photons varying in intensity as $\cos^2\theta$ and a transverse magnetic field (of transversely polarized $m=\pm 1$ photons) varying as $\sin^2\theta$ as in Figure 7.1. 1

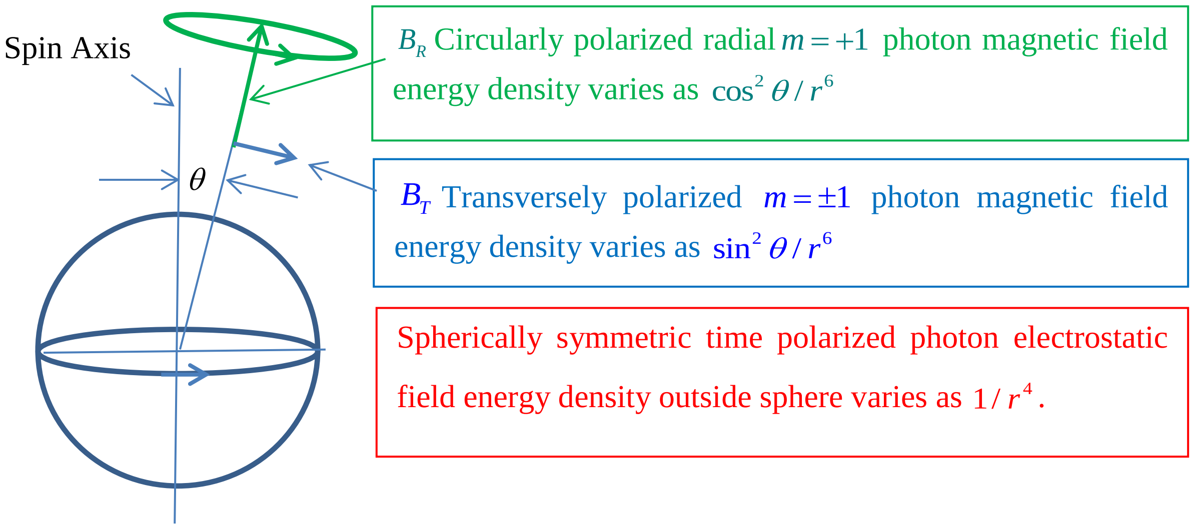


Figure 7.1. 1 Spinning electrically charged sphere. At same radius $B_R@(\theta=0)=2B_T@(\theta=\pi/2)$.

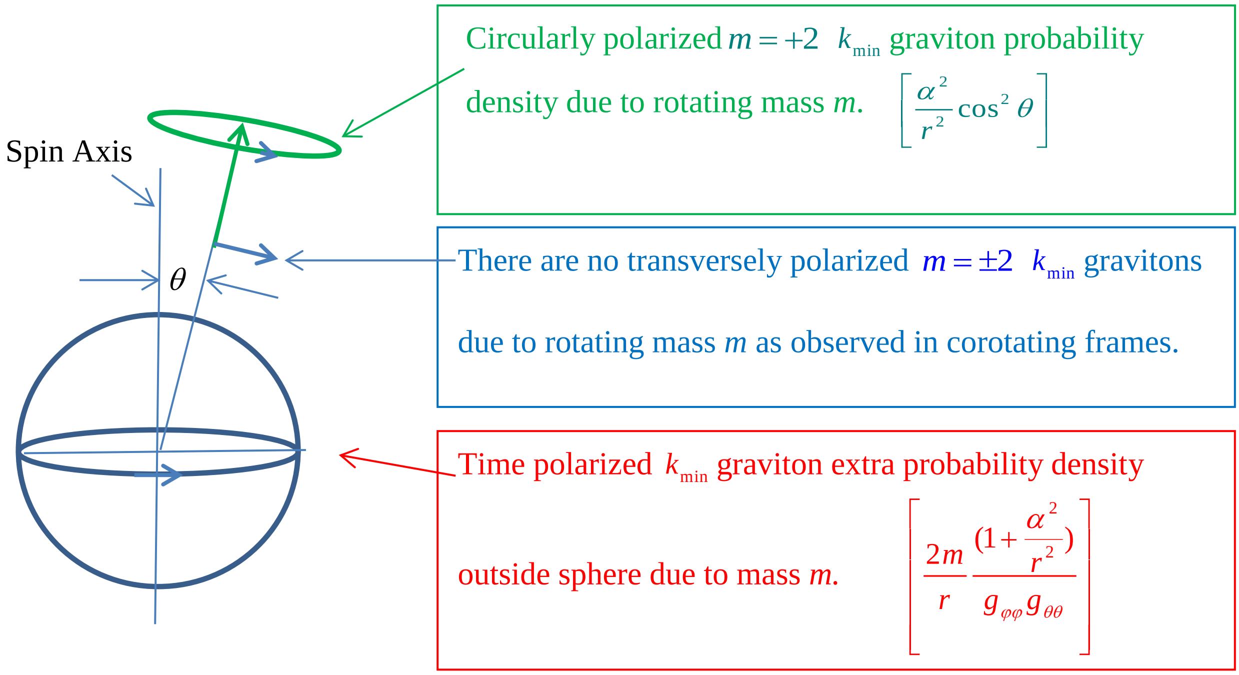


Figure 7.1. 2 Spinning mass m with angular momentum length parameter α as viewed in a corotating frame. There are circularly polarized $(m=\pm 2)\times k_{\min}$ gravitons due to the effect of frame dragging on the background time polarized $k_{\min}$ gravitons. There are no transversely polarized $(m=\pm 2)\times k_{\min}$ gravitons due to a rotating mass m as seen in a corotating frame. Radially polarized extra $k_{\min}$ gravitons due to mass m are distorted from spherical symmetry as $(1+\alpha^2/r^2)/(g_{\varphi\varphi}g_{\theta\theta})$. For $r >> r_{Sw}$ we can ignore the effects of $g_{\theta\theta}, g_{\varphi\varphi}$. as they rapidly tend to one, with the metric written in dimensionless form as in Equ's.(7.1. 2)

7.1.1 Stress tensor sources for spin 2 gravitons & 4 current sources for spin 1

Spin 1 particles behave like a 4 vector as they come from a 4 current source, transforming with velocity as in the Special Relativity transformations of Minkowski spacetime. Spin 2 gravitons in contrast come from mass/energy density sources. There are two factors in their transformations with velocity. One from the mass increase per source particle, and the second from the increase in particles per unit length due to length contraction. Thus Spin 2 particles transform as a 4x4 rank 2 tensor, which Einstein connected with spacetime curvature.

The rules of quantum mechanics tell us that spherically symmetric spin 2 particles should be equal $1/\sqrt{5}$ superpositions of $m=-2,-1,0,+1,+2$ states. But the shape of gravitational waves behaves like transversely polarized $m=\pm 2$ particles, suggesting the $k_{\min}$ gravitons surrounding mass concentrations may only consist of time polarized, plus $m=\pm 2$ circularly polarized, spin 2 particles. In Eq. (3.2. 9) we showed that a spherically symmetric massive (or infinitesimal mass) spin one state is built from equal $1/\sqrt{3}$ superpositions of $m=-2,0,+2$ states, and if this is true then we will conjecture that the same superposition of $m=-2,0,+2$

states must be able to build a spherically symmetric spin 2 state. When we looked at non rotating spherical masses it appeared that, even close to black holes, the spherical symmetry of the Schwarzchild metric suggested similarly spherically symmetric, time polarized, extra $k_{\min}$ gravitons down to the horizon; with space expanding only radially. Thus before we considered angular momentum we could treat all $k_{\min}$ gravitons as only time polarized. A stress tensor source with no angular momentum has spherically symmetric spacetime curvature with time polarized $k_{\min}$ gravitons. But angular momentum in the source produces cylindrically symmetric spacetime curvature. We still have radially polarized $k_{\min}$ gravitons (in co-rotating coordinates) due to the central mass, but distorted from spherical symmetry as $(1+\alpha^2/r^2)/(g_{\varphi\varphi}g_{\theta\theta})$ which only affects the close in region, disappearing as $\alpha \to 0$. But there are also circularly polarized $m = \pm 2$ $k_{\min}$ gravitons only related to angular momentum. These circularly polarized $k_{\min}$ gravitons do not have the $2m/r$ factor and must be very different. As we will discuss below it appears that they are generated from the background time polarized $k_{\min}$ gravitons by the swirling velocity of corotating space.

7.1.2 Circularly polarized gravitons from corotating space

The circularly polarized gravitons do not have a $2m/r$ factor. The Kerr metric is an exact solution to Einstein's field equations, which we conjecture (in an infinitesimally modified form as in Eq. (5.3. 23) are consistent with the $k_{\min}$ Graviton constant being invariant at all points in spacetime, or that Eq. (5.2. 11) is always true. If this is so then Eq. (7.1. 7) should be true also. We can perhaps just accept that it must be true, but at the same time we can look at whether it makes sense?

