

# Bipolar Complex Neutrosophic Graphs of Type 1

Said Broumi<sup>1,\*</sup>, Assia Bakali<sup>2</sup>, Mohamed Talea<sup>3</sup>, Florentin Smarandache<sup>4</sup>, V. Venkateswara Rao<sup>5</sup>

<sup>1,3</sup> Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco. E-mail: broumisaid78@gmail.com, taleamohamed@yahoo.fr

<sup>2</sup>Ecole Royale Navale-Boulevard Sour Jdid, B.P 16303 Casablanca, Morocco. E-mail: assiabakali@yahoo.fr

<sup>4</sup>Department of Mathematics, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA. E-mail: fsmarandache@gmail.com

<sup>5</sup>Department of S&H, VFSTR University, Vadlamudi - 522 207, Guntur, India  
E-mail: vunnamvenky@gmail.com

## ABSTRACT

*In this paper, we introduced a new neutrosophic graphs called bipolar complex neutrosophic graphs of type 1 (BCNG1) and presented a matrix representation for it and studied some properties of this new concept. The concept of BCNG1 is an extension of generalized fuzzy graphs of type 1 (GFG1), generalized single valued neutrosophic graphs of type 1 (GSVNG1), Generalized bipolar neutrosophic graphs of type 1 (GBNG1) and complex neutrosophic graph of type 1 (CNG1).*

**KEYWORDS:** Bipolar complex neutrosophic set; Bipolar complex neutrosophic graph of type 1; Matrix representation.

## 1. INTRODUCTION

In 1998, (Smarandache, 1998), introduced a new theory called Neutrosophy, which is basically a branch of philosophy that focus on the origin, nature, and scope of neutralities and their interactions with different ideational spectra. Based on the neutrosophy, Smarandache defined the concept of neutrosophic set which is characterized by a degree of truth membership  $T$ , a degree of indeterminate-membership  $I$  and a degree false-membership  $F$ . The concept of neutrosophic set theory is a generalization of the concept of classical sets, fuzzy sets (Zadeh, 1965), intuitionistic fuzzy sets (Atanassov, 1986), interval-valued fuzzy sets (Turksen, 1986). Neutrosophic sets is mathematical tool used to handle problems like imprecision, indeterminacy and inconsistency of data. Specially, the indeterminacy presented in the neutrosophic sets is independent on the truth and falsity values. To easily apply the neutrosophic sets to real scientific and engineering areas, (Smarandache, 1998) proposed the single valued neutrosophic sets as subclass of neutrosophic sets. Later on, (Wang et al., 2010) provided the set-theoretic operators and various properties of single valued neutrosophic sets. The concept of neutrosophic sets and their extensions such as bipolar neutrosophic sets, complex neutrosophic sets, bipolar complex neutrosophic sets (Broumi et al. 2017) and so on have been applied successfully in several fields (<http://fs.gallup.unm.edu/NSS/>).

Graphs are the most powerful tool used in representing information involving relationship between objects and concepts. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from  $[0, 1]$ . In (Shannon and Atanassov, 1994) introduced the concept of intuitionistic fuzzy graphs (IFGs) using five types of Cartesian products. The concept fuzzy graphs and their extensions have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects.

When description of the object or their relations or both is indeterminate and inconsistent, it cannot be handled by fuzzy graphs and their particular types (Sharma et al., 2013; Arindam et al., 2012, 2013). So, for this reason, (Smarandache, 2015) proposed the concept of neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. Then, (Smarandache, 2015, 2015a) introduced another version of neutrosophic graph theory using the neutrosophic truth-values (T, I, F) and proposed three structures of neutrosophic graphs: neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. Later on (Smarandache, 2016) proposed new version of neutrosophic graphs such as neutrosophic offgraph, neutrosophic bipolar/tripolar/multipolar graph. Presently, works on neutrosophic vertex-edge graphs and neutrosophic edge graphs are progressing rapidly. (Broumi et al., 2016) combined the concept of single valued neutrosophic sets and graph theory, and introduced certain types of single valued neutrosophic graphs (SVNG) such as strong single valued neutrosophic graph, constant single valued neutrosophic graph, complete single valued neutrosophic graph and investigate some of their properties with proofs and examples. Also, (Broumi et al., 2016a) also introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. In addition, (Broumi et al., 2016b) proved a necessary and sufficient condition for a single valued neutrosophic graph to be an isolated single valued neutrosophic graph. After Broumi, the studies on the single valued neutrosophic graph theory have been studied increasingly (Broumi et al., 2016c, 2016d, 2016e, 2016g, 2016h, 2016i; Samanta et al., 2016; Mehra, 2017; Ashraf et al., 2016; Fathi et al., 2016)

Recently, (Smarandache, 2017) initiated the idea of removal of the edge degree restriction of fuzzy graphs, intuitionistic fuzzy graphs and single valued neutrosophic graphs. (Samanta et al., 2016) introduced a new concept named the generalized fuzzy graphs (GFG) and defined two types of GFG, also the authors studied some major properties such as completeness and regularity with proved results. In this paper, the authors claims that fuzzy graphs and their extension defined by many researches are limited to represent for some systems such as social network. Later on (Broumi et al., 2017) have discussed the removal of the edge degree restriction of single valued neutrosophic graphs and defined a new class of single valued neutrosophic graph called generalized single valued neutrosophic graph of type 1, which is an extension of generalized fuzzy graph of type 1 (Samanta et al., 2016). Later on (Broumi et al., 2017a) introduced the concept of generalized bipolar neutrosophic of type 1. In addition, (Broumi et al., 2017b) combined the concept of complex neutrosophic sets with generalized single valued neutrosophic of type 1 (GSVNG1) and introduced the complex neutrosophic graph of type 1 (CNG1). Up to day, to our best knowledge, there is no research on bipolar complex neutrosophic graphs.

The main objective of this paper is to extend the concept of complex neutrosophic graph of type 1 (CNG1) introduced in (Broumi et al., 2017b) to bipolar complex neutrosophic graphs of type 1 and showed a matrix representation of BCNG1.

The remainder of this paper is organized as follows. In Section 2, we review some basic concepts about neutrosophic sets, single valued neutrosophic sets, complex neutrosophic sets, bipolar complex neutrosophic sets, generalized fuzzy graph, generalized single valued neutrosophic graphs of type 1, generalized bipolar neutrosophic graphs of type 1 and complex neutrosophic graph of type 1. In Section 3, the concept of complex neutrosophic graphs of type 1 is proposed with an illustrative example. In section 4 a representation matrix of complex neutrosophic graphs of type 1 is introduced. Finally, Section 5 outlines the conclusion of this paper and suggests several directions for future research.

## 2. PRELIMINARIES

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, complex neutrosophic sets, bipolar complex neutrosophic sets, generalized fuzzy graph, generalized single valued neutrosophic graphs of type 1, generalized bipolar neutrosophic graphs of type 1 and complex neutrosophic graph of type 1 relevant to the present work. See especially (Smarandache, 1998; Wang et al. 2010; Deli et al., 2015; Ali and Smarandache, 2015; Broumi et al., 2017, 2017b, 2017c; Samanta et al. 2016) for further details and background.

**Definition 2.1 (Smarandache, 1998).** Let  $X$  be a space of points and let  $x \in X$ . A neutrosophic set  $A$  in  $X$  is characterized by a truth membership function  $T$ , an indeterminacy membership function  $I$ , and a falsity membership function  $F$ .  $T, I, F$  are real standard or nonstandard subsets of  $]0, 1^+[$ , and  $T, I, F: X \rightarrow ]0, 1^+[$ . The neutrosophic set can be represented as

$$A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\} \quad (1)$$

There is no restriction on the sum of  $T, I, F$ , So

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+ \quad (2)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of  $]0, 1^+[$ . Thus it is necessary to take the interval  $[0, 1]$  instead of  $]0, 1^+[$ . For technical applications. It is difficult to apply  $]0, 1^+[$  in the real life applications such as engineering and scientific problems.

**Definition 2.2 (Wang et al. 2010).** Let  $X$  be a space of points (objects) with generic elements in  $X$  denoted by  $x$ . A single valued neutrosophic set  $A$  (SVNS  $A$ ) is characterized by truth-membership function  $T_A(x)$ , an indeterminate-membership function  $I_A(x)$ , and a false-membership function  $F_A(x)$ . For each point  $x$  in  $X$ ,  $T_A(x), I_A(x), F_A(x) \in [0, 1]$ . A SVNS  $A$  can be written as

$$A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\} \quad (3)$$

**Definition 2.3 (Deli et al., 2015).** A bipolar neutrosophic set  $A$  in  $X$  is defined as an object of the form

$A = \{ \langle x, (T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x)) \rangle : x \in X \}$ , where  $T_A^+, I_A^+, F_A^+ : X \rightarrow [1, 0]$  and  $T_A^-, I_A^-, F_A^- : X \rightarrow [-1, 0]$ . The positive membership degree  $T_A^+(x), I_A^+(x), F_A^+(x)$  denotes the truth membership, indeterminate membership and false membership of an element  $\in X$  corresponding to a bipolar neutrosophic set  $A$  and the negative membership degree  $T_A^-(x), I_A^-(x), F_A^-(x)$  denotes the truth membership, indeterminate membership and false membership of an element  $\in X$  to some implicit counter-property corresponding to a bipolar neutrosophic set  $A$ . For convenience a bipolar neutrosophic number is represented by

$$A = \langle (T_A^+, I_A^+, F_A^+, T_A^-, I_A^-, F_A^-) \rangle \quad (4)$$

**Definition 2.4 (Ali and Smarandache, 2015)**

A complex neutrosophic set  $A$  defined on a universe of discourse  $X$ , which is characterized by a truth membership function  $T_A(x)$ , an indeterminacy membership function  $I_A(x)$ , and a falsity membership function  $F_A(x)$  that assigns a complex-valued grade of  $T_A(x), I_A(x)$ , and  $F_A(x)$  in  $A$  for any  $x \in X$ . The values  $T_A(x), I_A(x)$ , and  $F_A(x)$  and their sum may all within the unit circle in the complex plane and so is of the following form,

$$T_A(x) = p_A(x) \cdot e^{j\mu_A(x)},$$

$$I_A(x) = q_A(x) \cdot e^{j\nu_A(x)} \text{ and}$$

$$F_A(x) = r_A(x) \cdot e^{j\omega_A(x)}$$

Where,  $p_A(x), q_A(x), r_A(x)$  and  $\mu_A(x), \nu_A(x), \omega_A(x)$  are respectively, real valued and  $p_A(x), q_A(x), r_A(x) \in [0, 1]$  such that

$$0 \leq p_A(x) + q_A(x) + r_A(x) \leq 3$$

The complex neutrosophic set A can be represented in set form as

$$A = \{(x, T_A(x) = a_T, I_A(x) = a_I, F_A(x) = a_F) : x \in X\}$$

where  $T_A: X \rightarrow \{a_T: a_T \in \mathbb{C}, |a_T| \leq 1\}$ ,

$I_A: X \rightarrow \{a_I: a_I \in \mathbb{C}, |a_I| \leq 1\}$ ,

$F_A: X \rightarrow \{a_F: a_F \in \mathbb{C}, |a_F| \leq 1\}$  and

$$|T_A(x) + I_A(x) + F_A(x)| \leq 3.$$

**Definition 2.5 (Ali and Smarandache, 2015)** The union of two complex neutrosophic sets as follows:

Let A and B be two complex neutrosophic sets in X, where  $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$  and

$$B = \{(x, T_B(x), I_B(x), F_B(x)) : x \in X\}.$$

Then, the union of A and B is denoted as  $A \cup_N B$  and is given as

$$A \cup_N B = \{(x, T_{A \cup B}(x), I_{A \cup B}(x), F_{A \cup B}(x)) : x \in X\}$$

