DERIVATION OF FORCES FROM MATTER WAVE

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Abstract: In this work we discuss the possibility that if matter wave is composed of two different physical fields, as in the case of the electromagnetic field which is composed of the electric field and a magnetic field, then it is possible to suggest that matter wave also produces forces, like the electric force and the magnetic force produced by the electromagnetic field. Furthermore, since the forms of forces that can be derived from matter wave has a Yukawa form and Coulomb form, it may be suggested that forces produced from matter wave are in fact related to the strong force and the electroweak force, respectively.

In classical physics, the concept of force is closely related to the concept of physical field, both of which may be defined intuitively as physical entities that we are able to qualitatively conceive and quantitatively measure. Despite the true nature of these physical entities still remains obscure, by devising a suitable mathematical framework, they can be formulated and expressed in terms of systems of differential equations that in turns can be applied into practical problems. For example, in Newton gravity the force of gravity is associated with the gravitational field and in Maxwell electromagnetism the electromagnetic force is associated with the electromagnetic field, both of these fields can be formulated in terms of differential equations and probably the most prominent feature that can be obtained from the resulted differential equations is the wave character of the fields that they describe. On the other hand, in quantum physics, there exists matter wave. If matter wave is considered as a manifestation of a physical field then the question of whether there is some kind of force associated with it may be raised. In particular, this type of question becomes obvious if matter wave can also be represented by similar mathematical formulation to the classical waves. In fact, it has been shown in our previous works that both Dirac equation and Maxwell field equations can be formulated from a general system of linear first order partial differential equations and the similarity between the mathematical structure of two systems of equations leads to the suggestion that the two physical fields may also have the same physical structure in the sense that they are composed of two different physical fields. Furthermore, if the matter wave is the result of the coupling of two different physical fields then there should also be two different types of force associated with them. In the following we will address this problem and show how to derive these forces from matter wave that is assumed to be described by Dirac equation. Most of the results given in this work have been presented in our previous works, however, for clarity we will briefly outline them first. In our previous works we showed that Dirac equation for a free particle, Dirac equation for an arbitrary field and Maxwell field equations can be formulated from a general system of linear first order partial differential equations that can be written as follows [1, 2, 3, 4]

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{r}\frac{\partial\psi_i}{\partial x_j}=\sum_{i=1}^{n}\left(\sum_{j=1}^{n} b_{ij}^{r}V_j+c_i^{r}\right)\psi_i+d^{r},\quad r=1,2,\dots,n \tag{1}$$

The system of equations given in Equation (1) can be rewritten in a matrix form as

$$\left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)\psi=-i\left(\sum_{i=1}^{n} qB_iV_i+m\sigma\right)\psi+J \tag{2}$$

where $\psi=(\psi_1,\psi_2,\dots,\psi_n)^T$, $\partial\psi/\partial x_i=(\partial\psi_1/\partial x_i,\partial\psi_2/\partial x_i,\dots,\partial\psi_n/\partial x_i)^T$ with A_i, B_i and σ are matrices representing the quantities a_{ij}^{r}, b_{ij}^{r}, c_j^{r}, which are assumed to be constant in this work, and J is a matrix that represents the quantity d^{r}. While the quantities q, m and J represent physical entities related directly to the physical properties of the particle under consideration, the quantities V_i represent the potentials of an external field, such as an electromagnetic field or the matter field of a quantum particle. In order to formulate a physical theory from the system of equations given in Equation (2), it is necessary to determine the unknown quantities A_i, B_i and σ, as well as the mathematical conditions that they must satisfy, such as commutation relations between them. The commutation relations between the matrices can be determined if we apply the operator $\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}$ on the left on both sides of Equation (2) as follows

$$\begin{aligned}\left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)&\left(\sum_{j=1}^{n} A_j\frac{\partial}{\partial x_j}\right)\psi\\ &=\left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)\left(-i\left(\sum_{j=1}^{n} qB_jV_j+m\sigma\right)\psi+J\right)\end{aligned} \tag{3}$$

