Interval Valued Neutrosophic Soft Graphs

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ABSTRACT

In this article, we combine the interval valued neutrosophic soft set and graph theory. We introduce the notions of interval valued neutrosophic soft graphs, strong interval valued neutrosophic graphs, complete interval valued neutrosophic graphs, and investigate some of their related properties. We study some operations on interval valued neutrosophic soft graphs. We also give an application of interval valued neutrosophic soft graphs into a decision making problem. We hold forth an algorithm to solve decision making problems by using interval valued neutrosophic soft graphs.

KEYWORDS: interval valued neutrosophic soft sets, interval valued neutrosophic soft sets, interval valued neutrosophic soft graphs, strong interval valued neutrosophic soft graphs, complete interval valued neutrosophic soft graphs, decision making.

1. INTRODUCTION

The neutrosophic set (NSs), proposed by (Smarandache, 2006, 2011), is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. Itis a generalization of the theory of fuzzy sets (Zadeh, 1965), intuitionistic fuzzy sets (Atanassov, 1986,1999) and interval-valued intuitionistic fuzzy sets (Atanassov, 1989). The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity-membership function (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. In order to conveniently employ NS in real life applications, (Wang et al., 2010) introduced the concept of single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors (Wang, Zhang, & Sunderraman, 2005) introduced the concept of interval valued neutrosophic set (IVNS), which is more precise and flexible than single valued neutrosophic set. The IVNS is a generalization of single valued neutrosophic set, in which three membership functions are independent and their value belong to the unit interval [0, 1]. Some more work on single valued neutrosophic set, interval valued neutrosophic set and their applications may be found in (Aydoğdu, 2015; Ansari et a.l, 2012; Ansari et al. 2013; Ansari et al. 2013a; Zhang et al., 2015; Zhang et al., 2015b; Deli et al. ,2015; Ye, 2014, 2014a; Şahin, 2015; Aggarwal et al.,2010; Broumi and Smarandache, 2014; Karaaslan and Davvaz, 2018).

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving a combinatorial problem in different areas, such as geometry, algebra, number theory, topology, optimization and computer science. Most important thing to be noted is that, when we have uncertainty regarding either the set of vertices or edges, or both, the model becomes a fuzzy graph. The extension of fuzzy graph theory (Nagoor and Basheer, 2003; Nagoor & Latha,2012; Bhattacharya,1987) have been developed by several researchers. Intuitionistic fuzzy graphs

(Nagoor & Shajitha, 2010; Akram, 2012) considered the vertex sets and edge sets as intuitionistic fuzzy sets. Interval valued fuzzy graphs (Akram & Dudek, 2011; Akram, 2012a) considered the vertex sets and edge sets as interval valued fuzzy sets. Interval valued intuitionistic fuzzy graphs (Akram, 2014; Hai-Long et.,2016) considered the vertex sets and edge sets as interval valued intuitionistic fuzzy sets. Bipolar fuzzy graphs (Akram, 2011, 2013) considered the vertex sets and edge sets as bipolar fuzzy sets. M-polar fuzzy graphs (Akram, 2016) considered the vertex sets and edge sets as m-polar fuzzy sets. But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and their extensions fail. For this purpose, (Smarandache, 2015,2015a,2015b; Vasantha and Smarandache,2013) defined four main categories of neutrosophic graphs. Two of them are based on literal indeterminacy (I), which are called I-edge neutrosophic graph and I-vertex neutrosophic graph; these concepts are studied deeply and gained popularity among the researchers due to their applications via real world problems (Devadoss et al., 2013, Jiang et al., 2010; Vasantha et al., 2015) The two others graphs arebased on (t, i, f) components and are called:(t, i, f)-edge neutrosophic graph and (t, i, f)-vertex neutrosophic graph; these concepts are not developed at all.

Later on, (Broumi et al., 2016a) introduced a third neutrosophic graph model, and investigated some of its properties. This model allows the attachment of truth-membership (t), indeterminacy–membership (i) and falsity- membership degrees (f) both to vertices and edges. The third neutrosophic graph model is called single valued neutrosophic graph (SVNG for short). The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionistic fuzzy graph. Also, the same authors (Broumi et al., 2016a, 2016e) introduced neighborhood degree of a vertex and closed neighborhood degree of a vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of a vertex in fuzzy graph and intuitionistic fuzzy graph. Also, (Broumi et al., 2016b) introduced the concept of interval valued neutrosophic graph as a generalization of fuzzy graph, intuitionistic fuzzy graph, interval valued fuzzy graph, interval valued intuitionistic fuzzy graph and single valued neutrosophic graph, and have discussed some of their properties with proofs and examples. In addition, (Broumi et al., 2016c) have introduced some operations, such as Cartesian product, composition, union and join on interval valued neutrosophic graphs, and investigate some their properties. On the other hand, (Broumi et al., 2016d) discussed a subclass of interval valued neutrosophic graph, called strong interval valued neutrosophic graph, and introduced some operations such as, Cartesian product, composition and join of two strong interval valued neutrosophic graph with proofs. Interval valued neutrosophic soft sets are the generalization of fuzzy soft sets (Maji, 2001), intuitionistic fuzzy soft sets (Maji, 2001a), interval valued intuitionistic fuzzy soft sets (Jiang, et al., 2010) and (Maji, 2013). (Thumbakara and George,2014) combined the concept of soft set theory with graph theory. (Irfan et al, 2016) proposed a method to represent a graph, which is based on adjacency of vertices and soft set theory and introduced some operations such as restricted intersection, restricted union, extended intersection and extended union for graphs. In addition, the authors defined a metric to find distances between graphs represented by soft sets. Later on, Mohinta (2015) extended the concept of soft graph to the case of fuzzy soft graph. Also, Akram et al. (2015) studied more properties on fuzzy soft graphs and some operations. Shahzadi and Akram (2016) presented different types of new concepts, including intuitionistic fuzzy soft graphs, complete intuitionistic fuzzy soft graph, strong intuitionistic fuzzy soft graph and self-complement of intuitionistic fuzzy soft graph. And described various methods of their construction, and investigated some of their related properties and discussed the applications of intuitionistic fuzzy soft graphs in communication network and decision making.

Recently, the notion of neutrosophic soft set has been extended in the graph theory and the concept of neutrosophic soft graph was provided by (Shah and Hussain, 2016) Later on, Shahzadi and Akram (2016) have applied the concept of neutrosophic soft sets to graphs and discussed various methods of construction of neutrosophic soft graphs. In the literature, the study of interval valued neutrosophic soft graphs (IVNS-graph) is still blank.

In the present paper, interval valued neutrosophic soft sets (Deli, 2015). are employed to study graphs and give rise to a new class of graphs called interval valued neutrosophic soft graphs. We have discussed different operations defined on neutrosophic soft graphs such as Cartesian product, composition, union and join with examples and proofs. The concepts of strong interval valued neutrosophic soft graphs, complete interval valued neutrosophic soft graphs and the complement of strong interval valued neutrosophic soft graphs a real so discussed. Interval valued neutrosophic soft graphs are pictorial representation in which each vertex and each edge is an element of interval valued neutrosophic soft sets.

This paper is organized as follows. In section 2, we give all the basic definitions related to interval valued neutrosophic graphs and interval valued neutrosophic soft sets which will be employed in later sections. In section 3, we introduce certain notions including interval valued neutrosophic soft graphs, strong interval valued neutrosophic soft graphs, complete interval valued neutrosophic soft graphs, the complement of strong interval valued neutrosophic soft graphs, and illustrate these notions by several examples, then we present some operations such as Cartesian product, composition, intersection, union and join on an interval valued neutrosophic soft graphs and investigate some of their related properties. In section 4, we present an application of interval valued neutrosophic soft graphs in decision making.

2. PRELIMINARIES

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets, neutrosophic soft sets, interval valued, soft sets, neutrosophic soft sets, single valued neutrosophic graphs, fuzzy graph, intuitionistic fuzzy graph, interval valued intuitionistic fuzzy graphs and interval valued neutrosophic graphs, relevant to the present work. See especially (Mohamed et al, 2014; Nagoor and Basheer, 2003; Nagoor and Shajitha2010; Molodtsov, 1999; Smarandache, 2006; Wang et al., 2005; Wang et al., 2010; Deli, 2015; Broumi et al., 2016a, 2016b) for further details and background.

Definition 2.1 (Smarandache, 2006). Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X}, where the functions T, I, F: X→]$^-$0,1$^+$[define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element x ∈ X to the set A, with the condition:

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]$^-$0,1$^+$[.

Since it is difficult to apply NSs to practical problems, (Wang et al., 2010). introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 (Wang et al., 2010). Let Xbe a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X$T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}. \quad (2)$$

Definition2.3 (Nagoor and Basheer, 2003)A fuzzy graph is a pair of functions G = (σ, μ) where σ is a fuzzy subset of a non-empty set V andμis a symmetric fuzzy relation on σ. i.eσ : V → [0,1] and μ:VxV→[0,1] such thatμ(uv) ≤ σ(u) ∧ σ(v)for all u, v ∈ V, where uv denotes the edge between u and v and σ(u) ∧ σ(v) denotes the minimum of σ(u) and σ(v). σ is called the fuzzy vertex set of V andμ is called the fuz... ...e set of E.

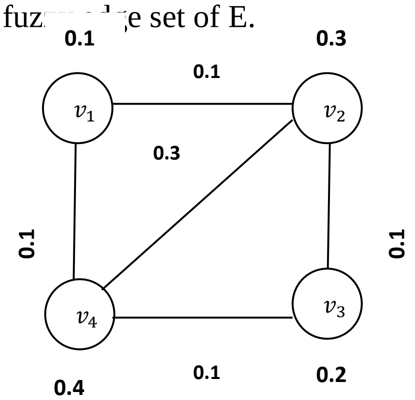


Fig.1:FuzzyGraph

Definition2.4 (Nagoor and Basheer, 2003) The fuzzy subgraph H = (τ, ρ) is called a fuzzy subgraph of G = (σ, μ)
If τ(u) ≤ σ(u) for all u ∈ V and ρ(u, v) ≤μ(u, v)for all u, v ∈ V.

Definition2.5 (Nagoor and Shajitha2010) An Intuitionistic fuzzy graph is of the form G = (V,E) where:

i. V={v_1,v_2,….,v_n} such that μ_1:V→ [0,1] andγ_1:V→ [0,1] denote the degree of membership and non-membership of the element $v_i \in$ V, respectively, and $0 \le \mu_1(v_i)+\gamma_1(v_i)) \le 1$for every$v_i \in$ V,(i=1, 2,……. n),
ii. E⊆ VxV whereμ_2:VxV→[0,1]andγ_2:VxV→ [0,1] are such that $\mu_2(v_i, v_j) \le \min[\mu_1(v_i),\mu_1(v_j)]$and $\gamma_2(v_i, v_j) \ge \max[\gamma_1(v_i),\gamma_1(v_j)]$and $0 \le \mu_2(v_i, v_j)+\gamma_2(v_i, v_j) \le 1$ for every$(v_i, v_j) \in$E,(i,j =1,2,……. n)

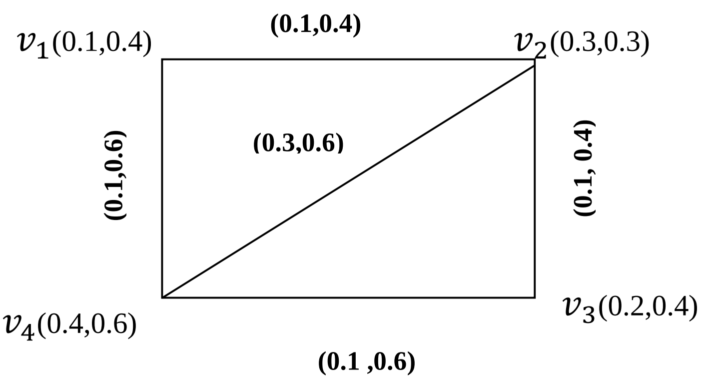


Fig.2: Intuitionistic Fuzzy Graph

Definition 2.6 (Broumi et al., 2016a).Let A = (T_A,I_A, F_A) and B = (T_B,I_B, F_B)be single valued neutrosophic sets on a set X. If A = (T_A,I_A, F_A) is a single valued neutrosophic relation on a set X, then A =(T_A,I_A, F_A) is called a single valued neutrosophic relation on B = (T_B,I_B, F_B) if

$T_B(x, y) \le \min(T_A(x),T_A(y))$
$I_B(x, y) \ge \max(I_A(x),I_A(y))$ and
$F_B(x, y) \ge \max(F_A x),F_A(y))$ for all x, y ∈ X.

A single valued neutrosophic relation A on X is called symmetric if $T_A(x, y) = T_A(y, x)$, $I_A(x, y) = I_A(y, x)$, $F_A(x, y) = F_A(y, x)$ and $T_B(x, y) = T_B(y, x)$, $I_B(x, y) = I_B(y, x)$ and $F_B(x, y) = F_B(y, x)$, for all $x, y \in X$.

Definition 2.7 (Broumi et al., 2016a). A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair G= (A, B) where:
1.The functions T_A:V→[0, 1], I_A:V→[0, 1] and F_A:V→[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in$ V, respectively,and

$0\le T_A(v_i) + I_A(v_i) + F_A(v_i) \le 3$ for all $v_i \in$ V (i=1, 2, …,n)

2. The functionsT_B: E ⊆ V x V →[0, 1],I_B:E ⊆ V x V →[0, 1] and F_B: E ⊆ V x V →[0, 1] are defined by

$T_B(\{v_i, v_j\}) \le \min [T_A(v_i), T_A(v_j)]$,
$I_B(\{v_i, v_j\}) \ge \max [I_A(v_i), I_A(v_j)]$, and
$F_B(\{v_i, v_j\}) \ge \max [F_A(v_i), F_A(v_j)]$,

Denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i,v_j) \in$ E respectively, where:

$0\le T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \le 3$for all$\{v_i, v_j\} \in$ E (i, j = 1, 2,…, n)

We call A the single valued neutrosophic vertex set of V, B the single valued neutrosophic edge set of E, respectively. Note that B is a symmetric single valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E. Thus, G = (A, B) is a single valued neutrosophic graph of G*= (V, E) if:

$T_B(v_i, v_j) \le \min [T_A(v_i), T_A(v_j)]$,
$I_B(v_i, v_j) \ge \max [I_A(v_i), I_A(v_j)]$ and
$F_B(v_i, v_j) \ge \max [F_A(v_i), F_A(v_j)]$, for all$(v_i, v_j) \in$ E.

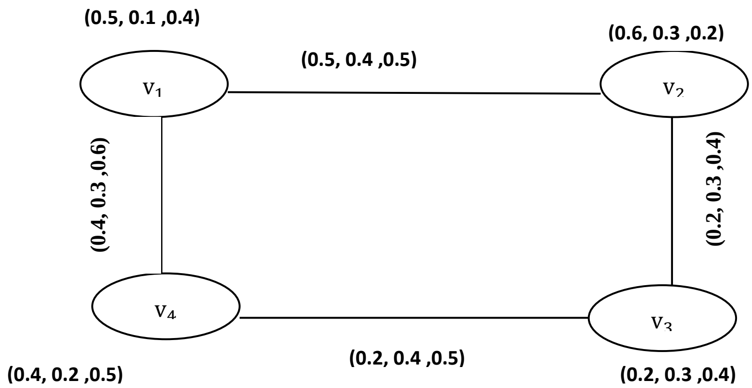


Fig.3: Single valued neutrosophic graph

Definition 2.9 (**Broumi et al., 2016a**).A partial SVN-subgraph of SVN-graph G= (A, B)is a SVN-graph H=($\boldsymbol{V'}$,$\boldsymbol{E'}$)such that

(i) $\boldsymbol{V' \subseteq V}$, where$\boldsymbol{T'_A(v_i) \le T_A(v_i)}$,$\boldsymbol{I'_A(v_i) \ge I_A(v_i)}$,$\boldsymbol{F'_A(v_i) \ge F_A(v_i)}$,for all$\boldsymbol{v_i \in V}$.

(ii) $\boldsymbol{E' \subseteq E}$, where $\boldsymbol{T'_B(v_i, v_j) \le T_B(v_i, v_j)}$,$\boldsymbol{I'_{Bij} \ge I_B(v_i, v_j)}$, $\boldsymbol{F'_B(v_i, v_j) \ge}$

$\boldsymbol{F_B(v_i, v_j)}$,for all $(\boldsymbol{v_i v_j}) \in \boldsymbol{E}$.