Where the truth membership function  $T_{A \cup B}(x)$ , the indeterminacy membership function  $I_{A \cup B}(x)$  and the falsehood membership function  $F_{A \cup B}(x)$  is defined by

$$T_{A \cup B}(x) = [(p_A(x) \vee p_B(x))] \cdot e^{j\mu_{T_{A \cup B}}(x)},$$

$$I_{A \cup B}(x) = [(q_A(x) \wedge q_B(x))] \cdot e^{j\nu_{I_{A \cup B}}(x)},$$

$$F_{A \cup B}(x) = [(r_A(x) \wedge r_B(x))] \cdot e^{j\omega_{F_{A \cup B}}(x)}$$

Where  $\vee$  and  $\wedge$  denotes the max and min operators respectively.

The phase term of complex truth membership function, complex indeterminacy membership function and complex falsity membership function belongs to  $(0, 2\pi)$  and, they are defined as follows:

a) Sum:

$$\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x),$$

$$\nu_{A \cup B}(x) = \nu_A(x) + \nu_B(x),$$

$$\omega_{A \cup B}(x) = \omega_A(x) + \omega_B(x).$$

b) Max:

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)),$$

$$\nu_{A \cup B}(x) = \max(\nu_A(x), \nu_B(x)),$$

$$\omega_{A \cup B}(x) = \max(\omega_A(x), \omega_B(x)).$$

c) Min:

$$\mu_{A \cup B}(x) = \min(\mu_A(x), \mu_B(x)),$$

$$v_{A \cup B}(x) = \min(v_A(x), v_B(x)),$$

$$\omega_{A \cup B}(x) = \min(\omega_A(x), \omega_B(x)).$$

d) “The game of winner, neutral, and loser”:

$$\mu_{A \cup B}(x) = \begin{cases} \mu_A(x) & \text{if } p_A > p_B \\ \mu_B(x) & \text{if } p_B > p_A \end{cases},$$

$$v_{A \cup B}(x) = \begin{cases} v_A(x) & \text{if } q_A < q_B \\ v_B(x) & \text{if } q_B < q_A \end{cases},$$

$$\omega_{A \cup B}(x) = \begin{cases} \omega_A(x) & \text{if } r_A < r_B \\ \omega_B(x) & \text{if } r_B < r_A \end{cases}.$$

The game of winner, neutral, and loser is the generalization of the concept “winner take all” introduced by Ramot et al. in (2002) for the union of phase terms.

**Definition 2.6 (Ali and Smarandache, 2015)** Intersection of complex neutrosophic sets

Let A and B be two complex neutrosophic sets in X,  $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$  and  $B = \{(x, T_B(x), I_B(x), F_B(x)) : x \in X\}$ .

Then the intersection of A and B is denoted as  $A \cap_N B$  and is define as

$$A \cap_N B = \{(x, T_{A \cap B}(x), I_{A \cap B}(x), F_{A \cap B}(x)) : x \in X\}$$

Where the truth membership function  $T_{A \cap B}(x)$ , the indeterminacy membership function  $I_{A \cap B}(x)$  and the falsehood membership function  $F_{A \cap B}(x)$  is given as:

$$T_{A \cap B}(x) = [(p_A(x) \wedge p_B(x))] \cdot e^{j \cdot \mu_{T_{A \cap B}}(x)},$$

$$I_{A \cap B}(x) = [(q_A(x) \vee q_B(x))] \cdot e^{j \cdot \nu_{I_{A \cap B}}(x)},$$

$$F_{A \cap B}(x) = [(r_A(x) \vee r_B(x))] \cdot e^{j \cdot \omega_{F_{A \cap B}}(x)}$$

Where  $\vee$  and  $\wedge$  denotes denotes the max and min operators respectively

The phase terms  $e^{j \cdot \mu_{T_{A \cap B}}(x)}$ ,  $e^{j \cdot \nu_{I_{A \cap B}}(x)}$  and  $e^{j \cdot \omega_{F_{A \cap B}}(x)}$  was calculated on the same lines by winner, neutral, and loser game.

**Definition 2.7 (Broumi et al., 2017c).** A bipolar complex neutrosophic set A in X is defined as an object of the form

$A = \{ \langle x, T_1^+ e^{iT_2^+}, I_1^+ e^{iI_2^+}, F_1^+ e^{iF_2^+}, T_1^- e^{iT_2^-}, I_1^- e^{iI_2^-}, F_1^- e^{iF_2^-} \rangle : x \in X \}$ , where  $T_1^+, I_1^+, F_1^+ : X \rightarrow [1, 0]$  and  $T_1^-, I_1^-, F_1^- : X \rightarrow [-1, 0]$ . The positive membership degree  $T_1^+(x), I_1^+(x), F_1^+(x)$  denotes the truth membership, indeterminate membership and false membership of an element  $x \in X$  corresponding to a bipolar complex neutrosophic set A and the negative membership degree  $T_1^-(x), I_1^-(x), F_1^-(x)$  denotes the truth membership, indeterminate membership and false membership of an element  $x \in X$  to some implicit counter-property corresponding to a bipolar complex neutrosophic set A. For convenience a bipolar complex neutrosophic number is represented by

$$A = \langle T_1^+ e^{iT_2^+}, I_1^+ e^{iI_2^+}, F_1^+ e^{iF_2^+}, T_1^- e^{iT_2^-}, I_1^- e^{iI_2^-}, F_1^- e^{iF_2^-} \rangle$$

**Definition 2.8 (Broumi et al., 2017c).** The union of two bipolar complex neutrosophic sets as follows:

Let A and B be two bipolar complex neutrosophic sets in X, where

$A = (T_1^+ e^{iT_2^+}, I_1^+ e^{iI_2^+}, F_1^+ e^{iF_2^+}, T_1^- e^{iT_2^-}, I_1^- e^{iI_2^-}, F_1^- e^{iF_2^-})$  and

$$B = (T_3^+ e^{iT_4^+}, I_3^+ e^{iI_4^+}, F_3^+ e^{iF_4^+}, T_3^- e^{iT_4^-}, I_3^- e^{iI_4^-}, F_3^- e^{iF_4^-})$$

Then the union of  $A$  and  $B$  is denoted as  $A \cup_{BN} B$  and is given as

$$A \cup_{BN} B = \{(x, T_{A \cup B}^+(x), I_{A \cup B}^+(x), F_{A \cup B}^+(x), T_{A \cup B}^-(x), I_{A \cup B}^-(x), F_{A \cup B}^-(x)) : x \in X\}$$

Where positive the truth membership function  $T_{A \cup B}^+(x)$ , positive the indeterminacy membership function  $I_{A \cup B}^+(x)$  and positive the falsehood membership function  $F_{A \cup B}^+(x)$ , negative the truth membership function  $T_{A \cup B}^-(x)$ , negative the indeterminacy membership function  $I_{A \cup B}^-(x)$  and negative the falsehood membership function  $F_{A \cup B}^-(x)$  is defined by

$$T_{A \cup B}^+(x) = (T_1^+ \vee T_3^+) e^{i(T_2^+ \cup T_4^+)},$$

$$T_{A \cup B}^-(x) = (T_1^- \wedge T_3^-) e^{i(T_2^- \cup T_4^-)},$$

$$I_{A \cup B}^+(x) = (I_1^+ \wedge I_3^+) e^{i(I_2^+ \cup I_4^+)},$$

$$I_{A \cup B}^-(x) = (I_1^- \vee I_3^-) e^{i(I_2^- \cup I_4^-)},$$

$$F_{A \cup B}^+(x) = (F_1^+ \wedge F_3^+) e^{i(F_2^+ \cup F_4^+)},$$

$$F_{A \cup B}^-(x) = (F_1^- \vee F_3^-) e^{i(F_2^- \cup F_4^-)}$$

Where  $\vee$  and  $\wedge$  denotes the max and min operators respectively

The phase term of bipolar complex truth membership function, bipolar complex indeterminate membership function and bipolar complex false -membership function belongs to  $(0, 2\pi)$  and, they are defined as follows:

e) Sum:

$$T_{A \cup B}^+(x) = T_A^+(x) + T_B^+(x)$$

$$T_{A \cup B}^-(x) = T_A^-(x) + T_B^-(x)$$

$$I_{A \cup B}^+(x) = I_A^+(x) + I_B^+(x)$$

$$I_{A \cup B}^-(x) = I_A^-(x) + I_B^-(x)$$

$$F_{A \cup B}^+(x) = F_A^+(x) + F_B^+(x)$$

$$F_{A \cup B}^-(x) = F_A^-(x) + F_B^-(x)$$

f) Max and min:

$$T_{A \cup B}^+(x) = \max(T_A^+(x), T_B^+(x))$$

$$T_{A \cup B}^-(x) = \min(T_A^-(x), T_B^-(x))$$

$$I_{A \cup B}^+(x) = \min(I_A^+(x), I_B^+(x))$$

$$I_{A \cup B}^-(x) = \max(I_A^-(x), I_B^-(x))$$

$$F_{A \cup B}^+(x) = \min(F_A^+(x), F_B^+(x))$$

$$F_{A \cup B}^-(x) = \max(F_A^-(x), F_B^-(x))$$

g) “The game of winner, neutral, and loser”:

$$T_{A \cup B}^+(x) = \begin{cases} T_A^+(x) & \text{if } p_A > p_B, \\ T_B^+(x) & \text{if } p_B > p_A \end{cases},$$

$$T_{A \cup B}^-(x) = \begin{cases} T_A^-(x) & \text{if } p_A < p_B \\ T_B^-(x) & \text{if } p_B < p_A \end{cases}$$

$$I_{A \cup B}^+(x) = \begin{cases} I_A^+(x) & \text{if } q_A < q_B, \\ I_B^+(x) & \text{if } q_B < q_A \end{cases},$$

$$I^-_{A \cup B}(x) = \begin{cases} I^-_A(x) & \text{if } q_A > q_B \\ I^-_B(x) & \text{if } q_B > q_A \end{cases}$$

$$F^+_{A \cup B}(x) = \begin{cases} F^+_A(x) & \text{if } r_A < r_B \\ F^+_B(x) & \text{if } r_B < r_A \end{cases}$$

$$F^-_{A \cup B}(x) = \begin{cases} F^-_A(x) & \text{if } r_A > r_B \\ F^-_B(x) & \text{if } r_B > r_A \end{cases}$$

