E8 Physics: Cayley-Dickson and Clifford Algebras - - Braids - Cellular Automata

Frank Dodd Tony Smith Jr - 2018

Louis H. Kauffman in arxiv 1710.04650 said:
"... Let Bn denote the Artin braid group on n strands ... Bn is generated by elementary braids { s1, ... , s(n-1) } with relations

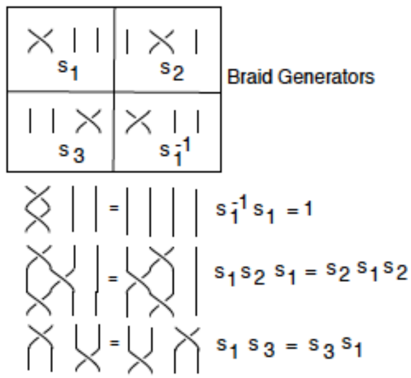


Figure 1: **Braid Generators**

1. $s_i s_j = s_j s_i$ for $|i - j| > 1$,
2. $s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}$ for $i = 1, \ldots, n-2$.

Braiding operators associated with Majorana operators are described as follows. Let $\{c_1, c_2, \cdots, c_n\}$ denote a collection of Majorana operators such that $c_k^2 = 1$ for $k = 1, \cdots, n$ and $c_i c_j + c_j c_i = 0$ when $i \neq j$. Take the indices $\{1, 2, ..., n\}$ as a set of residues modulo n so that $n + 1 = 1$. Define operators

$$\sigma_k = (1 + c_{k+1} c_k)/\sqrt{2}$$

for $k = 1, \cdots n$ where it is understood that $c_{n+1} = c_1$ since $n + 1 = 1$ modulo n. Then one can verify that

$$\sigma_i \sigma_j = \sigma_j \sigma_i$$

when $|i - j| \geq 2$ and that

$$\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$$

for all $i = 1, \cdots n$. Thus $\{\sigma_1, \cdots, \sigma_{n-1}\}$ describes a representation of the n-strand Artin braid group B_n.

... the three braid generators of B4 are shown, and ... the inverse of the first generator ...

Clifford Braiding Theorem. Let C be the Clifford algebra over the real numbers generated by linearly independent elements $\{c_1, c_2, \ldots c_n\}$ with $c_k^2 = 1$ for all k and $c_k c_l = -c_l c_k$ for $k \neq l$. Then the algebra elements $\tau_k = (1 + c_{k+1} c_k)/\sqrt{2}$, form a representation of the (circular) Artin braid group. That is, we have $\{\tau_1, \tau_2, \ldots \tau_{n-1}, \tau_n\}$ where $\tau_k = (1 + c_{k+1} c_k)/\sqrt{2}$ for $1 \leq k < n$ and $\tau_n = (1 + c_1 c_n)/\sqrt{2}$, and $\tau_k \tau_{k+1} \tau_k = \tau_{k+1} \tau_k \tau_{k+1}$ for all k and $\tau_i \tau_j = \tau_j \tau_i$ when $|i - j| > 2$. Note that each braiding generator τ_k has order 8.

Remark. It is worth noting that a triple of Majorana Fermions say x, y, z gives rise to a representation of the quaternion group.

...".

Therefore: **Braid Group B3 corresponds to the Clifford Algebra Cl(0,2)**
and to the Cayley-Dickson Quaternion Algebra H

Tao Cheng, Hua-Lin Huang, and Yuping Yang in arxiv 1510.04408 said "...

Many interesting algebras appear as twisted group algebras. Here we recall some examples presented in [1, 2, 15]. Let $\mathbb{R}$ denote the field of real numbers, $\mathbb{Z}_2 = \{0, 1\}$ the cyclic group of order 2, and $\mathbb{Z}_2^n$ the direct product of n copies of $\mathbb{Z}_2$. Elements of $\mathbb{Z}_2^n$ are written as n-tuples of $\{0, 1\}$ and the group product is written as $+$. Define functions $f_m \colon \mathbb{Z}_2^n \times \mathbb{Z}_2^n \to \mathbb{Z}_2$ for all $1 \le m \le 3$ by

$$f_1(x,y) = \sum_i x_i y_i, \qquad f_2(x,y) = \sum_{i<j} x_i y_j, \qquad f_3(x,y) = \sum_{\substack{\text{distinct } i,j,k \\ i<j}} x_i x_j y_k.$$