Definition 2.10 (Broumi et al., 2016a). ASVN-subgraph of SVN-graph G= (V, E)is a SVN-graph H=($\boldsymbol{V'}$,$\boldsymbol{E'}$)such that

(i) $\boldsymbol{V' = V}$, where$\boldsymbol{T'_A(v_i) = T_A(v_i)}$,$\boldsymbol{I'_A(v_i) = I_A(v_i)}$,$\boldsymbol{F'_A(v_i) = F_A(v_i)}$for all$\boldsymbol{v_i}$in the vertex set of$\boldsymbol{V'}$.

(ii) $\boldsymbol{E' = E}$, where$\boldsymbol{T'_B(v_i, v_j) = T_B(v_i, v_j)}$,$\boldsymbol{I'_B(v_i, v_j) = I_B(v_i, v_j)}$,$\boldsymbol{F'_B(v_i, v_j) = F_B(v_i, v_j)}$for every $(\boldsymbol{v_i v_j}) \in \boldsymbol{E}$ in the edge set of$\boldsymbol{E'}$.

Definition 2.10 (Broumi et al., 2016a). Let G= (A, B) be a single valued neutrosophic graph. Then the degree of any vertex v is sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those edges which are incident on vertex v denoted by d(v)= ($d_T(v)$, $d_I(v)$,$d_F(v)$) where:

$d_T(v)$=$\sum_{u \neq v} T_B(u, v)$ denotesdegree of truth-membership vertex.

$d_I(v)$=$\sum_{u \neq v} I_B(u, v)$ denotes degree of indeterminacy-membership vertex.

$d_F(v)$=$\sum_{u \neq v} F_B(u, v)$ denotes degree of falsity-membership vertex.

Definition 2.11(Broumi et al., 2016a). A single valued neutrosophic graph G = (A, B) of G^*= (V, E) is calledstrong single valued neutrosophic graph if:

$T_B(v_i, v_j)$ =min [$T_A(v_i)$,$T_A(v_j)$]

$I_B(v_i, v_j)$ =max [$I_A(v_i)$,$I_A(v_j)$]

$F_B(v_i, v_j)$ =max [$F_A(v_i)$, $F_A(v_j)$], for all $(v_i, v_j) \in$ E.

Definition 2.12(Broumi et al., 2016a). A single valued neutrosophic graph G= (A, B) is called complete if

$T_B(v_i, v_j)$ =min [$T_A(v_i)$,$T_A(v_j)$]

$I_B(v_i, v_j)$ =max [$I_A(v_i)$,$I_A(v_j)$]

$F_B(v_i, v_j)$ =max [$F_A(v_i)$, $F_A(v_j)$], for all $v_i, v_j \in$ V.

Definition 2.13(Broumi et al., 2016a)The complement of a single valued neutrosophic graph G (A, B) onG^* is a single valued neutrosophic graph $\bar{G}$ on G^* where:

1.$\bar{A}$ =A

2.$\overline{T_A}(v_i)$= $T_A(v_i)$,$\overline{I_A}(v_i)$= $I_A(v_i)$,$\overline{F_A}(v_i)$ = $F_A(v_i)$, for all $v_j \in$ V.

3.$\overline{T_B}(v_i, v_j)$= min $\left[T_A(v_i), T_A(v_j)\right]$-$T_B(v_i, v_j)$

$\overline{I_B}(v_i, v_j)$= max $\left[I_A(v_i), I_A(v_j)\right]$-$I_B(v_i, v_j)$ and

$\overline{F_B}(v_i, v_j)$= max $\left[F_A(v_i), F_A(v_j)\right]$-$F_B(v_i, v_j)$,for all $(v_i, v_j) \in$ E.

Definition 2.14 (Mohamed et al, 2014). An interval valued intuitionistic fuzzy graph with underlying set V is defined to be a pair G= (A, B) where

1)The functions M_A : V→ D [0, 1]and N_A : V→ D [0, 1] denote the degree of membership and non-membership of the element x ∈ V, respectively, such that 0 such that0≤M_A(x)+ N_A(x) ≤ 1 for all x ∈ V.

2) The functions M_B : E ⊆ $V \times V$ → D [0, 1]and N_B : : E ⊆ $V \times V$ → D [0, 1] are defined by

$$M_{BL}(x, y))\leq \min (M_{AL}(x), M_{AL}(y)) \text{ and } N_{BL}(x, y)) \geq \max (N_{AL}(x), N_{AL}(y))$$

$$M_{BU}(x, y))\leq \min (M_{AU}(x), M_{AU}(y)) \text{ and } N_{BU}(x, y)) \geq \max (N_{AU}(x), N_{AU}(y)),$$

such that

$$0\leq M_{BU}(x, y))+ N_{BU}(x, y)) \leq 1 \text{ for all } (x, y) \in E.$$

Définition 2.15 (Broumi et al., 2016b). By an interval-valued neutrosophic graph of a graph G* = (V, E) we mean a pair G = (A,B), where A =< [T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}]> is an interval-valued neutrosophic set on V and B =< [T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}]> is an interval-valued neutrosophic relation on E satisfies the following condition:

1. V= $\{v_1, v_2, \ldots, v_n\}$ such that T_{AL}:V→[0, 1],T_{AU}:V→[0, 1], I_{AL}:V→[0, 1],I_{AU}:V→[0, 1]and F_{AL}:V→[0, 1],F_{AU}:V→[0, 1] denote the degree of truth-membership, the degree ofindeterminacy- membership and falsity-membership of the element $y \in$ V, respectively,and0≤ $T_A(v_i)$ + $I_A(v_i)$ +$F_A(v_i)$ ≤3 for allv_i ∈ V (i=1, 2, …,n).
2. The functionsT_{BL}:V x V →[0, 1],T_{BU}:V x V →[0, 1],I_{BL}:V x V →[0, 1],I_{BU}:V x V →[0, 1]and F_{BL}:V x V →[0,1],F_{BU}:V x V →[0, 1] are such that:

$$T_{BL}(\{v_i, v_j\}) \leq \min [T_{AL}(v_i), T_{AL}(v_j)]$$

$$T_{BU}(\{v_i, v_j\}) \leq \min [T_{AU}(v_i), T_{AU}(v_j)]$$

$$I_{BL}(\{v_i, v_j\}) \geq \max[I_{BL}(v_i), I_{BL}(v_j)]$$

$$I_{BU}(\{v_i, v_j\}) \geq \max[I_{BU}(v_i), I_{BU}(v_j)]$$

$$F_{BL}(\{v_i, v_j\}) \geq \max[F_{BL}(v_i), F_{BL}(v_j)]$$

$$F_{BU}(\{v_i, v_j\}) \geq \max[F_{BU}(v_i), F_{BU}(v_j)],$$

Denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge (v_i,v_j) ∈ E respectively, where

$$0\leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3 \text{for all} \{v_i, v_j\} \in E \text{ (i, j = 1, 2, ., n)}.$$

they call A the interval valued neutrosophic vertex set of V, B the interval valued neutrosophic edge set of E, respectively, Note that B is a symmetric interval valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E Thus, G = (A, B) is an interval valued neutrosophic graph of G*= (V, E) if

$$T_{BL}(v_i, v_j) \leq \min [T_{AL}(v_i), T_{AL}(v_j)]$$

$T_{BU}(v_i, v_j)$ ≤min $[T_{AU}(v_i), T_{AU}(v_j)]$

$I_{BL}(v_i, v_j)$ ≥max$[I_{BL}(v_i), I_{BL}(v_j)]$

$I_{BU}(v_i, v_j)$ ≥max$[I_{BU}(v_i), I_{BU}(v_j)]$And

$F_{BL}(v_i, v_j)$ ≥max$[F_{BL}(v_i), F_{BL}(v_j)]$

$F_{BU}(v_i, v_j)$ ≥max$[F_{BU}(v_i), F_{BU}(v_j)]$,for all$(v_i, v_j)$ ∈ E.

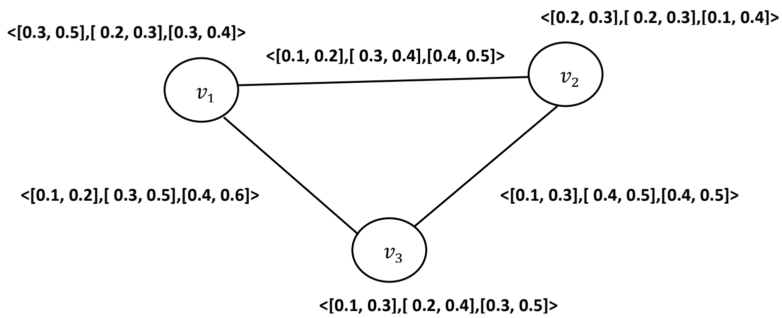


Fig. 4: G: Interval valued neutrosophic graph.

Definition 2.16 (Molodtsov, 1999). Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K: A → P(U).

As an illustration, let us consider the following example.

Example 2. Suppose that U is the set of houses under consideration, say U = $\{h_1, h_2. . .,h_5\}$. Let E be the set of some attributes of such houses, say E = $\{e_1, e_2, . . ., e_5\}$, where $e_1, e_2, . . ., e_5$ stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:

A=$\{e_1, e_2,e_3, e_4,e_5\}$;

K(e_1) = $\{h_2, h_3, h_5\}$, K(e_2) = $\{h_2, h_4\}$, K(e_3) = $\{h_1\}$, K(e_4) = U, K(e_5) = $\{h_3, h_5\}$.

Definition 2.17 (Wang et al., 2005**).**Let IVNS(X) denote the family of all the interval valued neutrosophic sets in universe X, assume A, B ∈ IVNS(X) such that

$$A = \{\langle x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)]\rangle: x \in X\}$$
$$B = \{\langle x, [T_B^L(x), T_B^U(x)], [I_B^L(x), I_B^U(x)], [F_B^L(x), F_B^U(x)]\rangle: x \in X\}$$

then some operations can be defined as follows:

(1) $A \cup B = \{\langle x, [\max\{T_A^L(x), T_B^L(x)\}, \max\{T_A^U(x), T_B^U(x)\}], [\min\{I_A^L(x), I_B^L(x)\}, \min\{I_A^U(x), I_B^U(x)\}], [\min\{F_A^L(x), F_B^L(x)\}, \min\{F_A^U(x), F_B^U(x)\}]\rangle: x \in X\}$;

(2) $A \cap B = \{\langle x, [\min\{T_A^L(x), T_B^L(x)\}, \min\{T_A^U(x), T_B^U(x)\}],$

$\left[\max\{I_A^L(x), I_B^L(x)\}, \max\{I_A^U(x), I_B^U(x)\}\right], \left[\max\{F_A^L(x), F_B^L(x)\}, \max\{F_A^U(x), F_B^U(x)\}\right]\rangle: x \in X\};$

(3) $A^c = \{\langle x, [F_A^L(x), F_A^U(x)], [1 - I_A^U(x), 1 - I_A^L(x)], [T_A^L(x), T_A^U(x)]\rangle: x \in X\};$

(4) $A \subseteq B$, iff $T_A^L(x) \le T_B^L(x)$, $T_A^U(x) \le T_B^U(x)$,$I_A^L(x) \ge I_B^L(x)$, $I_A^U(x) \ge I_B^U(x)$ and $F_A^L(x) \ge F_B^L(x)$, $F_A^U(x) \ge F_B^U(x)$ for all $x \in X$.

$A = B$, iff $A \subseteq$ Band $B \subseteq$A.

As an illustration, let us consider the following example.

Example 2.18.Assume that the universe of discourse U= $\{x_1, x_2, x_3, x_4\}$. Then, A is an interval valued neutrosophic set (IVNS) of U such that:

A = {<x_1, [0.1, 0.8], [0.2, 0.6], [0.8, 0.9] >, <x_2, [0.2, 0.5], [0.3, 0.5], [0.6, 0.8]>, <x_3, [0.5, 0.8], [0.4, 0.5], [0.5, 0.6] >, <x_4, [0.1, 0.4], [0.1, 0.5], [0.4, 0.8] >}.

Definition 2.19 (**Deli et al., 2015**).Let U be an initial universe set and $A \subset E$ be a set of parameters. Let IVNS (U) denote the set of all interval valued neutrosophic subsets of U. The collection (K, A) is termed to be the soft interval valued neutrosophic set over U, where K is a mapping given by K: A → IVNS(U).

The interval valued neutrosophic soft set defined over a universe is denoted by INSS. Here,

1. Υ is an ivn-soft subset of Ψ, denoted by $\Upsilon \Subset \Psi$, if K(e) ⊆L(e) for all e∈E.
2. Υ is an ivn-soft equals toΨ, denoted by $\Upsilon = \Psi$, if K(e)=L(e) for all e∈E.
3. The complement of Υ is denoted by Υ^c , and is defined by Υ^c = {(x, K^o (x)): x∈E}
4. The union of Υ and Ψ is denoted by $\Upsilon \cup'' \Psi$, if K(e) ∪L(e) for all e∈E.
5. The intersection of $\Upsilon$and Ψ is denoted by $\Upsilon \cap'' \Psi$,if K(e) ∪L(e) for all e∈E.

To illustrate let us consider the following example:

Let U be the set of houses under consideration and E is the set of parameters (or qualities). Each parameter is an interval valued neutrosophic word or sentence involving interval valued neutrosophic words. Consider E= {beautiful, costly, in the green surroundings, moderate, expensive}. In this case, to define an interval valued neutrosophic soft set means to point out beautiful houses, costly houses, and so on.

Suppose that there are five houses in the universe U, given by U = {h_1,h_2,h_3,h_4,h_5} and the set of parameters A = {e_1,e_2,e_3,e_4}, where each$e_i$is a specific criterion for houses:

e_1 stands for 'beautiful',
e_2 stands for 'costly',
e_3 stands for 'in the green surroundings',
e_4 stands for 'moderate'.

Suppose that,

K(beautiful)={<h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,<h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, <h3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,<h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,<h5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.

K(costly)={<h_1,[0.5, 0.6], [0.3, 0.7], [0.1, 0.4]>,<h_2,[0.3, 0.5], [0.6 ,0.8], [0.1, 0.3] >, <h_3,[0.3, 0.5],[0.2 ,0.6],[0.3, 0.4] >,<h_4,[0.2 ,0.5],[0.1, 0.2],[0.2, 0.4] >,<h_5,[0.2, 0.4] ,[0.1 ,0.5],[0.1,

0.4] >}.
K(in the green surroundings)= {<h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,<h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.5]>, <h_3,[0.2, 0.4],[0.2 ,0.3],[0.3, 0.5]>,<h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,<h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.2, 0.3] >},
K(moderate)={<h_1,[0.1, 0.6], [0.6, 0.7], [0.3, 0.4]>,<h_2,[0.2, 0.5], [0.4,0.8], [0.2, 0.4] >, <h_3,[0.3, 0.7],[0.2 ,0.4],[0.2, 0.5] >,<h_4,[0.7 ,0.8],[0.3, 0.4],[0.1, 0.2] >,<h_5,[0.3, 0.4] ,[0.2,0.6],[0.1, 0.2] >}.

3. INTERVAL VALUED NEUTROSOPHIC SOFT GRAPHS

Let U be an initial universe and P the set of all parameters, P(U) denoting the set of all interval neutrosophic sets of U. Let A be a subset of P. A pair (K, A) is called an interval valued neutrosophic soft set over U. Let P(V) denote the set of all interval valued neutrosophic sets of V and P(E) denote the set of all interval valued neutrosophic sets of E.

Definition3.1 An interval valued neutrosophics of the graph G=(G*,K, M,A) is a 4-tuple such that

a) G^*= (V, E) is a simple graph,
b) A is a nonempty set of parameters,
c) (K, A) is an interval valued neutrosophic soft set over V,
d) (M, A) is an interval valued neutrosophic over E,
e) (K(e), M(e)) is an interval valued neutrosophic (sub)graph of $G^*$for all e∈A.

That is,

$T^L_{M(e)}(xy) \leq$min [$T^L_{K(e)}(x)$, $T^L_{K(e)}(y)$], $T^U_{M(e)}(xy) \leq$ min [$T^U_{K(e)}(x)$, $T^U_{K(e)}(y)$],
$I^L_{M(e)}(xy) \geq$max [$I^L_{K(e)}(x)$, $I^L_{K(e)}(y)$], $I^U_{M(e)}(xy) \geq$ max [$T^U_{K(e)}(x)$, $T^U_{K(e)}(y)$]
and$F^L_{M(e)}(xy) \geq$max [$F^L_{K(e)}(x)$, $F^L_{K(e)}(y)$], $F^U_{M(e)}(xy) \geq$ max [$T^U_{K(e)}(x)$, $T^U_{K(e)}(y)$],

such that

0≤ $T_{M(e)}(xy)$ + $I_{M(e)}(xy)$+ $F(xy)$ ≤3 for all e∈ A and x, y ∈ V.