**Example 2.9:** Let  $X = \{x_1, x_2\}$  be a universe of discourse. Let  $A$  and  $B$  be two bipolar complex neutrosophic sets in  $X$  as shown below:

$$A = \left( \frac{0.5e^{i.0.7}, 0.2e^{i.\pi}, 0.4e^{i.0.1}, -0.7e^{i.-0.4}, -0.3e^{i.-\frac{\pi}{3}}, -0.2e^{i.0}}{x_1} \right), \left( \frac{0.6e^{i.0.8}, 0.3e^{i.\frac{\pi}{3}}, 0.1e^{i.0.3}, -0.8e^{i.-0.5}, -0.4e^{i.-\frac{2\pi}{3}}, -0.1e^{i.-0.1}}{x_2} \right)$$

And

$$B = \left( \frac{0.9e^{i.0.6}, 0.3e^{i.\pi}, 0.1e^{i.0.3}, -0.6e^{i.-0.6}, -0.2e^{i.-2\pi}, -0.3e^{i.-0.3}}{x_1} \right), \left( \frac{0.8e^{i.0.9}, 0.4e^{i.\frac{3\pi}{4}}, 0.2e^{i.0.2}, -0.5e^{i.-0.6}, -0.1e^{i.-\frac{\pi}{3}}, -0.2e^{i.-0.1}}{x_2} \right)$$

Then

$$A \cup_{BN} B = \left( \frac{0.9e^{i.0.7}, 0.2e^{i.\pi}, 0.1e^{i.0.1}, -0.7e^{i.-0.6}, -0.2e^{i.-\frac{\pi}{3}}, -0.2e^{i.0}}{x_1} \right), \left( \frac{0.8e^{i.0.9}, 0.3e^{i.\frac{\pi}{3}}, 0.1e^{i.0.2}, -0.8e^{i.-0.6}, -0.1e^{i.-\frac{\pi}{3}}, -0.1e^{i.-0.1}}{x_2} \right)$$

**Definition 2.10(Broumi et al., 2017c)** The intersection of two bipolar complex neutrosophic sets as follows:

Let  $A$  and  $B$  be two bipolar complex neutrosophic sets in  $X$ , where

$$A = (T_1^+ e^{iT_2^+}, I_1^+ e^{iI_2^+}, F_1^+ e^{iF_2^+}, T_1^- e^{iT_2^-}, I_1^- e^{iI_2^-}, F_1^- e^{iF_2^-}) \text{ and}$$

$$B = (T_3^+ e^{iT_4^+}, I_3^+ e^{iI_4^+}, F_3^+ e^{iF_4^+}, T_3^- e^{iT_4^-}, I_3^- e^{iI_4^-}, F_3^- e^{iF_4^-})$$

Then the intersection of  $A$  and  $B$  is denoted as  $A \cap_{BN} B$  and is given as

$$A \cap_{BN} B = \{(x, T^+_{A \cap B}(x), I^+_{A \cap B}(x), F^+_{A \cap B}(x), T^-_{A \cap B}(x), I^-_{A \cap B}(x), F^-_{A \cap B}(x)) : x \in X\}$$

Where positive the truth membership function  $T^+_{A \cap B}(x)$ , positive the indeterminacy membership function  $I^+_{A \cap B}(x)$  and positive the falsehood membership function  $F^+_{A \cap B}(x)$ , negative the truth membership function  $T^-_{A \cap B}(x)$ , negative the indeterminacy membership function  $I^-_{A \cap B}(x)$  and negative the falsehood membership function  $F^-_{A \cap B}(x)$  is defined by

$$T^+_{A \cap B}(x) = (T_1^+ \wedge T_3^+) e^{i(T_2^+ \cap T_4^+)},$$

$$T^-_{A \cap B}(x) = (T_1^- \vee T_3^-) e^{i(T_2^- \cap T_4^-)},$$

$$\begin{aligned} I_{A \cap B}^+(x) &= (I_1^+ \vee I_3^+) e^{i(I_2^+ \cap I_4^+)}, \\ T_{A \cap B}^-(x) &= (I_1^- \wedge I_3^-) e^{i(I_2^- \cap I_4^-)}, \\ F_{A \cap B}^+(x) &= (F_1^+ \vee F_3^+) e^{i(F_2^+ \cap F_4^+)}, \\ F_{A \cap B}^-(x) &= (F_1^- \wedge F_3^-) e^{i(F_2^- \cap F_4^-)} \end{aligned}$$

Where  $\vee$  and  $\wedge$  denotes the max and min operators respectively

The phase term of bipolar complex truth membership function, bipolar complex indeterminacy membership function and bipolar complex falsity membership function belongs to  $(0, 2\pi)$  and, they are defined as follows:

h) Sum:

$$\begin{aligned} T_{A \cap B}^+(x) &= T_A^+(x) + T_B^+(x) \\ T_{A \cap B}^-(x) &= T_A^-(x) + T_B^-(x) \\ I_{A \cap B}^+(x) &= I_A^+(x) + I_B^+(x) \\ I_{A \cap B}^-(x) &= I_A^-(x) + I_B^-(x) \\ F_{A \cap B}^+(x) &= F_A^+(x) + F_B^+(x) \\ F_{A \cap B}^-(x) &= F_A^-(x) + F_B^-(x) \end{aligned}$$

i) Max and min:

$$\begin{aligned} T_{A \cap B}^+(x) &= \min(T_A^+(x), T_B^+(x)) \\ T_{A \cap B}^-(x) &= \max(T_A^-(x), T_B^-(x)) \\ I_{A \cap B}^+(x) &= \max(I_A^+(x), I_B^+(x)) \\ I_{A \cap B}^-(x) &= \min(I_A^-(x), I_B^-(x)) \\ F_{A \cap B}^+(x) &= \max(F_A^+(x), F_B^+(x)) \\ F_{A \cap B}^-(x) &= \min(F_A^-(x), F_B^-(x)) \end{aligned}$$

j) “The game of winner, neutral, and loser”:

$$\begin{aligned} T_{A \cap B}^+(x) &= \begin{cases} T_A^+(x) & \text{if } p_A < p_B, \\ T_B^+(x) & \text{if } p_B < p_A \end{cases}, \\ T_{A \cap B}^-(x) &= \begin{cases} T_A^-(x) & \text{if } p_A > p_B \\ T_B^-(x) & \text{if } p_B > p_A \end{cases} \\ I_{A \cap B}^+(x) &= \begin{cases} I_A^+(x) & \text{if } q_A > q_B, \\ I_B^+(x) & \text{if } q_B > q_A \end{cases}, \\ I_{A \cap B}^-(x) &= \begin{cases} I_A^-(x) & \text{if } q_A < q_B \\ I_B^-(x) & \text{if } q_B < q_A \end{cases} \\ F_{A \cap B}^+(x) &= \begin{cases} F_A^+(x) & \text{if } r_A > r_B \\ F_B^+(x) & \text{if } r_B > r_A \end{cases} \\ F_{A \cap B}^-(x) &= \begin{cases} F_A^-(x) & \text{if } r_A < r_B \\ F_B^-(x) & \text{if } r_B < r_A \end{cases} \end{aligned}$$

**Example 2.11:** Let  $X = \{x_1, x_2\}$  be a universe of discourse. Let  $A$  and  $B$  be two bipolar complex neutrosophic sets in  $X$  as shown below:

$$A = \left( \frac{0.5e^{i.0.7}, 0.2e^{i.\pi}, 0.4e^{i.0.1}, -0.7e^{i.-0.4}, -0.3e^{i.-\frac{\pi}{3}}, -0.2e^{i.0}}{x_1} \right)$$

$$, \left( \frac{0.6e^{i.0.8}, 0.3e^{i.\frac{\pi}{3}}, 0.1e^{i.0.3}, -0.8e^{i.-0.5}, -0.4e^{i.-\frac{2\pi}{3}}, -0.1e^{i.-0.1}}{x_2} \right)$$

And

$$B = \left( \frac{0.9e^{i.0.6}, 0.3e^{i.\pi}, 0.1e^{i.0.3}, -0.6e^{i.-0.6}, -0.2e^{i.-2\pi}, -0.3e^{i.-0.3}}{x_1} \right), \left( \frac{0.8e^{i.0.9}, 0.4e^{i.\frac{3\pi}{4}}, 0.2e^{i.0.2}, -0.5e^{i.-0.6}, -0.1e^{i.-\frac{\pi}{3}}, -0.2e^{i.-0.1}}{x_2} \right)$$

Then

$$A \cap_{BN} B = \left( \frac{0.5e^{i.0.6}, 0.3e^{i.\pi}, 0.4e^{i.0.3}, -0.6e^{i.-0.4}, -0.3e^{i.-2\pi}, -0.3e^{i.-0.3}}{x_1} \right), \left( \frac{0.6e^{i.0.8}, 0.4e^{i.\frac{3\pi}{4}}, 0.2e^{i.0.3}, -0.5e^{i.-0.5}, -0.4e^{i.-\frac{2\pi}{3}}, -0.2e^{i.-0.1}}{x_2} \right)$$

**Definition 2.12 (Samanta et al.2016).** Let V be a non-void set. Two function are considered as follows:

$\rho:V \rightarrow [0, 1]$  and  $\omega:V \times V \rightarrow [0, 1]$ . We suppose

$A = \{(\rho(x), \rho(y)) \mid \omega(x, y) > 0\}$ ,

We have considered  $\omega_T, > 0$  for all set A

The triad  $(V, \rho, \omega)$  is defined to be generalized fuzzy graph of first type (GFG1) if there is function  $\alpha:A \rightarrow [0, 1]$  such that  $\omega(x, y) = \alpha((\rho(x), \rho(y)))$  Where  $x, y \in V$ .

The  $\rho(x)$ ,  $x \in V$  are the membership of the vertex x and  $\omega(x, y)$ ,  $x, y \in V$  are the membership values of the edge  $(x, y)$ .