The angular momentum parameter α has dimensions of length, and is defined as $\alpha = \dfrac{J}{mc}$.
Because angular momentum is the cross product of momentum by radius or $m\mathbf{v}\times\mathbf{r}$, we can think of this length parameter as a vector of length α, pointing along the axis of spin, with components $\alpha\cos\theta$ at any polar angle θ to the spin axis. Space corotates around spinning masses with angular velocity $\Omega = \dfrac{g_{t\phi}}{g_{\phi\phi}}$ which in the plane of the equator simplifies to

$$\Omega = \frac{r_S\alpha c}{r^3 + r\alpha^2 + r_S\alpha^2} \approx \frac{r_S\alpha c}{r^3} \text{ when } r >>> r_S \;\&\; \alpha .$$

$$\text{At large radii the corotating velocity} \quad V = \Omega\times\mathbf{r} \approx \frac{r_S\alpha c}{r^2} \tag{7.1. 8}$$

Because r_S & α have dimensions of length this equation has dimensions of velocity, and if we divide it by c it is dimensionless. We will call it $\beta_{Coratating} = \beta_C$

At large radii $\beta_{Coratating} = \beta_C = \frac{V}{c} = \frac{\Omega \times r}{c} \approx \frac{r_S \alpha}{r^2}$ a dimensionless number. (7.1. 9)

If we now think of $\alpha = \frac{J}{mc}$ as $\alpha = \frac{m}{m}\frac{\mathbf{v}\times\mathbf{r}}{c} = \frac{\mathbf{v}\times\mathbf{r}}{c}$ we can consider a similar vector along the spin axis consisting of the cross product of the corotating velocity of space $\frac{V}{c} \approx \frac{r_S \alpha}{r^2}$ by the radius r. The length along the spin axis of this cross product vector $\frac{\mathbf{V}\times\mathbf{r}}{c}$ is simply $\frac{r_S \alpha}{r}$.

At the equator: Length of vector $\frac{\mathbf{V}\times\mathbf{r}}{c}$ along the spin axis is $\approx \frac{r_S \alpha}{r}$ for $r >>> r_S$ (7.1. 10)

We need this vector length to be a dimensionless number representing the amplitude that a background time polarized $k_{\min}$ graviton generates a circularly polarized $k_{\min}$ graviton around the spin axis. If we divide Eq. (7.1. 10) by the Schwarzchild radius r_S , all rotating black holes with the same percentage of maximum spin look identical, and we get a dimensionless magnitude as required

Magnitude of normalized dimensionless vector $\left|\frac{\mathbf{V}\times\mathbf{r}}{r_S c}\right| \approx \frac{r_S \alpha}{r_S r} \approx \frac{\alpha}{r}$ (7.1. 11)

The whirling velocity of space is a maximum out from the equator, but circularly polarized gravitons generated in this region have to be distributed on this shell around the spin axis as the square of the component of angular momentum. We thus conjecture that the probability of *background* time polarized $k_{\min}$ gravitons, on a corotating thin spherical shell at large radius, generating circularly polarized $k_{\min}$ gravitons around the spin axis on the same shell is

$$\text{Probability of } \frac{\text{Extra circularly polarized } m = \pm 2 \times k_{\min}\text{gravitons}}{\text{Background time polarized } k_{\min}\text{gravitons}} \approx \frac{\alpha^2 \cos^2\theta}{r^2} \qquad (7.1.\ 12)$$

There is a background density of time polarized $k_{\min}$ gravitons on each corotating spherical shell. The swirling velocity of these $k_{\min}$ gravitons generates extra circularly polarized $k_{\min}$ gravitons around the spin axis with a $\cos^2\theta$ distribution around the spin axis on the same shell, in agreement with Figure 7.1. 2. For simple explanatory purposes, we approximated at large radii only. At small radii we must use $\Omega = \frac{g_{t\phi}}{g_{\phi\phi}} = \frac{r_S \alpha c}{r^3 g_{\phi\phi} g_{\theta\theta}}$. On the equator $g_{\theta\theta} = 1$, the co-rotation velocity $V = \Omega \times \mathbf{r}\sqrt{g_{\phi\phi}}$. The circumferential volume generating these circularly polarized gravitons also expands as $\sqrt{g_{\phi\phi}}$. Rederiving Eq's. (7.1. 9) and those following, the effective angular momentum term becomes $\Omega \times \mathbf{r} \times \mathbf{r} \cdot \sqrt{g_{\phi\phi}} \cdot \sqrt{g_{\phi\phi}} = \left[\frac{r_S \alpha c}{r^3 g_{\phi\phi}} \right] \cdot r^2 g_{\phi\phi} = \frac{r_S \alpha c}{r}$ just as before; our derivation applies down to the equatorial horizon. This circular polarization appears to be the result of the swirling or corotating velocity of space as it has no mass term, only angular momentum terms.

7.1.3 Why there are no transverse polarized gravitons in co-rotating coordinates?

In Section 5.1 we showed that the majority of $k_{\min}$ gravitons around any non rotating mass m is due to the interaction between that mass and the rest of the mass in the universe $\Delta\rho_{Gk\min} \propto (\psi_{\text{Universe}} * \psi_m + \psi_m * \psi_{\text{Universe}})$; and these were all time polarized $k_{\min}$ gravitons. Let us imagine that a rotating mass emits transversely polarized $k_{\min}$ gravitons, there will only be a small number unless there are also transversely polarized $k_{\min}$ gravitons from the rest of the universe for their amplitudes to interact with. But from what we have just done above there appears to be only circularly polarized $k_{\min}$ gravitons due to the corotation of space. Also if a rotating mass emits its own circularly polarized $k_{\min}$ gravitons, these would interact with the circularly polarized $k_{\min}$ gravitons due to the corotation of space. It thus appears that, when observed in corotating coordinates, a rotating mass does not itself emit either tranverse or circularly polarized $k_{\min}$ gravitons. This perhaps makes sense, as in corotating frames, we are effectively at rest above the horizon which is rotating in sync with us.

7.1.4 Does our time polarized $k_{\min}$ value in co-rotating coordinates make sense?

The inner horizon radius R is defined when $\Delta = 1 + \frac{\alpha^2}{r^2} - A = 1 + \frac{\alpha^2}{r^2} - \frac{2m}{r} = 0$ where we initially define the dimensionless number $A = \frac{2m}{r}$. Using $A = \frac{2m}{r}$ the inner horizon is where $r^2 - 2mr + \alpha^2 = 0$. So horizon radius $R = r = m + \sqrt{m^2 + \alpha^2} = 2m$ when $\alpha = 0$, and at maximum spin $r = R = m$ when $\alpha = m$. But we will, for generality, revert to the dimensionless A and look at what happens near the horizon for various spins.

Let A_H be the value of A at the horizon where $1 + \frac{\alpha^2}{R^2} - A_H = 0$ is always true

$$\text{So} \quad \frac{\alpha^2}{R^2} = A_H - 1 \quad \text{and} \quad A_H = 1 + \frac{\alpha^2}{R^2} \tag{7.1. 13}$$

At the horizon $g_{\theta\theta} g_{\phi\phi} = (1 + \frac{\alpha^2}{R^2}\cos^2\theta)(1 + \frac{\alpha^2}{R^2} + A_H \frac{\alpha^2}{R^2}\frac{\sin^2\theta}{g_{\theta\theta}}) = g_{\theta\theta}(A_H + A_H \frac{\alpha^2}{R^2}\frac{\sin^2\theta}{g_{\theta\theta}})$

$$= g_{\theta\theta} A_H (1 + \frac{\alpha^2}{R^2}\frac{\sin^2\theta}{g_{\theta\theta}}) \qquad = A_H (g_{\theta\theta} + \frac{\alpha^2}{R^2}\sin\theta)$$

At the horizon $g_{\phi\phi}g_{\theta\theta} = A_H(1+\frac{\alpha^2}{R^2}\cos^2\theta+\frac{\alpha^2}{R^2}\sin^2\theta) = A_H(1+\frac{\alpha^2}{R^2}) = A_H^2$ (7.1. 14)

and independant of angle θ near the horizon only. This is true regardless of spin from zero to maximum as the radius shrinks.

Near the horozon the black hole surface area is $4\pi R^2\sqrt{g_{\phi\phi}g_{\theta\theta}} = 4\pi R^2 A_H$ (7.1. 15)

We have shown in co-rotating frames the extra time polarized $k_{\min}$ graviton density due to a central mass is $\Delta\rho_{Gk\min} = A\cdot\frac{1+\frac{\alpha^2}{r^2}}{g_{\theta\theta}g_{\phi\phi}}K_{Gk\min}dk_{\min}$ where $A=\frac{2m}{r}$ so far, and dimensionless. Using Eq's. (7.1. 13) & (7.1. 14) above, near the horizon in a corotating frame, this becomes

Extra time polarized $k_{\min}$ graviton density near horizon for all Black holes

$$\Delta\rho_{Gk\min} = A_H\cdot\frac{1+\frac{\alpha^2}{R^2}}{g_{\theta\theta}g_{\phi\phi}}K_{Gk\min}dk_{\min} = \frac{A_H^2}{A_H^2}K_{Gk\min}dk_{\min} = K_{Gk\min}dk_{\min} \quad (7.1.\ 16)$$

Ignoring the factor $K_{Gk\min}dk_{\min}$ the extra radially polarized $k_{\min}$ graviton probability density *is always one in Planck unit*s regardless of spin and is spherically symmetric, but only near the horizon where the background density $K_{Gk\min}dk_{\min}/\gamma_M^2 \to 0$ as $\beta_M^2 \approx 1$ & $\gamma_M^2 \to \infty$, providing it is observed (somehow) in a corotating reference frame. It can also be shown that near the horizon of a black hole $\gamma_M^2 \approx \frac{4R^2}{s^2}$ and $\gamma_M \approx \frac{2R}{s}$ is always true regardless of the degree of spin, and the value of the dimensionless number A_H, where R is the horizon radius and s the proper distance from it, providing it is all measured in co-rotating coordinates. The region well above the horizon is not spherically symmetric until several Schwarzschild radii away where spherical symmetry is gradually retained as in the non-rotating case. Also near the horizon this density due to a central mass is so great that we can effectively ignore the background value, but the rotation of space generates circularly polarized $k_{\min}$ gravitons, of probability density $\frac{\alpha^2}{r^2}\cos^2\theta$ from this background. The extra radially, plus circularly, polarized $k_{\min}$ graviton probability density near the horizon ignoring the factor $K_{Gk\min}dk_{\min}$ is

Time polarized 1 + Circularly polarized $\frac{\alpha^2}{R^2}\cos^2\theta = (1+\frac{\alpha^2}{R^2}\cos^2\theta)$ (7.1. 17)

$= g_{\theta\theta}$ as in our original derivation, but ignoring the background, which is infinitesimal near the horizon.

