Multi-criteria Group Decision Making based on Linguistic Refined Neutrosophic Strategy

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ABSTRACT

Multi-criteria group decision making (MCGDM) strategy, which consists of a group of experts acting collectively for best selection among all possible alternatives with respect to some criteria, is focused on in this study. To develop the paper, we define linguistic neutrosophic refine set. We also define entropy to calculate unknown weights of the criteria and establish basic properties of entropy in linguistic neutrosophy refine set environment. In the developed strategy, the rating of all alternatives is expressed with linguistic variables. All linguistic variables are expressed as refined neutrosophic numbers which are characterized by truth-membership sequences, indeterminacy-membership sequences, and falsity-membership sequences. Linguistic refined neutrosophic score function (LRNSF) and linguistic refined neutrosophic accumulated function (LRNAF) are proposed. Weight of each criterion is unknown to decision maker. Finally, an illustrative example is provided to demonstrate the applicability of the proposed approach.

KEYWORDS: Linguistic variable, Neutrosophic set, Refined neutrosophic set, Linguistic refined neutrosophic set, Score function, Group decision making

1. INTRODUCTION

To deal uncertainty characterized by indeterminacy, Smarandache (1998) introduced neutrosophic sets. The concept of neutrosophic sets is the generalization fuzzy set (Zadeh, 1965) and intuitionistic fuzzy set (Atanassov, 1986). Wang et al. (2010) proposed the concept of single valued neutrosophic set (SVNS) to deal with practical problems. SVNS has been studied and applied in different fields such as medical diagnosis (Ye, 2015a, Ye & Fu, 2016) decision making problems (Sodenkamp, 2013; Kharal, 2014; Biswas et al. 2014a, 2014b, 2015a, 2015b, 2016a, 2016b; Mondal & Pramanik, 2014b, 2015a, 2015c; Şahin, 2017; Şahin & Liu, 2016; Ye, 2015b, Smarandache & Pramanik, 2016), social problems (Mondal & Pramanik, 2014; Pramanik & Chackrabarti, 2013), engineering problem (Ye, 2016), conflict resolution (Pramanik & Roy, 2014) and so on.

Different neutrosophic hybrid sets are proposed in the literature such as neutrosophic soft set (Maji, 2013), neutrosophic cubic set (Ali, Deli, & Smarandache, 2016), neutrosophic bipolar set (Deli, Ali, M., & Smarandache, 2015), rough bipolar neutrosophic set (Pramanik & Mondal, 2016), etc. Broumi et al. (2014a, 2014b) proposed rough neutrosophic set by combining rough set and neutrosophic set. Mondal and Pramanik (2015a) proved the basic properties of cosine similarity measure of rough neutrosophic sets and provided its application in medical diagnosis. Pramanik & Mondal (2015) proved the basic properties of cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Mondal & Pramanik (2015d) also proposed new rough neutrosophic multi-attribute decision-making strategy based on grey relational analysis. Mondal, Pramanik and Smarandache (2016a) proposed multi-attribute decision making based on rough neutrosophic variational coefficient similarity measure. Mondal, Pramanik and Smarandache (2016b) also established rough neutrosophic TOPSIS for multi-attribute group decision making. Pramanik, Roy, Roy and Smarandache (2017) proposed rough multi criteria decision making based on correlation coefficient.

Smarandache (2013) extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component T, I, F into respectively, T_1 ,T_2 , ... T_m, and I_1 ,I_2 , ... I_p and F_1 ,F_2 , ... F_r. Broumi & Smarandache (2014) presented an application of cosine similarity measure of neutrosophic refined sets in medical diagnosis problems. Ye & Ye (2014) introduced the concept of single valued neutrosophic multi-set (SVNM) and proved its basic operational relations. In the same study, Ye and Ye (2014) proposed the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigated their properties and they applied the Dice similarity measure of SVNMs to medical diagnosis. Broumi and Deli (2014) proposed correlation measure for neutrosophic refined sets and applied to medical diagnosis. Mondal and Pramanik (2015b) proposed neutrosophic refined similarity measure based on tangent function and applied it to multi-attribute decision making. In this paper, we propose a new multi-criteria group decision making method based on linguistic variables and refined neutrosophic sets. The proposed method is illustrated by solving an illustrative example.

Rest of the paper has been organized as follows: In section 2, some definitions of neutrosophic set, single valued neutrosophic set, refined neutrosophic set, refined neutrosophic number, and linguistic refined neutrosophic set have been presented briefly. In section 3, a new multi-criteria group decision making method has been presented. In section 4, the proposed method has been applied to deal with an illustrative example related to suitable spot selection for construction purpose. Section 5 presents the concluding remarks and future scope of research.