**Definition 2.13 (Broumi et al., 2017).** Let V be a non-void set. Two function are considered as follows:

$\rho = (\rho_T, \rho_I, \rho_F):V \rightarrow [0, 1]^3$  and

$\omega = (\omega_T, \omega_I, \omega_F):V \times V \rightarrow [0, 1]^3$ . Suppose

$A = \{(\rho_T(x), \rho_T(y)) \mid \omega_T(x, y) \geq 0\}$ ,

$B = \{(\rho_I(x), \rho_I(y)) \mid \omega_I(x, y) \geq 0\}$ ,

$C = \{(\rho_F(x), \rho_F(y)) \mid \omega_F(x, y) \geq 0\}$ ,

We have considered  $\omega_T, \omega_I$  and  $\omega_F \geq 0$  for all set A, B, C, since its is possible to have edge degree = 0 (for T, or I, or F).

The triad  $(V, \rho, \omega)$  is defined to be generalized single valued neutrosophic graph of type 1 (GSVNG1) if there are functions

$\alpha:A \rightarrow [0, 1]$ ,  $\beta:B \rightarrow [0, 1]$  and  $\delta:C \rightarrow [0, 1]$  such that

$\omega_T(x, y) = \alpha((\rho_T(x), \rho_T(y)))$

$\omega_I(x, y) = \beta((\rho_I(x), \rho_I(y)))$

$\omega_F(x, y) = \delta((\rho_F(x), \rho_F(y)))$  where  $x, y \in V$ .

Here  $\rho(x) = (\rho_T(x), \rho_I(x), \rho_F(x))$ ,  $x \in V$  are the truth-membership, indeterminate-membership and false-membership of the vertex x and  $\omega(x, y) = (\omega_T(x, y), \omega_I(x, y), \omega_F(x, y))$ ,  $x, y \in V$  are the truth-membership, indeterminate-membership and false-membership values of the edge  $(x, y)$ .

**Definition 2.14 (Broumi et al., 2017b)** Let  $V$  be a non-void set. Two functions are considered as follows:

$$\rho=(\rho_T, \rho_I, \rho_F):V \rightarrow [0, 1]^3 \text{ and}$$

$$\omega=(\omega_T, \omega_I, \omega_F):V \times V \rightarrow [0, 1]^3 . \text{ Suppose}$$

$$A= \{(\rho_T(x), \rho_T(y)) \mid \omega_T(x, y) \geq 0\},$$

$$B= \{(\rho_I(x), \rho_I(y)) \mid \omega_I(x, y) \geq 0\},$$

$$C= \{(\rho_F(x), \rho_F(y)) \mid \omega_F(x, y) \geq 0\},$$

We have considered  $\omega_T, \omega_I$  and  $\omega_F \geq 0$  for all set  $A, B, C$ , since its is possible to have edge degree = 0 (for  $T$ , or  $I$ , or  $F$ ).

The triad  $(V, \rho, \omega)$  is defined to be complex neutrosophic graph of type 1 (CNG1) if there are functions

$$\alpha:A \rightarrow [0, 1], \beta:B \rightarrow [0, 1] \text{ and } \delta:C \rightarrow [0, 1] \text{ such that}$$

$$\omega_T(x, y) = \alpha((\rho_T(x), \rho_T(y)))$$

$$\omega_I(x, y) = \beta((\rho_I(x), \rho_I(y)))$$

$$\omega_F(x, y) = \delta((\rho_F(x), \rho_F(y)))$$

Where  $x, y \in V$ .

Here  $\rho(x) = (\rho_T(x), \rho_I(x), \rho_F(x))$ ,  $x \in V$  are the complex truth-membership, complex indeterminate-membership and complex false-membership of the vertex  $x$  and  $\omega(x, y) = (\omega_T(x, y), \omega_I(x, y), \omega_F(x, y))$ ,  $x, y \in V$  are the complex truth-membership, complex indeterminate-membership and complex false-membership values of the edge  $(x, y)$ .

**Definition 2.15 (Broumi et al., 2017b)**. Let  $V$  be a non-void set. Two function are considered as follows:

$$\rho=(\rho_T^+, \rho_I^+, \rho_F^+, \rho_T^-, \rho_I^-, \rho_F^-):V \rightarrow [0, 1]^3 \times [-1, 0]^3 \text{ and}$$

$$\omega=(\omega_T^+, \omega_I^+, \omega_F^+, \omega_T^-, \omega_I^-, \omega_F^-):V \times V \rightarrow [0, 1]^3 \times [-1, 0]^3 . \text{ We suppose}$$

$$A= \{(\rho_T^+(x), \rho_T^+(y)) \mid \omega_T^+(x, y) \geq 0\},$$

$$B= \{(\rho_I^+(x), \rho_I^+(y)) \mid \omega_I^+(x, y) \geq 0\},$$

$$C= \{(\rho_F^+(x), \rho_F^+(y)) \mid \omega_F^+(x, y) \geq 0\},$$

$$D= \{(\rho_T^-(x), \rho_T^-(y)) \mid \omega_T^-(x, y) \leq 0\},$$

$$E= \{(\rho_I^-(x), \rho_I^-(y)) \mid \omega_I^-(x, y) \leq 0\},$$

$$F= \{(\rho_F^-(x), \rho_F^-(y)) \mid \omega_F^-(x, y) \leq 0\},$$

We have considered  $\omega_T^+, \omega_I^+, \omega_F^+ \geq 0$  and  $\omega_T^-, \omega_I^-, \omega_F^- \leq 0$  for all set  $A, B, C, D, E, F$  since its is possible to have edge degree = 0 (for  $T^+$  or  $I^+$  or  $F^+, T^-$  or  $I^-$  or  $F^-$ ).

The triad  $(V, \rho, \omega)$  is defined to be generalized bipolar neutrosophic graph of first type (GBNG1) if there are functions

$$\alpha:A \rightarrow [0, 1], \beta:B \rightarrow [0, 1], \delta:C \rightarrow [0, 1] \text{ and } \xi:D \rightarrow [-1, 0], \sigma:E \rightarrow [-1, 0], \psi:F \rightarrow [-1, 0] \text{ such that}$$

$$\omega_T^+(x, y) = \alpha((\rho_T^+(x), \rho_T^+(y))),$$

$$\omega_T^-(x, y) = \xi((\rho_T^-(x), \rho_T^-(y))),$$

$$\omega_I^+(x, y) = \beta((\rho_I^+(x), \rho_I^+(y))),$$

$$\omega_I^-(x, y) = \sigma((\rho_I^-(x), \rho_I^-(y))),$$

$$\omega_F^+(x, y) = \delta((\rho_F^+(x), \rho_F^+(y))),$$

$$\omega_F^-(x, y) = \psi((\rho_F^-(x), \rho_F^-(y)))$$

Where  $x, y \in V$ .

Here  $\rho(x) = (\rho_T^+(x), \rho_I^+(x), \rho_F^+(x), \rho_T^-(x), \rho_I^-(x), \rho_F^-(x))$ ,  $x \in V$  are the positive and negative membership, indeterminacy and non-membership of the vertex  $x$  and  $\omega(x, y) = (\omega_T^+(x, y),$

$\omega_I^+(x, y), \omega_F^+(x, y), \omega_T^-(x, y), \omega_I^-(x, y), \omega_F^-(x, y)$ ,  $x, y \in V$  are the positive and negative membership, indeterminacy membership and non-membership values of the edge  $(x, y)$ .

### 3. Bipolar Complex Neutrosophic Graph of Type 1

In this section, based on the concept of bipolar complex neutrosophic sets (Broumi et al., 2017c) and the concept of generalized single valued neutrosophic graph of type 1 (Broumi et al., 2017), we define the concept of bipolar complex neutrosophic graph of type 1 as follows:

**Definition 3.1.** Let  $V$  be a non-void set. Two function are considered as follows:

$$\rho = (\rho_T^+, \rho_I^+, \rho_F^+, \rho_T^-, \rho_I^-, \rho_F^-): V \rightarrow [-1, 1]^6 \text{ and}$$

$$\omega = (\omega_T^+, \omega_I^+, \omega_F^+, \omega_T^-, \omega_I^-, \omega_F^-): V \times V \rightarrow [-1, 1]^6. \text{ We suppose}$$

$$A = \{(\rho_T^+(x), \rho_T^+(y)) \mid \omega_T^+(x, y) \geq 0\},$$

$$B = \{(\rho_I^+(x), \rho_I^+(y)) \mid \omega_I^+(x, y) \geq 0\},$$

$$C = \{(\rho_F^+(x), \rho_F^+(y)) \mid \omega_F^+(x, y) \geq 0\},$$

$$D = \{(\rho_T^-(x), \rho_T^-(y)) \mid \omega_T^-(x, y) \leq 0\},$$

$$E = \{(\rho_I^-(x), \rho_I^-(y)) \mid \omega_I^-(x, y) \leq 0\},$$

$$F = \{(\rho_F^-(x), \rho_F^-(y)) \mid \omega_F^-(x, y) \leq 0\},$$

We have considered  $\omega_T^+, \omega_I^+, \omega_F^+ \geq 0$  and  $\omega_T^-, \omega_I^-, \omega_F^- \leq 0$  for all set  $A, B, C, D, E, F$  since its is possible to have edge degree = 0 (for  $T^+$  or  $I^+$  or  $F^+, T^-$  or  $I^-$  or  $F^-$ ).

The triad  $(V, \rho, \omega)$  is defined to be bipolar complex neutrosophic graph of first type (BCNG1) if there are functions

$$\alpha: A \rightarrow [0, 1], \beta: B \rightarrow [0, 1], \delta: C \rightarrow [0, 1] \text{ and } \xi: D \rightarrow [-1, 0], \sigma: E \rightarrow [-1, 0], \psi: F \rightarrow [-1, 0] \text{ such that}$$

$$\omega_T^+(x, y) = \alpha((\rho_T^+(x), \rho_T^+(y))),$$

$$\omega_T^-(x, y) = \xi((\rho_T^-(x), \rho_T^-(y))),$$

$$\omega_I^+(x, y) = \beta((\rho_I^+(x), \rho_I^+(y))),$$

$$\omega_I^-(x, y) = \sigma((\rho_I^-(x), \rho_I^-(y))),$$

$$\omega_F^+(x, y) = \delta((\rho_F^+(x), \rho_F^+(y))),$$

$$\omega_F^-(x, y) = \psi((\rho_F^-(x), \rho_F^-(y)))$$

Where  $x, y \in V$ .

Here  $\rho(x) = (\rho_T^+(x), \rho_I^+(x), \rho_F^+(x), \rho_T^-(x), \rho_I^-(x), \rho_F^-(x))$ ,  $x \in V$  are the positive and negative complex truth-membership, indeterminate and false-membership of the vertex  $x$  and  $\omega(x, y) = (\omega_T^+(x, y), \omega_I^+(x, y), \omega_F^+(x, y), \omega_T^-(x, y), \omega_I^-(x, y), \omega_F^-(x, y))$ ,  $x, y \in V$  are the positive and negative complex truth-membership, indeterminate and false-membership values of the edge  $(x, y)$ .