As the Kerr metric is an exact solution for rotating black holes we can say that if the extra $k_{\min}$ gravitons due to a rotating mass are consistent with X as in Eq. (7.1. 7) then it is also consistent with keeping the Graviton constant $K_{Gk\min}$ as in Eq. (5.2. 11) invariant in the spacetime surrounding it. We come back to this, and potential changes to the dimensionless term $A = 2m/r$ below. When we looked at non rotating black holes in section 5.2 we used simple first principles to show that the warping of spacetime around them is consistent with an invariant Graviton constant $K_{Gk\min}$. With rotating black holes we turned the argument around and assumed this invariance to derive the extra probability densities of time, and circular polarized $k_{\min}$ gravitons, before the density dilution from the expansion of space around the rotating mass. Equations (7.1. 16) & (7.1. 17) can perhaps increase our confidence that our hypothesis is possibly correct. If it is correct on the horizon, and also far from a rotating black hole, we will conjecture that it is also correct in all regions between, even if it might not initially appear to be so. It is important to remember that the Kerr metric is an exact solution for rotating black holes, not for rotating masses in general. We have only considered here the exact solution. We can thus perhaps summarize this section as follows:

Spherically symmetric spacetime curvature generates only time polarized $k_{\min}$ gravitons.

Cylindrically symmetric spacetime curvature, due to angular momentum, generates time polarized $k_{\min}$ gravitons and circularly polarized $m = \pm 2$ $k_{\min}$ gravitons in corotating coordinates.

We have not yet included the relatively small number of $k_{\min}$ gravitons emitted by the mass itself $(\psi_m * \psi_m)$, which mainly has significant effect close to black holes.

8 Messing up what appeared to be promising, or maybe not?

8.1.1 The *k*min virtual gravitons emitted by the mass interacting with itself

In section 5 we started out by finding the average $k_{\min}$ graviton probability density in a uniform universe. We then placed a mass concentration in it, and calculated the extra probability density of $k_{\min}$ gravitons (before the dilution due to local space expansion) due to the amplitude of this mass multiplied by the amplitude of the rest of the mass in the universe. This ended up being proportional to $2m/r$ in Planck units. (In this section we also use simple 3 volume $k_{\min}$ graviton probability densities, with no need for 4 volume superscripts.)

$$\Delta\rho_{Gk\min} = (\psi_{\text{Universe}} * \psi_m) + (\psi_m * \psi_{\text{Universe}}) \propto 2m/r \quad \text{as in Eq.}\textbf{(5.2. 6)}$$

And this is true in weak field metrics, except as we start approaching the Schwarzchild radius because of the extra $k_{\min}$ gravitons from the mass interacting with itself: $\psi_m * \psi_m$.

Using Eq.**(5.1. 5)** and coupling probability: $(1-e^{-0.61k^2/k_{\min}^2})\left[\frac{2\alpha_G}{\pi}m^2\frac{dk}{k}\right]$

$$\psi_m{}^*\psi_m=(1-e^{-0.61k^2/k_{\min}^2})\left[\frac{2\alpha_G}{\pi}m^2\frac{dk}{k}\right]\cdot\left[\frac{2k'}{4\pi}\frac{e^{-2k'r}}{r^2}\right]=(1-e^{-0.61k^2/k_{\min}^2})\alpha_G\frac{m^2}{r^2}\frac{k'e^{-2k'r}}{\pi^2}\frac{dk}{k}$$

Also using Eq. **(5.1. 4)** $k'=\sqrt{k^2+11.09k_{\min}^2}\approx 3.477k_{\min}$ when $k=k_{\min}$

$$\psi_m{}^*\psi_m=(1-e^{-0.61})\alpha_G\frac{m^2}{r^2}\frac{3.477k_{\min}e^{-2(3.477k_{\min}r)}}{\pi^2}\frac{dk_{\min}}{k_{\min}}$$

$$=\alpha_G\frac{m^2}{r^2}\frac{1.588e^{-6.95k_{\min}r}}{\pi^2}dk_{\min}\ \text{ when }\ k=k_{\min}$$

The radial exponential decay term $e^{-6.95k_{\min}r}\approx 1$ as we are only interested in radii r that are small in relation to the observable radius of the Universe $R_{OU}\approx k_{\min}^{-1}$, just as in the assumptions we made in section 5.2.1. Thus in these regions we can approximate this equation with good accuracy as

$$\psi_m{}^*\psi_m\approx\alpha_G\frac{m^2}{r^2}\frac{1.588}{\pi^2}dk_{\min}$$

$$\Delta\rho_{Gk\min}\ \text{due to self emission}\ \psi_m{}^*\psi_m\approx\alpha_G\frac{m^2}{r^2}0.161dk_{\min}$$

$$\approx 1.4\frac{m^2}{r^2}0.115\alpha_G dk_{\min}\ \text{ when }\ k=k_{\min}$$

$$\Delta\rho_{Gk\min}\ \text{due to}\ \psi_m{}^*\psi_m\approx 1.4\frac{m^2}{r^2}K_{Gk\min}dk_{\min}\ \text{ using Eq. (5.2. 11).} \tag{8.1. 1}$$

8.1.2 What does this extra term mean for non rotating black holes?

When deriving Eq. **(5.2. 14)** we found (about two equations previous) that due to interactions with the rest of the Universe $\Delta\rho_{Gk\min}\approx\frac{2m}{r}0.115\alpha_G dk_{\min}\approx 2\frac{m}{r}K_{Gk\min}dk_{\min}$

$$\text{Thus }\Delta\rho_{Gk\min}\ \text{total}\approx\left[2\frac{m}{r}+1.4\frac{m^2}{r^2}\right]K_{Gk\min}dk_{\min}\ \text{ in Planck units.} \tag{8.1. 2}$$

Staying on our current path appears to contradict General Relativity, but temporarily ignoring this, let us repeat section 5.2.2 which modifies a non rotating black hole metric to

$$g'_{00} = 1 - \frac{2m}{r} - 1.4\frac{m^2}{r^2} = \frac{1}{g'_{rr}}; \quad \gamma_M^2 = \frac{1}{1 - 2m/r - 1.4m^2/r^2} \tag{8.1. 3}$$

$$\beta_M^2 = \frac{2m}{r} + 1.4\frac{m^2}{r^2}$$

Where β_M is the velocity reached by a small test mass falling in from infinity in the same rest frame. Applying the same proceedures as in section 5.2.2 we can use Equ's.) to show that $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ in this new metric, and we will discuss how this relates with General Relativity in sections 0 & 8.1.6. The modified non rotating horizon radius occurs when $r^2 - 2mr - 1.4m^2 = 0$ or the:

$$\text{Modified non rotating horizon radius } r \approx 2.55m \tag{8.1. 4}$$

or $\approx$ 27.5% larger than the Schwarzchild value.

8.1.3 **What does it mean for rotating black holes?**

In section 7 when we looked at the Kerr Metric we used a dimensionless form of the metric in Equ's. **(7.1. 2**). We also used a dimensionless parameter A where we initially put $A = 2m/r$ We also showed that we could change A without changing $g'_{tt} = \Delta / g_{\varphi\varphi}$ the time component in the corotating frame, provided there is a modified $\Delta = 1 + \frac{\alpha^2}{r^2} - A$. So again temporarily ignoring potential conflicts with General Relativity let us change $A = \frac{2m}{r}$ to $A = \frac{2m}{r} + 1.4\frac{m^2}{r^2}$ and look at the consequences. Firstly from Equ's **(8.1. 3**) we can see that $A = \beta_M^2$ where β_M is the radial inward velocity, in a corotating rest frame, of a small test mass falling from infinity (in the rest frame of the rotating black hole centre). The inner event horizon is the radius where $g_{rr} \to \infty$ so using Equ's.**(7.1. 2**) let $g_{rr} = \frac{g_{\theta\theta}}{\Delta} \to \infty$ or put

$$\Delta = 1 + \frac{\alpha^2}{r^2} - A = 0$$

$$= 1 + \frac{\alpha^2}{r^2} - \frac{2m}{r} - 1.4\frac{m^2}{r^2} = 0$$

$$\text{or } r^2 + \alpha^2 - 2mr - 1.4m^2 = 0$$

$$r = \frac{2m \pm \sqrt{4m^2 + 5.6m^2 - 4\alpha^2}}{2}$$

Event Horizon radius $$r = \frac{2m \pm \sqrt{9.6m^2 - 4\alpha^2}}{2}$$

When $\alpha = 0$ $r = \frac{2m \pm \sqrt{9.6m^2}}{2} \approx \frac{2m + 3.1m}{2} \approx 2.55m$ as in the non rotating case. (8.1. 5)

Maximum spin is when $4\alpha^2 = 9.6m^2$ or $\alpha_{\max} \approx 1.55m$

At this maximum spin $r = m$ as in the usual Kerr Metric.

