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A novel approach to nano topology via neutrosophic sets

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Abstract: The main objective of this study is to introduce a new hybrid intelligent structure called Neutrosophic nano topology. Fuzzy nano topology and intuitionistic nano topology can also be deduced from the neutrosophic nano topology. Based on the neutrosophic nano approximations we have classified neutrosophic nano topology. Some properties like neutrosophic nano interior and neutrosophic nano closure are derived.
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1 INTRODUCTION

Nano topology explored by Thivagar et.al can be described as a collection of nano approximations, a non-empty finite universe and empty set for which equivalence classes are buliding blocks. It is named as nano topology because whatever may be the size of the universe it has at most five open sets. After this, there has been many models built upon different aspect, i.e, universe, relations, object and operators. One of the interesting generalizations of the theories of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F.Smarandache. Neutrosophic set is described by three functions : a membership function, indeterminacy function and a nonmembership function that are independently related. The theories of neutrosophic set have achieved greater success in various areas such as medical diagnosis, database, topology, image processing and decision making problem. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough set is a powerful mathematical tool to deal with incompleteness. Neutrosophic sets and rough sets are two different topics, none conflicts the other. The main objective of this study is to introduce a new hybrid intelligent structure called neutrosophic nano topology. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results. The rest of this paper is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3 , the concept of neutrosophic nano topology is investigated. Section 4 concludes the paper with some properties on neutrosophic nano interior and neutrosophic nano closure.

2 Preliminaries

The following recalls requisite ideas and preliminaries necessitated in the sequel of our work.

Definition 2.1 *[8]*: Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and R be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup\limits_{x \in \mathcal{U}} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup\limits_{x \in \mathcal{U}} \{R(x) : R(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Remark 2.2 *[8]*: If ($\mathcal{U}$, R) is an approximation space and X, $Y \subseteq \mathcal{U}$, then the following statements hold:

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(\mathcal{U}) = U_R(\mathcal{U}) = \mathcal{U}$.

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$.

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$.

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$.

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 *[8]*: Let $\mathcal{U}$ be an universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \phi, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:

(i) $\mathcal{U}$ and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $\mathcal{U}$ called the nano topology on $\mathcal{U}$ with respect to X. We call $(\mathcal{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

Proposition 2.4 *[8]*: Let $\mathcal{U}$ be a non-empty finite universe and $X \subseteq \mathcal{U}$. Then the following statements hold:

(i) If $L_R(X) = \phi$ and $U_R(X) = \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \phi\}$, is the indiscrete nano topology on $\mathcal{U}$.

(ii) If $L_R(X) = U_R(X) = X$, then the nano topology, $\tau_R(X) = \{\mathcal{U}, \phi, L_R(X)\}$.

(iii) If $L_R(X) = \phi$ and $U_R(X) \neq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \phi, U_R(X)\}$.

(iv) If $L_R(X) \neq \phi$ and $U_R(X) = \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \phi, L_R(X), B_R(X)\}$.

(v) If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \phi$ and $U_R(X) \neq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \phi, L_R(X), U_R(X), B_R(X)\}$ is the discrete nano topology on $\mathcal{U}$.

Definition 2.5 *[3]*: Let X be a non empty set. A fuzzy set A is an object having the form A $= \{< x : \mu_A(x), x \in X\}$, where $0 \leq \mu_A(x) \leq 1$ represent the degree of membership of each $x \in X$ to the set A.

Definition 2.6 *[2]*: Let X be a non empty set. An intuitionstic set A is of the form A $= \{< x : \mu_A(x), \nu_A(x), x \in X\}$, where $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership function and the degree of non membership respectively of each $x \in X$ to the set A and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

Definition 2.7 *[6]*: Let X be an universe of discourse with a generic element in X denoted by x, the neutrosophic set is an object having the form A $= \{< x : \mu_A(x), \sigma_A(x), \nu_A(x) >, x \in X\}$, where the functions $\mu, \sigma, \nu : X \to [0, 1]$ define respectively the degree of membership or truth , the degree of indeterminancy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition. $-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$.

3 Neutrosophic Nano Topological Space

In this section we introduce the notion of neutrosophic nano topology by means of nano neutrosophic nano approximations namely neutrosophic nano lower, neutrosophic nano upper and neutrosophic nano boundary. From Neutrosophic nano topology we have also defined and deduced intuitionistic nano topology and fuzzy nano topology.

