From E.G. Haug Escape Velocity
To the Golden Ratio at the Black Hole

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Abstract
Escape velocity from the E.G. Haug has been checked. It is compared with orbital velocity formula for an ideal circular path. The formulas are simplified so that we have only one variable that contains the Planck values and the mass of the central body. In the case of an arbitrary star, the values of these velocities are determined during its compression to the black hole. Unlike the standard and relativistic formulas that are approximations for a weak gravitational field, Haug's formula is exact for a weak and strong gravitational field. The relationships between formulas showed the importance of the golden ratio below the Schwarzschild radius.

Key words: Planck, Haug, escape velocity, golden ratio, black hole

1. Introduction

We will use the known formula for the orbital velocity for the ideal circular path of the radius r, body revolve around the central mass M, [1]:

$$V = \sqrt{\frac{GM}{r}} \qquad \textbf{(1)}$$

From the known relationship for G - The universal gravitational constant, c - light velocity, l_{pl} - Planck length and m_{pl} - Planck mass [2] we have:

$$G = c^2 * \frac{l_{pl}}{m_{pl}} \qquad \textbf{(2)}$$

From [3, formula 4] we take the formula here in the form (3).

$$V_{eh} = c * \frac{\sqrt{1 + 2c^2 r / GM}}{(1 + c^2 r / GM)} \qquad \textbf{(3)}$$

Where V_{eh} – is escape velocity by E. G. Haug. Note that in [3], the transition from Haug's formula 3 to 4, may be displayed in more steps. The accuracy of the pass, I checked thanks to Wolfram Alpha, and confirmed the correctness of formula. Substituting (2) into (3) we obtain:

$$V_{eh} = c * \frac{\sqrt{1+2rm_{pl} / l_{pl}M}}{(1+rm_{pl} / l_{pl}M)} \qquad \textbf{(4)}$$

To note that my attempts with other escape velocities, from literature, resulted in rejection of proposed formulas. Only Haug's formula (3) passed the test and also has a solid derivation and a rational explanation in [3].

2. Preparation for analysis

For the sake of simplicity we introduce (5):

$$x = \frac{c^2 r}{GM} = \frac{rm_{pl}}{l_{pl}M} \qquad \textbf{(5)}$$

It is obviously, x dimensionless. If we include (1) we get:

$$x = \frac{c^2}{V^2} \qquad \textbf{(6)}$$

That is, x is the ratio of the square of the speed of light to the velocity V. The ratio β = v / c is often used in literature, so that for v=V is: $x = c^2 / v^2 = 1/\beta^2$. Note that: "*v is the relative velocity between inertial reference frames"*, [4]. In our case, the velocity V is defined exclusively by equation (1), and therefore we use x instead β, so that no confusion arises.

In the case of the planet Earth, $x = 1.4 * 10^9$, while for the Sun and stars about four orders of magnitude are smaller. Now, (1) and (4) can be written in the form (7) and (8):

$$V = \frac{c}{\sqrt{x}} \qquad \textbf{(7)}$$

$$V_{eh} = c * \frac{\sqrt{1+2x}}{1+x} \qquad \textbf{(8)}$$

For celestial bodies, formula (7) multiplied by √2, gives a standard value for escape velocity, which is not very different from (8), until the radii of near Schwarzschild's reach, when in the standard approach there are speeds larger of the speed of light. At even smaller radii there is a collapse of equations and black hole singularities, which is not only a mathematically undefined state, but also a physical nonsense, see [3, Table 1]. Formulas (7) and (8) will be further analysed in the next section.

3. An example of a star

Let's take a definition from [5]: "*Black hole, cosmic body of extremely intense gravity from which nothing, not even light, can escape*."

There is no black hole in the above definition, because there is no situation "that light cannot escape." We will, for historical reasons, retain the term "black hole" in the meaning of "cosmic body of extremely intense gravity", which is in fact a body that tends toward a black hole.

We will analyse the results obtained by the preceding formulas in the case of a gravitational compression of a star having a mass of about 5 times the mass of the Sun. We will form Table 1, similarly to [3, Table 1], but for the sake of simplicity, instead of the Schwarzschild radius, we use half of that value, r_b:

$$r_b = \frac{GM}{c^2} \qquad \textbf{(9)}$$

Let's call this value the basic radius, r_b, so we will express other radii as the products of this radius. For example, Schwarzschild's radius $r_s = 2 * r_b$.

Let's emphasize that the calculations here are based on the idealized situation in which the mass of the central body does not change and there are no other bodies in its vicinity that make the calculations more complex. At real cosmological situations should take into account as many neighbouring influences as possible.

