LABELED TREES WITH FIXED NODE LABEL SUM

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Abstract. The non-cyclic graphs known as trees may be labeled by assigning positive integer numbers (weights) to their vertices or to their edges. We count the trees up to 10 vertices that have prescribed sums of weights, or, from the number-theoretic point of view, we count the compositions of positive integers that are constrained by the symmetries of trees.

1. Partitions and Compositions

Whereas partitions of s into some fixed number of parts regard the order of the parts as irrelevant and to not pay attention to the symmetry or order of the parts, the compositions of n take the opposite point of view and consider the group of all permutations of the parts to generate distinct objects. In between these extremes one may impose partial restrictions of symmetry on the arrangement of the parts.

The compositions of n into m parts might for example be counted only if inequivalent under cyclic shifts. The associated triangle with rows and columns enumerated by n and m is [2, A037306] in the Online Encyclopedia of Integer Sequences with row sums in [2, A008965].

The number of compositions of s into m positive parts is a variant of Pascal's triangle [5, §1.2],

$$\bar{C}(s,m) = \binom{s-1}{m-1} \tag{1}$$

with generating function

$$\bar{C}_m(x) \equiv \sum_{s\ge m} \bar{C}(s,m)x^s = \frac{x^m}{(1-x)^m}. \tag{2}$$

The number of partitions of s into m positive parts has the generating function [5, (1.77)]

$$P_m(x) = \frac{x^m}{\prod_{j=1}^{m}(1-x^j)}. \tag{3}$$

2. Compositions reduced by Tree's Symmetries

The main theme of this paper is to partition integers imposing the symmetry of the graphs known as trees; we distribute the parts of the composition across the vertices. We ask how many different labeled trees exist with a given vertex label sum (weight), i.e., how many different compositions of an integer exist that are different if their associated labeled trees are different. A complete table of

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these weighted trees up to 10 vertices is computed here by providing the (ordinary) generating functions.

We write $v_{n,i}(x)$ for the generating function of the number of vertex-labeled trees with n vertices and shape i. We write $v_n(x)$ for the generating function of the number of vertex-labeled trees with n vertices, $v_n(x) = \sum_i v_{n,i}(x)$. For $n \le 3$ there is only a single tree, and the shape indicator i is not needed.

The number of ways of partitioning a number over the labels for a tree with one vertex has the generating function

$$v_1 = \bar{C}_1(x) \mapsto 1, 1, 1, 1, 1, 1 \ldots (s \ge 1) \tag{4}$$

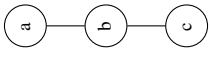
because, whatever the sum is, there is only the single option to put that sum onto the only existing vertex.

The number of ways of partitioning a number over the labels of the tree with two vertices is

$$v_2 = P_2(x) \mapsto 1, 1, 2, 2, 3, 3, 4, 4 \ldots (s \ge 2) \tag{5}$$

because there are $\lfloor n/2 \rfloor$ ways to push a fraction of the integer on one vertex (the other gets the remainder). Because the tree is symmetric with respect to swapping the two vertices all variants are exhausted then.

3. Three vertices

There is one tree with 3 vertices. 

Its symmetry is that it may be flipped over with an inert middle vertex, reading the labels from either direction, so the compositions $a+b+c$ and $c+b+a$ are considered the same. The generating function of the middle vertex indicates that there is one way to insert any positive integer [2, A000012],

$$\bar{C}_1(x) = \sum_{s \ge 1} 1 \times x^s = \frac{x}{1-x} \tag{6}$$

The generating functions of the two terminal elements is the same; because their order does not matter, they represent the number of partitions of a number into two parts [2, A004526]:

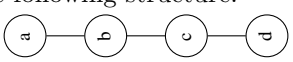
$$P_2(x) = \frac{x^2}{(1+x)(1-x)^2} \mapsto 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7 \ldots (s \ge 2). \tag{7}$$

Convolution with the inert vertex b gives [2, A002620]

$$v_3(x) = \bar{C}_1(x) P_2(x) = \frac{x^3}{(1+x)(1-x)^3} \mapsto 1, 2, 4, 6, 9, 12, 16, 20, 25, 30 \ldots (s \ge 3) \tag{8}$$

4. Four vertices

4.1. The linear graph with 4 vertices (with the Carbon backbone of n-butane) has the following structure:



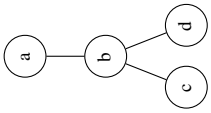
The entire string may be flipped over, equivalent to the symmetry of the group of order 2 with cycle index $(t_1^2 + t_2)/2$. The individual subgraphs of vertices ab

and vertices cd halves have generating functions $\bar{C}_2(x)$, so with $t_j = \bar{C}_2(x^j)$ [2, A005993]

(9)

$$v_{4,1}(x) = \frac{(\frac{x^2}{(1-x)^2})^2 + \frac{x^4}{(1-x^2)^2}}{2} = \frac{x^4(1+x^2)}{(1-x)^4(1+x)^2} \mapsto 1, 2, 6, 10, 19, 28, 44 \ldots (s \geq 4)$$

4.2. The star graph with 4 vertices has the following structure:



The center vertex b has the generating function $\bar{C}_1(x)$ and the outer 3 elements are counted irrespective of order, and represent the number of ways of partitioning s into 3 nonnegative parts [2, A069905]

(10)

$$P_3(x) = \frac{x^3}{(1+x)(1+x+x^2)(1-x)^3} \mapsto 1, 1, 2, 3, 4, 5, 7, 8, 10, 12, 14, 16, 19, 21, \ldots (s \geq 3)$$

The convolution of these two sequences is [2, A000601]

(11)

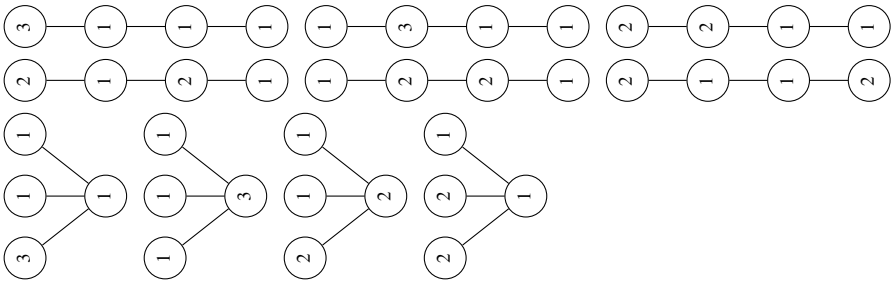
$$v_{4,2} = \bar{C}_1(x)P_3(x) = \frac{x^4}{(1+x)(1+x+x^2)(1-x)^4} \mapsto 1, 2, 4, 7, 11, 16, 23, 31, 41, 53 \ldots (s \geq 4)$$

The partitions with the symmetries either of the two trees with 4 vertices are [2, A301739]

(12)

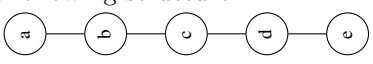
$$v_4 = v_{4,1} + v_{4,2} = \frac{x^4(2+2x+2x^2+x^3+x^4)}{(1+x+x^2)(1+x)^2(1-x)^4} \mapsto 2, 4, 10, 17, 30, 44, 67, 91, 126 \ldots (s \geq 4)$$

To illustrate the third nonzero term $v_4(s=6) = 10$ in that sequence: The 10 trees with labels that sum to 6=3+1+1+1=2+2+1+1 are



5. Five vertices

5.1. The linear graph with 5 vertices (the n-Pentane of Organic Chemistry) has the following structure:

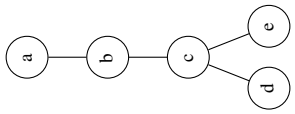


The middle vertex c is inert when symmetries are engaged; the outer elements ab and de have the "palindromic" C_2 symmetry of (9). The convolution of (6) and

(9) is [2, A005994]

$$v_{5,1} = \bar{C}_1(x) v_{4,1} = \frac{x^5(1+x^2)}{(1+x)^2(1-x)^5} \mapsto 1, 3, 9, 19, 38, 66, 110, 170 \ldots (s \ge 5) \tag{13}$$

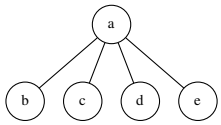
5.2. The bifurcating graph with 5 vertices (2-Methyl-Butane) has the following structure:



There is a backbone of 3 vertices *abc* with full compositorial symmetry as represented by $\bar{C}_3(x)$ [2, A000217], and the two twigs *d* and *e* at the end are the number of ways of partitioning into 2 parts (7). The convolution is [2, A001752]

$$v_{5,2} = \bar{C}_3(x) P_2(x) = \frac{x^5}{(1-x)^5(1+x)} \mapsto 1, 4, 11, 24, 46, 80, 130, 200, 295 \ldots (s \ge 5) \tag{14}$$

5.3. The star graph with 5 vertices has the following structure:



The inert vertex *a* is represented by (6) and the 4 outer vertices *bcde* mean partitioning numbers into 4 parts [2, A026810],

$$P_4(x) = \frac{x^4}{(1-x)^4(1+x)^2(1+x+x^2)(1+x^2)} \mapsto 1, 1, 2, 3, 5, 6, 9, 11, 15, 18, 23, 27, 34, 39, 47, 54, 64 \ldots (s \ge 4) \tag{15}$$

The convolution of the two sequences is [2, A002621]

$$v_{5,3} = \bar{C}_1(x) P_4(x) = \frac{x^5}{(1-x)^5(1+x)^2(1+x+x^2)(1+x^2)} \mapsto 1, 2, 4, 7, 12, 18, 27, 38, 53, 71, 94 \ldots (s \ge 5) \tag{16}$$

The sum of the contribution of the three shapes of trees on 5 vertices is [2, A301740]

$$v_5 = v_{5,1} + v_{5,2} + v_{5,3} = \frac{x^5(3+3x+6x^2+5x^3+5x^4+2x^5+x^6)}{(1-x)^5(1+x)^2(1+x+x^2)(1+x^2)} \mapsto 3, 9, 24, 50, 96, 164, 267 \ldots (s \ge 5) \tag{17}$$

6. Summary (Vertex Labeled)

The overview of Table 1 fills sequences (8), (12), (17) into the columns. The generating functions for columns $n \le 10$ are listed in Appendix C.1. Harary and Prins gave an explicit formula for the bivariate generating function in terms of the rooted variant [3, (9b)][2, A303911].

Its diagonal contains the number of unlabeled trees [2, A000055], because there is an obvious bijection between the unlabeled trees and the trees where the weight equals the number of vertices (each vertex has label 1). The first subdiagonal contains the number of unlabeled rooted trees [2, A000081], because there is a

$s\backslash n$	1	2	3	4	5	6	7	8	9	10	11	12
1	1											
2	1	1										
3	1	1	1									
4	1	2	2	2								
5	1	2	4	4	3							
6	1	3	6	10	9	6						
7	1	3	9	17	24	20	11					
8	1	4	12	30	50	63	48	23				
9	1	4	16	44	96	146	164	115	47			
10	1	5	20	67	164	315	437	444	286	106		
11	1	5	25	91	267	592	1022	1300	1204	719	235	
12	1	6	30	126	408	1059	2126	3331	3899	3328	1842	551
13	1	6	36	163	603	1754	4098	7511	10781	11692		4766
14	1	7	42	213	856	2805	7368	15619	26294	34844		
15	1	7	49	265	1186	4270	12590	30111	58485	91037		

TABLE 1. The trees with n vertices with positive integer vertex labels where that weight sum is s for each tree [2, A303841].

bijection between the trees where the weight is one larger than the number of vertices; the single vertex with label 2 marks the root of the tree.

The second subdiagonal are the trees where the vertex sums exceed the vertex count by two; there is either a vertex with label 3 or two vertices with label 2. The trees with a vertex of label 3 are the rooted trees; the trees with two vertices with label 2 are the 2-rooted trees. So the 2-rooted trees are the difference between the second and first subdiagonal and column $f = 2$ in Table 3 [2, A303833]: 0, 1, 2, 6, 15, 43, 116, 329, 918, 2609,... $(n \geq 1)$.

The row sums 1,2,3,7,14,... have been tabulated earlier [2, A036250]. The second column are repeated integers [2, A004526]. The third column is [2, A002620], the forth [2, A301739], and the fifth [2, A301740].

7. One or two edges

A number may as well be partitioned over the edges of trees, leading to edge-labeled trees. The symmetry considerations of the previous sections remain essentially intact; the compositions of n into as many parts as there are edges are roughly obtained by reducing the degrees of freedom by one (as the number of edges in a tree is one less than the number of vertices). Zero or one edges (suffix still counting the vertices in the tree) get all the weight:

$$e_1 = e_2 = \bar{C}_1(x) \mapsto 1, 1, 1, 1, 1, \ldots. \tag{18}$$

Two edges in the tree of 3 vertices have the C_2 symmetry:

$$e_3 = P_2(x) \mapsto 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, \ldots. \tag{19}$$

8. Three edges

8.1. The linear tree with 4 vertices contributes

$$e_{4,1} = \bar{C}_1(x)P_2(x) = v_3 \mapsto 1, 2, 4, 6, 9, \ldots (s \geq 3) \tag{20}$$

8.2. The star graph with 4 vertices partitions the weight over 3 symmetric edges as in (10):

$$e_{4,2} = P_3(x) \mapsto 1, 1, 2, 3, 4, 5, 7, 8, 10, 12 \ldots (s \ge 3) \tag{21}$$

The number of ways of partitioning n over the edges of trees with 4 vertices is

$$e_4 = e_{4,1}+e_{4,2} = \frac{x^3(2+x+x^2)}{(1-x)^3(1+x)(1+x+x^2)} \mapsto 2, 3, 6, 9, 13, 17, 23, 28, 35, 42, 50, 58, \ldots (s \ge 3) \tag{22}$$

9. Four Edges

9.1. On five vertices we get from the linear graph

$$e_{5,1} = v_{4,1} = \frac{x^4(1+x^2)}{(1-x)^4(1+x)^2} \mapsto 1, 2, 6, 10, 19, 28, 44, \ldots (s \ge 4). \tag{23}$$

9.2. In the bifurcating graph the two edges along the backbone contribute $\bar{C}_2(x)$ and the two edges in the twigs contribute $P_2(x)$ [2, A002623]:

$$e_{5,2} = \bar{C}_2(x)P_2(x) = \frac{x^4}{(1-x)^4(1+x)} \mapsto 1, 3, 7, 13, 22, 34, 50, 70, \ldots (s \ge 4) \tag{24}$$

9.3. The star graph contributes [2, A026810]

$$e_{5,3} = P_4(x) = \frac{x^4}{(1-x)^4(1+x)^2(1+x+x^2)(1+x^2)} \mapsto 1, 1, 2, 3, 5, 6, 9, 11, 15, 18, \ldots (s \ge 4) \tag{25}$$

The number of ways of partitioning n over the edges of trees with 5 vertices is

$$e_5 = e_{5,1} + e_{5,2} + e_{5,3} = \frac{x^4(3+3x+6x^2+5x^3+5x^4+2x^5+x^6)}{(1-x)^4(1+x)^2(1+x+x^2)(1+x^2)} \mapsto 3, 6, 15, 26, 46, 68, 103, 141, 195, 253, \ldots (s \ge 4) \tag{26}$$

10. Summary (Edge Labeled)

Table 2 distributes sequences (19), (22), (26) and so on along its columns. The generating functions for columns $n \le 10$ are shown in Appendix C.2.

The diagonal shows the number of unlabeled trees [2, A000055]. The subdiagonal is [2, A027852], explained by Bomfim's comment in the Encyclopedia.

11. Signed Vertices or Edges

Insertion of $t[j] \to 1 + x^j$ into the Cycle indices of the automorphism group of the trees gives [2, A294783], refining the number of bi-colored trees with respect to the abundancy of one color over the other: Table 3.

Insertion of $t[j] \to 1 + x^j$ into the Cycle indices of the automorphism group of the line graphs gives [2, A302939], refining the number of signed trees with respect to the abundancy of one sign over the other: Table 4.

$s\backslash n$	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1										
2	1	1	1									
3	1	1	1	2								
4	1	1	2	3	3							
5	1	1	2	6	6	6						
6	1	1	3	9	15	16	11					
7	1	1	3	13	26	43	37	23				
8	1	1	4	17	46	88	116	96	47			
9	1	1	4	23	68	169	273	329	239	106		
10	1	1	5	28	103	287	585	869	918	622	235	
11	1	1	5	35	141	467	1104	2031	2695	2609	1607	551
12	1	1	6	42	195	711	1972	4211	6882	8399		4235
13	1	1	6	50	253	1051	3270	8108	15513	23152		
14	1	1	7	58	330	1489	5222	14552	32191	56291		
15	1	1	7	68	412	2063	7958	24846	62014	124958		

TABLE 2. The trees with n vertices with positive integer edge labels where that sum is s for each tree [2, A303842]. The row sums are (discarding the 1 from the single vertex) 1, 2, 4, 9, 21, 55, 146, 415, 1212, 3653, 11246...

$n\backslash f$	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	1	1								
3	1	2	2	1							
4	2	4	6	4	2						
5	3	9	15	15	9	3					
6	6	20	43	51	43	20	6				
7	11	48	116	175	175	116	48	11			
8	23	115	329	573	698	573	329	115	23		
9	47	286	918	1866	2626	2626	1866	918	286	47	
10	106	719	2609	5978	9656	11241	9656	5978	2609	719	106
11	235	1842									

TABLE 3. The bicolored trees with n vertices, f vertices of the first color and $n - f$ vertices of the second color.

APPENDIX A. EDGE LISTS

To attach a unique shape index `i` to each tree up to 10 vertices in this article, its edge list is printed in this section. Each entry starts with the number of vertices `N` of the tree, our (essentially arbitrary) index `t` which runs from 1 up to the number of trees with `N` vertices, the diameter, the maximum vertex degree, and the Wiener index [1]. Each edge is represented by two vertex numbers connected by a hyphen.

The trees have been weakly sorted first by increasing number of vertices, then by decreasing diameter (largest distance between two vertices), then by increasing maximum degree, and finally by decreasing Wiener index (sum of the distances

$n\backslash p$	0	1	2	3	4	5	6	7	8	9
1	1									
2	1	1								
3	1	1	1							
4	2	3	3	2						
5	3	6	9	6	3					
6	6	16	27	27	16	6				
7	11	37	79	96	79	37	11			
8	23	96	233	349	349	233	96	23		
9	47	239	679	1187	1439	1187	679	239	47	
10	106	622	1987	4017	5639	5639	4017	1987	622	106
11	235	1607								

TABLE 4. The signed trees with n vertices, p positive and $n-1-p$ negative edges.

between vertex pairs). *Weakly* means that even by defining these four parameters, some trees may have the same parameter set. This happens for the first time with $n = 7$ vertices, where trees $i = 4$ and $i = 5$ have both diameter 4, maximum degree 3, and Wiener index 48 (Appendix A). With this sorting, the unbranched straight tree is number 1 in the list and the star graph is last.

These indices `t` are used to identify the trees by the second index of the cycle indices and generating function in Sections B and C.

