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Beyond Miner's Rule
Free Energy Damage Equivalence

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ABSTRACT

In this paper, we extend the concept of damage which was original developed empirically by Miner using the concept of thermodynamic work and free energy. We first explain the equivalency of free energy to damage. Then we develop ways to measure the free energy in products to help assess the amount of damage that a product can experience prior to failure. The main approach is to use product's ultimate work energy. Although Miner's rule and the cumulative damage concept is typically thought of as specific for cyclic work, we note that in most cases, damage accumulates during a product useful life prior to failure.

1. INTRODUCTION

Miner's Rule [1] was perhaps one of the most important original physics of failure equations to be formulated. It gave us the concept of damage. Miner wrote the rule (published in 1945) in terms of a ratio for n_i cycles performed to N_i cycles to failure per each i^{th} stress level as

$$Damage = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_k}{N_k} = \sum_{i=1}^{K} \frac{n_i}{N_i} \qquad (1)$$

At the time it was simple enough, it provided a means of assessing cumulative damage. However the concept of measurable damage was born whether it is cumulative or otherwise. Today we can derive Miner's empirical rule using an energy approach [2]. The energy approach goes beyond Miner's rule for it is more general and exact; and is reasonably practical and accurate approach at the measurable level. In the evolution of the energy approach we measure damage in thermodynamic work terms W [2] as

$$Damage = \frac{\sum W_{actual}(t)}{W_{actual-failure}} \qquad (2)$$

Thus, we have a physics of failure law for damage whose origin came from Miner's formulation. This law can be stated [2]

- <u>*The measurable work damage ratio:*</u> *consists of the actual work performed to the actual work needed to cause system failure. In system failure, we exhaust the maximum amount of useful system work. To consistently find this damage ratio, all work found must be taken over the same work path.*

Of course one of the key issues is in the denominator, what is the value of the work to failure? If we know this we are in a good position to assess damage.

2. FREE ENERGY AND DAMAGE

To understand the problems at hand we need to ask the question, is there a way to predict the work to failure based on a material property? Typically, to estimate the work to failure, one either uses historical information from test coupons or a reliability test is performed to obtain the work to failure at a particular stress level. To understand this approach consider Einstein famous equation as an example

$$E=mc^2 \qquad (3)$$

This equation allows us to predict how much energy we can theoretically obtain from a given mass. We can ask, is there a classical analogy for assessing the potential useful work that can be achieved related to a known material property.

In thermodynamics, a materials free energy provides an assessment of the amount of useful work that a product can perform. This is often cited, but is in reality not currently a listed material property. This is because it is considered hard to assess and is often treated for academic interest only. In reality, if we can assess a materials free energy for a particular work 'path', i.e. the type of work and how it will be performed, then it would be a very useful property to know. Since free energy is associated with the material useful work, it is also equivalent to the amount of thermodynamic accumulated damage that can be allowed by a product. Another way of saying this is how much work can we get out of a product before it fails?

- *The work that can be done on or by the system is then bounded by the system's free energy [2]*

$$Work \leq \Delta \text{ Free Energy Change of the system} \quad (4)$$

If the system's free energy is at its lowest state, then the system is in equilibrium with the environment so ΔFree energy=0 and the system is completely degraded.

For the reliability engineer assessing the maximum amount of accumulative damage depends on material properties, geometry etc. In the case of metals, it includes how the metal is treated, what kind of work is being done and so forth.

3. FREE ENERGY DAMAGE EQUIVALENCE

In this paper, we will propose that a materials Ultimate Work Energy (W_{UE}) for a given failure mode or mechanism is the most measurable and useful property to assess a materials free energy, in analogy to Einstein's equation

- $(\Delta\text{Free Energy})_j^{th} \geq (\text{Maximum Ultimate Work Energy})_j^{th} = (\text{Maximum Damage Amount})_k^{th}$

where 'j' is for the j^{th} type failure mode/mechanism of failure. Here when analytical means are too difficult to calculate, establishing criteria would be of practical importance.

As damage increase, the free energy decreases and so does the available work. If the system's initial free energy is denoted by F_i (before aging) and the final free energy is denoted by F_f (after aging), then $F_f < F_i$. The system is in thermal equilibrium with the environment, when the free energy is minimized. At that point, the system has failed and the maximum amount of damage occurs

$$F_i - F_f = (\Delta Free\, Energy)_{Max-damage} = W_{failure}(UE) \quad (5)$$

This is the free energy damage equivalence in terms of energy damage units. Then damage equivalency is a unitless quantity (that is commonly used) and yields the damage equivalence free energy written as

$$Damage = \frac{\Delta Free\;\; Energy}{(\Delta Free\;\; Energy)_{Max-damage}} = \frac{\Delta Free\;\; Energy}{W_{failure}(UE)},$$
$$and\ D = 1,\ when\ \ \Delta Free\ Energy = W_{failure}(UE) \quad (6)$$

We know that often there are many ways a product can fail; therefore, we need to be specific for which failure mechanism we are concerned about. For example, stainless steel can fatigue, corrode, or fail due to corrosion assisted fatigue and so forth.