1. Let $F_{\mathrm{Cl}} \colon \mathbb{Z}_2^n \times \mathbb{Z}_2^n \to \mathbb{R}^*$ be a function defined by

   $$F_{\mathrm{Cl}}(x,y) = (-1)^{f_1(x,y)+f_2(x,y)}.$$

   Then the associated twisted group algebra $\mathbb{R}_{F_{\mathrm{Cl}}}[\mathbb{Z}_2^n]$ is the well-known real Clifford algebra $\mathrm{Cl}_{0,n}$, see [2] for detail. This recovers the algebra of complex numbers $\mathbb{C}$ when $n = 1$ and the algebra of quaternions $\mathbb{H}$ when $n = 2$. Note that $\mathrm{Cl}_{0,n}$ is associative in the usual sense since the function F_{Cl} is a 2-cocycle.

2. Assume $n \ge 3$. Define the function $F_{\mathbb{O}} \colon \mathbb{Z}_2^n \times \mathbb{Z}_2^n \to \mathbb{R}^*$ by

   $$F_{\mathbb{O}}(x,y) = (-1)^{f_1(x,y)+f_2(x,y)+f_3(x,y)}.$$

   Then the twisted group algebra $\mathbb{R}_{F_{\mathbb{O}}}[\mathbb{Z}_2^n]$ is the algebra of higher octonions $\mathbb{O}_n$ introduced in [15] by generalizing the realization of octonions via twisted group algebras (i.e., $n = 3$)

...".

Note that Z2^n corresponds to Braid Group B(n+1) so

n = 1 gives B2 and Cl(0,1) and Complex Numbers and Sphere S1 = U(1)

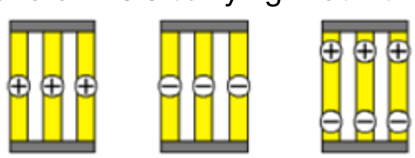
Photons can be represented by B2 Braids

γ

n = 2 gives B3 and Cl(0,2) and Quaternions and Sphere S3 = SU(2)

Sundance Bilson-Thompson in hep-ph/0503213 represents SU(2) Bosons by B3 Braids

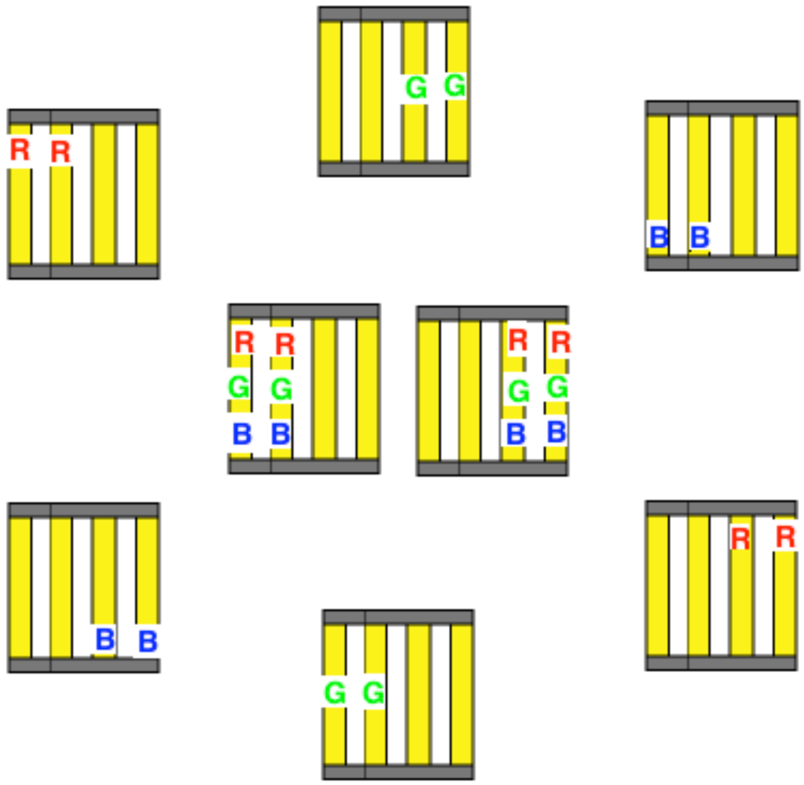
(+ and - denote twists carrying Electric Charge)