The outer horizon occurs when $g_{tt} = 1 - \frac{A}{g_{\theta\theta}} = 0$ or $g_{\theta\theta} - A = 0$ and using Equ's. (7.1. 2)

$$1 + \frac{\alpha^2}{r^2}\cos^2\theta - A = 1 + \frac{\alpha^2}{r^2}\cos^2\theta - \frac{2m}{r} - 1.4\frac{m^2}{r^2} = 0$$

$$r^2 - 2mr - 1.4m^2 + \alpha^2\cos^2\theta = 0$$

$$r = \frac{2m + \sqrt{4m^2 + 5.6m^2 - 4\alpha^2\cos^2\theta}}{2}$$

$$r = \frac{2m + \sqrt{9.6m^2 - 4\alpha^2\cos^2\theta}}{2}$$

$$\text{Ergosphere radius } r = \frac{2m + \sqrt{9.6m^2 - 4\alpha^2}}{2} \quad @\,\theta = 0\ \&\ \pi$$

$$= \frac{2m + \sqrt{9.6m^2}}{2} \approx 2.55m \quad @\,\theta = \frac{\pi}{2} \qquad (8.1.\ 6)$$

Figure 8.1. 1 illustrates these changes from the Kerr Metric. The main effect from changing A is to allow an increase in maximum spin from $\alpha = m$ to $\alpha \approx 1.55m$, and $\approx 27.5\%$ increase in the maximum ergosphere radius from $r = 2m$ to 2.55m. It appears to contradict General Relativity whch we discuss in sections 0 & 8.1.6, but provided the extra densities of time polarized and $m = \pm 2$ circular gravitons are as in Eq.(7.1. 7) with $A = \frac{2m}{r} + 1.4\frac{m^2}{r^2}$ then $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ is still true in rotating space outside black holes.

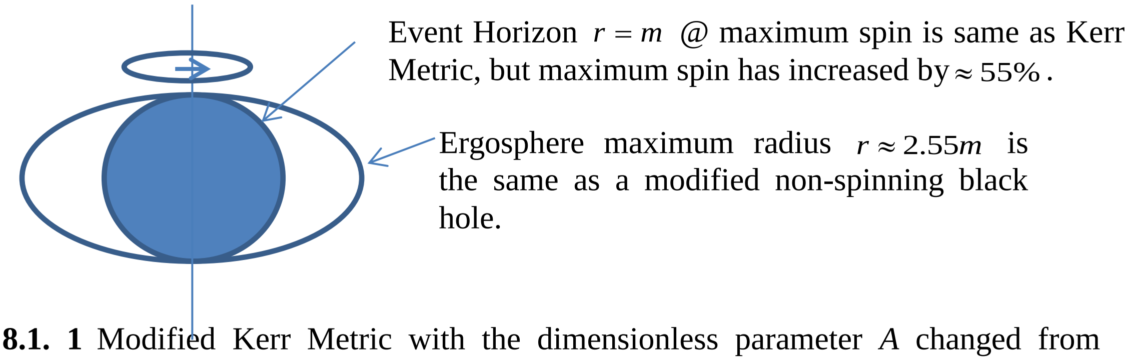


Figure 8.1. 1 Modified Kerr Metric with the dimensionless parameter A changed from $A = \frac{2m}{r} \to A = \frac{2m}{r} + 1.4\frac{m^2}{r^2}$. It initially appears to clash with GR near the horizon.

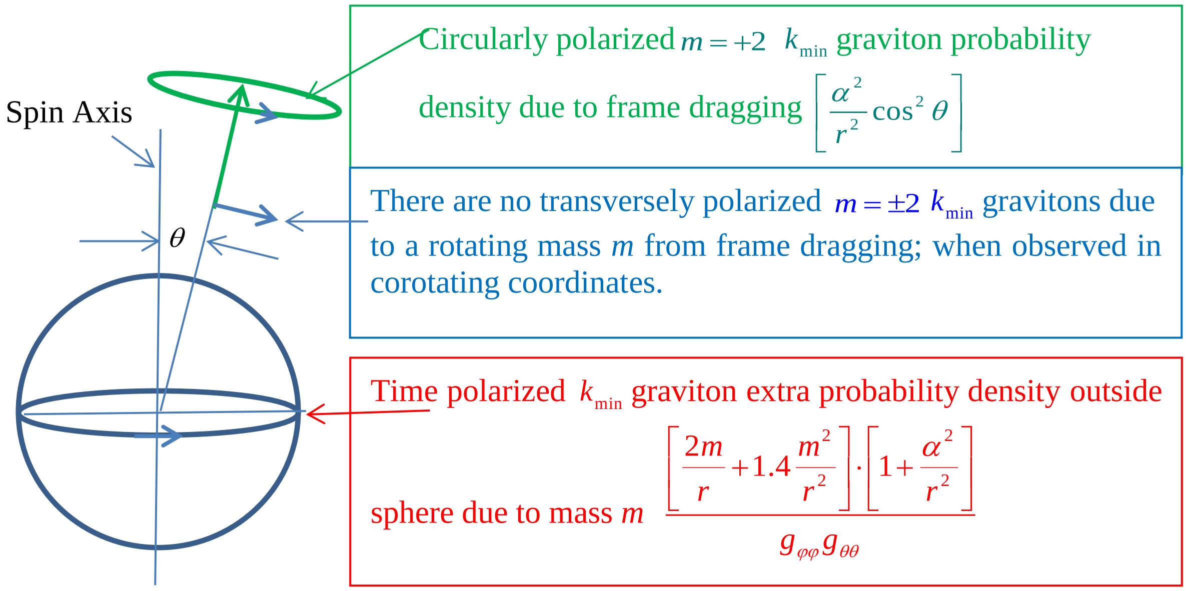


Figure 8.1. 2 Spinning black hole mass m with angular momentum length parameter α, but with the dimensionless parameter A changed from $A=\frac{2m}{r} \rightarrow A=\frac{2m}{r}+1.4\frac{m^2}{r^2}$. The determinant of the metric is independent of A. The denominator terms $g_{\theta\theta}$ & $g_{\phi\phi}$, in dimensionless form as in Equ's. (7.1. 2) rapidly tend to one for radii $r >> r_{Sw}$, and can then be ignored. It shows the probability densities of time polarized, and circularly polarized $m=\pm 2$ $k_{\min}$ gravitons as in Eq. (7.1. 7) in this modified metric which keeps the $k_{\min}$ graviton constant $K_{Gk\min}$ invariant outside the black hole. This is as observed in corotating coordinates.

8.1.4 Determinant of the metric and the $k_{\min}$ graviton constant $K_{Gk\min}$

Working in dimensionless form as in Equ's. (7.1. 2) using Eq. (7.1. 3) $g'_{tt}=\frac{\Delta}{g_{\phi\phi}}$ and the steps used in its derivation; the determinant of the metric is

$$\left|g_{\mu\nu}\right|=(g_{tt}g_{\phi\phi}-g_{t\phi}^2)g_{\theta\theta}g_{rr}=g'_{tt}g_{\phi\phi}g_{\theta\theta}g_{rr}=\frac{\Delta}{g_{\phi\phi}}g_{\phi\phi}g_{\theta\theta}\frac{g_{\theta\theta}}{\Delta}=g_{\theta\theta}^2=(1+\frac{\alpha^2}{r^2}\cos^2\theta)^2$$

As 4 volumes are invariant in relativity and $\rho_{Gk\min}=K_{Gk\min}dk_{\min}$ is true in corotating frames

If $\left|g_{\mu\nu}\right|=g_{\theta\theta}^2=(1+\frac{\alpha^2}{r^2}\cos^2\theta)^2$ then $\rho_{Gk\min}=K_{Gk\min}dk_{\min}$ is true in all frames, and is independent of the dimensionless parameter A. **8.1. 7)**

Despite what initially appears to be a conflict with General Relativity (which we discuss below), if the metric determinant Eq. 8.1. 7) is $g_{\theta\theta}^2$ then the $k_{\min}$ graviton probability density is always $\rho_{Gk\min}=K_{Gk\min}dk_{\min}$ in all frames outside the black hole, and this is also true if there is no rotation, regardless of the value of the dimensionless parameter A. (See section 9).

8.1.5 The Reissner-Nordstrom Metric and $\mathbf{m^2 / r^2}$ terms

Reissner [26] [27] solved the metric surrounding an electrically charged non-rotating mass not long after Schwarzchild had solved the metric around a static mass. He added the electromagnetic stress tensor surrounding a charge to the usual Einstein Energy-momentum tensor, in the region where the mass density term had previously been zero as in the Schwarzchild case. As before we will put $G = c = 1$ so we can work in Planck masses. The Schwarzchild radius $r_S = 2Gm / c^2$ has length dimension and thus $2Gm / rc^2$ becomes $2m / r$, and both $2m / r$ and m^2 / r^2 are effectively dimensionless as, in these units, mass effectively has a length dimension.

Reissner similarly used the characteristic length r_Q where $r_Q^2 = \dfrac{Q^2 G}{4\pi\varepsilon_0 c^4}$

Working in length units of charge with the Coulomb force constant $\dfrac{1}{4\pi\varepsilon_0} = 1$ (8.1. 8)

If $G = c = 1$ & these units of charge $\dfrac{r_Q^2}{r^2} \equiv \dfrac{Q^2}{r^2}$ are both dimensionless numbers.

Table 8.1. 1 Both parameters, mass m and charge Q, effectively have dimensions of length.

Metric	Schwarzchild	Modified Schwarzchild	Reissner-Nordstrom
$g_{00} = g_{rr}^{-1}$	$1 - \dfrac{2m}{r}$	$1 - \dfrac{2m}{r} - 1.4\dfrac{m^2}{r^2}$	$1 - \dfrac{2m}{r} + \dfrac{Q^2}{r^2}$

Using our modified Schwarzchild metric from Eq (8.1. 3) we can see the similarities to the Reissner-Nordstrom metric for a charged mass, providing we measure charge parameter Q in a similar manner to measuring mass in Planck units. The signs are reversed however.
We can crudely think of this another way as follows: In the units we have been working in, the electrostatic field energy outside any radius r is $Q^2 / 2r$. This mass/energy must be subtracted from the original central charged mass as work is done bringing these charged particles together to establish the field energy.

So we can very crudely say the original central mass m becomes $m' = m - \dfrac{Q^2}{2r}$ at radius r .

