A 'constant Lagrangian' model for galactic dynamics in a geodetic approach towards the galactic rotation Dark Matter issue.

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I start with a historical note on the galactic rotation curves issue. The problem with the virial theorem in observed galactic dynamics, lead to the Dark Matter hypothesis but also to Modified Newtonian Dynamics or MOND. Then I move (away) from MOND towards a relativistic, Lagrangian approach of orbital dynamics in a curved Schwarzschild metric. I propose a 'constant Lagrangian' model for galactic scale geodetic dynamics. I shown with four rotation fitting curves to what extend my proposed model galaxies 'constant Lagrangian' postulate works in these limited number of situations. The fitted galaxies are NGC 2403, NGC 3198, UGC 6614 and F571-8. In the paper I present a theoretical context in which the 'constant Lagrangian' postulate might replace the classical virial theorem on a galactic scale. But the proposed postulate isn't a 'general law of nature' because in the solar system and in the GNSS relativistic context, the classical virial theorem is proven accurate. Due to the limitations of the proposed postulate, a statement regarding Dark Matter can't be made. But the model might achieve within the GR-Schwarzschild paradigm what MOND achieves within the Newtonian paradigm, fitting the experimental galactic rotation curves.

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I. THE VIRIAL THEOREM IN TROUBLE ON THE GALACTIC SCALE.

In 1932 the Dutch astronomer Oort observed that the stars in the galactic vicinity of the Sun are moving peculiarly fast, almost 8 times as fast as could be inferred from the calculated Newtonian acceleration. Oort assumed that dark matter would be the cause of this apparent difference, with 'dark' referring to ordinary matter not seen by us due to various reasons (Oort, 1932).

In 1933 Dark Matter was mentioned as "dunkle Materie" in a paper by Zwicky. Fritz Zwicky was studying the Coma Cluster of galaxies and found that his calculations for orbital acceleration and stellar mass within it was off by a large factor. He concluded that there should be a much greater density of dark matter within the cluster than there was luminous matter. Zwicky concluded that this constituted an unsolved problem (Zwicky, 1933). In 1937 Zwicky regarded his study on the Coma Cluster a test of Newton's law of gravity on the largest cosmological scale possible, by applying the virial theorem on a cluster of galaxies. He also mentioned in his 1937 paper the possibility to test the virial theorem by applying it to the rotational velocities of the individual stars in the separate galaxies. But he concluded that this was technologically out of reach (Zwicky, 1937).

The breakthrough research of Rubin and Ford around 1970-1975 established beyond doubt the outer rotational velocity curves of individual galaxies, which turned out to be flat (Rubin et al., 1978). This was in conflict with velocity curves that resulted from the application of the virial theorem to the luminous mass of these galaxies. Rubin and Ford cited colleagues who suggested the existence of a large galactic halo of dark matter. In a 1980 paper presenting further research they concluded that the form of the rotation curves implied that significant non-luminous mass should be located at large distances beyond the optical galaxy. The total mass of a galaxy should, for large distances, increase at least as fast as the distance from the center (Rubin et al., 1980).

The third major evidence for Dark Matter was the gravitational lensing effect of clusters of galaxies. The mass of stars and hot gas in clusters who collectively act as a gravitational lens is too small to bend the light from the background galaxies as much as they actually do. A large density of dark matter in the center of these cluster is needed to explain the strength of the observed lensing effect (Koopmans et al., 2009).

In the course of decades it has become more and more clear that ordinary matter can't

be the cause of those observed phenomena. That realization caused the term 'dark matter' to evolve into 'Dark Matter', with the capital letters indicating its elusive character. Today it has been predominantly, but not unanimously, been accepted that non-baryonic particles must exist in the calculated densities. A range of different astrophysical measurements point in this direction. I quote:

> *Astrophysical observations have provided compelling evidence for the existence of a non-baryonic dark component of the universe: dark matter (DM). The currently most accurate, although somewhat indirect, determination of DM abundance comes from global fits of cosmological parameters to a variety of observations, while the nature of DM remains largely unknown. One of the candidates for a DM particle is a weakly interacting massive particle (WIMP). (The ATLAS Collaboration, 2018)*

II. MOND

One of the few non-particle approaches to the problem of Dark Matter is MOND or MOdified Newtonian Dynamics. MOND started in 1983 with two seminal paper of Milgrom. I quote from his papers:

> *All determinations of dynamical mass within galaxies and galaxy systems make use of a virial relation of the form $V^2 = MGr^{-1}$ where V is some typical velocity of particles in the system, r is of the order of the size of the system, M is the mass to be determined, and G is the gravitational constant. [...] It must have occurred to many that there may, in fact, not be much hidden mass in the universe and that the dynamical masses determined on the basis of the above virial relation are gross overestimates of the true gravitational masses.(Milgrom, 1983b)*

> *I have considered the possibility that Newton's second law does not describe the motion of objects under the conditions which prevail in galaxies and systems of galaxies. In particular I allowed for the inertia term not to be proportional to the acceleration of the object but rather be a more general function of it. With some simplifying assumptions I was led to the form*

$$m_g \mu \left(\frac{a}{a_0} \right) \mathbf{a} = \mathbf{F},$$

replacing $m_g\mathbf{a} = \mathbf{F}$. [...] For accelerations much larger than the acceleration constant (a_0), $\mu \approx 1$, and the Newtonian dynamics is restored. (Milgrom, 1983b)

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, assuming that galaxies contain no hidden mass, with the following main results. 1. The Keplerian, circular velocity around a finite galaxy becomes independent of r at large radii, thus resulting in asymptotically flat velocity curves. 2. The asymptotic circular velocity (V_∞) is determined only by the total mass of the galaxy (M): $V_\infty^4 = a_0 GM$, where a_0 is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass. (Milgrom, 1983a)

The original Tully-Fisher relation is a relation between the luminosity of a spiral galaxy and its, maximum, rotation velocity (Tully and Fisher, 1977). The physical basis of the Tully-Fisher relation is the relation between a galaxy's total baryonic mass and the velocity at the flat end of the rotation curve, the final velocity. According to McGaugh both stellar and gas mass of galaxies have to be taken into account in the relation that is referred to as the Baryonic Tully-Fisher (BTF) relation (McGaugh, 2005). In 2005 McGaugh determined the baryonic version of the LT relation as $M_d = 50v_f^4$, see (McGaugh, 2005) and Fig(1). In this form, M_d is expressed in solar mass $M_\odot = 1,99 \cdot 10^{30}$ kg units and the final velocity of the galactic rotation velocity curve v_f is expressed in km/s. If we express the galactic mass in kg and the velocity in m/s we get the total baryonic mass, final velocity relations in SI unit values as $M_b = 1,0 \cdot 10^{20}v_f^4$.

In 1983, Milgrom interpreted the BTF relation as an indication of a deviation from Newtonian gravity, making a modification of Newtonian dynamics or MOND necessary (Milgrom, 1983b). Using McGaug's 2005 values in SI units, Milgrom presented the BTF relation in the form $v_f^4 = 1,0 \cdot 10^{-20}M_b = Ga_0M_b$, resulting in an acceleration $a_0 = 1,5 \cdot 10^{-10}$ m/s^2 in McGaug's values. Milgrom hypothesized that this relation should hold exactly, thus interpreting it as an inductive law of nature instead of looking at it as just an empirical relation (Milgrom, 1983a). The resulting acceleration can be written as $5 \cdot a_0 \approx cH_0$, with the velocity of light c and the Hubble constant H_0. According to Milgrom, the deeper significance
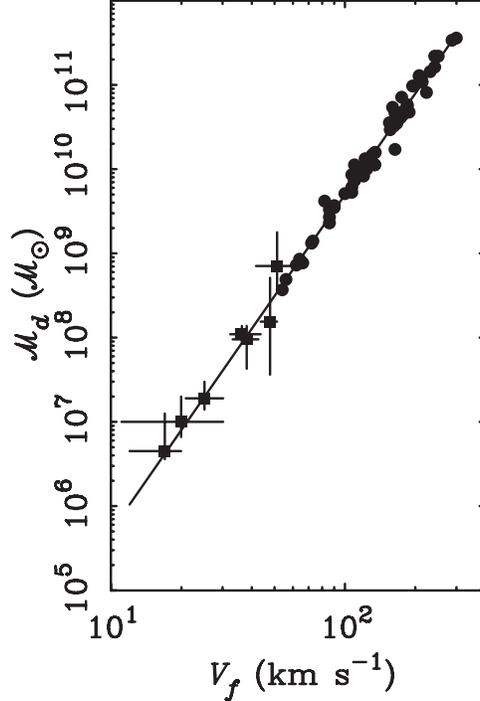
FIG. 1. The Baryonic Tully-Fisher relation. Reprint from McGaug 2005 (McGaugh, 2005).

of this relation between this special galactic acceleration and the Hubble acceleration should be revealed by future cosmological insights (Milgrom, 1983b).