w^+ w^- z°

and

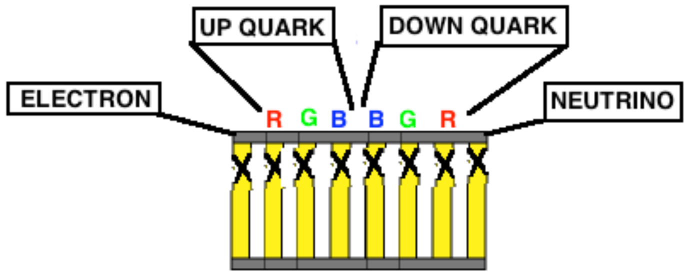
n = 3 gives B4 and Cl(0,3) and Octonions and Sphere S7
Octonions and Cl(0,3) both have 1 3 3 1 graded structure
SU(3) Color Force has 1+1 Neutral Gluons and 3+3 Colored Gluons
(R G B denote twists carrying Color Charge)



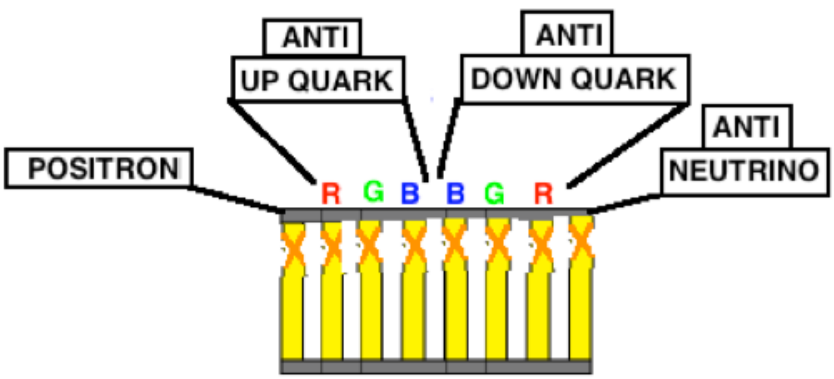
and

n = 7 gives B8 and Cl(0,7) and 21-dim Spin(7) and S7 + Spin(7) = 28-dim D4

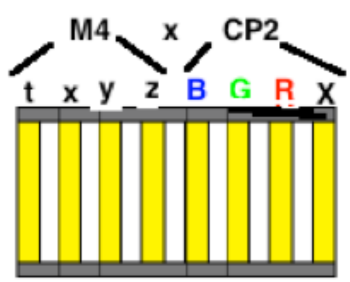
The 8 Strands of B8 represent 8 First-Generation Fermion Particles
(**X** denotes left-handed twist carrying no charge, but representing Octonion)
(right-handed massive electron and quarks emerge dynamically)



and by Triality 8 First=Generation Fermion Antipaticles
(**X** denotes right-handed twist carrying no charge, but representing Octonion)
(left-handed massive positron and antiquarks emerge dynamically)

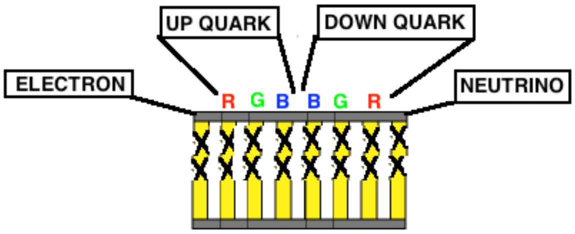


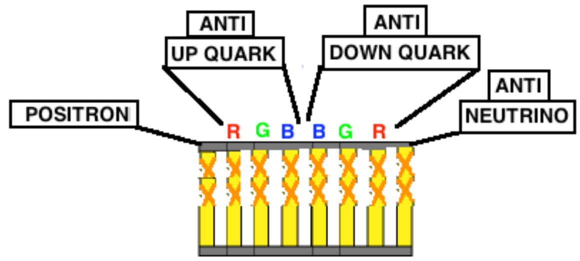
and also by Triality 8D Spacetime - M4 x CP2 Kaluza-Klein
M4 coordinates = {t,x,y,z} CP2 coordinates = {**R**,**G**,**B**,**X**}
(Spacetime Strands have no Twist)