2. PRELIMINARIES

2.1 Concepts of neutrosophic sets (Smarandache, 1998)

A neutrosophic set A in a universal set X, which is characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_A(x)$: $X \rightarrow]^-0, 1^+[$, $I_A(x)$: $X \rightarrow]^-0, 1^+[$, and $F_A(x)$: $X \rightarrow]^-0, 1^+[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

2.2 Some concepts of single valued neutrosophic sets (Wang et al., 2010)

Definition 1 A single valued neutrosophic set A in a universal set X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a single valued neutrosophic set A can be denoted by

$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in X\}$ where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Let $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle / x \in X\}$ be two single valued neutrosophic sets, and then there are the following relations.

- Complement: $A^c = \{x, \langle F_A(x), 1 - I_A(x), T_A(x)\rangle / x \in X\}$;
- Inclusion: $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X;

- Equality: $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$;
- Union: $A \cup B = \{\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x) \rangle / x \in X\}$
- Intersection: $A \cap B = \{\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x) \rangle / x \in X\}$
- Addition: $A \oplus B = \{\langle x, T_A(x) + T_B(x) - T_A(x).T_B(x),\ I_A(x).I_B(x),\ F_A(x).F_B(x) \rangle / x \in X\}$
- Multiplication: $A \otimes B = \{\langle x, T_A(x).T_B(x), I_A(x) + I_B(x) - I_A(x).I_B(x),\ F_A(x) + F_B(x) - F_A(x).F_B(x) \rangle / x \in X\}$

2.3 Refined neutrosophic sets (Smarandache, 2013)

Let A be a refined neutrosophic set in a universal set X. Then A can be expressed as
$A = \langle < x, (T_A^1(x), T_A^2(x), \cdots, T_A^p(x)), (I_A^1(x), I_A^2(x), \cdots, I_A^p(x)), (F_A^1(x), F_A^2(x), \cdots, F_A^p(x)), x \in X \rangle$,
where, $0 \le T_A^1(x), T_A^2(x), \cdots, T_A^p(x) \le 1$, $0 \le I_A^1(x), I_A^2(x), \cdots, I_A^p(x) \le 1$, $0 \le F_A^1(x), F_A^2(x), \cdots, F_A^p(x) \le 1$ such that $0 \le \sup T_A^i(x) + \sup T_A^i(x) + \sup T_A^i(x) \le 3$ for i = 1, 2, …, p, for any $x \in X$. $T_A^1(x), T_A^2(x), \cdots, T_A^p(x)$, $I_A^1(x), I_A^2(x), \cdots, I_A^p(x)$ and $F_A^1(x), F_A^2(x), \cdots, F_A^p(x)$ are the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, 'p' is called the dimension of neutrosophic refined sets A.

2.4 Linguistic refined neutrosophic set

Let X be a universal set and a linguistic term S represented by a refined neutrosophic set A on X. The set containing linguistic variables S which is characterized by the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence respectively is called a linguistic refined neutrosophic set. If the dimension of refined neutrosophic set is p, then the dimension of linguistic refined neutrosophic set is also p. Some linguistic variables and corresponding refined neutrosophic numbers are presented as follows (see Table 1).

Table 1: Refined neutrosophic sets corresponding to linguistic variables

Linguistic Variables	Refined neutrosophic set
Extremely Good (EG)	$\langle(1.00, 1.00, 1.00, \cdots \text{p times}), (0.00, 0.00, 0.00, \cdots \text{p times}), (0.00, 0.00, 0.00, \cdots \text{p times})\rangle$
Very Good(VG)	$\langle(0.90, 0.90, 0.90, \cdots \text{p times}), (0.08, 0.08, 0.08, \cdots \text{p times}), (0.08, 0.08, 0.08, \cdots \text{p times})\rangle$
Good (G)	$\langle(0.80, 0.80, 0.80, \cdots \text{p times}), (0.20, 0.20, 0.20, \cdots \text{p times}), (0.20, 0.20, 0.20, \cdots \text{p times})\rangle$
Medium Good (MG)	$\langle(0.60, 0.60, 0.60, \cdots \text{p times}), (0.40, 0.40, 0.40, \cdots \text{p times}), (0.30, 0.30, 0.30, \cdots \text{p times})\rangle$
Medium (M)	$\langle(0.50, 0.50, 0.50, \cdots \text{p times}), (0.50, 0.50, 0.50, \cdots \text{p times}), (0.40, 0.40, 0.40, \cdots \text{p times})\rangle$
Medium Bad (MB)	$\langle(0.40, 0.40, 0.40, \cdots \text{p times}), (0.40, 0.40, 0.40, \cdots \text{p times}), (0.\ 50, 0.50, 0.50, \cdots \text{p times})\rangle$
Bad (G)	$\langle(0.20, 0.20, 0.20, \cdots \text{p times}), (0.80, 0.80, 0.80, \cdots \text{p times}), (0.80, 0.80, 0.80, \cdots \text{p times})\rangle$
Very Bad (VB)	$\langle(0.10, 0.10, 0.10, \cdots \text{p times}), (0.80, 0.80, 0.80, \cdots \text{p times}), (0.90, 0.90, 0.90, \cdots \text{p times})\rangle$
Very very Bad (VVB)	$\langle(0.05, 0.05, 0.05, \cdots \text{p times}), (0.90, 0.90, 0.90, \cdots \text{p times}), (0.90, 0.90, 0.90, \cdots \text{p times})\rangle$