The interval valued neutrosophic graph (K(e), M (e)) is denoted by H(e) for convenience. An interval valued neutrosophic graph is a parametrized family of interval valued neutrosophic graphs. The class of all interval valued neutrosophic soft graphs of G^* is denoted by IVN(G^*). Note that

$T^L_{M(e)}(xy)$= $T^U_{M(e)}(xy)$ =$I^L_{M(e)}(xy)$= $I^U_{M(e)}(xy)$ = 0 and $F^L_{M(e)}(xy)$= $F^U_{M(e)}(xy)$ = 0 for all xy ∈ V− E, e ∉ A.

Definition 3.2 Let G_1=(K_1, M_1, A) and G_2=(K_2, M_2, B) be two interval valued neutrosophic graphs of G*. Then G_1 is an interval valued neutrosophic soft subgraph of G_2 if

(i) A⊆ B
(ii) H_1(e) is a partial subgraph of H_2(e) for all e ∈ A.

Example 3.3.Consider a simple graphG^*=(V, E) such that V={v_1, v_2,v_3} and E={v_1v_2, v_2v_3,v_3v_1}.

Let A= {e_1, e_2} be a set of parameter and let(K, A)bean interval valued neutrosophic soft set over V with its interval valued neutrosophic approximate function K : A →P(V) defined by

$K(e_1)$={v_1|([0.3, 0.5],[0.2, 0.3], [0.3, 0.4]), v_2|([0.2, 0.3], [0.2, 0.3], [0.1, 0.4]), v_3|([0.1, 0.3], [0.2, 0.4], [0.3, 0.5])},

$K(e_2)$={v_1|([0.1, 0.4], [0.1, 0.3], [0.2, 0.3]), v_2|([0.1, 0.3], [0.1, 0.2], [0.1, 0.4]), v_3|([0.1, 0.2], [0.2, 0.3], [0.2, 0.5])}.

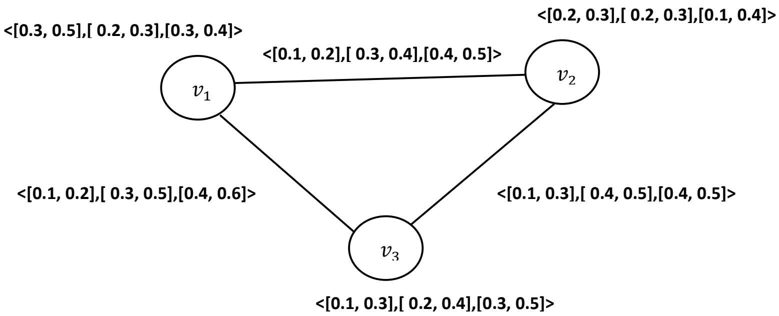
Let (M, A)be an interval valued neutrosophic soft set over E with its interval valued neutrosophic approximate function$M$: A →P(E) defined by

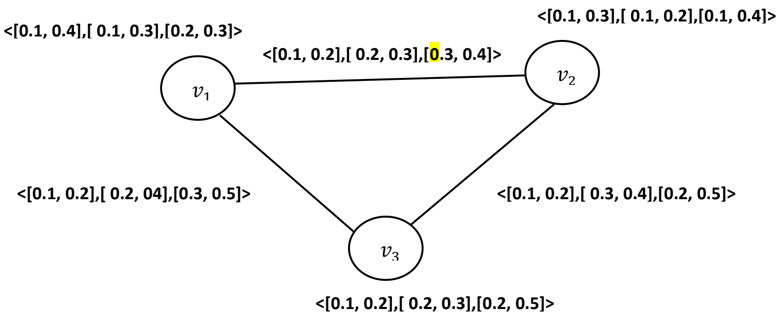
$M(e_1)$={v_1v_2|([0.1, 0.2], [0.3, 0.4], [0.4, 0.5]), v_2v_3|([0.1, 0.3], [0.4, 0.5], [0.4, 0.5]),v_3v_1 |([0.1, 0.2], [0.3, 0.5], [0.5, 0.6])},

$M(e_2)$= {v_1v_2|([0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), v_2v_3|([0.1, 0.2], [0.3, 0.4], [0.2, 0.5]),, v_3v_1 |([0.1, 0.2], [0.2, 0.4], [0.3, 0.5])}.

Thus, $H(e_1)$=($K(e_1)$, $M(e_1)$), $H(e_2)$=($K(e_2)$, $M(e_2)$) are interval valued neutrosophic graphs corresponding to the parameters $e_1$and $e_1$as shown below.

$H(e_1)$





$H(e_2)$

Fig. 3.1:Interval valued neutrosophic soft graph G= {$H(e_1)$, $H(e_2)$}.

Hence G= { $H(e_1)$, $H(e_2)$} is an interval valued neutrosophic soft graph of G^*.

Tabular representation of an interval valued neutrosophic soft graph is given in Table below.

Table 1: Tabular representation of an interval valued neutrosophic soft graph.

K	v_1	v_2	v_3
e_1	<[0.3,0.5],[0.2, 0.3][0.3, 0.4]>	<[0.2,0.3],[0.2, 0.3][0.1, 0.4]>	<[0.1,0.3],[0.2, 0.4][0.3, 0.5]>
e_2	<[0.1,0.4],[0.1, 0.3][0.2, 0.3]>	<[0.1,0.3],[0.1, 0.2][0.1, 0.4]>	<[0.1,0.2],[0.2, 0.3][0.2, 0.5]>

M	(v_1, v_2)	(v_2, v_3)	(v_1, v_3)
e_1	<[0.1,0.2],[0.3, 0.4][0.4, 0.5]>	<[0.1,0.3],[0.4, 0.5][0.4, 0.5]>	<[0.1,0.2],[0.3, 0.5][0.4, 0.6]>
e_2	<[0.1,0.2],[0.2, 0.3][0.3, 0.4]>	<[0.1,0.2],[0.3, 0.4][0.2, 0.5]>	<[0.1,0.2],[0.2, 0.4][0.3, 0.5]>

Definition 3.4 Let G_1=(K_1, M_1, A) and G_2=(K_2, M_2, B) be two interval valued neutrosophic graphs of G_1^* = (V_1, E_1) andG_1^* = (V_2, E_2) respectively. The Cartesian product of two graphs$G_1$and G_2 is an interval valued neutrosophic soft graph G= G_1×G_2 = (K,M, A×B), where (K=K_1×K_2, A×B) is an interval valued neutrosophic soft set over V= V_1×V_2, (M=M_1×M_2, A×B) is an interval valued neutrosophic soft set over E= $\{(x, x_2)\ (x, y_2)\ /x \in V_1,\ x_2y_2 \in E_2\}$ ∪$\{(x_1,z)$ $(y_1, z)\ /z \in V_2,\ x_1y_1 \in E_1\}$and(K, M, A×B) are interval valued neutrosophic soft graphs such that:

1) $(T^L_{K_1(a)} \times T^L_{K_2(b)})\ (x_1, x_2)$ = min $(T^L_{K_1(a)}(x_1), T^L_{K_2(b)}(x_2))$
$(T^U_{K_1(a)} \times T^U_{K_2(b)})\ (x_1, x_2)$ = min $(T^U_{K_1(a)}(x_1), T^U_{K_2(b)}(x_2))$
$(I^L_{K_1(a)} \times I^L_{K_2(b)})\ (x_1, x_2)$ = max $(I^L_{K_1(a)}(x_1), I^L_{K_2(b)}(x_2))$
$(I^U_{K_1(a)} \times I^U_{K_2(b)})\ (x_1, x_2)$ = max $(I^U_{K_1(a)}(x_1), I^U_{K_2(b)}(x_2))$
$(F^L_{K_1(a)} \times F^L_{K_2(b)})\ (x_1, x_2)$ = max $(F^L_{K_1(a)}(x_1), F^L_{K_2(b)}(x_2))$
$(F^U_{K_1(a)} \times F^U_{K_2(b)})\ (x_1, x_2)$ = max $(F^U_{K_1(a)}(x_1), F^U_{K_2(b)}(x_2))$ for all (x_1, x_2) ∈ A×B

2) $(T^L_{M_1(a)} \times T^L_{M_2(b)})\ ((x, x_2)(x, y_2))$ = min $(T^L_{K_1(a)}(x), T^L_{M_2(b)}(x_2y_2))$
$(T^U_{M_1(a)} \times T^U_{M_2(b)})\ ((x, x_2)(x, y_2))$ = min $(T^U_{K_1(a)}(x), T^U_{M_2(b)}(x_2y_2))$
$(I^L_{M_1(a)} \times I^L_{M_2(b)})\ ((x, x_2)(x, y_2))$ =max $(I^L_{K_1(a)}(x), I^L_{M_2(b)}(x_2y_2))$
$(I^U_{M_1(a)} \times I^U_{M_2(b)})\ ((x, x_2)(x, y_2))$ = max $(I^U_{K_1(a)}(x), I^U_{M_2(b)}(x_2y_2))$
$(F^L_{M_1(a)} \times F^L_{M_2(b)})\ ((x, x_2)\ (x, y_2))$ = max $(F^L_{K_1(a)}(x), F^L_{M_2(b)}(x_2y_2))$
$(F^U_{M_1(a)} \times F^U_{M_2(b)})\ ((x, x_2)(x, y_2))$ = max$(F^U_{K_1(a)}(x), F^U_{M_2(b)}(x_2y_2))$ ∀ x ∈ V_1
and ∀x_2y_2 ∈ E_2

3) $(T^L_{M_1(a)} \times T^L_{M_2(b)})\ ((x_1, z)\ (y_1, z))$ = min $(T^L_{M_1(a)}(x_1y_1), T^L_{K_2(b)}(z))$
$(T^U_{M_1(a)} \times T^U_{M_2(b)})\ ((x_1, z)\ (y_1, z))$ = min $(T^U_{M_1(a)}(x_1y_1), T^U_{K_2(b)}(z))$
$(I^L_{M_1(a)} \times I^L_{M_2(b)})\ ((x_1,z)\ (y_1, z))$ = max $(I^L_{M_1(a)}(x_1y_1), I^L_{K_2(b)}(z))$
$(I^U_{M_1(a)} \times I^U_{M_2(b)})\ ((x_1,z)\ (y_1,z))$ = max $(I^U_{M_1(a)}(x_1y_1), I^U_{K_2(b)}(z))$
$(F^L_{M_1(a)} \times F^L_{M_2(b)})((x_1,z)\ (y_1, z))$ = max $(F^L_{M_1(a)}(x_1y_1), F^L_{K_2(b)}(z))$
$(F^U_{M_1(a)} \times F^U_{M_2(b)})\ ((x_1,z)\ (y_1, z))$ = max $(F^U_{M_1(a)}(x_1y_1), F^U_{K_2(b)}(z))$ ∀ z ∈ V_2
and ∀x_1y_1 ∈ E_1

H(a, b) = $H_1(a) \times H_2(b)$ for all (a, b) ∈ A×B are interval valued neutrosophic graphs of G.

Example 3.5.Let A= $\{e_1, e_2\}$ and B= $\{e_3, e_4\}$ be a set ofparameters. Consider two interval valued neutrosophic soft graphs G_1=(H_1, A) =$\{H(e_1), H(e_2)\}$and G_2=(H_2, B) = $\{H(e_3), H(e_4)\}$such that

$H_1(e_1)$=({u_1|([0.3, 0.5], [0.2, 0.3], [0.3, 0.4]), u_2|([0.6, 0.7], [0.2, 0.4], [0.1, 0.3])}, {u_1u_2|([0.3, 0.6], [0.2, 0.4], [0.2, 0.5])}).

$H_1(e_2)$=({u_1|([0.3, 0.5], [0.2, 0.3], [0.3, 0.4]), u_2|([0.2, 0.3], [0.2, 0.3], [0.1, 0.4]), u_3|([0.1, 0.3], [0.2, 0.4], [0.3, 0.5])}, {u_1u_2|([0.1, 0.2], [0.3, 0.4], [0.4, 0.5]), u_2u_3|([0.1, 0.3], [0.4, 0.5], [0.4, 0.5]), u_3u_1 |([0.1, 0.2], [0.3, 0.5], [0.5, 0.6])}).

$H_2(e_3)$=({v_1|([0.4, 0.6], [0.2, 0.3], [0.1, 0.3]), v_2|([0.4, 0.7], [0.2, 0.4], [0.1, 0.3])}, {v_1v_2|([0.3, 0.5], [0.4, 0.5], [0.3, 0.5])}).

$H_2(e_4)$=({v_1|([0.1, 0.4], [0.1, 0.3], [0.2, 0.3]), v_2|([0.1, 0.3], [0.1, 0.2], [0.1, 0.4]), v_3|([0.1, 0.2], [0.2, 0.3], [0.2, 0.5])}, {v_1v_2|([0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), v_2v_3|([0.1, 0.2], [0.3, 0.4], [0.2, 0.5]),v_3v_1 |([0.1, 0.2], [0.2, 0.4], [0.3, 0.5])})

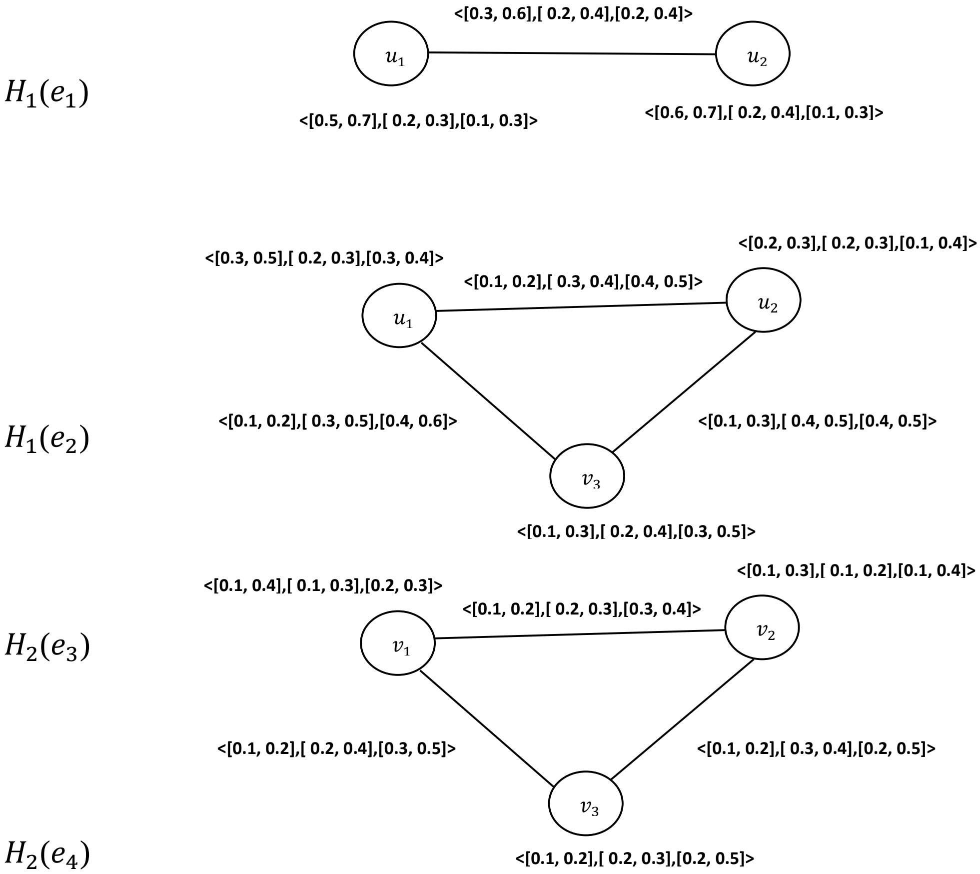


Fig. 3.2: Interval valued neutrosophic soft graph G_1= {$H_1(e_1)$,$H_1(e_2)$} and G_2= {$H_2(e_3)$,$H_2(e_4)$}

The Cartesian product ofG_1 and G_2 is $G_1 \times G_2$ = (H,$A \times B$), where A×B= {(e_1, e_3), (e_1, e_4), (e_2, e_3), (e_2, e_4)}, H(e_1, e_3) = $H_1(e_1)$ ×$H_2(e_3)$, H(e_1, e_4) = $H_1(e_1)$ ×$H_2(e_4)$, H(e_2, e_3) = $H_1(e_2)$ × $H_2(e_3)$ and H(e_2, e_4) = $H_1(e_2)$ ×$H_2(e_4)$ are interval valued neutrosophic graphs of G = $G_1 \times G_2$. H(e_1, e_3) = $H_1(e_1)$ ×$H_2(e_3)$ is shown in Fig. 3.3.

