Difference Sieve

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A Sieve that shows different properties of sequences of numbers

1. INTRODUCTION

This paper describes an analytical process called Difference Sieve, when used upon a sequence, equation, or number can produce unique patterns that cam actually identify the type of number being analyzed.

2. PATTERNS

There are five major patterns we see in the Difference Sieve. The first is absolute convergence. The second to last row is filled with the same number for all entries and the last row is all zeroes. This occurs for any polynomial term such as x^5 or x^2. Additional terms to the polynomial show no notable behaviour.

Then there is absolute divergence. In this case the numbers go on forever and never reach a convergence point of any kind. Third is rotation. This is only evident, so far, in the Fibonacci sequence, where in the sieve the sequence continually shifts to the right for each row, copying the prior. The fourth case is singular convergence. This happens when a pattern repeats, row by row, but does not absolutely converge. The fifth case is indeterminant. In this case we don't have enough digits available or the number doesn't have enough digits to make a concrete decision on it being either divergent or convergent.

One pattern we see is triangular zeroes. This occurs in all of the irrational functions examined. This is a pattern of zeroes forming a triangle in the grid. Other patterns we see are like in the primes, where there is a never-ending alternating pattern of zeroes and twos. The relation between the primes and the pattern of twos and zeroes has yet to be found. A note on the numbers used to determine these patterns. If a number starts with a zero or zeroes we truncate them. This is primarily because of the property of the triangular zeroes. If we were to allow for leading zeroes, every decimal with a zero in front of it has the potential of generating false triangular zeroes. The lack of zeroes in a decimal with leading zeroes does not effect the results of the sieve, merely produces those false triangles.

3. FORMULAS

The formula for the Difference Sieve is as follows and is recursive in nature:

$$x_i^a = \left|\left|x_{i+1}^{a-1} - x_i^{a-1}\right|\right|$$

Where the bars || · || is the following operator. For any given a, b, the ||a-b|| is the difference of a from b is taken if b > a and if the difference of b from a is taken if a > b. If a=b the ordering does not matter.

In the formula above, a is the iteration of the differences, a=0 the first row, a=1 the second row, etc. The index i is the element number in the row. For example, the value for the second element of the third row is x_1^2. The equation is zero referenced.

4. CONVERGENCE

Certain patterns such as successor function (1, 2, 3, 4, 5, ..) are said to converge when put in Difference Sieve. This is the most simple pattern and applies to any linear function. The example below is y=x.

1		2		3		4		5		6		7		8		9		10
	1		1		1		1		1		1		1		1		1	
		0		0		0		0		0		0		0		0		

This pattern is said to converge to 1. Polynomials will converge to the constant derivative before going to 0. For example, the last derivative of x is 1. This is a case of absolute convergence.

Another linear function is a linear term and a constant such as x + 2

2		3		4		5		6		7		8		9		10		11		12		13		14		15
	1		1		1		1		1		1		1		1		1		1		1		1		1	
		0		0		0		0		0		0		0		0		0		0		0		0		0

Again, the function is linear and converges to the last derivative. This is a case of absolute divergence.

For the polynomial x^2 the pattern is

1		4		9		16		25		36		49
	3		5		7		9		11		13	
		2		2		2		2		2		
			0		0		0		0			

x^2 converges at 2, because $d/dx(x^2) = 2x$ and $d/dx(2x) = 2$. Again, this is a case of absolute divergence.

Once again, a single x term to a power converges to the derivative of the function. $d/dx(x^3) = 3x^2$ and $d/dx(3x^2) = 6x$ then $d/dx(6x) = 6$ follows.

5. DIVERGENCE AND FIBONACCI

The Fibonacci sequence presents a very unique, repeating Difference Sieve pattern. The Fibonacci sequence, for each iteration, shifts one position to the right. We say that the Fibonacci sequence diverges, since it never settles down on one number. We will find more numbers and functions like these as we go on. This is a case of rotation.