Definition 2: Linguistic refined neutrosophic accumulated function (LRNAF)

Let $n_{ij}=\langle(a_{ij}^1, a_{ij}^2, \cdots, a_{ij}^p), (b_{ij}^1, b_{ij}^2, \cdots, b_{ij}^p), (c_{ij}^1, c_{ij}^2, \cdots, c_{ij}^p)\rangle$ i = 1, 2, …, m, j = 1, 2, …, n be a collection of refined neutrosophic sets of order p. Then linguistic refined neutrosophic accumulated function (LRNAF) is defined as follows:

$$\text{LRNAF}(n_{ij}) = \langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle = \left\langle \frac{a_{ij}^1 + a_{ij}^2 + \cdots + a_{ij}^p}{p}, \frac{b_{ij}^1 + b_{ij}^2 + \cdots + b_{ij}^p}{p}, \frac{c_{ij}^1 + c_{ij}^2 + \cdots + c_{ij}^p}{p} \right\rangle \quad (1)$$

i = 1, 2, …, m, j = 1, 2, …, n.

Definition 3: Linguistic refined neutrosophic score function (LRNSF)

Let $\langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle$ be a LRNAF, and then a score function of LRNAF can be defined as follows.

$$S\langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle = \frac{1}{3}(2 + \alpha_{ij} - \beta_{ij} - \gamma_{ij}),\ S\langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle \in [0, 1] \quad (2)$$

where the larger value of $S\langle \alpha_j, \beta_j, \gamma_j \rangle$ indicates the truth value of LRNAF is larger.

Definition 4: Weighted accumulation score value (WASV)

Weighted accumulation score value (WASV) of all criteria is presented as:

$$\text{WASV}(C_1, C_2, \ldots, C_n) = \sum_{j=1}^{n} w_j S(\alpha_j, \beta_j, \gamma_j) \quad (3)$$

$\sum_{j=1}^{n} w_j = 1$, j = 1, 2, . . ., n

3. DECISION MAKING METHODOLOGY

Assume that $L_1, L_2, \ldots, L_m$ be a discrete set of alternatives, $C_1, C_2, \ldots, C_n$ be the set of criteria and K_1, K_2,

..., K_k are the decision makers. The decision makers provide the rating of alternatives with respect to all criteria. The rating represents the performances of alternative L_i (i = 1, 2, ..., m) against the criterion C_j (j = 1, 2, ..., n). The values associated with the alternatives for MCGDM problem can be presented in the following decision matrix. The relation between alternatives and criteria is given in the Table 2.

Table 2: The relation between alternatives and criteria

	C_1	C_2	$\cdots$	C_n
L_1	$\left\langle \begin{matrix}(T_{11}^1,T_{11}^2,\cdots,T_{11}^p),\\(I_{11}^1,I_{11}^2,\cdots,T_{11}^p),\\(F_{11}^1,F_{11}^2,\cdots,T_{11}^p)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(T_{12}^1,T_{12}^2,\cdots,T_{12}^p),\\(I_{12}^1,I_{12}^2,\cdots,T_{11}^p),\\(F_{12}^1,F_{12}^2,\cdots,T_{11}^p)\end{matrix}\right\rangle$	$\cdots$	$\left\langle \begin{matrix}(T_{1n}^1,T_{1n}^2,\cdots,T_{1n}^p),\\(I_{1n}^1,I_{1n}^2,\cdots,T_{11}^p),\\(F_{1n}^1,F_{1n}^2,\cdots,T_{11}^p)\end{matrix}\right\rangle$
L_2	$\left\langle \begin{matrix}(T_{21}^1,T_{21}^2,\cdots,T_{11}^p),\\(I_{21}^1,I_{21}^2,\cdots,T_{11}^p),\\(F_{21}^1,F_{21}^2,\cdots,T_{11}^p)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(T_{22}^1,T_{22}^2,\cdots,T_{11}^p),\\(I_{22}^1,I_{22}^2,\cdots,T_{11}^p),\\(F_{22}^1,F_{22}^2,\cdots,T_{11}^p\end{matrix}\right\rangle$	$\cdots$	$\left\langle \begin{matrix}(T_{2n}^1,T_{2n}^2,\cdots,T_{11}^p),\\(I_{2n}^1,I_{2n}^2,\cdots,T_{11}^p),\\(F_{2n}^1,F_{2n}^2,\cdots,T_{11}^p\end{matrix}\right\rangle$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
L_m	$\left\langle \begin{matrix}(T_{m1}^1,T_{m1}^2,\cdots,T_{11}^p),\\(I_{m1}^1,I_{m1}^2,\cdots T_{11}^p),\\(F_{m1}^1,F_{m1}^2,\cdots,T_{11}^p)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(T_{m2}^1,T_{m2}^2,\cdots,T_{11}^p),\\(I_{m2}^1,I_{m2}^2,\cdots T_{11}^p),\\(F_{m2}^1,F_{m2}^2,\cdots,T_{11}^p\end{matrix}\right\rangle$	$\cdots$	$\left\langle \begin{matrix}(T_{mn}^1,T_{mn}^2,\cdots,T_{11}^p),\\(I_{mn}^1,I_{mn}^2,\cdots,T_{11}^p),\\(F_{mn}^1,F_{mn}^2,\cdots,T_{11}^p)\end{matrix}\right\rangle$