Definition 3.1 : Let $\mathcal{U}$ be a non-empty set and R be an equivalence relation on $\mathcal{U}$. Let F be a neutrosophic set in $\mathcal{U}$ with the membership function μ_F, the indeterminancy function σ_F and the non-membership function ν_F. The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of F in the approximation $(\mathcal{U}, R)$ denoted by $\underline{N}(F), \overline{N}(F) and\ BN(F)$ are respectively defined as follows:

(i) $\underline{N}(F) = \{< x, \mu_{\underline{R}(A)}(x), \sigma_{\underline{R}(A)}(x), \nu_{\underline{R}(A)}(x) > /y \in [x]_R, x \in \mathcal{U}\}$.

(ii) $\overline{N}(F) = \{< x, \mu_{\overline{R}(A)}(x), \sigma_{\overline{R}(A)}(x), \nu_{\overline{R}(A)}(x) > /y \in [x]_R, x \in \mathcal{U}\}$.

(iii) BN(F)= $\overline{N}(F) - \underline{N}(F)$.

where $\mu_{\underline{R}(A)}(x) = \bigwedge_{y\in[x]_R} \mu_A(y)$, $\sigma_{\underline{R}(A)}(x) = \bigwedge_{y\in[x]_R} \sigma_A(y)$, $\nu_{\underline{R}(A)}(x) = \bigvee_{y\in[x]_R} \nu_A(y)$.

$\mu_{\overline{R}(A)}(x) = \bigvee_{y\in[x]_R} \mu_A(y)$, $\sigma_{\overline{R}(A)}(x) = \bigvee_{y\in[x]_R} \sigma_A(y)$, $\nu_{\overline{R}(A)}(x) = \bigwedge_{y\in[x]_R} \nu_A(y)$.

Definition 3.2 : Let $\mathcal{U}$ be an universe, R be an equivalence relation on $\mathcal{U}$ and F be a neutrosophic set in $\mathcal{U}$ and if the collection $\tau_N(F) = \{0_N, 1_N, \underline{N}(F), \overline{N}(F), BN(F)\}$ forms a topology then it is said to be a neutrosophic nano topology. We call $(\mathcal{U}, \tau_N(F))$ as the neutrosophic nano topological space. The elements of $\tau_N(F)$ are called neutrosophic nano open sets.

Remark 3.3 : From **Neutrosophic nano topology** we can deduce and define the **fuzzy nano topology** and **intuitionistic nano topology**. Fuzzy nano topology is obtained by considering the membership values alone whereas in case of intuitionistic nano topology both membership and non member ship values are considered.

Definition 3.4 : Let $\mathcal{U}$ be a non-empty set and R be an equivalence relation on $\mathcal{U}$. Let F be an intuitionistic set in $\mathcal{U}$ with the membership function μ_F and the non-membership function ν_F. The intuitionistic nano lower, intuitionistic nano upper approximation and intuitionistic nano boundary of F in the approximation $(\mathcal{U}, R)$ denoted by $\underline{I}(F), \overline{I}(F) and\ B_I(F)$ are respectively defined as follows:

(i) $\underline{I}(F) = \{< x, \mu_{\underline{R}(A)}(x), \nu_{\underline{R}(A)}(x) > / y \in [x]_R, x \in \mathcal{U}\}$.

(ii) $\overline{I}(F) = \{< x, \mu_{\overline{R}(A)}(x), \nu_{\overline{R}(A)}(x) > / y \in [x]_R, x \in \mathcal{U}\}$.

(iii) $B_I(F) = \overline{I}(F) - \underline{I}(F)$.

where $\mu_{\underline{R}_{\mathcal{I}}(A)}(x) = \bigwedge_{y\in[x]_R} \mu_A(y)$, $\nu_{\underline{R}_{\mathcal{I}}(A)}(x) = \bigvee_{y\in[x]_R} \nu_A(y)$.

$\mu_{\overline{R}(A)}(x) = \bigvee_{y\in[x]_R} \mu_A(y)$, $\nu_{\overline{R}_{\mathcal{I}}(A)}(x) = \bigwedge_{y\in[x]_R} \nu_A(y)$.