III. CLASSICAL LAGRANGIAN DYNAMICS

The Lagrangian equation of motion reads

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0. \tag{1}$$

In classical gravitational dynamics I assume circular orbits with $\dot{q} = v$ and $q = r$. The Lagrangian itself is then given by $L = K - V$, with V the Newtonian potential gravitational energy and K the kinetic energy. One then gets

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{dp}{dt} = F. \tag{2}$$

The other part gives

$$\frac{\partial L}{\partial q} = -\frac{dV}{dr}, \tag{3}$$

so one gets Newton's equation of motion in a central field of gravity

$$F_g = -\frac{dV}{dr}. \tag{4}$$

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Further analysis of the context results in the identification of the Hamiltonian of the system, $H = K + V$, as being a constant of the orbital motion and the virial theorem as describing a relation between K and V in one single orbit but also between different orbits, $2K + V = 0$.

On the galactic scale it is assumed that velocities are so low and gravitational fields are so weak, that Newtonian mechanics suffices and not much of relativity is needed. The problem with the rotational velocities of stars in galaxies and galaxies in cluster of galaxies is thus supposed to be a Newtonian physics issue that can be dealt with in the dynamics described above. The Dark Matter solution to the too fast rotational galactic velocities has two faces. On the one hand one tries to describe the density distribution of Dark Matter, needed in order to match the measurements with classical dynamics, specifically the virial theorem. On the other hand one tries to identify the Dark Matter constituents, usually seen as an out-of-the-box extension of the known Standard Model of particle physics.

IV. A GEODETIC APPROACH OF GRAVITATIONAL ORBITS

The problem with the previous analysis is connected to the notion of geodetic motion in General Relativity. The problem can best be described in a semi-relativistic approach using the classical Lagrangian equation of motion for geodetic orbits. The most important aspect of geodetic motion in GR is that it requires no force to move on a geodetic. This has important implications for the Lagrangian equation of motion, because $F = 0$ on a geodetic. One gets

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = F_g = 0 \tag{5}$$

and as a consequence also

$$\frac{\partial L}{\partial q} = -\frac{dL}{dr} = 0. \tag{6}$$

As a result, one gets the crucial

$$L = K - V = constant \tag{7}$$

on geodetic orbits.

This result, the Lagrangian of the system as being the constant of the geodetic motion, is used on a daily basis by many of us because it is used by GNSS systems for the relativistic correction of atomic clocks in their satellites. Let's elaborate this a bit further. In General

Relativity, the proper time-rate $d\tau$ is defined through the metric distance ds as $ds \equiv cd\tau$. The square metric distance is defined through

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu. \tag{8}$$

Given coordinate world time-rate dt, which is the time-rate of a standard clock at a position where $d\tau = dt$ (in GR-Schwarzschild this implies a clock at rest at infinity), we get the general

$$\frac{ds^2}{dt^2} = \frac{c^2 d\tau^2}{dt^2} = g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt} = g_{\mu\nu}V^\mu V^\nu, \tag{9}$$

with the geodesic four-vector velocity V^μ. In this equation, $d\tau$ stands for the local proper clock-rate of a clock in a geodetic orbit in a field of gravity and dt is the universal clock-rate. Because of this interpretation of dt, the velocity V^μ is the velocity as seen from a position where $d\tau = dt$. See for example (Singer, 1956), (Weinberg, 1972, p. 79), (Misner et al., 1973, p. 1054-1055), (Straumann, 1984, p. 97), (Ohanian and Ruffini, 2013, p. 119).

In case of the Schwarzschild metric in polar coordinates, we have (Ruggiero et al., 2008)

$$ds^2 = \left(1 + \frac{2\Phi}{c^2}\right)c^2 dt^2 - \left(1 + \frac{2\Phi}{c^2}\right)^{-1}dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2. \tag{10}$$

In case of a clock on a circular geodesic on the equator of a central non-rotating mass M we have $\frac{dr}{dt} = 0$, $\frac{d\theta}{dt} = 0$, $sin\theta = 1$ and $\frac{d\phi}{dt} = \omega$. We thus get

$$\frac{ds^2}{dt^2} = \frac{c^2 d\tau^2}{dt^2} = \left(1 + \frac{2\Phi}{c^2}\right)c^2 - r^2\omega^2 \tag{11}$$

and

$$\frac{d\tau^2}{dt^2} = 1 + \frac{2\Phi}{c^2} - \frac{r^2\omega^2}{c^2}. \tag{12}$$

With $v_{orbit} = r\omega$ we have

$$\frac{d\tau^2}{dt^2} = 1 + \frac{2\Phi}{c^2} - \frac{v_{orbit}^2}{c^2}. \tag{13}$$

So finally we get the GR result

$$\frac{d\tau}{dt} = \sqrt{1 + \frac{2\Phi}{c^2} - \frac{v_{orbit}^2}{c^2}} \tag{14}$$

with $d\tau$ as the clock-rate of a standard clock A in a geodetic orbit and dt as the 'universal' clock-rate G of a standard clock at rest in infinity, the only condition for which $d\tau = dt$. The result of Eqn. (14) is the basic relativistic correction used in GNSS clock frequencies, with the first as the gravity effect or gravitational potential correction and the second as the

velocity effect or the correction due to Special Relativity (Ashby, 2002; Hećimović, 2013; Delva and Lodewyck, 2013).

Given the classical definitions of $K = \frac{mv_{orbit}^2}{2}$ and $V = m\Phi$, we get

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2L}{U_0}}. \tag{15}$$

All the satellites of a GNSS system are being installed on a similar orbit and thus syntonized relative to one another because they share the same high and velocity and have constant L and $\frac{d\tau}{dt}$ on those orbits. But different GNSS systems, as for example GPS compared to GALILEO, are functioning on different orbits with different velocities and those systems aren't syntonized relative to one another. This non-syntonization between satellites on orbits with different heights and virial theorem connected velocities is very annoying for the effort towards realizing an integration of the different GNSS systems into one single global network.

V. A RELATIVISTIC VIRIAL THEOREM FOR A MODEL GALAXY

When I connected

$$\frac{d\tau^2}{dt^2} = 1 + \frac{2\Phi}{c^2} - \frac{v_{orbit}^2}{c^2} = 1 - \frac{2L}{U_0} \tag{16}$$

to the problem of the galactic rotation curve, I realized that the flat rotation curve implies atomic clock synchronization in those areas. In those outer regions, the potential can be assumed to be zero and the velocity constant. Those flat rotation rate zones are the GNSS engineer's dream come true. This made me curious as to the clock-rate status in the inner regions. The intriguing thing is that you can jump from orbit to orbit and still encounter a constant clock-rate on all the orbiting satellites you encounter on an imaginary voyage through the outer regions of galaxies. This implies that precisely in those regions where the classical virial theorem seems in trouble, $L \simeq constant$, not just in one single orbit *but also between different orbits*. It should be clear that for those geodetic orbits the classical virial theorem, which in its most essential form states that $F_{gravity} = F_{centripetal}$, becomes meaningless because on circular geodetics this reduces to the empty expression $0 = 0$.

In order to study the relativistic clock-rate behavior in the inner regions of galaxies, I had to construct a model galaxy. Real galaxies are way to fussy, complex and messed up to get interpretable results. My model galaxy is build of a model bulge with mass M and radius

R and a Schwardschild metric emptiness around it. The model bulge has constant density $\rho_0 = \frac{M}{V} = \frac{3M}{4\pi R^3}$ and its composing stars rotate on geodetics in a quasi-solid way. So all those stars in the bulge have equal angular velocity on their geodetic orbits, with $v = \omega r$. On the boundary between the quasi solid spherical bulge and the emptiness outside of it, the orbital velocities are behaving smoothly. So the last star in the bulge and the first star in the Schwarzschild region have equal velocities and potentials. I also assume that the Newtonian potential itself is unchanged and unchallenged, remains classical in the whole galaxy and its surroundings. Such a model galaxy doesn't have a SMBH in the center of its bulge and it only has some on occasional and very lonely stars in the space outside the bulge.

Point	Relation	Expression
Outside the **bulge**	$r > R$	$-\dfrac{GM}{r}$
On the Surface	$r = R$	$-\dfrac{GM}{R}$
Inside the **bulge**	$r < R$	$-GM\left[\dfrac{3R^2 - r^2}{2R^3}\right]$
At the centre	$r = 0$	$-\dfrac{3}{2}\left(\dfrac{GM}{R}\right)$
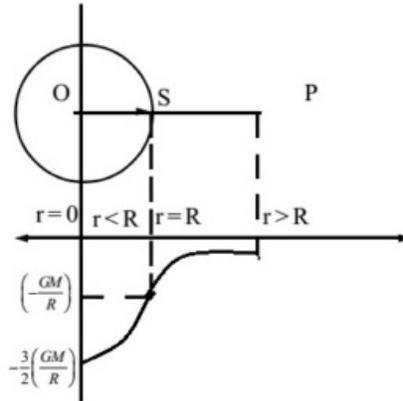
FIG. 2. The potential inside and out of a model bulge

The gravitational potential in such a case is well known, see Fig.(2). If this sphere would be in a condition where the classical virial theorem would hold, so $2K = -V$, then on the boundary $r = R$ we would have $K = \frac{GM}{2R}$ and $L = K - V = \frac{3GM}{2R}$. At the center of the rotating sphere, $K = 0$ and we also have $L = \frac{3GM}{2R}$.