The order of the vertices implied by the adjacency matrices has been vaguely aligned with the IUPAC rules of alkane names. A longest path along the vertices is labeled up to the diameter. The vertices where the branches start that divert from the backbone get small vertex numbers. (So an attempt is to obtain a matrix with a long tridiagonal upper left submatrix.)

```
N=1 i=1
dia=0 deg=0 Wie=0
N=2 i=1
dia=1 deg=1 Wie=1 0-1
N=3 i=1
dia=2 deg=2 Wie=4 0-1 1-2
N=4 i=1
dia=3 deg=2 Wie=10 0-1 1-2 2-3
N=4 i=2
dia=2 deg=3 Wie=9 0-1 1-2 1-3
N=5 i=1
dia=4 deg=2 Wie=20 0-1 1-2 2-3 3-4
N=5 i=2
dia=3 deg=3 Wie=18 0-1 1-2 1-4 2-3
N=5 i=3
dia=2 deg=4 Wie=16 0-1 1-2 1-3 1-4
N=6 i=1
dia=5 deg=2 Wie=35 0-1 1-2 2-3 3-4 4-5
N=6 i=2
dia=4 deg=3 Wie=32 0-1 1-2 1-5 2-3 3-4
N=6 i=3
dia=4 deg=3 Wie=31 0-1 1-2 2-3 2-5 3-4
```

```
N=6 i=4
dia=3 deg=3 Wie=29 0-1 1-2 1-4 2-3 2-5
N=6 i=5
dia=3 deg=4 Wie=28 0-1 1-2 1-4 1-5 2-3
N=6 i=6
dia=2 deg=5 Wie=25 0-1 1-2 1-3 1-4 1-5
N=7 i=1
dia=6 deg=2 Wie=56 0-1 1-2 2-3 3-4 4-5 5-6
N=7 i=2
dia=5 deg=3 Wie=52 0-1 1-2 1-6 2-3 3-4 4-5
N=7 i=3
dia=5 deg=3 Wie=50 0-1 1-2 2-3 2-6 3-4 4-5
N=7 i=4
dia=4 deg=3 Wie=48 0-1 1-2 2-3 2-5 3-4 5-6
N=7 i=5
dia=4 deg=3 Wie=48 0-1 1-2 1-5 2-3 3-4 3-6
N=7 i=6
dia=4 deg=3 Wie=46 0-1 1-2 1-5 2-3 2-6 3-4
N=7 i=7
dia=4 deg=4 Wie=46 0-1 1-2 1-5 1-6 2-3 3-4
N=7 i=8
dia=4 deg=4 Wie=44 0-1 1-2 2-3 2-5 2-6 3-4
N=7 i=9
dia=3 deg=4 Wie=42 0-1 1-2 1-4 1-5 2-3 2-6
N=7 i=10
dia=3 deg=5 Wie=40 0-1 1-2 1-4 1-5 1-6 2-3
N=7 i=11
dia=2 deg=6 Wie=36 0-1 1-2 1-3 1-4 1-5 1-6
N=8 i=1
dia=7 deg=2 Wie=84 0-1 1-2 2-3 3-4 4-5 5-6 6-7
N=8 i=2
dia=6 deg=3 Wie=79 0-1 1-2 1-7 2-3 3-4 4-5 5-6
N=8 i=3
dia=6 deg=3 Wie=76 0-1 1-2 2-3 2-7 3-4 4-5 5-6
N=8 i=4
dia=6 deg=3 Wie=75 0-1 1-2 2-3 3-4 3-7 4-5 5-6
N=8 i=5
dia=5 deg=3 Wie=74 0-1 1-2 1-6 2-3 3-4 4-5 4-7
N=8 i=6
dia=5 deg=3 Wie=72 0-1 1-2 2-3 2-6 3-4 4-5 6-7
N=8 i=7
dia=5 deg=3 Wie=71 0-1 1-2 1-6 2-3 3-4 3-7 4-5
N=8 i=8
dia=5 deg=3 Wie=70 0-1 1-2 1-6 2-3 2-7 3-4 4-5
N=8 i=9
dia=5 deg=3 Wie=68 0-1 1-2 2-3 2-6 3-4 3-7 4-5
N=8 i=10
dia=5 deg=4 Wie=71 0-1 1-2 1-6 1-7 2-3 3-4 4-5
N=8 i=11
dia=5 deg=4 Wie=67 0-1 1-2 2-3 2-6 2-7 3-4 4-5
N=8 i=12
dia=4 deg=3 Wie=67 0-1 1-2 1-5 2-3 2-6 3-4 6-7
N=8 i=13
```

```
dia=4 deg=3 Wie=65 0-1 1-2 1-5 2-3 2-6 3-4 3-7
N=8 i=14
dia=4 deg=4 Wie=66 0-1 1-2 1-5 1-6 2-3 3-4 3-7
N=8 i=15
dia=4 deg=4 Wie=64 0-1 1-2 2-3 2-5 2-7 3-4 5-6
N=8 i=16
dia=4 deg=4 Wie=63 0-1 1-2 1-5 1-6 2-3 2-7 3-4
N=8 i=17
dia=4 deg=4 Wie=62 0-1 1-2 1-5 2-3 2-6 2-7 3-4
N=8 i=18
dia=4 deg=5 Wie=62 0-1 1-2 1-5 1-6 1-7 2-3 3-4
N=8 i=19
dia=4 deg=5 Wie=59 0-1 1-2 2-3 2-5 2-6 2-7 3-4
N=8 i=20
dia=3 deg=4 Wie=58 0-1 1-2 1-4 1-5 2-3 2-6 2-7
N=8 i=21
dia=3 deg=5 Wie=57 0-1 1-2 1-4 1-5 1-6 2-3 2-7
N=8 i=22
dia=3 deg=6 Wie=54 0-1 1-2 1-4 1-5 1-6 1-7 2-3
N=8 i=23
dia=2 deg=7 Wie=49 0-1 1-2 1-3 1-4 1-5 1-6 1-7
N=9 i=1
dia=8 deg=2 Wie=120 0-1 1-2 2-3 3-4 4-5 5-6 6-7 7-8
N=9 i=2
dia=7 deg=3 Wie=114 0-1 1-2 1-8 2-3 3-4 4-5 5-6 6-7
N=9 i=3
dia=7 deg=3 Wie=110 0-1 1-2 2-3 2-8 3-4 4-5 5-6 6-7
N=9 i=4
dia=7 deg=3 Wie=108 0-1 1-2 2-3 3-4 3-8 4-5 5-6 6-7
N=9 i=5
dia=6 deg=3 Wie=108 0-1 1-2 1-7 2-3 3-4 4-5 5-6 5-8
N=9 i=6
dia=6 deg=3 Wie=104 0-1 1-2 2-3 2-7 3-4 4-5 5-6 7-8
N=9 i=7
dia=6 deg=3 Wie=104 0-1 1-2 1-7 2-3 3-4 4-5 4-8 5-6
N=9 i=8
dia=6 deg=3 Wie=102 0-1 1-2 2-3 3-4 3-7 4-5 5-6 7-8
N=9 i=9
dia=6 deg=3 Wie=102 0-1 1-2 1-7 2-3 3-4 3-8 4-5 5-6
N=9 i=10
dia=6 deg=3 Wie=102 0-1 1-2 1-7 2-3 2-8 3-4 4-5 5-6
N=9 i=11
dia=6 deg=3 Wie=100 0-1 1-2 2-3 2-7 3-4 4-5 4-8 5-6
N=9 i=12
dia=6 deg=3 Wie=98 0-1 1-2 2-3 2-7 3-4 3-8 4-5 5-6
N=9 i=13
dia=6 deg=4 Wie=104 0-1 1-2 1-7 1-8 2-3 3-4 4-5 5-6
N=9 i=14
dia=6 deg=4 Wie=98 0-1 1-2 2-3 2-7 2-8 3-4 4-5 5-6
N=9 i=15
dia=6 deg=4 Wie=96 0-1 1-2 2-3 3-4 3-7 3-8 4-5 5-6
N=9 i=16
dia=5 deg=3 Wie=98 0-1 1-2 1-6 2-3 3-4 3-7 4-5 7-8
```

```
N=9 i=17
dia=5 deg=3 Wie=96 0-1 1-2 1-6 2-3 2-7 3-4 4-5 7-8
N=9 i=18
dia=5 deg=3 Wie=96 0-1 1-2 1-6 2-3 2-7 3-4 4-5 4-8
N=9 i=19
dia=5 deg=3 Wie=94 0-1 1-2 2-3 2-6 3-4 3-8 4-5 6-7
N=9 i=20
dia=5 deg=3 Wie=92 0-1 1-2 1-6 2-3 2-7 3-4 3-8 4-5
N=9 i=21
dia=5 deg=4 Wie=98 0-1 1-2 1-6 1-7 2-3 3-4 4-5 4-8
N=9 i=22
dia=5 deg=4 Wie=94 0-1 1-2 1-6 1-7 2-3 3-4 3-8 4-5
N=9 i=23
dia=5 deg=4 Wie=92 0-1 1-2 2-3 2-6 2-7 3-4 4-5 7-8
N=9 i=24
dia=5 deg=4 Wie=92 0-1 1-2 2-3 2-6 2-7 3-4 4-5 4-8
N=9 i=25
dia=5 deg=4 Wie=92 0-1 1-2 1-6 1-7 2-3 2-8 3-4 4-5
N=9 i=26
dia=5 deg=4 Wie=90 0-1 1-2 1-6 2-3 2-7 2-8 3-4 4-5
N=9 i=27
dia=5 deg=4 Wie=88 0-1 1-2 2-3 2-6 3-4 3-7 3-8 4-5
N=9 i=28
dia=5 deg=5 Wie=92 0-1 1-2 1-6 1-7 1-8 2-3 3-4 4-5
N=9 i=29
dia=5 deg=5 Wie=86 0-1 1-2 2-3 2-6 2-7 2-8 3-4 4-5
N=9 i=30
dia=4 deg=3 Wie=90 0-1 1-2 1-5 2-3 2-6 3-4 3-8 6-7
N=9 i=31
dia=4 deg=4 Wie=88 0-1 1-2 2-3 2-5 2-7 3-4 5-6 7-8
N=9 i=32
dia=4 deg=4 Wie=88 0-1 1-2 1-5 1-6 2-3 2-7 3-4 7-8
N=9 i=33
dia=4 deg=4 Wie=88 0-1 1-2 1-5 1-6 2-3 3-4 3-7 3-8
N=9 i=34
dia=4 deg=4 Wie=86 0-1 1-2 2-3 2-5 2-6 3-4 3-8 6-7
N=9 i=35
dia=4 deg=4 Wie=86 0-1 1-2 1-5 1-6 2-3 2-7 3-4 3-8
N=9 i=36
dia=4 deg=4 Wie=84 0-1 1-2 1-5 2-3 2-6 2-7 3-4 3-8
N=9 i=37
dia=4 deg=4 Wie=82 0-1 1-2 1-5 1-6 2-3 2-7 2-8 3-4
N=9 i=38
dia=4 deg=5 Wie=86 0-1 1-2 1-5 1-6 1-7 2-3 3-4 3-8
N=9 i=39
dia=4 deg=5 Wie=82 0-1 1-2 2-3 2-5 2-6 2-7 3-4 7-8
N=9 i=40
dia=4 deg=5 Wie=82 0-1 1-2 1-5 1-6 1-7 2-3 2-8 3-4
N=9 i=41
dia=4 deg=5 Wie=80 0-1 1-2 1-5 2-3 2-6 2-7 2-8 3-4
N=9 i=42
dia=4 deg=6 Wie=80 0-1 1-2 1-5 1-6 1-7 1-8 2-3 3-4
N=9 i=43
```

```
dia=4 deg=6 Wie=76 0-1 1-2 2-3 2-5 2-6 2-7 2-8 3-4
N=9 i=44
dia=3 deg=5 Wie=76 0-1 1-2 1-4 1-5 1-6 2-3 2-7 2-8
N=9 i=45
dia=3 deg=6 Wie=74 0-1 1-2 1-4 1-5 1-6 1-7 2-3 2-8
N=9 i=46
dia=3 deg=7 Wie=70 0-1 1-2 1-4 1-5 1-6 1-7 1-8 2-3
N=9 i=47
dia=2 deg=8 Wie=64 0-1 1-2 1-3 1-4 1-5 1-6 1-7 1-8
N=10 i=1
dia=9 deg=2 Wie=165 0-1 1-2 2-3 3-4 4-5 5-6 6-7 7-8 8-9
N=10 i=2
dia=8 deg=3 Wie=158 0-1 1-2 1-9 2-3 3-4 4-5 5-6 6-7 7-8
N=10 i=3
dia=8 deg=3 Wie=153 0-1 1-2 2-3 2-9 3-4 4-5 5-6 6-7 7-8
N=10 i=4
dia=8 deg=3 Wie=150 0-1 1-2 2-3 3-4 3-9 4-5 5-6 6-7 7-8
N=10 i=5
dia=8 deg=3 Wie=149 0-1 1-2 2-3 3-4 4-5 4-9 5-6 6-7 7-8
N=10 i=6
dia=7 deg=3 Wie=151 0-1 1-2 1-8 2-3 3-4 4-5 5-6 6-7 6-9
N=10 i=7
dia=7 deg=3 Wie=146 0-1 1-2 1-8 2-3 3-4 4-5 5-6 5-9 6-7
N=10 i=8
dia=7 deg=3 Wie=145 0-1 1-2 2-3 2-8 3-4 4-5 5-6 6-7 8-9
N=10 i=9
dia=7 deg=3 Wie=143 0-1 1-2 1-8 2-3 3-4 4-5 4-9 5-6 6-7
N=10 i=10
dia=7 deg=3 Wie=143 0-1 1-2 1-8 2-3 2-9 3-4 4-5 5-6 6-7
N=10 i=11
dia=7 deg=3 Wie=142 0-1 1-2 1-8 2-3 3-4 3-9 4-5 5-6 6-7
N=10 i=12
dia=7 deg=3 Wie=141 0-1 1-2 2-3 3-4 3-8 4-5 5-6 6-7 8-9
N=10 i=13
dia=7 deg=3 Wie=141 0-1 1-2 2-3 2-8 3-4 4-5 5-6 5-9 6-7
N=10 i=14
dia=7 deg=3 Wie=138 0-1 1-2 2-3 2-8 3-4 4-5 4-9 5-6 6-7
N=10 i=15
dia=7 deg=3 Wie=137 0-1 1-2 2-3 2-8 3-4 3-9 4-5 5-6 6-7
N=10 i=16
dia=7 deg=3 Wie=135 0-1 1-2 2-3 3-4 3-8 4-5 4-9 5-6 6-7
N=10 i=17
dia=7 deg=4 Wie=146 0-1 1-2 1-8 1-9 2-3 3-4 4-5 5-6 6-7
N=10 i=18
dia=7 deg=4 Wie=138 0-1 1-2 2-3 2-8 2-9 3-4 4-5 5-6 6-7
N=10 i=19
dia=7 deg=4 Wie=134 0-1 1-2 2-3 3-4 3-8 3-9 4-5 5-6 6-7
N=10 i=20
dia=6 deg=3 Wie=138 0-1 1-2 2-3 3-4 3-7 4-5 5-6 7-8 8-9
N=10 i=21
dia=6 deg=3 Wie=138 0-1 1-2 1-7 2-3 3-4 4-5 4-8 5-6 8-9
N=10 i=22
dia=6 deg=3 Wie=136 0-1 1-2 1-7 2-3 2-8 3-4 4-5 5-6 5-9
```

```
N=10 i=23
dia=6 deg=3 Wie=135 0-1 1-2 1-7 2-3 3-4 3-8 4-5 5-6 5-9
N=10 i=24
dia=6 deg=3 Wie=134 0-1 1-2 1-7 2-3 3-4 3-8 4-5 5-6 8-9
N=10 i=25
dia=6 deg=3 Wie=134 0-1 1-2 1-7 2-3 2-8 3-4 4-5 5-6 8-9
N=10 i=26
dia=6 deg=3 Wie=133 0-1 1-2 2-3 2-7 3-4 4-5 4-9 5-6 7-8
N=10 i=27
dia=6 deg=3 Wie=131 0-1 1-2 2-3 3-4 3-7 4-5 5-6 7-8 7-9
N=10 i=28
dia=6 deg=3 Wie=131 0-1 1-2 1-7 2-3 2-8 3-4 4-5 4-9 5-6
N=10 i=29
dia=6 deg=3 Wie=130 0-1 1-2 2-3 2-7 3-4 3-9 4-5 5-6 7-8
N=10 i=30
dia=6 deg=3 Wie=130 0-1 1-2 1-7 2-3 3-4 3-8 4-5 4-9 5-6
N=10 i=31
dia=6 deg=3 Wie=129 0-1 1-2 2-3 2-7 3-4 3-8 4-5 5-6 8-9
N=10 i=32
dia=6 deg=3 Wie=128 0-1 1-2 1-7 2-3 2-8 3-4 3-9 4-5 5-6
N=10 i=33
dia=6 deg=3 Wie=125 0-1 1-2 2-3 2-7 3-4 3-8 4-5 4-9 5-6
N=10 i=34
dia=6 deg=4 Wie=139 0-1 1-2 1-7 1-8 2-3 3-4 4-5 5-6 5-9
N=10 i=35
dia=6 deg=4 Wie=134 0-1 1-2 1-7 1-8 2-3 3-4 4-5 4-9 5-6
N=10 i=36
dia=6 deg=4 Wie=131 0-1 1-2 1-7 2-3 3-4 4-5 4-8 4-9 5-6
N=10 i=37
dia=6 deg=4 Wie=131 0-1 1-2 1-7 1-8 2-3 3-4 3-9 4-5 5-6
N=10 i=38
dia=6 deg=4 Wie=130 0-1 1-2 1-7 1-8 2-3 2-9 3-4 4-5 5-6
N=10 i=39
dia=6 deg=4 Wie=129 0-1 1-2 2-3 2-7 2-9 3-4 4-5 5-6 7-8
N=10 i=40
dia=6 deg=4 Wie=127 0-1 1-2 1-7 2-3 3-4 3-8 3-9 4-5 5-6
N=10 i=41
dia=6 deg=4 Wie=127 0-1 1-2 1-7 2-3 2-8 2-9 3-4 4-5 5-6
N=10 i=42
dia=6 deg=4 Wie=126 0-1 1-2 2-3 3-4 3-7 3-9 4-5 5-6 7-8
N=10 i=43
dia=6 deg=4 Wie=126 0-1 1-2 2-3 2-7 2-8 3-4 4-5 4-9 5-6
N=10 i=44
dia=6 deg=4 Wie=123 0-1 1-2 2-3 2-7 2-8 3-4 3-9 4-5 5-6
N=10 i=45
dia=6 deg=4 Wie=122 0-1 1-2 2-3 2-7 3-4 3-8 3-9 4-5 5-6
N=10 i=46
dia=6 deg=5 Wie=131 0-1 1-2 1-7 1-8 1-9 2-3 3-4 4-5 5-6
N=10 i=47
dia=6 deg=5 Wie=122 0-1 1-2 2-3 2-7 2-8 2-9 3-4 4-5 5-6
N=10 i=48
dia=6 deg=5 Wie=119 0-1 1-2 2-3 3-4 3-7 3-8 3-9 4-5 5-6
N=10 i=49
```

```
dia=5 deg=3 Wie=127 0-1 1-2 1-6 2-3 2-7 3-4 4-5 4-9 7-8
N=10 i=50
dia=5 deg=3 Wie=125 0-1 1-2 2-3 2-6 3-4 3-8 4-5 6-7 8-9
N=10 i=51
dia=5 deg=3 Wie=124 0-1 1-2 1-6 2-3 2-7 3-4 4-5 7-8 7-9
N=10 i=52
dia=5 deg=3 Wie=123 0-1 1-2 1-6 2-3 2-7 3-4 3-8 4-5 8-9
N=10 i=53
dia=5 deg=3 Wie=122 0-1 1-2 1-6 2-3 2-7 3-4 3-9 4-5 7-8
N=10 i=54
dia=5 deg=3 Wie=121 0-1 1-2 1-6 2-3 2-7 3-4 3-8 4-5 4-9
N=10 i=55
dia=5 deg=4 Wie=127 0-1 1-2 1-6 1-7 2-3 3-4 4-5 4-8 4-9
N=10 i=56
dia=5 deg=4 Wie=126 0-1 1-2 1-6 1-7 2-3 3-4 3-8 4-5 8-9
N=10 i=57
dia=5 deg=4 Wie=124 0-1 1-2 1-6 1-7 2-3 3-4 3-8 4-5 4-9
N=10 i=58
dia=5 deg=4 Wie=123 0-1 1-2 1-6 1-7 2-3 2-8 3-4 4-5 4-9
N=10 i=59
dia=5 deg=4 Wie=122 0-1 1-2 1-6 2-3 3-4 3-7 3-9 4-5 7-8
N=10 i=60
dia=5 deg=4 Wie=122 0-1 1-2 1-6 1-7 2-3 2-8 3-4 4-5 8-9
N=10 i=61
dia=5 deg=4 Wie=121 0-1 1-2 2-3 2-6 2-8 3-4 4-5 6-7 8-9
N=10 i=62
dia=5 deg=4 Wie=120 0-1 1-2 1-6 2-3 2-7 2-8 3-4 4-5 4-9
N=10 i=63
dia=5 deg=4 Wie=119 0-1 1-2 1-6 2-3 2-7 2-9 3-4 4-5 7-8
N=10 i=64
dia=5 deg=4 Wie=119 0-1 1-2 1-6 1-7 2-3 3-4 3-8 3-9 4-5
N=10 i=65
dia=5 deg=4 Wie=118 0-1 1-2 2-3 2-6 3-4 3-8 3-9 4-5 6-7
N=10 i=66
dia=5 deg=4 Wie=118 0-1 1-2 1-6 1-7 2-3 2-8 3-4 3-9 4-5
N=10 i=67
dia=5 deg=4 Wie=117 0-1 1-2 2-3 2-6 2-8 3-4 3-9 4-5 6-7
N=10 i=68
dia=5 deg=4 Wie=116 0-1 1-2 1-6 2-3 2-7 3-4 3-8 3-9 4-5
N=10 i=69
dia=5 deg=4 Wie=115 0-1 1-2 1-6 2-3 2-7 2-8 3-4 3-9 4-5
N=10 i=70
dia=5 deg=4 Wie=115 0-1 1-2 1-6 1-7 2-3 2-8 2-9 3-4 4-5
N=10 i=71
dia=5 deg=4 Wie=111 0-1 1-2 2-3 2-6 2-7 3-4 3-8 3-9 4-5
N=10 i=72
dia=5 deg=5 Wie=124 0-1 1-2 1-6 1-7 1-8 2-3 3-4 4-5 4-9
N=10 i=73
dia=5 deg=5 Wie=119 0-1 1-2 1-6 1-7 1-8 2-3 3-4 3-9 4-5
N=10 i=74
dia=5 deg=5 Wie=116 0-1 1-2 1-6 1-7 1-8 2-3 2-9 3-4 4-5
N=10 i=75
dia=5 deg=5 Wie=115 0-1 1-2 1-6 2-3 3-4 3-7 3-8 3-9 4-5
```

```
N=10 i=76
dia=5 deg=5 Wie=114 0-1 1-2 2-3 2-6 2-8 2-9 3-4 4-5 6-7
N=10 i=77
dia=5 deg=5 Wie=112 0-1 1-2 1-6 2-3 2-7 2-8 2-9 3-4 4-5
N=10 i=78
dia=5 deg=5 Wie=110 0-1 1-2 2-3 2-6 2-7 2-8 3-4 3-9 4-5
N=10 i=79
dia=5 deg=6 Wie=115 0-1 1-2 1-6 1-7 1-8 1-9 2-3 3-4 4-5
N=10 i=80
dia=5 deg=6 Wie=107 0-1 1-2 2-3 2-6 2-7 2-8 2-9 3-4 4-5
N=10 i=81
dia=4 deg=3 Wie=117 0-1 1-2 1-5 2-3 2-6 3-4 3-9 6-7 6-8
N=10 i=82
dia=4 deg=4 Wie=115 0-1 1-2 1-5 1-6 2-3 2-7 3-4 3-9 7-8
N=10 i=83
dia=4 deg=4 Wie=114 0-1 1-2 1-5 2-3 2-6 2-8 3-4 6-7 8-9
N=10 i=84
dia=4 deg=4 Wie=112 0-1 1-2 1-5 2-3 2-6 2-8 3-4 3-9 6-7
N=10 i=85
dia=4 deg=4 Wie=111 0-1 1-2 1-5 1-6 2-3 2-7 3-4 3-8 3-9
N=10 i=86
dia=4 deg=4 Wie=110 0-1 1-2 1-5 1-6 2-3 2-7 2-9 3-4 7-8
N=10 i=87
dia=4 deg=4 Wie=108 0-1 1-2 1-5 1-6 2-3 2-7 2-8 3-4 3-9
N=10 i=88
dia=4 deg=5 Wie=112 0-1 1-2 1-5 1-6 1-7 2-3 3-4 3-8 3-9
N=10 i=89
dia=4 deg=5 Wie=111 0-1 1-2 1-5 1-6 1-7 2-3 2-8 3-4 8-9
N=10 i=90
dia=4 deg=5 Wie=109 0-1 1-2 2-3 2-5 2-7 2-9 3-4 5-6 7-8
N=10 i=91
dia=4 deg=5 Wie=109 0-1 1-2 1-5 1-6 1-7 2-3 2-8 3-4 3-9
N=10 i=92
dia=4 deg=5 Wie=107 0-1 1-2 1-5 2-3 2-6 2-8 2-9 3-4 6-7
N=10 i=93
dia=4 deg=5 Wie=105 0-1 1-2 1-5 2-3 2-6 2-7 2-8 3-4 3-9
N=10 i=94
dia=4 deg=5 Wie=104 0-1 1-2 1-5 1-6 1-7 2-3 2-8 2-9 3-4
N=10 i=95
dia=4 deg=5 Wie=103 0-1 1-2 1-5 1-6 2-3 2-7 2-8 2-9 3-4
N=10 i=96
dia=4 deg=6 Wie=108 0-1 1-2 1-5 1-6 1-7 1-8 2-3 3-4 3-9
N=10 i=97
dia=4 deg=6 Wie=103 0-1 1-2 1-5 1-6 1-7 1-8 2-3 2-9 3-4
N=10 i=98
dia=4 deg=6 Wie=102 0-1 1-2 2-3 2-5 2-7 2-8 2-9 3-4 5-6
N=10 i=99
dia=4 deg=6 Wie=100 0-1 1-2 1-5 2-3 2-6 2-7 2-8 2-9 3-4
N=10 i=100
dia=4 deg=7 Wie=100 0-1 1-2 1-5 1-6 1-7 1-8 1-9 2-3 3-4
N=10 i=101
dia=4 deg=7 Wie=95 0-1 1-2 2-3 2-5 2-6 2-7 2-8 2-9 3-4
N=10 i=102
```

```
dia=3 deg=5 Wie=97 0-1 1-2 1-4 1-5 1-6 2-3 2-7 2-8 2-9
N=10 i=103
dia=3 deg=6 Wie=96 0-1 1-2 1-4 1-5 1-6 1-7 2-3 2-8 2-9
N=10 i=104
dia=3 deg=7 Wie=93 0-1 1-2 1-4 1-5 1-6 1-7 1-8 2-3 2-9
N=10 i=105
dia=3 deg=8 Wie=88 0-1 1-2 1-4 1-5 1-6 1-7 1-8 1-9 2-3
N=10 i=106
dia=2 deg=9 Wie=81 0-1 1-2 1-3 1-4 1-5 1-6 1-7 1-8 1-9
```

Appendix B. Cycle indices

B.1. **Vertex labeled.** The cycle indices for the automorphism group of the vertex-labeled trees are detailed in this section as polynomials in the free variable t. `Zv[N,t]` is the cycle index for tree number `t` with `N` vertices as introduced in Section A.

After a group of `Zv[N,t]` a sum `Z4` is appended which illustrates which of these cycle indices contribute to the tree counts if only trees with maximum degree 4 are taken into account. This is mainly of interest if the theory is applied to carbon chemistry.