Thus $\dfrac{2m'}{r} = \dfrac{2m}{r} - \dfrac{Q^2}{r^2}$ and the new $g_{00} = 1 - \dfrac{2m'}{r} = 1 - \dfrac{2m}{r} + \dfrac{Q^2}{r^2}$.

It is very tempting from this, to think of our modified Schwarzchild metric, as somehow including the negative gravitational field energy; which in Planck units is $-m^2 / 2r$ outside radius r. Using the same logic as the electrostatic case, but reversing signs, as gravitational field energy is negative, the original central mass m becomes $m' = m + \dfrac{m^2}{2r}$ at radius r.

Thus $\dfrac{2m'}{r} = \dfrac{2m}{r} + \dfrac{m^2}{r^2}$ and the new $g_{00} = 1 - \dfrac{2m'}{r} = 1 - \dfrac{2m}{r} - \dfrac{m^2}{r^2}$.

Of course our coefficient of 1.4 for m^2/r^2 does not fit this scenario, but our analysis is full of approximations and we could have it wrong. Roger Penrose in Chapter 19 of his "Road to Reality" gives a very good discussion on the concerns of many eminent physicists early last century when General Relativity was first published. They worried that gravitational energy was not explicitly included in the stress tensor. But Einstein could not do this and maintain covariance. In the century since, many eminent physicists have tried unsuccessfully to include gravitational energy in a covariant manner. So we must conclude that it is probably not related to gravitational energy; and as we have shown in this section, it is really due to the small number of $k_{\min}$ gravitons (except close in) emitted by the mass itself.

The Maxwell stress tensor tells us in the the electrostatic case, that if the field is in the z direction, there is a tension or negative pressure $P_Z = -E^2/2$ along the z axis and transverse positive pressures $P_X = P_Y = +E^2/2$ such that $P_X + P_Y + P_Z = E^2/2$ and the mass/energy density $\rho = E^2/2$ if they are all in appropriate units. The stress tensor contracts to $\rho - P_X - P_Y - P_Z = 0$ and this is a property of massless particles. Thus the presence of an electromagnetic field does not alter field equation covariance. So if we simply reverse all these signs with a negative mass energy density of $\rho = -1.4m^2/2$ with transverse tensions $P_X = P_Y = -1.4m^2/2$ and in the field direction positive pressure $P_Z = 1.4m^2/2$ such that the stress tensor contracts again contracts to $\rho - P_X - P_Y - P_Z = 0$. We can thus include a negative energy massless particle in the stress tensor in the same way as in the positive energy electrostatic case, and similarly maintain covariance.

8.1.6 The Kerr-Newman Metric and $\mathbf{m^2/r^2}$ terms

In 1965 Newman [28] **[29]** solved the charged version of the axisymmetric rotating black hole solved earlier by Kerr **[30]** in 1962. In section 7 and Equ's. **(7.1. 2)**we introduced the dimensionless parameter $A = 2m/r$ where as above we have assumed a silent $G = 1$ in the numerator and a silent $c^2 = 1$ in the denominator and in section 8 modified this to get a dimensionless $A = \dfrac{2m}{r} + 1.4\dfrac{m^2}{r^2}$.We showed in section 7 that provided this A is dimensionless it does not change Equ's.**(7.1. 3)** If we look carefully at the Kerr-Newman metric we can see that it fits Equ's. **(7.1. 3)** provided we put $A = \dfrac{2m}{r} - \dfrac{r_Q^2}{r^2}$ which is equivalent to putting $A = \dfrac{2m}{r} - \dfrac{Q^2}{r^2}$ where $r_Q^2 = \dfrac{Q^2 G}{4\pi\varepsilon_0 c^4}$ and we have again measured charge Q in length units as in Equ's. (8.1. 8).

Thus our modified Kerr metric where $A = \dfrac{2m}{r} + 1.4\dfrac{m^2}{r^2}$ is again similar to:

The Kerr-Newman metric where $A = \dfrac{2m}{r} - \dfrac{Q^2}{r^2}$ but with opposite signs.

These two metrics are the rotating versions of our modified Schwarzchild metric and the Reissner-Nordstrom metrics. We can perhaps summarize this in the following two tables.

Table 8.1. 2 The non rotating metrics where dimensionless parameter A is as in Eq. (7.1. 2**)**

The modified Schwarzchild and Reissner-Nordrom metrics both have the same form of changes to the Reimannian curvature tensor but of opposite sign.

Schwarzchild	Modified Schwarzchild	Reissner-Nordstrom
$A=\frac{2m}{r}$	$A=\frac{2m}{r}+1.4\frac{m^2}{r^2}$	$A=\frac{2m}{r}-\frac{Q^2}{r^2}$

Table 8.1. 3 The rotating versions of the above. Again the modified Kerr and Kerr-Newman metrics both have the same form of changes to the Reimannian tensor but of opposite sign.

Kerr	Modified Kerr	Kerr-Newman
$A=\frac{2m}{r}$	$A=\frac{2m}{r}+1.4\frac{m^2}{r^2}$	$A=\frac{2m}{r}-\frac{Q^2}{r^2}$

Again massless particles in the electromagnetic field apply equally in the Reissner-Nordstrom and Kerr-Newmann metrics. The arguments we used above in the non rotating case using massless negative energy particles in our modified stress tensor apply equally in the rotating case. The small changes in the Riemannian curvature tensor, due to this m^2/r^2 term, are of opposite sign for both our modified Kerr and Schwarzchild metrics, when compared to the Kerr-Newman and Reissner-Nordstrom metrics, but of exactly the same form.

So, provided we include such an appropriate negative energy massless particle in the stress tensor, solutions to $R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R=\frac{8\pi G}{c^4}\left[T_{\mu\upsilon}-T_{\mu\upsilon}(\text{Background})\right]$ are consistent with $k_{\min}$ graviton probability density $\rho_{Gk\min}=K_{Gk\min}dk_{\min}$ where $K_{Gk\min}$ is invariant for all observers; whether they are near the horizon of black holes, or if they are at our current cosmic horizon. Also for any observers outside it, who are looking at their own cosmic horizons; and for all cosmic time since the big bang. But wavenumber $k_{\min}$ depends on the local metric and cosmic time. It is approximately the inverse of the causally connected radius at any cosmic time.

Einstein **[24]** based his remarkable equation on the "Equivalence Principle", or the same physics in all free falling frames as in empty space; with covariant tensor equations that apply in all coordinates throughout all spacetime. He wanted it to be similar to Gauss's law and Poisson's equation $\nabla^2\varphi=\rho$ ignoring constants, but in curved spacetime. This naturally leads to inverse square force laws with inverse potentials where masses are concerned, but the inclusion of an m^2/r^2 potential term in the metric due to $\psi_m{}^*\psi_m$ seems to mess all this up. But does it really? Could it be trying to tell us something that we need to know, but did not want to know? Quantum mechanics in the form of QED tells us that, close to the Compton wavelength, the normally simple inverse square force law starts to change, close in shielding makes fundamental electric charge appear to increase, and QED takes over with increadible accuracy. Simple inverse square electric force laws had ruled with remarkable accuracy for over a century before QED arrived on the scene. In fact it was the announcement of the Lamb Shift at the Long Island conference in 1947 that started the big breakthroughs in QED. World War II developments in radar had enabled these remarkably accurate experiments. Is it possible that similar developments today will allow improvements in Gravitational Wave observation accuracy? Developments that may see effects in gravity close to black holes with some parallels to QED changes inside the Compton wavelength of electric charges?

8.1.7 What is the effect of this term in the solar system?

The distance to Mars can be measured very precisely as we have instruments on the surface that can reflect radar from Earth at known locations. On the other hand we don't know the exact diameter of the Sun. If we look at the outer rim it will be deflected outwards by $\approx 1.75/2$ arc seconds (half that of the gravitational bending of starlight because it is coming from the rim). At the distance of the sun, $\approx 150\times 10^6$ KM this is roughly 640 KM in radius. Even if we optically measure the diameter precisely with no error the actual sun diameter will be about 1275 KM smaller so we only know the true diameter approximately. We also do not know the exact surface of radar reflection. The Astronomical unit is quoted as 149,597,870,700 metres, but this is really based on knowing interplanetary measurements accurately and then using Kepler's laws modified by the Schwarzschild metric to give us this level of accuracy. So, let us do a crude first order approximation of what happens if we include an m^2/r^2 term in the metric. Using low velocity (compared with light) Christoffel symbol approximations and circular orbits for simple comparisons the accelerations are:

$\omega^2 r \approx \frac{1}{2}\frac{d}{dr}g_{00} \approx \frac{1}{2}\frac{d}{dr}(1-\frac{2m}{r}) \approx \frac{m}{r^2}$ in the usual Schwarzschild case if $g_{00}\approx 1$ and

$\omega'^2 r \approx \frac{1}{2}\frac{d}{dr}g'_{00} \approx \frac{1}{2}\frac{d}{dr}(1-\frac{2m}{r}-\frac{1.4m^2}{r^2}) \approx \frac{m}{r^2}+\frac{1.4m^2}{r^3} \approx \frac{m}{r^2}(1+\frac{1.4m}{r})$ in the modified metric case. So $\omega^2 \approx \frac{m}{r^3}$ in the usual Schwarzschild case and $\omega'^2 \approx \frac{m}{r^3}(1+\frac{1.4m}{r})$ in the modified metric case. In weak gravitational field accelerations we can replace mass m with a new effective mass $m' = m(1+\frac{1.4m}{r})$ but orbital periods and angular velocities ω cannot change as we know them very precisely. So we will try the following modification to all planetary radii $\omega^2 = \frac{m}{r^3} = \frac{m}{(r+\Delta r)^3}(1+\frac{1.4m}{r+\Delta r}) = \frac{m}{r^3(1+\frac{\Delta r}{r})^3}(1+\frac{1.4m}{r+\Delta r}) \approx \frac{m}{r^3(1+\frac{3\Delta r}{r})}(1+\frac{1.4m}{r+\Delta r})$

$$\omega^2 = \frac{m}{r^3} \approx \frac{m}{r^3}(1-\frac{3\Delta r}{r})(1+\frac{1.4m}{r+\Delta r}) \approx \frac{m}{r^3}(1-\frac{3\Delta r}{r}+\frac{1.4m}{r+\Delta r}) \text{ and if } \omega \text{ is unchanged}$$

$$1-\frac{3\Delta r}{r}+\frac{1.4m}{r+\Delta r} \approx 1 \quad \text{and} \quad \frac{3\Delta r}{r} \approx \frac{1.4m}{r+\Delta r} \approx \frac{1.4m}{r} \quad \text{thus} \quad \Delta r \approx \frac{1.4m}{3}$$

The Schwarzschild radius $R_{SW} = 2m$ and the extra distance to the sun $\Delta r \approx \frac{1.4m}{3} \approx \frac{1.4R_{SW}}{6}$

The Schwarzschild radius of the Sun is $R_{SW} \approx 3$ km so $\Delta r \approx \frac{1.4R_{SW}}{6} \approx 0.7$ km.