Definition 3.5 : Let $\mathcal{U}$ be an universe, R be an equivalence relation on $\mathcal{U}$ and F be an intuitionistic set in $\mathcal{U}$ and if the collection $\tau_I(F) = \{0_N, 1_N, \underline{I}(F), \overline{I}(F), B_I(F)\}$ forms a topology then it is said to be a intuitionistic nano topology. We call $(\mathcal{U}, \tau_I(F))$ as the intuitionistic nano topological space. The elements of $\tau_I(F)$ are called intuitionistic nano open sets.

Definition 3.6 : Let $\mathcal{U}$ be a non-empty set and R be an equivalence relation on $\mathcal{U}$. Let F be a fuzzy set in $\mathcal{U}$ with the membership function μ_F. Then the fuzzy nano lower, fuzzy nano upper approximation of F and fuzzy nano boundary of F in the approximation $(\mathcal{U}, R)$ denoted by $\underline{\mathcal{F}}(F), \overline{\mathcal{F}}(F) and\ B_{\mathcal{F}}(F)$ are respectively defined as follows:

(i) $\underline{\mathcal{F}}(F) = \{< x, \mu_{\underline{R}(A)}(x) > / y \in [x]_R, x \in \mathcal{U}\}$.

(ii) $\overline{\mathcal{F}}(F) = \{< x, \mu_{\overline{R}(A)}(x) > / y \in [x]_R, x \in \mathcal{U}\}$.

(iii) $B_{\mathcal{F}}(F) = \overline{\mathcal{F}}(F) - \underline{\mathcal{F}}(F)$.

where $\mu_{\underline{R}(A)}(x) = \bigwedge_{y\in[x]_R} \mu_A(y)$, $\mu_{\overline{R}(A)}(x) = \bigvee_{y\in[x]_R} \mu_A(y)$

Definition 3.7 : Let $\mathcal{U}$ be an universe, R be an equivalence relation on $\mathcal{U}$ and F be a fuzzy set in $\mathcal{U}$ and if the collection $\tau_{\mathcal{F}}(F) = \{0_N, 1_N, \underline{\mathcal{F}}(F), \overline{\mathcal{F}}(F), B_{\mathcal{F}}(F)\}$ forms a topology then it is said to be a fuzzy nano topology. We call $(\mathcal{U}, \tau_{\mathcal{F}}(F))$ as the fuzzy nano topological space. The elements of $\tau_{\mathcal{F}}(F)$ are called fuzzy nano open sets.

Remark 3.8 : Thus from the above definitions of intuitionistic and fuzzy nano topologies we can assure that throughout this paper all the properties and examples also holds good when it is possible for neutrosophic nano topology.

Remark 3.9 : Since our main purpose is to construct tools for developing neutrosophic nano topological spaces, we must introduce 0_N , 1_N and certain neutrosophic set operations in X as follows:

Definition 3.10 : Let $\mathcal{U}$ be a nonempty set and the neutrosophic sets A and B in the form A $= \{< x : \mu_A(x), \sigma_A(x), \nu_A(x) >, x \in \mathcal{U}\}$, B $= \{< x : \mu_B(x), \sigma_B(x), \nu_B(x) >, x \in \mathcal{U}\}$. Then the following statements hold:

(i) $0_N = \{< x, 0, 0, 1 >: x \in \mathcal{U}\}$ and $1_N = \{< x, 1, 1, 0 >: x \in \mathcal{U}\}$.

(ii) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \nu_A(x) \geq \nu_B(x) for\ all\ x \in \mathcal{U}\}$.

(iii) A $=$ B iff $A \subseteq B$and $B \subseteq A$.

(iv) $A^C = \{< x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x) >: x \in \mathcal{U}\}$.

(v) $A \cap B = \{x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x) for\ all\ x \in \mathcal{U}\}$.

(vi) $A \cup B = \{x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x) for\ all\ x \in \mathcal{U}\}$.

Theorem 3.11 *[8]*: Let $\mathcal{U}$ be a non-empty finite universe and $X \subseteq \mathcal{U}$. Let $\tau_R(X)$ be the nano topology on $\mathcal{U}$ with respect to X. Then $[\tau_R(X)]^C$, whose elements are A^C for $A \in \tau_R(X)$, is a topology on $\mathcal{U}$.

Remark 3.12 : $[\tau_N(F)]^C$ is called the dual neutrosophic nano topology of $\tau_N(F)$. Elements of $[\tau_N(F)]^C$ are called neutrosophic nano closed sets. Thus, we note that a neutrosophic set N(G) of $\mathcal{U}$ is neutrosophic nano closed in $\tau_N(F)$ if and only if $\mathcal{U} - N(G)$ is neutrosophic nano open in $\tau_N(F)$.

