PHYSICAL INTERACTIONS AS GEOMETRIC PROCESSES

Vu B Ho

Advanced Study, 9 Adela Court, Mulgrave, Victoria 3170, Australia

Email: vubho@bigpond.net.au

Abstract: In this work we discuss the possibility to identify physical interactions with geometric processes whose revolutions can be described by the Ricci flow. In particular, we show that it is possible to suggest that the charge of an elementary particle does not exist as a physical quantity possessed by the elementary particle itself but rather a collective dynamical effect that is associated with the intermediate particles, which are the force carriers. These force carrying particles have the geometric structures of two-dimensional spheres or n-tori. Furthermore, since the Ricci flow on two-dimensional manifolds does not give rise to neckpinches and if such geometric flows can be shown to not exist then it can be stated that the force carrying particles are the only particles that are truly fundamental.

According to Einstein theory of general relativity, dynamical interactions are the results of the changes of the intrinsic geometric structures of spacetime which are governed by matter and energy that are supposed to be contained in the spacetime continuum. It is seen from such formulation that there is a clear distinction between the concept of geometric structures and physical entities [1]. On the other hand, it has been shown in recent developments in differential geometry that there are geometric processes that can also be used to describe the revolutions of the intrinsic geometric structures of differentiable manifolds, in particular the intrinsic geometric processes such as the Ricci flow [2,3]. Even though these two descriptions of the changes of the intrinsic geometric structures of spacetime continuum seem to be different, one from the physical point of view and one from the mathematical point of view, it is reasonable to suggest that physical interactions are in fact geometric processes. If this is the case then both of them should be derived from the same mathematical formulation, and as shown in our previous work that this mathematical formulation can be based on the Bianchi identities in differential geometry [4,5]. It should be emphasised here that the conception of formulation of physical phenomena put forward in general relativity is hold true not only for gravitation but also for all types of physical interactions [6].

It is shown in differential geometry that the Ricci curvature tensor $R^{\alpha\beta}$ satisfies the Bianchi identities [7]

$$\nabla_\beta R^{\alpha\beta} = \frac{1}{2} g^{\alpha\beta} \nabla_\beta R \tag{1}$$

If the quantity $\frac{1}{2} g^{\alpha\beta} \nabla_\beta R$ can be identified as a physical entity, such as a four-current of gravitational matter, then Equation (1) has the status of a dynamical law of a physical theory which can be postulated as the field equations of the gravitational field. In this case, as in the case of the electromagnetic field, the energy-momentum tensor $T_{\alpha\beta}$ for the gravitational field can be established in terms of the Ricci curvature tensor $R_{\alpha\beta}$ and the metric tensor $g_{\alpha\beta}$ as

$$T_{\alpha\beta} = k\left(R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R\right) \tag{2}$$

Equation (2) is Einstein field equations of general relativity. For a purely gravitational field in which $\frac{1}{2}g^{\alpha\beta}\nabla_{\beta}R = 0$, Equation (1) reduces to the equation

$$\nabla_{\beta}R^{\alpha\beta} = 0 \tag{3}$$

In the following we will show that solutions that are found from the original Einstein field equations given in Equation (2), such as the Schwarzschild solution, and the Ricci flow that is used to describe geometric processes of three-dimensional differentiable manifolds can be obtained from Equation (3). Since $\nabla_{\mu}g^{\alpha\beta} \equiv 0$, from Equation (3) we obtain

$$R_{\alpha\beta} = \Lambda g_{\alpha\beta} \tag{4}$$

where Λ is an undetermined constant. If we consider a centrally symmetric gravitational field with the metric

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{5}$$

then the Schwarzschild solution can be found as [8]

$$ds^2 = \left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}\right)c^2dt^2 - \left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{6}$$