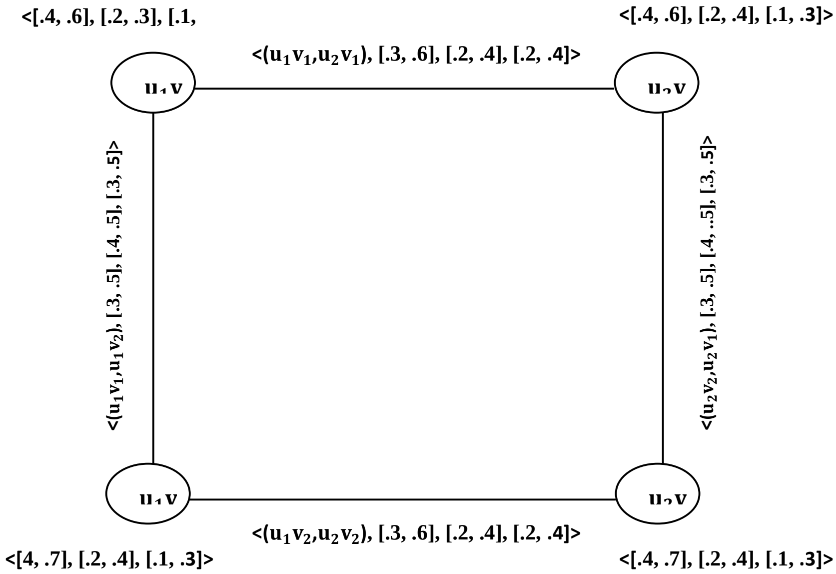


Fig. 3.3: Cartesian product

In the similar way, Cartesian product of H(e_1, e_4) = $H_1(e_1)$ ×$H_2(e_4)$, H(e_2, e_3) = $H_1(e_2)$ × $H_2(e_3)$ and H(e_2, e_4) = $H_1(e_2)$ ×$H_2(e_4)$ can be drawn.

Hence G =G_1×G_2= {H(e_1, e_3), H(e_1, e_4), H(e_2, e_3), H(e_2, e_4)} is an interval valued neutrosophic soft graph.

Theorem 3.6. The Cartesian product of two interval valued neutrosophic soft graph is an interval valued neutrosophic soft graph.

Proof. Let G_1=(K_1, M_1, A) and G_2=(K_2, M_2, B) be two interval valued neutrosophic graphs of G_1^* = (V_1, E_1) and G_1^* = (V_2, E_2) respectively. Let G= G_1×G_2 = (K,M, A×B) be the Cartesian product of two graphs$G_1$and G_2. We claim that G= G_1×G_2 = (K,M, A×B)is an interval valued neutrosophic soft graph G= G_1×G_2 = (K,M, A×B), where (K=K_1×K_2, A×B) is an interval valued neutrosophic soft graph and (H, A×B) = { (K_1×K_2) (a_i, b_j), (M_1×M_2)(a_i, b_j)} for all a_i ∈ A,b_i ∈ B for i= 1, 2,…, m, j= 1, 2,…,n are interval valued neutrosophic graphs of G.

Consider, (x, x_2) (x, y_2) ∈ E, we have

$T^L_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) = min ($T^L_{K_1(a_i)}(x)$, $T^L_{M_2(b_j)}(x_2y_2)$), for i= 1, 2,…, m, j= 1, 2,…,n

≤min {$T^L_{K_1(a_i)}(x)$, min{$T^L_{K_2(b_j)}(x_2)$,$T^L_{K_2(b_j)}(y_2)$}}

= min {min{$T^L_{K_1(a_i)}(x)$,$T^L_{K_2(b_j)}(x_2)$}, min{$T^L_{K_1(a_i)}(x)$,$T^L_{K_2(b_j)}(y_2)$}}

$T^L_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) ≤min{($T^L_{K_1(a_i)} \times T^L_{K_2(b_j)}$) (x, x_2), ($T^L_{K_1(a_i)} \times T^L_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

Similarly, we prove that

$T^{U}_{M(a_i,b_j)}$ $((x,\ x_2)(x,\ y_2))$ ≤min{($T^{U}_{K_1(a_i)} \times T^{U}_{K_2(b_j)}$) (x, x_2), ($T^{U}_{K_1(a_i)} \times T^{U}_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n.

$I^{L}_{M(a_i,b_j)}$ $((x,\ x_2)(x,\ y_2))$ = max ($I^{L}_{K_1(a_i)}(x)$, $I^{L}_{M_2(b_j)}(x_2 y_2)$), for i= 1, 2,…, m, j= 1, 2,…,n

≥max {$I^{L}_{K_1(a_i)}(x)$, max{$I^{L}_{K_2(b_j)}(x_2)$,$I^{L}_{K_2(b_j)}(y_2)$}}

= max {max{$I^{L}_{K_1(a_i)}(x)$,$I^{L}_{K_2(b_j)}(x_2)$}, max{$I^{L}_{K_1(a_i)}(x)$,$I^{L}_{K_2(b_j)}(y_2)$}}

$I^{L}_{M(a_i,b_j)}$ $((x,\ x_2)(x,\ y_2))$ ≥max {($I^{L}_{K_1(a_i)} \times I^{L}_{K_2(b_j)}$) (x, x_2), ($I^{L}_{K_1(a_i)} \times I^{L}_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

Similarly, we prove that

$I^{U}_{M(a_i,b_j)}$ $((x,\ x_2)(x,\ y_2))$ ≥max {($I^{U}_{K_1(a_i)} \times I^{U}_{K_2(b_j)}$) (x, x_2), ($I^{U}_{K_1(a_i)} \times I^{U}_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

$F^{L}_{M(a_i,b_j)}$ $((x,\ x_2)(x,\ y_2))$ = max ($F^{L}_{K_1(a_i)}(x)$, $F^{L}_{M_2(b_j)}(x_2 y_2)$), for i= 1, 2,…, m, j= 1, 2,…,n

≥ max {$F^{L}_{K_1(a_i)}(x)$, max {$F^{L}_{K_2(b_j)}(x_2)$,$F^{L}_{K_2(b_j)}(y_2)$}}

= max{ max {$F^{L}_{K_1(a_i)}(x)$,$F^{L}_{K_2(b_j)}(x_2)$}, max {$F^{L}_{K_1(a_i)}(x)$,$F^{L}_{K_2(b_j)}(y_2)$}}

$F^{L}_{M(a_i,b_j)}$ $((x,\ x_2)(x,\ y_2))$ ≥ max {($F^{L}_{K_1(a_i)} \times F^{L}_{K_2(b_j)}$) (x, x_2), ($F^{L}_{K_1(a_i)} \times F^{L}_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

Similarly, we prove that

$F^{U}_{M(a_i,b_j)}$ $((x,\ x_2)(x,\ y_2))$ ≥ max {($F^{U}_{K_1(a_i)} \times F^{U}_{K_2(b_j)}$) (x, x_2), ($F^{U}_{K_1(a_i)} \times F^{U}_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

Similarly, for (x_1,z) (y_1, z)∈ E, we have

$T^{L}_{M(a_i,b_j)}$((x_1, z) (y_1, z)) ≤min{($T^{L}_{K_1(a_i)} \times T^{L}_{K_2(b_j)}$) ($x_1$, z), ($T^{L}_{K_1(a_i)} \times T^{L}_{K_2(b_j)}$) ($y_1$, z),

$T^{U}_{M(a_i,b_j)}$((x_1, z) (y_1, z)) ≤min{($T^{U}_{K_1(a_i)} \times T^{U}_{K_2(b_j)}$) ($x_1$, z), ($T^{U}_{K_1(a_i)} \times T^{U}_{K_2(b_j)}$) ($y_1$, z),

$I^{L}_{M(a_i,b_j)}$((x_1, z) (y_1, z)) ≥ max {($I^{L}_{K_1(a_i)} \times I^{L}_{K_2(b_j)}$) ($x_1$, z), ($I^{L}_{K_1(a_i)} \times I^{L}_{K_2(b_j)}$) ($y_1$, z),

$I^{U}_{M(a_i,b_j)}$((x_1, z) (y_1, z)) ≥ max {($I^{U}_{K_1(a_i)} \times I^{U}_{K_2(b_j)}$) ($x_1$, z), ($I^{U}_{K_1(a_i)} \times I^{U}_{K_2(b_j)}$) ($y_1$, z),

$F^{L}_{M(a_i,b_j)}$((x_1, z) (y_1, z)) ≥ max {($F^{L}_{K_1(a_i)} \times F^{L}_{K_2(b_j)}$) ($x_1$, z), ($F^{L}_{K_1(a_i)} \times F^{L}_{K_2(b_j)}$) ($y_1$, z),

$F^{U}_{M(a_i,b_j)}$((x_1, z) (y_1, z)) ≥ max {($F^{U}_{K_1(a_i)} \times F^{U}_{K_2(b_j)}$) ($x_1$, z), ($F^{U}_{K_1(a_i)} \times F^{U}_{K_2(b_j)}$) ($y_1$, z),for i= 1, 2,…, m, j= 1, 2,…,n

Hence G = (K, M, $A\times B$) is an interval valued neutrosophic soft graph.

Definition 3.7 Let G_1=(K_1, M_1, A) and G_2=(K_2, M_2, B) be two interval valued neutrosophic graphs of G_1^* = (V_1, E_1) and G_1^* = (V_2, E_2) respectively. The strong product of two graphs$G_1$and G_2 is an interval valued neutrosophic soft graph G= $G_1 \otimes G_2$ = (K,M, $A\times B$), where (K=$K_1\times K_2$, $A\times B$) is an interval valued neutrosophic soft set over V= $V_1\times V_2$, (M, $A\times B$) is an interval valued neutrosophic soft set over E= $\{(x, x_2)\ (x, y_2)\ /x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1,z)\ (y_1, z)\ /z \in V_2, x_1y_1 \in E_1\} \cup \{(x_1,x_2)\ (y_1,y_2)\ /x_1y_1 \in E_1, x_2y_2 \in E_2\}$ and(K, M, $A\times B$) are interval valued neutrosophic soft graphs such that:

1) $(T^L_{K_1(a)} \times T^L_{K_2(b)})\ (x_1, x_2) = \min\ (T^L_{K_1(a)}(x_1), T^L_{K_2(b)}(x_2))$
   $(T^U_{K_1(a)} \times T^U_{K_2(b)})\ (x_1, x_2) = \min\ (T^U_{K_1(a)}(x_1), T^U_{K_2(b)}(x_2))$
   $(I^L_{K_1(a)} \times I^L_{K_2(b)})\ (x_1, x_2) = \max\ (I^L_{K_1(a)}(x_1), I^L_{K_2(b)}(x_2))$
   $(I^U_{K_1(a)} \times I^U_{K_2(b)})\ (x_1, x_2) = \max\ (I^U_{K_1(a)}(x_1), I^U_{K_2(b)}(x_2))$
   $(F^L_{K_1(a)} \times F^L_{K_2(b)})\ (x_1, x_2) = \max\ (F^L_{K_1(a)}(x_1), F^L_{K_2(b)}(x_2))$
   $(F^U_{K_1(a)} \times F^U_{K_2(b)})\ (x_1, x_2) = \max\ (F^U_{K_1(a)}(x_1), F^U_{K_2(b)}(x_2))$ for all (x_1, x_2) ∈ $A\times B$

2) $(T^L_{M_1(a)} \times T^L_{M_2(b)})\ ((x, x_2)(x, y_2)) = \min\ (T^L_{K_1(a)}(x), T^L_{M_2(b)}(x_2y_2))$
   $(T^U_{M_1(a)} \times T^U_{M_2(b)})\ ((x, x_2)(x, y_2)) = \min\ (T^U_{K_1(a)}(x), T^U_{M_2(b)}(x_2y_2))$
   $(I^L_{M_1(a)} \times I^L_{M_2(b)})\ ((x, x_2)(x, y_2)) =\max\ (I^L_{K_1(a)}(x), I^L_{M_2(b)}(x_2y_2))$
   $(I^U_{M_1(a)} \times I^U_{M_2(b)})\ ((x, x_2)(x, y_2)) = \max\ (I^U_{K_1(a)}(x), I^U_{M_2(b)}(x_2y_2))$
   $(F^L_{M_1(a)} \times F^L_{M_2(b)})\ ((x, x_2)\ (x, y_2)) = \max\ (F^L_{K_1(a)}(x), F^L_{M_2(b)}(x_2y_2))$
   $(F^U_{M_1(a)} \times F^U_{M_2(b)})\ ((x,x_2)(x, y_2))= \max(F^U_{K_1(a)}(x), F^U_{M_2(b)}(x_2y_2))$ $\forall$ x $\in V_1$and $\forall x_2y_2 \in E_2$.

3) $(T^L_{M_1(a)} \times T^L_{M_2(b)})\ ((x_1, z)\ (y_1, z)) = \min\ (T^L_{M_1(a)}(x_1y_1), T^L_{K_2(b)}(z))$
   $(T^U_{M_1(a)} \times T^U_{M_2(b)})\ ((x_1, z)\ (y_1, z)) = \min\ (T^U_{M_1(a)}(x_1y_1), T^U_{K_2(b)}(z))$
   $(I^L_{M_1(a)} \times I^L_{M_2(b)})\ ((x_1, z)\ (y_1, z)) = \max\ (I^L_{M_1(a)}(x_1y_1), I^L_{K_2(b)}(z))$
   $(I^U_{M_1(a)} \times I^U_{M_2(b)})\ ((x_1, z)\ (y_1,z)) = \max\ (I^U_{M_1(a)}(x_1y_1), I^U_{K_2(b)}(z))$
   $(F^L_{M_1(a)} \times F^L_{M_2(b)})((x_1, z)\ (y_1, z)) = \max\ (F^L_{M_1(a)}(x_1y_1), F^L_{K_2(b)}(z))$
   $(F^U_{M_1(a)} \times F^U_{M_2(b)})\ ((x_1, z)\ (y_1, z)) = \max\ (F^U_{M_1(a)}(x_1y_1), F^U_{K_2(b)}(z))$ $\forall$ z $\in V_2$ and $\forall x_1y_1 \in E_1$.

4) $(T^L_{M_1(a)} \times T^L_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \min\ (T^L_{K_1(a)}(x_1y_1), T^L_{K_2(b)}(x_2y_2))$
   $(T^U_{M_1(a)} \times T^U_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \min\ (T^U_{K_1(a)}(x_1y_1), T^U_{K_2(b)}(x_2y_2))$
   $(I^L_{M_1(a)} \times I^L_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \max\ (I^L_{K_1(a)}(x_1y_1), I^L_{K_2(b)}(x_2y_2))$
   $(I^U_{M_1(a)} \times I^U_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \max\ (I^U_{K_1(a)}(x_1y_1), I^U_{K_2(b)}(x_2y_2))$
   $(F^L_{M_1(a)} \times F^L_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \max\ (F^L_{K_1(a)}(x_1y_1), F^L_{K_2(b)}(x_2y_2))$
   $(F^U_{M_1(a)} \times F^U_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \max\ (F^U_{K_1(a)}(x_1y_1), F^U_{K_2(b)}(x_2y_2))$ for all (x_1, y_1) ∈ E_1, ((x_2, y_2) ∈ E_2.

H(a, b) = $H_1(a) \otimes H_2(b)$ for all (a, b) ∈ $A\times B$ are interval valued neutrosophic graphs of G.

Theorem 3.8. The strong product of two interval valued neutrosophic soft graph is an interval valued neutrosophic soft graph.