Here and in Section B.2 the cycle indices include already a factor `t[1]` for each vertex or edge that is inert (not moved) by the operations of the symmetry group. The cycle indices `Zv[5,1]`, `Zv([5,2])` and `Zv[5,3]`, for example contain common factors of `t[1]`, `t[1]^3` and `t[1]` which represent the factors $\bar{C}_1(x)$, $\bar{C}_3(x)$ and $\bar{C}_1(x)$ in equations (13), (14) and (16).

```
Zv[1,1] := t[1] :
Z4 := +Zv[1,1]:
Zv[2,1] := +( +t[1]^2 +t[2])/2 :
Z4 := +Zv[2,1]:
Zv[3,1] := +( +t[1]^3 +t[1]*t[2])/2 :
Z4 := +Zv[3,1]:
Zv[4,1] := +( +t[1]^4 +t[2]^2)/2 :
Zv[4,2] := +( +t[1]^4 +3*t[1]^2*t[2] +2*t[1]*t[3])/6 :
Z4 := +Zv[4,1] +Zv[4,2]:
Zv[5,1] := +( +t[1]^5 +t[1]*t[2]^2)/2 :
Zv[5,2] := +( +t[1]^5 +t[1]^3*t[2])/2 :
Zv[5,3] := +( +t[1]^5 +6*t[1]^3*t[2] +8*t[1]^2*t[3] +3*t[1]*t[2]^2
+6*t[1]*t[4])/24 :
Z4 := +Zv[5,1] +Zv[5,2] +Zv[5,3]:
Zv[6,1] := +( +t[1]^6 +t[2]^3)/2 :
Zv[6,2] := +( +t[1]^6 +t[1]^4*t[2])/2 :
Zv[6,3] := +( +t[1]^6 +t[1]^2*t[2]^2)/2 :
Zv[6,4] := +( +t[1]^6 +2*t[1]^4*t[2] +2*t[2]^3 +2*t[2]*t[4]
+t[1]^2*t[2]^2)/8 :
Zv[6,5] := +( +t[1]^6 +3*t[1]^4*t[2] +2*t[1]^3*t[3])/6 :
Zv[6,6] := +( +t[1]^6 +10*t[1]^4*t[2] +20*t[1]^3*t[3] +15*t[1]^2*t[2]^2
+30*t[1]^2*t[4] +20*t[1]*t[2]*t[3] +24*t[1]*t[5])/120 :
Z4 := +Zv[6,1] +Zv[6,2] +Zv[6,3] +Zv[6,4] +Zv[6,5]:
Zv[7,1] := +( +t[1]^7 +t[1]*t[2]^3)/2 :
Zv[7,2] := +( +t[1]^7 +t[1]^5*t[2])/2 :
Zv[7,3] := +( +t[1]^7)/1 :
Zv[7,4] := +( +t[1]^7 +3*t[1]^3*t[2]^2 +2*t[1]*t[3]^2)/6 :
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Zv[7,5] := +( +t[1]^7 +2*t[1]^5*t[2] +2*t[1]*t[2]^3 +2*t[1]*t[2]*t[4]
+t[1]^3*t[2]^2)/8 :
Zv[7,6] := +( +t[1]^7 +t[1]^5*t[2])/2 :
Zv[7,7] := +( +t[1]^7 +3*t[1]^5*t[2] +2*t[1]^4*t[3])/6 :
Zv[7,8] := +( +t[1]^7 +t[1]^5*t[2] +t[1]^3*t[2]^2 +t[1]*t[2]^3)/4 :
Zv[7,9] := +( +t[1]^7 +4*t[1]^5*t[2] +3*t[1]^3*t[2]^2 +2*t[1]^4*t[3]
+2*t[1]^2*t[2]*t[3])/12 :
Zv[7,10] := +( +t[1]^7 +6*t[1]^5*t[2] +8*t[1]^4*t[3] +3*t[1]^3*t[2]^2
+6*t[1]^3*t[4])/24 :
Zv[7,11] := +( +t[1]^7 +15*t[1]^5*t[2] +40*t[1]^4*t[3] +45*t[1]^3*t[2]^2
+90*t[1]^3*t[4] +120*t[1]^2*t[2]*t[3] +144*t[1]^2*t[5] +15*t[1]*t[2]^3
+90*t[1]*t[2]*t[4] +40*t[1]*t[3]^2 +120*t[1]*t[6])/720 :
Z4 := +Zv[7,1] +Zv[7,2] +Zv[7,3] +Zv[7,4] +Zv[7,5] +Zv[7,6] +Zv[7,7]
+Zv[7,8] +Zv[7,9]:
Zv[8,1] := +( +t[1]^8 +t[2]^4)/2 :
Zv[8,2] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Zv[8,3] := +( +t[1]^8)/1 :
Zv[8,4] := +( +t[1]^8 +t[1]^2*t[2]^3)/2 :
Zv[8,5] := +( +t[1]^8 +2*t[1]^6*t[2] +2*t[2]^4 +2*t[2]^2*t[4]
+t[1]^4*t[2]^2)/8 :
Zv[8,6] := +( +t[1]^8 +t[1]^4*t[2]^2)/2 :
Zv[8,7] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Zv[8,8] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Zv[8,9] := +( +t[1]^8 +t[2]^4)/2 :
Zv[8,10] := +( +t[1]^8 +3*t[1]^6*t[2] +2*t[1]^5*t[3])/6 :
Zv[8,11] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Zv[8,12] := +( +t[1]^8 +t[1]^4*t[2]^2 +t[1]^6*t[2] +t[1]^2*t[2]^3)/4 :
Zv[8,13] := +( +t[1]^8 +2*t[1]^6*t[2] +2*t[1]^2*t[2]^3 +2*t[1]^2*t[2]*t[4]
+t[1]^4*t[2]^2)/8 :
Zv[8,14] := +( +t[1]^8 +4*t[1]^6*t[2] +3*t[1]^4*t[2]^2 +2*t[1]^5*t[3]
+2*t[1]^3*t[2]*t[3])/12 :
Zv[8,15] := +( +t[1]^8 +3*t[1]^4*t[2]^2 +2*t[1]^2*t[3]^2)/6 :
Zv[8,16] := +( +t[1]^8 +3*t[1]^6*t[2] +2*t[1]^5*t[3])/6 :
Zv[8,17] := +( +t[1]^8 +2*t[1]^6*t[2] +t[1]^4*t[2]^2)/4 :
Zv[8,18] := +( +t[1]^8 +6*t[1]^6*t[2] +8*t[1]^5*t[3] +3*t[1]^4*t[2]^2
+6*t[1]^4*t[4])/24 :
Zv[8,19] := +( +t[1]^8 +3*t[1]^6*t[2] +2*t[1]^5*t[3] +t[1]^4*t[2]^2
+3*t[1]^2*t[2]^3 +2*t[1]*t[2]^2*t[3])/12 :
Zv[8,20] := +( +t[1]^8 +6*t[1]^6*t[2] +9*t[1]^4*t[2]^2 +4*t[1]^5*t[3]
+12*t[1]^3*t[2]*t[3] +6*t[2]^4 +18*t[2]^2*t[4] +12*t[2]*t[6]
+4*t[1]^2*t[3]^2)/72 :
Zv[8,21] := +( +t[1]^8 +7*t[1]^6*t[2] +8*t[1]^5*t[3] +9*t[1]^4*t[2]^2
+8*t[1]^3*t[2]*t[3] +6*t[1]^4*t[4] +3*t[1]^2*t[2]^3
+6*t[1]^2*t[2]*t[4])/48 :
Zv[8,22] := +( +t[1]^8 +10*t[1]^6*t[2] +20*t[1]^5*t[3] +15*t[1]^4*t[2]^2
+30*t[1]^4*t[4] +20*t[1]^3*t[2]*t[3] +24*t[1]^3*t[5])/120 :
Zv[8,23] := +( +t[1]^8 +21*t[1]^6*t[2] +70*t[1]^5*t[3] +105*t[1]^4*t[2]^2
+210*t[1]^4*t[4] +420*t[1]^3*t[2]*t[3] +504*t[1]^3*t[5] +105*t[1]^2*t[2]^3
+630*t[1]^2*t[2]*t[4] +280*t[1]^2*t[3]^2 +840*t[1]^2*t[6]
+210*t[1]*t[2]^2*t[3] +504*t[1]*t[2]*t[5] +420*t[1]*t[3]*t[4]
+720*t[1]*t[7])/5040 :
Z4 := +Zv[8,1] +Zv[8,2] +Zv[8,3] +Zv[8,4] +Zv[8,5] +Zv[8,6] +Zv[8,7]
+Zv[8,8] +Zv[8,9] +Zv[8,10] +Zv[8,11] +Zv[8,12] +Zv[8,13] +Zv[8,14]
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+Zv[8,15] +Zv[8,16] +Zv[8,17] +Zv[8,20]:
Zv[9,1] := +( +t[1]^9 +t[1]*t[2]^4)/2 :
Zv[9,2] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Zv[9,3] := +( +t[1]^9)/1 :
Zv[9,4] := +( +t[1]^9)/1 :
Zv[9,5] := +( +t[1]^9 +2*t[1]^7*t[2] +2*t[1]*t[2]^4 +2*t[1]*t[2]^2*t[4]
+t[1]^5*t[2]^2)/8 :
Zv[9,6] := +( +t[1]^9 +t[1]^5*t[2]^2)/2 :
Zv[9,7] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Zv[9,8] := +( +t[1]^9 +t[1]^3*t[2]^3)/2 :
Zv[9,9] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Zv[9,10] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Zv[9,11] := +( +t[1]^9 +t[1]*t[2]^4)/2 :
Zv[9,12] := +( +t[1]^9)/1 :
Zv[9,13] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Zv[9,14] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Zv[9,15] := +( +t[1]^9 +t[1]^7*t[2] +t[1]^3*t[2]^3 +t[1]*t[2]^4)/4 :
Zv[9,16] := +( +t[1]^9 +t[1]^5*t[2]^2 +t[1]^7*t[2] +t[1]^3*t[2]^3)/4 :
Zv[9,17] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Zv[9,18] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Zv[9,19] := +( +t[1]^9 +t[1]^5*t[2]^2)/2 :
Zv[9,20] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Zv[9,21] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
Zv[9,22] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Zv[9,23] := +( +t[1]^9 +t[1]^5*t[2]^2)/2 :
Zv[9,24] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Zv[9,25] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Zv[9,26] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Zv[9,27] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Zv[9,28] := +( +t[1]^9 +6*t[1]^7*t[2] +8*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+6*t[1]^5*t[4])/24 :
Zv[9,29] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Zv[9,30] := +( +t[1]^9 +2*t[1]^7*t[2] +2*t[1]^3*t[2]^3 +2*t[1]^3*t[2]*t[4]
+t[1]^5*t[2]^2)/8 :
Zv[9,31] := +( +t[1]^9 +6*t[1]^5*t[2]^2 +8*t[1]^3*t[3]^2 +3*t[1]*t[2]^4
+6*t[1]*t[4]^2)/24 :
Zv[9,32] := +( +t[1]^9 +3*t[1]^7*t[2] +t[1]^5*t[2]^2 +3*t[1]^3*t[2]^3
+2*t[1]^6*t[3] +2*t[1]^2*t[2]^2*t[3])/12 :
Zv[9,33] := +( +t[1]^9 +6*t[1]^7*t[2] +9*t[1]^5*t[2]^2 +4*t[1]^6*t[3]
+12*t[1]^4*t[2]*t[3] +6*t[1]*t[2]^4 +18*t[1]*t[2]^2*t[4]
+12*t[1]*t[2]*t[6] +4*t[1]^3*t[3]^2)/72 :
Zv[9,34] := +( +t[1]^9 +t[1]^7*t[2] +t[1]^5*t[2]^2 +t[1]^3*t[2]^3)/4 :
Zv[9,35] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
Zv[9,36] := +( +t[1]^9 +3*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +3*t[1]^3*t[2]^3
+2*t[1]*t[2]^4 +2*t[1]^3*t[2]*t[4] +2*t[1]*t[2]^2*t[4])/16 :
Zv[9,37] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
Zv[9,38] := +( +t[1]^9 +7*t[1]^7*t[2] +8*t[1]^6*t[3] +9*t[1]^5*t[2]^2
+8*t[1]^4*t[2]*t[3] +6*t[1]^5*t[4] +3*t[1]^3*t[2]^3
+6*t[1]^3*t[2]*t[4])/48 :
Zv[9,39] := +( +t[1]^9 +t[1]^7*t[2] +3*t[1]^5*t[2]^2 +3*t[1]^3*t[2]^3
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+2*t[1]^3*t[3]^2 +2*t[1]*t[2]*t[3]^2)/12 :
Zv[9,40] := +( +t[1]^9 +6*t[1]^7*t[2] +8*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+6*t[1]^5*t[4])/24 :
Zv[9,41] := +( +t[1]^9 +4*t[1]^7*t[2] +2*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+2*t[1]^4*t[2]*t[3])/12 :
Zv[9,42] := +( +t[1]^9 +10*t[1]^7*t[2] +20*t[1]^6*t[3] +15*t[1]^5*t[2]^2
+30*t[1]^5*t[4] +20*t[1]^4*t[2]*t[3] +24*t[1]^4*t[5])/120 :
Zv[9,43] := +( +t[1]^9 +6*t[1]^7*t[2] +8*t[1]^6*t[3] +4*t[1]^5*t[2]^2
+6*t[1]^5*t[4] +6*t[1]^3*t[2]^3 +8*t[1]^2*t[2]^2*t[3] +3*t[1]*t[2]^4
+6*t[1]*t[2]^2*t[4])/48 :
Zv[9,44] := +( +t[1]^9 +9*t[1]^7*t[2] +21*t[1]^5*t[2]^2 +10*t[1]^6*t[3]
+36*t[1]^4*t[2]*t[3] +16*t[1]^3*t[3]^2 +9*t[1]^3*t[2]^3 +6*t[1]^5*t[4]
+18*t[1]^3*t[2]*t[4] +6*t[1]^2*t[2]^2*t[3] +12*t[1]^2*t[3]*t[4])/144 :
Zv[9,45] := +( +t[1]^9 +11*t[1]^7*t[2] +20*t[1]^6*t[3] +25*t[1]^5*t[2]^2
+30*t[1]^5*t[4] +40*t[1]^4*t[2]*t[3] +15*t[1]^3*t[2]^3
+30*t[1]^3*t[2]*t[4] +24*t[1]^4*t[5] +20*t[1]^2*t[2]^2*t[3]
+24*t[1]^2*t[2]*t[5])/240 :
Zv[9,46] := +( +t[1]^9 +15*t[1]^7*t[2] +40*t[1]^6*t[3] +45*t[1]^5*t[2]^2
+90*t[1]^5*t[4] +120*t[1]^4*t[2]*t[3] +144*t[1]^4*t[5] +15*t[1]^3*t[2]^3
+90*t[1]^3*t[2]*t[4] +40*t[1]^3*t[3]^2 +120*t[1]^3*t[6])/720 :
Zv[9,47] := +( +t[1]^9 +28*t[1]^7*t[2] +112*t[1]^6*t[3] +210*t[1]^5*t[2]^2
+420*t[1]^5*t[4] +1120*t[1]^4*t[2]*t[3] +1344*t[1]^4*t[5]
+420*t[1]^3*t[2]^3 +2520*t[1]^3*t[2]*t[4] +1120*t[1]^3*t[3]^2
+3360*t[1]^3*t[6] +1680*t[1]^2*t[2]^2*t[3] +4032*t[1]^2*t[2]*t[5]
+3360*t[1]^2*t[3]*t[4] +5760*t[1]^2*t[7] +105*t[1]*t[2]^4
+1260*t[1]*t[2]^2*t[4] +1120*t[1]*t[2]*t[3]^2 +3360*t[1]*t[2]*t[6]
+2688*t[1]*t[3]*t[5] +1260*t[1]*t[4]^2 +5040*t[1]*t[8])/40320 :
Z4 := +Zv[9,1] +Zv[9,2] +Zv[9,3] +Zv[9,4] +Zv[9,5] +Zv[9,6] +Zv[9,7]
+Zv[9,8] +Zv[9,9] +Zv[9,10] +Zv[9,11] +Zv[9,12] +Zv[9,13] +Zv[9,14]
+Zv[9,15] +Zv[9,16] +Zv[9,17] +Zv[9,18] +Zv[9,19] +Zv[9,20] +Zv[9,21]
+Zv[9,22] +Zv[9,23] +Zv[9,24] +Zv[9,25] +Zv[9,26] +Zv[9,27] +Zv[9,30]
+Zv[9,31] +Zv[9,32] +Zv[9,33] +Zv[9,34] +Zv[9,35] +Zv[9,36] +Zv[9,37]:
Zv[10,1] := +( +t[1]^10 +t[2]^5)/2 :
Zv[10,2] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,3] := +( +t[1]^10)/1 :
Zv[10,4] := +( +t[1]^10)/1 :
Zv[10,5] := +( +t[1]^10 +t[1]^2*t[2]^4)/2 :
Zv[10,6] := +( +t[1]^10 +2*t[1]^8*t[2] +2*t[2]^5 +2*t[2]^3*t[4]
+t[1]^6*t[2]^2)/8 :
Zv[10,7] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,8] := +( +t[1]^10 +t[1]^6*t[2]^2)/2 :
Zv[10,9] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,10] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,11] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,12] := +( +t[1]^10)/1 :
Zv[10,13] := +( +t[1]^10 +t[2]^5)/2 :
Zv[10,14] := +( +t[1]^10)/1 :
Zv[10,15] := +( +t[1]^10)/1 :
Zv[10,16] := +( +t[1]^10 +t[2]^5)/2 :
Zv[10,17] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3])/6 :
Zv[10,18] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,19] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,20] := +( +t[1]^10 +3*t[1]^4*t[2]^3 +2*t[1]*t[3]^3)/6 :
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Zv[10,21] := +( +t[1]^10 +t[1]^6*t[2]^2 +t[1]^8*t[2] +t[1]^4*t[2]^3)/4 :
Zv[10,22] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2)/4 :
Zv[10,23] := +( +t[1]^10 +2*t[1]^8*t[2] +2*t[1]^2*t[2]^4
+2*t[1]^2*t[2]^2*t[4] +t[1]^6*t[2]^2)/8 :
Zv[10,24] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,25] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,26] := +( +t[1]^10 +t[1]^6*t[2]^2)/2 :
Zv[10,27] := +( +t[1]^10 +t[1]^8*t[2] +t[1]^4*t[2]^3 +t[1]^2*t[2]^4)/4 :
Zv[10,28] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,29] := +( +t[1]^10 +t[1]^6*t[2]^2)/2 :
Zv[10,30] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,31] := +( +t[1]^10)/1 :
Zv[10,32] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,33] := +( +t[1]^10 +t[1]^2*t[2]^4)/2 :
Zv[10,34] := +( +t[1]^10 +4*t[1]^8*t[2] +3*t[1]^6*t[2]^2 +2*t[1]^7*t[3]
+2*t[1]^5*t[2]*t[3])/12 :
Zv[10,35] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3])/6 :
Zv[10,36] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2)/4 :
Zv[10,37] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3])/6 :
Zv[10,38] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3])/6 :
Zv[10,39] := +( +t[1]^10 +t[1]^6*t[2]^2)/2 :
Zv[10,40] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2)/4 :
Zv[10,41] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2)/4 :
Zv[10,42] := +( +t[1]^10 +t[1]^4*t[2]^3)/2 :
Zv[10,43] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,44] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,45] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,46] := +( +t[1]^10 +6*t[1]^8*t[2] +8*t[1]^7*t[3] +3*t[1]^6*t[2]^2
+6*t[1]^6*t[4])/24 :
Zv[10,47] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3])/6 :
Zv[10,48] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3] +t[1]^4*t[2]^3
+3*t[1]^2*t[2]^4 +2*t[1]*t[2]^3*t[3])/12 :
Zv[10,49] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2)/4 :
Zv[10,50] := +( +t[1]^10 +2*t[1]^6*t[2]^2 +2*t[2]^5 +2*t[2]*t[4]^2
+t[1]^2*t[2]^4)/8 :
Zv[10,51] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2 +2*t[1]^4*t[2]^3
+2*t[1]^4*t[2]*t[4])/8 :
Zv[10,52] := +( +t[1]^10 +t[1]^6*t[2]^2 +t[1]^8*t[2] +t[1]^4*t[2]^3)/4 :
Zv[10,53] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,54] := +( +t[1]^10 +2*t[1]^8*t[2] +2*t[2]^5 +2*t[2]^3*t[4]
+t[1]^6*t[2]^2)/8 :
Zv[10,55] := +( +t[1]^10 +6*t[1]^8*t[2] +9*t[1]^6*t[2]^2 +4*t[1]^7*t[3]
+12*t[1]^5*t[2]*t[3] +6*t[2]^5 +18*t[2]^3*t[4] +12*t[2]^2*t[6]
+4*t[1]^4*t[3]^2)/72 :
Zv[10,56] := +( +t[1]^10 +3*t[1]^8*t[2] +t[1]^6*t[2]^2 +3*t[1]^4*t[2]^3
+2*t[1]^7*t[3] +2*t[1]^3*t[2]^2*t[3])/12 :
Zv[10,57] := +( +t[1]^10 +4*t[1]^8*t[2] +3*t[1]^6*t[2]^2 +2*t[1]^7*t[3]
+2*t[1]^5*t[2]*t[3])/12 :
Zv[10,58] := +( +t[1]^10 +4*t[1]^8*t[2] +3*t[1]^6*t[2]^2 +2*t[1]^7*t[3]
+2*t[1]^5*t[2]*t[3])/12 :
Zv[10,59] := +( +t[1]^10 +t[1]^6*t[2]^2 +t[1]^8*t[2] +t[1]^4*t[2]^3)/4 :
Zv[10,60] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3])/6 :
Zv[10,61] := +( +t[1]^10 +3*t[1]^6*t[2]^2 +2*t[1]^4*t[3]^2)/6 :
```

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Zv[10,62] := +( +t[1]^10 +3*t[1]^8*t[2] +3*t[1]^6*t[2]^2 +t[1]^4*t[2]^3)/8
:
Zv[10,63] := +( +t[1]^10 +t[1]^8*t[2])/2 :
Zv[10,64] := +( +t[1]^10 +4*t[1]^8*t[2] +3*t[1]^6*t[2]^2 +2*t[1]^7*t[3]
+2*t[1]^5*t[2]*t[3])/12 :
Zv[10,65] := +( +t[1]^10 +t[1]^8*t[2] +t[1]^6*t[2]^2 +t[1]^4*t[2]^3)/4 :
Zv[10,66] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3])/6 :
Zv[10,67] := +( +t[1]^10 +t[1]^6*t[2]^2)/2 :
Zv[10,68] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2)/4 :
Zv[10,69] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2)/4 :
Zv[10,70] := +( +t[1]^10 +4*t[1]^8*t[2] +3*t[1]^6*t[2]^2 +2*t[1]^7*t[3]
+2*t[1]^5*t[2]*t[3])/12 :
Zv[10,71] := +( +t[1]^10 +2*t[1]^8*t[2] +t[1]^6*t[2]^2 +2*t[2]^5
+2*t[2]^3*t[4])/8 :
Zv[10,72] := +( +t[1]^10 +7*t[1]^8*t[2] +8*t[1]^7*t[3] +9*t[1]^6*t[2]^2
+8*t[1]^5*t[2]*t[3] +6*t[1]^6*t[4] +3*t[1]^4*t[2]^3
+6*t[1]^4*t[2]*t[4])/48 :
Zv[10,73] := +( +t[1]^10 +6*t[1]^8*t[2] +8*t[1]^7*t[3] +3*t[1]^6*t[2]^2
+6*t[1]^6*t[4])/24 :
Zv[10,74] := +( +t[1]^10 +6*t[1]^8*t[2] +8*t[1]^7*t[3] +3*t[1]^6*t[2]^2
+6*t[1]^6*t[4])/24 :
Zv[10,75] := +( +t[1]^10 +4*t[1]^8*t[2] +2*t[1]^7*t[3] +3*t[1]^6*t[2]^2
+2*t[1]^5*t[2]*t[3])/12 :
Zv[10,76] := +( +t[1]^10 +t[1]^8*t[2] +t[1]^6*t[2]^2 +t[1]^4*t[2]^3)/4 :
Zv[10,77] := +( +t[1]^10 +4*t[1]^8*t[2] +2*t[1]^7*t[3] +3*t[1]^6*t[2]^2
+2*t[1]^5*t[2]*t[3])/12 :
Zv[10,78] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3])/6 :
Zv[10,79] := +( +t[1]^10 +10*t[1]^8*t[2] +20*t[1]^7*t[3] +15*t[1]^6*t[2]^2
+30*t[1]^6*t[4] +20*t[1]^5*t[2]*t[3] +24*t[1]^5*t[5])/120 :
Zv[10,80] := +( +t[1]^10 +6*t[1]^8*t[2] +8*t[1]^7*t[3] +3*t[1]^6*t[2]^2
+6*t[1]^6*t[4])/24 :
Zv[10,81] := +( +t[1]^10 +3*t[1]^8*t[2] +3*t[1]^6*t[2]^2 +7*t[1]^4*t[2]^3
+6*t[1]^4*t[2]*t[4] +6*t[1]^2*t[2]^4 +6*t[1]^2*t[2]^2*t[4] +8*t[1]*t[3]^3
+8*t[1]*t[3]*t[6])/48 :
Zv[10,82] := +( +t[1]^10 +4*t[1]^8*t[2] +3*t[1]^6*t[2]^2 +2*t[1]^7*t[3]
+2*t[1]^5*t[2]*t[3])/12 :
Zv[10,83] := +( +t[1]^10 +3*t[1]^6*t[2]^2 +2*t[1]^4*t[3]^2 +t[1]^8*t[2]
+3*t[1]^4*t[2]^3 +2*t[1]^2*t[2]*t[3]^2)/12 :
Zv[10,84] := +( +t[1]^10 +2*t[1]^8*t[2] +2*t[1]^4*t[2]^3
+2*t[1]^4*t[2]*t[4] +t[1]^6*t[2]^2)/8 :
Zv[10,85] := +( +t[1]^10 +6*t[1]^8*t[2] +9*t[1]^6*t[2]^2 +4*t[1]^7*t[3]
+12*t[1]^5*t[2]*t[3] +6*t[1]^2*t[2]^4 +18*t[1]^2*t[2]^2*t[4]
+12*t[1]^2*t[2]*t[6] +4*t[1]^4*t[3]^2)/72 :
Zv[10,86] := +( +t[1]^10 +3*t[1]^8*t[2] +t[1]^6*t[2]^2 +3*t[1]^4*t[2]^3
+2*t[1]^7*t[3] +2*t[1]^3*t[2]^2*t[3])/12 :
Zv[10,87] := +( +t[1]^10 +5*t[1]^8*t[2] +7*t[1]^6*t[2]^2 +3*t[1]^4*t[2]^3
+2*t[1]^7*t[3] +4*t[1]^5*t[2]*t[3] +2*t[1]^3*t[2]^2*t[3])/24 :
Zv[10,88] := +( +t[1]^10 +9*t[1]^8*t[2] +21*t[1]^6*t[2]^2 +10*t[1]^7*t[3]
+36*t[1]^5*t[2]*t[3] +16*t[1]^4*t[3]^2 +9*t[1]^4*t[2]^3 +6*t[1]^6*t[4]
+18*t[1]^4*t[2]*t[4] +6*t[1]^3*t[2]^2*t[3] +12*t[1]^3*t[3]*t[4])/144 :
Zv[10,89] := +( +t[1]^10 +6*t[1]^8*t[2] +8*t[1]^7*t[3] +4*t[1]^6*t[2]^2
+6*t[1]^4*t[2]^3 +8*t[1]^3*t[2]^2*t[3] +6*t[1]^6*t[4] +3*t[1]^2*t[2]^4
+6*t[1]^2*t[2]^2*t[4])/48 :
```

```
Zv[10,90] := +( +t[1]^10 +6*t[1]^6*t[2]^2 +8*t[1]^4*t[3]^2
+3*t[1]^2*t[2]^4 +6*t[1]^2*t[4]^2)/24 :
Zv[10,91] := +( +t[1]^10 +7*t[1]^8*t[2] +8*t[1]^7*t[3] +9*t[1]^6*t[2]^2
+8*t[1]^5*t[2]*t[3] +6*t[1]^6*t[4] +3*t[1]^4*t[2]^3
+6*t[1]^4*t[2]*t[4])/48 :
Zv[10,92] := +( +t[1]^10 +2*t[1]^8*t[2] +2*t[1]^6*t[2]^2 +2*t[1]^4*t[2]^3
+t[1]^2*t[2]^4)/8 :
Zv[10,93] := +( +t[1]^10 +5*t[1]^8*t[2] +2*t[1]^7*t[3] +7*t[1]^6*t[2]^2
+4*t[1]^5*t[2]*t[3] +5*t[1]^4*t[2]^3 +6*t[1]^2*t[2]^4 +4*t[1]*t[2]^3*t[3]
+2*t[1]^4*t[2]*t[4] +6*t[1]^2*t[2]^2*t[4] +4*t[1]*t[2]*t[3]*t[4]
+2*t[1]^3*t[2]^2*t[3])/48 :
Zv[10,94] := +( +t[1]^10 +7*t[1]^8*t[2] +9*t[1]^6*t[2]^2 +8*t[1]^7*t[3]
+8*t[1]^5*t[2]*t[3] +3*t[1]^4*t[2]^3 +6*t[1]^6*t[4]
+6*t[1]^4*t[2]*t[4])/48 :
Zv[10,95] := +( +t[1]^10 +6*t[1]^8*t[2] +4*t[1]^7*t[3] +9*t[1]^6*t[2]^2
+12*t[1]^5*t[2]*t[3] +4*t[1]^4*t[3]^2)/36 :
Zv[10,96] := +( +t[1]^10 +11*t[1]^8*t[2] +20*t[1]^7*t[3] +25*t[1]^6*t[2]^2
+30*t[1]^6*t[4] +40*t[1]^5*t[2]*t[3] +15*t[1]^4*t[2]^3
+30*t[1]^4*t[2]*t[4] +24*t[1]^5*t[5] +20*t[1]^3*t[2]^2*t[3]
+24*t[1]^3*t[2]*t[5])/240 :
Zv[10,97] := +( +t[1]^10 +10*t[1]^8*t[2] +20*t[1]^7*t[3] +15*t[1]^6*t[2]^2
+30*t[1]^6*t[4] +20*t[1]^5*t[2]*t[3] +24*t[1]^5*t[5])/120 :
Zv[10,98] := +( +t[1]^10 +3*t[1]^8*t[2] +2*t[1]^7*t[3] +3*t[1]^6*t[2]^2
+9*t[1]^4*t[2]^3 +6*t[1]^3*t[2]^2*t[3] +2*t[1]^4*t[3]^2
+6*t[1]^2*t[2]*t[3]^2 +4*t[1]*t[3]^3)/36 :
Zv[10,99] := +( +t[1]^10 +7*t[1]^8*t[2] +8*t[1]^7*t[3] +9*t[1]^6*t[2]^2
+6*t[1]^6*t[4] +8*t[1]^5*t[2]*t[3] +3*t[1]^4*t[2]^3
+6*t[1]^4*t[2]*t[4])/48 :
Zv[10,100] := +( +t[1]^10 +15*t[1]^8*t[2] +40*t[1]^7*t[3]
+45*t[1]^6*t[2]^2 +90*t[1]^6*t[4] +120*t[1]^5*t[2]*t[3] +144*t[1]^5*t[5]
+15*t[1]^4*t[2]^3 +90*t[1]^4*t[2]*t[4] +40*t[1]^4*t[3]^2
+120*t[1]^4*t[6])/720 :
Zv[10,101] := +( +t[1]^10 +10*t[1]^8*t[2] +20*t[1]^7*t[3]
+16*t[1]^6*t[2]^2 +30*t[1]^6*t[4] +20*t[1]^5*t[2]*t[3] +24*t[1]^5*t[5]
+10*t[1]^4*t[2]^3 +20*t[1]^3*t[2]^2*t[3] +15*t[1]^2*t[2]^4
+30*t[1]^2*t[2]^2*t[4] +20*t[1]*t[2]^3*t[3] +24*t[1]*t[2]^2*t[5])/240 :
Zv[10,102] := +( +t[1]^10 +12*t[1]^8*t[2] +16*t[1]^7*t[3]
+42*t[1]^6*t[2]^2 +96*t[1]^5*t[2]*t[3] +64*t[1]^4*t[3]^2 +12*t[1]^6*t[4]
+36*t[1]^4*t[2]^3 +72*t[1]^4*t[2]*t[4] +48*t[1]^3*t[2]^2*t[3]
+96*t[1]^3*t[3]*t[4] +24*t[2]^5 +144*t[2]^3*t[4] +192*t[2]^2*t[6]
+72*t[2]*t[4]^2 +144*t[2]*t[8] +9*t[1]^2*t[2]^4 +36*t[1]^2*t[2]^2*t[4]
+36*t[1]^2*t[4]^2)/1152 :
Zv[10,103] := +( +t[1]^10 +13*t[1]^8*t[2] +45*t[1]^6*t[2]^2
+22*t[1]^7*t[3] +100*t[1]^5*t[2]*t[3] +45*t[1]^4*t[2]^3 +30*t[1]^6*t[4]
+90*t[1]^4*t[2]*t[4] +40*t[1]^4*t[3]^2 +90*t[1]^3*t[2]^2*t[3]
+60*t[1]^3*t[3]*t[4] +24*t[1]^5*t[5] +72*t[1]^3*t[2]*t[5]
+40*t[1]^2*t[2]*t[3]^2 +48*t[1]^2*t[3]*t[5])/720 :
Zv[10,104] := +( +t[1]^10 +16*t[1]^8*t[2] +40*t[1]^7*t[3]
+60*t[1]^6*t[2]^2 +90*t[1]^6*t[4] +160*t[1]^5*t[2]*t[3] +144*t[1]^5*t[5]
+60*t[1]^4*t[2]^3 +180*t[1]^4*t[2]*t[4] +120*t[1]^3*t[2]^2*t[3]
+144*t[1]^3*t[2]*t[5] +40*t[1]^4*t[3]^2 +120*t[1]^4*t[6] +15*t[1]^2*t[2]^4
+90*t[1]^2*t[2]^2*t[4] +40*t[1]^2*t[2]*t[3]^2 +120*t[1]^2*t[2]*t[6])/1440 :
Zv[10,105] := +( +t[1]^10 +21*t[1]^8*t[2] +70*t[1]^7*t[3]
```

```
+105*t[1]^6*t[2]^2 +210*t[1]^6*t[4] +420*t[1]^5*t[2]*t[3] +504*t[1]^5*t[5]
+105*t[1]^4*t[2]^3 +630*t[1]^4*t[2]*t[4] +280*t[1]^4*t[3]^2
+840*t[1]^4*t[6] +210*t[1]^3*t[2]^2*t[3] +504*t[1]^3*t[2]*t[5]
+420*t[1]^3*t[3]*t[4] +720*t[1]^3*t[7])/5040 :
Zv[10,106] := +( +t[1]^10 +36*t[1]^8*t[2] +168*t[1]^7*t[3]
+378*t[1]^6*t[2]^2 +756*t[1]^6*t[4] +2520*t[1]^5*t[2]*t[3]
+3024*t[1]^5*t[5] +1260*t[1]^4*t[2]^3 +7560*t[1]^4*t[2]*t[4]
+3360*t[1]^4*t[3]^2 +10080*t[1]^4*t[6] +7560*t[1]^3*t[2]^2*t[3]
+18144*t[1]^3*t[2]*t[5] +15120*t[1]^3*t[3]*t[4] +25920*t[1]^3*t[7]
+945*t[1]^2*t[2]^4 +11340*t[1]^2*t[2]^2*t[4] +10080*t[1]^2*t[2]*t[3]^2
+30240*t[1]^2*t[2]*t[6] +24192*t[1]^2*t[3]*t[5] +11340*t[1]^2*t[4]^2
+45360*t[1]^2*t[8] +2520*t[1]*t[2]^3*t[3] +9072*t[1]*t[2]^2*t[5]
+15120*t[1]*t[2]*t[3]*t[4] +25920*t[1]*t[2]*t[7] +2240*t[1]*t[3]^3
+20160*t[1]*t[3]*t[6] +18144*t[1]*t[4]*t[5] +40320*t[1]*t[9])/362880 :
Z4 := +Zv[10,1] +Zv[10,2] +Zv[10,3] +Zv[10,4] +Zv[10,5] +Zv[10,6]
+Zv[10,7] +Zv[10,8] +Zv[10,9] +Zv[10,10] +Zv[10,11] +Zv[10,12] +Zv[10,13]
+Zv[10,14] +Zv[10,15] +Zv[10,16] +Zv[10,17] +Zv[10,18] +Zv[10,19]
+Zv[10,20] +Zv[10,21] +Zv[10,22] +Zv[10,23] +Zv[10,24] +Zv[10,25]
+Zv[10,26] +Zv[10,27] +Zv[10,28] +Zv[10,29] +Zv[10,30] +Zv[10,31]
+Zv[10,32] +Zv[10,33] +Zv[10,34] +Zv[10,35] +Zv[10,36] +Zv[10,37]
+Zv[10,38] +Zv[10,39] +Zv[10,40] +Zv[10,41] +Zv[10,42] +Zv[10,43]
+Zv[10,44] +Zv[10,45] +Zv[10,49] +Zv[10,50] +Zv[10,51] +Zv[10,52]
+Zv[10,53] +Zv[10,54] +Zv[10,55] +Zv[10,56] +Zv[10,57] +Zv[10,58]
+Zv[10,59] +Zv[10,60] +Zv[10,61] +Zv[10,62] +Zv[10,63] +Zv[10,64]
+Zv[10,65] +Zv[10,66] +Zv[10,67] +Zv[10,68] +Zv[10,69] +Zv[10,70]
+Zv[10,71] +Zv[10,81] +Zv[10,82] +Zv[10,83] +Zv[10,84] +Zv[10,85]
+Zv[10,86] +Zv[10,87]:
```

B.2. **Edge labeled.** `Ze[N,t]` is the cycle index for the automorphism group of the vertex-labeled tree number `t` with `N` vertices as introduced in Section A.

These cycle indices of the edge labeled trees have appeared in the ancillary directories of my earlier tabulation [4], because one way of applying the edge labels (or weights) is to replace the edge by a multi-edge of the equivalent multiplicity.