(4)

The steps of the group decision making method under linguistic refined neutrosophic environment are described as follows:

Step 1: Construction of the decision matrix with linguistic refined neutrosophic sets

For MCGDM, the rating of alternative L_i (i = 1, 2,…, m) with respect to criterion C_j (j = 1, 2,…n) is taken as refined neutrosophic environment. It can be represented with the following forms:

$$L_i = \begin{bmatrix} C_1 \big/ \left\langle (T_{i1}^1,T_{i1}^2,\cdots,T_{i1}^p),(I_{i1}^1,I_{i1}^2,\cdots,I_{i1}^p),\ (F_{i1}^1,F_{i1}^2,\cdots,F_{i1}^p)\right\rangle, \\ C_2 \big/ \left\langle (T_{i2}^1,T_{i2}^2,\cdots,T_{i2}^p),(I_{i2}^1,I_{i2}^2,\cdots,I_{i2}^p),\ (F_{i2}^1,F_{i2}^2,\cdots,F_{i2}^p)\right\rangle, \\ \ldots,\ C_n \big/ \left\langle (T_{in}^1,T_{in}^2,\cdots,T_{in}^p),(I_{in}^1,I_{in}^2,\cdots,I_{in}^p),\ (F_{in}^1,F_{in}^2,\cdots,F_{in}^p)\right\rangle \end{bmatrix}$$

$$L_i = \begin{bmatrix} C_j \big/ \left\langle (T_{ij}^1,T_{ij}^2,\cdots,T_{ij}^p),(I_{ij}^1,I_{ij}^2,\cdots,I_{ij}^p),\ (F_{ij}^1,F_{ij}^2,\cdots,F_{ij}^p)\right\rangle \end{bmatrix}, j=1,2,...,n \tag{5}$$

Here $\left\langle (T_{ij}^1,T_{ij}^2,\cdots,T_{ij}^p),(I_{ij}^1,I_{ij}^2,\cdots,I_{ij}^p),\ (F_{ij}^1,F_{ij}^2,\cdots,F_{ij}^p)\right\rangle$ denotes refined neutrosophic set.

The degrees of truth, indeterminacy and falsity membership of the alternative L_i satisfying the criterion C_j, respectively where

$0 \le T_{ij}^1, T_{ij}^2, \cdots, T_{ij}^p \le 1,\ 0 \le I_{ij}^1, I_{ij}^2, \cdots, I_{ij}^p \le 1,\ 0 \le F_{ij}^1, F_{ij}^2, \cdots, F_{ij}^p \le 1$

Step 2: Determination of the linguistic refined neutrosophic accumulated decision matrix

Assume that, a linguistic refined neutrosophic set is of the form

$\left\langle (T_{ij}^1,T_{ij}^2,\cdots,T_{ij}^p),\ (I_{ij}^1,I_{ij}^2,\cdots,I_{ij}^p),\ (F_{ij}^1,F_{ij}^2,\cdots,F_{ij}^p)\right\rangle$

The linguistic refined neutrosophic matrix is formed by utilizing equation (1) and it is presented in the Table 3.

Table3: The linguistic refined neutrosophic accumulated decision matrix for decision maker K_i

$$[\text{LRNAF}]_{m\times n}^{K_i} = \begin{array}{c|cccc} & C_1 & C_2 & \ldots & C_n \\ \hline L_1 & \langle \alpha_{11}, \beta_{11}, \gamma_{11}\rangle^{K_i} & \langle \alpha_{12}, \beta_{12}, \gamma_{12}\rangle^{K_i} & \ldots & \langle \alpha_{1n}, \beta_{1n}, \gamma_{1n}\rangle^{K_i} \\ L_2 & \langle \alpha_{21}, \beta_{21}, \gamma_{21}\rangle^{K_i} & \langle \alpha_{22}, \beta_{22}, \gamma_{22}\rangle^{K_i} & \ldots & \langle \alpha_{2n}, \beta_{2n}, \gamma_{2n}\rangle^{K_i} \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ L_m & \langle \alpha_{m1}, \beta_{m1}, \gamma_{m1}\rangle^{K_i} & \langle \alpha_{m2}, \beta_{m2}, \gamma_{m2}\rangle^{K_i} & \ldots & \langle \alpha_{mn}, \beta_{mn}, \gamma_{mn}\rangle^{K_i} \end{array} \quad (6)$$

