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On Neutrosophic Soft Topological Space

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Abstract: In this paper, the concept of connectedness and compactness on neutrosophic soft topological space have been introduced along with the investigation of their several characteristics. Some related theorems have been established also. Then, the notion of neutrosophic soft continuous mapping on a neutrosophic soft topological space and it's properties are developed here.

Keywords : Connectedness and compactness on neutrosophic soft topological space, Neutrosophic soft continuous mapping.

1 Introduction

Zadeh's [1] classical concept of fuzzy sets is a strong mathematical tool to deal with the complexity generally arising from uncertainty in the form of ambiguity in real life scenario. Researchers in economics, sociology, medical science and many other several fields deal daily with the vague, imprecise and occasionally insufficient information of modeling uncertain data. For different specialized purposes, there are suggestions for nonclassical and higher order fuzzy sets since from the initiation of fuzzy set theory. Among several higher order fuzzy sets, intuitionistic fuzzy sets introduced by Atanassov [2] have been found to be very useful and applicable. But each of these theories has it's different difficulties as pointed out by Molodtsov [3]. The basic reason for these difficulties is inadequacy of parametrization tool of the theories.

Molodtsov [3] presented soft set theory as a completely generic mathematical tool which is free from the parametrization inadequacy syndrome of different theory dealing with uncertainty. This makes the theory very convenient, efficient and easily applicable in practice. Molodtsov [3] successfully applied several directions for the applications of soft set theory, such as smoothness of functions, game theory, operation reaserch, Riemann integration, Perron integration and probability etc. Now, soft set theory and it's applications are progressing rapidly in different fields. Shabir and Naz [4] presented soft topological spaces and defined some concepts of soft sets on this spaces and separation axioms. Moreover, topological structure on fuzzy, fuzzy soft, intuitionistic fuzzy and intuitionistic fuzzy soft set was defined by Coker [5], Li and Cui [6], Chang [7], Tanay and Kandemir [8], Osmanoglu and Tokat [9], Neog et al. [10], Varol and Aygun [11], Bayramov and Gunduz [12,13]. Turanh and Es [14] defined compactness in intuitionistic fuzzy soft topological spaces.

The concept of **Neutrosophic Set (NS)** was first introduced by Smarandache [15,16] which is a generalisation of classical sets, fuzzy set, intuitionistic fuzzy set etc. Later, Maji [17] has introduced a combined concept **Neutrosophic soft set (NSS)**. Using this concept, several mathematicians have produced their research works in different mathematical structures for instance Arockiarani et al.[18,19], Bera and Mahapatra [20], Deli [21,22], Deli and Broumi [23], Maji [24], Broumi and Smarandache [25], Salama and Alblowi [26], Saroja and Kalaichelvi [27], Broumi [28], Sahin et al.[29]. Later, this concept has been modified by Deli and Broumi [30]. Accordingly, Bera and Mahapatra [31-36] have developed some algebraic structures over the neutrosophic soft set.

The present study introduces the notion of connectedness, compactness and neutrosophic soft continuous mapping on a neutrosophic soft topological space. Section 2 gives some preliminary necessary definitions which will be used in rest of this paper. The notion of connectedness and compactness on neutrosophic soft topological spaces along with investigation of related properties have been introduced in Section 3 and Section 4, respectively. The concept of neutrosophic soft continuous mapping has been developed in Section 5. Finally, the conclusion of the present work has been stated in Section 6.

2 Preliminaries

In this section, we recall some necessary definitions and theorems related to fuzzy set, soft set, neutrosophic set, neutrosophic soft set, neutrosophic soft topological space for the sake of completeness.
Unless otherwise stated, E is treated as the parametric set through out this paper and $e \in E$, an arbitrary parameter.

2.1 Definition [31]

1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t - norm if $*$ satisfies the following conditions :
(i) $*$ is commutative and associative.
(ii) $*$ is continuous.

(iii) $a * 1 = 1 * a = a,\ \forall a \in [0, 1]$.
(iv) $a * b \leq c * d$ if $a \leq c,\ b \leq d$ with $a, b, c, d \in [0, 1]$.

A few examples of continuous t-norm are $a * b = ab, a * b = \min\{a, b\}, a * b = \max\{a + b - 1, 0\}$.

2. A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t - conorm (s - norm) if $\diamond$ satisfies the following conditions :

(i) $\diamond$ is commutative and associative.
(ii) $\diamond$ is continuous.
(iii) $a \diamond 0 = 0 \diamond a = a,\ \forall a \in [0, 1]$.
(iv) $a \diamond b \leq c \diamond d$ if $a \leq c,\ b \leq d$ with $a, b, c, d \in [0, 1]$.

A few examples of continuous s-norm are $a \diamond b = a + b - ab, a \diamond b = \max\{a, b\}, a \diamond b = \min\{a + b, 1\}$.

2.2 Definition [15]

Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. That is $T_A, I_A, F_A : X \to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$ and so, $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

2.3 Definition [3]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F : A \to P(U)$ is a mapping.

2.4 Definition [17]

Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then for $A \subseteq E$, a pair (F, A) is called an NSS over U, where $F : A \to NS(U)$ is a mapping.

This concept has been modified by Deli and Broumi [30] as given below.

2.5 Definition [30]

Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping $f_N : E \to NS(U)$ where f_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $NS(U)$ and therefore it can be written as a set of ordered pairs,

$$N = \{(e, \{< x, T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) >: x \in U\}) : e \in E\}$$

where $T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) \in [0, 1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $f_N(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{f_N(e)}(x) + I_{f_N(e)}(x) + F_{f_N(e)}(x) \leq 3$ is obvious.

2.5.1 *Example*

Let $U = \{h_1, h_2, h_3\}$ be a set of houses and $E = \{e_1(\text{beautiful}), e_2(\text{wooden}), e_3(\text{costly})\}$ be a set of parameters with respect to which the nature of houses are described. Let,

$$f_N(e_1) = \{< h_1, (0.5, 0.6, 0.3) >, < h_2, (0.4, 0.7, 0.6) >, < h_3, (0.6, 0.2, 0.3) >\};$$
$$f_N(e_2) = \{< h_1, (0.6, 0.3, 0.5) >, < h_2, (0.7, 0.4, 0.3) >, < h_3, (0.8, 0.1, 0.2) >\};$$
$$f_N(e_3) = \{< h_1, (0.7, 0.4, 0.3) >, < h_2, (0.6, 0.7, 0.2) >, < h_3, (0.7, 0.2, 0.5) >\};$$

Then $N = \{[e_1, f_N(e_1)], [e_2, f_N(e_2)], [e_3, f_N(e_3)]\}$ is an NSS over (U, E). The tabular representation of the NSS N is as :

Table 1 : Tabular form of NSS N.

	$f_N(e_1)$	$f_N(e_2)$	$f_N(e_3)$
h_1	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)
h_2	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)
h_3	(0.6,0.2,0.3)	(0.8,0.1,0.2)	(0.7,0.2,0.5)

2.6 Definition [30]

1. The complement of a neutrosophic soft set N is denoted by N^c and is defined by

$$N^c = \{(e, \{< x, F_{f_N(e)}(x), 1 - I_{f_N(e)}(x), T_{f_N(e)}(x) >: x \in U\}) : e \in E\}$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then N_1 is said to be the neutrosophic soft subset of N_2 if $\forall e \in E$ and $\forall x \in U$,

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x),\ I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x),\ F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x).$$

We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

2.7 Definition [30]

1. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined as :

$$N_3 = \{(e, \{< x, T_{f_{N_3}(e)}(x), I_{f_{N_3}(e)}(x), F_{f_{N_3}(e)}(x) >: x \in U\}) : e \in E\}$$

where $T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) \diamond T_{f_{N_2}(e)}(x), I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) * I_{f_{N_2}(e)}(x), F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) * F_{f_{N_2}(e)}(x)$.

2. Their intersection is denoted by $N_1 \cap N_2 = N_4$ and is defined as :

$$N_4 = \{(e, \{< x, T_{f_{N_4}(e)}(x), I_{f_{N_4}(e)}(x), F_{f_{N_4}(e)}(x) >: x \in U\}) : e \in E\}$$

where $T_{f_{N_4}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x), I_{f_{N_4}(e)}(x) = I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x), F_{f_{N_4}(e)}(x) = F_{f_{N_1}(e)}(x) \diamond F_{f_{N_2}(e)}(x)$.

2.8 Definition [33]

1. Let M, N be two NSSs over (U, E). Then $M - N$ may be defined as, $\forall x \in U,\ e \in E$,

$$M - N = \{< x, T_{f_M(e)(x)} * F_{f_N(e)(x)}, I_{f_M(e)(x)} \diamond (1 - I_{f_N(e)}(x)), F_{f_M(e)}(x) \diamond T_{f_N(e)}(x) >\}$$

2. A neutrosophic soft set N over (U, E) is said to be null neutrosophic soft set if $T_{f_N(e)}(x) = 0, I_{f_N(e)}(x) = 1, F_{f_N(e)}(x) = 1, \forall e \in E, \forall x \in U$. It is denoted by ϕ_u.

A neutrosophic soft set N over (U, E) is said to be absolute neutrosophic soft set if $T_{f_N(e)}(x) = 1, I_{f_N(e)}(x) = 0, F_{f_N(e)}(x) = 0, \forall e \in E, \forall x \in U$. It is denoted by 1_u.

Clearly, $\phi_u^c = 1_u$ and $1_u^c = \phi_u$.

2.9 Definition [33]

Let *NSS*(U, E) be the family of all neutrosophic soft sets over U via parameters in E and $\tau_u \subset NSS(U, E)$. Then τ_u is called neutrosophic soft topology on (U, E) if the following conditions are satisfied.