From $r = 0$ to $r = R$, the potential Φ increased as r^2. The kinetic energy does the same

because $v^2 = \omega^2 r^2$. One can conclude that they increase identical and that $L = K - V$ is a constant inside the quasi-solid sphere. We can write for the region from $r = 0$ to $r = R$

$$\frac{L}{m} = \frac{v_{orbit}^2}{2} + \frac{GM}{r} = \frac{3GM}{2R} = constant. \tag{17}$$

As a result, in such a model bulge, L is a constant of the motion, not only in one orbit but also between orbits. All the clocks in such a model bulge would be syntonized.

Thus, in the model galaxy that I am about to construct, we have $L = constant$ inside the model bulge and we have $L = constant$ in the outer regions where the rotational velocity curve flattens and the Newtonian potential turns negligibly small. So let's be bold and declare $L = K - V = constant$ in the entire galaxy, without changing the Newtonian potential. What would that have as effects?
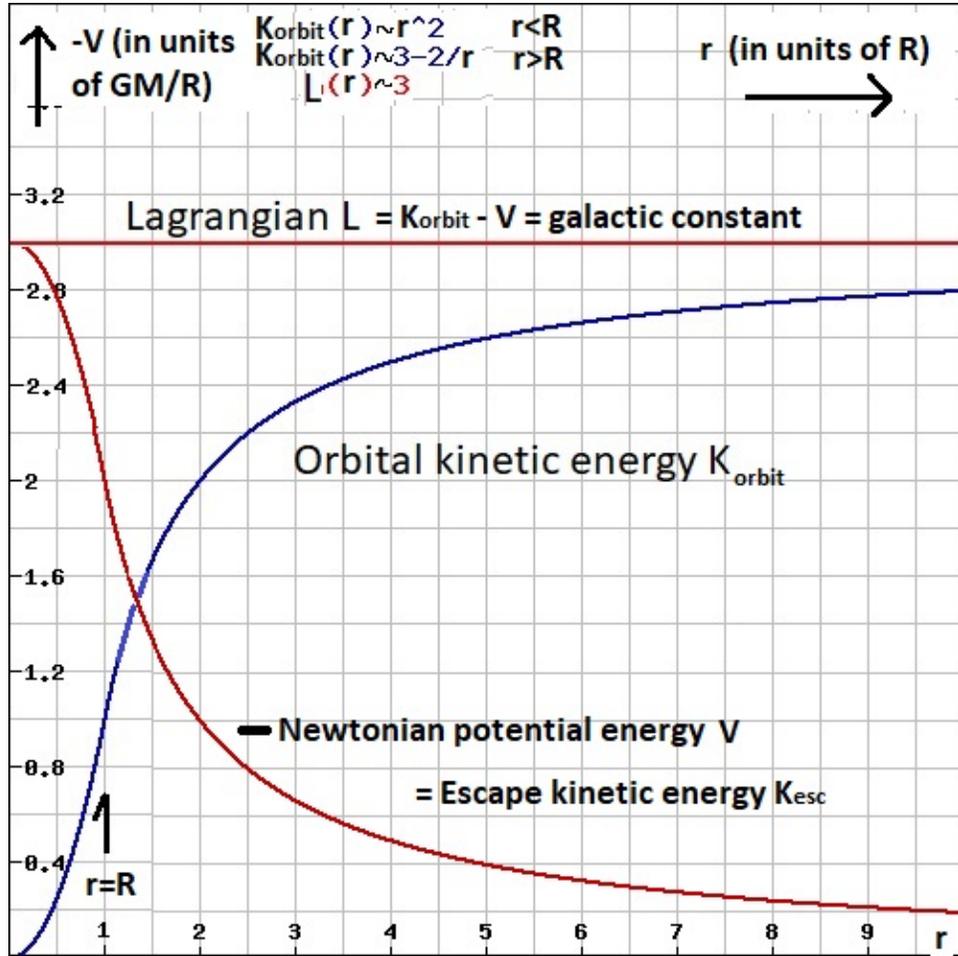
FIG. 3. The square of the orbital velocity profile in the model galaxy with $L = constant$.

We would get $K = L + V$ and $L = V(r = 0)$ so for the region $0 \leq r \leq R$ we get

$$v_{orbit}^2 = \frac{GM}{R} \cdot \frac{r}{R} \tag{18}$$

and outside the model bulge, where $R \leq r \leq \infty$, we have

$$v_{orbit}^2 = \frac{3GM}{R} - \frac{GM}{r}. \tag{19}$$

In Fig.(3) I sketched the result, with $-V = +K_{escape}$.

From the perspective of a free fall Einstein elevator observer, the free fall on a radial geodetic from infinity towards the center of the bulge, the other free fall tangential geodetics seem to abide the law of conservation of energy, because the escape kinetic energy plus the orbital kinetic energy is a constant on my model galaxy with galactic constant L. An Einstein elevator system with test mass m that would be put in an orbital collapse situation, magically descending from orbit to orbit in a process in thermodynamic equilibrium, would have constant total kinetic energy, from the radial free fall perspective. This can be expressed as $L = K_{orbit} - V = K_{orbit} + K_{escape} = K_{final}$.

Such a model galaxy would also be a GNSS engineer's dream come true because the whole model galaxy is in one single syntonized time-bubble.

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2L}{U_0}}. \tag{20}$$

Given the Baryonic Tully-Fisher relation in Milgrom's version $v_{final}^4 = Ga_0 M$ with $2\pi a_0 \approx cH_0$, with a_0 as Milgrom's galactic minimum acceleration and H_0 as the Hubble constant, we get as a galactic time bubble fix

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2L}{U_0}} = \sqrt{1 - \frac{v_{final}^2}{c^2}} = \sqrt{1 - \sqrt{\frac{v_{final}^4}{c^4}}} = \sqrt{1 - \sqrt{\frac{v_{final}^4}{c^4}}} = \tag{21}$$

$$\sqrt{1 - \sqrt{\frac{Ga_0 M}{c^4}}} = \sqrt{1 - \sqrt{\frac{GH_0 M}{2\pi c^3}}} = \sqrt{1 - \sqrt{\frac{M}{2\pi M_U}}}, \tag{22}$$

in which I used $L = 3GM/R = K_{final} = \frac{1}{2}mv_{final}^2$ and $M_U = \frac{c^3}{GH_0}$. This last constant can be referred to as an apparent mass of the Universe, a purely theoretical number constant, see (Mercier, 2015). In a model Universe, this would imply that my model galaxy would be in a proper time bubble with clock-rate $d\tau$ relative to the universal clock-rate dt in proportion to the masses of galaxy M and Universe M_U. In my model galaxy theoretical environment

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the Baryonic Tully-Fisher relationship implies that the galactic time bubble is fixed through the mass of my model galaxy and that this fix is a cosmological one. So what is a universal acceleration minimum a_0 in MOND can be interpreted as a universally correlated (through M_U) but still local (through M) time bubble fix in my model galaxy geodetic environment.

This doesn't imply that I can integrate my model galaxy approach into MOND, because you either take the perspective of geodetic motion without any force of gravity, or you don't have a curved metric and use the classical gravitational acceleration approach. My approach of $L = constant$ started with setting $F_g = 0$ in the Lagrangian equation of motion. Milgrom started by modifying Newton's second law, leading to an adapted $F_g \neq 0$. You can't have it both ways. My approach of $L = constant$ and Milgrom's $F_g = ma\mu$ are mutually exclusive.

VI. FITTING FOUR REAL GALACTIC ROTATION CURVES

Having determined the model galactic velocity rotation curve based on the Lagrangian as a galactic constant of orbital motion, the question is to what extend real galaxies can be modeled in this way. For this I used the experimental velocity rotation data of four galaxies: NGC 2403, NGC 3198, UGC 6614 and F571-8. I plotted them in Excell. The velocity rotation curve data come from different sources. The NGC 2403 data are from (Begeman, 2006, p. 51). The UGC 6614 and F571-8 data are from (McGaugh et al., 2001) and were retrieved from the data website of McGaugh. The NGC 2403 data are from (Karukes et al., 2015, p. 2) and brought to my attention by (Vossos and Vossos, 2017).

In this section I present the plots of V_{orb}^2 against r, with in each plot the experimental values in red stars and the theoretical values in black bars. The fitting plots are given in two versions. The first plot is with one single fit for M and R, this is the pure model. In the second plot the parameters M and R are used as 'free' parameters for every single measurement, within reasonable constraints given by the first plot and by some common sense. Thus in plot 2 the apparent model mass density of the bulge ρ_{bulge} decreases as r increases. With this parameter freedom of almost one single value, M and R in ρ_{bulge}, all four experimental curves could be fitted really nice. The most important cut in the model is the change from the model bulge to the model empty space around it. In the model bulge, $V_{orb}^2 \propto r^2$, outside the model bulge $V_{orb}^2 \propto -r^{-1}$. In the fixed fitting curve, the apparent mass density of the bulge is the main variable that changes due to more realistic circumstances.