On the other hand, as has also been shown in our previous works on the Ricci flow that within the group of coordinate transformations that are time-independent, from Equation (4) we can derive the Ricci flow [5,9]

$$\frac{\partial g_{\alpha\beta}}{\partial t} = kR_{\alpha\beta} \tag{7}$$

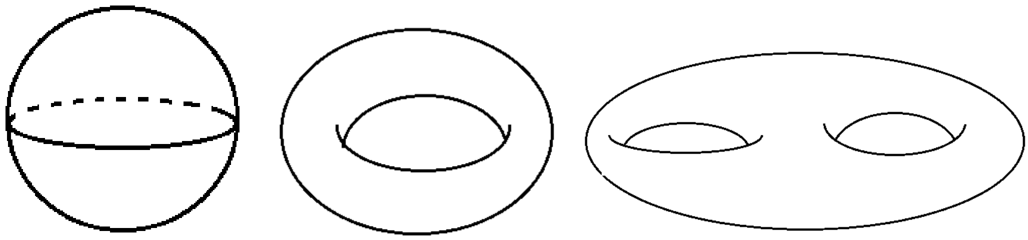
where k is a scaling factor.

The above results show that physical interactions and geometric processes are two different perceptions, which result in two different formulations of the dynamics of revolution, of the same intrinsic geometric structures of spacetime. With this view, we can identify quantum particles as three-dimensional differentiable manifolds, which may be subjected to intrinsic geometric flows described by the Ricci flow or physical interactions described by physical laws in order to smooth out irregularities of their geometric structures. If physical interactions are in fact geometric processes then the formulation of a composite physical system is equivalent to either the composition of a three-dimensional differentiable manifold from Thurston geometries or the decomposition into those geometries [10]. However, there is still a profound difference between the formulation of a physical law and the description of a geometric process. The mathematical descriptions of the geometric revolution of a differentiable manifold have been involved only with the possibility of composition and decomposition rather than a formulation that can be used to quantify a geometric process. For example, in Perelman works [11,12,13], all mathematical works out were for the purpose of proving the existence of a geometric structure rather than the quantity of work that is needed to be done to obtain the final geometric structure. This is the same as proving the existence of the gravitational field by showing the fall of an apple without the need to devise Newton's law of gravitation. In order to quantify the geometric processes we would need to formulate geometric laws in the form of physical laws. The starting point of formulating these geometric laws is to consider the composition and decomposition of three-dimensional differentiable manifolds, such as the formalism of the construction of composite spacetimes as described in the works of Yasuno *et al* [14]. They showed that a composite spacetime can be constructed from spatially compact locally homogeneous spacetimes (SCLHS) by gluing two different SCLHSs along two timelike shells. If each of the two SCLHSs admits at least one pair of two commuting local Killing vectors then each has a homogeneous torus section. Timelike hypersurface that can be cut from each SCLHS and then glued along the resulting boundaries is differomorphic to a homogeneous torus. In general, a compact three-dimensional manifold is composite if it can be decomposed along the embedded two-spheres. The decomposition of a three-dimensional manifold will produce three types of prime manifolds, which are the spherical types, $S^2 \times S^1$ and $K(\pi, 1)$. Of these three prime manifolds, only the prime manifold $K(\pi, 1)$ can be decomposed along embedded tori. We may speculate further from these geometric revolutions that the intermediate particles, which are the force carriers of physical fields and radiated from a physical system, may possess the geometric structures of the two-spheres and the n-tori. This speculation leads to a more profound speculation that physical properties assigned to an elementary particle, such as charge, are in fact manifestations due to the force carriers rather than physical quantities that are contained inside the elementary particle. If this is the case then the analysis of physical interactions will be reduced to the analysis of the geometric processes that are related to the geometric structures of the force carriers, which are the two-spheres and the n-tori for the above investigation. Therefore, for observable physical phenomena, the study of physical dynamics reduces to the study of the change of two-dimensional Riemannian surfaces.

It has been shown in differential geometry that all two-dimensional Riemannian manifolds admit a geometric structure which can be modelled on one of the two-sphere S^2, the Euclidean two-space E^2 or the hyperbolic two-space H^2. Therefore, in two dimensions there are only three geometries. Two-dimensional compact surfaces can also be classified according to their topological properties in which they are diffeomorphic to the two-sphere, the connected sum of n torus or the connected sum of m projective planes. Furthermore, the uniformisation theorem classifies closed orientable Riemannian two-dimensional manifolds of constant curvatures of values of -1, 0 and $+1$. On the other hand, in classical electrodynamics, as shown by experiments, the electric charge is a conserved property of elementary particles that can take integral values of -1, 0 and $+1$. Is this just a mere coincidence or the charge of an elementary particle is in fact a manifestation of the curvature of the geometric structure of the force carrying particles? It is worth mentioning here that the Ricci flow on two-dimensional differentiable manifolds can be represented as the Gauss Bonnet theorem as follows [15]. If M is a compact two-dimensional Riemannian manifold with boundary ∂M then we have