Definition 3.9 Let G_1=(K_1, M_1, A) and G_2=(K_2, M_2, B) be two interval valued neutrosophic graphs of G_1^* = (V_1, E_1) and G_1^* = (V_2, E_2) respectively. The composition of two graphs$G_1$and G_2 is an interval valued neutrosophic soft graph G= $G_1[G_2]$ = (K,M, $A \circ B$), where (K=$K_1 \circ K_2$, $A \circ B$) is an interval valued neutrosophic soft set over V= $V_1 \times V_2$, (M, $A \circ B$) is an interval valued neutrosophic soft set over E= $\{(x, x_2)\ (x, y_2)\ /x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1,z)\ (y_1, z)\ /z \in V_2, x_1y_1 \in E_1\} \cup \{(x_1,x_2)\ (y_1,y_2)\ /x_1y_1 \in E_1, x_2 \neq y_2\}$and(K, M, $A \circ B$) are interval valued neutrosophic soft graphs such that:

1) $(T^L_{K_1(a)} \circ T^L_{K_2(b)})\ (x_1, x_2) = \min\ (T^L_{K_1(a)}(x_1), T^L_{K_2(b)}(x_2))$
$(T^U_{K_1(a)} \circ T^U_{K_2(b)})\ (x_1, x_2) = \min\ (T^U_{K_1(a)}(x_1), T^U_{K_2(b)}(x_2))$
$(I^L_{K_1(a)} \circ I^L_{K_2(b)})\ (x_1, x_2) = \max\ (I^L_{K_1(a)}(x_1), I^L_{K_2(b)}(x_2))$
$(I^U_{K_1(a)} \circ I^U_{K_2(b)})\ (x_1, x_2) = \max\ (I^U_{K_1(a)}(x_1), I^U_{K_2(b)}(x_2))$
$(F^L_{K_1(a)} \circ F^L_{K_2(b)})\ (x_1, x_2) = \max\ (F^L_{K_1(a)}(x_1), F^L_{K_2(b)}(x_2))$
$(F^U_{K_1(a)} \circ F^U_{K_2(b)})\ (x_1, x_2) = \max\ (F^U_{K_1(a)}(x_1), F^U_{K_2(b)}(x_2))$ for all (x_1, x_2) $\in A \times B$

2) $(T^L_{M_1(a)} \circ T^L_{M_2(b)})\ ((x, x_2)(x, y_2)) = \min\ (T^L_{K_1(a)}(x), T^L_{M_2(b)}(x_2y_2))$
$(T^U_{M_1(a)} \circ T^U_{M_2(b)})\ ((x, x_2)(x, y_2)) = \min\ (T^U_{K_1(a)}(x), T^U_{M_2(b)}(x_2y_2))$
$(I^L_{M_1(a)} \circ I^L_{M_2(b)})\ ((x, x_2)(x, y_2)) = \max\ (I^L_{K_1(a)}(x), I^L_{M_2(b)}(x_2y_2))$
$(I^U_{M_1(a)} \circ I^U_{M_2(b)})\ ((x, x_2)(x, y_2)) = \max\ (I^U_{K_1(a)}(x), I^U_{M_2(b)}(x_2y_2))$
$(F^L_{M_1(a)} \circ F^L_{M_2(b)})\ ((x, x_2)\ (x, y_2)) = \max\ (F^L_{K_1(a)}(x), F^L_{M_2(b)}(x_2y_2))$
$(F^U_{M_1(a)} \circ F^U_{M_2(b)})\ ((x, x_2)(x, y_2)) = \max(F^U_{K_1(a)}(x), F^U_{M_2(b)}(x_2y_2))$ $\forall$ x $\in V_1$and $\forall x_2y_2 \in E_2$.

3) $(T^L_{M_1(a)} \circ T^L_{M_2(b)})\ ((x_1, z)\ (y_1, z)) = \min\ (T^L_{M_1(a)}(x_1y_1), T^L_{K_2(b)}(z))$
$(T^U_{M_1(a)} \circ T^U_{M_2(b)})\ ((x_1, z)\ (y_1, z)) = \min\ (T^U_{M_1(a)}(x_1y_1), T^U_{K_2(b)}(z))$
$(I^L_{M_1(a)} \circ I^L_{M_2(b)})\ ((x_1, z)\ (y_1, z)) = \max\ (I^L_{M_1(a)}(x_1y_1), I^L_{K_2(b)}(z))$
$(I^U_{M_1(a)} \circ I^U_{M_2(b)})\ ((x_1, z)\ (y_1,z)) = \max\ (I^U_{M_1(a)}(x_1y_1), I^U_{K_2(b)}(z))$
$(F^L_{M_1(a)} \circ F^L_{M_2(b)})((x_1, z)\ (y_1, z)) = \max\ (F^L_{M_1(a)}(x_1y_1), F^L_{K_2(b)}(z))$
$(F^U_{M_1(a)} \circ F^U_{M_2(b)})\ ((x_1, z)\ (y_1, z)) = \max\ (F^U_{M_1(a)}(x_1y_1), F^U_{K_2(b)}(z))$ $\forall$ z $\in V_2$ and $\forall x_1y_1 \in E_1$.

4) $(T^L_{M_1(a)} \circ T^L_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \min\ (T^L_{K_1(a)}(x_1y_1), T^L_{K_2(b)}(x_2),T^L_{K_2(b)}(y_2))$
$(T^U_{M_1(a)} \circ T^U_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \min\ (T^U_{K_1(a)}(x_1y_1), T^U_{K_2(b)}(x_2), T^U_{K_2(b)}(y_2))$
$(I^L_{M_1(a)} \circ I^L_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \max\ (I^L_{K_1(a)}(x_1y_1), I^L_{K_2(b)}(x_2),I^L_{K_2(b)}(y_2))$
$(I^U_{M_1(a)} \circ I^U_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \max\ (I^U_{K_1(a)}(x_1y_1), I^U_{K_2(b)}(x_2), I^U_{K_2(b)}(y_2))$
$(F^L_{M_1(a)} \circ F^L_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \max\ (F^L_{K_1(a)}(x_1y_1), F^L_{K_2(b)}(x_2),F^L_{K_2(b)}(y_2))$
$(F^U_{M_1(a)} \circ F^U_{M_2(b)})\ ((x_1, x_2), (y_1, y_2)) = \max\ (F^U_{K_1(a)}(x_1y_1), F^U_{K_2(b)}(x_2), F^U_{K_2(b)}(y_2))$ for all (x_1, y_1) $\in E_1$, and $x_2 \neq y_2$.

H(a, b) = $H_1(a)[H_2(b)]$ for all (a, b) $\in A \times B$ are interval valued neutrosophic graphs of G.

Example 3.10. Let A= $\{e_1\}$ A= $\{e_2, e_3\}$ be the parameters sets. Consider two interval valued neutrosophic soft graphs G_1=(H_1, A) ={$H_1(e_1)$}and G_2=(H_2, B) = {$H_2(e_2)$,$H_2(e_3)$} such that

$H_1(e_1)$=({u_1|([0.5, 0.7], [0.2, 0.3], [0.1, 0.3]), u_2|([0.6, 0.7], [0.2, 0.4], [0.1, 0.3])},{u_1u_2|([0.3, 0.6], [0.2, 0.4], [0.2, 0.4])})

$H_2(e_2)$=({v_1|([0.1, 0.4], [0.1, 0.3], [0.2, 0.3]), v_2|([0.1, 0.3], [0.1, 0.2], [0.1, 0.4]), v_3|([0.1, 0.2], [0.2, 0.3], [0.2, 0.5])}, {v_1v_2|([0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), v_2v_3|([0.1, 0.2], [0.3, 0.4], [0.2, 0.5]),v_3v_1 |([0.1, 0.2], [0.2, 0.4], [0.3, 0.5])})

$H_2(e_3)$=({v_1|([0.4, 0.6], [0.2, 0.3], [0.1, 0.3]), v_2|([0.4, 0.7], [0.2, 0.4], [0.1, 0.3])},{v_1v_2|([0.3, 0.5], [0.2, 0.5], [0.3, 0.5])})

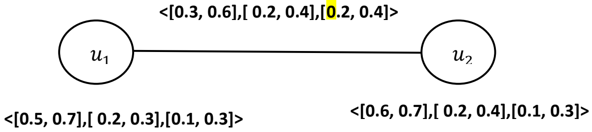


Fig.3.4:Interval valued neutrosophic soft graph corresponding to $H_1(e_1)$

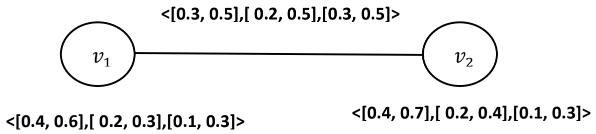


Fig. 3.5: Interval valued neutrosophic soft graph corresponding to$H_2(e_3)$.

The composition ofG_1 and G_2 is $G_1[G_2]$=(H,$A\times B$), where A$\times B$= {(e_1, e_2), (e_1, e_3), (e_2, e_3)}, H(e_1, e_2) = $H_1(e_1)$ [$H_2(e_2)$] and H(e_1, e_3) = $H_1(e_1)$ [$H_2(e_3)$] are interval valued neutrosophic graphs of $G_1[G_2]$. $H_1(e_1)$ [$H_2(e_3)$] is shown in Fig. 3.6.

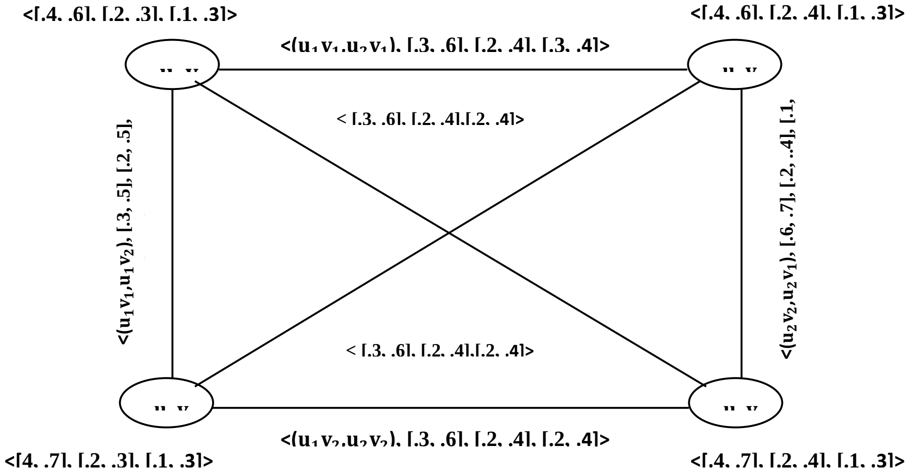


Fig. 3.6:Composition$H_1(e_1)$[$H_2(e_3)$]

Hence G=$G_1[G_2]$={$H_1(e_1)$ [$H_2(e_2)$], $H_1(e_1)$ [$H_2(e_3)$]}is an interval valued neutrosophic soft graph.

Theorem 3.11. The composition of two interval valued neutrosophic soft graph is an interval valued neutrosophic soft graph

Proof. Let G_1=(K_1, M_1, A) and G_2=(K_2, M_2, B) be two interval valued neutrosophic graphs of G_1^* = (V_1, E_1) and G_1^* = (V_2, E_2) respectively. Let G= $G_1[G_2]$ = (K,M, $A\times B$) be the Cartesian composition of two graphs G_1 and G_2. We claim that G= $G_1[G_2]$ = (K,M, $A \circ B$)I s an interval valued neutrosophic soft graph and (H, $A \circ B$) = {$K_1(a_i)[K_2(b_j)]$, $M_1(a_i)[M_2(b_j)]$} for all a_i ∈ A,b_i ∈ B for i= 1, 2,…, m, j= 1, 2,…,n are interval valued neutrosophic graphs of G.

Consider, (x, x_2) $(x, y_2) \in E$, we have

$T^L_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) = min ($T^L_{K_1(a_i)}(x)$, $T^L_{M_2(b_j)}(x_2 y_2)$), for i= 1, 2,…, m, j= 1, 2,…,n

≤min {$T^L_{K_1(a_i)}(x)$, min{$T^L_{K_2(b_j)}(x_2)$,$T^L_{K_2(b_j)}(y_2)$}}

= min { min{$T^L_{K_1(a_i)}(x)$,$T^L_{K_2(b_j)}(x_2)$}, min{$T^L_{K_1(a_i)}(x)$,$T^L_{K_2(b_j)}(y_2)$}}

$T^L_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) ≤min{($T^L_{K_1(a_i)} \circ T^L_{K_2(b_j)}$) (x, x_2), ($T^L_{K_1(a_i)} \circ T^L_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

Similarly, we prove that

$T^U_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) ≤min{($T^U_{K_1(a_i)} \circ T^U_{K_2(b_j)}$) (x, x_2), ($T^U_{K_1(a_i)} \circ T^U_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n.

$I^L_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) = max ($I^L_{K_1(a_i)}(x)$, $I^L_{M_2(b_j)}(x_2 y_2)$), for i= 1, 2,…, m, j= 1, 2,…,n

≥ max {$I^L_{K_1(a_i)}(x)$, max {$I^L_{K_2(b_j)}(x_2)$,$I^L_{K_2(b_j)}(y_2)$}}

= max{ max {$I^L_{K_1(a_i)}(x)$,$I^L_{K_2(b_j)}(x_2)$}, max {$I^L_{K_1(a_i)}(x)$,$I^L_{K_2(b_j)}(y_2)$}}

$I^L_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) ≥ max {($I^L_{K_1(a_i)} \circ I^L_{K_2(b_j)}$) (x, x_2), ($I^L_{K_1(a_i)} \circ I^L_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

Similarly, we prove that

$I^U_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) ≥ max {($I^U_{K_1(a_i)} \circ I^U_{K_2(b_j)}$) (x, x_2), ($I^U_{K_1(a_i)} \circ I^U_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

$F^L_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) = max ($F^L_{K_1(a_i)}(x)$, $F^L_{M_2(b_j)}(x_2 y_2)$), for i= 1, 2,…, m, j= 1, 2,…,n

≥ max {$F^L_{K_1(a_i)}(x)$, max {$F^L_{K_2(b_j)}(x_2)$,$F^L_{K_2(b_j)}(y_2)$}}

= max{ max {$F^L_{K_1(a_i)}(x)$,$F^L_{K_2(b_j)}(x_2)$}, max {$F^L_{K_1(a_i)}(x)$,$F^L_{K_2(b_j)}(y_2)$}}

$F^L_{M(a_i,b_j)}$ ((x, x_2)(x, y_2)) ≥ max {($F^L_{K_1(a_i)} \circ F^L_{K_2(b_j)}$) (x, x_2), ($F^L_{K_1(a_i)} \circ F^L_{K_2(b_j)}$) (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

Similarly, we prove that

$F^U_{M(a_i,b_j)}$ $((x,\ x_2)(x,\ y_2)) \geq$ max $\{(\ F^U_{K_1(a_i)} \circ F^U_{K_2(b_j)})$ (x, x_2), $(\ F^U_{K_1(a_i)} \circ F^U_{K_2(b_j)})$ (x, y_2),for i= 1, 2,…, m, j= 1, 2,…,n

Similarly, for (x_1,z) $(y_1, z) \in E$, we have

$T^L_{M(a_i,b_j)}$ $((x_1, z)$ $(y_1, z)) \leq$min$\{(\ T^L_{K_1(a_i)} \circ T^L_{K_2(b_j)})$ (x_1, z), $(\ T^L_{K_1(a_i)} \circ T^L_{K_2(b_j)})$ (y_1, z),

$T^U_{M(a_i,b_j)}$ $((x_1, z)$ $(y_1, z)) \leq$min$\{(\ T^U_{K_1(a_i)} \circ T^U_{K_2(b_j)})$ (x_1, z), $(\ T^U_{K_1(a_i)} \circ T^U_{K_2(b_j)})$ (y_1, z),

$I^L_{M(a_i,b_j)}$ $((x_1, z)$ $(y_1, z)) \geq$ max $\{(\ I^L_{K_1(a_i)} \circ I^L_{K_2(b_j)})$ (x_1, z), $(\ I^L_{K_1(a_i)} \circ I^L_{K_2(b_j)})$ (y_1, z),

$I^U_{M(a_i,b_j)}$ $((x_1, z)$ $(y_1, z)) \geq$ max $\{(\ I^U_{K_1(a_i)} \circ I^U_{K_2(b_j)})$ (x_1, z), $(\ I^U_{K_1(a_i)} \circ I^U_{K_2(b_j)})$ (y_1, z),

$F^L_{M(a_i,b_j)}$ $((x_1, z)$ $(y_1, z)) \geq$ max $\{(\ F^L_{K_1(a_i)} \circ F^L_{K_2(b_j)})$ (x_1, z), $(\ F^L_{K_1(a_i)} \circ F^L_{K_2(b_j)})$ $(y_1,$ $z)$,

$F^U_{M(a_i,b_j)}$ $((x_1, z)$ $(y_1, z)) \geq$ max $\{(\ F^U_{K_1(a_i)} \circ F^U_{K_2(b_j)})$ (x_1, z), $(\ F^U_{K_1(a_i)} \circ F^U_{K_2(b_j)})$ $(y_1,$ $z)$,for i= 1, 2,…, m, j= 1, 2,…,n

Let (x_1, x_2) $(y_1, y_2) \in E$, $(x_1, y_1) \in E_1$and $x_2 \neq y_2$. Then we have

$T^L_{M(a_i,b_j)}((x_1, x_2), (y_1, y_2))$ = min $(T^L_{K_1(a_i)}(x_1 y_1), T^L_{K_2(b_j)}(x_2),T^L_{K_2(b_j)}(y_2))$

$\leq$min{min$\{T^L_{K_1(a_i)}(x_1),T^L_{K_1(a_i)}(y_1)\},T^L_{K_2(b_j)}(x_2),T^L_{K_2(b_j)}(y_2))\}$

=min { min$\{T^L_{K_1(a_i)}(x_1), T^L_{K_2(b_j)}(x_2)\}$, min$\{T^L_{K_1(a_i)}(y_1), T^L_{K_2(b_j)}(y_2)\}\}$

$T^L_{M(a_i,b_j)}((x_1, x_2), (y_1, y_2)) \leq$min$\{T^L_{K(a_i,b_j)}(x_1, x_2), T^L_{K(a_i,b_j)}(y_1, y_2)\}$

We prove also that,

$T^U_{M(a_i,b_j)}((x_1, x_2), (y_1, y_2)) \geq$max$\{T^U_{K(a_i,b_j)}(x_1, x_2), T^U_{K(a_i,b_j)}(y_1, y_2)\}$.