```
Ze[1,1] := t[1] :
Z4 := +Ze[1,1]:
Ze[2,1] := +( +2*t[1])/2 :
Z4 := +Ze[2,1]:
Ze[3,1] := +( +t[1]^2 +t[2])/2 :
Z4 := +Ze[3,1]:
Ze[4,1] := +( +t[1]^3 +t[1]*t[2])/2 :
Ze[4,2] := +( +t[1]^3 +3*t[1]*t[2] +2*t[3])/6 :
Z4 := +Ze[4,1] +Ze[4,2]:
Ze[5,1] := +( +t[1]^4 +t[2]^2)/2 :
Ze[5,2] := +( +t[1]^4 +t[1]^2*t[2])/2 :
Ze[5,3] := +( +t[1]^4 +6*t[1]^2*t[2] +8*t[1]*t[3] +3*t[2]^2 +6*t[4])/24 :
Z4 := +Ze[5,1] +Ze[5,2] +Ze[5,3]:
Ze[6,1] := +( +t[1]^5 +t[1]*t[2]^2)/2 :
Ze[6,2] := +( +t[1]^5 +t[1]^3*t[2])/2 :
Ze[6,3] := +( +t[1]^5 +t[1]*t[2]^2)/2 :
Ze[6,4] := +( +t[1]^5 +2*t[1]^3*t[2] +3*t[1]*t[2]^2 +2*t[1]*t[4])/8 :
Ze[6,5] := +( +t[1]^5 +3*t[1]^3*t[2] +2*t[1]^2*t[3])/6 :
Ze[6,6] := +( +t[1]^5 +10*t[1]^3*t[2] +20*t[1]^2*t[3] +15*t[1]*t[2]^2
```

```
+30*t[1]*t[4] +20*t[2]*t[3] +24*t[5])/120 :
Z4 := +Ze[6,1] +Ze[6,2] +Ze[6,3] +Ze[6,4] +Ze[6,5]:
Ze[7,1] := +( +t[1]^6 +t[2]^3)/2 :
Ze[7,2] := +( +t[1]^6 +t[1]^4*t[2])/2 :
Ze[7,3] := +( +t[1]^6)/1 :
Ze[7,4] := +( +t[1]^6 +3*t[1]^2*t[2]^2 +2*t[3]^2)/6 :
Ze[7,5] := +( +t[1]^6 +2*t[1]^4*t[2] +2*t[2]^3 +2*t[2]*t[4]
+t[1]^2*t[2]^2)/8 :
Ze[7,6] := +( +t[1]^6 +t[1]^4*t[2])/2 :
Ze[7,7] := +( +t[1]^6 +3*t[1]^4*t[2] +2*t[1]^3*t[3])/6 :
Ze[7,8] := +( +t[1]^6 +t[1]^4*t[2] +t[1]^2*t[2]^2 +t[2]^3)/4 :
Ze[7,9] := +( +t[1]^6 +4*t[1]^4*t[2] +3*t[1]^2*t[2]^2 +2*t[1]^3*t[3]
+2*t[1]*t[2]*t[3])/12 :
Ze[7,10] := +( +t[1]^6 +6*t[1]^4*t[2] +8*t[1]^3*t[3] +3*t[1]^2*t[2]^2
+6*t[1]^2*t[4])/24 :
Ze[7,11] := +( +t[1]^6 +15*t[1]^4*t[2] +40*t[1]^3*t[3] +45*t[1]^2*t[2]^2
+90*t[1]^2*t[4] +120*t[1]*t[2]*t[3] +144*t[1]*t[5] +15*t[2]^3
+90*t[2]*t[4] +40*t[3]^2 +120*t[6])/720 :
Z4 := +Ze[7,1] +Ze[7,2] +Ze[7,3] +Ze[7,4] +Ze[7,5] +Ze[7,6] +Ze[7,7]
+Ze[7,8] +Ze[7,9]:
Ze[8,1] := +( +t[1]^7 +t[1]*t[2]^3)/2 :
Ze[8,2] := +( +t[1]^7 +t[1]^5*t[2])/2 :
Ze[8,3] := +( +t[1]^7)/1 :
Ze[8,4] := +( +t[1]^7 +t[1]*t[2]^3)/2 :
Ze[8,5] := +( +t[1]^7 +2*t[1]^5*t[2] +2*t[1]*t[2]^3 +2*t[1]*t[2]*t[4]
+t[1]^3*t[2]^2)/8 :
Ze[8,6] := +( +t[1]^7 +t[1]^3*t[2]^2)/2 :
Ze[8,7] := +( +t[1]^7 +t[1]^5*t[2])/2 :
Ze[8,8] := +( +t[1]^7 +t[1]^5*t[2])/2 :
Ze[8,9] := +( +t[1]^7 +t[1]*t[2]^3)/2 :
Ze[8,10] := +( +t[1]^7 +3*t[1]^5*t[2] +2*t[1]^4*t[3])/6 :
Ze[8,11] := +( +t[1]^7 +t[1]^5*t[2])/2 :
Ze[8,12] := +( +t[1]^7 +t[1]^3*t[2]^2 +t[1]^5*t[2] +t[1]*t[2]^3)/4 :
Ze[8,13] := +( +t[1]^7 +2*t[1]^5*t[2] +2*t[1]*t[2]^3 +2*t[1]*t[2]*t[4]
+t[1]^3*t[2]^2)/8 :
Ze[8,14] := +( +t[1]^7 +4*t[1]^5*t[2] +3*t[1]^3*t[2]^2 +2*t[1]^4*t[3]
+2*t[1]^2*t[2]*t[3])/12 :
Ze[8,15] := +( +t[1]^7 +3*t[1]^3*t[2]^2 +2*t[1]*t[3]^2)/6 :
Ze[8,16] := +( +t[1]^7 +3*t[1]^5*t[2] +2*t[1]^4*t[3])/6 :
Ze[8,17] := +( +t[1]^7 +2*t[1]^5*t[2] +t[1]^3*t[2]^2)/4 :
Ze[8,18] := +( +t[1]^7 +6*t[1]^5*t[2] +8*t[1]^4*t[3] +3*t[1]^3*t[2]^2
+6*t[1]^3*t[4])/24 :
Ze[8,19] := +( +t[1]^7 +3*t[1]^5*t[2] +2*t[1]^4*t[3] +t[1]^3*t[2]^2
+3*t[1]*t[2]^3 +2*t[2]^2*t[3])/12 :
Ze[8,20] := +( +t[1]^7 +6*t[1]^5*t[2] +9*t[1]^3*t[2]^2 +4*t[1]^4*t[3]
+12*t[1]^2*t[2]*t[3] +6*t[1]*t[2]^3 +18*t[1]*t[2]*t[4] +12*t[1]*t[6]
+4*t[1]*t[3]^2)/72 :
Ze[8,21] := +( +t[1]^7 +7*t[1]^5*t[2] +8*t[1]^4*t[3] +9*t[1]^3*t[2]^2
+8*t[1]^2*t[2]*t[3] +6*t[1]^3*t[4] +3*t[1]*t[2]^3 +6*t[1]*t[2]*t[4])/48 :
Ze[8,22] := +( +t[1]^7 +10*t[1]^5*t[2] +20*t[1]^4*t[3] +15*t[1]^3*t[2]^2
+30*t[1]^3*t[4] +20*t[1]^2*t[2]*t[3] +24*t[1]^2*t[5])/120 :
Ze[8,23] := +( +t[1]^7 +21*t[1]^5*t[2] +70*t[1]^4*t[3] +105*t[1]^3*t[2]^2
+210*t[1]^3*t[4] +420*t[1]^2*t[2]*t[3] +504*t[1]^2*t[5] +105*t[1]*t[2]^3
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+630*t[1]*t[2]*t[4] +280*t[1]*t[3]^2 +840*t[1]*t[6] +210*t[2]^2*t[3]
+504*t[2]*t[5] +420*t[3]*t[4] +720*t[7])/5040 :
Z4 := +Ze[8,1] +Ze[8,2] +Ze[8,3] +Ze[8,4] +Ze[8,5] +Ze[8,6] +Ze[8,7]
+Ze[8,8] +Ze[8,9] +Ze[8,10] +Ze[8,11] +Ze[8,12] +Ze[8,13] +Ze[8,14]
+Ze[8,15] +Ze[8,16] +Ze[8,17] +Ze[8,20]:
Ze[9,1] := +( +t[1]^8 +t[2]^4)/2 :
Ze[9,2] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Ze[9,3] := +( +t[1]^8)/1 :
Ze[9,4] := +( +t[1]^8)/1 :
Ze[9,5] := +( +t[1]^8 +2*t[1]^6*t[2] +2*t[2]^4 +2*t[2]^2*t[4]
+t[1]^4*t[2]^2)/8 :
Ze[9,6] := +( +t[1]^8 +t[1]^4*t[2]^2)/2 :
Ze[9,7] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Ze[9,8] := +( +t[1]^8 +t[1]^2*t[2]^3)/2 :
Ze[9,9] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Ze[9,10] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Ze[9,11] := +( +t[1]^8 +t[2]^4)/2 :
Ze[9,12] := +( +t[1]^8)/1 :
Ze[9,13] := +( +t[1]^8 +3*t[1]^6*t[2] +2*t[1]^5*t[3])/6 :
Ze[9,14] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Ze[9,15] := +( +t[1]^8 +t[1]^6*t[2] +t[1]^2*t[2]^3 +t[2]^4)/4 :
Ze[9,16] := +( +t[1]^8 +t[1]^4*t[2]^2 +t[1]^6*t[2] +t[1]^2*t[2]^3)/4 :
Ze[9,17] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Ze[9,18] := +( +t[1]^8 +2*t[1]^6*t[2] +t[1]^4*t[2]^2)/4 :
Ze[9,19] := +( +t[1]^8 +t[1]^4*t[2]^2)/2 :
Ze[9,20] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Ze[9,21] := +( +t[1]^8 +4*t[1]^6*t[2] +3*t[1]^4*t[2]^2 +2*t[1]^5*t[3]
+2*t[1]^3*t[2]*t[3])/12 :
Ze[9,22] := +( +t[1]^8 +3*t[1]^6*t[2] +2*t[1]^5*t[3])/6 :
Ze[9,23] := +( +t[1]^8 +t[1]^4*t[2]^2)/2 :
Ze[9,24] := +( +t[1]^8 +2*t[1]^6*t[2] +t[1]^4*t[2]^2)/4 :
Ze[9,25] := +( +t[1]^8 +3*t[1]^6*t[2] +2*t[1]^5*t[3])/6 :
Ze[9,26] := +( +t[1]^8 +2*t[1]^6*t[2] +t[1]^4*t[2]^2)/4 :
Ze[9,27] := +( +t[1]^8 +t[1]^6*t[2])/2 :
Ze[9,28] := +( +t[1]^8 +6*t[1]^6*t[2] +8*t[1]^5*t[3] +3*t[1]^4*t[2]^2
+6*t[1]^4*t[4])/24 :
Ze[9,29] := +( +t[1]^8 +3*t[1]^6*t[2] +2*t[1]^5*t[3])/6 :
Ze[9,30] := +( +t[1]^8 +2*t[1]^6*t[2] +2*t[1]^2*t[2]^3 +2*t[1]^2*t[2]*t[4]
+t[1]^4*t[2]^2)/8 :
Ze[9,31] := +( +t[1]^8 +6*t[1]^4*t[2]^2 +8*t[1]^2*t[3]^2 +3*t[2]^4
+6*t[4]^2)/24 :
Ze[9,32] := +( +t[1]^8 +3*t[1]^6*t[2] +t[1]^4*t[2]^2 +3*t[1]^2*t[2]^3
+2*t[1]^5*t[3] +2*t[1]*t[2]^2*t[3])/12 :
Ze[9,33] := +( +t[1]^8 +6*t[1]^6*t[2] +9*t[1]^4*t[2]^2 +4*t[1]^5*t[3]
+12*t[1]^3*t[2]*t[3] +6*t[2]^4 +18*t[2]^2*t[4] +12*t[2]*t[6]
+4*t[1]^2*t[3]^2)/72 :
Ze[9,34] := +( +t[1]^8 +t[1]^6*t[2] +t[1]^4*t[2]^2 +t[1]^2*t[2]^3)/4 :
Ze[9,35] := +( +t[1]^8 +4*t[1]^6*t[2] +3*t[1]^4*t[2]^2 +2*t[1]^5*t[3]
+2*t[1]^3*t[2]*t[3])/12 :
Ze[9,36] := +( +t[1]^8 +3*t[1]^6*t[2] +3*t[1]^4*t[2]^2 +3*t[1]^2*t[2]^3
+2*t[2]^4 +2*t[1]^2*t[2]*t[4] +2*t[2]^2*t[4])/16 :
Ze[9,37] := +( +t[1]^8 +4*t[1]^6*t[2] +3*t[1]^4*t[2]^2 +2*t[1]^5*t[3]
+2*t[1]^3*t[2]*t[3])/12 :
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Ze[9,38] := +( +t[1]^8 +7*t[1]^6*t[2] +8*t[1]^5*t[3] +9*t[1]^4*t[2]^2
+8*t[1]^3*t[2]*t[3] +6*t[1]^4*t[4] +3*t[1]^2*t[2]^3
+6*t[1]^2*t[2]*t[4])/48 :
Ze[9,39] := +( +t[1]^8 +t[1]^6*t[2] +3*t[1]^4*t[2]^2 +3*t[1]^2*t[2]^3
+2*t[1]^2*t[3]^2 +2*t[2]*t[3]^2)/12 :
Ze[9,40] := +( +t[1]^8 +6*t[1]^6*t[2] +8*t[1]^5*t[3] +3*t[1]^4*t[2]^2
+6*t[1]^4*t[4])/24 :
Ze[9,41] := +( +t[1]^8 +4*t[1]^6*t[2] +2*t[1]^5*t[3] +3*t[1]^4*t[2]^2
+2*t[1]^3*t[2]*t[3])/12 :
Ze[9,42] := +( +t[1]^8 +10*t[1]^6*t[2] +20*t[1]^5*t[3] +15*t[1]^4*t[2]^2
+30*t[1]^4*t[4] +20*t[1]^3*t[2]*t[3] +24*t[1]^3*t[5])/120 :
Ze[9,43] := +( +t[1]^8 +6*t[1]^6*t[2] +8*t[1]^5*t[3] +4*t[1]^4*t[2]^2
+6*t[1]^4*t[4] +6*t[1]^2*t[2]^3 +8*t[1]*t[2]^2*t[3] +3*t[2]^4
+6*t[2]^2*t[4])/48 :
Ze[9,44] := +( +t[1]^8 +9*t[1]^6*t[2] +21*t[1]^4*t[2]^2 +10*t[1]^5*t[3]
+36*t[1]^3*t[2]*t[3] +16*t[1]^2*t[3]^2 +9*t[1]^2*t[2]^3 +6*t[1]^4*t[4]
+18*t[1]^2*t[2]*t[4] +6*t[1]*t[2]^2*t[3] +12*t[1]*t[3]*t[4])/144 :
Ze[9,45] := +( +t[1]^8 +11*t[1]^6*t[2] +20*t[1]^5*t[3] +25*t[1]^4*t[2]^2
+30*t[1]^4*t[4] +40*t[1]^3*t[2]*t[3] +15*t[1]^2*t[2]^3
+30*t[1]^2*t[2]*t[4] +24*t[1]^3*t[5] +20*t[1]*t[2]^2*t[3]
+24*t[1]*t[2]*t[5])/240 :
Ze[9,46] := +( +t[1]^8 +15*t[1]^6*t[2] +40*t[1]^5*t[3] +45*t[1]^4*t[2]^2
+90*t[1]^4*t[4] +120*t[1]^3*t[2]*t[3] +144*t[1]^3*t[5] +15*t[1]^2*t[2]^3
+90*t[1]^2*t[2]*t[4] +40*t[1]^2*t[3]^2 +120*t[1]^2*t[6])/720 :
Ze[9,47] := +( +t[1]^8 +28*t[1]^6*t[2] +112*t[1]^5*t[3] +210*t[1]^4*t[2]^2
+420*t[1]^4*t[4] +1120*t[1]^3*t[2]*t[3] +1344*t[1]^3*t[5]
+420*t[1]^2*t[2]^3 +2520*t[1]^2*t[2]*t[4] +1120*t[1]^2*t[3]^2
+3360*t[1]^2*t[6] +1680*t[1]*t[2]^2*t[3] +4032*t[1]*t[2]*t[5]
+3360*t[1]*t[3]*t[4] +5760*t[1]*t[7] +105*t[2]^4 +1260*t[2]^2*t[4]
+1120*t[2]*t[3]^2 +3360*t[2]*t[6] +2688*t[3]*t[5] +1260*t[4]^2
+5040*t[8])/40320 :
Z4 := +Ze[9,1] +Ze[9,2] +Ze[9,3] +Ze[9,4] +Ze[9,5] +Ze[9,6] +Ze[9,7]
+Ze[9,8] +Ze[9,9] +Ze[9,10] +Ze[9,11] +Ze[9,12] +Ze[9,13] +Ze[9,14]
+Ze[9,15] +Ze[9,16] +Ze[9,17] +Ze[9,18] +Ze[9,19] +Ze[9,20] +Ze[9,21]
+Ze[9,22] +Ze[9,23] +Ze[9,24] +Ze[9,25] +Ze[9,26] +Ze[9,27] +Ze[9,30]
+Ze[9,31] +Ze[9,32] +Ze[9,33] +Ze[9,34] +Ze[9,35] +Ze[9,36] +Ze[9,37]:
Ze[10,1] := +( +t[1]^9 +t[1]*t[2]^4)/2 :
Ze[10,2] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,3] := +( +t[1]^9)/1 :
Ze[10,4] := +( +t[1]^9)/1 :
Ze[10,5] := +( +t[1]^9 +t[1]*t[2]^4)/2 :
Ze[10,6] := +( +t[1]^9 +2*t[1]^7*t[2] +2*t[1]*t[2]^4 +2*t[1]*t[2]^2*t[4]
+t[1]^5*t[2]^2)/8 :
Ze[10,7] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,8] := +( +t[1]^9 +t[1]^5*t[2]^2)/2 :
Ze[10,9] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,10] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,11] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,12] := +( +t[1]^9)/1 :
Ze[10,13] := +( +t[1]^9 +t[1]*t[2]^4)/2 :
Ze[10,14] := +( +t[1]^9)/1 :
Ze[10,15] := +( +t[1]^9)/1 :
Ze[10,16] := +( +t[1]^9 +t[1]*t[2]^4)/2 :
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Ze[10,17] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Ze[10,18] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,19] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,20] := +( +t[1]^9 +3*t[1]^3*t[2]^3 +2*t[3]^3)/6 :
Ze[10,21] := +( +t[1]^9 +t[1]^5*t[2]^2 +t[1]^7*t[2] +t[1]^3*t[2]^3)/4 :
Ze[10,22] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Ze[10,23] := +( +t[1]^9 +2*t[1]^7*t[2] +2*t[1]*t[2]^4 +2*t[1]*t[2]^2*t[4]
+t[1]^5*t[2]^2)/8 :
Ze[10,24] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,25] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,26] := +( +t[1]^9 +t[1]^5*t[2]^2)/2 :
Ze[10,27] := +( +t[1]^9 +t[1]^7*t[2] +t[1]^3*t[2]^3 +t[1]*t[2]^4)/4 :
Ze[10,28] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,29] := +( +t[1]^9 +t[1]^5*t[2]^2)/2 :
Ze[10,30] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,31] := +( +t[1]^9)/1 :
Ze[10,32] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,33] := +( +t[1]^9 +t[1]*t[2]^4)/2 :
Ze[10,34] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
Ze[10,35] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Ze[10,36] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Ze[10,37] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Ze[10,38] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Ze[10,39] := +( +t[1]^9 +t[1]^5*t[2]^2)/2 :
Ze[10,40] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Ze[10,41] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Ze[10,42] := +( +t[1]^9 +t[1]^3*t[2]^3)/2 :
Ze[10,43] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,44] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,45] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,46] := +( +t[1]^9 +6*t[1]^7*t[2] +8*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+6*t[1]^5*t[4])/24 :
Ze[10,47] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Ze[10,48] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3] +t[1]^3*t[2]^3
+3*t[1]*t[2]^4 +2*t[2]^3*t[3])/12 :
Ze[10,49] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Ze[10,50] := +( +t[1]^9 +2*t[1]^5*t[2]^2 +3*t[1]*t[2]^4 +2*t[1]*t[4]^2)/8 :
Ze[10,51] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2 +2*t[1]^3*t[2]^3
+2*t[1]^3*t[2]*t[4])/8 :
Ze[10,52] := +( +t[1]^9 +t[1]^5*t[2]^2 +t[1]^7*t[2] +t[1]^3*t[2]^3)/4 :
Ze[10,53] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,54] := +( +t[1]^9 +2*t[1]^7*t[2] +2*t[1]*t[2]^4 +2*t[1]*t[2]^2*t[4]
+t[1]^5*t[2]^2)/8 :
Ze[10,55] := +( +t[1]^9 +6*t[1]^7*t[2] +9*t[1]^5*t[2]^2 +4*t[1]^6*t[3]
+12*t[1]^4*t[2]*t[3] +6*t[1]*t[2]^4 +18*t[1]*t[2]^2*t[4]
+12*t[1]*t[2]*t[6] +4*t[1]^3*t[3]^2)/72 :
Ze[10,56] := +( +t[1]^9 +3*t[1]^7*t[2] +t[1]^5*t[2]^2 +3*t[1]^3*t[2]^3
+2*t[1]^6*t[3] +2*t[1]^2*t[2]^2*t[3])/12 :
Ze[10,57] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
Ze[10,58] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
```

```
Ze[10,59] := +( +t[1]^9 +t[1]^5*t[2]^2 +t[1]^7*t[2] +t[1]^3*t[2]^3)/4 :
Ze[10,60] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Ze[10,61] := +( +t[1]^9 +3*t[1]^5*t[2]^2 +2*t[1]^3*t[3]^2)/6 :
Ze[10,62] := +( +t[1]^9 +3*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +t[1]^3*t[2]^3)/8 :
Ze[10,63] := +( +t[1]^9 +t[1]^7*t[2])/2 :
Ze[10,64] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
Ze[10,65] := +( +t[1]^9 +t[1]^7*t[2] +t[1]^5*t[2]^2 +t[1]^3*t[2]^3)/4 :
Ze[10,66] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Ze[10,67] := +( +t[1]^9 +t[1]^5*t[2]^2)/2 :
Ze[10,68] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Ze[10,69] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2)/4 :
Ze[10,70] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
Ze[10,71] := +( +t[1]^9 +2*t[1]^7*t[2] +t[1]^5*t[2]^2 +2*t[1]*t[2]^4
+2*t[1]*t[2]^2*t[4])/8 :
Ze[10,72] := +( +t[1]^9 +7*t[1]^7*t[2] +8*t[1]^6*t[3] +9*t[1]^5*t[2]^2
+8*t[1]^4*t[2]*t[3] +6*t[1]^5*t[4] +3*t[1]^3*t[2]^3
+6*t[1]^3*t[2]*t[4])/48 :
Ze[10,73] := +( +t[1]^9 +6*t[1]^7*t[2] +8*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+6*t[1]^5*t[4])/24 :
Ze[10,74] := +( +t[1]^9 +6*t[1]^7*t[2] +8*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+6*t[1]^5*t[4])/24 :
Ze[10,75] := +( +t[1]^9 +4*t[1]^7*t[2] +2*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+2*t[1]^4*t[2]*t[3])/12 :
Ze[10,76] := +( +t[1]^9 +t[1]^7*t[2] +t[1]^5*t[2]^2 +t[1]^3*t[2]^3)/4 :
Ze[10,77] := +( +t[1]^9 +4*t[1]^7*t[2] +2*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+2*t[1]^4*t[2]*t[3])/12 :
Ze[10,78] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3])/6 :
Ze[10,79] := +( +t[1]^9 +10*t[1]^7*t[2] +20*t[1]^6*t[3] +15*t[1]^5*t[2]^2
+30*t[1]^5*t[4] +20*t[1]^4*t[2]*t[3] +24*t[1]^4*t[5])/120 :
Ze[10,80] := +( +t[1]^9 +6*t[1]^7*t[2] +8*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+6*t[1]^5*t[4])/24 :
Ze[10,81] := +( +t[1]^9 +3*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +7*t[1]^3*t[2]^3
+6*t[1]^3*t[2]*t[4] +6*t[1]*t[2]^4 +6*t[1]*t[2]^2*t[4] +8*t[3]^3
+8*t[3]*t[6])/48 :
Ze[10,82] := +( +t[1]^9 +4*t[1]^7*t[2] +3*t[1]^5*t[2]^2 +2*t[1]^6*t[3]
+2*t[1]^4*t[2]*t[3])/12 :
Ze[10,83] := +( +t[1]^9 +3*t[1]^5*t[2]^2 +2*t[1]^3*t[3]^2 +t[1]^7*t[2]
+3*t[1]^3*t[2]^3 +2*t[1]*t[2]*t[3]^2)/12 :
Ze[10,84] := +( +t[1]^9 +2*t[1]^7*t[2] +2*t[1]^3*t[2]^3
+2*t[1]^3*t[2]*t[4] +t[1]^5*t[2]^2)/8 :
Ze[10,85] := +( +t[1]^9 +6*t[1]^7*t[2] +9*t[1]^5*t[2]^2 +4*t[1]^6*t[3]
+12*t[1]^4*t[2]*t[3] +6*t[1]*t[2]^4 +18*t[1]*t[2]^2*t[4]
+12*t[1]*t[2]*t[6] +4*t[1]^3*t[3]^2)/72 :
Ze[10,86] := +( +t[1]^9 +3*t[1]^7*t[2] +t[1]^5*t[2]^2 +3*t[1]^3*t[2]^3
+2*t[1]^6*t[3] +2*t[1]^2*t[2]^2*t[3])/12 :
Ze[10,87] := +( +t[1]^9 +5*t[1]^7*t[2] +7*t[1]^5*t[2]^2 +3*t[1]^3*t[2]^3
+2*t[1]^6*t[3] +4*t[1]^4*t[2]*t[3] +2*t[1]^2*t[2]^2*t[3])/24 :
Ze[10,88] := +( +t[1]^9 +9*t[1]^7*t[2] +21*t[1]^5*t[2]^2 +10*t[1]^6*t[3]
+36*t[1]^4*t[2]*t[3] +16*t[1]^3*t[3]^2 +9*t[1]^3*t[2]^3 +6*t[1]^5*t[4]
+18*t[1]^3*t[2]*t[4] +6*t[1]^2*t[2]^2*t[3] +12*t[1]^2*t[3]*t[4])/144 :
Ze[10,89] := +( +t[1]^9 +6*t[1]^7*t[2] +8*t[1]^6*t[3] +4*t[1]^5*t[2]^2
```