The change Δr for our solar system is about 700 metres. But all interplanetary radial separations do not change. So we can still use the old metric and the astronomical unit unchanged with Kepler's laws to a first approximation, or the new metric and just add 700 metres to all the planetary radii from the sun. Orbital periods are identical to a very high accuracy. The gravitational constant does not change in both cases.

What we have done here is a bit like dipoles with the electrostatic field dropping as $\propto 1/r^2$ and the resultant field as $\propto \Delta r / r^3$ where $\frac{1}{r^2} - \frac{1}{(r+\Delta r)^2} \approx \frac{2\Delta r}{r^3}$. However, in the non-spinning gravity case there is spherical symmetry but not in an electric dipole.

8.1.8 Can we measure this difference?

We used circular orbits for a simple crude calculation but the same arguments apply in a slightly more complicated way for eccentric orbits; in a similar manner as Kepler's original arguments with elliptical orbits that sweep out equal angular segment areas with time. The orbit of Mars in particular is highly eccentric and Earth much less so. If the eccentricity of both Earth and Mars orbits were known, to better than say a hundred metres or so between max and min, we should be able to check this difference by measuring the distance (also to around a 100 metre or so accuracy) between Mars and Earth at various points around their orbits. It would seem however that this would be pushing at the very border of current technology, as radar measurements to the sun are inherently a little blurry due to surface variability. We need these to get a very precise value for the eccentricity of Earth's orbit. Even if we can measure Earth-Mars with complete accuracy we have to add in errors due to lack of accuracy in Earth's eccentricity. Also when Mars is on the opposite side of the sun to us, if the distance measuring signal grazes the sun there will a Shapiro type delay that is equivalent to roughly a 15 km error that reduces logarithmically with the minimum radial distance of the signal from the sun. Even if the beam passes through a half Earth-Sun radius there is still a few km error. All these effects introduce possible errors that make it difficult to measure a 700 metre difference in all planetary radii.

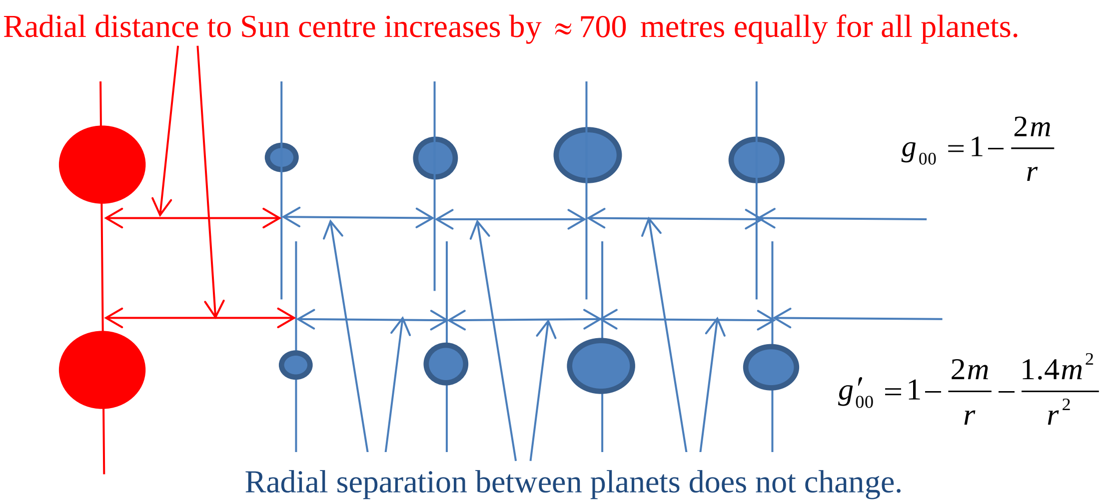


Figure 2.6. 1 Scales are grossly exaggerated for clarity. We have also assumed circular obits here for simplicity, and ignored errors due to the centre of mass of the solar system not being at centre of sun. We have also assumed infinitesimal planet masses so we can simply ignore the effect they have on each other.

8.1.9 What about the Hulse Taylor binary pulsar, can it show this change?

The timing of this pulsar is accurate to 14 significant figures and it would initially seem that this accuracy would show up such differences. However, the semi-major orbit of this binary is $\approx 2\times 10^9$ metres, with a decay rate of ≈ 3.5 metres per orbit, or a change $\Delta r \approx 100$ metres over 30 years due to gravitational radiated energy. If we totally ignore this change in the radius, treating it as effectively zero, the accumulated time delay is parabolic; or proportional to elapsed time squared. If we include the small effect of the change in radius, $2m/r + 1.4m^2/r^2$ increases minutely to $2m/(r-\Delta r) + m^2/(r-\Delta r)^2$ adding two minute cubic terms, both proportional to elapsed time cubed, where the m^2/r^2 contribution is about 10^{-7} of that due to the $2m/r$ term. Even the cubic effect of a $\Delta r \approx 100$ metres change in the usual m/r term (which is currently $m/r \approx 5\times 10^{-7}$) on the parabola over 30 years, is miniscule. The chances of measuring either the m/r, or the m^2/r^2 cubic terms are very small in the foreseeable future; let alone distinguish between them. The best chance of measuring any difference will almost certainly turn out to be gravitational wave observations.

8.1.10 Gravitational Wave observations of Black Hole mergers

Some of the mergers observed so far [25] have been suggesting relatively larger Black Hole masses than current astrophysics theory had expected. If we look at our new metric term we can write $g_{00} = 1 - \frac{2m}{r} - 1.4\frac{m^2}{r^2} = g_{00} \approx 1 - \frac{2m}{r}(1 + 0.7\frac{m}{r}) = 1 - \frac{2m'}{r}$ where $m' \approx 1 + 0.7\frac{m}{r}$

For a maximum spin black hole when $r = m$ we can say the effective mass at merger is $m' \approx 1.7m$ or about 70% greater. The addition of an m^2/r^2 term in our modified metrics increases the total merging energy, and hence that in the resulting gravitational waves. Inward radial accelerations would appear to be greater also. However computer simulations with these changed metrics would be required to model all this in detail, but our rough analysis above suggests that the masses of the black holes before merging could well be less than what they have so far seemed. In other words, a pair of smaller black holes merging might create the gravitational waves current theory predicts from the mergers of two, up to maybe 70% larger black holes. Spins had also been expected to be roughly perpendicular to their orbiting plane, but their merging speeds don't tie up with this. Is it possible that this unexpected behaviour is trying to tell us something is different; something different in the metric as we get close to Black Hole Horizons?

Finally in this section, does this extra m^2/r^2 term alter what we said in Eq.**(7.1. 16**) where we first used. Before we introduced the self emission term $1.4m^2/r^2$ we found that the extra time polarized $k_{\min}$ graviton density near the horizon, for all black holes is

$$A_H \cdot \frac{1+\dfrac{\alpha^2}{R^2}}{g_{\theta\theta}g_{\phi\phi}} = \frac{A_H^2}{A_H^2} = 1 \quad \text{And this is still true.} \qquad \text{But now } A_H = \frac{2m}{R} + 1.4\frac{m^2}{R^2} = 1 + \frac{\alpha'^2}{R^2}$$

Where α' is the increased spin parameter due to the extra $1.4m^2/R^2$ term and we have also reused Eq's.**(7.1. 13**) & (7.1. 14). Everything we did there is not affected by this extra term.

9 Four Vectors and Four Volume Action Densities

9.1.1 Graviton densities represented as invariant 4 velocities

Four velocity vectors have the property that $U_0^2 - U_1^2 = 1$ is invariant under local Lorentz transformations; where U_0 is the *time component* of the four velocity, and U_1 the *spatial component.* We will, as previously, use the notation

$$U_0^2 = \gamma_M^2 \quad \text{and} \quad U_1^2 = \gamma_M^2\beta_M^2 \quad \text{where}\ \gamma_M^2 = \frac{1}{1-\beta_M^2}$$

We can think of the spatial component U_1 as the four velocity $\gamma_M\beta_M$ of a free falling mass that came from rest at infinity, in the same coordinate frame as the black hole, and pointing radially inwards. We can also write

$$U_0^2 - U_1^2 = 1 \text{ as } 1 + U_1^2 = U_0^2 \text{ or } 1 + \gamma_M^2\beta_M^2 = \gamma_M^2 .$$

This was what we did for the Schwarzschild metric when we had temporarily multiplied both sides by γ_M^2 and normalized the background $k_{\min}$ graviton three volume probability density to 1 with $\gamma_M^2\beta_M^2$ the extra $k_{\min}$ graviton density due to a central mass, and γ_M^2 the total; this equation only applies before we have expanded the volume and changed time in the new metric. Because this is a 4 vector relationship it is true in all coordinates. Multiplying both sides temporarily by γ_M^2 does not change its validity.