The excell data sheets of the plots are in the appendix.
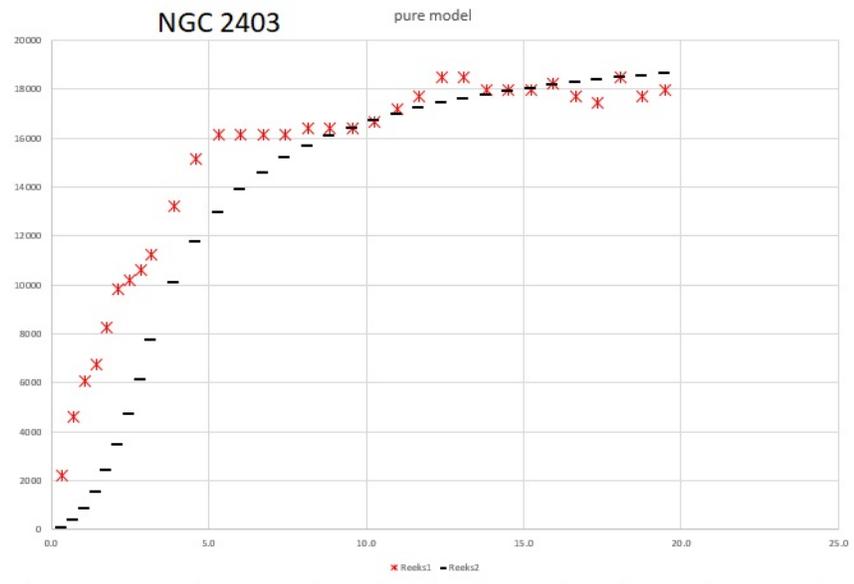
FIG. 4. UGC 2403 Plot1, V_{orb}^2 against r, pure model.
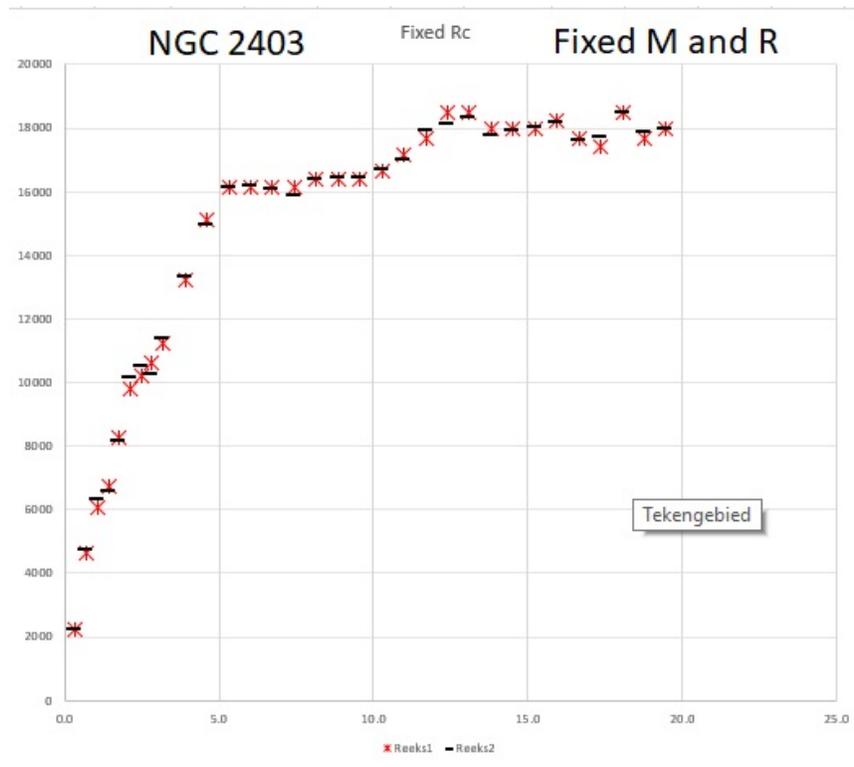
FIG. 5. UGC 2403 Plot1, V_{orb}^2 against r, fixed model.

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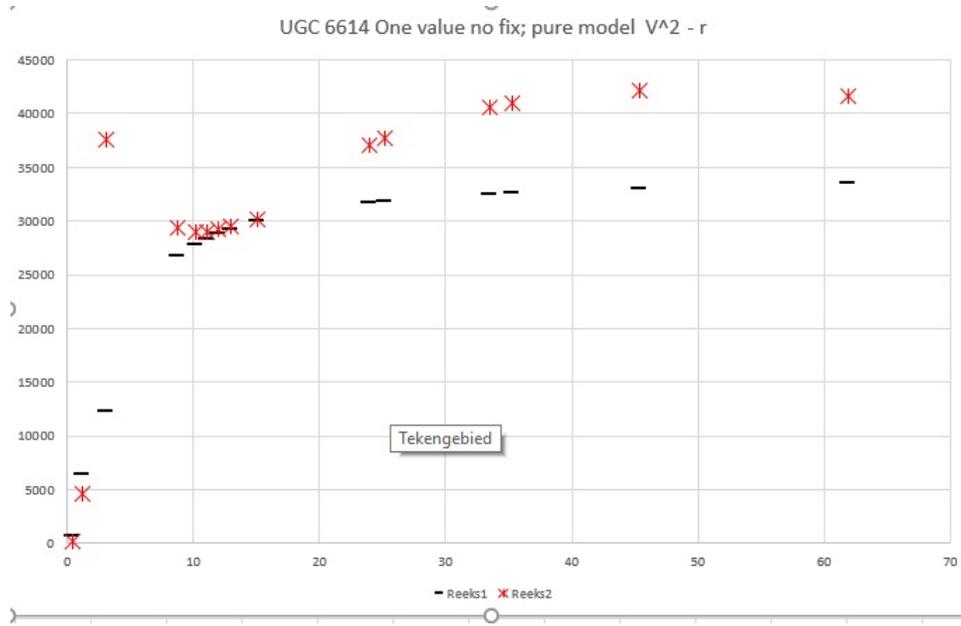
FIG. 6. UGC 6614 Plot1, V_{orb}^2 against r, pure model.
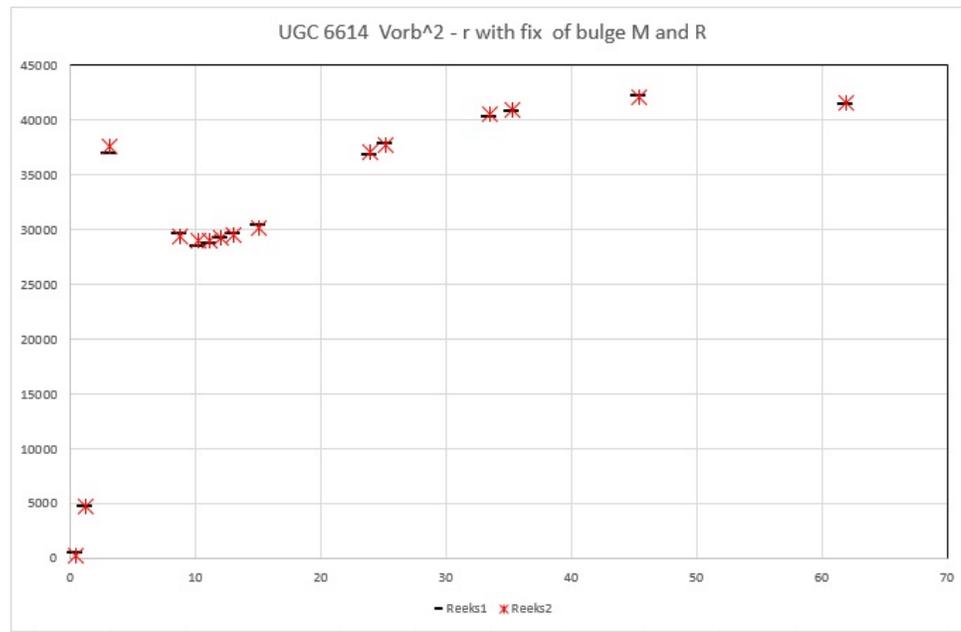
FIG. 7. UGC 6614 Plot1, V_{orb}^2 against r, fixed model.