Example 3.13 : Let $\mathcal{U} = \{p_1, p_2, p_3\}$ be the universe of discourse. Let $\mathcal{U}/R = \{\{p_1, p_2\}, \{p_3\}\}$ be an equivalence relation on $\mathcal{U}$ and $A = \{< p_1, (0.7, 0.6, 0.5) >, < p_2, (0.3, 0.4, 0.5) >, < p_3, (0.1, 0.5, 0.1) >\}$ be a neutrosophic set on $\mathcal{U}$ then $\underline{N}(A) = \{< p_1, (0.3, 0.4, 0.5) >, < p_2, (0.3, 0.4, 0.5) >, < p_3, (0.1, 0.5, 0.1) >\}$, $\overline{N}(A) = \{< p_1, (0.7, 0.6, 0.5) >, < p_2, (0.7, 0.6, 0.5) >, < p_3, (0.1, 0.5, 0.1) >\}$, $B(A) = \{< p_1, (0.5, 0.6, 0.5) >, < p_2, (0.5, 0.6, 0.5) >, < p_3, (0.1, 0.5, 0.1) >\}$. Then the collection $\tau_N(A) = \{0_N, 1_N, \{< p_1, (0.3, 0.4, 0.5) >, < p_2, (0.3, 0.4, 0.5) >, < p_3, (0.1, 0.5, 0.1) >\}, \{< p_1, (0.7, 0.6, 0.5) >, < p_2, (0.7, 0.6, 0.5) >, < p_3, (0.1, 0.5, 0.1) >\}, \{< p_1, (0.5, 0.6, 0.5) >, < p_2, (0.5, 0.6, 0.5) >, < p_3, (0.1, 0.5, 0.1) >\}\}$ is a neutrosophic nano topology on $\mathcal{U}$ and $[\tau_N(A)]^C$ is also a neutrosophic nano topology on $\mathcal{U}$. Thus $\tau_{\mathcal{I}}(A) = \{0_N, 1_N, \{< p_1, (0.3, 0.5) >, < p_2, (0.3, 0.5) >, < p_3, (0.1, 0.1) >\}, \{< p_1, (0.7, 0.5) >, < p_2, (0.7, 0.5) >, < p_3, (0.1, 0.1) >\}, \{< p_1, (0.5, 0.5) >, < p_2, (0.5, 0.5) >, < p_3, (0.1, 0.1) >\}\}$ and $\tau_{\mathcal{F}}(A) = \{0_N, 1_N, \{< p_1, (0.3) >, < p_2, (0.3) >, < p_3, (0.1) >\}, \{< p_1, (0.7) >, < p_2, (0.7) >, < p_3, (0.1) >\}, \{< p_1, (0.5) >, < p_2, (0.5) >, < p_3, (0.1) >\}\}$ are the intuitionistic nano topology and fuzzy nano topology.

Remark 3.14 : In neutrosophic nano topological space, the neutrosophic nano boundary cannot be empty. Since the difference between neutrosophic nano upper and neutrosophic nano lower approximations is defined here as the maximum and minimum of the values in the neutrosophic sets.

Proposition 3.15 : Let $\mathcal{U}$ be a non-empty finite universe and F be a neutrosophic set on $\mathcal{U}$. Then the following statements hold:

(i) The collection $\tau_N(F) = \{0_N, 1_N\}$, is the indiscrete neutrosophic nano topology on $\mathcal{U}$.

(ii) If $\underline{N}(F) = \overline{N}(F) = N(F)$, then the neutrosophic nano topology, $\tau_N(F) = \{0_N, 1_N, \underline{N}(F), BN(F)\}$.

(iii) If $\underline{N}(F) = BN(F)$, then $\tau_N(F) = \{0_N, 1_N, \underline{N}(F), \overline{N}(F)\}$ is a neutrosophic nano topology

(iv) If $\overline{N}(F) = BN(F)$ then $\tau_N(F) = \{0_N, 1_N, \overline{N}(F), BN(F)\}$.

(v) The collection $\tau_N(F) = \{0_N, 1_N, \underline{N}(F), \overline{N}(F), BN(F)\}$ is the discrete neutrosophic nano topology on $\mathcal{U}$.