```
+6*t[1]^3*t[2]^3 +8*t[1]^2*t[2]^2*t[3] +6*t[1]^5*t[4] +3*t[1]*t[2]^4
+6*t[1]*t[2]^2*t[4])/48 :
Ze[10,90] := +( +t[1]^9 +6*t[1]^5*t[2]^2 +8*t[1]^3*t[3]^2 +3*t[1]*t[2]^4
+6*t[1]*t[4]^2)/24 :
Ze[10,91] := +( +t[1]^9 +7*t[1]^7*t[2] +8*t[1]^6*t[3] +9*t[1]^5*t[2]^2
+8*t[1]^4*t[2]*t[3] +6*t[1]^5*t[4] +3*t[1]^3*t[2]^3
+6*t[1]^3*t[2]*t[4])/48 :
Ze[10,92] := +( +t[1]^9 +2*t[1]^7*t[2] +2*t[1]^5*t[2]^2 +2*t[1]^3*t[2]^3
+t[1]*t[2]^4)/8 :
Ze[10,93] := +( +t[1]^9 +5*t[1]^7*t[2] +2*t[1]^6*t[3] +7*t[1]^5*t[2]^2
+4*t[1]^4*t[2]*t[3] +5*t[1]^3*t[2]^3 +6*t[1]*t[2]^4 +4*t[2]^3*t[3]
+2*t[1]^3*t[2]*t[4] +6*t[1]*t[2]^2*t[4] +4*t[2]*t[3]*t[4]
+2*t[1]^2*t[2]^2*t[3])/48 :
Ze[10,94] := +( +t[1]^9 +7*t[1]^7*t[2] +9*t[1]^5*t[2]^2 +8*t[1]^6*t[3]
+8*t[1]^4*t[2]*t[3] +3*t[1]^3*t[2]^3 +6*t[1]^5*t[4]
+6*t[1]^3*t[2]*t[4])/48 :
Ze[10,95] := +( +t[1]^9 +6*t[1]^7*t[2] +4*t[1]^6*t[3] +9*t[1]^5*t[2]^2
+12*t[1]^4*t[2]*t[3] +4*t[1]^3*t[3]^2)/36 :
Ze[10,96] := +( +t[1]^9 +11*t[1]^7*t[2] +20*t[1]^6*t[3] +25*t[1]^5*t[2]^2
+30*t[1]^5*t[4] +40*t[1]^4*t[2]*t[3] +15*t[1]^3*t[2]^3
+30*t[1]^3*t[2]*t[4] +24*t[1]^4*t[5] +20*t[1]^2*t[2]^2*t[3]
+24*t[1]^2*t[2]*t[5])/240 :
Ze[10,97] := +( +t[1]^9 +10*t[1]^7*t[2] +20*t[1]^6*t[3] +15*t[1]^5*t[2]^2
+30*t[1]^5*t[4] +20*t[1]^4*t[2]*t[3] +24*t[1]^4*t[5])/120 :
Ze[10,98] := +( +t[1]^9 +3*t[1]^7*t[2] +2*t[1]^6*t[3] +3*t[1]^5*t[2]^2
+9*t[1]^3*t[2]^3 +6*t[1]^2*t[2]^2*t[3] +2*t[1]^3*t[3]^2
+6*t[1]*t[2]*t[3]^2 +4*t[3]^3)/36 :
Ze[10,99] := +( +t[1]^9 +7*t[1]^7*t[2] +8*t[1]^6*t[3] +9*t[1]^5*t[2]^2
+6*t[1]^5*t[4] +8*t[1]^4*t[2]*t[3] +3*t[1]^3*t[2]^3
+6*t[1]^3*t[2]*t[4])/48 :
Ze[10,100] := +( +t[1]^9 +15*t[1]^7*t[2] +40*t[1]^6*t[3] +45*t[1]^5*t[2]^2
+90*t[1]^5*t[4] +120*t[1]^4*t[2]*t[3] +144*t[1]^4*t[5] +15*t[1]^3*t[2]^3
+90*t[1]^3*t[2]*t[4] +40*t[1]^3*t[3]^2 +120*t[1]^3*t[6])/720 :
Ze[10,101] := +( +t[1]^9 +10*t[1]^7*t[2] +20*t[1]^6*t[3] +16*t[1]^5*t[2]^2
+30*t[1]^5*t[4] +20*t[1]^4*t[2]*t[3] +24*t[1]^4*t[5] +10*t[1]^3*t[2]^3
+20*t[1]^2*t[2]^2*t[3] +15*t[1]*t[2]^4 +30*t[1]*t[2]^2*t[4]
+20*t[2]^3*t[3] +24*t[2]^2*t[5])/240 :
Ze[10,102] := +( +t[1]^9 +12*t[1]^7*t[2] +16*t[1]^6*t[3] +42*t[1]^5*t[2]^2
+96*t[1]^4*t[2]*t[3] +64*t[1]^3*t[3]^2 +12*t[1]^5*t[4] +36*t[1]^3*t[2]^3
+72*t[1]^3*t[2]*t[4] +48*t[1]^2*t[2]^2*t[3] +96*t[1]^2*t[3]*t[4]
+33*t[1]*t[2]^4 +180*t[1]*t[2]^2*t[4] +192*t[1]*t[2]*t[6] +108*t[1]*t[4]^2
+144*t[1]*t[8])/1152 :
Ze[10,103] := +( +t[1]^9 +13*t[1]^7*t[2] +45*t[1]^5*t[2]^2 +22*t[1]^6*t[3]
+100*t[1]^4*t[2]*t[3] +45*t[1]^3*t[2]^3 +30*t[1]^5*t[4]
+90*t[1]^3*t[2]*t[4] +40*t[1]^3*t[3]^2 +90*t[1]^2*t[2]^2*t[3]
+60*t[1]^2*t[3]*t[4] +24*t[1]^4*t[5] +72*t[1]^2*t[2]*t[5]
+40*t[1]*t[2]*t[3]^2 +48*t[1]*t[3]*t[5])/720 :
Ze[10,104] := +( +t[1]^9 +16*t[1]^7*t[2] +40*t[1]^6*t[3] +60*t[1]^5*t[2]^2
+90*t[1]^5*t[4] +160*t[1]^4*t[2]*t[3] +144*t[1]^4*t[5] +60*t[1]^3*t[2]^3
+180*t[1]^3*t[2]*t[4] +120*t[1]^2*t[2]^2*t[3] +144*t[1]^2*t[2]*t[5]
+40*t[1]^3*t[3]^2 +120*t[1]^3*t[6] +15*t[1]*t[2]^4 +90*t[1]*t[2]^2*t[4]
+40*t[1]*t[2]*t[3]^2 +120*t[1]*t[2]*t[6])/1440 :
Ze[10,105] := +( +t[1]^9 +21*t[1]^7*t[2] +70*t[1]^6*t[3]
```

```
+105*t[1]^5*t[2]^2 +210*t[1]^5*t[4] +420*t[1]^4*t[2]*t[3] +504*t[1]^4*t[5]
+105*t[1]^3*t[2]^3 +630*t[1]^3*t[2]*t[4] +280*t[1]^3*t[3]^2
+840*t[1]^3*t[6] +210*t[1]^2*t[2]^2*t[3] +504*t[1]^2*t[2]*t[5]
+420*t[1]^2*t[3]*t[4] +720*t[1]^2*t[7])/5040 :
Ze[10,106] := +( +t[1]^9 +36*t[1]^7*t[2] +168*t[1]^6*t[3]
+378*t[1]^5*t[2]^2 +756*t[1]^5*t[4] +2520*t[1]^4*t[2]*t[3]
+3024*t[1]^4*t[5] +1260*t[1]^3*t[2]^3 +7560*t[1]^3*t[2]*t[4]
+3360*t[1]^3*t[3]^2 +10080*t[1]^3*t[6] +7560*t[1]^2*t[2]^2*t[3]
+18144*t[1]^2*t[2]*t[5] +15120*t[1]^2*t[3]*t[4] +25920*t[1]^2*t[7]
+945*t[1]*t[2]^4 +11340*t[1]*t[2]^2*t[4] +10080*t[1]*t[2]*t[3]^2
+30240*t[1]*t[2]*t[6] +24192*t[1]*t[3]*t[5] +11340*t[1]*t[4]^2
+45360*t[1]*t[8] +2520*t[2]^3*t[3] +9072*t[2]^2*t[5] +15120*t[2]*t[3]*t[4]
+25920*t[2]*t[7] +2240*t[3]^3 +20160*t[3]*t[6] +18144*t[4]*t[5]
+40320*t[9])/362880 :
Z4 := +Ze[10,1] +Ze[10,2] +Ze[10,3] +Ze[10,4] +Ze[10,5] +Ze[10,6]
+Ze[10,7] +Ze[10,8] +Ze[10,9] +Ze[10,10] +Ze[10,11] +Ze[10,12] +Ze[10,13]
+Ze[10,14] +Ze[10,15] +Ze[10,16] +Ze[10,17] +Ze[10,18] +Ze[10,19]
+Ze[10,20] +Ze[10,21] +Ze[10,22] +Ze[10,23] +Ze[10,24] +Ze[10,25]
+Ze[10,26] +Ze[10,27] +Ze[10,28] +Ze[10,29] +Ze[10,30] +Ze[10,31]
+Ze[10,32] +Ze[10,33] +Ze[10,34] +Ze[10,35] +Ze[10,36] +Ze[10,37]
+Ze[10,38] +Ze[10,39] +Ze[10,40] +Ze[10,41] +Ze[10,42] +Ze[10,43]
+Ze[10,44] +Ze[10,45] +Ze[10,49] +Ze[10,50] +Ze[10,51] +Ze[10,52]
+Ze[10,53] +Ze[10,54] +Ze[10,55] +Ze[10,56] +Ze[10,57] +Ze[10,58]
+Ze[10,59] +Ze[10,60] +Ze[10,61] +Ze[10,62] +Ze[10,63] +Ze[10,64]
+Ze[10,65] +Ze[10,66] +Ze[10,67] +Ze[10,68] +Ze[10,69] +Ze[10,70]
+Ze[10,71] +Ze[10,81] +Ze[10,82] +Ze[10,83] +Ze[10,84] +Ze[10,85]
+Ze[10,86] +Ze[10,87]:
```

Appendix C. Generating Functions

C.1. **Vertex labeled.** The generating functions for the number of labeled trees with `N` vertices and shape `i` are listed here, obtained from the cycle indices of Section B.1 by the substitution $t[j] -> x^j/(1-x^j)$. For each `[N,i]` the generating function is a rational polynomial. The first 30 integers obtained from the expansion are attached.

At the end of each group of `N` there is another, unnamed generating function which is the sum over all shapes `t` of the trees with `N` vertices, also followed by its expansion which enters Table 1. Furthermore a generating function named `Zv4_` is given which sums over trees with maximum degree 4.

```
Vertex labeled
Zv[1,1] := -x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]

-x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]

Zv4_1 := -x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]
```

```
Zv[2,1] := x^2 / (-1+x)^2 / (1+x)
[0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 10, 10, 11, 11, 12
, 12, 13, 13, 14, 14]

x^2 / (-1+x)^2 / (1+x)
[0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 10, 10, 11, 11, 12
, 12, 13, 13, 14, 14]

Zv4_2 := x^2 / (-1+x)^2 / (1+x)
[0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 10, 10, 11, 11, 12
, 12, 13, 13, 14, 14]


Zv[3,1] := -x^3 / (-1+x)^3 / (1+x)
[0, 0, 0, 1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49, 56, 64, 72, 81, 90,
100, 110, 121, 132, 144, 156, 169, 182, 196]

-x^3 / (-1+x)^3 / (1+x)
[0, 0, 0, 1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49, 56, 64, 72, 81, 90,
100, 110, 121, 132, 144, 156, 169, 182, 196]

Zv4_3 := -x^3 / (-1+x)^3 / (1+x)
[0, 0, 0, 1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49, 56, 64, 72, 81, 90,
100, 110, 121, 132, 144, 156, 169, 182, 196]


Zv[4,1] := (1+x^2)*x^4 / (-1+x)^4 / (1+x)^2
[0, 0, 0, 0, 1, 2, 6, 10, 19, 28, 44, 60, 85, 110, 146, 182, 231, 280, 344, 408
, 489, 570, 670, 770, 891, 1012, 1156, 1300, 1469, 1638]

Zv[4,2] := x^4 / (-1+x)^4 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 1, 2, 4, 7, 11, 16, 23, 31, 41, 53, 67, 83, 102, 123, 147, 174,
204, 237, 274, 314, 358, 406, 458, 514, 575, 640]

x^4*(2*x^2 +2*x +2+x^4+x^3) / (-1+x)^4 / (1+x)^2 / (x^2+x +1)
[0, 0, 0, 0, 2, 4, 10, 17, 30, 44, 67, 91, 126, 163, 213, 265, 333, 403, 491,
582, 693, 807, 944, 1084, 1249, 1418, 1614, 1814, 2044, 2278]

Zv4_4 := x^4*(2*x^2 +2*x +2+x^4+x^3) / (-1+x)^4 / (1+x)^2 / (x^2+x +1)
[0, 0, 0, 0, 2, 4, 10, 17, 30, 44, 67, 91, 126, 163, 213, 265, 333, 403, 491,
582, 693, 807, 944, 1084, 1249, 1418, 1614, 1814, 2044, 2278]


Zv[5,1] := -(1+x^2)*x^5 / (-1+x)^5 / (1+x)^2
[0, 0, 0, 0, 0, 1, 3, 9, 19, 38, 66, 110, 170, 255, 365, 511, 693, 924, 1204,
1548, 1956, 2445, 3015, 3685, 4455, 5346, 6358, 7514, 8814, 10283]

Zv[5,2] := -x^5 / (-1+x)^5 / (1+x)
[0, 0, 0, 0, 0, 1, 4, 11, 24, 46, 80, 130, 200, 295, 420, 581, 784, 1036, 1344,
1716, 2160, 2685, 3300, 4015, 4840, 5786, 6864, 8086, 9464, 11011]

Zv[5,3] := -x^5 / (-1+x)^5 / (1+x)^2 / (x^2+x +1) / (1+x^2)
```

```
[0, 0, 0, 0, 0, 1, 2, 4, 7, 12, 18, 27, 38, 53, 71, 94, 121, 155, 194, 241, 295
, 359, 431, 515, 609, 717, 837, 973, 1123, 1292]

-x^5*(6*x^2 +3*x +3 +5*x^4 +5*x^3+x^6 +2*x^5) / (-1+x)^5 / (1+x)^2 / (x^2+x +1) / (1+x^2)
[0, 0, 0, 0, 0, 3, 9, 24, 50, 96, 164, 267, 408, 603, 856, 1186, 1598, 2115,
2742, 3505, 4411, 5489, 6746, 8215, 9904, 11849, 14059, 16573, 19401, 22586]

Zv4_5 := -x^5*(6*x^2 +3*x +3 +5*x^4 +5*x^3+x^6 +2*x^5) / (-1+x)^5 / (1+x)^2 / (x^2+x +1) / (1
+x^2)
[0, 0, 0, 0, 0, 3, 9, 24, 50, 96, 164, 267, 408, 603, 856, 1186, 1598, 2115,
2742, 3505, 4411, 5489, 6746, 8215, 9904, 11849, 14059, 16573, 19401, 22586]


Zv[6,1] := (3*x^2 +1)*x^6 / (-1+x)^6 / (1+x)^3
[0, 0, 0, 0, 0, 0, 1, 3, 12, 28, 66, 126, 236, 396, 651, 1001, 1512, 2184, 3108
, 4284, 5832, 7752, 10197, 13167, 16852, 21252, 26598, 32890, 40404, 49140]

Zv[6,2] := x^6 / (-1+x)^6 / (1+x)
[0, 0, 0, 0, 0, 0, 1, 5, 16, 40, 86, 166, 296, 496, 791, 1211, 1792, 2576, 3612
, 4956, 6672, 8832, 11517, 14817, 18832, 23672, 29458, 36322, 44408, 53872]

Zv[6,3] := (1+x^2)*x^6 / (-1+x)^6 / (1+x)^2
[0, 0, 0, 0, 0, 0, 1, 4, 13, 32, 70, 136, 246, 416, 671, 1036, 1547, 2240, 3164
, 4368, 5916, 7872, 10317, 13332, 17017, 21472, 26818, 33176, 40690, 49504]

Zv[6,4] := x^6*(2*x^2-x +1) / (-1+x)^6 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 1, 2, 7, 14, 31, 54, 97, 154, 246, 364, 538, 756, 1058, 1428
, 1918, 2508, 3263, 4158, 5273, 6578, 8169, 10010, 12215, 14742]

Zv[6,5] := x^6 / (-1+x)^6 / (1+x) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 50, 91, 155, 250, 386, 575, 831, 1170, 1611,
2175, 2886, 3771, 4860, 6186, 7786, 9700, 11972, 14650, 17786, 21436]

Zv[6,6] := x^6 / (-1+x)^6 / (1+x)^2 / (x^2+x +1) / (1+x^2) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 1, 2, 4, 7, 12, 19, 29, 42, 60, 83, 113, 150, 197, 254, 324,
408, 509, 628, 769, 933, 1125, 1346, 1601, 1892]

x^6*(6 +14*x +31*x^2 +49*x^3 +69*x^4 +74*x^5 +72*x^6 +55*x^7 +37*x^8 +17*x^9 +7*x^10+x^11
) / (-1+x)^6 / (1+x)^3 / (1+x^2) / (x^2+x +1) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 6, 20, 63, 146, 315, 592, 1059, 1754, 2805, 4270, 6333, 9076
, 12750, 17465, 23548, 31143, 40663, 52288, 66529, 83607, 104140, 128394,
157104, 190586]

Zv4_6 := x^6*(5 +8*x +18*x^2 +18*x^3 +20*x^4 +10*x^5 +6*x^6+x^7) / (x^2+x +1) / (-1+x)^6 / (
1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 5, 18, 59, 139, 303, 573, 1030, 1712, 2745, 4187, 6220, 8926
, 12553, 17211, 23224, 30735, 40154, 51660, 65760, 82674, 103015, 127048,
155503, 188694]


Zv[7,1] := -(3*x^2 +1)*x^7 / (-1+x)^7 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 1, 4, 16, 44, 110, 236, 472, 868, 1519, 2520, 4032, 6216,
9324, 13608, 19440, 27192, 37389, 50556, 67408, 88660, 115258, 148148, 188552]
```

```
Zv[7,2] := -x^7 / (-1+x)^7 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 62, 148, 314, 610, 1106, 1897, 3108, 4900, 7476
, 11088, 16044, 22716, 31548, 43065, 57882, 76714, 100386, 129844, 166166,
210574]

Zv[7,3] := -x^7 / (-1+x)^7
[0, 0, 0, 0, 0, 0, 0, 1, 7, 28, 84, 210, 462, 924, 1716, 3003, 5005, 8008,
12376, 18564, 27132, 38760, 54264, 74613, 100947, 134596, 177100, 230230,
296010, 376740]

Zv[7,4] := -(x^6+x^4 +2*x^3+x^2 +1)*x^7 / (-1+x)^7 / (x^2+x +1)^2 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 1, 3, 9, 23, 51, 103, 196, 348, 590, 960, 1506, 2290,
3393, 4905, 6945, 9651, 13185, 17739, 23542, 30846, 39954, 51206, 64986]

Zv[7,5] := -x^7*(2*x^2-x +1) / (-1+x)^7 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 24, 55, 109, 206, 360, 606, 970, 1508, 2264,
3322, 4750, 6668, 9176, 12439, 16597, 21870, 28448, 36617, 46627, 58842]

Zv[7,6] := -x^7 / (-1+x)^7 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 62, 148, 314, 610, 1106, 1897, 3108, 4900, 7476
, 11088, 16044, 22716, 31548, 43065, 57882, 76714, 100386, 129844, 166166,
210574]

Zv[7,7] := -x^7 / (-1+x)^7 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 91, 182, 337, 587, 973, 1548, 2379, 3549,
5160, 7335, 10221, 13992, 18852, 25038, 32824, 42524, 54496, 69146, 86932]

Zv[7,8] := -(1+x^2)*x^7 / (-1+x)^7 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 36, 84, 172, 330, 588, 1001, 1624, 2548, 3864,
5712, 8232, 11628, 16104, 21945, 29436, 38962, 50908, 65780, 84084, 106470]

Zv[7,9] := -x^7 / (-1+x)^7 / (1+x)^2 / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 29, 62, 120, 217, 370, 603, 945, 1434, 2115,
3045, 4290, 5931, 8061, 10791, 14247, 18577, 23947, 30549, 38597, 48335]

Zv[7,10] := -x^7 / (-1+x)^7 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 51, 95, 166, 275, 437, 670, 997, 1445, 2048
, 2845, 3883, 5216, 6908, 9031, 11669, 14916, 18880, 23681, 29455]

Zv[7,11] := -x^7 / (-1+x)^7 / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1) / (1+x^2) / (x^4+x^3+x^2+x+
1)
[0, 0, 0, 0, 0, 0, 0, 1, 2, 4, 7, 12, 19, 30, 44, 64, 90, 125, 169, 227, 298,
388, 498, 634, 797, 996, 1231, 1513, 1844, 2235]

-x^7*(11 +26*x +68*x^2 +120*x^3 +196*x^4 +257*x^5 +320*x^6 +332*x^7 +327*x^8 +272*x^9+
211*x^10 +134*x^11 +80*x^12 +33*x^13 +12*x^14 +2*x^15) / (-1+x)^7 / (1+x)^3 / (x^2+x +1)^2 /
(x^2-x +1) / (1+x^2) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 11, 48, 164, 437, 1022, 2126, 4098, 7368, 12590, 20548,
32337, 49240, 72971, 105483, 149296, 207250, 282886, 380152, 503872, 659352,
852965, 1091675, 1383695]

Zv4_7 := -x^7*(9 +24*x +56*x^2 +83*x^3 +102*x^4 +89*x^5 +66*x^6 +31*x^7 +12*x^8 +2*x^9) /
```

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(-1+x)^7 / (1+x)^3 / (x^2+x +1)^2 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 9, 42, 149, 405, 959, 2012, 3902, 7049, 12089, 19788,
31215, 47626, 70696, 102340, 145025, 201536, 275344, 370324, 491207, 643205,
832572, 1066150, 1352005]


Zv[8,1] := (x^4 +6*x^2 +1)*x^8 / (-1+x)^8 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 20, 60, 170, 396, 868, 1716, 3235, 5720, 9752,
15912, 25236, 38760, 58200, 85272, 122661, 173052, 240460, 328900, 444158,
592020]

Zv[8,2] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Zv[8,3] := x^8 / (-1+x)^8
[0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 36, 120, 330, 792, 1716, 3432, 6435, 11440,
19448, 31824, 50388, 77520, 116280, 170544, 245157, 346104, 480700, 657800,
888030, 1184040]

Zv[8,4] := (3*x^2 +1)*x^8 / (-1+x)^8 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 21, 65, 175, 411, 883, 1751, 3270, 5790, 9822,
16038, 25362, 38970, 58410, 85602, 122991, 173547, 240955, 329615, 444873,
593021]

Zv[8,5] := x^8*(x^4-x^3 +4*x^2-x +1) / (-1+x)^8 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 13, 34, 89, 191, 397, 744, 1350, 2298, 3806,
6036, 9358, 14058, 20726, 29832, 42271, 58773, 80643, 108966, 145583, 192049]

Zv[8,6] := (1+x^2)*x^8 / (-1+x)^8 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 24, 74, 194, 450, 952, 1870, 3459, 6084, 10256,
16668, 26244, 40188, 60048, 87780, 125829, 177210, 245608, 335478, 452166,
602030]

Zv[8,7] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Zv[8,8] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Zv[8,9] := (x^4 +6*x^2 +1)*x^8 / (-1+x)^8 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 20, 60, 170, 396, 868, 1716, 3235, 5720, 9752,
15912, 25236, 38760, 58200, 85272, 122661, 173052, 240460, 328900, 444158,
592020]

Zv[8,10] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 154, 336, 673, 1260, 2233, 3781, 6160,
9709, 14869, 22204, 32425, 46417, 65269, 90307, 123131, 165655, 220151, 289297]
```

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Zv[8,11] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Zv[8,12] := (1+x^2)*x^8 / (-1+x)^8 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 55, 139, 311, 641, 1229, 2230, 3854, 6402,
10266, 15978, 24210, 35838, 51942, 73887, 103323, 142285, 193193, 258973,
343057]

Zv[8,13] := x^8*(2*x^2-x +1) / (-1+x)^8 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 38, 93, 202, 408, 768, 1374, 2344, 3852,
6116, 9438, 14188, 20856, 30032, 42471, 59068, 80938, 109386, 146003, 192630]

Zv[8,14] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 108, 228, 445, 815, 1418, 2363, 3797,
5912, 8957, 13247, 19178, 27239, 38030, 52277, 70854, 94801, 125350, 163947]

Zv[8,15] := (x^6+x^4 +2*x^3+x^2 +1)*x^8 / (-1+x)^8 / (x^2+x +1)^2 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 13, 36, 87, 190, 386, 734, 1324, 2284, 3790,
6080, 9473, 14378, 21323, 30974, 44159, 61898, 85440, 116286, 156240, 207446]

Zv[8,16] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 154, 336, 673, 1260, 2233, 3781, 6160,
9709, 14869, 22204, 32425, 46417, 65269, 90307, 123131, 165655, 220151, 289297]

Zv[8,17] := x^8 / (-1+x)^8 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 68, 171, 382, 781, 1488, 2678, 4596, 7578,
12072, 18666, 28116, 41382, 59664, 84447, 117546, 161161, 217932, 291005,
384098]

Zv[8,18] := x^8 / (-1+x)^8 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 92, 187, 353, 628, 1065, 1735, 2732,
4177, 6225, 9070, 12953, 18169, 25077, 34108, 45777, 60693, 79573, 103254]

Zv[8,19] := (1+x^2)*x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 37, 88, 186, 367, 676, 1187, 1991, 3224,
5051, 7703, 11456, 16679, 23807, 33401, 46115, 62769, 84309, 111895, 146853]

Zv[8,20] := (x^6-x^5 +2*x^4 +2*x^2-x +1)*x^8 / (-1+x)^8 / (1+x)^4 / (x^2+x +1)^2 / (x^2-x +1
) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 7, 15, 35, 66, 127, 218, 373, 596, 943, 1425,
2130, 3080, 4406, 6149, 8496, 11514, 15464, 20449, 26822, 34736]

Zv[8,21] := x^8 / (-1+x)^8 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 29, 63, 124, 229, 399, 666, 1069, 1663, 2514
, 3711, 5359, 7594, 10575, 14502, 19606, 26171, 34522, 45051, 58203]

Zv[8,22] := x^8 / (-1+x)^8 / (x^4+x^3+x^2+x +1) / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 51, 96, 170, 286, 462, 721, 1093, 1615,
2334, 3307, 4604, 6309, 8523, 11365, 14976, 19520, 25189, 32204]
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Zv[8,23] := x^8 / (-1+x)^8 / (x^6+x^5+x^4+x^3+x^2+x +1) / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1
) / (1+x^2) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 4, 7, 12, 19, 30, 45, 66, 94, 132, 181, 246, 328
, 433, 564, 728, 929, 1177, 1477, 1841, 2277]

x^8*(23 +92*x +283*x^2 +649*x^3 +1281*x^4 +2162*x^5 +3305*x^6 +4545*x^7 +5773*x^8 +6731*
x^9 +7311*x^10 +7335*x^11 +6872*x^12 +5938*x^13 +4766*x^14 +3493*x^15 +2348*x^16 +1402*
x^17 +746*x^18 +328*x^19 +119*x^20 +29*x^21 +5*x^22) / (-1+x)^8 / (1+x)^4 / (1+x^2) / (x^2+x
 +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1) / (x^6+x^5+x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 23, 115, 444, 1300, 3331, 7511, 15619, 30111, 54957,
95357, 159058, 255817, 399375, 606531, 899952, 1306744, 1862273, 2608073,
3596928, 4889909, 6562960, 8702891]

Zv4_8 := x^8*(18 +42*x +117*x^2 +168*x^3 +259*x^4 +263*x^5 +305*x^6 +240*x^7 +208*x^8+
118*x^9 +66*x^10 +19*x^11 +5*x^12) / (-1+x)^8 / (1+x)^4 / (x^2+x +1)^2 / (x^2-x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 18, 96, 387, 1161, 3025, 6899, 14470, 28077, 51511,
89747, 150214, 242279, 379156, 577011, 857689, 1247320, 1780042, 2495950,
3446058, 4689388, 6299411, 8360100]