Since the quantities A_i, B_i, σ, q and m are assumed to be constant, Equation (3) becomes

$$\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{n}\sum_{j>i}^{n}\left(A_iA_j + A_jA_i\right)\frac{\partial^2}{\partial x_i \partial x_j}\right)\psi$$
$$= \left(-i\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\left(\sum_{j=1}^{n} qB_jV_j + m\sigma\right)\right)\psi - i\left(\sum_{i=1}^{n} qB_iV_i + m\sigma\right)\left(\left(\sum_{j=1}^{n} A_j \frac{\partial}{\partial x_j}\right)\psi\right)$$
$$+ \sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i}$$
$$= -i\left(\sum_{i=1}^{n}\sum_{j=1}^{n} qA_iB_j \frac{\partial V_j}{\partial x_i}\right)\psi$$
$$-\left(\sum_{i=1}^{n}\sum_{j>i}^{n} q^2\left(B_iB_j + B_jB_i\right)V_iV_j - 2i\sum_{i=1}^{n} qmB_iV_i\,\sigma - m^2\sigma^2\right)\psi - i\left(\sum_{i=1}^{n} qB_iV_i + m\sigma\right)J$$
$$+ \sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \tag{4}$$

Maxwell field equations that can be obtained from the system of equations given in Equation (2) can be rewritten the following matrix form

$$\left(A_0\frac{\partial}{\partial t} + A_1\frac{\partial}{\partial x} + A_2\frac{\partial}{\partial y} + A_3\frac{\partial}{\partial z}\right)\psi = A_4 J \tag{5}$$

where $\psi = (\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\,)^T,\ J = (j_1, j_2, j_3, 0,0,0\,)^T$ with the matrices A_i given as

$$A_0 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_4 = \begin{pmatrix} \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{6}$$

The commutation relations between these matrices can be obtained as follows

$$A_0^2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad A_1^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad A_2^2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$
$$A_3^2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_4^2 = \begin{pmatrix} \mu^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_1A_2+A_2A_1=\begin{pmatrix}0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&1&0&0\\0&0&0&0&0&0\end{pmatrix}\quad A_1A_3+A_3A_1=\begin{pmatrix}0&0&1&0&0&0\\0&0&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&0&0\\0&0&0&1&0&0\end{pmatrix}\quad A_2A_3+A_3A_2=\begin{pmatrix}0&0&0&0&0&0\\0&0&1&0&0&0\\0&1&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\end{pmatrix}$$

$$A_0A_i+A_iA_0=0 \quad \text{for} \quad i=1,2,3 \tag{7}$$

From Equations (4), (6) and (7) we obtain the following equations for the electric field $\mathbf{E}=\left(E_x,E_y,E_z\right)=(\psi_1,\psi_2,\psi_3)$

$$\frac{\partial^2\psi_1}{\partial t^2}-\frac{\partial^2\psi_1}{\partial y^2}-\frac{\partial^2\psi_1}{\partial z^2}+\frac{\partial}{\partial x}\left(\frac{\partial\psi_2}{\partial y}+\frac{\partial\psi_3}{\partial z}\right)=-\mu\frac{\partial j_1}{\partial t} \tag{8}$$

$$\frac{\partial^2\psi_2}{\partial t^2}-\frac{\partial^2\psi_2}{\partial x^2}-\frac{\partial^2\psi_2}{\partial z^2}+\frac{\partial}{\partial y}\left(\frac{\partial\psi_1}{\partial x}+\frac{\partial\psi_3}{\partial z}\right)=-\mu\frac{\partial j_2}{\partial t} \tag{9}$$

$$\frac{\partial^2\psi_3}{\partial t^2}-\frac{\partial^2\psi_3}{\partial x^2}-\frac{\partial^2\psi_3}{\partial y^2}+\frac{\partial}{\partial z}\left(\frac{\partial\psi_1}{\partial x}+\frac{\partial\psi_2}{\partial y}\right)=-\mu\frac{\partial j_3}{\partial t} \tag{10}$$

