Speculations on π in a finite Universe.

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Research indicating a finite Universe could imply a value of π that terminates to a measurable length. Experimental observations of approximate sub-Planck scale spacetime pixel length can be used to approximate the length of π in a finite Universe and subsequent philosophical ramifications are discussed.

The relationship between mathematics and physics has often been discussed with irrational numbers as infinite quantities being a primary challenge of overlaying a mathematical model of reality with the physical world in our Universe. Recent research holds confidence that our Universe is finite. Thus the measure of any literal circumference would involve a π, or π approximation, that terminates even perhaps the circumference of the observable universe if we assume it to be a spherical form with a cosmological horizon. The key argument for a finite Universe comes from the recent work of Yasunori Nomura. This is clearly communicated in text taken directly from his article The Quantum Multiverse in *Scientific American*, June 2017 (30-35):

> *"In quantum mechanics, the so-called no-cloning theorem prohibits faithful, full copying of information. …. If you are a distant observer, then the information is outside. You need not describe the interior of the black hole, because you can never access it even in principle; in fact, to avoid cloning information, you must think of the interior spacetime as nonexistent. On the other hand, if you are an observer falling into the hole, then the interior is all you have, and it contains the book and its information. … This problem implies that any description of the quantum state of the universe should include only the region within (and on) the horizon—in particular, there can be no infinite space in any single, consistent description of the cosmos. … Because each of these universes is finite, we avoid the problem of predictability that was raised by the prospect of an infinitely large space that encompasses all possible outcomes…. The multiple universes in this case do not all exist simultaneously in real space—they coexist only in "probability space," that is, as possible outcomes of observations made by people living inside each world. Thus, each universe—each possible outcome—retains a specific probability of coming into being." [1]*

Quantum mechanics calculates an initial maximum for the pixel size of an "atom of spacetime." Historically the pixel size of the Universe has been compared to the Planck length which is calculated to be approximately 1.616 x 10^{-35} meters. However recent observations indicate the likely smallest possible length of spacetime to be much smaller from gamma ray measurements from the ESA Integral spacecraft. "Integral's observations are about 10,000 times more accurate than any previous and show that any quantum graininess must be at a level of 10^{-48} m or smaller." [2] If the limit of granularity of the universe is 10^{-48} m, then this physical world boundary could be used to calculate large scale dimensions of the Universe to approximate values. Using cosmological values for the diameter of the Universe at 8.8 x 10^{26} m [3] we can approximate the scale of the length of the physical representation of the length of π from the circumference of the Universe. If we divide this circumference derived π value by the miniscule value of a pixel length of the Universe, we should obtain a maximum possible length of "digits" or physical pixels for the length of π in the physical Universe, i.e. the point or length at which π terminates. Simple arithmetic shows that the length of π corresponding to pixilation sizes of 10^{-48}, and $^{-50}$ and $^{-53}$ meters. At 10^{-53} m our length value of 10^{80} pixels is interesting in it similarity of scale to the estimate of the number of total particles in the Universe (10^{80}) [4].

Figure 1: Estimated circumference of observable Universe over possible sub-Planck length spacetime "pixel" lengths to calculate length of pi in the physical Universe.

$$\frac{(27.632 \cdot 10^{26})}{(1.616 \cdot 10^{-48})} = 1.709900990099 \times 10^{75}$$

$$\frac{(27.632 \cdot 10^{26})}{(1.616 \cdot 10^{-50})} = 1.709900990099 \times 10^{77}$$

$$\frac{(27.632 \cdot 10^{26})}{(1.616 \cdot 10^{-53})} = 1.709900990099 \times 10^{80}$$

Philosophical Ramifications

The cosmological horizon as the spherical boundary of our Universe could represent not only the boundary to ensure the no-cloning theorem of quantum mechanics is maintained but this very size or diameter of the spherical horizon could be used to redefine the origin of the speed of light. In essence, the speed of light "c" would be the maximum velocity to ensure quantum information that is identical remains separate (invisible) in a separate or distinct Universe; a larger velocity of light could allow the communication of quantum information that must remain beyond the cosmological horizon. Quantum entanglement and special relativistic effects must be factored into any working theory here.

If we make an extreme assumption that the diameters of Universes are the same or similar in length, then we have the possibility of using the number of possible combinations of a base 10 length of π of this length to estimate the number of possible Universes where if 10^{80} as the length of π (base 10) then each normal possible combination of π digits (0,1, 2, …9) could represent a universe in which case we have $(10^{79})^{10}$ or 10^{790} universes in our probability space Cosmos or multi-verse.

Additional areas of philosophical focus would include the validity and output of the BBP (Bailey-Borwein-Plouffe) algorithm given a terminating physical world π as well as Euler's constant as seen in the equation $ln(-1) = i\pi$. Additional areas of research could involve the surface area of the sphere of the Universe (cosmological horizon) versus the holographic paradigm or even versus computer science concepts of digital memory compared to the total number of "pixels" (or the numerical capacity therein in binary) of this spherical surface area versus the number of total atoms or particles in the Universe (approximated at 10^{80}) [4].

If we envision at the foundations of the Universe a root reality of "structure and randomness," then perhaps the length of a terminating π in a finite Universe, since the arrangement of the digits of π have often been used as a measure of randomness (digits are considered to have a normal distribution), could represent the degree of "complexity" or a measure of randomness specific to, or required for, the complexity of our Universe. The length of π could be the minimum "granularity" required for the physics and chemistry of our Universe, not to mention the scale at which the curve of a circle ceases to be smooth in the physical Universe.

[1] Y. Nomura, The Quantum Multiverse, *Scientific American*, June (2017).
[2] P. Laurent, et al, arXiv:1106.1068v1 [astro-ph.HE].
[3] I. Bars, J. Terning, *Extra Dimensions in Space and Time*, Springer. (2011).
[4] S. Lloyd, Universe is a Computer, *Nature,* June (2002).