Zv[9,1] := -(x^4 +6*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 25, 85, 255, 651, 1519, 3235, 6470, 12190,
21942, 37854, 63090, 101850, 160050, 245322, 367983, 541035, 781495, 1110395,
1554553]

Zv[9,2] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Zv[9,3] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Zv[9,4] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Zv[9,5] := -x^9*(x^4-x^3 +4*x^2-x +1) / (-1+x)^9 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 17, 51, 140, 331, 728, 1472, 2822, 5120, 8926
, 14962, 24320, 38378, 59104, 88936, 131207, 189980, 270623, 379589, 525172]

Zv[9,6] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 299, 749, 1701, 3571, 7030, 13114,
23370, 40038, 66282, 106470, 166518, 254298, 380127, 557337, 802945, 1138423,
1590589]

Zv[9,7] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]
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Zv[9,8] := -(3*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 27, 92, 267, 678, 1561, 3312, 6582, 12372,
22194, 38232, 63594, 102564, 160974, 246576, 369567, 543114, 784069, 1113684,
1558557]

Zv[9,9] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Zv[9,10] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Zv[9,11] := -(x^4 +6*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 25, 85, 255, 651, 1519, 3235, 6470, 12190,
21942, 37854, 63090, 101850, 160050, 245322, 367983, 541035, 781495, 1110395,
1554553]

Zv[9,12] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Zv[9,13] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Zv[9,14] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Zv[9,15] := -(3*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 22, 70, 197, 481, 1080, 2232, 4350, 8022,
14172, 24060, 39534, 63030, 97944, 148632, 220935, 322179, 461890, 651794,
906763]

Zv[9,16] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 80, 219, 530, 1171, 2400, 4630, 8484,
14886, 25152, 41130, 65340, 101178, 153120, 227007, 330330, 472615, 665808,
924781]

Zv[9,17] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Zv[9,18] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
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17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Zv[9,19] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 299, 749, 1701, 3571, 7030, 13114,
23370, 40038, 66282, 106470, 166518, 254298, 380127, 557337, 802945, 1138423,
1590589]

Zv[9,20] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Zv[9,21] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Zv[9,22] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Zv[9,23] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 299, 749, 1701, 3571, 7030, 13114,
23370, 40038, 66282, 106470, 166518, 254298, 380127, 557337, 802945, 1138423,
1590589]

Zv[9,24] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Zv[9,25] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Zv[9,26] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Zv[9,27] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Zv[9,28] := -x^9 / (-1+x)^9 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 155, 342, 695, 1323, 2388, 4123, 6855
, 11032, 17257, 26327, 39280, 57449, 82526, 116634, 162411, 223104, 302677]

Zv[9,29] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
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14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Zv[9,30] := -x^9*(2*x^2-x +1) / (-1+x)^9 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 57, 150, 352, 760, 1528, 2902, 5246, 9098
, 15214, 24652, 38840, 59696, 89728, 132199, 191267, 272205, 381591, 527594]

Zv[9,31] := -x^9*(x^12+x^10 +2*x^9 +4*x^8 +2*x^7 +4*x^6 +2*x^5 +4*x^4 +2*x^3+x^2 +1) / (-\
1+x)^9 / (1+x)^4 / (1+x^2)^2 / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 9, 23, 56, 120, 247, 475, 879, 1551, 2651,
4375, 7036, 11010, 16859, 25261, 37172, 53728, 76479, 107251, 148449]

Zv[9,32] := -(1+x^2)*x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 56, 144, 330, 697, 1373, 2560, 4551, 7775
, 12826, 20529, 31985, 48664, 72471, 105872, 151987, 214756, 299065, 410960]

Zv[9,33] := -(x^6-x^5 +2*x^4 +2*x^2-x +1)*x^9 / (-1+x)^9 / (1+x)^4 / (x^2+x +1)^2 / (x^2-x+
1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 25, 60, 126, 253, 471, 844, 1440, 2383,
3808, 5938, 9018, 13424, 19573, 28069, 39583, 55047, 75496, 102318]

Zv[9,34] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 80, 219, 530, 1171, 2400, 4630, 8484,
14886, 25152, 41130, 65340, 101178, 153120, 227007, 330330, 472615, 665808,
924781]

Zv[9,35] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Zv[9,36] := -x^9*(2*x^2-x +1) / (-1+x)^9 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 15, 42, 108, 244, 516, 1012, 1890, 3356, 5742
, 9472, 15180, 23660, 36036, 53692, 78507, 112760, 159445, 222146, 305448]

Zv[9,37] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Zv[9,38] := -x^9 / (-1+x)^9 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 109, 233, 462, 861, 1527, 2596, 4259,
6773, 10484, 15843, 23437, 34012, 48514, 68120, 94291, 128813, 173864]

Zv[9,39] := -(x^6+x^4 +2*x^3+x^2 +1)*x^9 / (-1+x)^9 / (x^2+x +1)^2 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 40, 101, 230, 487, 964, 1811, 3248, 5601,
9328, 15074, 23706, 36397, 54680, 80556, 116578, 165996, 232864, 322236]

Zv[9,40] := -x^9 / (-1+x)^9 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 155, 342, 695, 1323, 2388, 4123, 6855
, 11032, 17257, 26327, 39280, 57449, 82526, 116634, 162411, 223104, 302677]

Zv[9,41] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]
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Zv[9,42] := -x^9 / (-1+x)^9 / (x^4+x^3+x^2+x +1) / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 92, 188, 358, 644, 1106, 1827, 2920,
4535, 6869, 10176, 14780, 21089, 29612, 40977, 55953, 75473, 100662]

Zv[9,43] := -x^9 / (-1+x)^9 / (1+x)^4 / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 37, 89, 190, 381, 713, 1276, 2181, 3605,
5764, 8979, 13637, 20284, 29571, 42380, 59752, 83053, 113880, 154275]

Zv[9,44] := -x^9 / (-1+x)^9 / (1+x)^3 / (1+x^2) / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 30, 67, 136, 259, 466, 802, 1328, 2129,
3316, 5039, 7488, 10910, 15614, 21990, 30516, 41785, 56512, 75567]

Zv[9,45] := -x^9 / (-1+x)^9 / (x^4+x^3+x^2+x +1) / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 29, 63, 125, 233, 411, 695, 1132, 1788,
2747, 4122, 6054, 8726, 12363, 17249, 23728, 32225, 43248, 57414]

Zv[9,46] := -x^9 / (-1+x)^9 / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1) / (1+x^2) / (x^4+x^3+x^2+x+
1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 51, 96, 171, 290, 473, 746, 1144,
1711, 2505, 3597, 5077, 7055, 9667, 13076, 17481, 23117, 30266]

Zv[9,47] := -x^9 / (-1+x)^9 / (1+x)^4 / (1+x^2)^2 / (1+x^4) / (x^6+x^5+x^4+x^3+x^2+x +1) / (
x^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 4, 7, 12, 19, 30, 45, 67, 96, 136, 188, 258,
347, 463, 609, 795, 1025, 1313, 1665, 2099]

-x^9*(47 +192*x +632*x^2 +1538*x^3 +3269*x^4 +5983*x^5 +10035*x^6 +15275*x^7 +21754*x^8
 +28780*x^9 +35999*x^10 +42297*x^11 +47248*x^12 +49831*x^13 +50065*x^14 +47555*x^15+
42999*x^16 +36659*x^17 +29646*x^18 +22447*x^19 +15997*x^20 +10525*x^21 +6419*x^22+
3509*x^23 +1724*x^24 +708*x^25 +242*x^26 +57*x^27 +9*x^28) / (-1+x)^9 / (1+x)^4 / (1+x^2)^
2 / (1+x^4) / (x^6+x^5+x^4+x^3+x^2+x +1) / (x^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 47, 286, 1204, 3899, 10781, 26294, 58485, 120499,
233524, 429496, 756045, 1280919, 2099701, 3342533, 5185483, 7860292, 11670354,
17004259, 24357547, 34350894, 47758861]

Zv4_9 := -x^9*(35 +89*x +267*x^2 +435*x^3 +742*x^4 +885*x^5 +1107*x^6 +1031*x^7 +1003*x
^8 +727*x^9 +534*x^10 +277*x^11 +137*x^12 +38*x^13 +9*x^14) / (-1+x)^9 / (1+x)^4 / (1+x^2)^
2 / (x^2+x +1)^2 / (x^2-x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 35, 229, 1008, 3357, 9464, 23406, 52609, 109279,
213160, 394121, 696821, 1184940, 1948478, 3110201, 4836416, 7346312, 10927151,
15947247, 22876671, 32304701, 44967210]


Zv[10,1] := (5*x^4 +10*x^2 +1)*x^10 / (-1+x)^10 / (1+x)^5
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 30, 110, 365, 1001, 2520, 5720, 12190,
24310, 46252, 83980, 147070, 248710, 408760, 653752, 1021735, 1562275, 2343770,
3453450]

Zv[10,2] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]
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Zv[10,3] := x^10 / (-1+x)^10
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 10, 55, 220, 715, 2002, 5005, 11440, 24310,
48620, 92378, 167960, 293930, 497420, 817190, 1307504, 2042975, 3124550,
4686825, 6906900]

Zv[10,4] := x^10 / (-1+x)^10
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 10, 55, 220, 715, 2002, 5005, 11440, 24310,
48620, 92378, 167960, 293930, 497420, 817190, 1307504, 2042975, 3124550,
4686825, 6906900]

Zv[10,5] := (x^4 +6*x^2 +1)*x^10 / (-1+x)^10 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 31, 116, 371, 1022, 2541, 5776, 12246,
24436, 46378, 84232, 147322, 249172, 409222, 654544, 1022527, 1563562, 2345057,
3455452]

Zv[10,6] := x^10*(4*x^4-3*x^3 +7*x^2-x +1) / (-1+x)^10 / (1+x)^5 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 21, 68, 208, 528, 1256, 2704, 5526, 10600,
19526, 34408, 58728, 96976, 156080, 244816, 376023, 565708, 836331, 1215500]

Zv[10,7] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,8] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 39, 144, 443, 1192, 2893, 6464, 13494,
26608, 49978, 90016, 156298, 262768, 429286, 683584, 1063711, 1621048, 2423993,
3562416]

Zv[10,9] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,10] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,11] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,12] := x^10 / (-1+x)^10
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 10, 55, 220, 715, 2002, 5005, 11440, 24310,
48620, 92378, 167960, 293930, 497420, 817190, 1307504, 2042975, 3124550,
4686825, 6906900]

Zv[10,13] := (5*x^4 +10*x^2 +1)*x^10 / (-1+x)^10 / (1+x)^5
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 30, 110, 365, 1001, 2520, 5720, 12190,
24310, 46252, 83980, 147070, 248710, 408760, 653752, 1021735, 1562275, 2343770,
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3453450]

Zv[10,14] := x^10 / (-1+x)^10
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 10, 55, 220, 715, 2002, 5005, 11440, 24310,
48620, 92378, 167960, 293930, 497420, 817190, 1307504, 2042975, 3124550,
4686825, 6906900]

Zv[10,15] := x^10 / (-1+x)^10
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 10, 55, 220, 715, 2002, 5005, 11440, 24310,
48620, 92378, 167960, 293930, 497420, 817190, 1307504, 2042975, 3124550,
4686825, 6906900]

Zv[10,16] := (5*x^4 +10*x^2 +1)*x^10 / (-1+x)^10 / (1+x)^5
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 30, 110, 365, 1001, 2520, 5720, 12190,
24310, 46252, 83980, 147070, 248710, 408760, 653752, 1021735, 1562275, 2343770,
3453450]

Zv[10,17] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 129, 375, 957, 2212, 4727, 9475, 18004
, 32693, 57091, 96358, 157829, 251725, 392038, 597620, 893509, 1312529, 1897204
]

Zv[10,18] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,19] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,20] := (3*x^7 +10*x^6 +6*x^5 +6*x^4 +7*x^3 +3*x^2 +1)*x^10 / (-1+x)^10 / (x^2+x +1)^
3 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 16, 54, 156, 405, 967, 2134, 4426, 8698,
16306, 29344, 50953, 85687, 140071, 223216, 347626, 530215, 793535, 1167194]

Zv[10,21] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 32, 112, 331, 861, 2032, 4432, 9062, 17546
, 32432, 57584, 98714, 164054, 265232, 418352, 645359, 975689, 1448304, 2114112
]

Zv[10,22] := x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 38, 136, 405, 1056, 2488, 5408, 11006,
21200, 38972, 68816, 117326, 193952, 311960, 489632, 751751, 1131416, 1672242,
2431000]

Zv[10,23] := x^10*(x^4-x^3 +4*x^2-x +1) / (-1+x)^10 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 22, 73, 213, 544, 1272, 2744, 5566, 10686,
19612, 34574, 58894, 97272, 156376, 245312, 376519, 566499, 837122, 1216711]

Zv[10,24] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
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32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,25] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,26] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 39, 144, 443, 1192, 2893, 6464, 13494,
26608, 49978, 90016, 156298, 262768, 429286, 683584, 1063711, 1621048, 2423993,
3562416]

Zv[10,27] := (3*x^2 +1)*x^10 / (-1+x)^10 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 28, 98, 295, 776, 1856, 4088, 8438, 16460,
30632, 54692, 94226, 157256, 255200, 403832, 624767, 946946, 1408836, 2060630]

Zv[10,28] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,29] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 39, 144, 443, 1192, 2893, 6464, 13494,
26608, 49978, 90016, 156298, 262768, 429286, 683584, 1063711, 1621048, 2423993,
3562416]

Zv[10,30] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,31] := x^10 / (-1+x)^10
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 10, 55, 220, 715, 2002, 5005, 11440, 24310,
48620, 92378, 167960, 293930, 497420, 817190, 1307504, 2042975, 3124550,
4686825, 6906900]

Zv[10,32] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,33] := (x^4 +6*x^2 +1)*x^10 / (-1+x)^10 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 31, 116, 371, 1022, 2541, 5776, 12246,
24436, 46378, 84232, 147322, 249172, 409222, 654544, 1022527, 1563562, 2345057,
3455452]

Zv[10,34] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 99, 276, 681, 1531, 3196, 6279, 11725,
20968, 36123, 60235, 97594, 154131, 237907, 359713, 533796, 778733, 1118471]

Zv[10,35] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 129, 375, 957, 2212, 4727, 9475, 18004
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, 32693, 57091, 96358, 157829, 251725, 392038, 597620, 893509, 1312529, 1897204
]

Zv[10,36] := x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 38, 136, 405, 1056, 2488, 5408, 11006,
21200, 38972, 68816, 117326, 193952, 311960, 489632, 751751, 1131416, 1672242,
2431000]

Zv[10,37] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 129, 375, 957, 2212, 4727, 9475, 18004
, 32693, 57091, 96358, 157829, 251725, 392038, 597620, 893509, 1312529, 1897204
]

Zv[10,38] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 129, 375, 957, 2212, 4727, 9475, 18004
, 32693, 57091, 96358, 157829, 251725, 392038, 597620, 893509, 1312529, 1897204
]

Zv[10,39] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 39, 144, 443, 1192, 2893, 6464, 13494,
26608, 49978, 90016, 156298, 262768, 429286, 683584, 1063711, 1621048, 2423993,
3562416]

Zv[10,40] := x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 38, 136, 405, 1056, 2488, 5408, 11006,
21200, 38972, 68816, 117326, 193952, 311960, 489632, 751751, 1131416, 1672242,
2431000]

Zv[10,41] := x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 38, 136, 405, 1056, 2488, 5408, 11006,
21200, 38972, 68816, 117326, 193952, 311960, 489632, 751751, 1131416, 1672242,
2431000]

Zv[10,42] := (3*x^2 +1)*x^10 / (-1+x)^10 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 34, 126, 393, 1071, 2632, 5944, 12526,
24898, 47092, 85324, 148918, 251482, 412456, 659032, 1028599, 1571713, 2355782,
3469466]

Zv[10,43] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,44] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,45] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]
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Zv[10,46] := x^10 / (-1+x)^10 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 247, 589, 1284, 2607, 4995, 9118,
15973, 27005, 44262, 70589, 109869, 167318, 249844, 366478, 528889, 751993]

Zv[10,47] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 129, 375, 957, 2212, 4727, 9475, 18004
, 32693, 57091, 96358, 157829, 251725, 392038, 597620, 893509, 1312529, 1897204
]

Zv[10,48] := (3*x^2 +1)*x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 22, 71, 202, 503, 1151, 2434, 4853, 9173,
16606, 28913, 48707, 79636, 126857, 197339, 300571, 449036, 659229, 952365]

Zv[10,49] := x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 38, 136, 405, 1056, 2488, 5408, 11006,
21200, 38972, 68816, 117326, 193952, 311960, 489632, 751751, 1131416, 1672242,
2431000]

Zv[10,50] := x^10*(3*x^8-3*x^7 +7*x^6 +6*x^4-3*x^3 +7*x^2-2*x +1) / (-1+x)^10 / (1+x)^5
 / (1+x^2)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 15, 46, 140, 351, 840, 1816, 3750, 7258,
13522, 24076, 41552, 69322, 112736, 178552, 276871, 420277, 626737, 918346]

Zv[10,51] := x^10*(2*x^2-x +1) / (-1+x)^10 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 82, 232, 584, 1344, 2872, 5774, 11020,
20118, 35332, 59984, 98824, 158520, 248248, 380447, 571714, 843919, 1225510]

Zv[10,52] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 32, 112, 331, 861, 2032, 4432, 9062, 17546
, 32432, 57584, 98714, 164054, 265232, 418352, 645359, 975689, 1448304, 2114112
]

Zv[10,53] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,54] := x^10*(4*x^4-3*x^3 +7*x^2-x +1) / (-1+x)^10 / (1+x)^5 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 21, 68, 208, 528, 1256, 2704, 5526, 10600,
19526, 34408, 58728, 96976, 156080, 244816, 376023, 565708, 836331, 1215500]

Zv[10,55] := (3*x^6-2*x^5 +4*x^4-x^3 +4*x^2-x +1)*x^10 / (-1+x)^10 / (1+x)^5 / (x^2+x +1)
^2 / (x^2-x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 13, 35, 95, 214, 467, 924, 1768, 3183,
5566, 9333, 15271, 24225, 37649, 57127, 85196, 124643, 179690, 254997]

Zv[10,56] := (1+x^2)*x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 81, 225, 555, 1252, 2625, 5185, 9736,
17511, 30337, 50866, 82851, 131515, 203986, 309858, 461845, 676601, 975666]

Zv[10,57] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 99, 276, 681, 1531, 3196, 6279, 11725,
20968, 36123, 60235, 97594, 154131, 237907, 359713, 533796, 778733, 1118471]
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Zv[10,58] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 99, 276, 681, 1531, 3196, 6279, 11725,
20968, 36123, 60235, 97594, 154131, 237907, 359713, 533796, 778733, 1118471]

Zv[10,59] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 32, 112, 331, 861, 2032, 4432, 9062, 17546
, 32432, 57584, 98714, 164054, 265232, 418352, 645359, 975689, 1448304, 2114112
]

Zv[10,60] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 129, 375, 957, 2212, 4727, 9475, 18004
, 32693, 57091, 96358, 157829, 251725, 392038, 597620, 893509, 1312529, 1897204
]

Zv[10,61] := (x^6+x^4 +2*x^3+x^2 +1)*x^10 / (-1+x)^10 / (x^2+x +1)^2 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 24, 78, 219, 550, 1267, 2718, 5493, 10552,
19401, 34330, 58732, 97512, 157615, 248692, 383928, 581062, 863636, 1262496]

Zv[10,62] := x^10 / (-1+x)^10 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 300, 756, 1732, 3676, 7330, 13870
, 25102, 43714, 73612, 120340, 191620, 298012, 453739, 677677, 994565, 1436435]

Zv[10,63] := x^10 / (-1+x)^10 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 46, 174, 541, 1461, 3544, 7896, 16414,
32206, 60172, 107788, 186142, 311278, 505912, 801592, 1241383, 1883167, 2803658
, 4103242]

Zv[10,64] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 99, 276, 681, 1531, 3196, 6279, 11725,
20968, 36123, 60235, 97594, 154131, 237907, 359713, 533796, 778733, 1118471]

Zv[10,65] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 32, 112, 331, 861, 2032, 4432, 9062, 17546
, 32432, 57584, 98714, 164054, 265232, 418352, 645359, 975689, 1448304, 2114112
]

Zv[10,66] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 129, 375, 957, 2212, 4727, 9475, 18004
, 32693, 57091, 96358, 157829, 251725, 392038, 597620, 893509, 1312529, 1897204
]

Zv[10,67] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 39, 144, 443, 1192, 2893, 6464, 13494,
26608, 49978, 90016, 156298, 262768, 429286, 683584, 1063711, 1621048, 2423993,
3562416]

Zv[10,68] := x^10 / (-1+x)^10 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 38, 136, 405, 1056, 2488, 5408, 11006,
21200, 38972, 68816, 117326, 193952, 311960, 489632, 751751, 1131416, 1672242,
2431000]

Zv[10,69] := x^10 / (-1+x)^10 / (1+x)^2
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[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 38, 136, 405, 1056, 2488, 5408, 11006,
21200, 38972, 68816, 117326, 193952, 311960, 489632, 751751, 1131416, 1672242,
2431000]

Zv[10,70] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 99, 276, 681, 1531, 3196, 6279, 11725,
20968, 36123, 60235, 97594, 154131, 237907, 359713, 533796, 778733, 1118471]

Zv[10,71] := x^10*(4*x^4-3*x^3 +7*x^2-x +1) / (-1+x)^10 / (1+x)^5 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 21, 68, 208, 528, 1256, 2704, 5526, 10600,
19526, 34408, 58728, 96976, 156080, 244816, 376023, 565708, 836331, 1215500]

Zv[10,72] := x^10 / (-1+x)^10 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 178, 411, 873, 1734, 3261, 5857,
10116, 16889, 27373, 43216, 66653, 100665, 149179, 217299, 311590, 440403]

Zv[10,73] := x^10 / (-1+x)^10 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 247, 589, 1284, 2607, 4995, 9118,
15973, 27005, 44262, 70589, 109869, 167318, 249844, 366478, 528889, 751993]

Zv[10,74] := x^10 / (-1+x)^10 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 247, 589, 1284, 2607, 4995, 9118,
15973, 27005, 44262, 70589, 109869, 167318, 249844, 366478, 528889, 751993]

Zv[10,75] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 99, 276, 681, 1531, 3196, 6279, 11725,
20968, 36123, 60235, 97594, 154131, 237907, 359713, 533796, 778733, 1118471]

Zv[10,76] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 32, 112, 331, 861, 2032, 4432, 9062, 17546
, 32432, 57584, 98714, 164054, 265232, 418352, 645359, 975689, 1448304, 2114112
]

Zv[10,77] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 99, 276, 681, 1531, 3196, 6279, 11725,
20968, 36123, 60235, 97594, 154131, 237907, 359713, 533796, 778733, 1118471]

Zv[10,78] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 129, 375, 957, 2212, 4727, 9475, 18004
, 32693, 57091, 96358, 157829, 251725, 392038, 597620, 893509, 1312529, 1897204
]

Zv[10,79] := x^10 / (-1+x)^10 / (x^4+x^3+x^2+x +1) / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 155, 343, 701, 1345, 2451, 4278,
7198, 11733, 18602, 28778, 43558, 64647, 94259, 135236, 191189, 266662]

Zv[10,80] := x^10 / (-1+x)^10 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 247, 589, 1284, 2607, 4995, 9118,
15973, 27005, 44262, 70589, 109869, 167318, 249844, 366478, 528889, 751993]

Zv[10,81] := (x^9 +5*x^8-x^7 +5*x^6 +2*x^5 +2*x^4 +2*x^3 +2*x^2-x +1)*x^10 / (-1+x)^10 / (
1+x)^4 / (x^2+x +1)^3 / (x^2-x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 28, 71, 163, 355, 719, 1391, 2569,
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4567, 7835, 13052, 21136, 33412, 51644, 78242, 116356, 170177, 245033]

Zv[10,82] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 99, 276, 681, 1531, 3196, 6279, 11725,
20968, 36123, 60235, 97594, 154131, 237907, 359713, 533796, 778733, 1118471]

Zv[10,83] := (x^6+x^4 +2*x^3+x^2 +1)*x^10 / (-1+x)^10 / (x^2+x +1)^2 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 59, 160, 390, 877, 1841, 3652, 6900,
12501, 21829, 36903, 60609, 97006, 151686, 232242, 348820, 514816, 747680]

Zv[10,84] := x^10*(2*x^2-x +1) / (-1+x)^10 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 82, 232, 584, 1344, 2872, 5774, 11020,
20118, 35332, 59984, 98824, 158520, 248248, 380447, 571714, 843919, 1225510]

Zv[10,85] := (x^6-x^5 +2*x^4 +2*x^2-x +1)*x^10 / (-1+x)^10 / (1+x)^4 / (x^2+x +1)^2 / (x^2-
x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 39, 99, 225, 478, 949, 1793, 3233,
5616, 9424, 15362, 24380, 37804, 57377, 85446, 125029, 180076, 255572]

Zv[10,86] := (1+x^2)*x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 81, 225, 555, 1252, 2625, 5185, 9736,
17511, 30337, 50866, 82851, 131515, 203986, 309858, 461845, 676601, 975666]

Zv[10,87] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 24, 75, 201, 480, 1051, 2145, 4134, 7591,
13377, 22746, 37489, 60105, 94026, 143881, 215832, 317964, 460769, 657702]

Zv[10,88] := x^10 / (-1+x)^10 / (1+x)^3 / (1+x^2) / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 47, 114, 250, 509, 975, 1777, 3105,
5234, 8550, 13589, 21077, 31987, 47601, 69591, 100107, 141892, 198404]

Zv[10,89] := x^10 / (-1+x)^10 / (x^2+x +1) / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 56, 145, 335, 716, 1429, 2705, 4886,
8491, 14255, 23234, 36871, 57155, 86726, 129106, 188858, 271911, 385791]

Zv[10,90] := x^10*(x^12+x^10 +2*x^9 +4*x^8 +2*x^7 +4*x^6 +2*x^5 +4*x^4 +2*x^3+x^2 +1) / (
-1+x)^10 / (1+x)^4 / (x^2+x +1)^2 / (1+x^2)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 13, 36, 92, 212, 459, 934, 1813, 3364,
6015, 10390, 17426, 28436, 45295, 70556, 107728, 161456, 237935, 345186]

Zv[10,91] := x^10 / (-1+x)^10 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 178, 411, 873, 1734, 3261, 5857,
10116, 16889, 27373, 43216, 66653, 100665, 149179, 217299, 311590, 440403]

Zv[10,92] := (1+x^2)*x^10 / (-1+x)^10 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 26, 86, 245, 616, 1416, 3016, 6046, 11500,
20932, 36652, 62062, 101992, 163240, 255112, 390247, 585442, 862862, 1251250]

Zv[10,93] := x^10*(2*x^2-x +1) / (-1+x)^10 / (1+x)^4 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 15, 43, 112, 259, 559, 1124, 2149, 3915,
6866, 11621, 19095, 30526, 47657, 72787, 109033, 160417, 232232, 331179]

Zv[10,94] := x^10 / (-1+x)^10 / (1+x)^3 / (1+x^2) / (x^2+x +1)
```

```
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 178, 411, 873, 1734, 3261, 5857,
10116, 16889, 27373, 43216, 66653, 100665, 149179, 217299, 311590, 440403]

Zv[10,95] := x^10 / (-1+x)^10 / (x^2+x +1)^2 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 70, 183, 428, 920, 1848, 3511, 6366,
11091, 18666, 30478, 48450, 75203, 114254, 170256, 249286, 359191, 509994]

Zv[10,96] := x^10 / (-1+x)^10 / (x^4+x^3+x^2+x +1) / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 109, 234, 467, 878, 1573, 2705,
4493, 7240, 11362, 17416, 26142, 38505, 55754, 79482, 111707, 154955]

Zv[10,97] := x^10 / (-1+x)^10 / (x^4+x^3+x^2+x +1) / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 155, 343, 701, 1345, 2451, 4278,
7198, 11733, 18602, 28778, 43558, 64647, 94259, 135236, 191189, 266662]