$I^L_{M(a_i,b_j)}((x_1, x_2), (y_1, y_2))$ = max $(I^L_{K_1(a_i)}(x_1 y_1), I^L_{K_2(b_j)}(x_2),I^L_{K_2(b_j)}(y_2))$

$\geq$max{max$\{I^L_{K_1(a_i)}(x_1),I^L_{K_1(a_i)}(y_1)\},I^L_{K_2(b_j)}(x_2),I^L_{K_2(b_j)}(y_2))\}$

=max{ max$\{I^L_{K_1(a_i)}(x_1), I^L_{K_2(b_j)}(x_2)\}$, max$\{I^L_{K_1(a_i)}(y_1), I^L_{K_2(b_j)}(y_2)\}\}$

$I^L_{M(a_i,b_j)}((x_1, x_2), (y_1, y_2)) \geq$max$\{I^L_{K(a_i,b_j)}(x_1, x_2), I^L_{K(a_i,b_j)}(y_1, y_2)\}$

We prove also that,

$I^U_{M(a_i,b_j)}((x_1, x_2), (y_1, y_2)) \geq$max$\{I^U_{K(a_i,b_j)}(x_1, x_2), I^U_{K(a_i,b_j)}(y_1, y_2)\}$

Similarly, we prove also that

$F^L_{M(a_i,b_j)}((x_1, x_2), (y_1, y_2)) \geq$max$\{F^L_{K(a_i,b_j)}(x_1, x_2), F^L_{K(a_i,b_j)}(y_1, y_2)\}$
$F^U_{M(a_i,b_j)}((x_1, x_2), (y_1, y_2)) \geq$max$\{F^U_{K(a_i,b_j)}(x_1, x_2), F^U_{K(a_i,b_j)}(y_1, y_2)\}$

Hence G= (K, M, A∘ B) is an interval valued neutrosophic graph.

Definition 3.12 Let G_1=(K_1, M_1, A) and G_2=(K_2, M_2, B) be two interval valued neutrosophic graphs of G_1^* = (V_1, E_1) and G_1^* = (V_2, E_2) respectively. The intersection of two graphs$G_1$and G_2 is an interval valued neutrosophic soft graph G= $G_1 \cap G_2$ = (K,M, $A \cup B$), where (K, $A \cup B$) is an interval valued neutrosophic soft set over V= $V_1 \cap V_2$, (M, $A \cup B$) is an interval valued neutrosophic soft set over E= $E_1 \cap E_2$, truth-membership, indeterminacy–membership, and falsity-membership function of G for all x, z ∈V defined by

1) $$T^L_{K(e)}(x) = \begin{cases} T^L_{K_1(e)}(x) & \text{if } e \in A-B \\ T^L_{K_2(e)}(x) & \text{if } e \in A-B \\ \min(T^L_{K_1(e)}(x), T^L_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$T^U_{K(e)}(x) = \begin{cases} T^U_{K_1(e)}(x) & \text{if } e \in A-B \\ T^U_{K_2(e)}(x) & \text{if } e \in A-B \\ \min(T^U_{K_1(e)}(x), T^U_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$I^L_{K(e)}(x) = \begin{cases} I^L_{K_1(e)}(x) & \text{if } e \in A-B \\ I^L_{K_2(e)}(x) & \text{if } e \in A-B \\ \max(I^L_{K_1(e)}(x), I^L_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$I^U_{K(e)}(x) = \begin{cases} I^U_{K_1(e)}(x) & \text{if } e \in A-B \\ I^U_{K_2(e)}(x) & \text{if } e \in A-B \\ \max(I^L_{K_1(e)}(x), I^U_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$F^L_{K(e)}(x) = \begin{cases} F^L_{K_1(e)}(x) & \text{if } e \in A-B \\ F^L_{K_2(e)}(x) & \text{if } e \in A-B \\ \max(F^L_{K_1(e)}(x), F^L_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$F^U_{K(e)}(x) = \begin{cases} F^U_{K_1(e)}(x) & \text{if } e \in A-B \\ F^U_{K_2(e)}(x) & \text{if } e \in A-B \\ \max(F^L_{K_1(e)}(x), F^U_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

2) $$T^L_{M(e)}(xz) = \begin{cases} T^L_{M_1(e)}(xz) & \text{if } e \in A-B \\ T^L_{M_2(e)}(xz) & \text{if } e \in A-B \\ \min(T^L_{M_1(e)}(xz), T^L_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$T^U_{M(e)}(xz) = \begin{cases} T^U_{M_1(e)}(xz) & \text{if } e \in A-B \\ T^U_{M_2(e)}(xz) & \text{if } e \in A-B \\ \min(T^U_{M_1(e)}(xz), T^U_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$I^L_{M(e)}(xz) = \begin{cases} I^L_{M_1(e)}(xz) & \text{if } e \in A - B \\ I^L_{M_2(e)}(xz) & \text{if } e \in A - B \\ \max(I^L_{M_1(e)}(xz), I^L_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$I^U_{M(e)}(xz) = \begin{cases} I^U_{M_1(e)}(xz) & \text{if } e \in A - B \\ I^U_{M_2(e)}(xz) & \text{if } e \in A - B \\ \max(I^L_{M_1(e)}(xz), I^U_{K_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$F^L_{M(e)}(x) = \begin{cases} F^L_{M_1(e)}(xz) & \text{if } e \in A - B \\ F^L_{M_2(e)}(xz) & \text{if } e \in A - B \\ \max(F^L_{M_1(e)}(xz), F^L_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$F^U_{M(e)}(xz) = \begin{cases} F^U_{M_1(e)}(xz) & \text{if } e \in A - B \\ F^U_{M_2(e)}(xz) & \text{if } e \in A - B \\ \max(F^L_{M_1(e)}(xz), F^U_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

Example 3.13. Let A= $\{e_1, e_2\}$ and B= $\{e_1, e_4\}$ be a set ofparameters. Consider two interval valued neutrosophic soft graphs G_1=(H_1, A) =$\{H_1(e_1), H_1(e_2)\}$ and G_2=(H_2, B) = $\{H_2(e_1), H_2(e_4)\}$such that

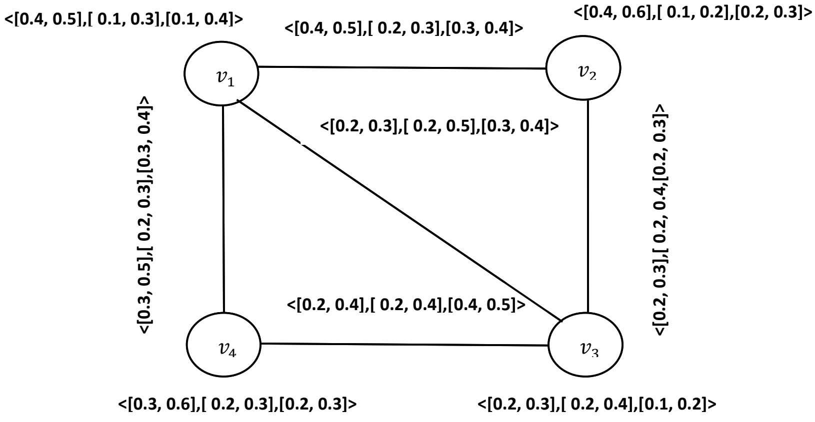
$H_1(e_1)$=($\{v_1$|([0.4, 0.5], [0.1, 0.3], [0.1, 0.4]), v_2|([0.4, 0.6], [0.1, 0.2], [0.2, 0.3]), v_3|([0.2, 0.3], [0.2, 0.4], [0.1, 0.2]), v_4|([0.3, 0.6], [0.2, 0.3], [0.2, 0.3])$\}$, $\{v_1v_2$|([0.4, 0.5], [0.2, 0.3], [0.3, 0.4]), v_2v_3|([0.2, 0.3], [0.2, 0.4], [0.4, 0.5]), v_3v_4 |([0.2, 0.4], [0.2, 0.4], [0.4, 0.5]), v_1v_4 |([0.3, 0.5], [0.2, 0.3], [0.3, 0.4]),v_1v_3 |([0.2, 0.3], [0.2, 0.5], [0.3, 0.4])$\}$).

$H_1(e_2)$=($\{v_1$|([0.4, 0.6], [0.2, 0.3], [0.1, 0.3]), v_2|([0.4, 0.7], [0.2, 0.4], [0.1, 0.3])$\}$, $\{v_1v_2$|([0.3, 0.5], [0.4, 0.5], [0.3, 0.5])$\}$).

$H_2(e_1)$=($\{v_1$|([0.3, 0.5], [0.2, 0.3], [0.3, 0.4]), v_2|([0.2, 0.3], [0.2, 0.3], [0.1, 0.4]), v_3|([0.1, 0.3], [0.2, 0.4], [0.3, 0.5])$\}$, $\{v_1v_2$|([0.1, 0.2], [0.3, 0.4], [0.4, 0.5]), v_2v_3|([0.1, 0.3], [0.4, 0.5], [0.4, 0.5]), v_3v_1 |([0.1, 0.2], [0.3, 0.5], [0.5, 0.6])$\}$).

$H_2(e_4)$=($\{u_1$|([0.4, 0.6], [0.2, 0.3], [0.2, 0.4]), u_2|([0.4, 0.5], [0.1, 0.4], [0.2, 0.3])$\}$, $\{u_1u_2$|([0.3, 0.5], [0.4, 0.5], [0.3, 0.5])$\}$).

$H_1(e_1)$



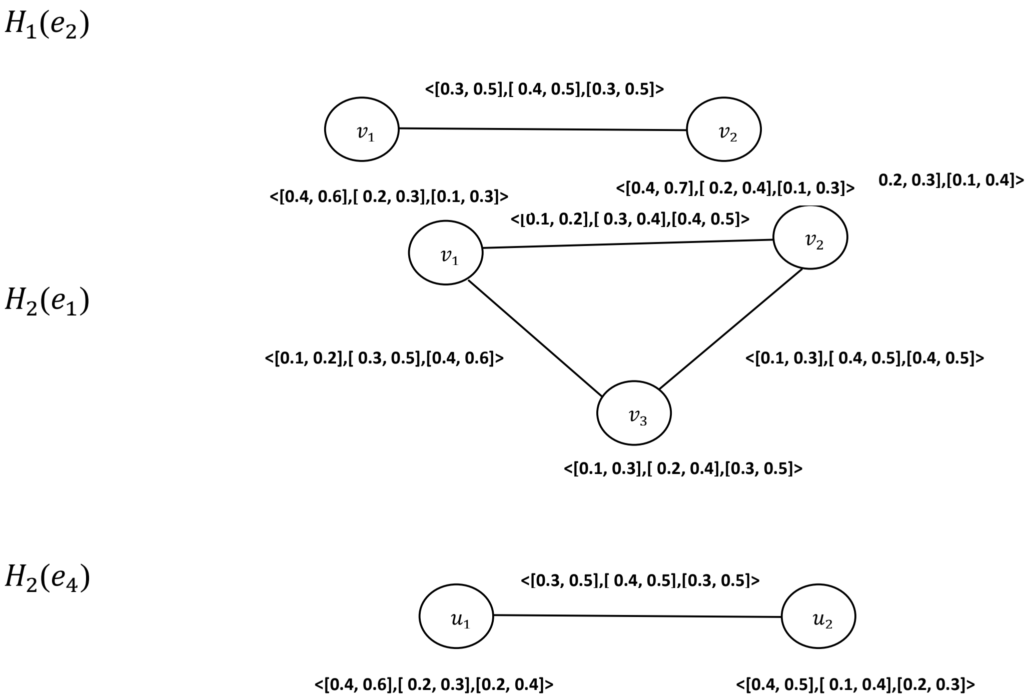


Fig. 3.7: Interval valued neutrosophic soft graph G_1= {$H_1(e_1)$,$H_1(e_2)$} and G_2= {$H_2(e_1)$,$H_2(e_4)$}

The intersection of G_1 and G_2 is $G_1 \cap G_2$ = (H,$A \cup B$), whereA$\cup$ B= {e_1, e_2, e_3,e_4 }, H(e_1) = $H_1(e_1) \cap H_2(e_1)$, H(e_2) and H(e_4) are interval valued neutrosophic graphs of G = $G_1 \cap G_2$. are shown in Fig. 3.8.

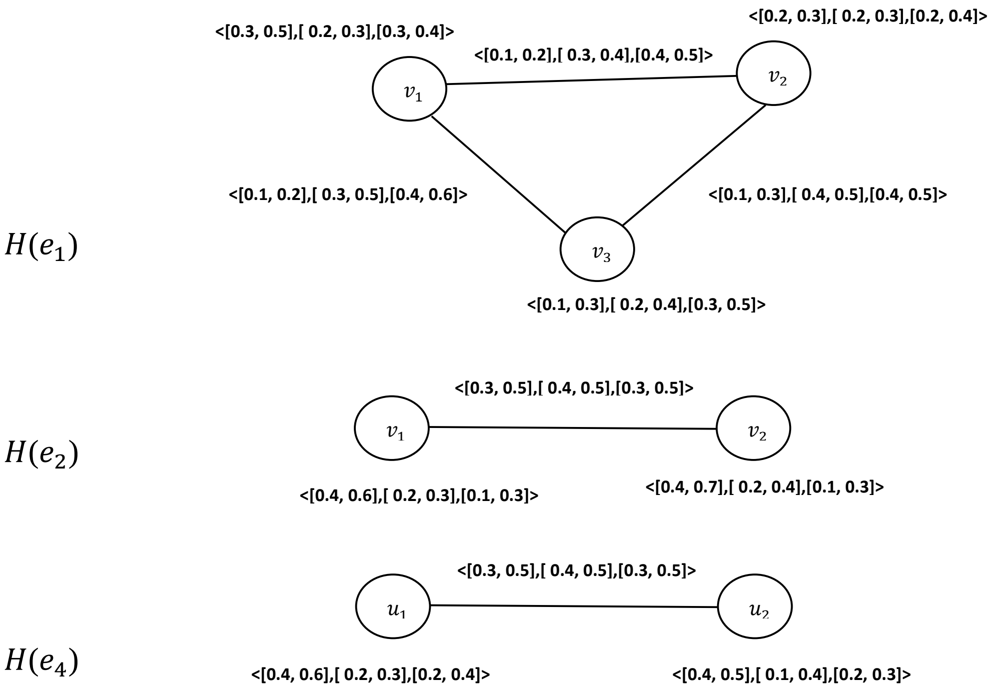


Fig. 3.8: Interval valued neutrosophic soft graph G = $G_1 \cap G_2$.