4 Neutrosophic nano closure and interior

In this section we have defined neutrosophic nano closure and neutrosophic nano interior on neutrosophic nano topological space. Based on this we also prove some properties.

Definition 4.1 : If $(\mathcal{U}, \tau_N(F))$ is a neutrosophic nano topological space with respect to neutrosophic subset of $\mathcal{U}$ and if A be any neutrosophic subset of $\mathcal{U}$, then the neutrosophic nano interior of A is defined as the union of all neutrosophic nano open subsets of A and it is denoted by $N_{\mathcal{F}}int(A)$. That is, $N_{\mathcal{F}}int(A)$ is the largest neutrosophic nano open subset of A. The neutrosophic nano closure of A is defined as the intersection of all neutrosophic nano closed sets containing A and it is denoted by $N_{\mathcal{F}}cl(A)$. That is, $N_{\mathcal{F}}cl(A)$ is the smallest neutrosophic nano closed set containing A.

Remark 4.2 : Let $(\mathcal{U}, \tau_N(F))$ be a neutrosophic nano topological space with respect to F where F is a neutrosophic subset of $\mathcal{U}$. The neutrosophic nano closed sets in $\mathcal{U}$ are 0_N,1_N, $(\underline{N}(F))^C$, $(\overline{N}(F))^C$ and $(B_N(F))^C$.

Theorem 4.3 *[8]*: Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space with respect to $X \subseteq \mathcal{U}$ then $\mathcal{N}cl(X) = \mathcal{U}$.

Remark 4.4 : The above theorem need not be true for all neutrosophic nano topological space $(\mathcal{U}, \tau_N(F))$ with respect to F where F is a neutrosophic subset of $\mathcal{U}$. That is $N_{\mathcal{F}}cl(A)$ *need not be equal to* $\mathcal{U}$ which can be shown by the following example.

Example 4.5 : Let $\mathcal{U} = \{p_1, p_2, p_3, p_4, p_5\}$ be the universe of discourse. Let $\mathcal{U}/R = \{\{p_1, p_4\}, \{p_2, p_3\}, \{p_5\}\}$ be an equivalence relation on $\mathcal{U}$ and $A = \{< p_1, (0.2, 0.3, 0.4) >, < p_4, (0.2, 0.3, 0.4) >, < p_5, (0.4, 0.6, 0.2) >\}$ be a neutrosophic set on $\mathcal{U}$. Then $\underline{N}(A) = \{< p_1, (0.2, 0.3, 0.4) >, < p_4, (0.2, 0.3, 0.4) >, < p_5, (0.4, 0.6, 0.2) >\}$, $\overline{N}(A) = \{< p_1, (0.2, 0.3, 0.4) >, < p_4, (0.2, 0.3, 0.4) >, < p_5, (0.4, 0.6, 0.2) >\}$ $B(A) = \{< p_1, (0.2, 0.3, 0.4) >, < p_4, (0.2, 0.3, 0.4) >, < p_5, (0.2, 0.4, 0.4) >\}$. Now we have $\tau_N(A) = \{0_N, 1_N, \{< p_1, (0.2, 0.3, 0.4) >, < p_4, (0.2, 0.3, 0.4) >, < p_5, (0.4, 0.6, 0.2) >\}, \{< p_1, (0.2, 0.3, 0.4) >, < p_4, (0.2, 0.3, 0.4) >, < p_5, (0.2, 0.4, 0.4) >\}\}$ which is a neutrosophic nano topology on $\mathcal{U}$. $[\tau_N(A)]^c = \{0_N, 1_N, \{< p_1, (0.2, 0.3, 0.4) >, < p_4, (0.2, 0.3, 0.4) >, < p_5, (0.4, 0.6, 0.2) >\}, \{< p_1, (0.2, 0.3, 0.4) >, < p_4, (0.2, 0.3, 0.4) >, < p_5, (0.2, 0.4, 0.4) >\}$. Here $N_{\mathcal{F}}cl(A) \neq \mathcal{U}$

Theorem 4.6 : Let $(\mathcal{U}, \tau_N(F))$ be a neutrosophic nano topological space with respect to F where F is a neutrosophic subset of $\mathcal{U}$. Let A and B be neutrosophic subsets of $\mathcal{U}$. Then the following statements hold:

(i) $A \subseteq N_{\mathcal{F}}cl(A)$.

(ii) A is nano closed if and only if $N_{\mathcal{F}}cl(A) = A$.