(i) $\phi_u, 1_u \in \tau_u$
(ii) the intersection of any finite number of members of τ_u also belongs to τ_u.
(iii) the union of any collection of members of τ_u belongs to τ_u.

Then the triplet (U, E, τ_u) is called a neutrosophic soft topological space. Every member of τ_u is called τ_u-open neutrosophic soft set. An NSS is called τ_u-closed iff it's complement is τ_u-open. There may be a number of topologies on (U, E). If τ_{u^1} and τ_{u^2} are two topologies on (U, E) such that $\tau_{u^1} \subset \tau_{u^2}$, then τ_{u^1} is called neutrosophic soft strictly weaker (coarser) than τ_{u^2} and in that case τ_{u^2} is neutrosophic soft strict finer than τ_{u^1}. Moreover *NSS*(U, E) is a neutrosophic soft topology on (U, E).

2.9.1 *Example*

1. Let $U = \{h_1, h_2\}, E = \{e_1, e_2\}$ and $\tau_u = \{\phi_u, 1_u, N_1, N_2, N_3, N_4\}$ where N_1, N_2, N_3, N_4 being NSSs are defined as following :

$$\begin{aligned} f_{N_1}(e_1) &= \{< h_1, (1,0,1) >, < h_2, (0,0,1) >\}, \\ f_{N_1}(e_2) &= \{< h_1, (0,1,0) >, < h_2, (1,0,0) >\}; \end{aligned}$$

$$\begin{aligned} f_{N_2}(e_1) &= \{< h_1, (0,1,0) >, < h_2, (1,1,0) >\}, \\ f_{N_2}(e_2) &= \{< h_1, (1,0,1) >, < h_2, (0,1,1) >\}; \end{aligned}$$

$$\begin{aligned} f_{N_3}(e_1) &= \{< h_1, (1,1,1) >, < h_2, (0,1,1) >\}, \\ f_{N_3}(e_2) &= \{< h_1, (0,1,0) >, < h_2, (0,1,1) >\}; \end{aligned}$$

$$\begin{aligned} f_{N_4}(e_1) &= \{< h_1, (1,1,0) >, < h_2, (1,1,0) >\}, \\ f_{N_4}(e_2) &= \{< h_1, (1,0,0) >, < h_2, (0,1,1) >\}; \end{aligned}$$

Here $N_1 \cap N_1 = N_1, N_1 \cap N_2 = \phi_u, N_1 \cap N_3 = N_3, N_1 \cap N_4 = N_3, N_2 \cap N_2 = N_2, N_2 \cap N_3 = \phi_u, N_2 \cap N_4 = N_2, N_3 \cap N_3 = N_3, N_3 \cap N_4 = N_3, N_4 \cap N_4 = N_4$ and $N_1 \cup N_1 = N_1, N_1 \cup N_2 = 1_u, N_1 \cup N_3 = N_1, N_1 \cup N_4 = 1_u, N_2 \cup N_2 = N_2, N_2 \cup N_3 = N_4, N_2 \cup N_4 = N_4, N_3 \cup N_3 = N_3, N_3 \cup N_4 = N_4, N_4 \cup N_4 = N_4$;

Corresponding t-norm and s-norm are defined as $a * b = \max\{a + b - 1, 0\}$ and $a \diamond b = \min\{a + b, 1\}$. Then τ_u is a neutrosophic soft topology on (U, E) and so (U, E, τ_u) is a neutrosophic soft topological space over (U, E).

2. Let $U = \{x_1, x_2, x_3\},\ E = \{e_1, e_2\}$ and $\tau_u = \{\phi_u, 1_u, N_1, N_2, N_3\}$ where N_1, N_2, N_3 being NSSs over (U, E) are defined as follow :

$$f_{N_1}(e_1) = \{< x_1, (1.0, 0.5, 0.4) >, < x_2, (0.6, 0.6, 0.6) >, < x_3, (0.5, 0.6, 0.4) >\},$$
$$f_{N_1}(e_2) = \{< x_1, (0.8, 0.4, 0.5) >, < x_2, (0.7, 0.7, 0.3) >, < x_3, (0.7, 0.5, 0.6) >\};$$
$$f_{N_2}(e_1) = \{< x_1, (0.8, 0.5, 0.6) >, < x_2, (0.5, 0.7, 0.6) >, < x_3, (0.4, 0.7, 0.5) >\},$$
$$f_{N_2}(e_2) = \{< x_1, (0.7, 0.6, 0.5) >, < x_2, (0.6, 0.8, 0.4) >, < x_3, (0.5, 0.8, 0.6) >\};$$
$$f_{N_3}(e_1) = \{< x_1, (0.6, 0.6, 0.7) >, < x_2, (0.4, 0.8, 0.8) >, < x_3, (0.3, 0.8, 0.6) >\},$$
$$f_{N_3}(e_2) = \{< x_1, (0.5, 0.8, 0.6) >, < x_2, (0.5, 0.9, 0.5) >, < x_3, (0.2, 0.9, 0.7) >\};$$

The t-norm and s-norm are defined as $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$. Here $N_1 \cap N_1 = N_1, N_1 \cap N_2 = N_2, N_1 \cap N_3 = N_3, N_2 \cap N_2 = N_2, N_2 \cap N_3 = N_3, N_3 \cap N_3 = N_3$ and $N_1 \cup N_1 = N_1, N_1 \cup N_2 = N_1, N_1 \cup N_3 = N_1, N_2 \cup N_2 = N_2, N_2 \cup N_3 = N_2, N_3 \cup N_3 = N_3$. Then τ_u is a neutrosophic soft topology on (U, E) and so (U, E, τ_u) is a neutrosophic soft topological space over (U, E).

3. Let *NSS*(U, E) be the family of all neutrosophic soft sets over (U, E). Then $\{\phi_u, 1_u\}$ and *NSS*(U, E) are two examples of the neutrosophic soft topology over (U, E). They are called, respectively, indiscrete (trivial) and discrete neutrosophic soft topology. Clearly, they are the smallest and largest neutrosophic soft topology on (U, E), respectively.

2.10 Definition [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M \in NSS(U, E)$ be arbitrary. Then the interior of M is denoted by M^o and is defined as :

$$M^o = \cup\{N_1 : N_1 \text{ is neutrosophic soft open and } N_1 \subset M\}$$

i.e., it is the union of all open neutrosophic soft subsets of M.

2.10.1 Theorem [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M, P \in NSS(U, E)$. Then,
(i) $M^o \subset M$ and M^o is the largest open set.
(ii)$M \subset P \Rightarrow M^o \subset P^o$.
(iii) M^o is an open neutrosophic soft set i.e., $M^o \in \tau_u$.
(iv) M is neutrosophic soft open set iff $M^o = M$.
(v) $(M^o)^o = M^o$.
(vi)$(\phi_u)^o = \phi_u$ and $1_u^o = 1_u$.
(vii) $(M \cap P)^o = M^o \cap P^o$.
(viii) $M^o \cup P^o \subset (M \cup P)^o$.

2.11 Definition [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M \in NSS(U, E)$ be arbitrary. Then the closure of M is denoted by $\overline{M}$ and is defined as :

$$\overline{M} = \cap\{N_1 : N_1 \text{ is neutrosophic soft closed and } N_1 \supset M\}$$

i.e., it is the intersection of all closed neutrosophic soft supersets of M.

2.11.1 Theorem [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M, P \in NSS(U, E)$. Then,
(i) $M \subset \overline{M}$ and $\overline{M}$ is the smallest closed set.
(ii) $M \subset P \Rightarrow \overline{M} \subset \overline{P}$.
(iii) $\overline{M}$ is closed neutrosophic soft set i.e., $\overline{M} \in \tau_u^c$.
(iv) M is neutrosophic soft closed set iff $\overline{M} = M$.
(v) $\overline{\overline{M}} = \overline{M}$.
(vi) $\overline{\phi_u} = \phi_u$ and $\overline{1_u} = 1_u$.
(vii) $\overline{M \cup P} = \overline{M} \cup \overline{P}$.
(viii) $\overline{M \cap P} \subset \overline{M} \cap \overline{P}$.

2.11.2 Theorem [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M \in NSS(U, E)$. Then, (i) $(\overline{M})^c = (M^c)^o$ (ii)$(M^o)^c = \overline{(M^c)}$

2.12 Definition [33]

1. A neutrosophic soft point in an NSS N is defined as an element $(e, f_N(e))$ of N, for $e \in E$ and is denoted by e_N, if $f_N(e) \notin \phi_u$ and $f_N(e') \in \phi_u, \forall e' \in E - \{e\}$.
2. The complement of a neutrosophic soft point e_N is another neutrosophic soft point e_N^c such that $f_N^c(e) = (f_N(e))^c$.
3. A neutrosophic soft point $e_N \in M$, M being an NSS if for the element $e \in E$, $f_N(e) \leq f_M(e)$.

2.12.1 *Example*

Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then,
$e_{1N} = \{< x_1, (0.6, 0.4, 0.8) >, < x_2, (0.8, 0.3, 0.5) >, < x_3, (0.3, 0.7, 0.6) >\}$
is a neutrosophic soft point whose complement is
$e_{1N}^c = \{< x_1, (0.8, 0.6, 0.6) >, < x_2, (0.5, 0.7, 0.8) >, < x_3, (0.6, 0.3, 0.3) >\}$.
For another NSS M defined on same (U, E), let,
$f_M(e_1) = \{< x_1, (0.7, 0.4, 0.7) >, < x_2, (0.8, 0.2, 0.4) >, < x_3, (0.5, 0.6, 0.5) >\}$.
Then, $f_N(e_1) \leq f_M(e_1)$ i.e., $e_{1N} \in M$.