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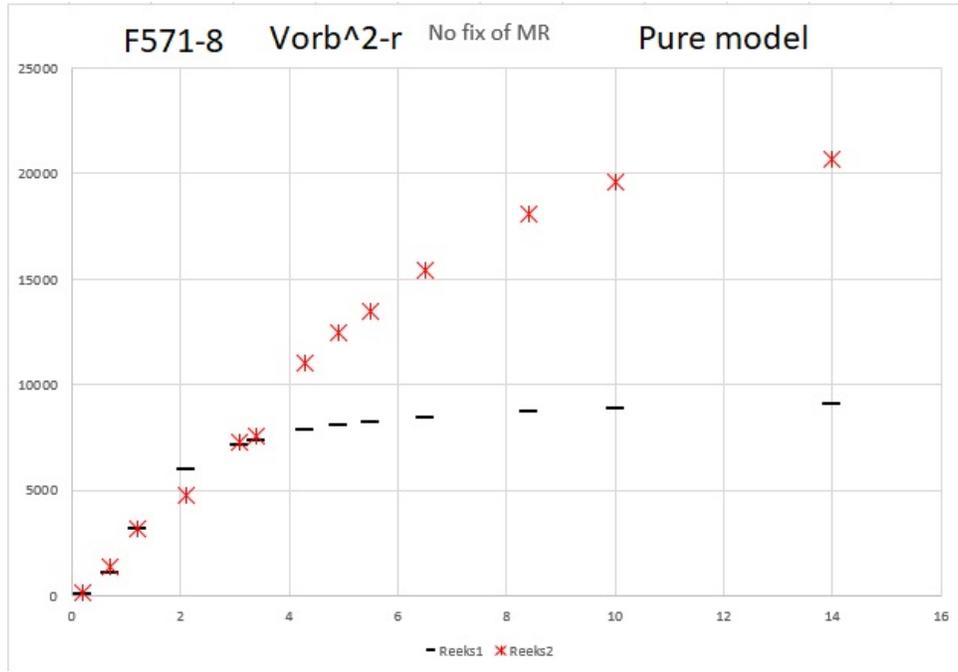
FIG. 8. F571-8 Plot1, V_{orb}^2 against r, pure model.
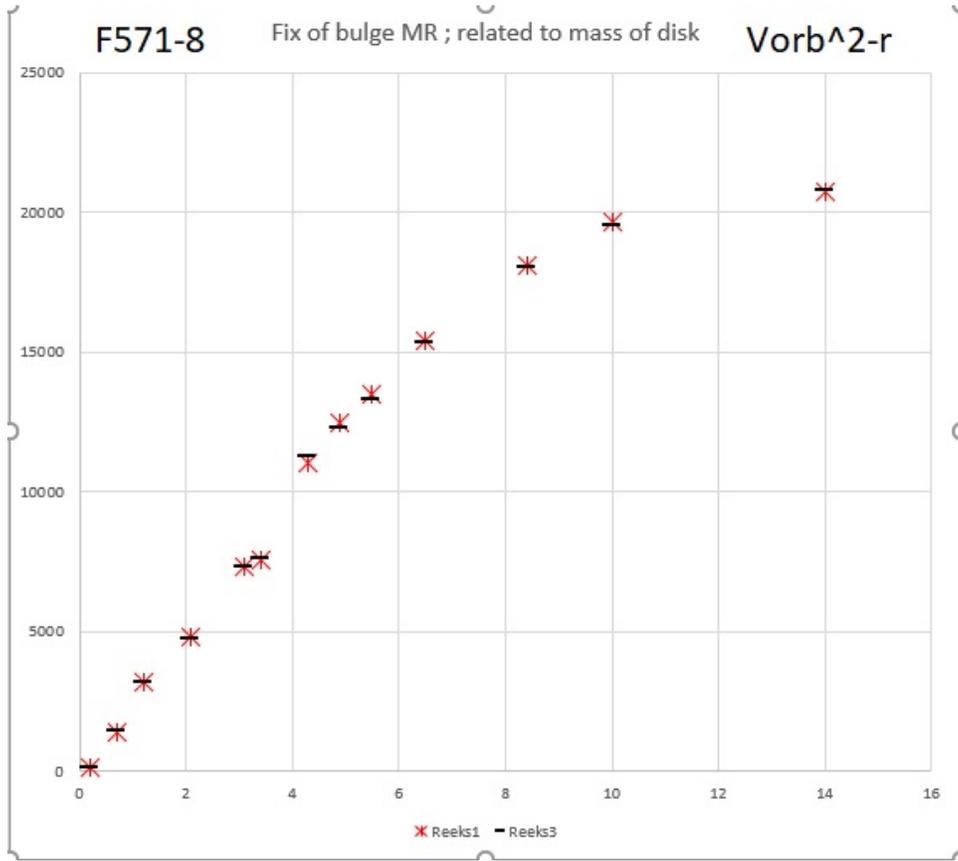
FIG. 9. F571-8 Plot1, V_{orb}^2 against r, fixed model.

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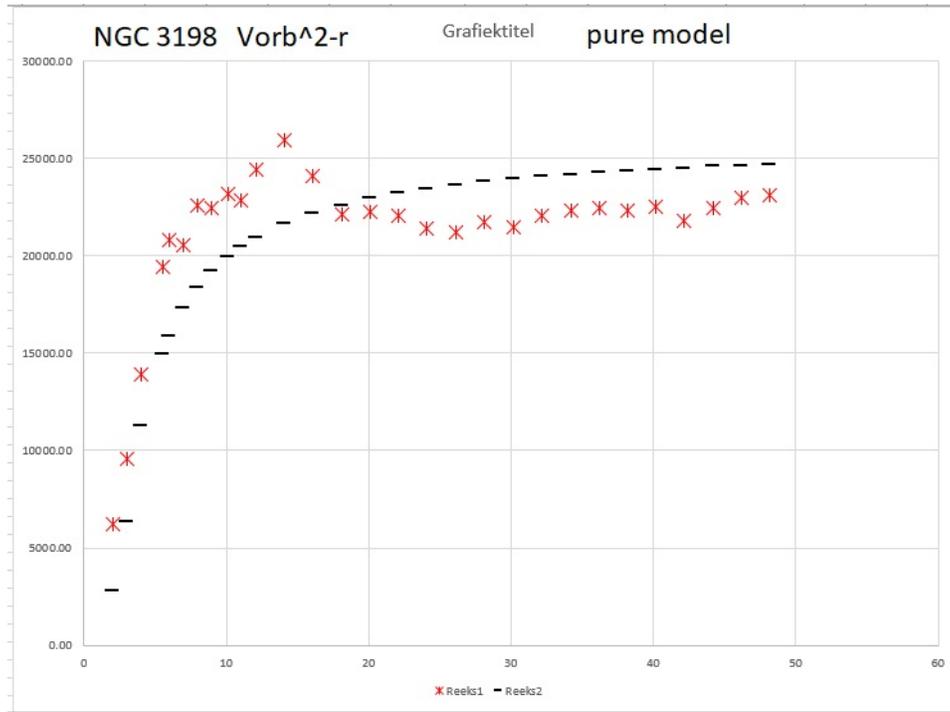
FIG. 10. NGC 3198 Plot1, V_{orb}^2 against r, pure model.
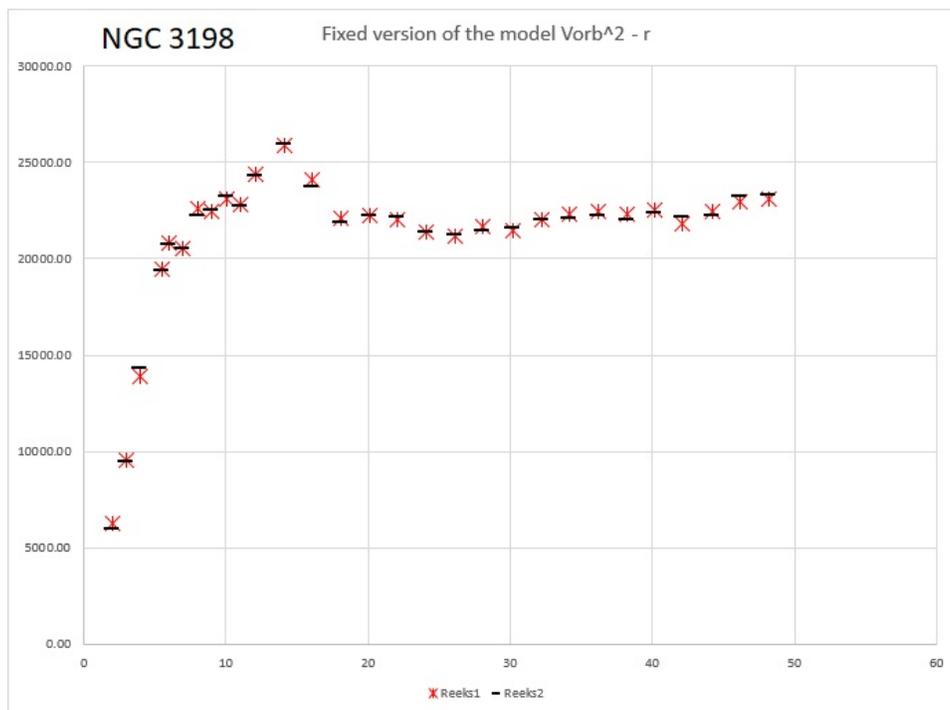
FIG. 11. NGC 3198 Plot1, V_{orb}^2 against r, fixed model.

VII. DARK MATTER, AN UNRESOLVED ISSUE

What about 'dark matter' and Dark Matter? Well, the early assumption of Oort and Zwicky that the astronomers where not seeing a lot of ordinary matter, the 'dark matter' postulate, turned out to be falsified. The attention then turned towards Dark Matter in the sense on non-baryonic (=non-ordinary Standard Model) stuff. The search for Dark Matter continues at ever increasing strengths.

My model galaxy, obeying to the 'constant Lagrangian' condition, just focuses on one aspect related to Dark Matter, the galactic rotation velocity curves. There are more issues leading to the Dark Matter hypothesis, unrelated to my model galaxy approach. Such as gravitational lensing, the galaxy cluster virial problem and cosmology related issues.