(iii) $N_{\mathcal{F}}cl(0_N) = 0_N$ and $N_{\mathcal{F}}cl(1_N) = 1_N$.

(iv) $A \subseteq B \Rightarrow N_{\mathcal{F}}cl(A) \subseteq N_{\mathcal{F}}cl(B)$.

(v) $N_{\mathcal{F}}cl(A \cup B) = N_{\mathcal{F}}cl(A) \cup N_{\mathcal{F}}cl(B)$.

(vi) $N_{\mathcal{F}}cl(A \cap B) \subseteq N_{\mathcal{F}}cl(A) \cap N_{\mathcal{F}}cl(B)$.

(vii) $N_{\mathcal{F}}cl(N_{\mathcal{F}}cl(A)) = N_{\mathcal{F}}cl(A)$.

Proof:

(i) By definition of neutrosophic nano closure, $A \subseteq N_{\mathcal{F}}cl(A)$.

(ii) If A is neutrosophic nano closed, then A is the smallest neutrosophic nano closed set containing itself and hence $N_{\mathcal{F}}$cl(A) = A. Conversely, if $N_{\mathcal{F}}$cl(A) = A, then A is the smallest neutrosophic nano closed set containing itself and hence A is neutrosophic nano closed.

(iii) Since 0_N and 1_N are neutrosophic nano closed in $(\mathcal{U}, \tau_N(F))$, $N_{\mathcal{F}}cl(0_N) = 0_N$ and $N_{\mathcal{F}}cl(1_N) = 1_N$.

(iv) If $A \subseteq B$, since $B \subseteq N_{\mathcal{F}}cl(B)$, then $A \subseteq N_{\mathcal{F}}cl(B)$. That is, $N_{\mathcal{F}}$cl(B) is a Neutrosophic nano closed set containing A. But $N_{\mathcal{F}}$cl(A) is the smallest Neutrosophic nano closed set containing A. Therefore, $N_{\mathcal{F}}cl(A) \subseteq N_{\mathcal{F}}cl(B)$.

(v) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, $N_{\mathcal{F}}cl(A) \subseteq N_{\mathcal{F}}cl(A \cup B)$ and $N_{\mathcal{F}}cl(B) \subseteq N_{\mathcal{F}}cl(A \cup B)$. Therefore, $N_{\mathcal{F}}cl(A) \cup N_{\mathcal{F}}cl(B) \subseteq N_{\mathcal{F}}cl(A \cup B)$. By the fact that $A \cup B \subseteq N_{\mathcal{F}}cl(A) \cup N_{\mathcal{F}}cl(B)$, and since $N_{\mathcal{F}}cl(A \cup B)$ is the smallest nano closed set containing $A \cup B, soN_{\mathcal{F}}cl(A \cup B) \subseteq N_{\mathcal{F}}cl(A) \cup N_{\mathcal{F}}cl(B)$. Thus, $N_{\mathcal{F}}cl(A \cup B) = N_{\mathcal{F}}cl(A) \cup N_{\mathcal{F}}cl(B)$.

(vi) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B, N_{\mathcal{F}}cl(A \cap B) \subseteq N_{\mathcal{F}}cl(A) \cap N_{\mathcal{F}}cl(B)$.

(vii) Since $N_{\mathcal{F}}cl(A)$ is nano closed, $N_{\mathcal{F}}cl(N_{\mathcal{F}}cl(A)) = N_{\mathcal{F}}cl(A)$.

Theorem 4.7 : $(\mathcal{U}, \tau_N(F))$ be a neutrosophic nano topological space with respect to F where F is a neutrosophic subset of $\mathcal{U}$. Let A be a neutrosophic subset of $\mathcal{U}$. Then

(i) $1_N - N_{\mathcal{F}}Int(A) = N_{\mathcal{F}}cl(1_N - A)$.

(ii) $1_N - N_{\mathcal{F}}cl(A) = N_{\mathcal{F}}Int(1_N - A)$.

Remark 4.8 : Taking complements on either side of(i) and (ii) Theorem 4.8, we get $(N_{\mathcal{F}}Int(A)) = 1_N - N_{\mathcal{F}}cl(1_N - A))$ and $(N_{\mathcal{F}}cl(A)) = 1_N - (N_{\mathcal{F}}Int(1_N - A))$.