2.13 Definition [33]

Hausdorff space : Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E). For two distinct neutrosophic soft points e_K, e_S, if there exists disjoint neutrosophic soft open sets M, P such that $e_K \in M$ and $e_S \in P$ then (U, E, τ_u) is called T_2 space or Hausdorff space.

2.13.1 *Example*

Let $U = \{h_1, h_2\}$, $E = \{e\}$ and $\tau_u = \{\phi_u, 1_u, M, P\}$ where M, P being neutrosophic soft subsets of N are defined as following :

$$f_M(e) = \{< h_1, (1, 0, 1) >, < h_2, (0, 0, 1) >\};$$
$$f_P(e) = \{< h_1, (0, 1, 0) >, < h_2, (1, 1, 0) >\};$$

Then τ_u is a neutrosophic soft topology on (U, E) with respect to the t-norm and s-norm defined as $a * b = \max\{a + b - 1, 0\}$ and $a \diamond b = \min\{a + b, 1\}$. Here $e_M \in M$ and $e_P \in P$ with $e_M \neq e_P$ and $M \cap P = \phi_u$.

2.14 Definition [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) where τ_u is a topology on (U, E) and $M \in NSS(U, E)$ an arbitrary NSS. Suppose $\tau_M = \{M \cap N_i : N_i \in \tau_u\}$. Then τ_M forms also a topology on (U, E). Thus (U, E, τ_M) is a neutrosophic soft topological subspace of (U, E, τ_u).

2.14.1 *Example*

Let us consider the example (2) in [2.9.1]. We define $M \in NSS(U, E)$ as following :

$$f_M(e_1) = \{< x_1, (0.4, 0.6, 0.8) >, < x_2, (0.7, 0.3, 0.2) >, < x_3, (0.5, 0.5, 0.7) >\};$$
$$f_M(e_2) = \{< x_1, (0.6, 0.3, 0.5) >, < x_2, (0.4, 0.7, 0.6) >, < x_3, (0.8, 0.3, 0.5) >\};$$

We denote $M \cap \phi_u = \phi_M, M \cap 1_u = 1_M, M \cap N_1 = M_1, M \cap N_2 = M_2, M \cap N_3 = M_3$; Then M_1, M_2, M_3 are given as following :

$f_{M_1}(e_1) = \{< x_1, (0.4, 0.6, 0.8) >, < x_2, (0.6, 0.6, 0.6) >, < x_3, (0.5, 0.6, 0.7) >\};$
$f_{M_1}(e_2) = \{< x_1, (0.6, 0.4, 0.5) >, < x_2, (0.4, 0.7, 0.6) >, < x_3, (0.7, 0.5, 0.6) >\};$
$f_{M_2}(e_1) = \{< x_1, (0.4, 0.6, 0.8) >, < x_2, (0.5, 0.7, 0.6) >, < x_3, (0.4, 0.7, 0.7) >\};$
$f_{M_2}(e_2) = \{< x_1, (0.6, 0.6, 0.5) >, < x_2, (0.4, 0.8, 0.6) >, < x_3, (0.5, 0.8, 0.6) >\};$
$f_{M_3}(e_1) = \{< x_1, (0.4, 0.6, 0.8) >, < x_2, (0.4, 0.8, 0.8) >, < x_3, (0.3, 0.8, 0.7) >\};$
$f_{M_3}(e_2) = \{< x_1, (0.5, 0.8, 0.6) >, < x_2, (0.4, 0.9, 0.6) >, < x_3, (0.2, 0.9, 0.7) >\};$

Here $M_1 \cap M_2 = M_2, M_1 \cap M_3 = M_3, M_2 \cap M_3 = M_3$ and $M_1 \cup M_2 = M_2, M_1 \cup M_3 = M_3, M_2 \cup M_3 = M_3$. Then $\tau_M = \{\phi_M, 1_M, M_1, M_2, M_3\}$ is neutrosophic soft subspace topology on (U, E).

2.15 Theorem [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M, N \in NSS(U, E)$. Then,
(i) If $\ss_u$ is a base of τ_u then $\ss_M = \{B \cap M : B \in \ss_u\}$ is a base for the topology τ_M.
(ii) If Q is closed NSS in M and M is closed NSS in N, then Q is closed in N.
(iii) Let $Q \subset M$. If $\overline{Q}$ is the closure of Q then $\overline{Q} \cap M$ is the closure of Q in M.
(iv) An NSS $M \in NSS(U, E)$ is an open NSS iff M is a neighbourhood of each NSS N contained in M.

2.16 Proposition (De-Morgan's law)[33]

Let N_1, N_2 be two neutrosophic soft sets over (U, E). Then,
(i) $(N_1 \cup N_2)^c = N_1{}^c \cap N_2{}^c$ (ii) $(N_1 \cap N_2)^c = N_1{}^c \cup N_2{}^c$.

3 Connectedness

In this section, the concept of connectedness on neutrosophic soft topological space has been introduced with suitable example. Some related theorems have been developed in continuation.

3.1 Definition

Two neutrosophic soft sets N_1, N_2 of a neutrosophic soft topological space (U, E, τ_u) over (U, E) are said to be separated if
(i) $N_1 \cap N_2 = \phi_u$ and (ii) $\overline{N_1} \cap N_2 = \phi_u$ or $N_1 \cap \overline{N_2} = \phi_u$.

3.2 Definition

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E). Then a pair of nonempty neutrosophic soft open sets N_1, N_2 is called a neutrosophic soft separation of (U, E, τ_u) if $1_u = N_1 \cup N_2$ and $N_1 \cap N_2 = \phi_u$.

In the Example (1) of [2.9.1], the pair N_1, N_2 is a neutrosophic soft separation of (U, E, τ_u) as $1_u = N_1 \cup N_2$ and $N_1 \cap N_2 = \phi_u$.

3.3 Definition

A neutrosophic soft topological space (U, E, τ_u) is said to be neutrosophic soft connected if there does not exist a neutrosophic soft separation of (U, E, τ_u). Otherwise, (U, E, τ_u) is called neutrosophic soft disconnected.

The topological space in the Example (2) of [2.9.1] is connected but (1) of [2.9.1] is disconnected.

3.4 Theorem

A neutrosophic soft topological space (U, E, τ_u) is said to be neutrosophic soft disconnected iff there exists a nonempty proper neutrosophic soft subset of 1_u which is both neutrosophic soft open and neutrosophic soft closed.

Proof. Let $M \subset 1_u, M \neq \phi_u$ and M is both neutrosophic soft open and closed. Then $M^c \subset 1_u, M^c \neq \phi_u$ and M^c is both neutrosophic soft open and closed, also. Let $P = M^c$. Then $\overline{M} = M$ and $\overline{P} = P$. Thus 1_u can be expressed as the union of two separated neutrosophic soft sets M, P and so, is neutrosophic soft disconnected.

Conversely, let 1_u be neutrosophic soft disconnected. Then there exists nonempty neutrosophic soft open sets N_1, N_2 such that $1_u = N_1 \cup N_2$ and $N_1 \cap N_2 = \phi_u$. Then $N_1 = N_2^c$ i.e., N_1 is closed, also. Similarly, $N_2 = N_1^c$ and so, N_2 is closed.

3.5 Theorem

A neutrosophic soft topological space (U, E, τ_u) is said to be neutrosophic soft connected iff there exists neutrosophic soft sets in $NSS(U, E)$ which are both neutrosophic soft open and neutrosophic soft closed, are ϕ_u and 1_u.

Proof. Let (U, E, τ_u) be a connected neutrosophic soft topological space. For contrary, we suppose that M is both neutrosophic soft open and closed different from $\phi_u, 1_u$. Then M^c is also both neutrosophic soft open and closed different from $\phi_u, 1_u$. Also $M \cap M^c = \phi_u$ and $M \cup M^c = 1_u$. Therefore M, M^c is a neutrosophic soft separation of 1_u. This is a contradiction. So, the only neutrosophic soft closed and open sets in $NSS(U, E)$ are ϕ_u and 1_u.

Conversely, let M, P be a neutrosophic soft separation of (U, E, τ_u). Then $M \neq N$ i.e., $M = P^c$, otherwise $M = 1_u$ implies $P = \phi_u$, a contradiction. This shows that M is both neutrosophic soft open and neutrosophic soft closed different from $\phi_u, 1_u$. This is a contradiction. Hence, (U, E, τ_u) is connected.

3.6 Theorem

If the neutrosophic soft sets N_1, N_2 form a neutrosophic soft separation of (U, E, τ_u) and if (U, E, τ_M) is a neutrosophic soft connected subspace of (U, E, τ_u), then $M \subset N_1$ or $M \subset N_2$.

Proof. Here $N_1, N_2 \in \tau_u$ such that $N_1 \cap N_2 = \phi_u$ and $N_1 \cup N_2 = 1_u$. Then $N_1 \cap M, N_2 \cap M \in \tau_M$ as (U, E, τ_M) is a neutrosophic soft topological subspace of (U, E, τ_u). Now $(N_1 \cap M) \cap (N_2 \cap M) = (N_1 \cap N_2) \cap M = \phi_u \cap M = \phi_u$ and $(N_1 \cap M) \cup (N_2 \cap M) = (N_1 \cup N_2) \cap M = 1_u \cap M = M$. Thus the pair $N_1 \cap M, N_2 \cap M$ would constitute a neutrosophic soft separation of (U, E, τ_M), a contradiction.

Hence, one of $N_1 \cap M$ and $N_2 \cap M$ is empty and so M is entirely contained in one of them.

3.7 Theorem

Let (U, E, τ_M) be a neutrosophic soft topological subspace of (U, E, τ_u). A separation of (U, E, τ_M) is a pair of disjoint nonempty neutrosophic soft sets M_1, M_2 whose union is M such that $M_1 \cap \overline{M_2} = \phi_u$ and $M_2 \cap \overline{M_1} = \phi_u$.