Step 3: Determination of linguistic refined neutrosophic score matrix for decision makers

Using the equation (2), aggregated transferred neutrosophic score matrix for alternative L_i (i = 1, 2, ..., n) is defined as follows:

Table 4: Aggregated transferred neutrosophic score matrix for alternatives

$$\langle S\langle \alpha_{ij}, \beta_{ij}, \gamma_{ij}\rangle\rangle_{m\times n}^{K_i}$$

$$\begin{array}{c|cccc} . & C_1 & C_2 & \cdots & C_n \\ \hline L_1 & S\langle \alpha_{11}, \beta_{11}, \gamma_{11}\rangle^{K_i} & S\langle \alpha_{12}, \beta_{12}, \gamma_{12}\rangle^{K_i} & \cdots & S\langle \alpha_{1n}, \beta_{1n}, \gamma_{1n}\rangle^{K_i} \\ L_2 & S\langle \alpha_{21}, \beta_{21}, \gamma_{21}\rangle^{K_i} & S\langle \alpha_{22}, \beta_{22}, \gamma_{22}\rangle^{K_i} & \cdots & S\langle \alpha_{2n}, \beta_{2n}, \gamma_{2n}\rangle^{K_i} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ L_m & S\langle \alpha_{m1}, \beta_{m1}, \gamma_{m1}\rangle^{K_i} & S\langle \alpha_{m2}, \beta_{m2}, \gamma_{m2}\rangle^{K_i} & \cdots & S\langle \alpha_{mn}, \beta_{mn}, \gamma_{mn}\rangle^{K_i} \end{array} \quad (7)$$

Step 4: Determination of geometric mean of score matrices for decision makers

To fuse the opinions of all decision makers, we determine geometric mean of all corresponding linguistic refined neutrosophic score values (see Table 5).

Table 5: Geometric mean of score matrix for decision makers

$$\langle S\langle \alpha_{ij}, \beta_{ij}, \gamma_{ij}\rangle\rangle_{m\times n}$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline L_1 & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{11}, \beta_{11}, \gamma_{11}\rangle^{K_i}\right)} & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{12}, \beta_{12}, \gamma_{12}\rangle^{K_i}\right)} & \cdots & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{1n}, \beta_{1n}, \gamma_{1n}\rangle^{K_i}\right)} \\ L_2 & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{21}, \beta_{21}, \gamma_{21}\rangle^{K_i}\right)} & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{22}, \beta_{22}, \gamma_{22}\rangle^{K_i}\right)} & \cdots & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{2n}, \beta_{2n}, \gamma_{2n}\rangle^{K_i}\right)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ L_m & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{m1}, \beta_{m1}, \gamma_{m1}\rangle^{K_i}\right)} & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{m2}, \beta_{m2}, \gamma_{m2}\rangle^{K_i}\right)} & \cdots & \sqrt[n]{\left(\prod_{r=1}^{k} S\langle \alpha_{mn}, \beta_{mn}, \gamma_{mn}\rangle^{K_i}\right)} \end{array} \quad (8)$$

Step 5: Determination of weights criteria

In practical decision making situation, criteria weights may be unknown to decision makers. Also, the importance of the criteria may be different.

3.1 Method of Entropy in linguistic refined neutrosophic environment

Entropy is an important method to measure uncertain information (Shannon, 1951). Kosko (1986) proposed fuzzy entropy and conditioning. Szmidt and Kacprzyk (2001) proposed entropy function for intuitionistic fuzzy sets. Majumdar and Samanta (2014) developed entropy measures for SVNSs. Biswas et al. (2014a) also studied entropy measures for SVNSs. The entropy measure can be used to calculate the criteria weights when it is completely unknown to decision maker.

In this paper we propose an entropy method for linguistic refined neutrosophic sets to determining unknown criteria weight. Assume that, $N=\langle (T_{i1}^1,T_{i1}^2,\cdots,T_{i1}^p),(I_{i1}^1,I_{i1}^2,\cdots,I_{i1}^p),(F_{i1}^1,F_{i1}^2,\cdots,F_{i1}^p)\rangle$ be a refined neutrosophic set. We define entropy function in linguistic refined neutrosophic environment as follows.

$$\mathrm{ENT}_i(N)=1-\frac{1}{n}\sum_{i=1}^{m}\left(\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p}+\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right)\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right| \quad (9)$$

The function has the following properties:

P1. $\mathrm{ENT}_i(N)=0\Rightarrow N \textit{ is a crisp set and } \frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}=0.$

Proof. $N \textit{ is a crisp set and } \frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}=0$

$$\Rightarrow \frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p}=1,\ \frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}=0,\ \frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}=0$$