We can also add a term $\gamma_M^2 X^2$ to both sides to get $1+\gamma_M^2\beta_M^2+\gamma_M^2 X^2=\gamma_M^2+\gamma_M^2 X^2$ and still maintain covariance as $(\gamma_M^2+\gamma_M^2 X^2)-(\gamma_M^2\beta_M^2+\gamma_M^2 X^2)=1$, and we can put $X^2=\dfrac{\alpha^2}{r^2}\cos^2\theta$

so that: $$1+\gamma_M^2\beta_M^2+\gamma_M^2\frac{\alpha^2}{r^2}\cos^2\theta=\gamma_M^2+\gamma_M^2\frac{\alpha^2}{r^2}\cos^2\theta=g_{\theta\theta}\gamma_M^2 .$$

We are not adding another 4 vector here; we are simply adding squared terms, which are equal on each side, so that Lorenz invariance is not affected. This is still an invariant equation in any coordinates. In the above the local metric clock rate is always $1/\gamma_M$.

The three volume probability density of circularly polarized $k_{\min}$ gravitons due to rotation, before volume expansion and time changes in the new metic, always obeys $\gamma_M^2 X^2=\gamma_M^2\dfrac{\alpha^2}{r^2}$ and the remaining $k_{\min}$ graviton three volume probability density is $\gamma_M^2\beta_M^2$.

This section on invariant 4 vectors, is really just saying the same thing as our invariant 4 volume densities of $k_{\min}$ gravitons.

9.1.2 **Four volumes in changing metrics**

Using our dimensionless form of the metric tensor, the nonrotating space metric determinant has magnitude $\left|\mathrm{Det}\, g\right|=\left|g\right|=\left|g_{tt}g_{rr}g_{\theta\theta}g_{\phi\phi}\right|=1$, but we want the square root of this $\sqrt{\left|g\right|}=1$.

However in rotating space this becomes $\sqrt{\left|g\right|}=g_{\theta\theta}=1+\cos^2\dfrac{\alpha^2}{r^2}$ which reverts to $\sqrt{\left|g\right|}=g_{\theta\theta}=1$ when the angular momentum length parameter $\alpha=0$. At a large radius from any mass concentration let us start with a unit four volume such that $\Delta^4 x=\Delta t\Delta x\Delta y\Delta z=1$ when $g_{\mu\nu}=\eta_{\mu\nu}$, where for simplicity we use $x, y\,\&\,z$ for the space components. As we approach the central mass in the new metric, this four volume becomes

$$\Delta^4 x'=\sqrt{\left|g\right|}\Delta t\Delta x\Delta y\Delta z=g_{\theta\theta}=1+\frac{\alpha^2}{r^2}\cos^2\theta=\Delta t'\Delta x'\Delta y'\Delta z'$$

Four volumes at a fixed point in spacetime are invariant as coordinates change, and also as the metric changes if in nonrotating space. In rotating space however it increases as $g_{\theta\theta}$.

$$\frac{\text{Curved spacetime 4 volume}}{\text{Flat spacetime 4 volume}}=\frac{\Delta^4 x'}{\Delta^4 x}=\frac{\Delta t'\Delta x\Delta y'\Delta z'}{\Delta t\Delta x\Delta y\Delta z}=g_{\theta\theta}=1 \text{ when angular momentim is zero}.$$

We also know that clocks change as $\Delta t'=\sqrt{g_{tt}}\Delta t=\dfrac{\Delta t}{\gamma_M}$ in curved spacetime so that

$$\frac{\Delta t'\Delta x'\Delta y'\Delta z'}{\Delta t\Delta x\Delta y\Delta z}=\frac{\Delta t\cdot\Delta x'\Delta y'\Delta z'}{\gamma_M\cdot\Delta t\Delta x\Delta yz}=g_{\theta\theta}=\frac{\Delta x'\Delta y'\Delta z'}{\gamma_M\cdot\Delta x\Delta y\Delta z}$$

The expanded spatial volume in the new metric $\Delta^3 x'=\Delta x'\Delta y'\Delta z'=\gamma_M g_{\theta\theta}\Delta x\Delta y\Delta z=\gamma_M g_{\theta\theta}\Delta^3 x$.

Spatial volume in any metric expands as $\dfrac{V'}{V}=\dfrac{\Delta^3 x'}{\Delta^3 x}=\dfrac{d^3 x'}{d^3 x}=\gamma_M g_{\theta\theta}=(1+\dfrac{\alpha^2}{r^2}\cos^2\theta)\gamma_M$

Where as above we have defined $\gamma_M=g_{tt}^{-1/2}$ as the local metric clock rate.

The 4 volume density invariance of $k_{\min}$ graviton still applies, as the extra circularly polarized gravitons due to angular momentum, occupy this expanded volume.

10 Finishing up and some loose ends

10.1.1 Preferred Frames

It might seem that we have been arguing for a preferred frame. But there is really no difference in what we are proposing compared to current physics. In comoving frames the cosmic microwave background is isotropic. At peculiar velocity β_P it is no longer isotropic, and the average background temperature increases by γ_P, exactly the same increase as $k_{\min}$ to $k'_{\min}=\gamma_P k_{\min}$, and that is if we could measure it, which is most unlikely. We have frequently talked in this paper about local observers measuring $k_{\min}$, but only as a thought experiment, and the average (over all directions) background temperature can be used to measure either $\gamma_{peculiar}$ or γ_{metric} at any particular cosmic time, provided we already know its value in flat comoving coordinates (see section 10.1.10). There are no other changes in physics in this comoving frame; it is exactly as Einstein originally postulated, an important experimentally verified feature of General Relativity **[32].** However it does make everything we did here much simpler if we work in comoving coordinates. All the mass moving at peculiar velocities in random directions does not affect the average universe density of either $k_{\min}$ gravitons or the $k_{\min}$ action density that they require. We calculated the average density of $k_{\min}$ action from the horizon in these comoving coordinates. But if we think in terms of spherically symmetric four volume $k_{\min}$ action density invariance, then whether we are in a non comoving frame, or in a non-flat metric, it makes no difference; and is why we can use 4 vector notation for the extra $k_{\min}$ gravitons around mass concentrations.

10.1.2 **Solar System Constraints and do our proposed changes fit?**

See "The Confrontation between General Relativity and Experiment" Clifford M. Will **[32].** Probably the most important constraint mentioned in this review is the Cassini Time Delay data that gives a fit with GR of $\approx 10^{-5}$ for signals passing close to the solar horizon, where our extra $1.4m^2/r^2$ term is $\approx 3\times 10^{-6}$. So it should be within the Cassini Constraint and also within the light deflection constraint. The remaining changes are discussed in section 8.1.7.

10.1.3 **Action Principles and the Einstein Field Equations**

The field equations of GR can be derived from an invariant action principle $\delta I = 0$ where $I = \dfrac{1}{16\pi G}\int R\sqrt{-g}\cdot d^4x + I_m(\psi_m, g_{\mu\nu})$ and R is the Ricci scalar, with I_m the matter action which depends on matter fields ψ_m coupled to the metric g. Varying the action with respect to $g_{\mu\nu}$ we obtain Einstein's field equations $G_{\mu\nu} \equiv R_{\mu\nu} - \dfrac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$. This paper suggests however, that an "Invariant spherically symmetric 4 volume cosmic wavelength graviton action density" applies to the solutions of a modified stress tensor. There is obviously a connection between these two ways of looking at all this.

10.1.4 **Gravitational Waves and 4 volume invariance**

We showed in section 5.3.6 that the 4 volume $k_{\min}$ graviton density at any cosmic time $\rho^{4V}_{Gk\min}$ is invariant in all coordinates and in any metric. But the metric can oscillate not changing this invariance, and such disturbances will travel at the speed of light. We can imagine extra gravitons around a mass concentration and the background gravitons as in section 5.2 (if they are accelerating as in binary pairs) generating real transversely polarized $m = \pm 2$, gravitons. This has some parallels to what we found in the Kerr metric, but now with real gravitons. But the intensity, or probability density, of these real gravitons will drop as the inverse radius squared, at least when far away. We can also show from Equ's.**(5.1. 9) & (5.2. 5)** that most of these gravitons are close to the locally measured value of the $k_{\min}$ wavenumber, about 96% are between $k_{\min}$ & $5k_{\min}$. Thus most of this radiated energy is near $k_{\min}$. The frequency of the radiated wave is twice the orbital frequency of the binary pair source. As most of the energy in the wave is in quanta near $k_{\min}$ there is no connection with the frequency of the radiated wave as in spin 1 photons in electromagnetism. In the recently observed gravitational waves the wave frequency was ≈ 250 cycles per second just before merger with wavelengths ≈ 1200 kilometres or approximately 10^{41} Planck lengths, whereas the wavelength of $k_{\min}$ gravitons is $1/k_{\min} \approx R_{OU} \approx 10^{62}$ Planck lengths. The ratio between them is $\approx 10^{21}$. This ratio is inverse to the binary pair orbital frequency. It could only approach one if the orbital period is approximately twice the age of the universe.

10.1.5 **Black Holes, the Firewall Paradox and possible Spacetime Boundaries**

Several recent papers [15] [16] [17] [18] [19] have discussed the BH firewall paradox. In section 5.2.2 we use the fact that outside observers see infalling mass remaining on the horizon. In fact if we look carefully at the analyses in sections 5.2.1 & 5.2.2 we see they may suggest that GR cutsoff at the BH horizon; one of the possible firewall paradox implications. The equations we derived do not appear to work inside the horizon. Our argument that a constant graviton scalar $K_{Gk\min}$ is consistent with GR is questionable inside the horizon. ($k_{\min}$ quanta that go in to build superpositions would not return in time $\Delta T \approx k_{\min}^{-1} \approx T$). Is it possible that the horizon of a Black Hole could be some sort of spacetime boundary?