$$\int KdA + \int k_g ds = 2\pi\chi(M) \tag{8}$$

where K is the Gaussian curvature of M, k_g is the geodesic curvature of ∂M, dA is the area element of the surface, ds is the line element along the boundary ∂M and $\chi(M)$ is the Euler characteristic of M. If M is a compact manifold without boundary then $\int k_g ds = 0$. It states that the total Gaussian curvature of such a closed surface is equal to 2π times the Euler characteristic of the surface. For compact surfaces without boundary, the Euler characteristic equals $2 - 2g$, where g is the genus which counts the number of holes the surface has. Since any compact surface without boundary is topologically equivalent to a sphere with attached handles and in this case g counts the number of handles. For a two-dimensional sphere $g = 0$, a torus $g = 1$ and a double torus $g = 2$. Therefore, besides the conventional signs, the force carrying particles of an electron could be associated with a sphere, those of a neutron with a torus and those of a proton with a double torus.



As shown in our previous works [16], the above associations can be formulated in terms of the Feynman integral method in quantum mechanics [17]. The Feynman's method of sum over random paths can be extended to higher-dimensional spaces to formulate physical theories in which the transition amplitude between states of a quantum mechanical system is the sum over random hypersurfaces. This generalisation of the path integral method in quantum mechanics has been developed and applied to other areas of physics, such as condensed matter physics, quantum field theories and quantum gravity theories, mainly for the purpose of field quantisation. In the following, however, we focus attention on the general idea of a sum over random surfaces. This formulation is based on surface integral methods by generalising the differential formulation as discussed for the Bohr's model of a hydrogen-like atom. Consider a surface defined by the relation $x^3 = f(x^1, x^2)$. The Gaussian curvature is given by $K = (f_{11}f_{22} - (f_{12})^2)/(1 + f_1^2 + f_2^2)^2$, where $f_\mu = \partial f/\partial x^\mu$ and $f_{\mu\nu} = \partial^2 f/\partial x^\mu \partial x^\nu$ [18]. Let P be a 3-dimensional physical quantity which plays the role of the momentum p in the 2-dimensional space action integral. The quantity P can be identified with the surface density of a physical quantity, such as charge. Since the momentum p is proportional to the curvature κ, which determines the planar path of a particle, it is seen that in the 3-dimensional space the quantity P should be proportional to the Gaussian curvature K, which is used to characterise a surface. If we consider a surface action integral of the form $S = \int PdA = \int (q/2\pi)\, KdA$, where q is a universal constant, which plays the role of Planck's constant, then we have

$$S = \frac{q}{2\pi} \int \frac{f_{11}f_{22} - (f_{12})^2}{(1 + f_1^2 + f_2^2)^{3/2}} dx^1 dx^2 \tag{9}$$

According to the calculus of variations, similar to the case of path integral, to extremise the action integral $S = \int L(f, f_\mu, f_{\mu\nu}, x^\mu) dx^1 dx^2$, the functional $L(f, f_\mu, f_{\mu\nu}, x^\mu)$ must satisfy the differential equations

$$\frac{\partial L}{\partial f} - \frac{\partial}{\partial x^\mu}\frac{\partial L}{\partial f_\mu} + \frac{\partial^2}{\partial x^\mu \partial x^\nu}\frac{\partial L}{\partial f_{\mu\nu}} = 0 \tag{10}$$