**Example 3.2:** Let the vertex set be  $V = \{x, y, z, t\}$  and edge set be  $E = \{(x, y), (x, z), (x, t), (y, t)\}$

	x	y	z	t
$\rho_T^+$	$0.5e^{i.0.8}$	$0.9e^{i.0.9}$	$0.3e^{i.0.3}$	$0.8e^{i.0.1}$
$\rho_I^+$	$0.3e^{i.\frac{3\pi}{4}}$	$0.2e^{i.\frac{\pi}{4}}$	$0.1e^{i.2\pi}$	$0.5e^{i.\pi}$
$\rho_F^+$	$0.1e^{i.0.3}$	$0.6e^{i.0.5}$	$0.8e^{i.0.5}$	$0.4e^{i.0.7}$
$\rho_T^-$	$-0.6e^{i.-0.6}$	$-1e^{i.-\pi}$	$-0.4e^{i.-0.1}$	$-0.9e^{i.-0.1}$
$\rho_I^-$	$-0.4e^{i.-2\pi}$	$-0.3e^{i.0}$	$-0.2e^{i.-0.3}$	$-0.6e^{i.-0.2}$
$\rho_F^-$	$-0.2e^{i.-0.3}$	$-0.7e^{i.-0.6}$	$-0.9e^{i.-2\pi}$	$-0.5e^{i.-\pi}$

Table 1: Bipolar complex truth-membership, bipolar complex indeterminate-membership and bipolar complex false-membership of the vertex set.

Let us consider the function

$$\alpha(m, n) = (m_T^+ \vee n_T^+) \cdot e^{j \cdot \mu_{Tm \cup n}}$$

$$\beta(m, n) = (m_I^+ \wedge n_I^+) \cdot e^{j \cdot \mu_{Im \cup n}}$$

$$\delta(m, n) = (m_F^+ \wedge n_F^+) \cdot e^{j \cdot \mu_{Fm \cup n}}$$

$$\xi(m, n) = (m_T^- \wedge n_T^-) \cdot e^{j \cdot \mu_{Tm \cup n}}$$

$$\sigma(m, n) = (m_I^- \vee n_I^-) \cdot e^{j \cdot \mu_{Im \cup n}} \text{ and}$$

$$\psi(m, n) = (m_F^- \vee n_F^-) \cdot e^{j \cdot \mu_{Fm \cup n}}$$

Here,

$$A = \{(0.5e^{i \cdot 0.8}, 0.9e^{i \cdot 0.9}), (0.5e^{i \cdot 0.8}, 0.3e^{i \cdot 0.3}), (0.5e^{i \cdot 0.8}, 0.8e^{i \cdot 0.1}), (0.9e^{i \cdot 0.9}, 0.8e^{i \cdot 0.1})\}$$

$$B = \{(0.3e^{i \cdot \frac{3\pi}{4}}, 0.2e^{i \cdot \frac{\pi}{4}}), (0.3e^{i \cdot \frac{3\pi}{4}}, 0.1e^{i \cdot 2\pi}), (0.3e^{i \cdot \frac{3\pi}{4}}, 0.5e^{i \cdot \pi}), (0.2e^{i \cdot \frac{\pi}{4}}, 0.5e^{i \cdot \pi})\}$$

$$C = \{(0.1e^{i \cdot 0.3}, 0.6e^{i \cdot 0.5}), (0.1e^{i \cdot 0.3}, 0.8e^{i \cdot 0.5}), (0.1e^{i \cdot 0.3}, 0.4e^{i \cdot 0.7}), (0.6e^{i \cdot 0.5}, 0.4e^{i \cdot 0.7})\}$$

$$D = \{(-0.6e^{i \cdot -0.6}, -1e^{i \cdot -\pi}), (-0.6e^{i \cdot -0.6}, -0.4e^{i \cdot -0.1}), (-0.6e^{i \cdot -0.6}, -0.9e^{i \cdot -0.1}), (-1e^{i \cdot -\pi}, -0.9e^{i \cdot -0.1})\}$$

$$E = \{(-0.4e^{i \cdot -2\pi}, -0.3e^{i \cdot 0}), (-0.4e^{i \cdot -2\pi}, -0.2e^{i \cdot -0.3}), (-0.4e^{i \cdot -2\pi}, -0.6e^{i \cdot -0.2}), (-0.3e^{i \cdot 0}, -0.6e^{i \cdot -0.2})\}$$

$$F = \{(-0.2e^{i \cdot -0.3}, -0.7e^{i \cdot -0.6}), (-0.2e^{i \cdot -0.3}, -0.9e^{i \cdot -2\pi}), (-0.2e^{i \cdot -0.3}, -0.5e^{i \cdot -\pi}), (-0.7e^{i \cdot -0.6}, -0.5e^{i \cdot -\pi})\}$$

Then

$\omega$	$(x, y)$	$(x, z)$	$(x, t)$	$(y, t)$
$\omega_T^+(x, y)$	$0.9e^{i \cdot 0.9}$	$0.5e^{i \cdot 0.8}$	$0.8e^{i \cdot 0.8}$	$0.9e^{i \cdot 0.9}$
$\omega_I^+(x, y)$	$0.2e^{i \cdot \frac{\pi}{4}}$	$0.1e^{i \cdot \frac{3\pi}{4}}$	$0.3e^{i \cdot \frac{3\pi}{4}}$	$0.2e^{i \cdot \frac{\pi}{4}}$
$\omega_F^+(x, y)$	$0.1e^{i \cdot 0.3}$	$0.1e^{i \cdot 0.3}$	$0.1e^{i \cdot 0.3}$	$0.4e^{i \cdot 0.5}$
$\omega_T^-(x, y)$	$-1e^{i \cdot -\pi}$	$-0.6e^{i \cdot -0.6}$	$-0.9e^{i \cdot -0.6}$	$-1e^{i \cdot -\pi}$
$\omega_I^-(x, y)$	$-0.3e^{i \cdot 0}$	$-0.2e^{i \cdot -2\pi}$	$-0.4e^{i \cdot -2\pi}$	$-0.3e^{i \cdot 0}$
$\omega_F^-(x, y)$	$-0.2e^{i \cdot -0.3}$	$-0.2e^{i \cdot -0.3}$	$-0.2e^{i \cdot -0.3}$	$-0.5e^{i \cdot -0.6}$

Table 2: Bipolar complex truth-membership, bipolar complex indeterminate-membership and bipolar complex false-membership of the edge set.

The corresponding complex neutrosophic graph is shown in Fig.2

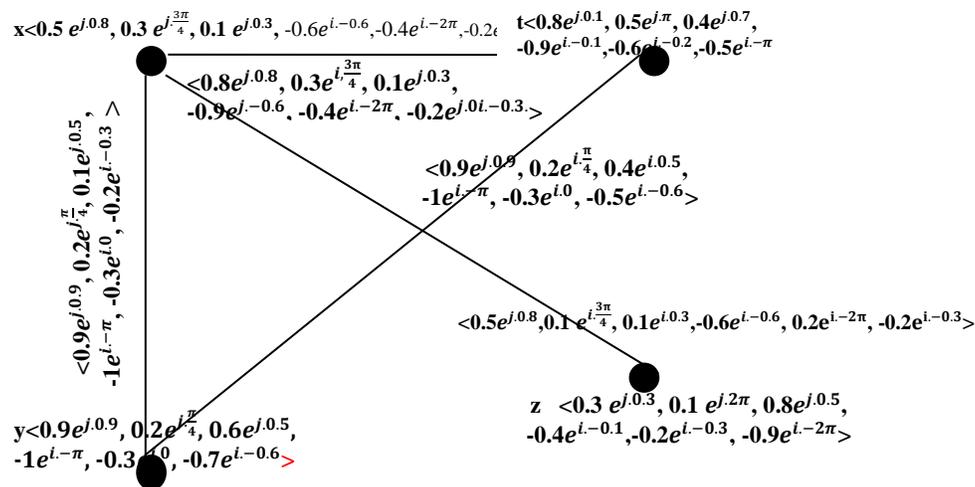


Fig 2. BCNG of type 1.

### 4. Matrix Representation of Bipolar Complex Neutrosophic Graph of Type 1

In this section, bipolar complex truth-membership, bipolar complex indeterminate-membership, and bipolar complex false-membership are considered independent. So, we adopted the representation matrix of complex neutrosophic graphs of type 1 presented in (Broumi et al., 2017b).

The bipolar complex neutrosophic graph (BCNG1) has one property that edge membership values ( $T^+, I^+, F^+, T^-, I^-, F^-$ ) depends on the membership values ( $T^+, I^+, F^+, T^-, I^-, F^-$ ) of adjacent vertices. Suppose  $\zeta=(V, \rho, \omega)$  is a BCNG1 where vertex set  $V=\{v_1, v_2, \dots, v_n\}$ . The functions

$\alpha : A \rightarrow [0, 1]$  is taken such that  $\omega_T^+(x, y) = \alpha ((\rho_T^+(x), \rho_T^+(y)))$ , where  $x, y \in V$  and  $A = \{(\rho_T^+(x), \rho_T^+(y)) \mid \omega_T^+(x, y) \geq 0\}$ ,

$\beta : B \rightarrow [0, 1]$  is taken such that  $\omega_I^+(x, y) = \beta ((\rho_I^+(x), \rho_I^+(y)))$ , where  $x, y \in V$  and  $B = \{(\rho_I^+(x), \rho_I^+(y)) \mid \omega_I^+(x, y) \geq 0\}$ ,

$\delta : C \rightarrow [0, 1]$  is taken such that  $\omega_F^+(x, y) = \delta ((\rho_F^+(x), \rho_F^+(y)))$ , where  $x, y \in V$  and  $C = \{(\rho_F^+(x), \rho_F^+(y)) \mid \omega_F^+(x, y) \geq 0\}$ ,

$\xi : D \rightarrow [-1, 0]$  is taken such that  $\omega_T^-(x, y) = \xi ((\rho_T^-(x), \rho_T^-(y)))$ , where  $x, y \in V$  and  $D = \{(\rho_T^-(x), \rho_T^-(y)) \mid \omega_T^-(x, y) \leq 0\}$ ,

$\sigma : E \rightarrow [-1, 0]$  is taken such that  $\omega_I^-(x, y) = \sigma ((\rho_I^-(x), \rho_I^-(y)))$ , where  $x, y \in V$  and  $E = \{(\rho_I^-(x), \rho_I^-(y)) \mid \omega_I^-(x, y) \leq 0\}$ , and

$\psi : F \rightarrow [-1, 0]$  is taken such that  $\omega_F^-(x, y) = \psi ((\rho_F^-(x), \rho_F^-(y)))$ , where  $x, y \in V$  and  $F = \{(\rho_F^-(x), \rho_F^-(y)) \mid \omega_F^-(x, y) \leq 0\}$ ,

The BCNG1 can be represented by  $(n+1) \times (n+1)$  matrix  $M_{G_1}^{T,I,F} = [a^{T,I,F}(i,j)]$  as follows:

The positive and negative bipolar complex truth-membership ( $T^+, T^-$ ), indeterminate-membership ( $I^+, I^-$ ) and false-membership ( $F^+, F^-$ ), values of the vertices are provided in the first row and first column. The  $(i+1, j+1)$ -th-entry are the bipolar complex truth-membership ( $T^+, T^-$ ), indeterminate-membership ( $I^+, I^-$ ) and the false-membership ( $F^+, F^-$ ) values of the edge  $(x_i, x_j)$ ,  $i, j=1, \dots, n$  if  $i \neq j$ .