10.1.6 **Dark Matter possibilities**

Table 4.3. 1 shows a spin 2, $N = 1$ neutral massive graviton type superposition that emitts infinitesimal mass $N = 2$ graviton superpositions with about 70 times the probability that $N = 2$ gravitons emitt $N = 2$ gravitons. It may possibly be only detected via these graviton interactions. (Dark Matter Wimp searches would not see these as spin 2 is not subject to the weak force.)

10.1.7 **Higgs Boson**

It is not clear if the Higgs boson is a spin zero superposition so it is not in Table 2.2. 1; but if it is, it would be some superposition of infinite superpositions with a total angular momentum vector summing to zero just as two spin ½ fermion superpositions can for example.

10.1.8 **Constancy of fundamental charge**

It has always been fundamental that the electromagnetic charge of protons and electrons is precisely equal and opposite to get a neutral universe. In section 4.2 we showed that the probability of superpositions was $sN \cdot dk(1+\varepsilon)/k$ where the infinitesimal ε is proportional to rest mass squared and thus different for various particles. We used this probability to determine interaction coupling strengths in section 3.3. This suggests that the probability of virtual photon emission is also proportional to the probability $sN \cdot dk(1+\varepsilon)/k$ of each superposition, and would not be precisely equal for electrons and protons due to small variations in ε of the order of $\approx 10^{-45}$ between electrons and quarks. If however we look closely at Eq.(4.2. 3) and the following equations, by adding the amplitude for gravity at right angles we effectively added the probabilities of spin 2 gravity generated superpositions to those of spin 1 colour and electromagnetic superpositions. If somehow only those superpositions generated by spin 1 electromagnetic and colour interact with spin 1 photons this would cancel any minute difference in charge. If this is not so then there are infinitesimal differences in charge of the order of $\approx 10^{-45}$ which would surely have shown up in some form by now unless there are minute differences in the total number of electrons and protons.

10.1.9 **Superpositions and Feynman's Strings**

Over a century ago there were various models of the electron. The Abraham-Lorenz was probably the most well-known [20] [21]. All these models suffered from the problem that the electromagnetic mass in the field was 4/3 times the relativistic mass. In 1906 Poincare showed that if the bursting forces due to charge were balanced by stresses (or forces) in the same rest frame as the particle, these would cancel the extra 1/3 figure restoring covariance [22]. In chapter 29 Volume II of his famous lectures on physics, Feynman, probably jokingly, suggested that if the electron is held together by strings, that their resonances could explain the muon mass [23]. He just may have been right. The equations for infinite superpositions in this paper apply equally to all massive particles. As infinite superpositions are held together by interactions with zero point forces in the same rest frame, *could these same zero point interactions possibly be Feynman's strings*? If they can hold virtual preons in their imaginary orbits, it would seem they should be able to balance any bursting forces due to electric charge. However this paper suffers the same problem as the Standard Model. There is nothing to suggest the quantization of mass of massive particles; only the infinitesimal mass particles.

10.1.10 **Some Conjectured Space-time Symmetries and Invariants**

Equation **(5.2. 10)** $\rho_U \approx (0.8823)k_{\min}^2$ implies $k_{\min}^2 \propto \rho_U \propto 1/a^3$ in the matter dominated era, where a is the scale factor, with $k_{\min}$ measured in flat comoving coordinates.

$$\text{So } k_{\min} \propto a^{-3/2} \text{ in the matter era} \qquad \text{(a)}$$

$$\& \ k_{\min} \propto a^{-2} \text{ in the radiation era.} \qquad \text{(b)}$$

For example if $a(t) = t^{2/3}$ in the matter era of what the $\Lambda - CMB$ model calls critical density with no Dark energy, implies $k_{\min} \propto \dfrac{1}{a^{3/2}} \propto \dfrac{1}{t} \propto \dfrac{1}{R_{OU}}$ in a flat universe. This contrasts with the CMB temperature where $t_{CMB} \propto 1/a$. But at any particular cosmic time $t'_{CMB} \propto k'_{\min}$ in any metric and any peculiar velocity. Using Eq's. **(5.2. 11)**, **(5.3. 21)**, **(5.3. 22)** plus (a) & (b)

SS 4 Volume Graviton Density $\rho_{Gk\min}^{4VSS}$ = 3 Volume Graviton Density $\rho_{Gk\min}^{3V} = K_{Gk\min} dk_{\min} \propto k_{\min}$

The SS 4 volume $k_{\min}$ graviton density $\rho_{Gk\min}^{4VSS} \propto k_{\min}$ and from Eq.**(5.3. 22)** $\rho_{Qk\min}^{4VSS} \propto \rho_{Gk\min}^{4VSS}$

SS 4 volume $k_{\min}$ graviton density $a^{3/2}\rho_{Gk\min}^{4VSS}$ is invariant in all space-time of the matter era.

SS 4 volume $k_{\min}$ graviton density $a^2\rho_{Gk\min}^{4VSS}$ is invariant in all space-time of the radiation era.

SS 4 volume $k_{\min}$ action density $a^{3/2}\rho_{Qk\min}^{4VSS}$ is invariant in all space-time of the matter era.

SS 4 volume $k_{\min}$ action density $a^2\rho_{Qk\min}^{4VSS}$ is invariant in all space-time of the radiation era.

(The perhaps over used label SS signifies that the action has to Spherically Symmetric)

11 Conclusions

Over the last few decades, the focus of many physicists has mainly been to somehow unite all the forces at high energy somewhere near Planck scale. It was hoped that Supersymmetry would facilitate this. In complete contrast, the focus of this paper is at the other extreme of the minimum possible frequency. Einstein always felt that gravity was not a force, and all particles travelled on geodesics controlled by the local metric. This paper supports that view.

Many physicists also think that in any contest between Quantum Mechanics and General Relativity that QM would most likely be the ultimate survivor. This paper has tampered with GR but only near black holes and at cosmic radii. The error, at say 1% of the horizon radius, will also be of the same order. At the scale of galaxies and even superclusters of galaxies the error is completely insignificant. In our solar system the differences are still probably inside our current measurement ability. The effect near black holes may relate with masses slightly greater than expected and the unexpected non-alignment of spins **[25].**

We have not tampered with either QM or Special Relativity. SR always applies locally, but who knows whether it may break down near black hole horizons, in line with some of the current paradoxes. Such a breakdown would probably not affect black hole merger observations. We have certainly tampered with fundamental particles suggesting a completely different approach. The most important difference is the infinitesimal rest mass particles, which is also key to what we suggest is behind gravity. At Planck energy our approach meshes with the Standard Model, but only the most basic SM model with three families of fermions. It suggests that the three forces of the SM do not unite near Planck energy; in fact they behave exactly as that very basic model predicts.

But many physicists will no doubt ask: If fundamental particles are built from infinite superpositions why do we not see signs of them? Well perhaps we already do. The components of infinite superpositions are virtual, and only complete infinite superpositions can behave as real particles. But we assumed in this paper that what we have always called virtual particles are single wavenumber k superpositions only, and thus components of an infinite superposition representing that particle. When we calculate Feynman transition amplitudes, we are effectively summing over all the k values of these virtual superposition components. Also the distinction between virtual and real can be blurred. It can even depend on the frame of reference, such as accelerating or not. And, all recent experiments continue to confirm the strange, and counterintuitive, behaviour of the outcome depending on the actual act of measurement. The behaviour of the superpositions in this paper, following this strange principle, is no different. If our arguments are correct it could turn out that the only real, but still indirect, evidence of infinite superpositions we will ever see; is the change in the metric around mass concentrations, and the exponential expansion of space.

We started our introduction to this paper by saying that the greatest theories; perhaps some would say the older great theories of the last few centuries, all had their roots in experimental science. This paper claims that if in fact all the fundamental particles are built from infinite superpositions, and at cosmic wavelength borrow redshifted Planck scale action, from zero point vector fields on the receding horizon, then space has to expand exponentially; starting at virtually the big bang. This exponential expansion does not need dark energy, its behaviour is quite different to what happens with dark energy and the requirement for $\Omega = 1$. In this sense this paper proposes a new idea that is experimentally testable **[31]** and in a manner that should satisfy all. But some will possibly say that the ideas presented here, bear little resemblance to the usual rules in modern quantum field theory. The rules for primary interactions are very simple in comparison to the complicated secondary interaction rules of QED/QCD, where the weak force is involved, coupling constants change with interaction energy, and there are Feynmann loops etc. This has allowed us to keep the mathematics simple. Exotic maths can be extremely powerfull, but it can also hide the wood for the trees, so to speak. At the end of the day, the goal of science should be, to at least where possible, try and express the behaviour of "Mother Nature" in the simplest manner.

For the last thirty to forty years or so since the Standard Model came on the world stage, there have been many different ideas suggested to make further progress. The ideas presented here will no doubt contain many errors, and need much polishing to put them on a more solid foundation. But they do comply with special relativity and basic quantum mechanics. The whole package is consistent, and may just possibly suggest a different path forward.

Finally, when we calculated the $k_{\min}$ quanta available from the horizon we focussed on the vector potential half (Eq. **(5.3. 10)**) but Figure 5.3. 1 shows a surplus of the time component available. Could it be possible that the scalar half is the source of the borrowed mass for all superpositions? Could this be the source of the Higg's scalar field? The total available could be in the right paddock. It meets the $\approx 10^{-33} eV$ rest mass requirement of all virtual gravitons. They outnumber by far all other long lasting particles in the Universe. A crude analysis suggests that this may in fact be possible.

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