$\Rightarrow \mathrm{ENT}_i(N)=0$

P2. $\mathrm{ENT}_i(N)=1\Rightarrow\left\langle\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p},\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p},\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right\rangle=\langle 0.5,0.5,0.5\rangle.$

Proof. $\left\langle\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p},\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p},\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right\rangle=\langle 0.5,0.5,0.5\rangle$

$\Rightarrow \mathrm{ENT}_i(N)=1-\frac{1}{n}.0=0.$

3. $\mathrm{ENT}_i(N_1)\geq \mathrm{ENT}_i(N_2)\Rightarrow$

$$\left(\frac{(T_{i1}^1,+T_{i1}^2+\cdots+T_{i1}^p)}{p}+\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right)_{\text{for }N_1}\leq\left(\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p}+\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right)_{\text{for }N_2}$$

and $\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right|_{\text{for }N_1}\leq\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right|_{\text{for }N_2}$

Proof.

$$\left(\frac{(T_{i1}^1,+T_{i1}^2+\cdots+T_{i1}^p)}{p}+\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right)_{\text{for }N_1}\leq\left(\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p}+\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right)_{\text{for }N_2}$$

and $\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right|_{\text{for }N_1}\leq\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right|_{\text{for }N_2}$

$$\Rightarrow\left[1-\frac{1}{n}\sum_{i=1}^{m}\left(\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p}+\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right)\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right|\right]_{\text{for }N_1}$$

$$\geq\left[1-\frac{1}{n}\sum_{i=1}^{m}\left(\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p}+\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right)\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right|\right]_{\text{for }N_2}$$

$\Rightarrow \mathrm{ENT}_i(N_1)\geq \mathrm{ENT}_i(N_2)$

P4. $\mathrm{ENT}_i(N_1)=\mathrm{ENT}_i(N_1^c).$

Proof.

$$\mathrm{ENT}_i(N_1)=1-\frac{1}{n}\sum_{i=1}^{m}\left(\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p}+\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}\right)\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right|_{\text{for } N_1}$$

$$\mathrm{ENT}_i(N_1^c)=1-\frac{1}{n}\sum_{i=1}^{m}\left(\frac{(F_{i1}^1+F_{i1}^2+\cdots+F_{i1}^p)}{p}+\frac{(T_{i1}^1+T_{i1}^2+\cdots+T_{i1}^p)}{p}\right)\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right|_{\text{for } N_1^C}$$

Hence, $\mathrm{ENT}_i(N_1)=\mathrm{ENT}_i(N_1^c)$.

In order to obtain the entropy value ENT_j of the j-th criterion C_j (j = 1, 2,…, n), equation (16) can be written as :

$$\mathrm{ENT}_i(N)=1-\frac{1}{n}\sum_{i=1}^{m}\left(\frac{(T_{i1}^1,T_{i1}^2,\cdots,T_{i1}^p)}{p}+\frac{(F_{i1}^1,F_{i1}^2,\cdots,F_{i1}^p)}{p}\right)\left|1-2.\frac{(I_{i1}^1+I_{i1}^2+\cdots+I_{i1}^p)}{p}\right| \tag{10}$$

For i = 1, 2, …, n; j = 1, 2, …, m

It is observed that $\mathrm{ENT}_j \in [0,1]$. The entropy weight of the j-th criterion C_j in refined neutrosophic environment is presented as:

$$w_j=\frac{1-\mathrm{ENT}_j}{\sum_{j=1}^{n}\left(1-\mathrm{ENT}_j\right)} \tag{11}$$

We have weight vector $W=(w_1, w_2,\ldots,w_n)^T$ of n criteria C_j (j = 1, 2, …, n) with $w_j \geq 0$ and $\sum_{i=1}^{n} w_j = 1$

Step 6: Determination of weighted accumulation score values (WASV)

Using equation (3), weighted accumulation score values (WASVs) for all alternatives corresponding to each criterion are defined as following matrix (see Table 6).

Table 6: Weighted accumulated score matrix

$$\begin{array}{c|cccc} . & C_1 & C_2 & \cdots & C_n \\ \hline L_1 & WASV_{11} & WASV_{12} & \cdots & WASV_{1n} \\ L_2 & WASV_{21} & WASV_{22} & \cdots & WASV_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ L_m & WASV_{m1} & WASV_{m2} & \cdots & WASV_{mn} \end{array} \tag{12}$$

Step 7: Calculate extreme averaging score values

We define extreme averaging score values (EASVs) to aggregate all weighted accumulated score values as follows.

$$\mathrm{EASV}(L_i) = \sum_{j=1}^{n} \mathrm{WASV}_{ij} \quad i = 1, 2, \ldots, n. \tag{13}$$

Step 8: Rank the priority

The set of alternatives then can be preference ranked according to the descending order of the extreme averaging score value $\mathrm{EASV}(L_i)$.
The alternative corresponding to the highest extreme averaging score value reflects the best choice.

Step 9: End.