The  $(i, i)$ -th entry is  $\rho(x_i) = (\rho_T^+(x_i), \rho_I^+(x_i), \rho_F^+(x_i), \rho_T^-(x_i), \rho_I^-(x_i), \rho_F^-(x_i))$  where  $i=1, 2, \dots, n$ . The positive and negative bipolar complex truth-membership ( $T^+, T^-$ ), indeterminate-membership ( $I^+, I^-$ ) and false-membership ( $F^+, F^-$ ), values of the edge can be computed easily using the functions  $\alpha, \beta, \delta, \xi, \sigma$  and  $\psi$  which are in  $(1,1)$ -position of the matrix. The matrix representation of BCNG1, denoted by  $M_{G_1}^{T,I,F}$ , can be written as sixth matrix representation  $M_{G_1}^{T^+}, M_{G_1}^{I^+}, M_{G_1}^{F^+}, M_{G_1}^{T^-}, M_{G_1}^{I^-}, M_{G_1}^{F^-}$ .

The  $M_{G_1}^{T^+}$  is represented in Table 3.

Table 3. Matrix representation of  $T^+$ -BCNG1

$\alpha$	$v_1(\rho_T^+(v_1))$	$v_2(\rho_T^+(v_2))$	$v_n(\rho_T^+(v_n))$
$v_1(\rho_T^+(v_1))$	$\rho_T^+(v_1)$	$\alpha(\rho_T^+(v_1), \rho_T^+(v_2))$	$\alpha(\rho_T^+(v_1), \rho_T^+(v_n))$
$v_2(\rho_T^+(v_2))$	$\alpha(\rho_T^+(v_2), \rho_T^+(v_1))$	$\rho_T^+(v_2)$	$\alpha(\rho_T^+(v_2), \rho_T^+(v_n))$
...	....	...	...
$v_n(\rho_T^+(v_n))$	$\alpha(\rho_T^+(v_n), \rho_T^+(v_1))$	$\alpha(\rho_T^+(v_n), \rho_T^+(v_2))$	$\rho_T^+(v_n)$

The  $M_{G_1}^{I^+}$  is presented in Table 4. Table4. Matrix representation of  $I^+$ - BCNG

$\beta$	$v_1(\rho_I^+(v_1))$	$v_2(\rho_I^+(v_2))$	$v_n(\rho_I^+(v_n))$
$v_1(\rho_I^+(v_1))$	$\rho_I^+(v_1)$	$\beta(\rho_I^+(v_1), \rho_I^+(v_2))$	$\beta(\rho_I^+(v_1), \rho_I^+(v_n))$
$v_2(\rho_I^+(v_2))$	$\beta(\rho_I^+(v_2), \rho_I^+(v_1))$	$\rho_I^+(v_2)$	$\beta(\rho_I^+(v_2), \rho_I^+(v_2))$
...	....	...	...
$v_n(\rho_I^+(v_n))$	$\beta(\rho_I^+(v_n), \rho_I^+(v_1))$	$\beta(\rho_I^+(v_n), \rho_I^+(v_2))$	$\rho_I^+(v_n)$

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The  $M_{G_1}^{F^+}$  is presented in Table 5.

Table5. Matrix representation of  $F^+$ - BCNG1

$\delta$	$v_1(\rho_F^+(v_1))$	$v_2(\rho_F^+(v_2))$	$v_n(\rho_F^+(v_n))$
$v_1(\rho_F^+(v_1))$	$\rho_F^+(v_1)$	$\delta(\rho_F^+(v_1), \rho_F^+(v_2))$	$\delta(\rho_F^+(v_1), \rho_F^+(v_n))$
$v_2(\rho_F^+(v_2))$	$\delta(\rho_F^+(v_2), \rho_F^+(v_1))$	$\rho_F^+(v_2)$	$\delta(\rho_F^+(v_2), \rho_F^+(v_2))$
...	....	...	...
$v_n(\rho_F^+(v_n))$	$\delta(\rho_F^+(v_n), \rho_F^+(v_1))$	$\delta(\rho_F^+(v_n), \rho_F^+(v_2))$	$\rho_F^+(v_n)$

The  $M_{G_1}^{T^-}$  is shown in table 6.

Table 6. Matrix representation of  $T^-$ - BCNG1

$\xi$	$v_1(\rho_T^-(v_1))$	$v_2(\rho_T^-(v_2))$	$v_n(\rho_T^-(v_n))$
$v_1(\rho_T^-(v_1))$	$\rho_T^-(v_1)$	$\xi(\rho_T^-(v_1), \rho_T^-(v_2))$	$\xi(\rho_T^-(v_1), \rho_T^-(v_n))$
$v_2(\rho_T^-(v_2))$	$\xi(\rho_T^-(v_2), \rho_T^-(v_1))$	$\rho_T^-(v_2)$	$\xi(\rho_T^-(v_2), \rho_T^-(v_2))$
...	....	...	...
$v_n(\rho_T^-(v_n))$	$\xi(\rho_T^-(v_n), \rho_T^-(v_1))$	$\xi(\rho_T^-(v_n), \rho_T^-(v_2))$	$\rho_T^-(v_n)$

The  $M_{G_1}^{I^-}$  is shown in Table 7.

Table 7. Matrix representation of  $I^-$ - BCNG1

$\sigma$	$v_1(\rho_I^-(v_1))$	$v_2(\rho_I^-(v_2))$	$v_n(\rho_I^-(v_n))$
$v_1(\rho_I^-(v_1))$	$\rho_I^-(v_1)$	$\sigma(\rho_I^-(v_1), \rho_I^-(v_2))$	$\sigma(\rho_I^-(v_1), \rho_I^-(v_n))$
$v_2(\rho_I^-(v_2))$	$\sigma(\rho_I^-(v_2), \rho_I^-(v_1))$	$\rho_I^-(v_2)$	$\sigma(\rho_I^-(v_2), \rho_I^-(v_2))$
...	....	...	...
$v_n(\rho_I^-(v_n))$	$\sigma(\rho_I^-(v_n), \rho_I^-(v_1))$	$\sigma(\rho_I^-(v_n), \rho_I^-(v_2))$	$\rho_I^-(v_n)$

The  $M_{G_1}^{F^-}$  is presented in Table 8.

Table8. Matrix representation of  $F^-$  - BCNG1

$\psi$	$v_1(\rho_F^-(v_1))$	$v_2(\rho_F^-(v_2))$	$v_n(\rho_F^-(v_n))$
$v_1(\rho_F^-(v_1))$	$\rho_F^-(v_1)$	$\psi(\rho_F^-(v_1), \rho_F^-(v_2))$	$\psi(\rho_F^-(v_1), \rho_F^-(v_n))$
$v_2(\rho_F^-(v_2))$	$\psi(\rho_F^-(v_2), \rho_F^-(v_1))$	$\rho_F^-(v_2)$	$\psi(\rho_F^-(v_2), \rho_F^-(v_2))$
...	....	...	...
$v_n(\rho_F^-(v_n))$	$\psi(\rho_F^-(v_n), \rho_F^-(v_1))$	$\psi(\rho_F^-(v_n), \rho_F^-(v_2))$	$\rho_F^-(v_n)$

**Remark1:**if  $\rho_T^-(x) = \rho_I^-(x) = \rho_F^-(x) = 0$ , the bipolar complex neutrosophic graphs of type 1 is reduced to complex neutrosophic graph of type 1 (CNG1).

**Remark2:**if  $\rho_T^-(x) = \rho_I^-(x) = \rho_F^-(x) = 0$ , and  $\rho_I^+(x) = \rho_F^+(x) = 0$ , the bipolar complex neutrosophic graphs of type 1 is reduced to generalized fuzzy graph of type 1 (GFG1).

**Remark3:**if the phase terms of bipolar complex neutrosophic values of the vertices equals 0, the bipolar complex neutrosophic graphs of type 1 is reduced to generalized bipolar neutrosophic graph of type 1 (GBNG1).

**Remark4:**if  $\rho_T^-(x) = \rho_I^-(x) = \rho_F^-(x) = 0$ , and the phase terms of positive truth-membership, indeterminate-membership and false-membership of the vertices equals 0, the bipolar complex neutrosophic graphs of type 1 is reduced to generalized single valued neutrosophic graph of type 1 (GSVNG1).

Here the bipolar complex neutrosophic graph of type 1 (BCNG1) can be represented by the matrix representation depicted in table 15. The matrix representation can be written as six matrices one containing the entries as  $T^+, I^+, F^+, T^-, I^-, F^-$  (see table 9, 10, 11, 12, 13 and 14).