1		1		2		3		5		8		13		21		24		45
	0		1		1		2		3		5		8		13		24	
		1		1		1		1		2		3		5		8		
			0		1		1		1		1		2		3			
				1		0		1		1		1		1				

6. CONVERGENCE AND DIVERGENCE OVER THE REAL, RATONAL NUMBERS

The first number we will analyze is $0.\overline{33}$

3		3		3		3		3		3		3		3		3		3
	0		0		0		0		0		0		0		0		0	

So as we can expect, the number $0.\overline{33}$ converges infinitely to itself. This is a case of absolute convergence.

A simple real number: $0.2525\overline{25}$

2		5		2		5		2		5
	3		3		3		3		3	
		0		0		0		0		

Converges to the value 3. Again, this is a case of absolute divergence.

The following is for the rational number 0.12345671234567

1		2		3		4		5		6		7		1		2		3		4		5		6		7
	1		1		1		1		1		1		6		1		1		1		1		1		1	
		0		0		0		0		0		5		5		0		0		0		0		0		0
			0		0		0		0		5		0		5		0		0		0		0		0	
				0		0		0		5		5		5		5		0		0		0		0		
					0		0		5		0		0		0		5		0		0		0			
						0		5		5		0		0		5		5		0		0				
							5		0		5		0		5		0		5							
								5		5		5		5		5		5								
									0		0		0		0		0									

As you can see, the number 0.12345671234567 converges to 5.This is a case of absolute convergence as well.

Another rational number is 6/356 = 0.0168539

1		6		8		5		3		9		3		0		0		0
	5		2		3		2		6		6		3		0		0	
		3		1		1		4		0		3		3		0		
			2		0		3		4		3		0		3			
				2		3		1		1		3		3				
					1		2		0		2		0					
						1		0		2		2						
							1		2		0							
								1		2								
									1									

This number appears to converge to 1 but to be certain we can add 0 terms to the right hand side.

1		6		8		5		3		9		3		0		0		0		0		0		0
	5		2		3		2		6		6		3		0		0		0		0		0	
		3		1		1		4		0		3		3		0		0		0		0		
			2		0		3		4		3		0		3		0		0		0		0	
				2		3		1		1		3		3		3		0		0		0		
					1		2		0		2		0		0		3		0		0			
						1		0		2		2		0		3		3		0				
							1		2		0		2		3		0		3					
								1		2		2		1		3		3						
									1		0		1		2		0							
										1		1		1		2								
											0		0		1									
												0		1										
													1											

It is mostly certain this sieve converges to 1. We call this a case of singular convergence.For every zero we add to the end of the sieve it will still converge eventually to 1.

The following is another example of a real, rational number 0.00398406

3		9		8		4		0		6
	6		1		4		4		6	
		5		3		0		2		
			2		3		2			
				1		1				
					0					

We say that this number converges on 1 and is a case of absolute convergence.

Another rational number, for sake of being thorough,

The number is 33/26 = 126923076

1		2		6		9		2		3		0		7		6
	1		4		3		7		1		3		7		1	
		3		1		4		6		2		4		6		
			2		3		2		4		2		2			
				1		1		2		2		0				
					0		1		0		2					
						1		1		2						
							0		1							
								1								

I'm not sure if this diverges or converges due to the lack of extra digits. We may find out more if we add 0s to the end to see if it is singular convergent or not.

1		2		6		9		2		3		0		7		6		0		0		0
	1		4		3		7		1		3		7		1		6		0		0	
		3		1		4		6		2		4		6		5		6		0		
			2		3		2		4		2		2		1		5		6			
				1		1		2		2		0		1		4		1				
					0		1		0		2		1		3		3					
						1		1		2		1		2		0						
							0		1		1		1		2							
								1		1		0		1								
									0		1		1									
										1		0										
											1											

Again, with the extra digits the number still converges on 1. We’ll call this singular convergence.

Another rational number is 89/102 = 87254902

8		7		2		5		4		9		0		2
	1		5		3		1		5		9		2	
		4		2		2		4		4		7		
			2		0		2		0		3			
				2		2		2		3				
					0		0		1					
						0		1						
							1							

This number again, is hard to tell if it diverges or converges with the few decimal digits we have to work with. With more digits it may be more clear.