4. ULTIMATE WORK ENERGY

Sometime free energy is simple to calculate such as the Gibbs free energy for batteries so that it is easier to predict the useful energy-hours of work that a battery can perform. What if we short-circuited a battery? Looking at the ultimate energy of a battery for a short period time? This in theory should be equivalent to the batteries ultimate work energy. This is a fast-destructive way to estimate a batteries free energy

Battery Ultimate Work Energy (short circuit)
=Voltage x max-current x time *(6)*

It would be more accurate rather than using a short circuit to say test at 5 ohms, which is a bit less destructive and likely more accurate measurement of the batteries ultimate work energy, than a true short circuit. We can denote $W(UE)_{0+}$ as a measurement of the ultimate work energy for a very short time, so that

$$W(UE)_{0+} \approx W(UE) \quad (7)$$

The concept is to measure the ultimate work energy in a short time so that it is reasonably accurate and representative of the actual ultimate work energy. Using this method, we can predict the free energy for many failure modes.

- Unfortunately, tables of ultimate work energy do not currently exist for materials and would be a proposal of this paper as an essential reliability property. Ways to obtain this are discussed here.

5. REMAINING WORK-A SIMPLE EQUATION

Once we know the W(UE) for a particular failure mode, then energy can be subtracted when work is accomplished as damage accumulates.

If interim work is denoted by Wi, then the work remaining, Wr, in a product is

$$Wr = W(UE) - Wi \quad (8)$$

We can define Damage 'D' and undamage 'Du' portion of the work. The undamaged portion is simply

$$Du = (W_{ue} - W_i)/W_{us} = 1 - w_i/W_{ue} \quad (9)$$

and the damage amount is of course $D = 1 - D_u = w_i/W_{ue}$

- These seemingly simple equations are perhaps hard to appreciate their importance.

First of all, if you wanted to predict the damage assessed from one stress level to another, you typically need to look at stress acceleration factor, which is often a non linear relationship. Here we simply subtract off the thermodynamic work to establish the amount of work left in the product. This is the energy approach and in terms of work, is a simple linear relationship. Of course, this still depends on the stress level and the work path. Once established, such measurements are far less complex then using the traditional non-energy approaches.

6. EXAMPLE 1: PRIMARY BATTERY LIFE

Battery life is a simple first example as batteries are actually rated in amp-hours. This becomes an energy unit for any particular battery, as the voltage level is assumed.

For example, let's say a battery 9V is rated for 0.5 amp-hours. We see that the maximum work that can be at 0.5 amps for 1 hour is

$$\text{Max Work} = 9v \times 0.5A \times 1hr\ (3600\ sec.) = 16{,}200\ joules \quad (10)$$

If we did not know that actual Max Work and needed to measure it in a short period of time, we might select a 2 ohm resistor and short the battery with it. Then the current would be I=V/R=4.5 Amps. The actual time to measure the batteries W(US) is

$$W(UE)_{0+} = 16{,}200\ J/(9V \times 4.5A) = 400\ seconds = 6.7\ Minutes \quad (11)$$

The criteria for failure might be when the batteries voltage drops by 20% as an example. We can from experiments, decide if 2 ohms is reasonable or not. Here we know the ultimate work energy so in fact

$$W(UE)_{0+} = W(UE) \quad (12)$$

If the battery does work for ¼ of an hour at a rate of 0.1A the energy used is

$$(Work)_i = 9V \times 0.1A \times \tfrac{1}{4}\ hr\ (900\ sec.) = 810\ Joules \quad (13)$$

Then the work remaining in the battery is

$$W_r = W_{max} - W_i = 16{,}200 - 810 = 15{,}390\ joules$$

The Undamaged D_U rating is

$$D_u = 1 - w_i/W_{us} = 1 - 810/16{,}200 = 0.95\ or\ 95\% \quad (14)$$

and the damage amount D is of course,

$$D = 1 - D_u = w_i/W_{ue} = 0.05\ or\ 5\% \quad (15)$$

Note that the damage was a simple subtraction and we did not have to concern ourselves with the stress level that changed from 0.5 amps to 0.1 amp.