Table 9.  $T^+$  - matrix representation of BCNG1

$\alpha$	$x(0.5 e^{j.0.8})$	$y(0.9 e^{j.0.9})$	$z(0.3 e^{j.0.3})$	$t(0.8 e^{j.0.1})$
$x(0.5 e^{i.0.8})$	$0.5 e^{j.0.8}$	$0.9 e^{j.0.9}$	$0.5 e^{j.0.8}$	$0.8 e^{i.0.8}$
$y(0.9 e^{i.0.9})$	$0.9 e^{j.0.9}$	$0.9 e^{j.0.9}$	0	$0.9 e^{j.0.9}$
$z(0.3 e^{i.0.3})$	$0.5 e^{j.0.8}$	0	$0.3 e^{j.0.3}$	0
$t(0.8 e^{i.0.1})$	$0.8 e^{j.0.8}$	$0.9 e^{j.0.9}$	0	$0.8 e^{j.0.1}$

Table 10.  $I^+$  - matrix representation of BCNG1

$\beta$	$x(0.3 e^{j.\frac{3\pi}{4}})$	$y(0.2 e^{j.\frac{\pi}{4}})$	$z(0.1 e^{j.2\pi})$	$t(0.5 e^{j.\pi})$
$x(0.3 e^{j.\frac{3\pi}{4}})$	$0.3 e^{j.\frac{3\pi}{4}}$	$0.2 e^{j.\frac{\pi}{4}}$	$0.1 e^{j.\frac{3\pi}{4}}$	$0.1 e^{j.\frac{3\pi}{4}}$
$y(0.2 e^{j.\frac{\pi}{4}})$	$0.2 e^{j.\frac{\pi}{4}}$	$0.2 e^{j.\frac{\pi}{4}}$	0	$0.2 e^{j.\frac{\pi}{4}}$
$z(0.1 e^{j.2\pi})$	$0.1 e^{j.\frac{3\pi}{4}}$	0	$0.1 e^{j.2\pi}$	0
$t(0.5 e^{j.\pi})$	$0.3 e^{j.2\pi}$	$0.2 e^{j.\frac{\pi}{4}}$	0	$0.5 e^{j.\pi}$

Table 11: $F^+$ - matrix representation of BCNG1

$\delta$	$x(0.1 e^{i.0.3})$	$y(0.6 e^{j.0.5})$	$z(0.8 e^{j.0.5})$	$t(0.4e^{j.0.7})$
$x(0.1 e^{j.0.3})$	$0.1 e^{i.0.3}$	$0.1 e^{i.0.3}$	$0.1 e^{j.0.3}$	$0.1 e^{j.0.3}$
$y(0.6 e^{j.0.5})$	$0.1 e^{i.0.3}$	$0.6 e^{j.0.5}$	0	$0.4e^{j.0.5}$
$z(0.8 e^{j.0.5})$	$0.1 e^{i.0.3}$	0	$0.8 e^{j.0.5}$	0
$t(0.4e^{j.0.7})$	$0.1 e^{i.0.3}$	$0.4e^{j.0.5}$	0	$0.4e^{j.0.7}$

Table 12: $T^-$ - matrix representation of BCNG1

$\xi$	$x(-0.6 e^{i.-0.6})$	$y(-1 e^{i.-\pi})$	$z(-0.4 e^{i.-0.1})$	$t(-0.9e^{i.-0.1})$
$x(-0.6 e^{i.-0.6})$	$-0.6 e^{i.-0.6}$	$-1 e^{i.-\pi}$	$-0.6 e^{i.-0.6}$	$-0.9 e^{i.-0.6}$
$y(-1 e^{i.-\pi})$	$-1 e^{i.-\pi}$	$-1 e^{i.-\pi}$	0	$-1 e^{i.-\pi}$
$z(-0.4 e^{i.-0.1})$	$-0.6 e^{i.-0.6}$	0	$-0.4 e^{i.-0.1}$	0
$t(-0.9e^{i.-0.1})$	$-0.9 e^{i.-0.6}$	$-1 e^{i.-\pi}$	0	$-0.9e^{i.-0.1}$

Table 13: $I^-$ - matrix representation of BCNG1

$\sigma$	$x(-0.4 e^{i.-2\pi})$	$y(-0.3 e^{i.0})$	$z(-0.2 e^{i.-0.3})$	$t(-0.6e^{i.-0.2})$
$x(-0.4e^{i.-2\pi})$	$-0.4 e^{i.-2\pi}$	$-0.3e^{i.0}$	$-0.2e^{i.-2\pi}$	$-0.4e^{i.-2\pi}$
$y(-0.3e^{i.0})$	$-0.3 e^{i.0}$	$-0.3 e^{i.0}$	0	$-0.3 e^{i.0}$
$z(-0.2 e^{i.-0.3})$	$-0.2 e^{i.-2\pi}$	0	$-0.2 e^{i.-0.3}$	0
$t(-0.6e^{i.-0.2})$	$-0.4 e^{i.-2\pi}$	$-0.3 e^{i.0}$	0	$-0.6e^{i.-0.2}$

Table 14: $F^-$ - matrix representation of BCNG1

$\psi$	$x(-0.2 e^{i.-2\pi})$	$y(-0.7 e^{i.-0.6})$	$z(-0.9 e^{i.-2\pi})$	$t(-0.5e^{i.-\pi})$
$x(-0.2e^{i.-2\pi})$	$-0.2 e^{i.-2\pi}$	$-0.2e^{i.-0.3}$	$-0.2 e^{i.-0.3}$	$-0.2e^{i.-0.3}$
$y(-0.7 e^{i.-0.6})$	$-0.2 e^{i.-0.3}$	$-0.7 e^{i.-0.6}$	0	$-0.5e^{i.-0.6}$
$z(-0.9 e^{i.-2\pi})$	$-0.2 e^{i.-0.3}$	0	$-0.9 e^{i.-2\pi}$	0
$t(-0.5e^{i.-\pi})$	$-0.2 e^{i.-0.3}$	$-0.3 e^{i.0}$	0	$-0.5e^{i.-\pi}$

The matrix representation of GBNG1 is shown in Table 15.

Table 15. Matrix representation of BCNG1.

$(\alpha, \beta, \delta, \xi, \sigma, \psi)$	$x < 0.5 e^{j.0.8}, 0.3 e^{j.\frac{3\pi}{4}}, 0.1 e^{j.0.3}, -0.6 e^{i.-0.6}, -0.4 e^{i.-2\pi}, -0.2 e^{i.-0.3} >$	$y < 0.9 e^{j.0.9}, 0.2 e^{j.\frac{\pi}{4}}, 0.6 e^{j.0.5}, -1 e^{i.-\pi}, -0.3 e^{i.0}, -0.7 e^{i.-0.6} >$	$z < 0.3 e^{j.0.3}, 0.1 e^{j.2\pi}, 0.8 e^{j.0.5}, -0.4 e^{i.-0.1}, -0.2 e^{i.-0.3}, -0.9 e^{i.-2\pi} >$	$t < 0.8 e^{j.0.1}, 0.5 e^{j.\pi}, 0.4 e^{j.0.7}, -0.9 e^{i.-0.1}, -0.6 e^{i.-0.2}, -0.5 e^{i.-\pi}, -0.7 e^{i.-0.6} >$
$x < 0.5 e^{j.0.8}, 0.3 e^{j.\frac{3\pi}{4}}, 0.1 e^{j.0.3}, -0.6 e^{i.-0.6}, -0.4 e^{i.-2\pi}, -0.2 e^{i.-0.3} >$	$< 0.5 e^{j.0.8}, 0.3 e^{j.\frac{3\pi}{4}}, 0.1 e^{j.0.3}, -0.6 e^{i.-0.6}, -0.4 e^{i.-2\pi}, -0.2 e^{i.-0.3} >$	$< 0.9 e^{j.0.9}, 0.2 e^{j.\frac{\pi}{4}}, 0.1 e^{j.0.5}, -1 e^{i.-\pi}, -0.3 e^{i.0}, -0.2 e^{i.-0.3} >$	$< 0.5 e^{j.0.8}, 0.1 e^{j.\frac{3\pi}{4}}, 0.1 e^{j.0.3}, -0.6 e^{i.-0.6}, -0.2 e^{i.-2\pi}, -0.2 e^{i.-0.3} >$	$< 0.8 e^{j.0.8}, 0.3 e^{j.\frac{3\pi}{4}}, 0.1 e^{j.0.3}, -0.9 e^{i.-0.6}, -0.4 e^{i.-2\pi}, -0.2 e^{i.0i.-0.3} >$
$y < 0.9 e^{j.0.9}, 0.2 e^{j.\frac{\pi}{4}}, 0.6 e^{j.0.5}, -1 e^{i.-\pi}, -0.3 e^{i.0}, -0.7 e^{i.-0.6} >$	$< 0.9 e^{j.0.9}, 0.2 e^{j.\frac{\pi}{4}}, 0.1 e^{j.0.5}, -1 e^{i.-\pi}, -0.3 e^{i.0}, -0.2 e^{i.-0.3} >$	$< 0.9 e^{j.0.9}, 0.2 e^{j.\frac{\pi}{4}}, 0.6 e^{j.0.5}, -1 e^{i.-\pi}, -0.3 e^{i.0}, -0.7 e^{i.-0.6} >$	$(0, 0, 0, 0, 0, 0)$	$< 0.9 e^{j.0.9}, 0.2 e^{j.\frac{\pi}{4}}, 0.4 e^{j.0.5}, -1 e^{i.-\pi}, -0.3 e^{i.0}, -0.5 e^{i.-0.6} >$
$z < 0.3 e^{j.0.3}, 0.1 e^{j.2\pi}, 0.8 e^{j.0.5}, -0.4 e^{i.-0.1}, -0.2 e^{i.-0.3}, -0.9 e^{i.-2\pi} >$	$< 0.5 e^{j.0.8}, 0.1 e^{j.\frac{3\pi}{4}}, 0.1 e^{j.0.3}, -0.6 e^{i.-0.6}, -0.2 e^{i.-2\pi}, -0.2 e^{i.-0.3} >$	$(0, 0, 0, 0, 0, 0)$	$z < 0.3 e^{j.0.3}, 0.1 e^{j.2\pi}, 0.8 e^{j.0.5}, -0.4 e^{i.-0.1}, -0.2 e^{i.-0.3}, -0.9 e^{i.-2\pi} >$	$(0, 0, 0, 0, 0, 0)$
$t < 0.8 e^{j.0.1}, 0.5 e^{j.\pi}, 0.4 e^{j.0.7}, -0.9 e^{i.-0.1}, -0.6 e^{i.-0.2}, -0.5 e^{i.-\pi}, -0.7 e^{i.-0.6} >$	$< 0.8 e^{j.0.8}, 0.3 e^{j.\frac{3\pi}{4}}, 0.1 e^{j.0.3}, -0.9 e^{i.-0.6}, -0.4 e^{i.-2\pi}, -0.2 e^{i.0i.-0.3} >$	$< 0.9 e^{j.0.9}, 0.2 e^{j.\frac{\pi}{4}}, 0.4 e^{j.0.5}, -1 e^{i.-\pi}, -0.3 e^{i.0}, -0.5 e^{i.-0.6} >$	$(0, 0, 0, 0, 0, 0)$	$< 0.8 e^{j.0.1}, 0.5 e^{j.\pi}, 0.4 e^{j.0.7}, -0.9 e^{i.-0.1}, -0.6 e^{i.-0.2}, -0.5 e^{i.-\pi}, -0.7 e^{i.-0.6} >$

Table 15: Matrix representation of BCNG1.

**Theorem 1.** Let  $M_{G_l}^{T^+}$  be matrix representation of  $T^+$ -BCNG1, then the degree of vertex  $D_{T^+}(x_k) = \sum_{j=1, j \neq k}^n a_{T^+}(k+1, j+1), x_k \in V$  or  $D_{T^+}(x_p) = \sum_{i=1, i \neq p}^n a_{T^+}(i+1, p+1), x_p \in V$ .