Proof. Suppose M_1, M_2 forms a separation of (U, E, τ_M). Then M_1 is both neutrosophic soft open and closed subset of M by Theorem [3.4]. The neutrosophic soft closure of M_1 in M is $\overline{M_1} \cap M$ by Theorem [2.19]. Since M_1 is neutrosophic soft closed in M then $M_1 = \overline{M_1} \cap M$. It implies $\overline{M_1} \cap M_2 = (\overline{M_1} \cap M) \cap M_2 = M_1 \cap M_2 = \phi_u$. Similarly, $\overline{M_2} \cap M_1 = \phi_u$.

Conversely, let $M = M_1 \cup M_2$ with $M_1 \cap M_2 = \phi_u$ such that $\overline{M_1} \cap M_2 = \phi_u$ and $\overline{M_2} \cap M_1 = \phi_u$. Then $M \cap \overline{M_1} = \phi_u$ and $M \cap \overline{M_2} = \phi_u \Rightarrow M_1, M_2$ are neutrosophic soft closed in M. Also $M_1 = M_2^c$ implies both are neutrosophic soft open in M.

3.8 Theorem

Let (U, E, τ_M) be a connected neutrosophic soft subspace of (U, E, τ_u). If (U, E, τ_P) be any neutrosophic soft subspace of (U, E, τ_u) such that $M \subset P \subset \overline{M}$, then (U, E, τ_P) is also neutrosophic soft connected.

Proof. Let the neutrosophic soft set P satisfy the hypothesis. If possible, let P_1, P_2 form a neutrosophic soft separation of (U, E, τ_P). Then $M \subset P_1$ or $M \subset P_2$. Let $M \cap P_1 = \phi_u$. So $M \subset P_1^c$ and P_1^c is closed NSS. It implies $M \subset P \subset \overline{M} \subset P_1^c \Rightarrow P \subset P_1^c \Rightarrow P \cap P_1 = \phi_u$. This is a contradiction to the fact that $P_1 \cup P_2 = P$. Hence, (U, E, τ_P) is neutrosophic soft connected.

3.9 Theorem

Arbitrary union of connected neutrosophic soft subspaces of (U, E, τ_u) having nonempty intersection is also neutrosophic soft connected.

Proof. Let $\{(U, E, \tau_{N_i}) : i \in \Gamma\}$ be a class of connected neutrosophic soft subspaces of (U, E, τ_u) with nonempty intersection. Let $\tau_M = \cup_i(\tau_{N_i})$. If possible, we take a neutrosophic soft separation P, Q of (U, E, τ_M). For each i, $P \cap N_i$ and $Q \cap N_i$ are disjoint neutrosophic soft open sets in the subspace such that their union is N_i. Since each (U, E, τ_{N_i}) is connected, any of $P \cap N_i$ and $Q \cap N_i$ must be empty. Let $P \cap N_i = \phi_u \Rightarrow Q \cap N_i = N_i \Rightarrow N_i \subset Q, \forall i \in \Gamma \Rightarrow \cup_i N_i \subset Q \Rightarrow M \subset Q \Rightarrow P \cup Q \subset Q \Rightarrow P$ is empty, a contradiction. So, (U, E, τ_M) is neutrosophic soft connected.

3.10 Theorem

Arbitrary union of a family of connected neutrosophic soft subspaces of (U, E, τ_u) such that one of the members of the family has nonempty intersection with every member of the family, is neutrosophic soft connected.

Proof. Let $\{(U, E, \tau_{N_i}) : i \in \Gamma\}$ be a class of connected neutrosophic soft subspaces of (U, E, τ_u) and N_k be a fixed member such that $N_k \cap N_i \neq \phi_u$ for each $i \in \Gamma$. Let $M_i = N_k \cup N_i$. Then by Theorem [3.9], (U, E, τ_{M_i}) is a neutrosophic soft connected for each $i \in \Gamma$. Now, $\cup_i M_i = \cup_i(N_k \cup N_i) = (N_k \cup N_1) \cup (N_k \cup N_2) \cup \cdots = N_k \cup (N_1 \cup N_2 \cup \cdots) = \cup_i N_i$ and $\cap_i M_i = \cap_i(N_k \cup N_i) = (N_k \cup N_1) \cap (N_k \cup N_2) \cap \cdots = N_k \cup (N_1 \cap N_2 \cap \cdots) \neq \phi_u$.

This completes the theorem.

4 Compactness

Here, the notion of compactness on neutrosophic soft topological space is developed with some basic theorems.

4.1 Definition

Let (U, E, τ_u) be a neutrosophic soft topological space and $M \in \tau_u$. A family $\Omega = \{Q_i : i \in \Gamma\}$ of neutrosophic soft sets is said to be a cover of M if $M \subset \cup Q_i$.

If every member of that family which covers M is neutrosophic soft open then it is called open cover of M. A subfamily of Ω which also covers M is called a subcover of M.

4.1.1 Definition

Let (U, E, τ_u) be a neutrosophic soft topological space and $M \in \tau_u$. Suppose Ω be an open cover of M. If Ω has a finite subcover which also covers M then M is called neutrosophic soft compact.

4.1.2 *Example*

In the Example (1) of [2.9.1], $1_u = \cup_{i=1}^4 N_i$. So $\{N_1, N_2, N_3, N_4\}$ is an open cover of (U, E, τ_u). Also, $1_u = N_1 \cup N_2$ or $1_u = N_1 \cup N_4$. So (U, E, τ_u) is neutrosophic soft compact topological space.

4.2 Theorem

Let (U, E, τ_u) be a neutrosophic soft compact topological space and M be a neutrosophic soft closed set of that space. Then M is also compact.

Proof. Let $\Omega = \{Q_i : i \in \Gamma\}$ be an open cover of M.

Then $\{Q_i\} \cup M^c$ is an open cover of (U, E, τ_u), obviously. Since (U, E, τ_u) is compact so there exists a finite subcover of $\{Q_i\} \cup M^c$ such that

$$\begin{aligned} & 1_u = Q_1 \cup Q_2 \cup \cdots \cup Q_n \cup M^c \\ \Rightarrow \quad & M \subset 1_u = Q_1 \cup Q_2 \cup \cdots \cup Q_n \cup M^c \\ \Rightarrow \quad & M \subset Q_1 \cup Q_2 \cup \cdots \cup Q_n \text{ as } M \cap M^c = \phi_u. \end{aligned}$$

Hence, M has a finite subcover and so is compact.

4.3 Theorem

Let (U, E, τ_u) be a neutrosophic soft Hausdorff topological space and M be a neutrosophic soft compact set belonging to that space. Then M is a closed NSS.

Proof. Let $e_K \in M^c$ be a neutrosophic soft point. Then for each $e_S \in M$, we have $e_K \neq e_S$. So by definition of Hausdorff space, there are disjoint neutrosophic soft open sets N_K, N_S so that $e_K \in N_K$ and $e_S \in N_S$. Let $\{N_S : e_S \in M\}$ be a neutrosophic soft open cover of M. Since M is neutrosophic soft compact so it has a finite subcover, say, $\{N_{S_1}, N_{S_2}, \cdots N_{S_n}\}$ i.e., $M \subset N_{S_1} \cup N_{S_2} \cup \cdots \cup N_{S_n} = P$, say. Then P is neutrosophic soft open.

Let $Q = N_{K_1} \cap N_{K_2} \cap \cdots \cap N_{K_n}$ where each N_{K_i} is open NSS corresponding to $e_{K_i} \in M^c$. Now, $N_{S_i} \cap N_{K_i} = \phi_u \Rightarrow N_{S_i} \cap Q = \phi_u$ for each i. Then $P \cap Q = (N_{S_1} \cup N_{S_2} \cup \cdots \cup N_{S_n}) \cap Q = (N_{S_1} \cap Q) \cup (N_{S_2} \cap Q) \cup \cdots \cup (N_{S_n} \cap Q) = \phi_u$. Since $M \subset P$ and $P \cap Q = \phi_u$, so $M \cap Q = \phi_u \Rightarrow Q \subset M^c$ and Q is open NSS. This implies M^c is open NSS i.e., M is closed.

4.4 Theorem

A neutrosophic soft topological space is compact iff each family of neutrosophic soft closed sets with the finite intersection property has a nonempty intersection.

Proof. Let (U, E, τ_u) be a compact neutrosophic soft topological space. Consider $\Omega = \{Q_i : i \in \Gamma\}$ be a family of closed NSSs such that $\cap_i Q_i = \phi_u$. We show Ω can not have finite intersection property. Let $\Delta = \{Q_i^c : Q_i \in \Omega, i \in \Gamma\}$. Then Δ is an open cover of (U, E, τ_u) such that there exists a finite subcover $\{Q_1^c, Q_2^c, \cdots, Q_n^c\}$. Now $\cap_{i=1}^n Q_i = 1_u - (Q_1^c \cup Q_2^c \cup \cdots \cup Q_n^c) = 1_u - 1_u = \phi_u$ by Definition [2.8]. Hence, the 'if part' holds.

Next assume that (U, E, τ_u) is not compact. Then, a neutrosophic soft open cover $\{Q_i : i \in \Gamma\}$, say, of (U, E, τ_u) has no finite subcover i.e., $Q_1 \cup Q_2 \cup \cdots \cup Q_n \neq 1_u$. This implies $Q_1^c \cap Q_2^c \cap \cdots \cap Q_n^c \neq \phi_u$ by Definition [2.8] and Proposition [2.16]. Thus $\{Q_i^c : i \in \Gamma\}$ has finite intersection property. Then by hypothesis, $\cap_i Q_i^c \neq \phi_u$ and $\cup_i Q_i \neq 1_u$ which is a contradiction. Hence, (U, E, τ_u) is compact.