I have shown in four rotation fitting curves that my proposed model galaxies 'constant Lagrangian' postulate works in a limited number of situations. I also gave a theoretical context in which the 'constant Lagrangian' postulate might replace the classical virial theorem on a galactic scale. But it isn't a 'general law of nature' because in the solar system and in the GNSS relativistic context, the classical virial theorem is proven accurate.

The question regarding Dark Matter depends on the status of the 'constant Lagrangian' postulate. If it is a cosmological law of nature, then we don't need Dark Matter. But then we do need to explain why the classical virial theorem is valid in the context of the solar system. If Einstein's theory is fundamental, then Newton's needs to be justified. If the 'constant Lagrangian' postulate only functions (if it functions in that context in the first place) at the scale of individual galaxies, then the postulate needs further justification. A Dark Matter density distribution curve can easily provide such a justification.

The problem with deriving such a density distribution function is that one then mixes two mutually exclusive axiomatic systems. I would have to add the classical energy situation, as expected according to the classical virial theorem, to my $L = K - V$ plot, drawn in a geodetic context where F_g is supposed to be zero. It can be done quite easily, but at the price of mixing mutually exclusive theoretical axiomatic approaches. This mixing of axioms results for $r > R$ in

$$\rho_{DM} = \frac{3M}{4\pi}\left(\frac{2}{Rr} - \frac{2}{r^2} - \frac{1}{r^2}ln\left(\frac{r}{R}\right)\right) \tag{23}$$

or, with $\rho_0 = M/V = 3M/4\pi R$, in

$$\rho_{DM} = \rho_0 \left(\frac{2R^2}{r} - \frac{2R^3}{r^2} - \frac{R^3}{r^2} ln\left(\frac{r}{R}\right) \right), \tag{24}$$

with M and R referring to the mass and the radius of the pure model galactic bulge.

I can conclude that the 'constant Lagrangian' postulate leads to an interesting model galaxy and that the pure model can be adjusted using the two key parameters, the model bulge's mass M and radius R, to match the four galaxy rotation curves to which is was exposed. The model itself doesn't decide on the existence of Dark Matter, because the postulate doesn't justify itself but is in need of external justification. On that level will the Dark Matter discussion play out. The model is presented in the context of Special and General relativity, it is a metric approach with the Schwarzschild metric and the related time dilation formula at its core. As such, it might be an interesting addition to the MOND approach towards galactic and cosmological virial theorem issues.

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	G =	6.674E-11		GMunit/Runit =	#### (m/s)^2 Vunit^2
Runit=	1kpc =	###### meter		Vunit =	208 km/s
Munit=	1E+10*M	2.00E+40 kg		Vunit^2 =	43227 (km/s)^2
	Mdisk =	11.4	Disk:	GM/R = 4.11E+10 (m/s)^2	#### (km/s)^2
	R disk =	12	Vdisk =	203 km/s	
	Mbulge	0.8	Bulge:	GM/R = 1.73E+10 (m/s)^2	#### (km/s)^2
	R bulge	2	Vbulge:	132 km/s	

ρbulge=3M/(4*3.14*R^3)*1000
ρfixed

1 arcmin	0.345	R^2 kpc	R arcsec	R kpc	V km/s	Vmodel	V (km/s)'	V^2mod	unit GM	Mfix	Rfix	fix GM/R	ρbulge	3MG/R	Vfinal km/s
voor NG kpc		0.13	22.5	0.4	47	10	2209	97	####	0.48	3	6.92E+03	4.2	2.08E+04	144
		0.50	45	0.7	68	20	4624	386	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		1.13	67.5	1.1	78	29	6084	869	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		2.01	90	1.4	82	39	6724	1545	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		3.14	112.5	1.8	91	49	8281	2414	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		4.52	135	2.1	99	59	9801	3476	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		6.15	157.5	2.5	101	69	10201	4732	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		8.04	180	2.8	103	78	10609	6115	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		10.17	202.5	3.2	106	88	11236	7742	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		15.20	247.5	3.9	115	101	13225	10110	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		21.22	292.5	4.6	123	108	15129	11748	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		28.26	337.5	5.3	127	114	16129	12950	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		36.29	382.5	6.0	127	118	16129	13869	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		45.33	427.5	6.7	127	121	16129	14595	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		55.38	472.5	7.4	127	123	16129	15182	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		66.43	517.5	8.2	128	125	16384	15667	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		78.49	562.5	8.9	128	127	16384	16075	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		91.55	607.5	9.6	128	128	16384	16422	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		105.61	652.5	10.3	129	129	16641	16721	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		120.68	697.5	11.0	131	130	17161	16982	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		136.76	742.5	11.7	133	131	17689	17211	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		153.84	787.5	12.4	136	132	18496	17414	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		171.92	832.5	13.1	136	133	18496	17535	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		191.01	877.5	13.8	134	133	17956	17757	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		211.10	922.5	14.5	134	134	17956	17904	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		232.20	967.5	15.2	134	134	17956	18037	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		254.30	1012.5	15.9	135	135	18225	18158	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		277.41	1057.5	16.7	133	135	17689	18269	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		301.52	1102.5	17.4	132	136	17424	18370	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		326.64	1147.5	18.1	136	136	18496	18464	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		352.76	1192.5	18.8	133	136	17689	18551	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		379.88	1237.5	19.5	134	136	17956	18631	43253	0.48	3	6.92E+03	4.2	2.08E+04	144

FIG. 12. UGC 2403 Excell datasheet 1, V_{orb}^2 against r, pure model.

		Mdisk =	11.4	Disk:	GM/R =	4.11E+10	(m/s)^2 =	4.11E+04	(km/s)^2			
		R disk =	12		Vdisk =	203	km/s					
		Mbulge =	0.8	Bulge:	GM/R =	1.73E+10	(m/s)^2 =	1.73E+04	(km/s)^2			
		R bulge =	2		Vbulge =	132	km/s					ρbulge=3M/(4*3.14*R^3)*1000

1 arcmin =	0.945	R^2 kpc^	R arcsec	R kpc	V km/s	Vmodel	V (km/s)'	V^2mode	unit GM/F	Mfix	Rfix	fix GM/R	ρbulge	3MG/R	Vfinal km/s
voor NGC 24	kpc	0.13	22.5	0.4	47	47	2209	2252	4.33E+04	0.48	1.05	1.98E+04	99.0	5.93E+04	244
		0.50	45	0.7	68	69	4624	4747	43253	0.48	1.3	1.60E+04	52.2	4.79E+04	219
		1.13	67.5	1.1	78	79	6084	6301	43253	0.48	1.55	1.34E+04	30.8	4.02E+04	200
		2.01	90	1.4	82	81	6724	6589	43253	0.48	1.85	1.12E+04	18.1	3.37E+04	183
		3.14	112.5	1.8	91	90	8281	8148	43253	0.48	2	1.04E+04	14.3	3.11E+04	176
		4.52	135	2.1	99	101	9801	10135	43253	0.48	2.1	9.89E+03	12.4	2.97E+04	172
		6.15	157.5	2.5	101	102	10201	10500	43253	0.48	2.3	9.03E+03	9.4	2.71E+04	165
		8.04	180	2.8	103	101	10609	10267	43253	0.48	2.5	8.30E+03	7.3	2.49E+04	158
		10.17	202.5	3.2	106	107	11236	11406	43253	0.48	2.55	8.14E+03	6.9	2.44E+04	156
		15.20	247.5	3.9	115	115	13225	13304	43253	0.48	2.6	7.99E+03	6.5	2.40E+04	155
		21.22	292.5	4.6	123	122	15129	14942	43253	0.48	2.6	7.99E+03	6.5	2.40E+04	155
		28.26	337.5	5.3	127	127	16129	16144	43253	0.48	2.6	7.99E+03	6.5	2.40E+04	155
		36.29	382.5	6.0	127	127	16129	16176	43253	0.48	2.7	7.69E+03	5.8	2.31E+04	152
		45.33	427.5	6.7	127	127	16129	16078	43253	0.48	2.8	7.41E+03	5.2	2.22E+04	149
		55.38	472.5	7.4	127	126	16129	15898	43253	0.48	2.9	7.16E+03	4.7	2.15E+04	147
		66.43	517.5	8.2	128	128	16384	16383	43253	0.48	2.9	7.16E+03	4.7	2.15E+04	147
		78.49	562.5	8.9	128	128	16384	16427	43253	0.48	2.95	7.04E+03	4.5	2.11E+04	145
		91.55	607.5	9.6	128	128	16384	16422	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		105.61	652.5	10.3	129	129	16641	16721	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		120.68	697.5	11.0	131	130	17161	16982	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		136.76	742.5	11.7	133	134	17689	17927	43253	0.48	2.9	7.16E+03	4.7	2.15E+04	147
		153.84	787.5	12.4	136	135	18496	18130	43253	0.48	2.9	7.16E+03	4.7	2.15E+04	147
		171.92	832.5	13.1	136	135	18496	18311	43253	0.48	2.9	7.16E+03	4.7	2.15E+04	147
		191.01	877.5	13.8	134	133	17956	17757	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		211.10	922.5	14.5	134	134	17956	17904	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		232.20	967.5	15.2	134	134	17956	18037	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		254.30	1012.5	15.9	135	135	18225	18158	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		277.41	1057.5	16.7	133	133	17689	17599	43253	0.48	3.1	6.70E+03	3.8	2.01E+04	142
		301.52	1102.5	17.4	132	133	17424	17701	43253	0.48	3.1	6.70E+03	3.8	2.01E+04	142
		326.64	1147.5	18.1	136	136	18496	18464	43253	0.48	3	6.92E+03	4.2	2.08E+04	144
		352.76	1192.5	18.8	133	134	17689	17881	43253	0.48	3.1	6.70E+03	3.8	2.01E+04	142
		379.88	1237.5	19.5	134	134	17956	17961	43253	0.48	3.1	6.70E+03	3.8	2.01E+04	142
					(+- 2-3)										

FIG. 13. UGC 2403 Excell datasheet 1, V_{orb}^2 against r, fixed model.