7. FATIGUE AND ULTIMATE WORK ENERGY

Fatigue life estimation is a more interesting and difficult problem for this approach. Fatigue strength of materials is a function of size, material properties, metal treatment (such as annealed) surface condition etc. In addition, data in the literature are not always in agreement.

To obtain an expression for fatigue cyclic work, we will look at plastic strain (ε) caused by a sinusoidal vibration level G stress (σ) in the material. A common model for the strain in this case is [1]

$$\varepsilon = \beta_o n^p G^j \quad (16)$$

The cyclic work is found as

$$w = \oint_{\Delta L} \sigma\, d\varepsilon = \oint_{\Delta L} G \frac{d\varepsilon}{dn} dn = A G^{j+1} n^P = A G^Y n^P \quad (17)$$

where Y=j+1. Similar to the above arguments, to assess the damage we need to have some knowledge of the critical damage at a repetitive vibration stress. Let's assume this occurs at N_1 cycles at stress level G_1. Then the thermodynamic damage ratio at any other stress G_2 level at n_2 cycle is [2]

$$\text{Vibration Damage} = \frac{w}{W_F} = \left(\frac{n_2}{N_1}\right)^P \left(\frac{G_2}{G_1}\right)^Y \quad (18)$$

If damage is represented by 1, $n_2 = N_2$, and failure occurs. We note the time acceleration factor is obtain in terms of cycles (N=f T, f is the frequency is considered constant, T is the time) as [2]

$$AF_D = \frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right) = \left(\frac{G_2}{G_1}\right)^b \quad (19)$$

where b=Y/P. AF_D is commonly used relationship for cyclic compression where we assumed the frequencies $f_1=f_2$.

This is a commonly used for the acceleration factor in sinusoidal testing. For random vibration above, we substitute for G the random vibration Grms level [2]. It is helpful to write the linear form for cycles to failure, for a particular stress. This is deduced to within a constant from the above equation (19)

$$N_1 = A(G_1)^{-b} \equiv A(G_{rms-1})^{-b} \qquad (20)$$

where $A = N_2 / G_2^{-b}$ treated as a constant. This is essentially the relation that holds for what is called the S-N curves. Note that if we write the cyclic equation with $G \propto S$ where S is the stress, we have

$$N_1 = C\,S_1^{-b} \text{ or } S_1 = K\,N_1^{-B} \qquad (21)$$

where B=1/b and C is the proportionality constant when going from G to S and K is a constant similar to C. The relationship is generally used to analyze S-N data, this is formally known as Basquin's equation which is used in the area of high cyclic fatigue.

8. *EXAMPLE 2: ULTIMATE WORK ENERGY - STAINLESS STEEL FATIGUE LIFE*

We are now in a position to look at an example. To use this approach for fatigue, first we need to understand that fatigue is dominated by tensile force rather than compressive force. That is, most of the damage in fatigue is due to tensile (rather than compressive) work. This helps us to identify the material's key property that we would need to know. Therefore, in this case, stainless steels ultimate tensile work strength for the material would be the key identified property of interest. Data on materials does not currently provide ultimate work energy (i.e. energy units). However, the ultimate strength (stress units) is provided which is a conjugate work variable (i.e. work=stress x strain). As an example, we will use 316L stainless steel. The properties of which are shown below

Table 1 Typical Properties of 316L Stainless Steel [4]

Properties	Stainless 316L
Yield strength	290 MPa
Ultimate Tensile Strength	560 MPa [4]
Fatigue/endurance limit*	309 MPa [4]

In our theory, we might be tempted to use N=1 cycle. However, experience has shown that for steel the S-N curve ultimate strength is closer to 1000 Cycles for 0.9 of the ultimate strength. This is similar to finding the ultimate work energy at a reasonable amount of time on a battery; we might use 5 ohms instead of a short circuit.

Furthermore it is well known that the endurance limit occurs around at 10^7 cycles. Therefore, we need to use some of the knowledge base since material properties do not actually give the ultimate work energy. Therefore from these two points we have

S_1=560 x 0.9=504 MPa at N_1=1000 Cycles, S_2=309 MPa at $N_2=10^7$ cycles

Then from our equations we can write

$$N_1 = N_2\left(\frac{G_1}{G_2}\right)^{-b}_{Sinusoidal} \equiv N_2\left(\frac{S_1}{S_2}\right)^{-b}_{Sinusoidal} \qquad (22)$$

where the slope is

1/b=-(logS1-logS2)/(logN2-LogN1)= 18.8 *(23)*

Next we did a S-N literature search of the material shown in Table 1 and plotted these results and compared them to that of the literature search shown in Figure 1. Comparisons are in good agreement for values and reasonable for the slope. The literature slope was 11.8.