5 Neutrosophic soft continuous mappings

In this section, first we define neutrosophic soft mapping, then define image and pre-image of an NSS under a neutrosophic soft mapping. In continuation, we introduce the notion of neutrosophic soft continuous mapping in a neutrosophic soft topological space along with some of it's properties.

In rest of the paper, if M be an NSS over U via parameter set E, we write (M, E), an NSS over U i.e., $(M, E) = \{< e, f_M(e) >: e \in E\}$.

5.1 Definition

Let, $\varphi : U \to V$ and $\psi : E \to E$ be two functions where E is the parameter set for each of the crisp sets U and V. Then the pair (φ, ψ) is called an NSS function from (U, E) to (V, E). We write, $(\varphi, \psi) : (U, E) \to (V, E)$.

5.1.1 Definition

Let (M, E) and (N, E) be two NSSs defined over U and V, respectively and (φ, ψ) be an NSS function from (U, E) to (V, E). Then,

(1) The image of (M, E) under (φ, ψ), denoted by $(\varphi, \psi)(M, E)$, is an NSS over V and is defined as :
$(\varphi, \psi)(M, E) = (\varphi(M), \psi(E)) = \{< \psi(a), f_{\varphi(M)}(\psi(a)) >: a \in E\}$ where $\forall b \in \psi(E), \forall y \in V$.

$$T_{f_{\varphi(M)}(b)}(y) = \begin{cases} \max_{\varphi(x)=y} \max_{\psi(a)=b} [T_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 0 \quad , & \text{otherwise.} \end{cases}$$

$$I_{f_{\varphi(M)}(b)}(y) = \begin{cases} \min_{\varphi(x)=y} \min_{\psi(a)=b} [I_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 1 \quad , & \text{otherwise.} \end{cases}$$

$$F_{f_{\varphi(M)}(b)}(y) = \begin{cases} \min_{\varphi(x)=y} \min_{\psi(a)=b} [F_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 1 \quad , & \text{otherwise.} \end{cases}$$

(2) The pre-image of (N, E) under (φ, ψ), denoted by $(\varphi, \psi)^{-1}(N, E)$, is an NSS over U and is defined by :
$(\varphi, \psi)^{-1}(N, E) = (\varphi^{-1}(N), \psi^{-1}(E))$ where $\forall a \in \psi^{-1}(E), \forall x \in U$.

$$\begin{aligned} T_{f_{\varphi^{-1}(N)}(a)}(x) &= T_{f_N(\psi(a))}(\varphi(x)) \\ I_{f_{\varphi^{-1}(N)}(a)}(x) &= I_{f_N(\psi(a))}(\varphi(x)) \\ F_{f_{\varphi^{-1}(N)}(a)}(x) &= F_{f_N(\psi(a))}(\varphi(x)) \end{aligned}$$

If ψ and φ are injective (surjective), then (φ, ψ) is injective (surjective).

5.1.2 Proposition

Let, $(\varphi, \psi) : (U, E) \to (V, E)$ be a neutrosophic soft mapping and (M_1, E) and (M_2, E) be two NSSs defined over U. Then the followings hold.

(1) $(M_1, E) \subseteq (\varphi, \psi)^{-1}[(\varphi, \psi)(M_1, E)]$
(2) $[(\varphi, \psi)(M_1, E)]^c \subseteq (\varphi, \psi)(M_1, E)^c$, if φ is surjective.
(3) $(\varphi, \psi)[(M_1, E) \cup (M_2, E)] = (\varphi, \psi)(M_1, E) \cup (\varphi, \psi)(M_2, E)$
(4) $(\varphi, \psi)[(M_1, E) \cap (M_2, E)] = (\varphi, \psi)(M_1, E) \cap (\varphi, \psi)(M_2, E)$

Proof.
(1) $(\varphi, \psi)^{-1}[(\varphi, \psi)(M_1, E)] = (\varphi, \psi)^{-1}[\varphi(M_1), \psi(E)] = [\varphi^{-1}(\varphi(M_1)), \psi^{-1}(\psi(E))]$. Then for $a \in \psi^{-1}(\psi(E))$ and $x \in U$, we have, $T_{f_{\varphi^{-1}(\varphi(M_1))}(a)}(x) = T_{f_{\varphi(M_1)}(\psi(a))}(\varphi(x)) = \max_{\varphi(x)} \max_{\psi(a)}[T_{f_M(a)}(x)]$. Now, $T_{f_M(a)}(x) \leq \max_{\varphi(x)} \max_{\psi(a)}[T_{f_M(a)}(x)] = T_{f_{\varphi^{-1}(\varphi(M_1))}(a)}(x)$.
Similarly, $I_{f_M(a)}(x) \geq I_{f_{\varphi^{-1}(\varphi(M_1))}(a)}(x)$ and $F_{f_M(a)}(x) \geq F_{f_{\varphi^{-1}(\varphi(M_1))}(a)}(x)$.
Hence, $(M_1, E) \subseteq (\varphi, \psi)^{-1}[(\varphi, \psi)(M_1, E)]$.

(2) Suppose, φ is surjective mapping. Here, $[(\varphi, \psi)(M_1, E)]^c = [(\varphi(M_1))^c, \psi(E)]$ and $(\varphi, \psi)(M_1, E)^c = [\varphi(M_1^c), \psi(E)]$. For $b \in \psi(E)$ and $y \in V$, we have, $T_{f_{(\varphi(M_1))^c}(b)}(y) = F_{f_{(\varphi(M_1))}(b)}(y) = \min_{\varphi(x)=y} \min_{\psi(a)=b}[F_{f_{M_1}(a)}(x)]$. But, $T_{f_{\varphi(M_1^c)}(b)}(y) = \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1^c}(a)}(x)] = \max_{\varphi(x)=y} \max_{\psi(a)=b}[F_{f_{M_1}(a)}(x)]$. Thus, $T_{f_{(\varphi(M_1))^c}(b)}(y) \leq T_{f_{\varphi(M_1^c)}(b)}(y) \cdots\cdots\cdots$ (i)
Similarly, $F_{f_{(\varphi(M_1))^c}(b)}(y) \geq F_{f_{\varphi(M_1^c)}(b)}(y) \cdots\cdots\cdots$ (ii)
Finally, $I_{f_{(\varphi(M_1))^c}(b)}(y) = 1 - I_{f_{(\varphi(M_1))}(b)}(y) = 1 - \min_{\varphi(x)=y} \min_{\psi(a)=b}[I_{f_{M_1}(a)}(x)]$ and $I_{f_{\varphi(M_1^c)}(b)}(y) = \min_{\varphi(x)=y} \min_{\psi(a)=b}[I_{f_{M_1^c}(a)}(x)] = \min_{\varphi(x)=y} \min_{\psi(a)=b}[1 - I_{f_{M_1}(a)}(x)]$.
This shows, $I_{f_{(\varphi(M_1))^c}(b)}(y) \geq I_{f_{\varphi(M_1^c)}(b)}(y) \cdots\cdots\cdots$ (iii)

This completes the 2nd part.

(3) Let, $(M_1, E) \cup (M_2, E) = (M, E)$.
Then, $(\varphi, \psi)[(M_1, E) \cup (M_2, E)] = (\varphi, \psi)(M, E) = [\varphi(M), \psi(E)]$. So, for $b \in \psi(E)$ and $y \in V$, we have,

$$\begin{aligned} T_{f_{\varphi(M)}(b)}(y) &= \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_M(a)}(x)] \\ &= \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x) \diamond T_{f_{M_2}(a)}(x)] \end{aligned}$$

Next, $(\varphi, \psi)(M_1, E) \cup (\varphi, \psi)(M_2, E) = [\varphi(M_1) \cup \varphi(M_2), \psi(E)] = [P, \psi(E)]$, say. Then,

$$\begin{aligned} & T_{f_P(b)}(y) \\ = \; & T_{f_{\varphi(M_1)}(b)}(y) \diamond T_{f_{\varphi(M_2)}(b)}(y) \\ = \; & \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x)] \diamond \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_2}(a)}(x)] \\ = \; & \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x) \diamond T_{f_{M_2}(a)}(x)] \end{aligned}$$

Thus, $T_{f_{\varphi(M)}(b)}(y) = T_{f_P(b)}(y)$. Similar results also hold for I, F.

This completes the proof of part (3).

(4) Let, $(M_1, E) \cap (M_2, E) = (M, E)$.
Then, $(\varphi, \psi)[(M_1, E) \cap (M_2, E)] = (\varphi, \psi)(M, E) = [\varphi(M), \psi(E)]$. So, for $b \in \psi(E)$ and $y \in V$, we have,

$$\begin{aligned} T_{f_{\varphi(M)}(b)}(y) &= \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_M(a)}(x)] \\ &= \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x) * T_{f_{M_2}(a)}(x)] \end{aligned}$$

Next, $(\varphi, \psi)(M_1, E) \cap (\varphi, \psi)(M_2, E) = [\varphi(M_1) \cap \varphi(M_2), \psi(E)] = [Q, \psi(E)]$, say. Then,

$$\begin{aligned} & T_{f_Q(b)}(y) \\ = \; & T_{f_{\varphi(M_1)}(b)}(y) * T_{f_{\varphi(M_2)}(b)}(y) \\ = \; & \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x)] * \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_2}(a)}(x)] \\ = \; & \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x) * T_{f_{M_2}(a)}(x)] \end{aligned}$$

Thus, $T_{f_{\varphi(M)}(b)}(y) = T_{f_Q(b)}(y)$. Similar results also hold for I, F.