**Proof:** It is similar as in theorem 1 of (Broumi et al., 2017b).

**Theorem 2.** Let  $M_{G_l}^{I^+}$  be matrix representation of  $I^+$ -BCNG1, then the degree of vertex  $D_{I^+}(x_k) = \sum_{j=1, j \neq k}^n a_{I^+}(k+1, j+1), x_k \in V$  or  $D_{I^+}(x_p) = \sum_{i=1, i \neq p}^n a_{I^+}(i+1, p+1), x_p \in V$ .

**Proof:** It is similar as in theorem 1 of (Broumi et al., 2017b).

**Theorem 3.** Let  $M_{G_l}^{F^+}$  be matrix representation of  $F^+$ - BCNG1, then the degree of vertex

$$D_{F^+}(x_k) = \sum_{j=1, j \neq k}^n a_{F^+}(k+1, j+1), x_k \in V \text{ or}$$

$$D_{F^+}(x_p) = \sum_{i=1, i \neq p}^n a_{F^+}(i+1, p+1), x_p \in V.$$

**Proof:** It is similar as in theorem 1 of (Broumi et al., 2017b)

**Theorem 4.** Let  $M_{G_l}^{T^-}$  be matrix representation of  $T^-$ - BCNG1, then the degree of vertex

$$D_{T^-}(x_k) = \sum_{j=1, j \neq k}^n a_{T^-}(k+1, j+1), x_k \in V \text{ or}$$

$$D_{T^-}(x_p) = \sum_{i=1, i \neq p}^n a_{T^-}(i+1, p+1), x_p \in V.$$

**Proof:** It is similar as in theorem 1 of (Broumi et al., 2017b).

**Theorem 5.** Let  $M_{G_l}^{I^-}$  be matrix representation of  $I^-$ - BCNG1, then the degree of vertex

$$D_{I^-}(x_k) = \sum_{j=1, j \neq k}^n a_{I^-}(k+1, j+1), x_k \in V \text{ or}$$

$$D_{I^-}(x_p) = \sum_{i=1, i \neq p}^n a_{I^-}(i+1, p+1), x_p \in V.$$

**Proof:** It is similar as in theorem 1 of (Broumi et al., 2017b).

**Theorem 6.** Let  $M_{G_l}^{F^-}$  be matrix representation of  $F^-$ - BCNG1, then the degree of vertex

$$D_{F^-}(x_k) = \sum_{j=1, j \neq k}^n a_{F^-}(k+1, j+1), x_k \in V \text{ or}$$

$$D_{F^-}(x_p) = \sum_{i=1, i \neq p}^n a_{F^-}(i+1, p+1), x_p \in V.$$

**Proof:** It is similar as in theorem 1 of (Broumi et al., 2017b)

## 5. CONCLUSION

In this article, we have extended the concept of complex neutrosophic graph of type 1 (CNG1) to bipolar complex neutrosophic graph of type 1(BCNG1) and presented a matrix representation of it. The concept of BCNG1 is a generalization of Generalized fuzzy graph of type 1 (GFG1), generalized bipolar neutrosophic graph of type 1 (GBNG1), generalized single valued neutrosophic graph of type 1 (GSVNG1) and complex neutrosophic graph of type 1(CNG1). This concept can be applied to the case of tri-polar neutrosophic graphs and multi-polar neutrosophic graphs. In the future works, we plan to study the concept of completeness, the concept of regularity and to define the concept of bipolar complex neutrosophic graphs of type 2.

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## REFERENCES.

- Ali, M., & Smarandache, F. (2015). Complex neutrosophic set. *Neural Computing and Applications*. doi:10.1007/s00521-015-2154-y.
- Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 1986, 87-96.
- Arindam, D. & Anita, P. (2012). Vertex coloring of a fuzzy graph using alpha cut. *International Journal of Management, IT and Engineering*, 2(8), 340-352.
- Arindam, D. and Anita, P. (2013). Fuzzy graph coloring technique to classify the accidental zone of a traffic control. *Annals of Pure and Applied Mathematics*, 3(2),169-178.

- Ashraf, S., Naz, S., Rashmanlou, H., & Malik, M. A. (2016). Regularity of graphs in single valued neutrosophic environment, in press
- Broumi, S., Talea, M., Bakali, A., & Smarandache, F. (2016). Single valued neutrosophic graphs. *Journal of New Theory*, 10, 86-101.
- Broumi, S., Talea, M., Smarandache, F. & Bakali, A. (2016a). Single Valued Neutrosophic Graphs: Degree, order and size. IEEE International Conference on Fuzzy Systems (FUZZ), 2444-2451.
- Broumi, S., Bakali, A., Talea, M., & Smarandache, F. (2016b). Isolated single valued neutrosophic graphs. *Neutrosophic Sets and Systems*, 11, 2016, 74-78.
- Broumi, S., Bakali, A., Talea, M., Smarandache, F. & Kishore Kumar, P. K. (2016c). A new concept of matrix algorithm for mst in undirected interval valued neutrosophic graph. Chapter in book- Neutrosophic Operational Research- Volume II-Florentin Smarandache, Mohamed Abdel-Basset and Victor Chang(Editors), 2017, pp. 54-69. ISBN 978-1-59973-537-
- Broumi, S., Smarandache, F., Talea, M. & Bakali, A. (2016d). Decision-making method based on the interval valued neutrosophic graph. *Future Technologie, IEEE*, 44-50.
- Broumi, S., Bakali, A., Talea, M., Smarandache, F. & Ali, M. (2016e). Shortest path problem under bipolar neutrosophic setting. *Applied Mechanics and Materials*, 859, 59-66.
- Broumi, S., Bakali, A., Talea, M., Smarandache, F. & Vladareanu, L.(2016f). Computation of shortest path problem in a network with sv-trapezoidal neutrosophic numbers. Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, 417-422.
- Broumi, S., Bakali, A., Talea, M., Smarandache, F. & Vladareanu, L. (2016g). Applying Dijkstra algorithm for solving neutrosophic shortest path problem. Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, November 30 - December 3, 412-416.
- Broumi, S., Talea, M., Bakali, A., & Smarandache, F. (2016h). On bipolar single valued neutrosophic graphs, *Journal of New Theory*, 11, 84-102.
- Broumi, S., Smarandache, F., Talea, M. & Bakali, A. (2016i). An introduction to bipolar single valued neutrosophic graph theory. *Applied Mechanics and Materials*, 841, 184-191.
- Broumi, S., Bakali, A., Talea, M., Hassan, A. & Smarandache, F. (2017). Generalized single valued neutrosophic graphs of type 1. 2017, submitted.
- Broumi, S., Bakali, A., Talea, M., & Smarandache, F. (2017a). Generalized bipolar neutrosophic graph of type 1. 2017, submitted.
- Broumi, S., Bakali, A., Talea, M., & Smarandache, F. (2017b). Complex neutrosophic graph of type 1, submitted.
- Broumi, S., Bakali, A., Talea, M., Smarandache, F., Ulucay, V., Khan, & M, Mumtaz, A. (2017c). Bipolar complex neutrosophic sets, submitted.
- Deli, I., Ali, M., & Smarandache, F. (2015). Bipolar neutrosophic sets and their application based on multi-criteria decision making problems. *Advanced Mechatronic Systems (ICAMechS)*, 249- 254, DOI: 10.1109/ICAMechS.2015.7287068.
- Fathi, S., Elchawalby, H. & Salama, A. A. (2016). A neutrosophic graph similarity measures, chapter in book- New Trends in Neutrosophic Theory and Applications- Florentin Smarandache and Surapati Pramanik (Editors), 223-230. ISBN 978-1-59973-498-9.
- Mehra, S. & Singh, M. (2017). Single valued neutrosophic signed graphs. *International Journal of Computer Applications*, 157(9), 31-34.
- Ramot, D., Menahem, F., Gideon, L. & Abraham, K. (2002). Complex fuzzy set. *IEEE Transactions on Fuzzy Systems*, 10 (2), 171-186. doi: 10.1109/91.995119 .
- Shannon, A., Atanassov, K. (1994). A first step to a theory of the intuitionistic fuzzy graphs. Proc. of the First Workshop on Fuzzy Based Expert Systems (D. akov, Ed.), Sofia, 59-61.
- Sharma, A.K., Padamwar, B.V., Dewangan, C. L. (2013). Trends in fuzzy graphs. *International Journal of Innovative Research in Science. Engineering and Technology*, 2(9), 4636-4640.
- Smarandache, F. (2015). Refined literal indeterminacy and the multiplication law of sub-indeterminacies. *Neutrosophic Sets and Systems*, 9, 58.63.
- Smarandache, F. (2015a). Types of Neutrosophic Graphs and neutrosophic Algebraic Structures together with their Applications in Technology, seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produsii Mediu, Brasov, Romania 06 June 2015.
- Smarandache, F. (1998). Neutrosophy. neutrosophic probability, set, and logic. ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998; <http://fs.gallup.unm.edu/eBook-neutrosophics6.pdf> (last edition online)

- Abdel-Basset, M., Mohamed, M., Smarandache, F., & Chang, V. (2018). Neutrosophic Association Rule Mining Algorithm for Big Data Analysis. *Symmetry*, 10(4), 106.
- Abdel-Basset, M., & Mohamed, M. (2018). The Role of Single Valued Neutrosophic Sets and Rough Sets in Smart City: Imperfect and Incomplete Information Systems. *Measurement*. Volume 124, August 2018, Pages 47-55
- Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Smarandache, F. A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing and Applications*, 1-11.
- Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, 1-22.
- Abdel-Basset, M., Mohamed, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. *Future Generation Computer Systems*, 86, 12-29.
- Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. *Journal of Intelligent & Fuzzy Systems*, 33(6), 4055-4066.
- Abdel-Basset, M.; Mohamed, M.; Smarandache, F. An Extension of Neutrosophic AHP–SWOT Analysis for Strategic Planning and Decision-Making. *Symmetry* 2018, 10, 116.
- Smarandache, F. (2016). Neutrosophic overset, neutrosophic underset, Neutrosophic off set, Similarly for Neutrosophic Over-/Under-/OffLogic, Probability, and Statistic, Pons Editions, Brussels, 170p.
- Smarandache, F. (2017). Nidusidearum. Scilogs, III: Viva la Neutrosophia!, Brussels, ,134p.
- Smarandache, F. (2015). Symbolic neutrosophic theory. Europanova, asbl, Brussels, 195 p., Belgium.
- Samanta, S., Sarkar, B., Shin, D. & Pal, M. (2016). Completeness and regularity of generalized fuzzy graphs. Springer Plus. doi: 10.1186/s40064-016-3558-6.
- Turksen, I. (1986). Interval valued fuzzy sets based on normal forms. *Fuzzy Sets and Systems*, 20, 191-210.
- Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure*, 4, 410 - 413.
- Zadeh, L. A. (1965). Fuzzy Sets. *Information and Control*, 8 (3), 338-353.