8		7		2		5		4		9		0		2		0		0		0		0		0
	1		5		3		1		5		9		2		2		0		0		0		0	
		4		2		2		4		4		7		0		2		0		0		0		
			2		0		2		0		3		5		0		2		0		0			
				2		2		2		3		2		5		2		2		0				
					0		0		1		1		3		3		0		2					
						0		1		0		2		0		3		2						
							1		1		2		2		3		1							
								0		1		0		1		2								
									1		1		1		1									
										0		0												

This number actually appears to converge absolutely at 1.

Another rational is 6/984

6		0		6		6		7		3
	6		6		0		1		4	
		0		6		1		3		
			6		5		2			
				1		3				
					2					

Again, this begs the question, does this number converge at 2 or if we were to add more decimals would we find another convergence point.

6		0		6		6		7		3		0		0		0		0		0		0		0		0		0		0		0		0		0
	6		6		0		1		4		3		0		0		0		0		0		0		0		0		0		0		0		0	
		0		6		1		3		1		3		0		0		0		0		0		0		0		0		0		0		0		
			6		5		2		2		2		3		0		0		0		0		0		0		0		0		0		0			
				1		3		0		0		1		3		0		0		0		0		0		0		0		0		0				
					2		3		0		1		2		3		0		0		0		0		0		0		0		0					
						1		3		1		1		1		3		0		0		0		0		0		0		0						
							2		2		0		0		2		3		0		0		0		0		0		0							
								0		2		0		2		1		3		0		0		0		0		0								
									2		2		2		1		2		3		0		0		0		0									
										0		0		1		1		1		3		0		0		0										
											0		0		1		1		2		3		0		0											
												0-		1		0		1		1		3		0												
													1		1		1		0		2		3													
														0		0		1		2		1														

It is safe to say that this sieve is either divergent or indeterminant, that is, we don’t have the resources (or space) to calculate a convergence point.

7/689.

1		0		1		5		9		6		5
	1		1		4		4		3		1	
		0		3		0		1		2		
			3		3		1		1			
				0		2		0				
					2		2					

This last rational clearly converges absolutely on 2.

20/485

4		1		2		3		7		1		1
	3		1		1		4		6		0	
		2		0		3		2		6		
			2		3		1		4			
				1		2		3				
					1		1					
						0						

Again, this rational converges on the number 1 absolutely.

7. POLYNOMIALS

Polynomial’s have an interesting behavior as well. For polynomials with more than one x term the behavior of converging at the final derivative isn’t evident.

We will start with an analyses of x^3

1		16		27		64		125		216		343		512		729		1000		1331		1728		2197		2744		3375		4096		4913
	15		11		37		61		91		127		169		217		271		331		397		469		547		631		721		817	
		4		26		24		30		36		42		48		54		60		66		72		78		84		90		96		
			22		2		6		6		6		6		6		6		6		6		6		6		6		6			
				20		4		0		0		0		0		0		0		0		0		0		0		0				
					16		4		0		0		0		0		0		0		0		0		0		0					
						12		4		0		0		0		0		0		0		0		0		0						
							8		4		0		0		0		0		0		0		0		0							
								4		4		0		0		0		0		0		0		0								
									0		0		0		0		0		0		0		0									

x^3 is one of those unique numbers that converges but only partially. It reaches a row of zeroes, but the row prior is not made up of the same integer but that integer “flows” down from the left hand side while the rest of the number cancels out.

