Generalized Lorentz Transformations

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This article presents the generalized Lorentz transformations of time, space, velocity and acceleration which can be applied in any inertial or non-inertial (uniform circular motion) frame.

Introduction

If we consider an inertial or non-inertial (uniform circular motion) frame S and another inertial frame Σ then the time (t), the position ($\mathbf{r}$), the velocity ($\mathbf{v}$) and the acceleration ($\mathbf{a}$) of a (massive or non-massive) particle relative to the inertial frame Σ are given by:

$$t = \int_0^{\mathtt{t}} \gamma \, \mathtt{dt} - \gamma \, \frac{\vec{r} \cdot \mathbf{V}}{c^2} + \mathtt{k}$$

$$\mathbf{r} = \vec{r} + \frac{\gamma^2}{\gamma + 1} \frac{(\vec{r} \cdot \mathbf{V})\,\mathbf{V}}{c^2} - \mathbf{R} - \frac{\gamma^2}{\gamma + 1} \frac{(\mathbf{R} \cdot \mathbf{V})\,\mathbf{V}}{c^2}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

where ($\mathtt{t}$, $\vec{r}$) are the time and the position of the particle relative to the frame S ($\mathbf{R}, \mathbf{V}, \mathbf{A}$) are the position, the velocity and the acceleration of the origin of the frame Σ relative to the frame S, ($\mathtt{k}$) is a particular constant between frames Σ and S, (c) is the speed of light in vacuum, and $\gamma = (1 - \mathbf{V} \cdot \mathbf{V}/c^2)^{-1/2}$

- $\dfrac{d\mathbf{r}}{dt} \;=\; \left(\dfrac{d\mathbf{r}}{\mathtt{dt}} + \Omega \times \mathbf{r} \right) \left(\dfrac{1}{dt \,/\, \mathtt{dt}} \right)$

- $\dfrac{d\mathbf{v}}{dt} \;=\; \left(\dfrac{d\mathbf{v}}{\mathtt{dt}} + \Omega \times \mathbf{v} \right) \left(\dfrac{1}{dt \,/\, \mathtt{dt}} \right)$

- $\Omega \;=\; \dfrac{\gamma^2}{\gamma + 1} \, \dfrac{(\mathbf{A} \times \mathbf{V})}{c^2}$

- $\dfrac{\gamma^2}{\gamma + 1} \dfrac{1}{c^2} \;=\; \dfrac{\gamma - 1}{\mathbf{V}^2} \qquad (\, \mathbf{V}^2 = \mathbf{V} \cdot \mathbf{V} \,)$

- $\vec{r} \,+\, \dfrac{\gamma^2}{\gamma + 1} \, \dfrac{(\vec{r} \cdot \mathbf{V})\, \mathbf{V}}{c^2} \;=\; \gamma\, \vec{r} \,+\, \dfrac{\gamma^2}{\gamma + 1} \, \dfrac{(\vec{r} \times \mathbf{V}) \times \mathbf{V}}{c^2}$

- $\mathbf{R} \,+\, \dfrac{\gamma^2}{\gamma + 1} \, \dfrac{(\mathbf{R} \cdot \mathbf{V})\, \mathbf{V}}{c^2} \;=\; \gamma\, \mathbf{R} \,+\, \dfrac{\gamma^2}{\gamma + 1} \, \dfrac{(\mathbf{R} \times \mathbf{V}) \times \mathbf{V}}{c^2}$

General Observations

If the frame S is inertial then the observer S must use an origin O’ such that $(\, \mathbf{R} \times \mathbf{V} \;=\; 0 \,)$

If the frame S is non-inertial (uniform circular motion) then the observer S must use an origin O’ such that $(\, \mathbf{R} \cdot \mathbf{V} \;=\; 0 \,)$

If the frame S is inertial then $(\, \mathbf{A} \;=\; 0 \,)$, $(\, \mathbf{V} \;=\; \text{constant} \,)$, $(\, \gamma \;=\; \text{constant} \,)$ $(\, \int_0^{\mathtt{t}} \gamma \, \mathtt{dt} = \gamma \, \mathtt{t} \,)$, $(\, \mathbf{R} \;=\; \mathbf{V}\, \mathtt{t} + \text{constant} \,)$, $(\, \mathbf{R} \times \mathbf{V} \;=\; 0 \,)$ & $(\, \Omega \;=\; 0 \,)$

If the frame S is non-inertial (uniform circular motion) then $(\, \mathbf{A} \;\neq\; 0 \,)$ $(\, \mathbf{A} \cdot \mathbf{V} \;=\; 0 \,)$, $(\, \gamma \;=\; \text{constant} \,)$, $(\, \int_0^{\mathtt{t}} \gamma \, \mathtt{dt} = \gamma \, \mathtt{t} \,)$, $(\, \mathbf{R} \cdot \mathbf{V} \;=\; 0 \,)$ & $(\, \Omega \;\neq\; 0 \,)$

Bibliography

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