However, it is straightforward to verify that with the functional of the form $L = (q/2\pi)\,(f_{11}f_{22} - (f_{12})^2)/(1 + f_1^2 + f_2^2)^{3/2}$ the differential equations given by Equation (10) are satisfied by any surface. Hence, we can generalise Feynman's postulate to formulate a quantum theory in which the transition amplitude between states of a quantum mechanical system is a sum over random surfaces, provided the functional P in the action integral $S = \int PdA$ is taken to be proportional to the Gaussian curvature K of a surface. Consider a closed surface and assume that we have many such different surfaces which are described by the higher dimensional homotopy groups. As in the case of the fundamental homotopy group of paths, we choose from among the homotopy class a representative spherical surface, in which case we can write

$$\oint PdA = \frac{q}{4\pi} \oint d\Omega, \tag{11}$$

where $d\Omega$ is an element of solid angle. Since $\oint d\Omega$ depends on the homotopy class of the sphere that it represents, we have $\oint d\Omega = 4\pi n$, where n is the topological winding number of the homotopy group. From this result we obtain a generalised Bohr quantum condition

$$\oint PdA = nq \tag{12}$$

From the result obtained in Equation (12), as in the case of Bohr's theory of quantum mechanics, we may consider a quantum process in which a physical entity transits from one surface to another with some radiation-like quantum created in the process. Since this kind of physical process can be considered as a transition from one homotopy class to another, the radiation-like quantum may be the result of a change of the topological structure of the physical system, and so it can be regarded as a topological effect. Furthermore, it is interesting to note that the action integral $(q/4\pi) \oint KdA$ is identical to Gauss's law in electrodynamics. In this case the constant q can be identified with the charge of a particle. In particular, the charge q represents the topological structure of a physical system must exist in multiples of q. Hence, the charge of a physical system, such as an elementary particle, may depend on the topological structure of the system and is classified by the homotopy group of closed surfaces. This result may shed some light on why charge is quantised even in classical physics.

References

[1] A. Einstein, *The Principle of Relativity*, Dover Publications, New York, 1952.

[2] Richard S. *Hamilton, Three-Manifolds with Positive Ricci Curvature*, J. Diff. Geo., **17** (1982) 255-306.

[3] Huai-Dong Cao and Xi-Ping Zhu, *A Complete Proof of the Poincaré and Geometrization Conjectures-Application of the Hamilton-Perelman Theory of the Ricci Flow,* Asian J. Math., Vol **10**, No. 2, 165-492, June 2006.

[4] Vu B Ho, *Spacetime Structures of Quantum Particles* (Preprint, ResearchGate, 2017), viXra 1708.0192v1.

[5] Vu B Ho, *A Derivation of the Ricci Flow* (Preprint, ResearchGate, 2017), viXra 1708.0191v1.

[6] E. Schrödinger, *Space-Time Structure*, Cambridge University Press, New York, 1950.

[7] Ray D'Inverno, *Introducing Einstein's Relativity*, Clarendon Press, Oxford, 1992.

[8] D. Kramer, H. Stephanie, E. Hert and M. MacCallum, *Exact solutions of Einstein's field equations* (Cambridge University Press, 1980).

[9] Vu B Ho, *A Covariant Ricci Flow* (Preprint, ResearchGate, 2017), viXra 1708.0424v1.

[10] P. Scott, *The Geometries of 3-Manifolds,* Bull. London Math. Soc. **15**, 1983.

[11] G. Perelman, *The entry formula for the Ricci flow and its geometric applications*, arXiv: math/0211159v1, 2002.

[12] G. Perelman, *Ricci flow with surgery on three-manifold*, arXiv: math/0303109v1, 2003.

[13] G. Perelman, *Finite extinction time for the solutions to the Ricci flow on certain three-manifolds*, arXiv: math/0307245v1, 2003.

[14] K. Yasuno, T. Koike and M. Siino, Thurston's Geometrization Conjecture and cosmological models, arXiv:gr-qc/0010002v1, 2000.

[15] M. Spivak, *A Comprehensive Introduction to Differential Geometry* (Publish or Perish, Berkerley, 1979).

[16] Vu B Ho, *On the Principle of Least Action*, ResearchGate (2016), viXra 1708.0226v2, Int. J. Phys., **6**, 2 (2018).

[17] R. P. Feynman, *Rev. Mod. Phys*. **20**, 367 (1948).

[18] E. Kreyszig, *Introduction to Differential Geometry and Riemannian Geometry* (University of Toronto Press, 1975).