Now we will analyze $x^3 + x^2$

0		2		12		36		70		150		252		392		576		810		1100		1452		1872		2366		2940
	2		10		24		34		80		102		140		184		234		290		352		420		494		574	
		8		14		10		46		22		38		44		50		56		62		68		74		80		
			6		4		36		24		16		6		6		6		6		6		6		6			
				2		32		12		8		10		0		0		0		0		0		0				
					30		20		4		2		10		0		0		0		0		0					
						10		16		2		8		10		0		0		0		0						
							6		14		6		2		10		0		0		0							
								8		8		4		8		10		0		0								
									0		4		4		2		10		0									
										4		0		2		8		10										
											4		2		6		2											
												2		4		4												
													2		0													
														2														

For this polynomial there is not a clear cut evaluation. It appears to have converged to two and the derivative property has fallen through. We will classify this as indeterminant since there is no clear cut convergent point but there is a general pattern ruling out divergence.

8. PRIMES

The primes have a very unique pattern that may serve useful.

There isn’t enough space here to show the entire pattern, but we will have to make do. The primes have a property that only typically shows up in the irrational numbers, a triangular pattern of 0s. This is because if we were to take each prime and add it to the end of the decimal, since the primes are never repeating and are infinite, the primes form an irrational number.

2		3		5		7		9		11		13		17		19		23		29		31		37
	1		2		2		2		2		2		4		2		4		6		2		6	
		1		0		0		0		0		2		2		2		2		4		4		
			1		0		0		0		2		0		0		0		2		0			
				1		0		0		2		2		0		0		2		2				
					1		0		2		0		2		0		2		0					
						1		2		2		2		2		2		2						
							1		0		0		0		0		0							
								1		0		0		0		0								
									1		0		0		0									

Continuing on….

41		43		47		53		59		61		67		71		73		79		83		89		97
	2		4		6		6		2		6		4		2		6		4		6		8	
		2		2		0		4		4		2		2		4		2		2		2		
			0		2		4		0		2		0		2		2		0		0			
				2		2		4		2		2		2		0		2		0				
					0		2		2		0		0		2		2		2					
						2		0		2		0		2		0		0						
							2		2		2		2		2		0							
								0		0		0		0		2								

It is difficult to illustrate above the pattern that the primes take. If we were to expand the number of primes, say up to the primes up to 200, we would see it isn't really converging, but there is an attractor centered around that row. The seemingly random pattern of 2s and 0s continues on, ad infinitum. The wider we make the triangle, the more 2 and 0 terms appear without any sort of order. The fact that the triangle of 0s above further strengthens the case that the sequence of prime numbers is irrational.

Again, to be more rigorous, this is the bottom of the triangle when I took it to the prime terms up to 200. Again, with another random placement of 2s and 0s. I cannot be certain, but the behavior of the triangle as it is expanded indicates that the triangle diverges.

```
1   0   2   0   2   0   2   0   2   2   2   0   0   2   2
  1   2   2   2   2   2   2   2   0   0   2   0   2   0
    1   0   0   0   0   0   0   2   0   2   2   2   2
      1   0   0   0   0   0   0   2   2   0   0   0
        1   0   0   0   0   0   2   0   2   0   0
          1   0   0   0   0   0   2   2   2   0
            1   0   0   0   0   2   0   0   2
              1   0   0   0   0   2   0   2
                1   0   0   0   0   2   2
                  1   0   0   0   2   0
                    1   0   0   0   2
                      1   0   0   2
```

9. PI AND THE IRRATIONALS

The next set of numbers we will examine is the irrationals, starting with the two transcendentals PI and e. Counterintuitively, triangles of 0s appear in the irrationals, numbers whose decimals are never supposed to repeat or have any sort of pattern. PI is the first one.

3		1		4		1		5		9		2		6		5		3		5		8		9		3		2		3		8		4		6
	2		3		3		4		4		7		4		1		2		2		3		1		6		1		1		5		4		2	
		1		0		1		0		3		3		3		1		0		1		3		5		5		0		4		1		2		
			1		1		1		3		0		0		2		1		1		2		2		0		5		4		3		1			
				0		0		2		3		0		2		1		0		1		0		2		5		1		1		2				
					0		2		1		3		2		1		1		1		1		2		3		4		0		1					
						2		1		2		1		1		0		0		0		1		1		1		4		1						
							1		1		1		0		1		0		0		1		0		0		3		3							
								0		0		1		1		1		0		1		1		0		3		0								
									0		1		0		0		1		1		0		1		3		3									
										1		1		0		1		0		1		1		2		0										
											0		1		1		1		1		0		1		2											
												1		0		0		0		1		1		1												
													1		0		0		1		0		0													
														1		0		1		1		0														
															1		1		0		1															
																0		1		1																
																	1		0																	
																		1																		