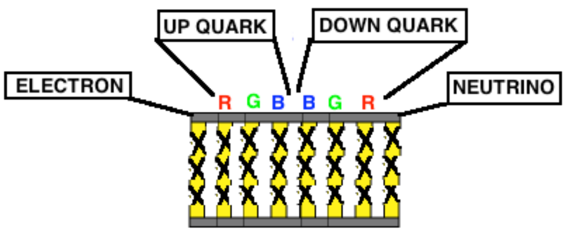
Fermions of Second and Third Generations have 2 or 3 Twists representing Pairs or Triples of Octonions

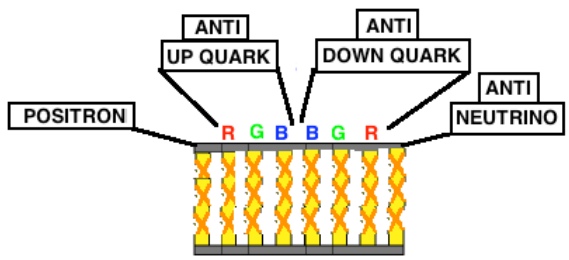
Second Generation:





Third Generation:





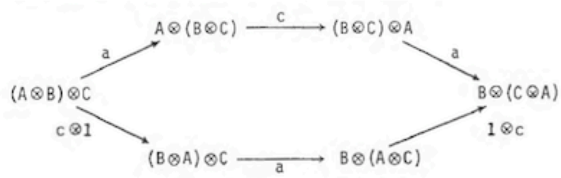
Further:

2 copies of 28D D4 + 64D D8 / D4xD4 + (64+64)D /e8 / D8 = 248D E8
that lives in Cl(0,16) = tensor product Cl(0,8) x Cl(0,8)
for E8-Cl(16) Physics (see viXra 1602.0319)

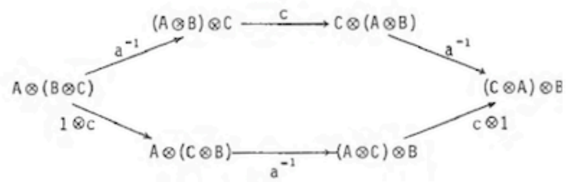
Also:

Andre Joyal and Rose Street in Macquarie Mathematics Report NO.860081 Nov 1986 gave diagrams for Braid Groups B1 - B7 and structures in Braids B5-B7

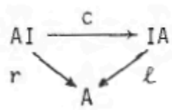
B1.


(A⊗B)⊗C
a
A⊗(B⊗C)
c
(B⊗C)⊗A
a
B⊗(C⊗A)
c⊗1
(B⊗A)⊗C
a
B⊗(A⊗C)
1⊗c

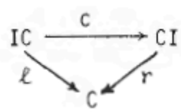
B2.

