Duality transform between black and white psychological profiles.

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September 19, 2018

Abstract

It is shown that the Fourier transformation is the appropriate defining characteristic of black and white polarization states in psychological archetypes.

1 Introduction.

In previous work, I have described the necessity for at least a black-white theory of psychological profiles. In principle, psychological archetypes are described by means of a real vectorspace $V = \mathbb{R}^n$ given that no obvious extremal states need to a priori exist. Such assumption would lead to a description by means of convex spaces. Black is an effective charge described by a delta peak distribution on V whereas a pure white state is described by its Fourier transform $\mathcal{F}$. They are extremal weak distributional states in $L^2(V, \mu)$ the Hilbertspace of square integrable functions on V with respect to the measure μ. Circularly polarized states are then defined as "black content equals white content" which is the space of eigenstates of the Fourier transform.'

2 Elaboration.

In what follows, we take the pure theory and assume $n = 1$. V then is spanned by means of the black states given by $\delta(x - a)$, white states are of the form e^{ika}. As such, no information loss occurs and black-white are just different configurations of the same substance. Hence, given

$$g(x) = \int dy g(y)\delta(y - x)$$

we have that

$$(\mathcal{F}g)(k) = \int dy g(y)e^{iky}.$$

We now look for states for which $|g(x)|^2 \sim |(\mathcal{F}g)(x)|^2$ or, more precisely,

$$(\mathcal{F}g)(x) \sim e^{idx}g(x)$$

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for some d. Taking into account that

$$(\mathcal{F}(g))(k) = \int dx e^{ikx} g(x)$$

the aforementioned class is given by the Gaussian functions

$$e^{-a(x-b)^2}$$

since

$$\int dx e^{ikx} e^{-a(x-b)^2} = \int dx e^{ikb} e^{-a((x-b)-\frac{ik}{2a})^2} e^{-\frac{k^2}{4a}} = \frac{\pi}{\sqrt{a}} e^{-ab^2} e^{-\frac{1}{4a}(k-2iab)^2}$$

and for this function to satisfy our criterion it is necessary and sufficient that $a = \frac{1}{2}$ and b is freely chosen. Hence, the diagonal in the white-black plane is one dimensional and parametrized by b just as the black line interval is. Taking $n > 1$ would entail a definition of blackness given by $\delta(|x| - a)$ thereby surpressing $n - 1$ dimensions. Taking a to zero and scaling the Gaussian functions appropriately leads to the pure black states whereas taking a to infinity provides for the pure white states. The appropriate duality is therefore $a \to \frac{1}{4a}$. Hence one has to consider the operators

$$P_{a,b} = e^{x2ab(1-i)} \circ \mathcal{F} \circ S_{\frac{1}{2a}}$$

and notice that

$$(P_{a,b})^2 = e^{x2ab(1-i)} \circ \mathcal{F} \circ S_{\frac{1}{2a}} \circ e^{x2ab(1-i)} \circ \mathcal{F} \circ S_{\frac{1}{2a}}$$

$$= 2ae^{x2ab(1-i)} \circ \mathcal{F} \circ e^{xb(1-i)} \circ \mathcal{F} = 2ae^{x2ab(1-i)} \circ T_{-b(1+i)} \circ \mathcal{F}^2$$

and consider eigenvectors $v_{d,\alpha}$ with the appropriate eigenvalue $2\pi\sqrt{a}$ to be calculated above (hint: $\alpha = \frac{1}{2a}$ for the diagonal states). All these functions are eigenvalues of the latter operator with eigenvalue $4\pi^2 a$. Here, e^{ixd} is the multiplication operator and S_α the scaling operator defined by

$$(S_\alpha g)(x) = g(\alpha x).$$

Generally, $d = (i + 1)2ab$ and there are some interesting commutation relations

$$e^{-ixd}\mathcal{F} = \mathcal{F}T_d$$

where

$$(T_d g)(x) = g(x + d)$$

and

$$(\mathcal{F}S_\alpha g)(x) = \frac{1}{\alpha}(S_{\frac{1}{\alpha}}\mathcal{F}g)(x).$$

Hence,

$$P^\dagger_{d,\alpha} P_{d,\alpha} \sim 1$$

given that e^{idx} is unitary and

$$S^\dagger_\alpha S_\alpha = \frac{1}{\alpha} 1.$$

Notice that

$$x\mathcal{F}x + \partial_x \mathcal{F} \partial_x = i\mathcal{F}^\dagger$$

and therefore, the Heisenberg algebra is equivalent to

$$X^\dagger X - P^\dagger P = i1$$

on inproduct spaces with a complex valued bilinear form gauged by $X^\dagger = P$. The Fourier transform is then recuperated by finding a unitary operator $\mathcal{F}$ on a Hilbert space representation of X, P such that $X^\dagger = \mathcal{F} X^H \mathcal{F}$ with $X^H = X$. In general, the angle α between the mixed states and the "axis" of black extremal states should be given by a self dual function, that is

$$\frac{1}{f(x)} = f\left(\frac{1}{4x}\right)$$

giving rise to

$$f(x) = \frac{1}{\sqrt{2x}}.$$

Therefore,

$$\tan\left(\frac{\pi}{2} - \alpha\right) = f(x)$$

and the magnitude is determined by the shift parameter b. Henceforth, a compactification of the infinite dimensional function space to a real two dimensional Hilbert space with black-white extremal states is given by

$$|a, b\rangle = b\left(\cos(\alpha)|b\rangle + \sin(\alpha)|w\rangle\right).$$

There is no information loss but the correspondence is not a linear one for sure given that any linear combination on the right hand side determines a Gaussian but not so for linear combinations of the left hand side. It remains a task to find other suitable, higher, "pictorial" charges such as shapes, including tigers, cats, dogs and sharks, to further specify the psychological mapping to relative moral values.