Similarly for the magnetic field $\mathbf{B}=\left(B_x,B_y,B_z\right)=(\psi_4,\psi_5,\psi_6)$ we obtain the following equations

$$\frac{\partial^2\psi_4}{\partial t^2}-\frac{\partial^2\psi_4}{\partial y^2}-\frac{\partial^2\psi_4}{\partial z^2}+\frac{\partial}{\partial x}\left(\frac{\partial\psi_5}{\partial y}+\frac{\partial\psi_6}{\partial z}\right)=0 \tag{11}$$

$$\frac{\partial^2\psi_5}{\partial t^2}-\frac{\partial^2\psi_5}{\partial x^2}-\frac{\partial^2\psi_5}{\partial z^2}+\frac{\partial}{\partial y}\left(\frac{\partial\psi_4}{\partial x}+\frac{\partial\psi_6}{\partial z}\right)=0 \tag{12}$$

$$\frac{\partial^2\psi_6}{\partial t^2}-\frac{\partial^2\psi_6}{\partial x^2}-\frac{\partial^2\psi_6}{\partial y^2}+\frac{\partial}{\partial z}\left(\frac{\partial\psi_4}{\partial x}+\frac{\partial\psi_5}{\partial y}\right)=0 \tag{13}$$

Using Gauss's laws for the electric field and the magnetic field

$$\boldsymbol{\nabla}.\,\mathbf{E}=\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=\frac{\partial\psi_1}{\partial x}+\frac{\partial\psi_2}{\partial y}+\frac{\partial\psi_3}{\partial z}=\frac{\rho_e}{\epsilon} \tag{14}$$

$$\boldsymbol{\nabla}.\,\mathbf{B}=\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=\frac{\partial\psi_4}{\partial x}+\frac{\partial\psi_5}{\partial y}+\frac{\partial\psi_6}{\partial z}=0 \tag{15}$$

the wave equations for the electric field **E** and the magnetic field **B** are obtained

$$\frac{\partial^2\mathbf{E}}{\partial t^2}-\nabla^2\mathbf{E}=\nabla\left(\frac{\rho_e}{\epsilon}\right)-\mu\frac{\partial\mathbf{J}_e}{\partial t} \tag{16}$$

$$\frac{\partial^2\mathbf{B}}{\partial t^2}-\nabla^2\mathbf{B}=0 \tag{17}$$

As shown in classical electrodynamics, there are two different types of force that can be associated with the electric field and the magnetic field.

Now, in quantum mechanics, matter wave can be described by Dirac equation. Dirac equation for an arbitrary field can be formulated from the system of linear first order partial differential equations given in Equation (2) by setting $B_i = A_i = \gamma_i,\ \sigma = 1,\ J = 0$ and $A_iA_j + A_jA_i = 0$. In this case Equation (2) becomes

$$\left(\sum_{i=1}^{4} \gamma_i \frac{\partial}{\partial x_i}\right)\psi = -i\left(\sum_{i=1}^{4} q\gamma_i V_i + m\right)\psi \tag{18}$$

where the matrices A_i have been identified with Dirac matrices γ_i [5]

$$\gamma^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Equation (18) can be rewritten in a covariant form as Dirac equation for an arbitrary field as

$$\left(\gamma^{\mu}\left(i\partial_{\mu} - qV_{\mu}\right) - m\right)\psi = 0 \tag{19}$$

Equation (4) also reduces to the following equation

$$\left(\sum_{i=1}^{4} {\gamma_i}^2 \frac{\partial^2}{\partial x_i^2}\right)\psi = \left(-i\sum_{i=1}^{4}\sum_{j>i}^{4} q\gamma_i\gamma_j\left(\frac{\partial V_j}{\partial x_i} - \frac{\partial V_i}{\partial x_j}\right) + 2i\sum_{i=1}^{4} qm\gamma_i V_i - m^2\right)\psi \tag{20}$$