This ends the last part.

5.1.3 Proposition

Let, $(\varphi, \psi) : (U, E) \to (V, E)$ be a neutrosophic soft mapping and (N_1, E) and (N_2, E) be two NSSs defined over V. Then the followings hold.
(1) $(\varphi, \psi)[(\varphi, \psi)^{-1}(N_1, E)] = (N_1, E)$, if (φ, ψ) is surjective.
(2) $[(\varphi, \psi)^{-1}(N_1, E)]^c = (\varphi, \psi)^{-1}(N_1, E)^c$
(3) $(\varphi, \psi)^{-1}[(N_1, E) \cup (N_2, E)] = (\varphi, \psi)^{-1}(N_1, E) \cup (\varphi, \psi)^{-1}(N_2, E)$
(4) $(\varphi, \psi)^{-1}[(N_1, E) \cap (N_2, E)] = (\varphi, \psi)^{-1}(N_1, E) \cap (\varphi, \psi)^{-1}(N_2, E)$

Proof. We shall prove (2) and (3), only. The others can be proved similarly.
(2) Here, $[(\varphi, \psi)^{-1}(N_1, E)]^c = [(\varphi^{-1}(N))^c, \psi^{-1}(E)]$. Then, for $a \in \psi^{-1}(E), x \in U$,

$$\begin{aligned} T_{f_{(\varphi^{-1}(N))^c}(a)}(x) = F_{f_{\varphi^{-1}(N)}(a)}(x) = F_{f_N(\psi(a))}(\varphi(x)), \\ I_{f_{(\varphi^{-1}(N))^c}(a)}(x) = 1 - I_{f_{\varphi^{-1}(N)}(a)}(x) = 1 - I_{f_N(\psi(a))}(\varphi(x)), \\ F_{f_{(\varphi^{-1}(N))^c}(a)}(x) = T_{f_{\varphi^{-1}(N)}(a)}(x) = T_{f_N(\psi(a))}(\varphi(x)). \end{aligned}$$

Next, $(\varphi, \psi)^{-1}(N_1, E)^c = [\varphi^{-1}(N_1^c), \psi)^{-1}(E)]$. Then,

$$\begin{aligned} T_{f_{\varphi^{-1}(N^c)}(a)}(x) = T_{f_{N^c}(a)}(x) = F_{f_N(\psi(a))}(\varphi(x)), \\ I_{f_{\varphi^{-1}(N^c)}(a)}(x) = I_{f_{N^c}(a)}(x) = 1 - I_{f_N(\psi(a))}(\varphi(x)), \\ F_{f_{\varphi^{-1}(N^c)}(a)}(x) = F_{f_{N^c}(a)}(x) = T_{f_N(\psi(a))}(\varphi(x)). \end{aligned}$$

Hence, the result is proved.

(3) Let, $(N_1, E) \cup (N_2, E) = (N, E)$.
Then, $(\varphi, \psi)^{-1}[(N_1, E) \cup (N_2, E)] = (\varphi, \psi)^{-1}(N, E) = [\varphi^{-1}(N), \psi^{-1}(E)]$. So, for $a \in \psi^{-1}(E)$ and $x \in U$, we have,

$$\begin{aligned} T_{f_{\varphi^{-1}(N)}(a)}(x) &= T_{f_N(\psi(a))}(\varphi(x)) \\ &= T_{f_{N_1}(\psi(a))}(\varphi(x)) \diamond T_{f_{N_2}(\psi(a))}(\varphi(x)) \end{aligned}$$

Next, $(\varphi,\psi)^{-1}(N_1,E) \cup (\varphi,\psi)^{-1}(N_2,E) = [\varphi^{-1}(N_1) \cup \varphi^{-1}(N_2), \psi^{-1}(E)] = [R, \psi^{-1}(E)]$, say. Then,

$$\begin{aligned} T_{f_{R(a)}}(x) &= T_{f_{\varphi^{-1}(N_1)}(a)}(x) \diamond T_{f_{\varphi^{-1}(N_2)}(a)}(x) \\ &= T_{f_{N_1}(\psi(a))}(\varphi(x)) \diamond T_{f_{N_2}(\psi(a))}(\varphi(x)) \end{aligned}$$

Thus, $T_{f_{\varphi^{-1}(N)}(a)}(x) = T_{f_{R(a)}}(x)$. Similar results also hold for I, F.

This completes the proof of part (3).

5.2 Definition

Let $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ be a mapping where (U,E,τ_u) and (V,E,τ_v) be two neutrosophic soft topological spaces.

(1) For each neutrosophic soft open set $(M,E) \in (U,E,\tau_u)$, if the image $(\varphi,\psi)(M,E)$ is open in (V,E,τ_v) then (φ,ψ) is said to be neutrosophic soft open mapping.

(2) For each neutrosophic soft closed set $(Q,E) \in (U,E,\tau_u)$, if the image $(\varphi,\psi)(Q,E)$ is closed in (V,E,τ_v) then (φ,ψ) is said to be neutrosophic soft closed mapping.

5.3 Theorem

Let, (U,E,τ_u) and (V,E,τ_v) be two neutrosophic soft topological spaces and $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ be a mapping. Then,

(1) (φ,ψ) is a neutrosophic soft open mapping iff for each neutrosophic soft set $(M,E) \in (U,E,\tau_u)$, there be hold $(\varphi,\psi)(M,E)^o \subset [(\varphi,\psi)(M,E)]^o$.

(2) (φ,ψ) is a neutrosophic soft closed mapping iff for each neutrosophic soft set $(Q,E) \in (U,E,\tau_u)$, there be hold $\overline{[(\varphi,\psi)(Q,E)]} \subset (\varphi,\psi)\overline{(Q,E)}$.

Proof. **(1)** Let (φ,ψ) is a neutrosophic soft open mapping and $(M,E) \in (U,E,\tau_u)$. Then $(M,E)^o$ is a neutrosophic soft open set and $(M,E)^o \subset (M,E)$. Since (φ,ψ) is a neutrosophic soft open mapping, $(\varphi,\psi)(M,E)^o$ is neutrosophic soft open in (V,E,τ_v). Then $(\varphi,\psi)(M,E)^o \subset (\varphi,\psi)(M,E)$. But $[(\varphi,\psi)(M,E)]^o$ is the largest open NSS contained in $(\varphi,\psi)(M,E)$. Hence, $(\varphi,\psi)(M,E)^o \subset [(\varphi,\psi)(M,E)]^o$ is obtained.

Conversely, suppose (M,E) be an open NSS in (U,E,τ_u) such that the given condition holds. Then $(M,E) = (M,E)^o$ and so $(\varphi,\psi)(M,E) = (\varphi,\psi)(M,E)^o \subset [(\varphi,\psi)(M,E)]^o \subset (\varphi,\psi)(M,E)$. Hence, $[(\varphi,\psi)(M,E)]^o = (\varphi,\psi)(M,E)$. This ends the proof.

(2) Let (φ,ψ) is a neutrosophic soft closed mapping and $(Q,E) \in (U,E,\tau_u)$. Then $\overline{(Q,E)}$ is a neutrosophic soft closed set and $(Q,E) \subset \overline{(Q,E)}$. Since (φ,ψ) is a neutrosophic soft closed mapping, $(\varphi,\psi)\overline{(Q,E)}$ is neutrosophic soft closed in (V,E,τ_v). Then $(\varphi,\psi)(Q,E) \subset (\varphi,\psi)\overline{(Q,E)}$. But $\overline{[(\varphi,\psi)(Q,E)]}$ is the smallest closed NSS containing $(\varphi,\psi)(Q,E)$. Hence, $\overline{[(\varphi,\psi)(Q,E)]} \subset (\varphi,\psi)\overline{(Q,E)}$ is obtained.

Conversely, suppose (Q,E) be a closed NSS in (U,E,τ_u) such that the given condition holds. Then $\overline{(Q,E)} = (Q,E)$ and so $(\varphi,\psi)(Q,E) \subset \overline{[(\varphi,\psi)(Q,E)]} \subset (\varphi,\psi)\overline{(Q,E)} = (\varphi,\psi)(Q,E)$. Hence, $\overline{[(\varphi,\psi)(Q,E)]} = (\varphi,\psi)(Q,E)$. This completes the proof.

5.4 Definition

Let, (U,E,τ_u) and (V,E,τ_v) be two neutrosophic soft topological spaces. Then $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ is said to be a neutrosophic soft continuous mapping if for each $(N,E) \in \tau_v$, the inverse image $(\varphi,\psi)^{-1}(N,E) \in \tau_u$ i.e., the inverse image of each open NSS in (V,E,τ_v) is also open in (U,E,τ_u).

5.4.1 *Example*

For two neutrosophic soft topological spaces (U,E,τ_u) and (V,E,τ_v), let $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ be a mapping.

(1) If τ_v is the neutrosophic soft indiscrete topology on V, then (φ,ψ) is a neutrosophic soft continuous mapping.

(2) If τ_u is the neutrosophic soft discrete topology on U, then (φ,ψ) is a neutrosophic soft continuous mapping.