The future of a star is a gravitational compression, or smaller radii. The star also had its own history, that is, larger radii, but here is the theme of work, the behaviour of the star in its final stage of development, near the state of the black hole. Therefore, let us examine the equalization of the formulas (7) and (8), ie $V = V_{eh}$, that is:

$$\frac{c}{\sqrt{x}} = c * \frac{\sqrt{1+2x}}{1+x} \qquad \textbf{(10)}$$

Or by shortening with c:

$$\frac{\sqrt{(1+2x}}{1+x} = \frac{1}{\sqrt{x}} \qquad \textbf{(11)}$$

We got the equation with one dimensionless variable x, which thanks to the software "Wolfram Alpha" we easily find the solution:

$$x = \frac{\sqrt{5}+1}{2} = \varphi \qquad \textbf{(12)}$$

This is a well-known value of the "Golden ratio", which we mark with φ. To summarize: the orbital velocity from (1) and Haug's escape velocity from (3) are equal at the radius which is φ times greater than the basic radius, r_b, or:

$$V_{eh} = V, \qquad \text{for} \qquad r = \varphi * r_b \qquad \textbf{(13)}$$

Or, if we include (9):

$$V_{eh} = V, \qquad \text{for} \qquad r = \varphi * \frac{GM}{c^2} \qquad \textbf{(14)}$$

The radius that satisfies (14), let's call the "Golden radius" and mark it with r_φ. We can say that each mass has its own Golden radius, if $r = GM / c^2$ is greater than the lower limit for the length, for that mass. If in the preceding equation we take the Planck mass than: $r = Gm_{pl} / c^2 = l_{pl}$. That is, Planck's mass is the smallest mass that can tends to the black hole and its golden radius is also the minimum radius of the black hole:

$$r_\varphi(planckmass) = \varphi * \frac{Gm_{pl}}{c^2} = \varphi * l_{pl} = 2.6151*10^{-35} m \qquad \textbf{(15)}$$

The radius r_φ(*Planck mass*) is φ times larger than Plank's length, which is the lower limit for the length. These black holes are called "mini black holes" [6]. At this radius is also the maximum density ($\rho_{max} = \rho_{rb} / \varphi^3$) and the maximum gravity ($a_{max} = a_{rb} / \varphi^2$) for black holes. It is easy to show that especially for the Planck mass, ρ_{rb} and a_{rb} are Planck's values for density and acceleration ($\rho_{rb}=\rho_{pl}$ and $a_{rb}=a_{pl}$).

In Table 1. we choose the exponents, k = 1 to 6 and k = 1.4404 to cover the Schwarzschild radius, where x = 2. Note, the basic radius is for k=0, x=1, $r_b=GM/c^2=7.42564*10^3$m.

Table 1. Velocities near the black hole for a star of 10^{31} kg

$x=\varphi^k$	$r=x*r_b$	V	V_{eh}	$(V_{eh}/V)^2$	$(c/V_{eh})^2$
17,944	1,3325E+05	7,077E+07	9,611E+07	1,844427191	9,72891
11,090	8,2352E+04	9,002E+07	1,194E+08	1,758705776	6,30587
6,854	5,0896E+04	1,145E+08	1,464E+08	1,634244881	4,194048
4,236	3,1456E+04	1,457E+08	1,762E+08	1,463525492	2,894427
2,618	1,9441E+04	1,853E+08	2,069E+08	1,247213595	2,099106
2,000	1,4851E+04	2,120E+08	2,235E+08	1,111111111	1,800000
1,618	1,2015E+04	**2,357E+08**	**2,357E+08**	1,000000000	**1,618034**
1,000	**7,42564E+03**	2,998E+08	2,596E+08	0,750000000	1,333333

The velocities near to the black hole are shown graphically too.

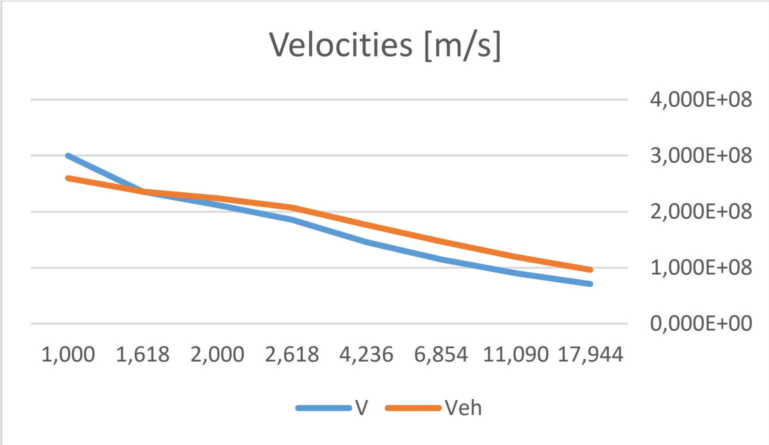

Figure 1. Velocities near to the black hole
(At abscise are the products of the radius $r_b = GM / c^2$)