If you extend the triangle beyond the last digit shown more 0 triangles appear, order in what should be disorder. Just as with PI, it doesn’t appear that this triangle converges. It appears to diverge. To be certain it would be necessary to take the triangle out to at least another 50 iterations.

The next irrational, transcendental, we will review is e. We will see the same triangular zero effect and the divergence we saw of PI.

2		7		1		8		2		8		1		8		2		8		4		5		9		0		4		5		2		3		5		3		6		0		3		8		7		4		7		1		3		5
	5		6		7		6		6		7		7		6		6		4		1		4		9		4		1		3		1		2		2		3		6		2		6		1		2		3		6		2		2	
		1		1		1		0		1		0		1		0		2		1		3		5		5		3		2		2		1		0		1		3		4		4		5		1		1		3		4		0		
			0		0		1		1		1		1		1		2		1		2		2		0		2		1		0		1		1		1		2		1		0		1		4		0		2		1		4			
				0		1		0		0		0		0		1		1		1		0		2		2		1		1		1		0		0		1		1		1		1		3		0		2		1		3				
					1		1		0		0		0		1		0		0		1		2		0		1		0		0		1		0		1		0		0		0		2		3		2		1		2					
						0		1		0		0		1		1		0		1		1		2		1		1		0		1		1		1		0		0		0		2		1		1		1		1						
							1		1		0		1		0		1		1		0		1		1		0		1		1		0		0		1		0		0		2		1		0		0		0							
								0		1		1		1		1		0		1		1		0		1		1		0		1		0		1		1		0		2		1		0		0		0								
									1		0		0		0		0		1		0		1		1		0		1		1		1		1		0		1		2		1		1		0		0									
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As you can see, the triangles of zeroes occur many times in the Difference Sieve applied to the constant e. We will test further irrationals and we will see that every irrational we test has this pattern of zeroes in its analyses.

Theorem
All irrational numbers on a Difference Sieve exhibit the effect of triangular zeroes.

Next we will examine e $+\pi$, a number that is conjectured to be irrational but there hasn't been a sufficient proof. We will see that e $+\pi$ has the same behavior as both pi and e individually. There is good evidence that e $+\pi$is irrational.

e = 2.71828459045235360287
π = 3.14159265358979323846264 3
e $+\pi$ = 5.85987448204883847382293

5		8		5		9		8		7		4		4		8		2		0		4		8		8		3		8		4		7		3		8		2		2		9		3
	3		3		4		1		1		3		0		4		6		2		4		4		0		5		5		4		3		4		5		6		0		7		6	
		0		1		3		0		2		3		4		2		4		2		0		4		5		0		1		1		1		1		1		6		7		1		
			1		2		3		2		1		1		2		2		2		2		4		1		5		1		0		0		0		0		5		1		6			
				1		1		1		1		0		1		0		0		0		2		3		4		4		1		0		0		0		5		4		5				
					0		0		0		1		1		1		0		0		2		1		1		0		3		1		0		0		5		1		1					
						0		0		1		0		0		1		0		2		1		0		1		3		2		1		0		5		4		0						
							0		1		1		0		1		1		2		1		1		1		2		1		1		1		5		1									
								1		0		1		1		0		0		1		0		0		1		1		0		0		4		4										
									1		1		0		1		0		1		1		0		1		0		1		0		4		0											
										0		1		1		1		1		0		1		1		1		1		1		4														
											1		0		0		0		1		1		0		0		0		0		4															
												1		0		0		1		0		1		0		0		0		4																
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As you can see, e $+\pi$ is littered with triangular zeroes. There is no doubt in my mind that the number is irrational.