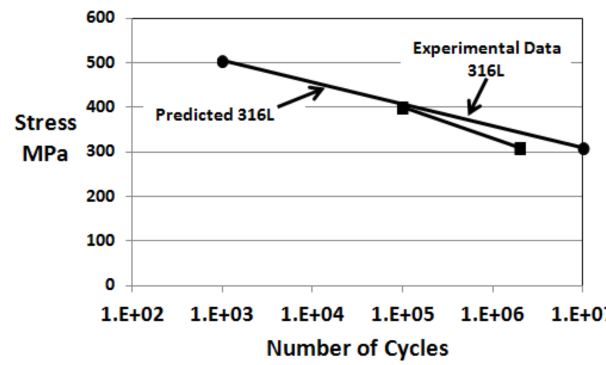

Figure 1 S-N Curve for 316L stainless 1) top curve predicted here, 2) from Reference 4

From this theory, what is missing is the work energy (or free energy) property of the material in order to fully appreciate the practical aspect of the energy approach.

9. *HIGH AND LOW CYCLIC FATIGUE THEORIES AND STRESS CONCENTRATION FACTORS*

Currently, cyclic fatigue is modeled in different regions of high cyclic fatigue (>10^5 cycles) and low cyclic fatigue (< 10^4 or 10^5 cycles). In theory, when using the energy approach, it is less likely that one would need to this type of modeling for high and low cyclic fatigue. This is another anticipated advantage in the area of fatigue when using the energy approach. Experimental work would help verify this anticipated result. Furthermore, fatigue theory often uses stress intensity k-factors (called knock down factors, such as surface finish, grain size, and/or corrosion effects) that affect fatigue strength in materials. This of course diminishes the free energy in the material and likely this k-factor would fit well with the energy approach since stress is a conjugate work variable [2]. For example with a notch stress 0<k<1

$$F_k = Free\ Energy_{without\ notch} \times k \qquad (24)$$

Therefore,

$$F_k < \text{Initial Free energy in the material} \quad (25)$$

10. FATIGUE/ENDURANCE LIMIT AND K1AC

Some materials are known to have a fatigue or endurance limit where below a certain stress level, fatigue does not occur or cannot easily be measured. In thermodynamic terms, the closest explanation to this is called, reversible work. To understand reversible work, its counterpart, irreversible work, may be helpful to first describe. Irreversible work is likely what one would imagine it to be, when we do irreversible work, we create damage that cannot be undone without at least providing some effort to make a repair. Therefore, reversible work is work done but no measurable damage has occurred! Since many purist would argue that reversible work is a conceptual term and in practice does not exist, we can simple describe reversible work as work that is done where no damage could be measured. This is along the same lines as a fatigue limit. Below the fatigue limit, the cyclic process is in the elastic region and plastic strain does not occur.

If we are below the fatigue limit, and we consider that work done below that limit as reversible, then in terms of cyclic work ***the total change of the internal energy of a material is zero with no damage so that the work is just converted to heat [1]***

$$\oint dU = \oint \delta W + \oint \delta Q = 0, \; or \quad -\oint \delta W = \oint \delta Q \quad (24)$$

Not all materials have a fatigue limit. In fact, most materials do not. Below is a partial list.

Reversible Limit Endurance/Fatigue Limit (Reversibility)	**No Reversible Limit No Endurance/Fatigue Limit**
-Low Strength Carbon & Alloy Steel -Some Stainless Steels & Irons -1045 Steel, Titanium Alloys -Some Polymers	-Aluminum, Magnesium, Copper, Nickel -Some Stainless Steels - Some high strength Carbons & Alloy steels

In fracture mechanics theory, the stress intensity factor can be related to the minimum stress needed for crack growth. This is similar to the irreversibility concept, below this stress or energy level, no irreversible damage occurs.

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Alec Feinberg is the founder of DfRSoft. He has a Ph.D. in Physics and is the principal author of the books, Design for Reliability and Thermodynamic Degradation Science: Physics of Failure, Accelerated Testing, Fatigue, and Reliability Applications. DfRSoft provides consulting in reliability and shock and vibration, training classes and DfRSoftware. DfRSoftware, used by numerous companies, helps solve problems in HALT, accelerated test, Weibull analysis, physics of failure, predictions, reliability growth, quality, etc. and is also used to accelerate learning in his training classes. Alec has provided reliability engineering services in diverse industries (AT&T Bell Labs, Tyco Electronics, HP, NASA, etc) for over 35 years in aerospace, automotive and electrical and mechanical systems. He has provided training classes in Design for Reliability & Quality, Shock and Vibration, HALT and ESD. Alec has presented numerous technical papers and won the 2003 RAMS best tutorial award for the topic, "Thermodynamic Reliability Engineering."