4. AN ILLUSTRATIVE EXAMPLE

A financial grand for Birnagar High School, West Bengal, India has been sanctioned from West Bengal

State Government to construct a modern sanitary system. For this purpose, school managing committee call for a meeting to select best spot for sanitary system construction. Three decision makers of the school are Headmaster (K_1), Assistant headmaster (K_2) and President (K_3). There are three potential spots in school boundary (marked as L_1, L_2, L_3) are chosen for final selection. Decision makers intended to select the best spot among L_1, L_2, L_3 with respect to six criteria namely,

- Distance form students (C_1),
- Water supply (C_2),
- Future maintenance (C_3),
- Costs for construction (C_4),
- Governmental Regulations and Laws (C_5),
- Environmental Impact (C_6).

Three alternatives (L_1, L_2, L_3) with respect to the six criteria (C_1, C_2, C_3, C_4, C_5, C_6) are evaluated by three decision makers (K_1, K_2, K_3) under the linguistic refined neutrosophic environment, thus we can establish the linguistic variables in terms of refined neutrosophic sets (LRNS) (see Table 7):

Table 7: Assessments of alternatives and criteria given by three decision makers in terms of linguistic variables

Alternatives	Decision Makers	C_1	C_2	C_3	C_4	C_5	C_6
L_1	K_1	EG	VG	G	EG	VG	G
	K_2	VG	G	G	EG	G	VG
	K_3	VG	VG	G	EG	VG	G
L_2	K_1	VG	VG	VG	VG	G	G
	K_2	VG	G	G	VG	VG	G
	K_3	VG	G	G	EG	G	G
L_3	K_1	EG	G	VG	VG	VG	VG
	K_2	VG	G	G	VG	G	VG
	K_3	VG	VG	G	VG	G	VG

Step 1: Construction of the decision matrix with linguistic refined neutrosophic sets

Three decision makers form decision matrix in terms of refined neutrosophic number corresponding to each logistic center. The decision matrices are described in Table 4, Table 5, and Table 6.

Table 8: Decision matrix for K_1

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	$\begin{pmatrix}(1.00, 1.00, \cdots \text{p times}),\\(0.00, 0.00, \cdots \text{p times}),\\(0.00, 0.00, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.90, 0.90, \cdots \text{p times}),\\(0.80, 0.80, \cdots \text{p times}),\\(0.08, 0.08, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.80, 0.80, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(1.00, 1.00, \cdots \text{p times}),\\(0.00, 0.00, \cdots \text{p times}),\\(0.00, 0.00, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.90, .090, \cdots \text{p times}),\\(0.80, 0.80, \cdots \text{p times}),\\(0.08, 0.08, \cdots, \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.80, 0.80, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times})\end{pmatrix}$
L_2	$\begin{pmatrix}(0.90, 0.90, \cdots \text{p times}),\\(0.80, 0.80, \cdots \text{p times}),\\(0.08, 0.08, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.80, 0.80, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.80, 0.80, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(1.00, 1.00, \cdots \text{p times}),\\(0.00, 0.00, \cdots \text{p times}),\\(0.00, 0.00, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.80, 0.80, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times}\end{pmatrix}$	$\begin{pmatrix}(0.90, 0.90, \cdots \text{p times}),\\(0.80, 0.80, \cdots \text{p times}),\\(0.08, 0.08, \cdots \text{p times})\end{pmatrix}$
L_3	$\begin{pmatrix}(0.90, 0.90, \cdots \text{p times}),\\(0.80, 0.80, \cdots \text{p times}),\\(0.08, 0.08, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.90, 0.90, \cdots \text{p times}),\\(0.80, 0.80, \cdots \text{p times}),\\(0.08, 0.08, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.80, 0.80, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(1.00, 1.00, \cdots \text{p times}),\\(0.00, 0.00, \cdots \text{p times}),\\(0.00, 0.00, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.90, 0.90, \cdots \text{p times}),\\(0.80, 0.80, \cdots \text{p times}),\\(0.08, 0.08, \cdots \text{p times})\end{pmatrix}$	$\begin{pmatrix}(0.80, 0.80, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times}),\\(0.20, 0.20, \cdots \text{p times})\end{pmatrix}$

Table 9: Decision matrix for *K*₂

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)
L_2	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times
L_3	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(1.00, 1.00, ⋯p times), (0.00, 0.00, ⋯p times), (0.00, 0.00, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)

Table 10: Decision matrix for *K*₃

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	(1.00, 1.00, ⋯p times), (0.00, 0.00, ⋯p times), (0.00, 0.00, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)
L_2	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)
L_3	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)	(0.80, 0.80, ⋯p times), (0.20, 0.20, ⋯p times), (0.20, 0.20, ⋯p times)	(0.90, 0.90, ⋯p times), (0.80, 0.80, ⋯p times), (0.08, 0.08, ⋯p times)

Step 2: Determination of the linguistic refined neutrosophic accumulated decision matrix

Form decision matrices (Table 4, Table 5 and Table 6), the aggregated transferred neutrosophic matrix for each alternative is formed by utilizing equation (1) and is presented in the Table 7, Table 8 and Table 9.