Example 4.9 : Let $\mathcal{U} = \{a, b, c\}$ and $\mathcal{U}/R = \{\{a, b\}, \{c\}\}$. Let $F = \{< a, (0.4, 0.5, 0.5) >, < b, (0.4, 0.5, 0.5) >, < c, (0.5, 0.5, 0.5) >\}$ be a neutrosophic set on $\mathcal{U}$ then the $\tau_N(A) = \{0_N, 1_N, \{< a, (0.4, 0.5, 0.5) >, < b, (0.4, 0.5, 0.5) >, < c, (0.5, 0.5, 0.5) >\}\}$ is a neutrosophic nano topology on $\mathcal{U}$. $[\tau_N(A)]^c = \{0_N, 1_N, \{< a, (0.5, 0.5, 0.4) >, < b, (0.5, 0.5, 0.4) >, < c, (0.5, 0.5, 0.5) >\}\}$. If $A = \{< a, (0.7, 0.6, 0.5) >, < b, (0.3, 0.4, 0.5) >, < c, (0.7, 0.5, 0.5) > \}$, then $(N_{\mathcal{F}}Int(A))^C = 1_N \ N_{\mathcal{F}}cl(1_N - A) = 1_N$. That is, $1_N - N_{\mathcal{F}}Int(A) = N_{\mathcal{F}}cl(1_N - A)$ Also, $1_N - N_{\mathcal{F}}cl(A) = \mathcal{N}_{\mathcal{F}}Int(1_N - A) = 0_N$

Theorem 4.10 : Let $(\mathcal{U}, \tau_N(F))$ be a neutrosophic nano topological space with respect to F where F is a neutrosophic subset of $\mathcal{U}$. Let A and B be neutrosophic subsets of $\mathcal{U}$, then the following statements hold:

(i) A is neutrosophic nano open if and only if $N_{\mathcal{F}}Int(A) = A$.

(iii) $N_{\mathcal{F}}Int(0_N) = 0_N$ and $N_{\mathcal{F}}Int(1_N) = 1_N$.

(iv) $A \subseteq B \Rightarrow N_{\mathcal{F}}Int(A) \subseteq N_{\mathcal{F}}Int(B)$.

(v) $N_{\mathcal{F}}Int(A) \cup N_{\mathcal{F}}Int(B) \subseteq N_{\mathcal{F}}Int(A \cup B)$.

(vi) $N_{\mathcal{F}}Int(A \cap B) = N_{\mathcal{F}}Int(A) \cap N_{\mathcal{F}}Int(B)$.

(vii) $N_{\mathcal{F}}Int(N_{\mathcal{F}}Int(A)) = N_{\mathcal{F}}Int(A)$.

Proof:

(i) A is neutrosophic nano open if and only if $1_N - A$ is neutrosophic nano closed, if and only if $N_{\mathcal{F}}cl(1_N - A) = 1_N - A$, if and only if $1_N - N_{\mathcal{F}}cl(1_N - A) = A$ if and only if $N_{\mathcal{F}}Int(A) = A$, by Remark 4.8.

(ii) Since 0_N and 1_N are neutrosophic nano open, $N_{\mathcal{F}}Int(0_N) = 0_N$ and $N_{\mathcal{F}}Int(1_N) = 1_N$.

(iii) $A \subseteq B \Rightarrow 1_N - B \subseteq 1_N - A$. Therefore, $N_{\mathcal{F}}cl(1_N - B) \subseteq N_{\mathcal{F}}cl(1_N - A)$. That is, $1_N - N_{\mathcal{F}}cl(1_N - A) \subseteq 1_N - N_{\mathcal{F}}cl(1_N - B)$. That is, $N_{\mathcal{F}}IntA \subseteq N_{\mathcal{F}}IntB$.

Proof of (iv), (v) and (vi) follow similarly from Theorem 4.7 and Remark 4.8.
Conclusion: Neutrosophic set is a general formal framework, which generalizes the concept of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and interval intuitionistic fuzzy set. Since the world is full of indeterminacy, the neutrosophic nano topology found its place into contemporary research world. This paper can be further developed into several possible such as Geographical Information Systems (GIS) field including remote sensing, object reconstruction from airborne laser scanner, real time tracking, routing applications and modeling cognitive agents. In GIS there is a need to model spatial regions with indeterminate boundary and under indeterminacy. Hence this neutrosophic nano topological spaces can also be extended to a neutrosophic spatial region.

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