		Mdisk =	12	Disk:	GM/R =	4.33E+10	(m/s)^2 =	4.33E+04	(km/s)^2	
		R disk =	12		Vdisk =	208	km/s			
		Mbulge =	0.3	Bulge:	GM/R =	8.11E+09	(m/s)^2 =	8.11E+03	(km/s)^2	
		R bulge =	1.6		Vbulge =	90	km/s			ρbulge=3M/(4*3.14*R^3)*1000

Rkpc	R^2	fix GM/R	V^2 mode	Vexp^2	Vmodel	Vexp	Vdisk^2	Vbulge^2	unit GM/F	Mfix	Rfix	fix GM/R	ρbulge	3MG/R	Vfinal km/s
0.4	0.16	1.15E+04	721	172	26.8	13.1	98	484	4.33E+04	0.8	3	1.15E+04	7.1	3.46E+04	186
1.2	1.44	1.15E+04	6488	4651	80.5	68.2	767	3831.61	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
3.1	9.61	1.15E+04	12278	37636	110.8	194	1706	9781.21	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
8.8	77.44	1.15E+04	26738	29412	163.5	171.5	1962	5461.21	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
10.2	104.04	1.15E+04	27818	28968	166.8	170.2	2088	4774.81	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
11.1	123.21	1.15E+04	28368	28934	168.4	170.1	2314	4395.69	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
12	144	1.15E+04	28836	29207	170	170.9	2611	4096	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
13	169	1.15E+04	29279	29515	171	171.8	3025	3782.25	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
15.1	228.01	1.15E+04	30020	30137	173	173.6	3745	3249	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
23.9	571.21	1.15E+04	31707	37095	178	192.6	3552	2052.09	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
25.2	635.04	1.15E+04	31856	37675	178	194.1	3329	1953.64	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
33.5	1122.25	1.15E+04	32537	40522	180	201.3	2970	1459.24	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
35.3	1246.09	1.15E+04	32642	40925	181	202.3	3170	1383.84	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
45.4	2061.16	1.15E+04	33078	42107	182	205.2	2884	1082.41	43253	0.8	3	1.15E+04	7.1	3.46E+04	186
61.9	3831.61	1.15E+04	33485	41575	183	203.9	1858	795.24	43253.4	0.8	3	1.15E+04	7.1	3.46E+04	186

FIG. 14. UGC 6614 Excell datasheet 1, V_{orb}^2 against r, pure model.

FIG. 15 — UGC 6614 Excell datasheet 1

		G =	6.674E-11			GMunit/Runit =	4.33E+10	(m/s)^2 =	Vunit^2
	Runit=	1kpc =	3.086E+19	meter		Vunit =	208	km/s	
	Munit=	1E+10*Mo	2.00E+40	kg		Vunit^2 =	43227	(km/s)^2	
		Mdisk =	11.4	Disk:	GM/R =	4.11E+10	(m/s)^2 =	4.11E+04	(km/s)^2
		R disk =	12	Vdisk =	203	km/s			
		Mbulge =	0.3	Bulge:	GM/R =	8.11E+09	(m/s)^2 =	8.11E+03	(km/s)^2
		R bulge =	1.6	Vbulge =	90	km/s			ρbulge=3M/(4*3.14*R^3)*1000

Rkpc	R^2	GM/R	V^2 mode	Vexp^2	Vmodel	Vexp	Vdisk^2	Vbulge^2	unit GM/R	Mfix	Rfix	fix GM/R	ρbulge	3MG/R	Vfinal km/s
0.4	0.16	8.32E+03	520	172	22.8	13.1	98	484	4.33E+04	0.25	1.3	8.32E+03	27.2	2.50E+04	158
1.2	1.44	8.32E+03	4679	4651	68.4	68.2	767	3831.61	43253	0.25	1.3	8.32E+03	27.2	2.50E+04	158
3.1	9.61	1.98E+04	36995	37636	192.3	194	1706	9781.21	43253	0.8	1.75	1.98E+04	35.7	5.93E+04	244
8.8	77.44	1.14E+04	29640	29412	172.2	171.5	1962	5461.21	43253	0.79	3	1.14E+04	7.0	3.42E+04	185
10.2	104.04	1.18E+04	28513	28968	168.9	170.2	2088	4774.81	43253	0.82	3	1.18E+04	7.3	3.55E+04	188
11.1	123.21	1.17E+04	28723	28934	169.5	170.1	2314	4395.69	43253	0.81	3	1.17E+04	7.2	3.50E+04	187
12	144	1.17E+04	29196	29207	171	170.9	2611	4096	43253	0.81	3	1.17E+04	7.2	3.50E+04	187
13	169	1.17E+04	29645	29515	172	171.8	3025	3782.25	43253	0.81	3	1.17E+04	7.2	3.50E+04	187
15.1	228.01	1.17E+04	30395	30137	174	173.6	3745	3249	43253	0.81	3	1.17E+04	7.2	3.50E+04	187
23.9	571.21	1.34E+04	36859	37095	192	192.6	3552	2052.09	43253	0.93	3	1.34E+04	8.2	4.02E+04	201
25.2	635.04	1.37E+04	37830	37675	194	194.1	3329	1953.64	43253	0.95	3	1.37E+04	8.4	4.11E+04	203
33.5	1122.25	1.43E+04	40264	40522	201	201.3	2970	1459.24	43253	0.99	3	1.43E+04	8.8	4.28E+04	207
35.3	1246.09	1.44E+04	40803	40925	202	202.3	3170	1383.84	43253	1	3	1.44E+04	8.8	4.33E+04	208
45.4	2061.16	1.47E+04	42175	42107	205	205.2	2884	1082.41	43253	1.02	3	1.47E+04	9.0	4.41E+04	210
61.9	3831.61	1.43E+04	41437	41575	204	203.9	1858	795.24	43253.4	0.99	3	1.43E+04	8.8	4.28E+04	207

FIG. 15. UGC 6614 Excell datasheet 1, V_{orb}^2 against r, fixed model.

FIG. 16 — F571 8 Excell datasheet 1

		G =	6.674E-11			GMunit/Runit =	4.33E+10	(m/s)^2 =	Vunit^2
	Runit=	1kpc =	3.086E+19	meter		Vunit =	208	km/s	
	Munit=	1E+10*Mo	2.00E+40	kg		Vunit^2 =	43227	(km/s)^2	
		Mdisk =	9.7	Disk:	GM/R =	2.69E+10	(m/s)^2 =	2.69E+04	(km/s)^2
		R disk =	15.6	Vdisk =	164	km/s			
		Mbulge =	0.098	Bulge:	GM/R =	3.53E+09	(m/s)^2 =	3.53E+03	(km/s)^2
		R bulge =	1.2	Vbulge =	59	km/s			ρbulge=3M/(4*3.14*R^3)*1000

Rkpc	GM/R	V^2 mode	V^2	Vmodel	Vexp	Vdisk^2	Vbulge^2	unit GM/R	Mfix	Rfix	fix GM/R	ρbulge	3MG/R	Vfinal km/s
0.2	3.21E+03	89	154	9.4	12.4	31	289	4.33E+04	0.089	1.2	3.21E+03	12.3	9.62E+03	98
0.7	3.21E+03	1092	1406	33.0	37.5	250	2052	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
1.2	3.21E+03	3208	3192	56.6	56.5	548	3376	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
2.1	3.21E+03	5958	4775	77.2	69.1	1056	2959	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
3.1	3.21E+03	7140	7310	84.5	85.5	1505	2228	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
3.4	3.21E+03	7359	7586	85.8	87.1	1576	2061	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
4.3	3.21E+03	7833	11025	89	105	1798	1560	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
4.9	3.21E+03	8053	12455	90	111.6	1866	1362	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
5.5	3.21E+03	8224	13456	91	116	1910	1176	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
6.5	3.21E+03	8439	15426	92	124.2	1910	986	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
8.4	3.21E+03	8707	18117	93	134.6	1781	745	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
10	3.21E+03	8854	19628	94	140.1	1616	625	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
14	3.21E+03	9074	20707	95	143.9	1190	441	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98

FIG. 16. F571 8 Excell datasheet 1, V_{orb}^2 against r, pure model.