(3) Let, $U = \{u_1,u_2,u_3\}, V = \{v_1,v_2,v_3\}, E = \{e_1,e_2\}, \tau_v = \{\phi_v, 1_v, (N_1,E), (N_2,E)\}, \tau_u = \{\phi_u, 1_u, (M_1,E), (M_2,E), (M_3,E)\}$, where $(N_1,E), (N_2,E)$ are as follows :

$$\begin{aligned} f_{N_1}(e_1) &= \{< v_1,(0.8,0.5,0.6) >, < v_2,(0.5,0.7,0.6) >, < v_3,(0.4,0.7,0.5) >\}; \\ f_{N_1}(e_2) &= \{< v_1,(0.7,0.6,0.5) >, < v_2,(0.6,0.8,0.4) >, < v_3,(0.5,0.8,0.6) >\}; \\ f_{N_2}(e_1) &= \{< v_1,(0.6,0.6,0.7) >, < v_2,(0.4,0.8,0.8) >, < v_3,(0.3,0.8,0.6) >\}; \\ f_{N_2}(e_2) &= \{< v_1,(0.5,0.8,0.6) >, < v_2,(0.5,0.9,0.5) >, < v_3,(0.2,0.9,0.7) >\}; \end{aligned}$$

and $(M_1,E), (M_2,E), (M_3,E)$ are given as followings :

$$\begin{aligned} f_{M_1}(e_1) &= \{< u_1,(0.8,0.4,0.5) >, < u_2,(0.7,0.5,0.6) >, < u_3,(0.7,0.7,0.3) >\}; \\ f_{M_1}(e_2) &= \{< u_1,(1.0,0.5,0.4) >, < u_2,(0.5,0.6,0.4) >, < u_3,(0.6,0.6,0.6) >\}; \\ f_{M_2}(e_1) &= \{< u_1,(0.5,0.8,0.6) >, < u_2,(0.2,0.9,0.7) >, < u_3,(0.5,0.9,0.5) >\}; \\ f_{M_2}(e_2) &= \{< u_1,(0.6,0.6,0.7) >, < u_2,(0.3,0.8,0.6) >, < u_3,(0.4,0.8,0.8) >\}; \\ f_{M_3}(e_1) &= \{< u_1,(0.7,0.6,0.5) >, < u_2,(0.5,0.8,0.6) >, < u_3,(0.6,0.8,0.4) >\}; \\ f_{M_3}(e_2) &= \{< u_1,(0.8,0.5,0.6) >, < u_2,(0.4,0.7,0.5) >, < u_3,(0.5,0.7,0.6) >\}; \end{aligned}$$

The t-norm and s-norm in both τ_u, τ_v are defined as $a * b = \min\{a,b\}$ and $a \diamond b = \max\{a,b\}$. Consider the mapping (φ,ψ)

as : $\varphi(u_1) = v_1, \varphi(u_2) = v_3, \varphi(u_3) = v_2$ and $\psi(e_1) = e_2, \psi(e_2) = e_1$. Then $(\varphi,\psi)^{-1}(N_1,E), (\varphi,\psi)^{-1}(N_2,E) \in \tau_u$.

For convenience, the calculation of $(\varphi,\psi)^{-1}(N_1,E)$ is provided for one parameter. The others are in similar way.

$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = T_{f_{N_1}(e_2)}(v_1) = 0.7$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = I_{f_{N_1}(e_2)}(v_1) = 0.6$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = F_{f_{N_1}(e_2)}(v_1) = 0.5$$
$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = T_{f_{N_1}(e_2)}(v_3) = 0.5$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = I_{f_{N_1}(e_2)}(v_3) = 0.8$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = F_{f_{N_1}(e_2)}(v_3) = 0.6$$
$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = T_{f_{N_1}(e_2)}(v_2) = 0.6$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = I_{f_{N_1}(e_2)}(v_2) = 0.8$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = F_{f_{N_1}(e_2)}(v_2) = 0.4$$

5.4.2 Proposition

Let $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ be a neutrosophic soft continuous mapping. Then for each $e \in E$, $(\varphi,\psi) : (U,\tau_u^e) \to (V,\tau_v^e)$ is a neutrosophic continuous mapping.

Proof. Let, $(N,E) \in \tau_v$. Since (φ,ψ) be a neutrosophic soft continuous mapping, so $(\varphi,\psi)^{-1}(N,E) \in \tau_u$. It implies $(\varphi,\psi)^{-1}(\{< e, f_N(e) >: e \in E\}) \in \tau_u$ i.e., $(\varphi,\psi)^{-1}(< e, f_N(e) >) \in \tau_u^e$ for $< e, f_N(e) > \in \tau_v^e$. This follows the theorem.

But the converse does not hold. The following example shows the fact.

Let, $U = \{u_1,u_2,u_3\}, V = \{v_1,v_2,v_3\}, E = \{e_1,e_2\}, \tau_v = \{\phi_v, 1_v, (N_1,E), (N_2,E)\}, \tau_u = \{\phi_u, 1_u, (M_1,E), (M_2,E), (M_3,E)\}$, where $(N_1,E), (N_2,E)$ are as follows :

$$f_{N_1}(e_1) = \{< v_1, (0.8,0.5,0.6) >, < v_2, (0.5,0.7,0.6) >, < v_3, (0.4,0.7,0.5) >\};$$
$$f_{N_1}(e_2) = \{< v_1, (0.7,0.6,0.5) >, < v_2, (0.6,0.8,0.4) >, < v_3, (0.5,0.8,0.6) >\};$$
$$f_{N_2}(e_1) = \{< v_1, (1.0,0.5,0.4) >, < v_2, (0.6,0.6,0.6) >, < v_3, (0.5,0.6,0.4) >\};$$
$$f_{N_2}(e_2) = \{< v_1, (0.8,0.4,0.5) >, < v_2, (0.7,0.7,0.3) >, < v_3, (0.7,0.5,0.6) >\};$$

and $(M_1,E), (M_2,E), (M_3,E)$ are given as follows :

$$f_{M_1}(e_1) = \{< u_1, (0.6,0.6,0.6) >, < u_2, (0.5,0.6,0.4) >, < u_3, (1.0,0.5,0.4) >\};$$
$$f_{M_1}(e_2) = \{< u_1, (0.7,0.7,0.3) >, < u_2, (0.7,0.5,0.6) >, < u_3, (0.8,0.4,0.5) >\};$$
$$f_{M_2}(e_1) = \{< u_1, (0.5,0.7,0.6) >, < u_2, (0.4,0.7,0.5) >, < u_3, (0.8,0.5,0.6) >\};$$
$$f_{M_2}(e_2) = \{< u_1, (0.5,0.9,0.5) >, < u_2, (0.2,0.9,0.7) >, < u_3, (0.5,0.8,0.6) >\};$$
$$f_{M_3}(e_1) = \{< u_1, (0.5,0.6,0.6) >, < u_2, (0.4,0.7,0.4) >, < u_3, (0.9,0.5,0.5) >\};$$
$$f_{M_3}(e_2) = \{< u_1, (0.6,0.8,0.4) >, < u_2, (0.5,0.8,0.6) >, < u_3, (0.7,0.6,0.5) >\};$$

The t-norm and s-norm in both τ_u, τ_v are defined as $a * b = \min\{a,b\}$ and $a \diamond b = \max\{a,b\}$. Define a neutrosophic soft mapping (φ,ψ) as : $\varphi(u_1) = v_2, \varphi(u_2) = v_3, \varphi(u_3) = v_1$ and $\psi(e_1) = e_1, \psi(e_2) = e_2$. We now calculate $(\varphi,\psi)^{-1}(N_1,E)$.

$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = T_{f_{N_1}(e_1)}(v_2) = 0.5$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = I_{f_{N_1}(e_1)}(v_2) = 0.7$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = F_{f_{N_1}(e_1)}(v_2) = 0.6$$
$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = T_{f_{N_1}(e_1)}(v_3) = 0.4$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = I_{f_{N_1}(e_1)}(v_3) = 0.7$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = F_{f_{N_1}(e_1)}(v_3) = 0.5$$
$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = T_{f_{N_1}(e_1)}(v_1) = 0.8$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = I_{f_{N_1}(e_1)}(v_1) = 0.5$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = F_{f_{N_1}(e_1)}(v_1) = 0.6$$
$$T_{f_{\varphi^{-1}(N_1)}(e_2)}(u_1) = T_{f_{N_1}(\psi(e_2))}(\varphi(u_1)) = T_{f_{N_1}(e_2)}(v_2) = 0.6$$
$$I_{f_{\varphi^{-1}(N_1)}(e_2)}(u_1) = I_{f_{N_1}(\psi(e_2))}(\varphi(u_1)) = I_{f_{N_1}(e_2)}(v_2) = 0.8$$
$$F_{f_{\varphi^{-1}(N_1)}(e_2)}(u_1) = F_{f_{N_1}(\psi(e_2))}(\varphi(u_1)) = F_{f_{N_1}(e_2)}(v_2) = 0.4$$
$$T_{f_{\varphi^{-1}(N_1)}(e_2)}(u_2) = T_{f_{N_1}(\psi(e_2))}(\varphi(u_2)) = T_{f_{N_1}(e_2)}(v_3) = 0.5$$
$$I_{f_{\varphi^{-1}(N_1)}(e_2)}(u_2) = I_{f_{N_1}(\psi(e_2))}(\varphi(u_2)) = I_{f_{N_1}(e_2)}(v_3) = 0.8$$
$$F_{f_{\varphi^{-1}(N_1)}(e_2)}(u_2) = F_{f_{N_1}(\psi(e_2))}(\varphi(u_2)) = F_{f_{N_1}(e_2)}(v_3) = 0.6$$
$$T_{f_{\varphi^{-1}(N_1)}(e_2)}(u_3) = T_{f_{N_1}(\psi(e_2))}(\varphi(u_3)) = T_{f_{N_1}(e_2)}(v_1) = 0.7$$
$$I_{f_{\varphi^{-1}(N_1)}(e_2)}(u_3) = I_{f_{N_1}(\psi(e_2))}(\varphi(u_3)) = I_{f_{N_1}(e_2)}(v_1) = 0.6$$
$$F_{f_{\varphi^{-1}(N_1)}(e_2)}(u_3) = F_{f_{N_1}(\psi(e_2))}(\varphi(u_3)) = F_{f_{N_1}(e_2)}(v_1) = 0.5$$