From (7) and (8) we get:

$$\frac{V_{eh}^2}{V^2} = \frac{x + 2x^2}{(1+x)^2} \qquad \textbf{(16)}$$

From where we analyse the ratio of velocities to show the results for some characteristic radii:

- For large x, $(V_{eh} / V)^2 = (x+2x^2) / (1+x)^2 \approx 2x^2 / x^2$ tend to 2, that is: V_{eh} tends to $\sqrt{2} * V$ In other words, for a weak gravitational field, exact Haug's solution gives same result as the standard and GR approaches see [3, Chapter 2].

- For x = 2 ie. Schwarzschild's radius, $(V_{eh} / V)^2 = (2 + 8) / (1 + 2)^2 = 10/9$, respectively: $V_{eh} = \sqrt{(10/9)} * V$ or $(c / V_{eh})^2 = 9/5$ (see Table 1) or at [3] $V_{eh} = c * \sqrt{5} / 3$.

- For $x = \varphi$, $r_\varphi = \varphi * r_b$, $(Veh / V)^2 = 1/1$, respectively:

$$r_\varphi = \varphi * \frac{GM}{c^2} \qquad \textbf{(17)}$$

$V_{eh} = V = 2.357 * 10^8$ m/s, $(c/V_{eh})^2 = 1{,}618034 = \varphi$, or V_{eh} is always less than the speed of light, and therefore there is no black hole. r_φ is the smallest radius that can reach some mass, similar to (15) for the mini-black hole. At this radius is also the maximum density and the maximum gravity for the used mass. This is due to the fact that in (17) the mass and its gold radius are proportional, and if in mass M there are n Planck masses, then there is also n radii from (15):

$$r_\varphi(M) = n * r_\varphi(m_{pl}) = \varphi * \frac{GM}{c^2} = n * \varphi * \frac{Gm_{pl}}{c^2} = n * \varphi * l_{pl} \qquad \textbf{(18)}$$

It follows that $r_\varphi(M)=n*\varphi*l_{pl}$ is the minimum radius for the black hole generated by the mass M, because for a smaller radius it is necessary that some ingredient in a black hole is smaller than the minimum, r_φ(*Planckmass*). From the previous we conclude that the golden radius $r_\varphi=\varphi*GM/c^2$ is more significant than the Scwarzchild radius $r_s=2*GM/c^2$.

Haug claim [3, page 3]: "***at a radius considerably below the Schwarzschild radius, the escape velocity is approaching c*. *This is in sharp contrast to the standard approximate escape velocity of modern physics that predicts that escape velocity at the Schwarzschild radius is c and that the escape velocity inside the Schwarzschild radius is > c."*** Although the Haug's statement is much more accurate than the dominant position, it is even more precise to say that the radius $r_\varphi = \varphi * r_b$, $V_{eh} = V = 2.357 * 10^8$ m/s, or we can say: at a radius below the Schwarzschild radius, the escape velocity approaches $c / \sqrt{\varphi}$. It can be seen in Figure 1. As well as a mini-black hole such as in the example of the star in Table 1 the maximum density and a maximum of gravitational attraction are related to the radius of r_φ.

- For $x < 1$, the radius is $r < \varphi * r_b$ and $V_{eh} < V$. Than the radius is smaller than the smallest radius r_φ, such a black hole cannot exist.

It is interesting to note that these relationships are the same for all bodies. These results would be interesting to compare with existing knowledge about the stars.

4. Conclusion

Haug's formula (3) was derived and explained in [3], while further possibilities of this formula are shown here. In contrast to an irrational modern understanding involving speeds greater than the speed of light, black hole singularities and equations that breaks, we have a rational approach in [3]. Here it is supported by the mechanism shown by formula (14) which shows the restriction of growth of escape velocity.

Two new radii at black holes are defined: the basic radius, $r_b=GM/c^2$ and the golden radius, $r_\varphi=\varphi*GM/c^2$ and the significance of these radii is shown.

Golden ratio obtained in formulas related to the black hole is probably the way to shift processes in nature. Here dominant gravity gives way to radiation and the process that threatens to pass into infinite gravity and infinite density moves in the opposite direction.

In order to come up with previous results, I have been helped by a philosophical approach expressed in several of my papers, for example in [7] with the following views:

Parts are dependent on the whole (Universe) and are also an integral part of the whole, therefore, the whole is also dependent on the parts!

And:

Matter dominant Universe and radiation dominant Universe coexist in every point in time!

That can all be combined under relational approach [8].

References:

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