Definition 3.14 Let G_1=(K_1, M_1, A) and G_2=(K_2, M_2, B) be two interval valued neutrosophic graphs of G_1^* = (V_1, E_1) and G_1^* = (V_2, E_2) respectively. The union of two graphs$G_1$and G_2 is an interval valued neutrosophic soft graph G= $G_1 \cup G_2$ = (K,M, $A \cup B$), where (K, $A \cup B$) is an interval valued neutrosophic soft set over V= $V_1 \cup V_2$, (M, $A \cup B$) is an interval valued neutrosophic soft set over E= $E_1 \cap E_2$, truth-membership, indeterminacy-membership, and falsity-membership function of G for all x, z ∈V defined by:

1) $$T^L_{K(e)}(x) = \begin{cases} T^L_{K_1(e)}(x) & \text{if } e \in A - B \\ T^L_{K_2(e)}(x) & \text{if } e \in A - B \\ \max(T^L_{K_1(e)}(x), T^L_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$T^U_{K(e)}(x) = \begin{cases} T^U_{K_1(e)}(x) & \text{if } e \in A - B \\ T^U_{K_2(e)}(x) & \text{if } e \in A - B \\ \max(T^U_{K_1(e)}(x), T^U_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$I^L_{K(e)}(x) = \begin{cases} I^L_{K_1(e)}(x) & \text{if } e \in A - B \\ I^L_{K_2(e)}(x) & \text{if } e \in A - B \\ \min(I^L_{K_1(e)}(x), I^L_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$I^U_{K(e)}(x) = \begin{cases} I^U_{K_1(e)}(x) & \text{if } e \in A - B \\ I^U_{K_2(e)}(x) & \text{if } e \in A - B \\ \min(I^L_{K_1(e)}(x), I^U_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$F^L_{K(e)}(x) = \begin{cases} F^L_{K_1(e)}(x) & \text{if } e \in A - B \\ F^L_{K_2(e)}(x) & \text{if } e \in A - B \\ \min(F^L_{K_1(e)}(x), F^L_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

$$F^U_{K(e)}(x) = \begin{cases} F^U_{K_1(e)}(x) & \text{if } e \in A - B \\ F^U_{K_2(e)}(x) & \text{if } e \in A - B \\ \min(F^L_{K_1(e)}(x), F^U_{K_2(e)}(x)) & \text{if } e \in A \cap B \end{cases}$$

2) $$T^L_{M(e)}(xz) = \begin{cases} T^L_{M_1(e)}(xz) & \text{if } e \in A - B \\ T^L_{M_2(e)}(xz) & \text{if } e \in A - B \\ \max(T^L_{M_1(e)}(xz), T^L_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$T^U_{M(e)}(xz) = \begin{cases} T^U_{M_1(e)}(xz) & \text{if } e \in A - B \\ T^U_{M_2(e)}(xz) & \text{if } e \in A - B \\ \max(T^U_{M_1(e)}(xz), T^U_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$I^L_{M(e)}(xz) = \begin{cases} I^L_{M_1(e)}(xz) & \text{if } e \in A - B \\ I^L_{M_2(e)}(xz) & \text{if } e \in A - B \\ \min(I^L_{M_1(e)}(xz), I^L_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$I^U_{M(e)}(xz) = \begin{cases} I^U_{M_1(e)}(xz) & \text{if } e \in A - B \\ I^U_{M_2(e)}(xz) & \text{if } e \in A - B \\ \min(I^L_{M_1(e)}(xz), I^U_{K_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$F^L_{M(e)}(x) = \begin{cases} F^L_{M_1(e)}(xz) & \text{if } e \in A - B \\ F^L_{M_2(e)}(xz) & \text{if } e \in A - B \\ \min(F^L_{M_1(e)}(xz), F^L_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

$$F^U_{M(e)}(xz) = \begin{cases} F^U_{M_1(e)}(xz) & \text{if } e \in A - B \\ F^U_{M_2(e)}(xz) & \text{if } e \in A - B \\ \min(F^L_{M_1(e)}(xz), F^U_{M_2(e)}(xz)) & \text{if } e \in A \cap B \end{cases}$$

Definition 3.16. Let G_1 and G_2 be two interval valued neutrosophic soft graphs denoted by G_1 + G_2 =(K_1 + K_2, M_1 + M_2,A ∪B), Where (K_1 + K_2, A ∪ B) is an interval valued neutrosophic soft set over $V_1 \cup V_2$, (M_1 + M_2 , A ∪ B) is an interval valued neutrosophic soft set over $E_1 \cup E_2 \cup E'$ defined by

(K_1 + K_2 , A ∪B) =(K_1, A) ∪ (K_2, B)

(M_1 + M_2 , A ∪B) =(M_1, A) ∪ (M_2, B) if xz ∈ $E_1 \cup E_2$,

when e ∈ A ∩ B, xz∈ E', where E' is the set of all edge joining the vertices of V_1 and V_2.

Definition 3.17 The complement of an interval valued neutrosophic soft graph G=(K, M, A) denoted by $\bar{G}$=($\overline{K}$, $\overline{M}$, $\bar{A}$).

1. $\bar{A}$ =A
2. $\overline{K(e)}$ =K(e),
3. $T^L_{\overline{M(e)}}$(x, z) = min($T^L_{K(e)}$(x) ,$T^L_{K(e)}$(z)) $-T^L_{M(e)}$(x,z),
   $T^U_{\overline{M(e)}}$(x, z) = min($T^L_{K(e)}$(x) ,$T^L_{K(e)}$(z)) $-T^U_{M(e)}$(x,z),
   $I^L_{\overline{M(e)}}$(x, z) = min($I^L_{K(e)}$(x) ,$I^L_{K(e)}$(z)) $-I^L_{M(e)}$(x,z),
   $I^U_{\overline{M(e)}}$(x, z) = min($T^L_{K(e)}$(x) ,$I^L_{K(e)}$(z)) $-I^U_{M(e)}$(x,z),
   $F^L_{\overline{M(e)}}$(x, z) = min($F^L_{K(e)}$(x) ,$F^L_{K(e)}$(z)) $-F^L_{M(e)}$(x,z),
   $F^U_{\overline{M(e)}}$(x, z) = min($T^L_{K(e)}$(x) ,$F^L_{K(e)}$(z)) $-F^U_{M(e)}$(x,z), for all x, z ∈V,e ∈ A.

Definition 3.18 An interval valued neutrosophic soft graph G is a complete interval valued neutrosophic soft graph if H(e) is a complete interval valued neutrosophic graph of G for all e ∈A, i.e.

$T^L_{M(e)}$(x,z)= min($T^L_{K(e)}$(x) ,$T^L_{K(e)}$(z))
$T^U_{M(e)}$(x,z)= min($T^L_{K(e)}$(x) ,$T^L_{K(e)}$(z))

$I^L_{M(e)}$(x,z)= min($I^L_{K(e)}$(x) ,$I^L_{K(e)}$(z))
$I^U_{M(e)}$(x,z)= min($T^L_{K(e)}$(x) ,$I^L_{K(e)}$(z))
$F^L_{M(e)}$(x,z) = min($F^L_{K(e)}$(x) ,$F^L_{K(e)}$(z))
$F^U_{M(e)}$(x,z) = min($T^L_{K(e)}$(x) ,$F^L_{K(e)}$(z)), For all x, z ∈V, e ∈ A.

Example 3.19.Consider a simple graphG^*=(V, E) such that V={u_1, u_2,u_3, u_4} and E={u_1u_2, u_2u_3,u_3u_1}.

Let A= {e_1, e_2, e_3}be a set ofparameters. Let (K, A) be an interval valued neutrosophic graph soft sets over V with its approximation function. K:A ⟶P(V) defined by

$K(e_1)$=({u_1|([0.1, 0.4], [0.1, 0.3], [0.2, 0.3]), u_2|([0.1, 0.3], [0.1, 0.2], [0.1, 0.4]), u_3|([0.1, 0.2], [0.2, 0.3], [0.2, 0.5])}.

$K(e_2)$=({u_1|([0.3, 0.5], [0.2, 0.3], [0.3, 0.4]), u_2|([0.2, 0.3], [0.2, 0.3], [0.1, 0.4]), u_3|([0.1, 0.3], [0.2, 0.4], [0.3, 0.5])}.

$K(e_3)$=({u_1|([0.4, 0.5], [0.1, 0.3], [0.1, 0.4]), u_2|([0.4, 0.6], [0.1, 0.2], [0.2 0.3]), u_3|([0.2, 0.3], [0.2, 0.4], [0.1, 0.2]), u_4|([0.3, 0.6], [0.2, 0.3], [0.2, 0.3])}.

Let (M, A) be an interval valued neutrosophic graph soft sets over E with its approximation function. M:A⟶P(E) defined by

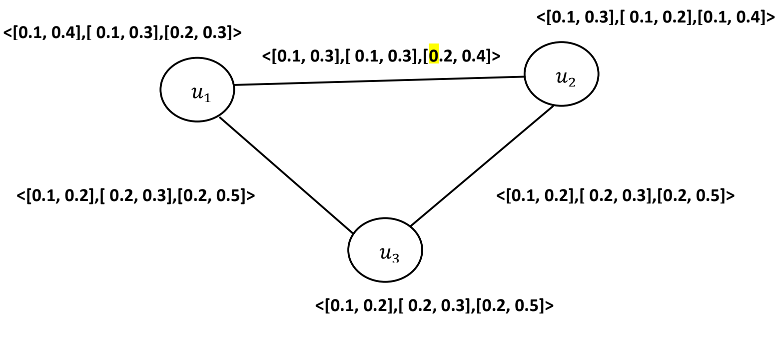
$M(e_1)$={u_1u_2|([0.1, 0.3], [0.1, 0.3], [0.2, 0.4]), u_2u_3|([0.1, 0.2], [0.2, 0.3], [0.2, 0.5]), u_3u_1 |([0.1, 0.2], [0.2, 0.3], [0.2, 0.5])}.

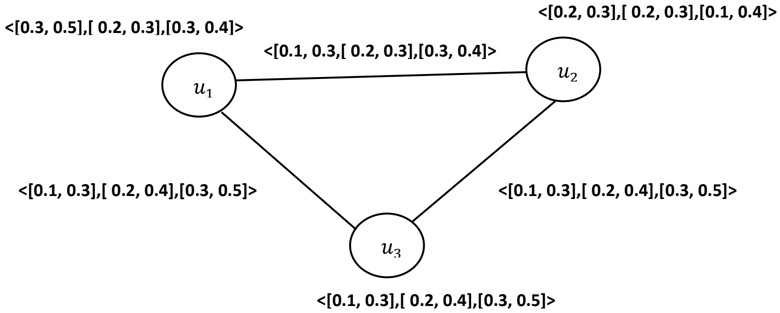
$M(e_2)$={u_1u_2|([0.1, 0.3], [0.2, 0.3], [0.3, 0.4]), u_2u_3|([0.1, 0.3], [0.2, 0.4], [0.3 0.5]), u_3u_1 |([0.1, 0.3], [0.2, 0.4], [0.3, 0.5])}.

$M(e_3)$={u_1u_2|([0.4, 0.5], [0.1, 0.3], [0.2, 0.4]), u_2u_3|([0.2, 0.3], [0.2, 0.4], [0.2, 0.3]),u_3u_4|([0.2, 0.3], [0.2, 0.4], [0.2, 0.3]), u_4u_1|([0.3, 0.5], [0.2, 0.3], [0.2, 0.4]),u_1u_3|([0.2, 0.3], [0.2, 0.4], [0.1, 0.4]), u_2u_4|([0.2, 0.6], [0.2, 0.4], [0.2, 0.3])}

It is easy to see that $H(e_1)$, $H(e_2)$, $H(e_3)$ are complete interval valued neutrosophic graphs of G corresponding to the parameterse_1, e_2, e_3 respectively as shown in Fig. 3.9.

$H(e_1)$





$H(e_2)$

$H(e_3)$

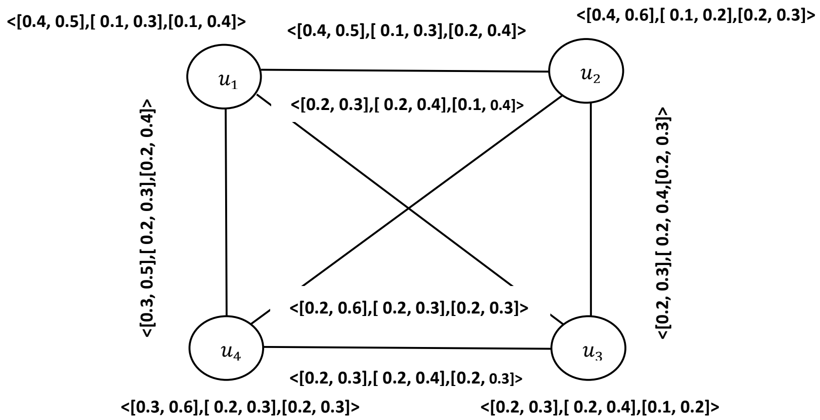


Fig. 3.9: Complete interval valued neutrosophic soft graph G={ $H(e_1)$, $H(e_2)$, $H(e_3)$}.

Definition 3.20: An interval valued neutrosophic soft graph G is a strong interval valued neutrosophic soft graph if H(e) is a strong interval valued neutrosophic graph of G for all e ∈A, i.e.

$T^L_{M(e)}$(x, z)= min($T^L_{K(e)}$(x) ,$T^L_{K(e)}$(z))
$T^U_{M(e)}$(x, z)= min($T^L_{K(e)}$(x) ,$T^L_{K(e)}$(z))
$I^L_{M(e)}$(x, z)= min($I^L_{K(e)}$(x) ,$I^L_{K(e)}$(z))
$I^U_{M(e)}$(x, z)= min($T^L_{K(e)}$(x) ,$I^L_{K(e)}$(z))
$F^L_{M(e)}$(x, z) = min($F^L_{K(e)}$(x) ,$F^L_{K(e)}$(z))
$F^U_{M(e)}$(x, z) = min($T^L_{K(e)}$(x) ,$F^L_{K(e)}$(z)), for all x, z ∈V, e ∈ A.

Example 3.21.Consider a simple graphG^*=(V, E) such that V={u_1, u_2,u_3, u_4} and E={u_1u_2, u_2u_3,u_3u_1}.

Let A= {e_1, e_2, e_3}be a set ofparameters. Let (K, A) be an interval valued neutrosophic graph soft sets over V with its approximation function. K:A⟶P(V) defined by

$K(e_1)$=({u_1|([0.1, 0.4], [0.1, 0.3], [0.2, 0.3]), u_2|([0.1, 0.3], [0.1, 0.2], [0.1, 0.4]), u_3|([0.1, 0.2], [0.2, 0.3], [0.2, 0.5])}.

$K(e_2)$=({u_1|([0.3, 0.5], [0.2, 0.3], [0.3, 0.4]), u_2|([0.2, 0.3], [0.2, 0.3], [0.1, 0.4]), u_3|([0.1, 0.3], [0.2, 0.4], [0.3, 0.5])}.

$K(e_3)$=({u_1|([0.4, 0.5], [0.1, 0.3], [0.1, 0.4]), u_2|([0.4, 0.6], [0.1, 0.2], [0.2 0.3]), u_3|([0.2, 0.3], [0.2, 0.4], [0.1, 0.2]), u_4|([0.3, 0.6], [0.2, 0.3], [0.2, 0.3])}.

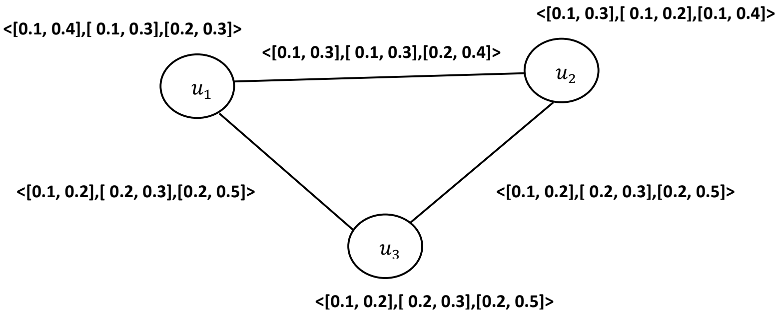
Let (M, A) be an interval valued neutrosophic graph soft sets over E with its approximation function. M:A⟶P(E) defined by

$M(e_1)$={u_1u_2|([0.1, 0.3], [0.1, 0.3], [0.2, 0.4]), u_2u_3|([0.1, 0.2], [0.2, 0.3], [0.2, 0.5]), u_3u_1 |([0.1, 0.2], [0.2, 0.3], [0.2, 0.5])}.

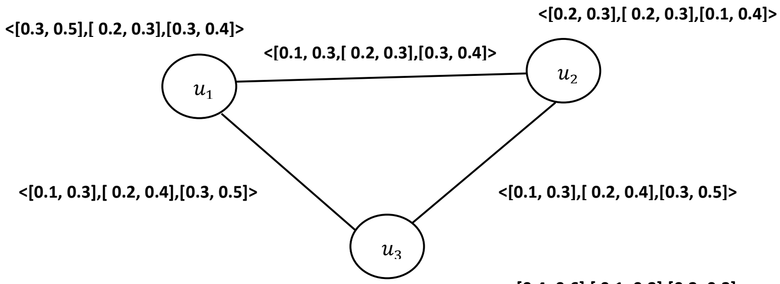
$M(e_2)$={u_1u_2|([0.1, 0.3], [0.2, 0.3], [0.3, 0.4]), u_2u_3|([0.1, 0.3], [0.2, 0.4], [0.3 0.5]), u_3u_1 |([0.1, 0.3], [0.2, 0.4], [0.3, 0.5])}.

$M(e_3)$={u_1u_2|([0.4, 0.6], [0.1, 0.3], [0.2, 0.4]), u_2u_3|([0.2, 0.3], [0.2, 0.4], [0.2, 0.3]),u_3u_4|([0.2, 0.3], [0.2, 0.4], [0.2, 0.3]), u_4u_1|([0.3, 0.5], [0.2, 0.3], [0.2, 0.4])}

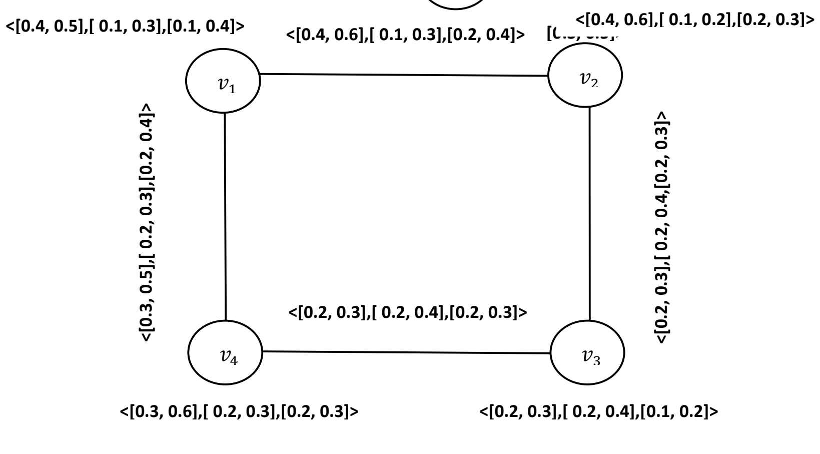
It is easy to see that $H(e_1)$, $H(e_2)$, $H(e_3)$ are strong interval valued neutrosophic graphs of G corresponding to the parameterse_1, e_2, e_3 respectively as shown in Fig. 3.10.