If the quantities V_i are the four-potential of an electromagnetic field given by the identification $(V_1, V_2, V_3, V_4) = \left(V, A_x, A_y, A_z\right)$ then Equation (20) can be used to determine the dynamics of the components of the wavefunction $\psi = (\psi_1, \psi_2, \psi_3, \psi_3)^T$, where the term $\frac{\partial V_j}{\partial x_i} - \frac{\partial V_i}{\partial x_j}$ are the components of the electric field **E** and the magnetic field **B**. For the case of free particle, we set $\sum_{i=1}^{n} qB_iV_i = 0$ and Equations (2) and (4) reduce to

$$\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\psi = -im\sigma\psi + J \tag{21}$$

$$\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{n}\sum_{j>i}^{n}\left(A_iA_j + A_jA_i\right)\frac{\partial^2}{\partial x_i \partial x_j}\right)\psi = -m^2\sigma^2\psi - im\sigma J + \sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \tag{22}$$

For the case of Dirac equation, the matrices A_i satisfy the following conditions

$$A_i^2 = \pm 1 \tag{23}$$

$$A_iA_j + A_jA_i = 0 \quad for \quad i \neq j \tag{24}$$

and Equation (22) reduces to the following equation

$$\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2}\right)\psi = -m^2\sigma^2\psi - im\sigma J + \sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \tag{25}$$

In particular, for $\sigma = 1$ and $J = 0$, Equation (21) reduces to Dirac equation for a free particle

$$\gamma^{\mu}\partial_{\mu}\psi = -im\psi \tag{26}$$

Dirac equation given in Equation (26) can be written out in full form for the wavefunction $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ as

$$\frac{\partial\psi_1}{\partial t} + im\psi_1 = -\frac{\partial\psi_3}{\partial z} - \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_4 \tag{27}$$

$$\frac{\partial\psi_2}{\partial t} + im\psi_2 = -\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_3 + \frac{\partial\psi_4}{\partial z} \tag{28}$$

$$\frac{\partial\psi_3}{\partial t} - im\psi_3 = -\frac{\partial\psi_1}{\partial z} - \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_2 \tag{29}$$

$$\frac{\partial\psi_4}{\partial t} - im\psi_4 = -\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_1 - \frac{\partial\psi_2}{\partial z} \tag{30}$$

Dirac equation written as a system of linear first order partial differential equations given in Equations (27-30) suggests that matter wave can be interpreted as a coupling of two different physical fields represented by the field (ψ_1, ψ_2) and the field (ψ_3, ψ_4) whose temporal rates of change will convert one field to the other.

For massive particles, all components of Dirac wavefunction $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ satisfy the Klein-Gordon equation

$$\frac{\partial^2\psi_\mu}{\partial t^2} - \frac{\partial^2\psi_\mu}{\partial x^2} - \frac{\partial^2\psi_\mu}{\partial y^2} - \frac{\partial^2\psi_\mu}{\partial z^2} = -m^2\psi_\mu \tag{31}$$

In particular, if the wavefunctions are time-independent then we obtain

$$\frac{\partial^2\psi_\mu}{\partial x^2} + \frac{\partial^2\psi_\mu}{\partial y^2} + \frac{\partial^2\psi_\mu}{\partial z^2} = m^2\psi_\mu \tag{32}$$

In this case the wavefunctions ψ_μ can be viewed as static Yukawa potential

$$\psi_\mu(r) = \frac{g}{4\pi r}e^{-r/R} \tag{33}$$

where g is an undetermined dimensional constant [6]. The wavefunctions given in Equation (33) can be associated with the strong interaction between nuclear particles.

On the other hand, for massless time-independent particles, the Klein-Gordon equation given in Equation (31) reduces to Laplace equation

$$\frac{\partial^2\psi_\mu}{\partial x^2} + \frac{\partial^2\psi_\mu}{\partial y^2} + \frac{\partial^2\psi_\mu}{\partial z^2} = 0 \tag{34}$$

Solutions to Laplace equation can be written in the form

$$\psi_\mu(x,y,z) = \frac{k}{\sqrt{x^2+y^2+z^2}}, \quad \mu = 1,2,3,4 \tag{35}$$

In this case the wavefunctions ψ_μ can be viewed as static Coulomb potential where k is an undetermined dimensional constant. The wavefunctions given in Equation (35) can be associated with the electroweak interaction between elementary particles.

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