	G =	6.674E-11			GMunit/Runit =	4.33E+10 (m/s)^2 =	Vunit^2						
Runit=	1kpc =	3.086E+19	meter		Vunit =	208 km/s							
Munit=	1E+10*M☉ =	2.00E+40	kg		Vunit^2 =	43227 (km/s)^2							
	Mdisk =	9.7		Disk:	GM/R =	2.69E+10 (m/s)^2 =	2.69E+04 (km/s)^2						
	R disk =	15.6		Vdisk =	164 km/s								
	Mbulge =	0.098		Bulge:	GM/R =	3.53E+09 (m/s)^2 =	3.53E+03 (km/s)^2						
	R bulge =	1.2		Vbulge =	59 km/s						ρbulge=3M/(4*3.14*R^3)*1000		

Rkpc	V^2 model	V^2	Vmodel	Vexp	Vdisk^2	2Vbulge^2	unit GM/R	Mfix	Rfix	fix GM/R	ρbulge	3MG/R	Vfinal km/s
0.2	156	154	12.5	12.4	31		4.33E+04	0.13	1	5.62E+03	31.1	1.69E+04	130
0.7	1472	1406	38.4	37.5	250		43253	0.11	1.1	4.33E+03	19.7	1.30E+04	114
1.2	3208	3192	56.6	56.5	548	6751.22	43253	0.089	1.2	3.21E+03	12.3	9.62E+03	98
2.1	4766	4775	69.0	69.1	1056	5918.72	43253	0.089	1.5	2.57E+03	6.3	7.70E+03	88
3.1	7317	7310	85.5	85.5	1505	4455.68	43253	0.114	1.5	3.29E+03	8.1	9.86E+03	99
3.4	7608	7586	87.2	87.1	1576	4122.32	43253	0.115	1.5	3.32E+03	8.1	9.95E+03	100
4.3	11266	11025	106	105	1798	3120.5	43253	0.16	1.5	4.61E+03	11.3	1.38E+04	118
4.9	12305	12455	111	111.6	1866	2723.22	43253	0.17	1.5	4.90E+03	12.0	1.47E+04	121
5.5	13306	13456	115	116	1910	2352.98	43253	0.18	1.5	5.19E+03	12.7	1.56E+04	125
6.5	15324	15426	124	124.2	1910	1971.92	43253	0.202	1.5	5.82E+03	14.3	1.75E+04	132
8.4	18002	18117	134	134.6	1781	1490.58	43253	0.23	1.5	6.63E+03	16.3	1.99E+04	141
10	19499	19628	140	140.1	1616	1250	43253	0.245	1.5	7.06E+03	17.3	2.12E+04	146
14	20799	20707	144	143.9	1190	882	43253	0.255	1.5	7.35E+03	18.0	2.21E+04	149

FIG. 17. F571 8 Excell datasheet 1, V_{orb}^2 against r, fixed model.

R^2 kpc^2	R kpc	V km/s	Vmodel	V (km/s)^	V^2model	unit GM/R	Mfix	Rfix	fix GM/R	ρbulge	3MG/R	Vfinal km/s
4.00	2	79.00	53	6241.00	2825	4.33E+04	0.7	3.5	8.65E+03	3.90	2.60E+04	161
9.00	3	97.80	80	9564.84	6356	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
16.00	4	118.00	106	13924	11299	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
30.25	5.5	139.40	122	19432.4	14942	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
36.00	6	144.20	126	20793.6	15860	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
49.00	7	143.30	132	20534.9	17301	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
64.00	8	150.30	136	22590.1	18383	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
81.00	9	149.90	139	22470	19224	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
102.01	10.1	152.10	141	23134.4	19957	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
121.00	11	151.10	143	22831.2	20447	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
146.41	12.1	156.20	145	24398.4	20948	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
198.81	14.1	161.00	147	25921	21657	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
259.21	16.1	155.30	149	24118.1	22191	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
327.61	18.1	148.70	150	22111.7	22606	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
404.01	20.1	149.10	151	22230.8	22939	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
488.41	22.1	148.40	152	22022.6	23212	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
580.81	24.1	146.20	153	21374.4	23439	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
681.21	26.1	145.50	154	21170.3	23632	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
789.61	28.1	147.30	154	21697.3	23797	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
912.04	30.2	146.50	155	21462.3	23947	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
1036.84	32.2	148.40	155	22022.6	24071	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
1169.64	34.2	149.30	156	22290.5	24181	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
1310.44	36.2	149.90	156	22470	24279	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
1459.24	38.2	149.30	156	22290.5	24367	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
1616.04	40.2	150.00	156	22500	24446	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
1772.41	42.1	147.60	157	21785.8	24514	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
1953.64	44.2	149.80	157	22440	24582	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
2134.44	46.2	151.50	157	22952.3	24641	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161
2323.24	48.2	151.90	157	23073.6	24696	43253	0.7	3.5	8.65E+03	3.90	2.60E+04	161

FIG. 18. NGC 3198 Excell datasheet 1, V_{orb}^2 against r, pure model.

R^2 kpc^2		R kpc	V km/s	Vmodel	V (km/s)^2	V^2model	unit GM/R	Mfix	Rfix	fix GM/R	ρbulge	3MG/R	Vfinal km/s
4.00		2	79.00	78	6241.00	6007	4.33E+04	0.48	2.4	8.65E+03	8.29	2.60E+04	161
9.00		3	97.80	97	9564.84	9493	43253	0.48	2.7	7.69E+03	5.82	2.31E+04	152
16.00		4	118.00	120	13924	14350	43253	0.48	2.85	7.28E+03	4.95	2.19E+04	148
30.25		5.5	139.40	139	19432.36	19371	43253	0.65	2.85	9.86E+03	6.71	2.96E+04	172
36.00		6	144.20	144	20793.64	20751	43253	0.65	2.8	1.00E+04	7.07	3.01E+04	174
49.00		7	143.30	143	20534.89	20558	43253	0.65	2.95	9.53E+03	6.05	2.86E+04	169
64.00		8	150.30	149	22590.09	22226	43253	0.67	2.95	9.82E+03	6.23	2.95E+04	172
81.00		9	149.90	150	22470.01	22540	43253	0.67	3	9.66E+03	5.93	2.90E+04	170
102.01		10.1	152.10	152	23134.41	23241	43253	0.67	3	9.66E+03	5.93	2.90E+04	170
121.00		11	151.10	151	22831.21	22776	43253	0.67	3.1	9.35E+03	5.37	2.80E+04	167
146.41		12.1	156.20	156	24398.44	24296	43253	0.7	3.1	9.77E+03	5.61	2.93E+04	171
198.81		14.1	161.00	161	25921	25983	43253	0.7	3	1.01E+04	6.19	3.03E+04	174
259.21		16.1	155.30	154	24118.09	23764	43253	0.7	3.3	9.17E+03	4.65	2.75E+04	166
327.61		18.1	148.70	148	22111.69	21886	43253	0.7	3.6	8.41E+03	3.58	2.52E+04	159
404.01		20.1	149.10	149	22230.81	22218	43253	0.7	3.6	8.41E+03	3.58	2.52E+04	159
488.41		22.1	148.40	149	22022.56	22145	43253	0.7	3.65	8.30E+03	3.44	2.49E+04	158
580.81		24.1	146.20	146	21374.44	21391	43253	0.7	3.8	7.97E+03	3.05	2.39E+04	155
681.21		26.1	145.50	146	21170.25	21273	43253	0.7	3.85	7.86E+03	2.93	2.36E+04	154
789.61		28.1	147.30	146	21697.29	21438	43253	0.7	3.85	7.86E+03	2.93	2.36E+04	154
912.04		30.2	146.50	147	21462.25	21588	43253	0.7	3.85	7.86E+03	2.93	2.36E+04	154
1036.84		32.2	148.40	148	22022.56	22023	43253	0.7	3.8	7.97E+03	3.05	2.39E+04	155
1169.64		34.2	149.30	149	22290.49	22133	43253	0.7	3.8	7.97E+03	3.05	2.39E+04	155
1310.44		36.2	149.90	149	22470.01	22230	43253	0.7	3.8	7.97E+03	3.05	2.39E+04	155
1459.24		38.2	149.30	148	22290.49	22008	43253	0.7	3.85	7.86E+03	2.93	2.36E+04	154
1616.04		40.2	150.00	150	22500	22397	43253	0.7	3.8	7.97E+03	3.05	2.39E+04	155
1772.41		42.1	147.60	149	21785.76	22154	43253	0.7	3.85	7.86E+03	2.93	2.36E+04	154
1953.64		44.2	149.80	149	22440.04	22223	43253	0.7	3.85	7.86E+03	2.93	2.36E+04	154
2134.44		46.2	151.50	152	22952.25	23239	43253	0.7	3.7	8.18E+03	3.30	2.45E+04	157
2323.24		48.2	151.90	153	23073.61	23293	43253	0.7	3.7	8.18E+03	3.30	2.45E+04	157

FIG. 19. NGC 3198 Excell datasheet 1, V_{orb}^2 against r, fixed model.