Thus $(\varphi,\psi)^{-1}(N_1,E) \notin \tau_u$ though $(\varphi,\psi)^{-1}(N_2,E) = (M_1,E)$. So $(\varphi,\psi)^{-1}$ is not neutrosophic soft continuous. Now,

$$\tau_u^{e_1} = \{(0,1,1),(1,0,0), f_{M_1}(e_1), f_{M_2}(e_1), f_{M_3}(e_1)\},$$
$$\tau_u^{e_2} = \{(0,1,1),(1,0,0), f_{M_1}(e_2), f_{M_2}(e_2), f_{M_3}(e_2)\};$$
$$\tau_v^{e_1} = \{(0,1,1),(1,0,0), f_{N_1}(e_1), f_{N_2}(e_1)\},$$
$$\tau_v^{e_2} = \{(0,1,1),(1,0,0), f_{N_1}(e_2), f_{N_2}(e_2)\};$$

Then, $(\varphi,\psi) : (U,\tau_u^{e_1}) \to (V,\tau_v^{e_1})$ is neutrosophic continuous mapping because $(\varphi,\psi)^{-1}[f_{N_1}(e_1)] = f_{M_2}(e_1)$ and $(\varphi,\psi)^{-1}[f_{N_2}(e_1)] = f_{M_1}(e_1)$.
Similarly, $(\varphi,\psi) : (U,\tau_u^{e_2}) \to (V,\tau_v^{e_2})$ is neutrosophic continuous mapping as : $(\varphi,\psi)^{-1}[f_{N_1}(e_2)] = f_{M_3}(e_2)$ and $(\varphi,\psi)^{-1}[f_{N_2}(e_2)] = f_{M_1}(e_2)$.

5.5 Theorem

For two neutrosophic soft topological spaces (U, E, τ_u) and (V, E, τ_v), let $(\varphi, \psi) : (U, E, \tau_u) \to (V, E, \tau_v)$ be a neutrosophic soft mapping. Then the following conditions are equivalent.

(1) (φ, ψ) is neutrosophic soft continuous mapping.
(2) The inverse image of a closed NSS in (V, E, τ_v) is closed in (U, E, τ_u).
(3) For each $(M, E) \in NSS(U, E)$, $(\varphi, \psi)\overline{(M, E)} \subset \overline{(\varphi, \psi)(M, E)}$.
(4) For each $(N, E) \in NSS(V, E)$, $\overline{(\varphi, \psi)^{-1}(N, E)} \subset (\varphi, \psi)^{-1}\overline{(N, E)}$.
(5) For each $(N, E) \in NSS(V, E)$, $(\varphi, \psi)^{-1}(N, E)^o \subset [(\varphi, \psi)^{-1}(N, E)]^o$.

Proof. $(1) \Rightarrow (2)$
Let, (Q, E) be a closed NSS in (V, E, τ_v). Then $(Q, E)^c \in \tau_v$ and so by (1), $(\varphi, \psi)^{-1}(Q, E)^c \in \tau_u$. But $(\varphi, \psi)^{-1}(Q, E)^c = ((\varphi, \psi)^{-1}(Q, E))^c$. So $(\varphi, \psi)^{-1}(Q, E)$ is a closed NSS in (U, E, τ_u).

$(2) \Rightarrow (3)$
Let, $(M, E) \in NSS(U, E)$. Since $(M, E) \subset (\varphi, \psi)^{-1}((\varphi, \psi)(M, E))$ and $(\varphi, \psi)(M, E) \subset \overline{(\varphi, \psi)(M, E)}$, we have $(M, E) \subset (\varphi, \psi)^{-1}((\varphi, \psi)(M, E)) \subset (\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$. Obviously, $\overline{(\varphi, \psi)(M, E)}$ is closed in (V, E, τ_v). Then by (2), $(\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$ is closed in (U, E, τ_u). But, since $(M, E) \subset \overline{(M, E)}$ and $\overline{(M, E)}$ is the smallest closed NSS, so $(M, E) \subset \overline{(M, E)} \subset (\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$. This implies $(\varphi, \psi)\overline{(M, E)} \subset (\varphi, \psi)[(\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})]$ i.e., $(\varphi, \psi)\overline{(M, E)} \subset \overline{(\varphi, \psi)(M, E)}$ is obtained.

$(3) \Rightarrow (4)$
Let, $(N, E) \in NSS(V, E)$ and $(\varphi, \psi)^{-1}(N, E) = (M, E)$. Then $\overline{(\varphi, \psi)^{-1}(N, E)} = \overline{(M, E)}$. But by (3), we have $\overline{(M, E)} \subset (\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$ i.e., $\overline{(\varphi, \psi)^{-1}(N, E)} \subset (\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$. This shows $\overline{(\varphi, \psi)^{-1}(N, E)} \subset (\varphi, \psi)^{-1}[\overline{(\varphi, \psi)((\varphi, \psi)^{-1}(N, E))}]$ i.e., $\overline{(\varphi, \psi)^{-1}(N, E)} \subset (\varphi, \psi)^{-1}\overline{(N, E)}$.

$(4) \Rightarrow (5)$
Let, $(N, E) \in NSS(V, E)$. Replacing (N, E) by $(N, E)^c$ and applying (4), we have $\overline{(\varphi, \psi)^{-1}(N, E)^c} \subset (\varphi, \psi)^{-1}(\overline{(N, E)^c})$ i.e., $[(\varphi, \psi)^{-1}(\overline{(N, E)^c})]^c \subset [\overline{(\varphi, \psi)^{-1}(N, E)^c}]^c$. By Theorem (ii) of [2.15.2], since $(N, E)^o = [\overline{(N, E)^c}]^c$, so $(\varphi, \psi)^{-1}(N, E)^o = (\varphi, \psi)^{-1}(\overline{(N, E)^c})^c = [(\varphi, \psi)^{-1}(\overline{(N, E)^c})]^c \subset [\overline{(\varphi, \psi)^{-1}(N, E)^c}]^c = [(\varphi, \psi)^{-1}(N, E)]^o$.

$(5) \Rightarrow (1)$
Let, (N, E) be an open NSS in (V, E, τ_v). Then $(N, E)^o = (N, E)$. Since $[(\varphi, \psi)^{-1}(N, E)]^o \subset (\varphi, \psi)^{-1}(N, E) = (\varphi, \psi)^{-1}(N, E)^o \subset [(\varphi, \psi)^{-1}(N, E)]^o$, so $[(\varphi, \psi)^{-1}(N, E)]^o = (\varphi, \psi)^{-1}(N, E)$ is obtained. Thus, $(\varphi, \psi)^{-1}(N, E)$ is an open NSS in (U, E, τ_u) and so (φ, ψ) is neutrosophic soft continuous mapping.

5.6 Theorem

Let, (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces. Also let, $(\varphi, \psi) : (U, E, \tau_u) \to (V, E, \tau_v)$ be a continuous neutrosophic soft mapping. If (M, E) is neutrosophic soft compact in (U, E, τ_u), then $(\varphi, \psi)(M, E)$ is so in (V, E, τ_v).

Proof. Let $\{(N_i, E) : i \in \Gamma\}$ be a neutrosophic soft open covering of $(\varphi, \psi)(M, E)$ i.e., $(\varphi, \psi)(M, E) \subset \cup_i (N_i, E)$. Since, (φ, ψ) is neutrosophic soft continuous, $\{(\varphi, \psi)^{-1}(N_i, E) : i \in \Gamma\}$ is a neutrosophic soft open cover of (M, E). But, (M, E) is neutrosophic soft compact. So, there exists a finite subcover $\{(\varphi, \psi)^{-1}(N_i, E) : 1 \leq i \leq k\}$ such that $(M, E) \subset \cup_{i=1}^{k} (\varphi, \psi)^{-1}(N_i, E)$ hold. Hence, $(\varphi, \psi)(M, E) \subset (\varphi, \psi)[\cup_{i=1}^{k} (\varphi, \psi)^{-1}(N_i, E)] = \cup_{i=1}^{k} (\varphi, \psi)[(\varphi, \psi)^{-1}(N_i, E)] = \cup_{i=1}^{k} (N_i, E)$.

This shows that $(\varphi, \psi)(M, E)$ is covered by a finite number of member of $\{(N_i, E) : i \in \Gamma\}$. Hence, $(\varphi, \psi)(M, E)$ is neutrosophic soft compact also.

5.7 Theorem

Let, (U, E, τ_u) be a neutrosophic soft topological space and (V, E, τ_v) be a neutrosophic soft Hausdorff space. Then, a neutrosophic soft function $(\varphi, \psi) : (U, E, \tau_u) \to (V, E, \tau_v)$ is closed if it is continuous.

Proof. Let (Q, E) be any neutrosophic soft closed set in (U, E, τ_u). Then by Theorem [4.2], (Q, E) is compact NSS. Since (φ, ψ) is continuous neutrosophic soft function then $(\varphi, \psi)(Q, E)$ is compact NSS in (V, E, τ_v). As (V, E, τ_v) is neutrosophic soft Hausdorff space, so $(\varphi, \psi)(Q, E)$ is closed by Theorem [4.3].

6 Conclusion

Topology is a major sector in mathematics and it can give many relationships between other scientific area and mathematical models. The motivation of the present paper is to extend the concept of topological structure on neutrosophic soft set introduced in the paper [33]. Here, we have defined connectedness and compactness on neutrosophic soft topological space, neutrosophic soft continuous mappings. These are illustrated by suitable examples. Their several related properties and structural characteristics have been investigated. We expect, this paper will promote the future study on neutrosophic soft topological groups and many other general frameworks.

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