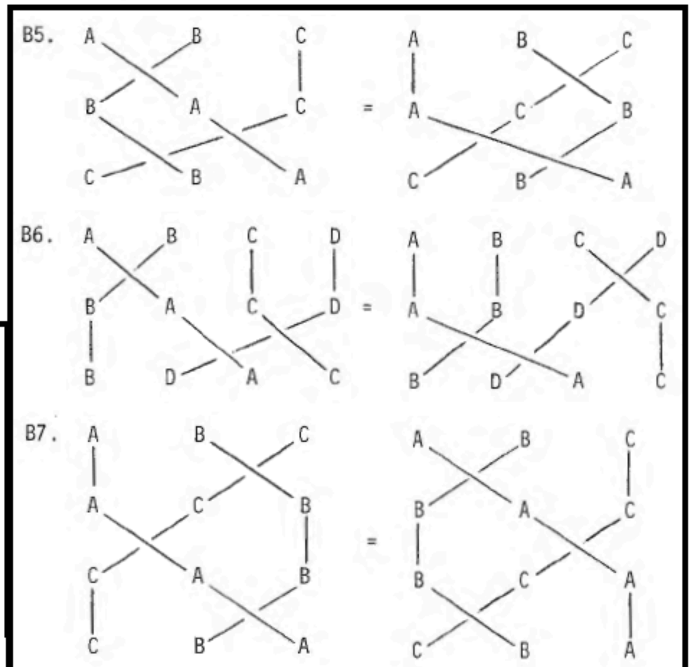

A⊗(B⊗C)
a⁻¹
(A⊗B)⊗C
c
C⊗(A⊗B)
a⁻¹
(C⊗A)⊗B
1⊗c
A⊗(C⊗B)
a⁻¹
(A⊗C)⊗B
c⊗1

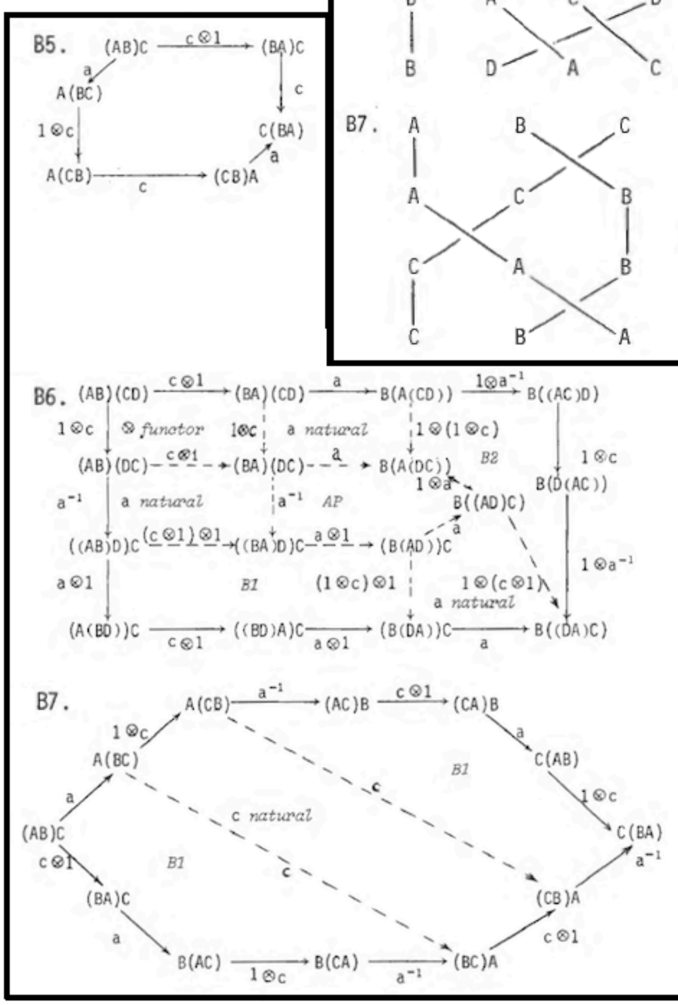
B3.


AI
c
IA
r
ℓ
A

B4.


IC
c
CI
ℓ
r
C


B5.
B6.
B7.
=


B5.
(AB)C
c⊗1
(BA)C
a
A(BC)
c
1⊗c
C(BA)
A(CB)
c
(CB)A
a
B6.
(AB)(CD)
c⊗1
(BA)(CD)
a
B(A(CD))
1⊗a⁻¹
B((AC)D)
1⊗c
⊗ functor
1⊗c
a natural
1⊗(1⊗c)
(AB)(DC)
c⊗1
(BA)(DC)
a
B(A(DC))
B2
1⊗c
B(D(AC))
a⁻¹
a natural
a⁻¹
AP
1⊗a
B((AD)C)
((AB)D)C
(c⊗1)⊗1
((BA)D)C
a⊗1
(B(AD))C
a
a⊗1
B1
(1⊗c)⊗1
1⊗(c⊗1)
1⊗a⁻¹
a natural
(A(BD))C
c⊗1
((BD)A)C
a⊗1
(B(DA))C
a
B((DA)C)
B7.
A(CB)
a⁻¹
(AC)B
c⊗1
(CA)B
1⊗c
a
A(BC)
C(AB)
B1
a
c
1⊗c
c natural
(AB)C
C(BA)
c⊗1
B1
c
a⁻¹
(BA)C
(CB)A
a
c⊗1
B(AC)
1⊗c
B(CA)
a⁻¹
(BC)A