Table11: The linguistic refined neutrosophic accumulated decision matrix for decision maker *K*₁

$[\text{LRNAF}]_{3\times 6}^{K_1} =$

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	$\langle 1.00, 0.00, 0.00\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 1.00, 0.00, 0.00\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$
L_2	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 1.00, 0.00, 0.00\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.90, 0.08, 0.08\rangle$
L_3	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 1.00, 0.00, 0.00\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$

Table12: The linguistic refined neutrosophic accumulated decision matrix for decision maker *K*₂

$[\text{LRNAF}]_{3\times 6}^{K_2} =$

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.80, 0.20, 0.20\rangle$
L_2	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$
L_3	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 1.00, 0.00, 0.00\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.80, 0.20, 0.20\rangle$

Table13: The linguistic refined neutrosophic accumulated decision matrix for decision maker *K*₃

$[\text{LRNAF}]_{3\times 6}^{K_3} =$

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	$\langle 1.00, 0.00, 0.00\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$
L_2	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.90, 0.08, 0.08\rangle$
L_3	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.90, 0.08, 0.08\rangle$	$\langle 0.80, 0.20, 0.20\rangle$	$\langle 0.90, 0.08, 0.08\rangle$

Step 3: Determination of linguistic refined neutrosophic score matrix for decision makers

Using the equation (2), linguistic refined neutrosophic score matrix for alternative L_i (i = 1, 2, 3) is presented as follows (see Table 10, Table 11, and Table 12):

Table 14: Linguistic refined neutrosophic score matrix for decision maker K_1

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	1.00	0.91	0.80	1.00	0.91	0.80
L_2	0.91	0.80	0.80	1.00	0.80	0.91
L_3	0.91	0.91	0.80	1.00	0.91	0.80

Table15: Linguistic refined neutrosophic score matrix for decision maker K_2

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	0.91	0.91	0.91	0.91	0.80	0.80
L_2	0.91	0.80	0.80	0.91	0.91	0.80
L_3	0.91	0.80	0.80	1.00	0.80	0.80

Table 16: Linguistic refined neutrosophic score matrix for decision maker K_3

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	1.00	0.80	0.91	0.91	0.91	0.91
L_2	0.91	0.80	0.80	0.91	0.80	0.91
L_3	0.91	0.91	0.80	0.91	0.80	0.91

Step 4: Determination of geometric mean of score matrices for decision makers

Using equation (8), we calculate geometric mean of score values as follows.

Table 17: Geometric mean of score matrix for decision makers

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	0.9691	0.8717	0.8717	0.9391	0.8717	0.8351
L_2	0.9100	0.8000	0.8000	0.9391	0.8351	0.8717
L_3	0.9100	0.8717	0.8000	0.9691	0.8351	0.8351

Step 5: Determination of weights criteria

Using equation (11), weight structure is calculated as follows:

$w_1 = 0.15$, $w_2 = 0.20$, $w_3 = 0.15$, $w_4 = 0.20$, $w_5 = 0.10$ and $w_6 = 0.20$

Step 6: Determination of weighted accumulation score values (WASV)

Using equation (3), weighted accumulation score values (WASV) of all decision makers corresponding to each alternative is presented in Table 18.

Table 18: Weighted accumulated score matrix

	C_1	C_2	C_3	C_4	C_5	C_6
L_1	0.1454	0.1740	0.1308	0.1878	0.0872	0.1670
L_2	0.1365	0.1600	0.1200	0.1878	0.0835	0.1743
L_3	0.1365	0.1740	0.1200	0.1938	0.0835	0.1670

Step 7: Calculate extreme averaging score values

According to the weighted accumulated score values, extreme averaging score values (EASV) are calculated as follows.

EASV(L_1) = 0.8922, EASV(L_2) = 0.8548, EASV(L_3) = 0.8748;

Step 8: Rank the priority

All the extreme averaging score values are arranged in descending order. Alternatives then can be preference ranked as follows: EASV(L_1) > EASV(L_3) > EASV(L_2).

So, L_1 is the best potential spot to construct a modern sanitary system for students for Birnagar High School.

Step 9: End

5. CONCLUSION

Linguistic values are rational and direct tools for decision makers to express qualitative evaluations under uncertainty characterized by indeterminacy. We employed refined neutrosophic set to express linguistic variables. Linguistic refined neutrosophic set is proposed. We have developed a multi-criteria decision making method based on linguistic refined neutrosophic set. We also proposed an entropy method to determine unknown weights of the criteria in linguistic neutrosophic refined set environment. An illustrative example of constructional spot selection has also been provided. The proposed concept can be used other practical decision making problems such as medical diagnosis, cluster analysis, pattern recognition, etc.

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