$H(e_1)$



$H(e_2)$



$H(e_3)$

Fig. 3.10: Strong interval valued neutrosophic soft graph G={ $H(e_1)$, $H(e_2)$, $H(e_3)$}.

4. APPLICATION

Interval valued neutrosophic soft set has several applications in decision making problems and can be used to deal with uncertainties from our different daily life problems. In this section, we apply the concept of interval valued neutrosophic soft sets in a decision making problem and then give an algorithm for the selection of optimal object based upon given sets of information.

Suppose that V={h_1,h_2,h_3,h_4,h_5} is the set of five houses under consideration. Mr. X is going to buy one of the houses on the basis of wishing parameters or attributes set A={e_1= large,e_2= beautiful, e_3= green surrounding}.(K, A) is the interval valued neutrosophic soft set on V which describes the value of the houses based upon the given parameters e_1= large,e_2= beautiful, e_3= green surrounding, respectively.

$K(e_1)$=({h_1|([0.3, 0.4], [0.2, 0.3], [0.3, 0.4]), h_3|([0.2, 0.3], [0.2, 0.3], [0.1, 0.4]), h_4|([0.2, 0.3], [0.2, 0.4], [0.3, 0.5])}.

$K(e_2)$=({h_1|([0.2, 0.5], [0.1, 0.3], [0.1, 0.3]), h_2|([0.3, 0.4], [0.1, 0.2], [0.2, 0.3]), h_3|([0.2, 0.3], [0.2, 0.3], [0.3, 0.4]),h_4|([0.3, 0.4], [0.2, 0.3], [0.1, 0.2]), h_5|([0.3, 0.4], [0.1, 0.2], [0.2, 0.4])}.

$K(e_3)$ =({h_1|([0.4, 0.5], [0.1, 0.3], [0.1, 0.4]), h_2|([0.4, 0.6], [0.1, 0.2], [0.2, 0.3]), h_3|([0.2, 0.3], [0.2, 0.4], [0.1, 0.2]),h_4|([0.3, 0.6], [0.2, 0.3], [0.2, 0.3]), h_5|([0.2, 0.3], [0.2, 0.3], [0.2, 0.4])}.

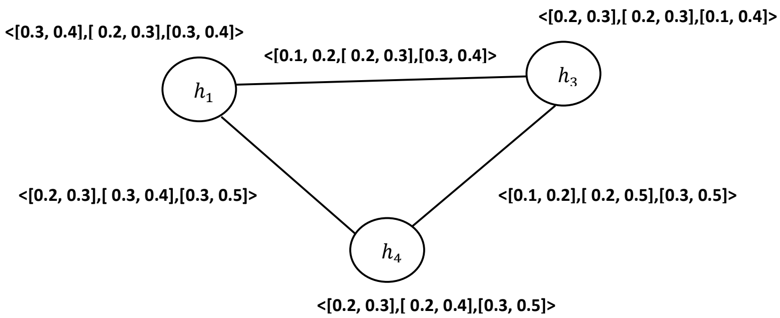
(M, A) is an interval valued neutrosophic soft sets on E= {h_1h_2, h_1h_3, h_1h_4,h_1h_5,h_2h_3,h_2h_4, h_2h_5, h_3h_4,h_4h_5} which describe the value of two houses corresponding to the given parameters e_1, e_2 and e_3.

$M(e_1)$={h_1h_3|([0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), h_3h_4|([0.1, 0.2], [0.2, 0.5], [0.3, 0.5]), h_1h_4 |([0.2, 0.3], [0.3, 0.4], [0.3, 0.5])}.

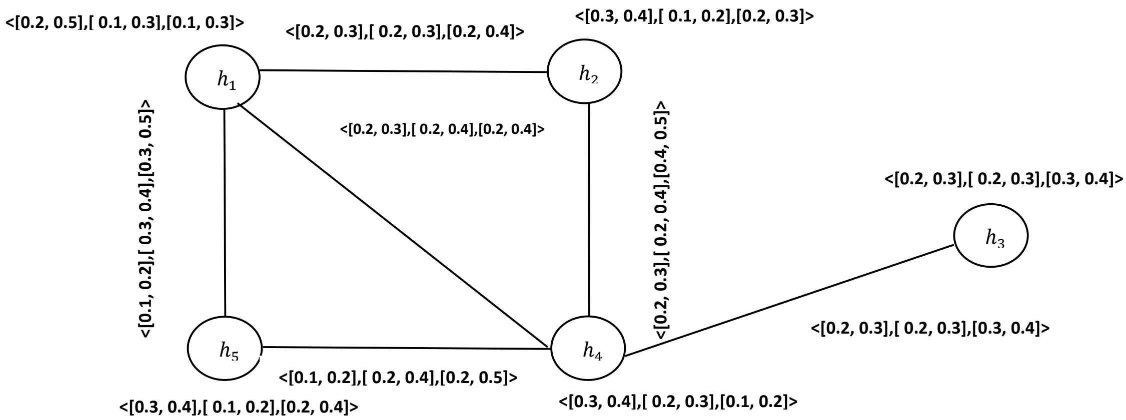
$M(e_2)$={h_1h_2|([0.2, 0.3], [0.2, 0.3], [0.2, 0.4]), h_1h_4|([0.2, 0.3], [0.2, 0.4], [0.2, 0.4]), h_1h_5 |([0.1, 0.3], [0.3, 0.4], [0.3, 0.5]), h_2h_4|([0.2, 0.3], [0.2, 0.4], [0.4, 0.5]), h_4h_5|([0.1, 0.2], [0.2, 0.4], [0.2, 0.5]), h_4h_3 |([0.2, 0.3], [0.2, 0.3], [0.3, 0.4])}.

$M(e_3)$= {h_1h_2|([0.4, 0.6], [0.2, 0.3], [0.3, 0.4]), h_1h_4|([0.3, 0.5], [0.3, 0.4], [0.2, 0.4]), h_2h_3 |([0.2, 0.3], [0.2, 0.5], [0.3, 0.4]), h_2h_5|([0.1, 0.2], [0.3, 0.4], [0.4, 0.5]), h_2h_4|([0.2, 0.4], [0.3, 0.4], [0.5, 0.6]), h_3h_4 |([0.2, 0.3], [0.4, 0.5], [0.2, 0.3])}.

The interval valued neutrosophic soft sets$H(e_1)$, $H(e_2)$, $H(e_3)$of interval valued neutrosophic graphs of G =(K, M, A) corresponding to the parameterse_1, e_2, e_3 respectively, as shown in Fig. 3.11.



$H(e_1)$



$H(e_2)$

$H(e_3)$

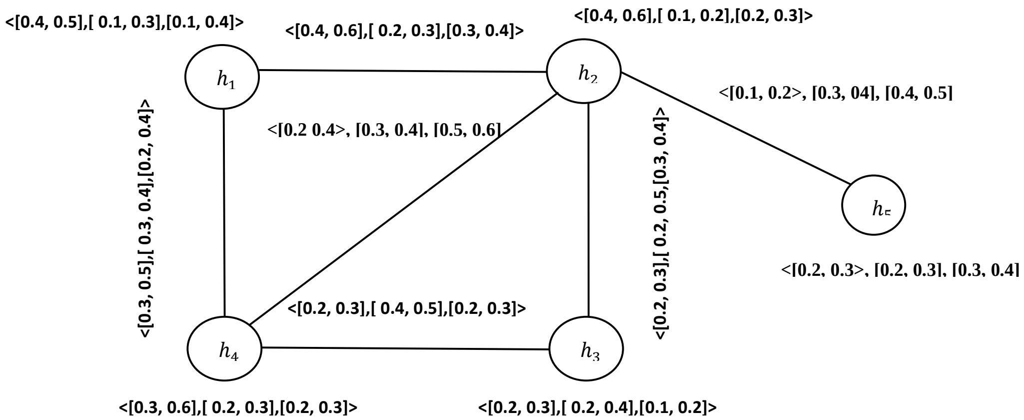


Fig. 3.11: Interval valued neutrosophic soft graph G={ $H(e_1)$, $H(e_2)$, $H(e_3)$}.

The interval valued neutrosophic graphs $H(e_1)$, $H(e_2)$, $H(e_3)$ corresponding to the parameters "large", "beautiful" and "green surrounding", respectively are represented by the following incidence matrix.

$H(e_1)$=

$$\begin{bmatrix} <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> & <[0.1,0.2],[0.2,0.3],[0.3,0.4]> & <[0.2,0.3],[0.3,0.4],[0.3,0.5]> & <[0,0],[0,0],[0,0]> \\ <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> \\ <[0.1,0.2],[0.2,0.3],[0.3,0.4]> & <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> & <[0.1,0.2],[0.2,0.5],[0.3,0.5]> & <[0,0],[0,0],[0,0]> \\ <[0.2,0.3],[0.3,0.4],[0.3,0.5]> & <[0,0],[0,0],[0,0]> & <[0.1,0.2],[0.2,0.5],[0.3,0.5]> & <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> \\ <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> & <[0,0],[0,0],[0,0]> \end{bmatrix}$$

$$H(e_2)=\begin{bmatrix} \langle[0,0],[0,0],[0,0]\rangle & \langle[0.2,0.3],[0.2,0.3],[0.2,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.2,0.3],[0.2,0.4],[0.2,0.4]\rangle & \langle[0.1,0.3],[0.3,0.4],[0.3,0.5]\rangle \\ \langle[0.2,0.3],[0.2,0.3],[0.2,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.2,0.3],[0.2,0.4],[0.4,0.5]\rangle & \langle[0,0],[0,0],[0,0]\rangle \\ \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.2,0.3],[0.2,0.3],[0.3,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle \\ \langle[0.2,0.3],[0.2,0.4],[0.2,0.4]\rangle & \langle[0.2,0.3],[0.2,0.4],[0.4,0.5]\rangle & \langle[0.2,0.3],[0.2,0.3],[0.3,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.1,0.2],[0.2,0.4],[0.2,0.5]\rangle \\ \langle[0.1,0.3],[0.3,0.4],[0.3,0.5]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.1,0.2],[0.2,0.4],[0.2,0.5]\rangle & \langle[0,0],[0,0],[0,0]\rangle \end{bmatrix}$$

And $H(e_3)$=

$$\begin{bmatrix} \langle[0,0],[0,0],[0,0]\rangle & \langle[0.4,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.3,0.5],[0.3,0.4],[0.2,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle \\ \langle[0.4,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.2,0.3],[0.2,0.5],[0.3,0.4]\rangle & \langle[0.2,0.4],[0.3,0.4],[0.5,0.6]\rangle & \langle[0.1,0.2],[0.3,0.4],[0.4,0.5]\rangle \\ \langle[0,0],[0,0],[0,0]\rangle & \langle[0.2,0.3],[0.2,0.5],[0.3,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.2,0.3],[0.4,0.5],[0.2,0.3]\rangle & \langle[0,0],[0,0],[0,0]\rangle \\ \langle[0.3,0.5],[0.3,0.4],[0.2,0.4]\rangle & \langle[0.2,0.4],[0.3,0.4],[0.5,0.6]\rangle & \langle[0.2,0.3],[0.4,0.5],[0.2,0.3]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0,0],[0,0]\rangle \\ \langle[0,0],[0,0],[0,0]\rangle & \langle[0.1,0.2],[0.3,0.4],[0.4,0.5]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0,0],[0,0]\rangle \end{bmatrix}$$

After performing some operation (AND or OR); we obtain the resultant interval valued neutrosophic graph H(e), where e= $e_1 \wedge e_2 \wedge e_3$. The incidence matrix of resultant interval neutrosophic soft graph is

$H(e_3)$=

$$\begin{bmatrix} \langle[0,0],[0,0],[0,0]\rangle & \langle[0.2,0.3],[0.2,0.3],[0.3,0.4]\rangle & \langle[0,0],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.2,0.3],[0.3,0.4],[0.3,0.5]\rangle & \langle[0,0],[0.3,0.4],[0.4,0.5]\rangle \\ \langle[0,0],[0.3,0.4],[0.4,0.5]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0.2,0.5],[0.3,0.4]\rangle & \langle[0,0],[0.3,0.4],[0.5,0.6]\rangle & \langle[0,0],[0.3,0.4],[0.4,0.5]\rangle \\ \langle[0,0],[0.2,0.3],[0.3,0.4]\rangle & \langle[0,0],[0.2,0.5],[0.3,0.4]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0.1,0.2],[0.4,0.5],[0.2,0.3]\rangle & \langle[0,0],[0,0],[0,0]\rangle \\ \langle[0,0],[0.3,0.4],[0.3,0.5]\rangle & \langle[0.2,0.3],[0.3,0.4],[0.5,0.6]\rangle & \langle[0.1,0.2],[0.4,0.5],[0.3,0.5]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0.2,0.4],[0.2,0.5]\rangle \\ \langle[0,0],[0.3,0.4],[0.3,0.5]\rangle & \langle[0,0],[0.3,0.4],[0.4,0.5]\rangle & \langle[0,0],[0,0],[0,0]\rangle & \langle[0,0],[0.2,0.4],[0.2,0.5]\rangle & \langle[0,0],[0,0],[0,0]\rangle \end{bmatrix}$$

Sahin (2015) defined the average possible membership degree of element x to interval valued neutrosophic set$A = \langle [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] \rangle$ as follows:

$$S_k(x) = \frac{1}{3}\left[\frac{T_A^L(x)+T_A^U(x)}{2} + 1 - \frac{I_A^L(x)+I_A^U(x)}{2} + 1 - \frac{F_A^L(x)+Fv_A^U(x)}{2}\right]$$
$$= \frac{T_A^L(x)+T_A^U(x)+4-I_A^L(x)-I_A^U(x)-F_A^L(x)-F_A^U(x)}{6}$$

Based on $S_k(x)$ we depictedthe Tabular representation of score value of incidence matrix of resultant interval valued neutrosophic graph H(e)with $S_k$andchoice valuefor each house h_k for k= 1, 2, 3, 4.

Table 2. Tabular representation of score values with choice values.

	h_1	h_2	h_3	h_4	h_5	h'_k
h_1	0.666	0.55	0.466	0.5	0.4	2,582
h_2	0.4	0.666	0.433	0.366	0.4	2,265
h_3	0.466	0.433	0.666	0.483	0.666	2,714
h_4	0.416	0.45	0.433	0.666	0.45	2,415
h_5	0.416	0.383	0.666	0.45	0.666	2,581

Clearly, the maximum score value is 2,714, scored by the h_3 Mr. X, will buy the house h_3.

We present our method as an algorithm that is used in our application.

Algorithm

1. Input the set P of choice of parameters of Mr. X, A is subset of P.
2. Input the interval valued neutrosophic soft sets (K, A) and (M, A).
3. Construct the interval valued neutrosophic soft graph G = (K, M, A).
4. Compute the resultant interval valued neutrosophic soft graph
   H(e) $=\bigcap_k H(e_k)$ fore = $\wedge_k e_k$ $\forall$ k.
5. Consider the interval valued neutrosophic graph H(e) and its incidence matrix form.
6. Compute the score S_k of $h_k \forall$ k.
7. The decision is h_k if $h'_k = \max_i h_k$.
8. If k has more than one value then any one of h_k may be chosen.

5. CONCLUSION

Interval valued neutrosophic soft sets is a generalization of fuzzy soft sets, intuitionistic fuzzy soft sets and neutrosophic soft sets. The neutrosophic set model is an important tool for dealing with real scientific and engineering applications; it can handle not only incomplete information, but also the inconsistent information and indeterminate information which exists in real situations. Interval valued neutrosophic models give more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy and/or intuitionistic fuzzy and single valued neutrosophic models. In this paper, we have introduced certain types of interval valued neutrosophic soft graphs, such as strong interval valued neutrosophic soft graph, complete interval valued neutrosophic soft graphs and complement of strong interval valued neutrosophic soft graphs. We introduced some operations such as Cartesian product, composition, intersection, union and join on an interval valued neutrosophic soft graphs. We presented an application of interval valued neutrosophic soft graphs in decision making. In future studies, we plan to extend our research to regular interval valued neutrosophic soft graphs and irregular interval valued neutrosophic soft graphs.

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