Zv[10,98] := (x^6+x^4 +2*x^3+x^2 +1)*x^10 / (-1+x)^10 / (x^2+x +1)^3 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 41, 105, 244, 528, 1069, 2055, 3776,
6670, 11383, 18850, 30376, 47780, 73530, 110932, 164358, 239526, 343796]

Zv[10,99] := x^10 / (-1+x)^10 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 178, 411, 873, 1734, 3261, 5857,
10116, 16889, 27373, 43216, 66653, 100665, 149179, 217299, 311590, 440403]

Zv[10,100] := x^10 / (-1+x)^10 / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1) / (1
+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 92, 188, 359, 649, 1122, 1868,
3012, 4723, 7228, 10825, 15902, 22957, 32624, 45700, 63181, 86298]

Zv[10,101] := x^10 / (-1+x)^10 / (1+x)^4 / (x^2+x +1) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 37, 89, 191, 385, 727, 1313, 2270,
3796, 6149, 9706, 14950, 22554, 33367, 48529, 69458, 98003, 136434]

Zv[10,102] := x^10*(2*x^6-3*x^5 +3*x^4-2*x^3 +3*x^2-2*x +1) / (-1+x)^10 / (1+x)^5 / (1+x
^2)^2 / (1+x^4) / (x^2+x +1)^2 / (x^2-x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 7, 15, 36, 70, 139, 248, 441, 734, 1209,
1906, 2968, 4478, 6674, 9713, 13976, 19736, 27583, 37969]

Zv[10,103] := x^10 / (-1+x)^10 / (x^4+x^3+x^2+x +1) / (1+x)^3 / (1+x^2) / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 30, 67, 137, 263, 478, 832, 1395, 2266
, 3579, 5517, 8320, 12305, 17880, 25569, 36033, 50105, 68817]

Zv[10,104] := x^10 / (-1+x)^10 / (1+x)^4 / (x^2+x +1)^2 / (x^2-x +1) / (1+x^2) / (x^4+x^3+x^2
+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 29, 63, 125, 234, 415, 707, 1161, 1851
, 2872, 4356, 6469, 9433, 13524, 19100, 26600, 36581, 49717]

Zv[10,105] := x^10 / (-1+x)^10 / (x^6+x^5+x^4+x^3+x^2+x +1) / (1+x)^3 / (x^2+x +1)^2 / (x^2
-x +1) / (1+x^2) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 51, 96, 171, 291, 477, 757, 1169,
1762, 2601, 3768, 5368, 7532, 10424, 14245, 19243, 25718]

Zv[10,106] := x^10 / (-1+x)^10 / (x^6+x^5+x^4+x^3+x^2+x +1) / (1+x)^4 / (x^2+x +1)^3 / (x^2
-x +1) / (x^6+x^3 +1) / (1+x^2)^2 / (1+x^4) / (x^4+x^3+x^2+x +1)
```

```
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 4, 7, 12, 19, 30, 45, 67, 97, 138, 192,
265, 359, 482, 639, 840, 1092, 1410, 1803]

x^10*(106 +613*x +2397*x^2 +7032*x^3 +17321*x^4 +36962*x^5 +71009*x^6 +124517*x^7+
202805*x^8 +309146*x^9 +445056*x^10 +607619*x^11 +790922*x^12 +983773*x^13 +1173101*x
^14 +1342720*x^15 +1478076*x^16 +1565409*x^17 +1596855*x^18 +1568379*x^19 +1483689*x^
20 +1350505*x^21 +1182361*x^22 +993735*x^23 +800867*x^24 +616945*x^25 +453242*x^26+
315976*x^27 +208171*x^28 +128495*x^29 +73774*x^30 +38778*x^31 +18402*x^32 +7637*x^33+
2695*x^34 +744*x^35 +151*x^36 +15*x^37) / (-1+x)^10 / (1+x)^5 / (x^2+x +1)^3 / (x^6+x^5+x^4
+x^3+x^2+x +1) / (x^2-x +1) / (x^6+x^3 +1) / (1+x^2)^2 / (1+x^4) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 106, 719, 3328, 11692, 34844, 91037, 215995,
472722, 969944, 1882764, 3488567, 6205662, 10656970, 17736207, 28713045,
45339623, 70014467, 105945454, 157394494, 229916506]

Zv4_10 := x^10*(75 +324*x +1096*x^2 +2444*x^3 +4702*x^4 +7209*x^5 +10030*x^6 +11780*x^
7 +12827*x^8 +12021*x^9 +10413*x^10 +7694*x^11 +5141*x^12 +2796*x^13 +1321*x^14 +450*x^
15 +119*x^16 +14*x^17) / (-1+x)^10 / (1+x)^5 / (x^2+x +1)^3 / (1+x^2)^2 / (x^2-x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 75, 549, 2668, 9674, 29484, 78307, 188141,
415830, 860007, 1680232, 3130222, 5593755, 9643882, 16104582, 26148863,
41398133, 64075552, 97158269, 144606781, 211588460]
```

C.2. **Edge labeled.** The generating functions for the number of labeled trees with `N` vertices and shape `t` are listed here, obtained from the cycle indices of Section B.2 by the substitution $t[j] -> x^j/(1-x^j)$.

```
Ze[1,1] := -x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]

-x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]

Ze4_1 := -x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]


Ze[2,1] := -x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]

-x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]

Ze4_2 := -x / (-1+x)
[0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]
```

```
Ze[3,1] := x^2 / (-1+x)^2 / (1+x)
[0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 10, 10, 11, 11, 12
, 12, 13, 13, 14, 14]

x^2 / (-1+x)^2 / (1+x)
[0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 10, 10, 11, 11, 12
, 12, 13, 13, 14, 14]

Ze4_3 := x^2 / (-1+x)^2 / (1+x)
[0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 10, 10, 11, 11, 12
, 12, 13, 13, 14, 14]


Ze[4,1] := -x^3 / (-1+x)^3 / (1+x)
[0, 0, 0, 1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49, 56, 64, 72, 81, 90,
100, 110, 121, 132, 144, 156, 169, 182, 196]

Ze[4,2] := -x^3 / (-1+x)^3 / (x^2+x +1) / (1+x)
[0, 0, 0, 1, 1, 2, 3, 4, 5, 7, 8, 10, 12, 14, 16, 19, 21, 24, 27, 30, 33, 37,
40, 44, 48, 52, 56, 61, 65, 70]

-x^3*(x^2+x +2) / (-1+x)^3 / (x^2+x +1) / (1+x)
[0, 0, 0, 2, 3, 6, 9, 13, 17, 23, 28, 35, 42, 50, 58, 68, 77, 88, 99, 111, 123,
137, 150, 165, 180, 196, 212, 230, 247, 266]

Ze4_4 := -x^3*(x^2+x +2) / (-1+x)^3 / (x^2+x +1) / (1+x)
[0, 0, 0, 2, 3, 6, 9, 13, 17, 23, 28, 35, 42, 50, 58, 68, 77, 88, 99, 111, 123,
137, 150, 165, 180, 196, 212, 230, 247, 266]


Ze[5,1] := (1+x^2)*x^4 / (-1+x)^4 / (1+x)^2
[0, 0, 0, 0, 1, 2, 6, 10, 19, 28, 44, 60, 85, 110, 146, 182, 231, 280, 344, 408
, 489, 570, 670, 770, 891, 1012, 1156, 1300, 1469, 1638]

Ze[5,2] := x^4 / (-1+x)^4 / (1+x)
[0, 0, 0, 0, 1, 3, 7, 13, 22, 34, 50, 70, 95, 125, 161, 203, 252, 308, 372, 444
, 525, 615, 715, 825, 946, 1078, 1222, 1378, 1547, 1729]

Ze[5,3] := x^4 / (-1+x)^4 / (1+x)^2 / (x^2+x +1) / (1+x^2)
[0, 0, 0, 0, 1, 1, 2, 3, 5, 6, 9, 11, 15, 18, 23, 27, 34, 39, 47, 54, 64, 72,
84, 94, 108, 120, 136, 150, 169, 185]

x^4*(6*x^2 +3*x +3 +5*x^4 +5*x^3+x^6 +2*x^5) / (-1+x)^4 / (1+x)^2 / (x^2+x +1) / (1+x^2)
[0, 0, 0, 0, 3, 6, 15, 26, 46, 68, 103, 141, 195, 253, 330, 412, 517, 627, 763,
906, 1078, 1257, 1469, 1689, 1945, 2210, 2514, 2828, 3185, 3552]

Ze4_5 := x^4*(6*x^2 +3*x +3 +5*x^4 +5*x^3+x^6 +2*x^5) / (-1+x)^4 / (1+x)^2 / (x^2+x +1) / (1+
x^2)
[0, 0, 0, 0, 3, 6, 15, 26, 46, 68, 103, 141, 195, 253, 330, 412, 517, 627, 763,
906, 1078, 1257, 1469, 1689, 1945, 2210, 2514, 2828, 3185, 3552]


Ze[6,1] := -(1+x^2)*x^5 / (-1+x)^5 / (1+x)^2
```

```
[0, 0, 0, 0, 0, 1, 3, 9, 19, 38, 66, 110, 170, 255, 365, 511, 693, 924, 1204,
1548, 1956, 2445, 3015, 3685, 4455, 5346, 6358, 7514, 8814, 10283]

Ze[6,2] := -x^5 / (-1+x)^5 / (1+x)
[0, 0, 0, 0, 0, 1, 4, 11, 24, 46, 80, 130, 200, 295, 420, 581, 784, 1036, 1344,
1716, 2160, 2685, 3300, 4015, 4840, 5786, 6864, 8086, 9464, 11011]

Ze[6,3] := -(1+x^2)*x^5 / (-1+x)^5 / (1+x)^2
[0, 0, 0, 0, 0, 1, 3, 9, 19, 38, 66, 110, 170, 255, 365, 511, 693, 924, 1204,
1548, 1956, 2445, 3015, 3685, 4455, 5346, 6358, 7514, 8814, 10283]

Ze[6,4] := -x^5*(x^2-x +1) / (-1+x)^5 / (1+x)^2 / (1+x^2)
[0, 0, 0, 0, 0, 1, 2, 5, 9, 17, 27, 43, 63, 92, 127, 174, 230, 302, 386, 490,
610, 755, 920, 1115, 1335, 1591, 1877, 2205, 2569, 2982]

Ze[6,5] := -x^5 / (-1+x)^5 / (1+x) / (x^2+x +1)
[0, 0, 0, 0, 0, 1, 3, 7, 14, 25, 41, 64, 95, 136, 189, 256, 339, 441, 564, 711,
885, 1089, 1326, 1600, 1914, 2272, 2678, 3136, 3650, 4225]

Ze[6,6] := -x^5 / (-1+x)^5 / (1+x)^2 / (x^2+x +1) / (1+x^2) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 1, 1, 2, 3, 5, 7, 10, 13, 18, 23, 30, 37, 47, 57, 70, 84, 101,
119, 141, 164, 192, 221, 255, 291, 333]

-x^5*(6 +10*x +21*x^2 +29*x^3 +38*x^4 +36*x^5 +33*x^6 +22*x^7 +14*x^8 +5*x^9 +2*x^10) / (-1
+x)^5 / (1+x)^2 / (x^2+x +1) / (1+x^2) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 6, 16, 43, 88, 169, 287, 467, 711, 1051, 1489, 2063, 2776, 3674
, 4759, 6083, 7651, 9520, 11695, 14241, 17163, 20533, 24356, 28710, 33602,
39117]

Ze4_6 := -x^5*(11*x^2 +5*x +5 +9*x^4 +8*x^3 +2*x^6 +3*x^5) / (1+x^2) / (x^2+x +1) / (-1+x)^5
 / (1+x)^2
[0, 0, 0, 0, 0, 5, 15, 41, 85, 164, 280, 457, 698, 1033, 1466, 2033, 2739, 3627
, 4702, 6013, 7567, 9419, 11576, 14100, 16999, 20341, 24135, 28455, 33311,
38784]


Ze[7,1] := (3*x^2 +1)*x^6 / (-1+x)^6 / (1+x)^3
[0, 0, 0, 0, 0, 0, 1, 3, 12, 28, 66, 126, 236, 396, 651, 1001, 1512, 2184, 3108
, 4284, 5832, 7752, 10197, 13167, 16852, 21252, 26598, 32890, 40404, 49140]

Ze[7,2] := x^6 / (-1+x)^6 / (1+x)
[0, 0, 0, 0, 0, 0, 1, 5, 16, 40, 86, 166, 296, 496, 791, 1211, 1792, 2576, 3612
, 4956, 6672, 8832, 11517, 14817, 18832, 23672, 29458, 36322, 44408, 53872]

Ze[7,3] := x^6 / (-1+x)^6
[0, 0, 0, 0, 0, 0, 1, 6, 21, 56, 126, 252, 462, 792, 1287, 2002, 3003, 4368,
6188, 8568, 11628, 15504, 20349, 26334, 33649, 42504, 53130, 65780, 80730,
98280]

Ze[7,4] := x^6*(x^6+x^4 +2*x^3+x^2 +1) / (-1+x)^6 / (1+x)^2 / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 1, 2, 6, 14, 28, 52, 93, 152, 242, 370, 546, 784, 1103, 1512
, 2040, 2706, 3534, 4554, 5803, 7304, 9108, 11252, 13780, 16744]
```

```
Ze[7,5] := x^6*(2*x^2-x +1) / (-1+x)^6 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 1, 2, 7, 14, 31, 54, 97, 154, 246, 364, 538, 756, 1058, 1428
, 1918, 2508, 3263, 4158, 5273, 6578, 8169, 10010, 12215, 14742]

Ze[7,6] := x^6 / (-1+x)^6 / (1+x)
[0, 0, 0, 0, 0, 0, 1, 5, 16, 40, 86, 166, 296, 496, 791, 1211, 1792, 2576, 3612
, 4956, 6672, 8832, 11517, 14817, 18832, 23672, 29458, 36322, 44408, 53872]

Ze[7,7] := x^6 / (-1+x)^6 / (1+x) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 50, 91, 155, 250, 386, 575, 831, 1170, 1611,
2175, 2886, 3771, 4860, 6186, 7786, 9700, 11972, 14650, 17786, 21436]

Ze[7,8] := (1+x^2)*x^6 / (-1+x)^6 / (1+x)^3
[0, 0, 0, 0, 0, 0, 1, 3, 10, 22, 48, 88, 158, 258, 413, 623, 924, 1316, 1848,
2520, 3396, 4476, 5841, 7491, 9526, 11946, 14872, 18304, 22386, 27118]

Ze[7,9] := x^6 / (-1+x)^6 / (1+x)^2 / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 1, 3, 8, 17, 33, 58, 97, 153, 233, 342, 489, 681, 930, 1245,
1641, 2130, 2730, 3456, 4330, 5370, 6602, 8048, 9738, 11698]

Ze[7,10] := x^6 / (-1+x)^6 / (1+x)^2 / (x^2+x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 1, 3, 7, 14, 26, 44, 71, 109, 162, 233, 327, 448, 603, 797,
1038, 1333, 1692, 2123, 2638, 3247, 3964, 4801, 5774, 6897]

Ze[7,11] := x^6 / (-1+x)^6 / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1) / (1+x^2
)
[0, 0, 0, 0, 0, 0, 1, 1, 2, 3, 5, 7, 11, 14, 20, 26, 35, 44, 58, 71, 90, 110,
136, 163, 199, 235, 282, 331, 391, 454]

x^6*(11 +26*x +68*x^2 +120*x^3 +196*x^4 +257*x^5 +320*x^6 +332*x^7 +327*x^8 +272*x^9 +211
*x^10 +134*x^11 +80*x^12 +33*x^13 +12*x^14 +2*x^15) / (-1+x)^6 / (1+x)^3 / (x^2+x +1)^2 / (x^
2-x +1) / (x^4+x^3+x^2+x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 11, 37, 116, 273, 585, 1104, 1972, 3270, 5222, 7958, 11789,
16903, 23731, 32512, 43813, 57954, 75636, 97266, 123720, 155480, 193613, 238710
, 292020, 354253]

Ze4_7 := x^6*(9 +24*x +56*x^2 +83*x^3 +102*x^4 +89*x^5 +66*x^6 +31*x^7 +12*x^8 +2*x^9) / (
-1+x)^6 / (1+x)^3 / (x^2+x +1)^2 / (1+x^2)
[0, 0, 0, 0, 0, 0, 9, 33, 107, 256, 554, 1053, 1890, 3147, 5040, 7699, 11427,
16411, 23070, 31644, 42685, 56511, 73808, 94980, 120883, 151998, 189367, 233578
, 285855, 346902]


Ze[8,1] := -(3*x^2 +1)*x^7 / (-1+x)^7 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 1, 4, 16, 44, 110, 236, 472, 868, 1519, 2520, 4032, 6216,
9324, 13608, 19440, 27192, 37389, 50556, 67408, 88660, 115258, 148148, 188552]

Ze[8,2] := -x^7 / (-1+x)^7 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 62, 148, 314, 610, 1106, 1897, 3108, 4900, 7476
, 11088, 16044, 22716, 31548, 43065, 57882, 76714, 100386, 129844, 166166,
210574]

Ze[8,3] := -x^7 / (-1+x)^7
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[0, 0, 0, 0, 0, 0, 0, 1, 7, 28, 84, 210, 462, 924, 1716, 3003, 5005, 8008,
12376, 18564, 27132, 38760, 54264, 74613, 100947, 134596, 177100, 230230,
296010, 376740]

Ze[8,4] := -(3*x^2 +1)*x^7 / (-1+x)^7 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 1, 4, 16, 44, 110, 236, 472, 868, 1519, 2520, 4032, 6216,
9324, 13608, 19440, 27192, 37389, 50556, 67408, 88660, 115258, 148148, 188552]

Ze[8,5] := -x^7*(2*x^2-x +1) / (-1+x)^7 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 24, 55, 109, 206, 360, 606, 970, 1508, 2264,
3322, 4750, 6668, 9176, 12439, 16597, 21870, 28448, 36617, 46627, 58842]

Ze[8,6] := -(1+x^2)*x^7 / (-1+x)^7 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 1, 5, 18, 50, 120, 256, 502, 918, 1589, 2625, 4172, 6412,
9576, 13944, 19860, 27732, 38049, 51381, 68398, 89870, 116688, 149864, 190554]

Ze[8,7] := -x^7 / (-1+x)^7 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 62, 148, 314, 610, 1106, 1897, 3108, 4900, 7476
, 11088, 16044, 22716, 31548, 43065, 57882, 76714, 100386, 129844, 166166,
210574]

Ze[8,8] := -x^7 / (-1+x)^7 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 62, 148, 314, 610, 1106, 1897, 3108, 4900, 7476
, 11088, 16044, 22716, 31548, 43065, 57882, 76714, 100386, 129844, 166166,
210574]

Ze[8,9] := -(3*x^2 +1)*x^7 / (-1+x)^7 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 1, 4, 16, 44, 110, 236, 472, 868, 1519, 2520, 4032, 6216,
9324, 13608, 19440, 27192, 37389, 50556, 67408, 88660, 115258, 148148, 188552]

Ze[8,10] := -x^7 / (-1+x)^7 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 91, 182, 337, 587, 973, 1548, 2379, 3549,
5160, 7335, 10221, 13992, 18852, 25038, 32824, 42524, 54496, 69146, 86932]

Ze[8,11] := -x^7 / (-1+x)^7 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 62, 148, 314, 610, 1106, 1897, 3108, 4900, 7476
, 11088, 16044, 22716, 31548, 43065, 57882, 76714, 100386, 129844, 166166,
210574]

Ze[8,12] := -(1+x^2)*x^7 / (-1+x)^7 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 36, 84, 172, 330, 588, 1001, 1624, 2548, 3864,
5712, 8232, 11628, 16104, 21945, 29436, 38962, 50908, 65780, 84084, 106470]

Ze[8,13] := -x^7*(2*x^2-x +1) / (-1+x)^7 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 24, 55, 109, 206, 360, 606, 970, 1508, 2264,
3322, 4750, 6668, 9176, 12439, 16597, 21870, 28448, 36617, 46627, 58842]

Ze[8,14] := -x^7 / (-1+x)^7 / (1+x)^2 / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 29, 62, 120, 217, 370, 603, 945, 1434, 2115,
3045, 4290, 5931, 8061, 10791, 14247, 18577, 23947, 30549, 38597, 48335]

Ze[8,15] := -(x^6+x^4 +2*x^3+x^2 +1)*x^7 / (-1+x)^7 / (x^2+x +1)^2 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 1, 3, 9, 23, 51, 103, 196, 348, 590, 960, 1506, 2290,
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3393, 4905, 6945, 9651, 13185, 17739, 23542, 30846, 39954, 51206, 64986]

Ze[8,16] := -x^7 / (-1+x)^7 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 91, 182, 337, 587, 973, 1548, 2379, 3549,
5160, 7335, 10221, 13992, 18852, 25038, 32824, 42524, 54496, 69146, 86932]

Ze[8,17] := -x^7 / (-1+x)^7 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 45, 103, 211, 399, 707, 1190, 1918, 2982, 4494,
6594, 9450, 13266, 18282, 24783, 33099, 43615, 56771, 73073, 93093, 117481]

Ze[8,18] := -x^7 / (-1+x)^7 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 51, 95, 166, 275, 437, 670, 997, 1445, 2048
, 2845, 3883, 5216, 6908, 9031, 11669, 14916, 18880, 23681, 29455]

Ze[8,19] := -(1+x^2)*x^7 / (-1+x)^7 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 23, 51, 98, 181, 309, 511, 804, 1233, 1827,
2652, 3753, 5223, 7128, 9594, 12714, 16654, 21540, 27586, 34958, 43926]

Ze[8,20] := -(x^4+x^2-x +1)*x^7 / (-1+x)^7 / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 1, 2, 5, 10, 20, 35, 61, 98, 155, 234, 347, 498, 705, 973
, 1326, 1774, 2347, 3059, 3950, 5038, 6373, 7981, 9922]

Ze[8,21] := -x^7 / (-1+x)^7 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 1, 3, 8, 17, 34, 61, 105, 170, 267, 403, 594, 851, 1197,
1648, 2235, 2981, 3927, 5104, 6565, 8351, 10529, 13152, 16303]

Ze[8,22] := -x^7 / (-1+x)^7 / (x^4+x^3+x^2+x +1) / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 1, 3, 7, 14, 26, 45, 74, 116, 176, 259, 372, 522, 719,
973, 1297, 1705, 2214, 2842, 3611, 4544, 5669, 7015, 8616]

Ze[8,23] := -x^7 / (-1+x)^7 / (x^6+x^5+x^4+x^3+x^2+x +1) / (1+x)^3 / (x^2+x +1)^2 / (x^2-x+
1) / (x^4+x^3+x^2+x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 3, 5, 7, 11, 15, 21, 28, 38, 49, 65, 82, 105,
131, 164, 201, 248, 300, 364, 436, 522]

-x^7*(23 +73*x +210*x^2 +444*x^3 +833*x^4 +1334*x^5 +1962*x^6 +2585*x^7 +3174*x^8 +3561*
x^9 +3738*x^10 +3606*x^11 +3259*x^12 +2693*x^13 +2071*x^14 +1434*x^15 +910*x^16 +499*x^
17 +242*x^18 +88*x^19 +26*x^20 +3*x^21) / (-1+x)^7 / (x^6+x^5+x^4+x^3+x^2+x +1) / (1+x)^3 /
(x^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 23, 96, 329, 869, 2031, 4211, 8108, 14552, 24846, 40503,
63701, 96921, 143558, 207397, 293421, 407133, 555529, 746266, 988855, 1293599,
1673051, 2140731, 2712810]

Ze4_8 := -x^7*(18 +28*x +81*x^2 +96*x^3 +149*x^4 +132*x^5 +154*x^6 +103*x^7 +91*x^8 +39*
x^9 +20*x^10 +3*x^11) / (-1+x)^7 / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 18, 82, 291, 787, 1864, 3905, 7571, 13667, 23434, 38339,
60467, 92227, 136877, 198096, 280678, 389972, 532722, 716374, 950108, 1243948,
1610023, 2061489, 2613988]


Ze[9,1] := (x^4 +6*x^2 +1)*x^8 / (-1+x)^8 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 20, 60, 170, 396, 868, 1716, 3235, 5720, 9752,
15912, 25236, 38760, 58200, 85272, 122661, 173052, 240460, 328900, 444158,
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592020]

Ze[9,2] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Ze[9,3] := x^8 / (-1+x)^8
[0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 36, 120, 330, 792, 1716, 3432, 6435, 11440,
19448, 31824, 50388, 77520, 116280, 170544, 245157, 346104, 480700, 657800,
888030, 1184040]

Ze[9,4] := x^8 / (-1+x)^8
[0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 36, 120, 330, 792, 1716, 3432, 6435, 11440,
19448, 31824, 50388, 77520, 116280, 170544, 245157, 346104, 480700, 657800,
888030, 1184040]

Ze[9,5] := x^8*(x^4-x^3 +4*x^2-x +1) / (-1+x)^8 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 13, 34, 89, 191, 397, 744, 1350, 2298, 3806,
6036, 9358, 14058, 20726, 29832, 42271, 58773, 80643, 108966, 145583, 192049]

Ze[9,6] := (1+x^2)*x^8 / (-1+x)^8 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 24, 74, 194, 450, 952, 1870, 3459, 6084, 10256,
16668, 26244, 40188, 60048, 87780, 125829, 177210, 245608, 335478, 452166,
602030]

Ze[9,7] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Ze[9,8] := (3*x^2 +1)*x^8 / (-1+x)^8 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 21, 65, 175, 411, 883, 1751, 3270, 5790, 9822,
16038, 25362, 38970, 58410, 85602, 122991, 173547, 240955, 329615, 444873,
593021]

Ze[9,9] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Ze[9,10] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Ze[9,11] := (x^4 +6*x^2 +1)*x^8 / (-1+x)^8 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 20, 60, 170, 396, 868, 1716, 3235, 5720, 9752,
15912, 25236, 38760, 58200, 85272, 122661, 173052, 240460, 328900, 444158,
592020]

Ze[9,12] := x^8 / (-1+x)^8
[0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 36, 120, 330, 792, 1716, 3432, 6435, 11440,
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19448, 31824, 50388, 77520, 116280, 170544, 245157, 346104, 480700, 657800,
888030, 1184040]

Ze[9,13] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 154, 336, 673, 1260, 2233, 3781, 6160,
9709, 14869, 22204, 32425, 46417, 65269, 90307, 123131, 165655, 220151, 289297]

Ze[9,14] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Ze[9,15] := (3*x^2 +1)*x^8 / (-1+x)^8 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 17, 48, 127, 284, 599, 1152, 2118, 3672, 6150,
9888, 15474, 23496, 34914, 50688, 72303, 101244, 139711, 189904, 254969, 338052
]

Ze[9,16] := (1+x^2)*x^8 / (-1+x)^8 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 55, 139, 311, 641, 1229, 2230, 3854, 6402,
10266, 15978, 24210, 35838, 51942, 73887, 103323, 142285, 193193, 258973,
343057]

Ze[9,17] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Ze[9,18] := x^8 / (-1+x)^8 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 68, 171, 382, 781, 1488, 2678, 4596, 7578,
12072, 18666, 28116, 41382, 59664, 84447, 117546, 161161, 217932, 291005,
384098]

Ze[9,19] := (1+x^2)*x^8 / (-1+x)^8 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 24, 74, 194, 450, 952, 1870, 3459, 6084, 10256,
16668, 26244, 40188, 60048, 87780, 125829, 177210, 245608, 335478, 452166,
602030]

Ze[9,20] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Ze[9,21] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 108, 228, 445, 815, 1418, 2363, 3797,
5912, 8957, 13247, 19178, 27239, 38030, 52277, 70854, 94801, 125350, 163947]

Ze[9,22] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 154, 336, 673, 1260, 2233, 3781, 6160,
9709, 14869, 22204, 32425, 46417, 65269, 90307, 123131, 165655, 220151, 289297]

Ze[9,23] := (1+x^2)*x^8 / (-1+x)^8 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 24, 74, 194, 450, 952, 1870, 3459, 6084, 10256,
16668, 26244, 40188, 60048, 87780, 125829, 177210, 245608, 335478, 452166,
```

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602030]

Ze[9,24] := x^8 / (-1+x)^8 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 68, 171, 382, 781, 1488, 2678, 4596, 7578,
12072, 18666, 28116, 41382, 59664, 84447, 117546, 161161, 217932, 291005,
384098]

Ze[9,25] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 154, 336, 673, 1260, 2233, 3781, 6160,
9709, 14869, 22204, 32425, 46417, 65269, 90307, 123131, 165655, 220151, 289297]

Ze[9,26] := x^8 / (-1+x)^8 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 68, 171, 382, 781, 1488, 2678, 4596, 7578,
12072, 18666, 28116, 41382, 59664, 84447, 117546, 161161, 217932, 291005,
384098]

Ze[9,27] := x^8 / (-1+x)^8 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 91, 239, 553, 1163, 2269, 4166, 7274, 12174,
19650, 30738, 46782, 69498, 101046, 144111, 201993, 278707, 379093, 508937,
675103]

Ze[9,28] := x^8 / (-1+x)^8 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 92, 187, 353, 628, 1065, 1735, 2732,
4177, 6225, 9070, 12953, 18169, 25077, 34108, 45777, 60693, 79573, 103254]

Ze[9,29] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 154, 336, 673, 1260, 2233, 3781, 6160,
9709, 14869, 22204, 32425, 46417, 65269, 90307, 123131, 165655, 220151, 289297]

Ze[9,30] := x^8*(2*x^2-x +1) / (-1+x)^8 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 38, 93, 202, 408, 768, 1374, 2344, 3852,
6116, 9438, 14188, 20856, 30032, 42471, 59068, 80938, 109386, 146003, 192630]

Ze[9,31] := x^8*(x^12+x^10 +2*x^9 +4*x^8 +2*x^7 +4*x^6 +2*x^5 +4*x^4 +2*x^3+x^2 +1) / (-1
+x)^8 / (1+x)^4 / (1+x^2)^2 / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 6, 14, 33, 64, 127, 228, 404, 672, 1100, 1724,
2661, 3974, 5849, 8402, 11911, 16556, 22751, 30772, 41198, 54436]

Ze[9,32] := (1+x^2)*x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 37, 88, 186, 367, 676, 1187, 1991, 3224,
5051, 7703, 11456, 16679, 23807, 33401, 46115, 62769, 84309, 111895, 146853]

Ze[9,33] := (x^6-x^5 +2*x^4 +2*x^2-x +1)*x^8 / (-1+x)^8 / (1+x)^4 / (x^2+x +1)^2 / (x^2-x +1
) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 7, 15, 35, 66, 127, 218, 373, 596, 943, 1425,
2130, 3080, 4406, 6149, 8496, 11514, 15464, 20449, 26822, 34736]

Ze[9,34] := (1+x^2)*x^8 / (-1+x)^8 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 55, 139, 311, 641, 1229, 2230, 3854, 6402,
10266, 15978, 24210, 35838, 51942, 73887, 103323, 142285, 193193, 258973,
343057]

Ze[9,35] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)^2
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[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 108, 228, 445, 815, 1418, 2363, 3797,
5912, 8957, 13247, 19178, 27239, 38030, 52277, 70854, 94801, 125350, 163947]

Ze[9,36] := x^8*(2*x^2-x +1) / (-1+x)^8 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 11, 27, 66, 136, 272, 496, 878, 1466, 2386, 3730
, 5708, 8480, 12376, 17656, 24815, 34253, 46685, 62701, 83302, 109328]

Ze[9,37] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 108, 228, 445, 815, 1418, 2363, 3797,
5912, 8957, 13247, 19178, 27239, 38030, 52277, 70854, 94801, 125350, 163947]

Ze[9,38] := x^8 / (-1+x)^8 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 29, 63, 124, 229, 399, 666, 1069, 1663, 2514
, 3711, 5359, 7594, 10575, 14502, 19606, 26171, 34522, 45051, 58203]

Ze[9,39] := (x^6+x^4 +2*x^3+x^2 +1)*x^8 / (-1+x)^8 / (x^2+x +1)^2 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 26, 61, 129, 257, 477, 847, 1437, 2353, 3727
, 5746, 8632, 12691, 18283, 25876, 36022, 49418, 66868, 89372, 118074]

Ze[9,40] := x^8 / (-1+x)^8 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 92, 187, 353, 628, 1065, 1735, 2732,
4177, 6225, 9070, 12953, 18169, 25077, 34108, 45777, 60693, 79573, 103254]

Ze[9,41] := x^8 / (-1+x)^8 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 108, 228, 445, 815, 1418, 2363, 3797,
5912, 8957, 13247, 19178, 27239, 38030, 52277, 70854, 94801, 125350, 163947]

Ze[9,42] := x^8 / (-1+x)^8 / (x^4+x^3+x^2+x +1) / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 51, 96, 170, 286, 462, 721, 1093, 1615,
2334, 3307, 4604, 6309, 8523, 11365, 14976, 19520, 25189, 32204]

Ze[9,43] := x^8 / (-1+x)^8 / (1+x)^4 / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 23, 52, 101, 191, 332, 563, 905, 1424, 2159,
3215, 4658, 6647, 9287, 12809, 17372, 23301, 30827, 40395, 52330]

Ze[9,44] := x^8 / (-1+x)^8 / (1+x)^3 / (1+x^2) / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 8, 18, 37, 69, 123, 207, 336, 526, 801, 1187,
1723, 2449, 3422, 4704, 6376, 8526, 11269, 14727, 19055, 24421]

Ze[9,45] := x^8 / (-1+x)^8 / (x^4+x^3+x^2+x +1) / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 8, 17, 34, 62, 108, 178, 284, 437, 656, 959,
1375, 1932, 2672, 3637, 4886, 6479, 8497, 11023, 14166, 18038]

Ze[9,46] := x^8 / (-1+x)^8 / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1) / (1+x^2
)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 7, 14, 26, 45, 75, 119, 183, 273, 398, 567, 794,
1092, 1480, 1978, 2612, 3409, 4405, 5636, 7149, 8993]

Ze[9,47] := x^8 / (-1+x)^8 / (1+x)^4 / (1+x^2)^2 / (1+x^4) / (x^6+x^5+x^4+x^3+x^2+x +1) / (x
^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 3, 5, 7, 11, 15, 22, 29, 40, 52, 70, 89, 116,
146, 186, 230, 288, 352, 434, 525]
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x^8*(47 +192*x +632*x^2 +1538*x^3 +3269*x^4 +5983*x^5 +10035*x^6 +15275*x^7 +21754*x^8+
28780*x^9 +35999*x^10 +42297*x^11 +47248*x^12 +49831*x^13 +50065*x^14 +47555*x^15+
42999*x^16 +36659*x^17 +29646*x^18 +22447*x^19 +15997*x^20 +10525*x^21 +6419*x^22+
3509*x^23 +1724*x^24 +708*x^25 +242*x^26 +57*x^27 +9*x^28) / (-1+x)^8 / (1+x)^4 / (1+x^2)^
2 / (1+x^4) / (x^6+x^5+x^4+x^3+x^2+x +1) / (x^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 47, 239, 918, 2695, 6882, 15513, 32191, 62014, 113025,
195972, 326549, 524874, 818782, 1242832, 1842950, 2674809, 3810062, 5333905,
7353288, 9993347, 13407967, 17774859]

Ze4_9 := x^8*(35 +89*x +267*x^2 +435*x^3 +742*x^4 +885*x^5 +1107*x^6 +1031*x^7 +1003*x^
8 +727*x^9 +534*x^10 +277*x^11 +137*x^12 +38*x^13 +9*x^14) / (-1+x)^8 / (1+x)^4 / (x^2+x +1)
^2 / (1+x^2)^2 / (x^2-x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 35, 194, 779, 2349, 6107, 13942, 29203, 56670, 103881,
180961, 302700, 488119, 763538, 1161723, 1726215, 2509896, 3580839, 5020096,
6929424, 9428030, 12662509, 16802319]


Ze[10,1] := -(x^4 +6*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 25, 85, 255, 651, 1519, 3235, 6470, 12190,
21942, 37854, 63090, 101850, 160050, 245322, 367983, 541035, 781495, 1110395,
1554553]

Ze[10,2] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,3] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Ze[10,4] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Ze[10,5] := -(x^4 +6*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 25, 85, 255, 651, 1519, 3235, 6470, 12190,
21942, 37854, 63090, 101850, 160050, 245322, 367983, 541035, 781495, 1110395,
1554553]

Ze[10,6] := -x^9*(x^4-x^3 +4*x^2-x +1) / (-1+x)^9 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 17, 51, 140, 331, 728, 1472, 2822, 5120, 8926
, 14962, 24320, 38378, 59104, 88936, 131207, 189980, 270623, 379589, 525172]

Ze[10,7] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,8] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 299, 749, 1701, 3571, 7030, 13114,
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23370, 40038, 66282, 106470, 166518, 254298, 380127, 557337, 802945, 1138423,
1590589]

Ze[10,9] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,10] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,11] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,12] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Ze[10,13] := -(x^4 +6*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 25, 85, 255, 651, 1519, 3235, 6470, 12190,
21942, 37854, 63090, 101850, 160050, 245322, 367983, 541035, 781495, 1110395,
1554553]

Ze[10,14] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Ze[10,15] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Ze[10,16] := -(x^4 +6*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 25, 85, 255, 651, 1519, 3235, 6470, 12190,
21942, 37854, 63090, 101850, 160050, 245322, 367983, 541035, 781495, 1110395,
1554553]

Ze[10,17] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Ze[10,18] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]
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Ze[10,19] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,20] := -(3*x^7 +10*x^6 +6*x^5 +6*x^4 +7*x^3 +3*x^2 +1)*x^9 / (-1+x)^9 / (x^2+x +1)^3
 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 12, 38, 102, 249, 562, 1167, 2292, 4272, 7608
, 13038, 21609, 34734, 54384, 83145, 124410, 182589, 263320, 373659, 522522]

Ze[10,21] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 80, 219, 530, 1171, 2400, 4630, 8484,
14886, 25152, 41130, 65340, 101178, 153120, 227007, 330330, 472615, 665808,
924781]

Ze[10,22] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Ze[10,23] := -x^9*(x^4-x^3 +4*x^2-x +1) / (-1+x)^9 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 17, 51, 140, 331, 728, 1472, 2822, 5120, 8926
, 14962, 24320, 38378, 59104, 88936, 131207, 189980, 270623, 379589, 525172]

Ze[10,24] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,25] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,26] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 299, 749, 1701, 3571, 7030, 13114,
23370, 40038, 66282, 106470, 166518, 254298, 380127, 557337, 802945, 1138423,
1590589]

Ze[10,27] := -(3*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 22, 70, 197, 481, 1080, 2232, 4350, 8022,
14172, 24060, 39534, 63030, 97944, 148632, 220935, 322179, 461890, 651794,
906763]

Ze[10,28] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,29] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 299, 749, 1701, 3571, 7030, 13114,
23370, 40038, 66282, 106470, 166518, 254298, 380127, 557337, 802945, 1138423,
1590589]
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Ze[10,30] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,31] := -x^9 / (-1+x)^9
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 9, 45, 165, 495, 1287, 3003, 6435, 12870, 24310,
43758, 75582, 125970, 203490, 319770, 490314, 735471, 1081575, 1562275, 2220075
, 3108105]

Ze[10,32] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,33] := -(x^4 +6*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 25, 85, 255, 651, 1519, 3235, 6470, 12190,
21942, 37854, 63090, 101850, 160050, 245322, 367983, 541035, 781495, 1110395,
1554553]

Ze[10,34] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Ze[10,35] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Ze[10,36] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Ze[10,37] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Ze[10,38] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Ze[10,39] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 299, 749, 1701, 3571, 7030, 13114,
23370, 40038, 66282, 106470, 166518, 254298, 380127, 557337, 802945, 1138423,
1590589]

Ze[10,40] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
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1049763]

Ze[10,41] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Ze[10,42] := -(3*x^2 +1)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 27, 92, 267, 678, 1561, 3312, 6582, 12372,
22194, 38232, 63594, 102564, 160974, 246576, 369567, 543114, 784069, 1113684,
1558557]

Ze[10,43] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,44] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,45] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,46] := -x^9 / (-1+x)^9 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 155, 342, 695, 1323, 2388, 4123, 6855
, 11032, 17257, 26327, 39280, 57449, 82526, 116634, 162411, 223104, 302677]

Ze[10,47] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Ze[10,48] := -(3*x^2 +1)*x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 17, 49, 131, 301, 648, 1283, 2419, 4320, 7433
, 12307, 19794, 30929, 47221, 70482, 103232, 148465, 210193, 293136, 403434]

Ze[10,49] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Ze[10,50] := -x^9*(x^2-x +1)*(x^6-x^5 +3*x^4 +2*x^3 +3*x^2-x +1) / (-1+x)^9 / (1+x)^4 / (1
+x^2)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 12, 34, 94, 220, 489, 995, 1934, 3546, 6264,
10620, 17476, 27880, 43414, 65986, 98319, 143661, 206460, 291974, 407162]

Ze[10,51] := -x^9*(2*x^2-x +1) / (-1+x)^9 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 57, 150, 352, 760, 1528, 2902, 5246, 9098
, 15214, 24652, 38840, 59696, 89728, 132199, 191267, 272205, 381591, 527594]
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Ze[10,52] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 80, 219, 530, 1171, 2400, 4630, 8484,
14886, 25152, 41130, 65340, 101178, 153120, 227007, 330330, 472615, 665808,
924781]

Ze[10,53] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,54] := -x^9*(x^4-x^3 +4*x^2-x +1) / (-1+x)^9 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 17, 51, 140, 331, 728, 1472, 2822, 5120, 8926
, 14962, 24320, 38378, 59104, 88936, 131207, 189980, 270623, 379589, 525172]

Ze[10,55] := -(x^6-x^5 +2*x^4 +2*x^2-x +1)*x^9 / (-1+x)^9 / (1+x)^4 / (x^2+x +1)^2 / (x^2-x
 +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 25, 60, 126, 253, 471, 844, 1440, 2383,
3808, 5938, 9018, 13424, 19573, 28069, 39583, 55047, 75496, 102318]

Ze[10,56] := -(1+x^2)*x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 56, 144, 330, 697, 1373, 2560, 4551, 7775
, 12826, 20529, 31985, 48664, 72471, 105872, 151987, 214756, 299065, 410960]

Ze[10,57] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Ze[10,58] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Ze[10,59] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 80, 219, 530, 1171, 2400, 4630, 8484,
14886, 25152, 41130, 65340, 101178, 153120, 227007, 330330, 472615, 665808,
924781]

Ze[10,60] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Ze[10,61] := -(x^6+x^4 +2*x^3+x^2 +1)*x^9 / (-1+x)^9 / (x^2+x +1)^2 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 18, 54, 141, 331, 717, 1451, 2775, 5059, 8849
, 14929, 24402, 38780, 60103, 91077, 135236, 197134, 282574, 398860, 555100]

Ze[10,62] := -x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 24, 74, 195, 456, 976, 1944, 3654, 6540,
11232, 18612, 29898, 46728, 71280, 106392, 155727, 223938, 316888, 441870,
607893]

Ze[10,63] := -x^9 / (-1+x)^9 / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 8, 37, 128, 367, 920, 2083, 4352, 8518, 15792,
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27966, 47616, 78354, 125136, 194634, 295680, 439791, 641784, 920491, 1299584,
1808521]

Ze[10,64] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Ze[10,65] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 80, 219, 530, 1171, 2400, 4630, 8484,
14886, 25152, 41130, 65340, 101178, 153120, 227007, 330330, 472615, 665808,
924781]

Ze[10,66] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Ze[10,67] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 31, 105, 299, 749, 1701, 3571, 7030, 13114,
23370, 40038, 66282, 106470, 166518, 254298, 380127, 557337, 802945, 1138423,
1590589]

Ze[10,68] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Ze[10,69] := -x^9 / (-1+x)^9 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 30, 98, 269, 651, 1432, 2920, 5598, 10194,
17772, 29844, 48510, 76626, 118008, 177672, 262119, 379665, 540826, 758758,
1049763]

Ze[10,70] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Ze[10,71] := -x^9*(x^4-x^3 +4*x^2-x +1) / (-1+x)^9 / (1+x)^4 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 17, 51, 140, 331, 728, 1472, 2822, 5120, 8926
, 14962, 24320, 38378, 59104, 88936, 131207, 189980, 270623, 379589, 525172]

Ze[10,72] := -x^9 / (-1+x)^9 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 109, 233, 462, 861, 1527, 2596, 4259,
6773, 10484, 15843, 23437, 34012, 48514, 68120, 94291, 128813, 173864]

Ze[10,73] := -x^9 / (-1+x)^9 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 155, 342, 695, 1323, 2388, 4123, 6855
, 11032, 17257, 26327, 39280, 57449, 82526, 116634, 162411, 223104, 302677]

Ze[10,74] := -x^9 / (-1+x)^9 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 155, 342, 695, 1323, 2388, 4123, 6855
, 11032, 17257, 26327, 39280, 57449, 82526, 116634, 162411, 223104, 302677]

Ze[10,75] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
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[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Ze[10,76] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 25, 80, 219, 530, 1171, 2400, 4630, 8484,
14886, 25152, 41130, 65340, 101178, 153120, 227007, 330330, 472615, 665808,
924781]

Ze[10,77] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Ze[10,78] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 7, 29, 92, 246, 582, 1255, 2515, 4748, 8529,
14689, 24398, 39267, 61471, 93896, 140313, 205582, 295889, 419020, 584675,
804826]

Ze[10,79] := -x^9 / (-1+x)^9 / (x^4+x^3+x^2+x +1) / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 92, 188, 358, 644, 1106, 1827, 2920,
4535, 6869, 10176, 14780, 21089, 29612, 40977, 55953, 75473, 100662]

Ze[10,80] := -x^9 / (-1+x)^9 / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 22, 63, 155, 342, 695, 1323, 2388, 4123, 6855
, 11032, 17257, 26327, 39280, 57449, 82526, 116634, 162411, 223104, 302677]

Ze[10,81] := -(x^9 +5*x^8-x^7 +5*x^6 +2*x^5 +2*x^4 +2*x^3 +2*x^2-x +1)*x^9 / (-1+x)^9 / (1
+x)^4 / (x^2+x +1)^3 / (x^2-x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 7, 18, 43, 92, 192, 364, 672, 1178, 1998,
3268, 5217, 8084, 12276, 18232, 26598, 38114, 53821, 74856, 102817]

Ze[10,82] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 6, 23, 69, 177, 405, 850, 1665, 3083, 5446, 9243
, 15155, 24112, 37359, 56537, 83776, 121806, 174083, 244937, 339738, 465088]

Ze[10,83] := -(x^6+x^4 +2*x^3+x^2 +1)*x^9 / (-1+x)^9 / (x^2+x +1)^2 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 40, 101, 230, 487, 964, 1811, 3248, 5601,
9328, 15074, 23706, 36397, 54680, 80556, 116578, 165996, 232864, 322236]

Ze[10,84] := -x^9*(2*x^2-x +1) / (-1+x)^9 / (1+x)^3 / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 57, 150, 352, 760, 1528, 2902, 5246, 9098
, 15214, 24652, 38840, 59696, 89728, 132199, 191267, 272205, 381591, 527594]

Ze[10,85] := -(x^6-x^5 +2*x^4 +2*x^2-x +1)*x^9 / (-1+x)^9 / (1+x)^4 / (x^2+x +1)^2 / (x^2-x
 +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 25, 60, 126, 253, 471, 844, 1440, 2383,
3808, 5938, 9018, 13424, 19573, 28069, 39583, 55047, 75496, 102318]

Ze[10,86] := -(1+x^2)*x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 19, 56, 144, 330, 697, 1373, 2560, 4551, 7775
, 12826, 20529, 31985, 48664, 72471, 105872, 151987, 214756, 299065, 410960]

Ze[10,87] := -x^9 / (-1+x)^9 / (x^2+x +1) / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 18, 51, 126, 279, 571, 1094, 1989, 3457, 5786
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, 9369, 14743, 22616, 33921, 49855, 71951, 102132, 142805, 196933, 268155]

Ze[10,88] := -x^9 / (-1+x)^9 / (1+x)^3 / (1+x^2) / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 30, 67, 136, 259, 466, 802, 1328, 2129,
3316, 5039, 7488, 10910, 15614, 21990, 30516, 41785, 56512, 75567]

Ze[10,89] := -x^9 / (-1+x)^9 / (1+x)^4 / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 14, 37, 89, 190, 381, 713, 1276, 2181, 3605,
5764, 8979, 13637, 20284, 29571, 42380, 59752, 83053, 113880, 154275]

Ze[10,90] := -x^9*(x^12+x^10 +2*x^9 +4*x^8 +2*x^7 +4*x^6 +2*x^5 +4*x^4 +2*x^3+x^2 +1) / (
-1+x)^9 / (1+x)^4 / (1+x^2)^2 / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 9, 23, 56, 120, 247, 475, 879, 1551, 2651,
4375, 7036, 11010, 16859, 25261, 37172, 53728, 76479, 107251, 148449]

Ze[10,91] := -x^9 / (-1+x)^9 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 109, 233, 462, 861, 1527, 2596, 4259,
6773, 10484, 15843, 23437, 34012, 48514, 68120, 94291, 128813, 173864]

Ze[10,92] := -(1+x^2)*x^9 / (-1+x)^9 / (1+x)^4
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 20, 60, 159, 371, 800, 1600, 3030, 5454, 9432
, 15720, 25410, 39930, 61248, 91872, 135135, 195195, 277420, 388388, 536393]

Ze[10,93] := -x^9*(2*x^2-x +1) / (-1+x)^9 / (1+x)^4 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 11, 28, 69, 147, 300, 565, 1025, 1766, 2951,
4755, 7474, 11431, 17131, 25130, 36246, 51384, 71815, 98947, 134686]

Ze[10,94] := -x^9 / (-1+x)^9 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 109, 233, 462, 861, 1527, 2596, 4259,
6773, 10484, 15843, 23437, 34012, 48514, 68120, 94291, 128813, 173864]

Ze[10,95] := -x^9 / (-1+x)^9 / (x^2+x +1)^2 / (1+x)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 47, 113, 245, 492, 928, 1663, 2855, 4725,
7575, 11812, 17972, 26753, 39051, 56002, 79030, 109905, 150803, 204380]

Ze[10,96] := -x^9 / (-1+x)^9 / (x^4+x^3+x^2+x +1) / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 12, 29, 63, 125, 233, 411, 695, 1132, 1788,
2747, 4122, 6054, 8726, 12363, 17249, 23728, 32225, 43248, 57414]

Ze[10,97] := -x^9 / (-1+x)^9 / (x^4+x^3+x^2+x +1) / (1+x)^2 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 16, 41, 92, 188, 358, 644, 1106, 1827, 2920,
4535, 6869, 10176, 14780, 21089, 29612, 40977, 55953, 75473, 100662]

Ze[10,98] := -(x^6+x^4 +2*x^3+x^2 +1)*x^9 / (-1+x)^9 / (x^2+x +1)^3 / (1+x)^3
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 27, 64, 139, 284, 541, 986, 1721, 2894,
4713, 7467, 11526, 17404, 25750, 37402, 53426, 75168, 104270, 142798]

Ze[10,99] := -x^9 / (-1+x)^9 / (1+x)^3 / (1+x^2) / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 5, 17, 46, 109, 233, 462, 861, 1527, 2596, 4259,
6773, 10484, 15843, 23437, 34012, 48514, 68120, 94291, 128813, 173864]

Ze[10,100] := -x^9 / (-1+x)^9 / (1+x)^3 / (x^2+x +1)^2 / (x^2-x +1) / (1+x^2) / (x^4+x^3+x^2+
x +1)
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[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4, 11, 25, 51, 96, 171, 290, 473, 746, 1144,
1711, 2505, 3597, 5077, 7055, 9667, 13076, 17481, 23117, 30266]

Ze[10,101] := -x^9 / (-1+x)^9 / (x^4+x^3+x^2+x +1) / (1+x)^4 / (x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 10, 23, 52, 102, 194, 342, 586, 957, 1526,
2353, 3557, 5244, 7604, 10813, 15162, 20929, 28545, 38431, 51208]

Ze[10,102] := -x^9*(x^6-2*x^5 +2*x^4-x^3 +2*x^2-2*x +1) / (-1+x)^9 / (1+x)^4 / (1+x^2)^2
 / (1+x^4) / (x^2+x +1)^2 / (x^2-x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 5, 10, 21, 38, 69, 116, 193, 305, 475, 715,
1062, 1537, 2196, 3077, 4263, 5813, 7847, 10457, 13811]

Ze[10,103] := -x^9 / (-1+x)^9 / (x^4+x^3+x^2+x +1) / (1+x)^3 / (1+x^2) / (x^2+x +1)^2
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 8, 18, 37, 70, 126, 215, 354, 563, 871, 1313,
1938, 2803, 3985, 5575, 7689, 10464, 14072, 18712, 24630]

Ze[10,104] := -x^9 / (-1+x)^9 / (1+x)^4 / (x^2+x +1)^2 / (x^2-x +1) / (x^4+x^3+x^2+x +1) / (1+
x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 8, 17, 34, 62, 109, 181, 292, 454, 690, 1021,
1484, 2113, 2964, 4091, 5576, 7500, 9981, 13136, 17130]

Ze[10,105] := -x^9 / (-1+x)^9 / (x^6+x^5+x^4+x^3+x^2+x +1) / (1+x)^3 / (x^2+x +1)^2 / (x^2-
x +1) / (x^4+x^3+x^2+x +1) / (1+x^2)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, 7, 14, 26, 45, 75, 120, 186, 280, 412, 593,
839, 1167, 1600, 2164, 2892, 3821, 4998, 6475, 8316]

Ze[10,106] := -x^9 / (-1+x)^9 / (1+x)^4 / (1+x^2)^2 / (1+x^4) / (x^2+x +1)^3 / (x^2-x +1) / (x^
6+x^3 +1) / (x^6+x^5+x^4+x^3+x^2+x +1) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 41, 54, 73, 94,
123, 157, 201, 252, 318, 393, 488]

-x^9*(106 +516*x +1881*x^2 +5168*x^3 +12144*x^4 +24846*x^5 +46137*x^6 +78410*x^7+
124341*x^8 +184838*x^9 +260134*x^10 +347524*x^11 +443299*x^12 +540541*x^13 +632465*x^
14 +710356*x^15 +767640*x^16 +797905*x^17 +798883*x^18 +769638*x^19 +713980*x^20+
636648*x^21 +545625*x^22 +448194*x^23 +352574*x^24 +264425*x^25 +188722*x^26 +127288*
x^27 +80812*x^28 +47715*x^29 +26018*x^30 +12790*x^31 +5595*x^32 +2066*x^33 +625*x^34+
132*x^35 +19*x^36) / (-1+x)^9 / (1+x)^4 / (1+x^2)^2 / (1+x^4) / (x^2+x +1)^3 / (x^2-x +1) / (x^6
+x^3 +1) / (x^6+x^5+x^4+x^3+x^2+x +1) / (x^4+x^3+x^2+x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 106, 622, 2609, 8399, 23152, 56291, 124958, 256944,
497222, 913243, 1605803, 2717838, 4451308, 7080458, 10976838, 16628471,
24674844, 35933801, 51449040, 72526042, 100795125]

Ze4_10 := -x^9*(75 +257*x +823*x^2 +1646*x^3 +3006*x^4 +4272*x^5 +5668*x^6 +6221*x^7+
6490*x^8 +5653*x^9 +4652*x^10 +3135*x^11 +1932*x^12 +913*x^13 +374*x^14 +93*x^15 +18*x^
16) / (-1+x)^9 / (1+x)^4 / (1+x^2)^2 / (x^2+x +1)^3 / (x^2-x +1)
[0, 0, 0, 0, 0, 0, 0, 0, 0, 75, 482, 2119, 7039, 19810, 48917, 109834, 227899,
444177, 820636, 1449990, 2464258, 4050127, 6461894, 10044281, 15251125,
22677419, 33085478, 47448512, 66985638, 93219783]
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URL: `http://www.mpia.de/~mathar`

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