Tao Cheng, Hua-Lin Huang, and Yuping Yang in arxiv 1510.04408
gave 168 braidings such that Octonions are an Azumaya algebra

Theorem 1.4. *There are exactly 168 braidings $\mathcal{R}$ such that $\mathbb{O}$ is an Azumaya algebra in $(Vec_{\mathbb{Z}_2^3}^{\partial F}, \mathcal{R})$, where*

$$\mathcal{R}(x,y) = (-1)^{x_1x_2y_3+x_1y_2x_3+y_1x_2x_3+y_1y_2x_3+y_1x_2y_3+x_1y_2y_3+\sum_{i,j=1}^{3} a_{ij}x_iy_j}, \quad \forall x,y \in \mathbb{Z}_2^3$$

with $(a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33})$ listed in the following table.

0, 0, 0, 0, 0, 0, 0, 0, 1	0, 0, 0, 0, 0, 0, 0, 1, 0	0, 0, 0, 0, 0, 0, 1, 0, 0	0, 0, 0, 0, 0, 0, 1, 1, 1	0, 0, 0, 0, 0, 1, 0, 0, 0
0, 0, 0, 0, 0, 1, 0, 1, 1	0, 0, 0, 0, 0, 1, 1, 0, 0	0, 0, 0, 0, 0, 1, 1, 1, 1	0, 0, 0, 0, 1, 0, 0, 0, 0	0, 0, 0, 0, 1, 0, 0, 1, 1
0, 0, 0, 0, 1, 0, 1, 0, 0	0, 0, 0, 0, 1, 0, 1, 1, 1	0, 0, 0, 0, 1, 1, 0, 0, 1	0, 0, 0, 0, 1, 1, 0, 1, 0	0, 0, 0, 0, 1, 1, 1, 0, 0
0, 0, 0, 0, 1, 1, 1, 1, 1	0, 0, 0, 1, 0, 0, 0, 0, 0	0, 0, 0, 1, 0, 0, 0, 0, 1	0, 0, 0, 1, 0, 0, 0, 1, 0	0, 0, 0, 1, 0, 0, 0, 1, 1
0, 0, 0, 1, 1, 1, 0, 0, 0	0, 0, 0, 1, 1, 1, 0, 0, 1	0, 0, 0, 1, 1, 1, 0, 1, 0	0, 0, 0, 1, 1, 1, 0, 1, 1	0, 0, 1, 0, 0, 0, 0, 0, 0
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0, 1, 0, 1, 1, 0, 0, 0, 1	0, 1, 0, 1, 1, 0, 0, 1, 0	0, 1, 0, 1, 1, 0, 0, 1, 1	0, 1, 0, 1, 1, 1, 0, 0, 0	0, 1, 0, 1, 1, 1, 0, 0, 1
0, 1, 0, 1, 1, 1, 0, 1, 0	0, 1, 0, 1, 1, 1, 0, 1, 1	1, 0, 0, 0, 0, 0, 0, 0, 0	1, 0, 0, 0, 0, 0, 0, 1, 0	1, 0, 0, 0, 0, 0, 1, 0, 1
1, 0, 0, 0, 0, 0, 1, 1, 1	1, 0, 0, 0, 0, 1, 0, 0, 0	1, 0, 0, 0, 0, 1, 0, 1, 1	1, 0, 0, 0, 0, 1, 1, 0, 0	1, 0, 0, 0, 0, 1, 1, 1, 1
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1, 1, 1, 1, 1, 1, 0, 1, 1	1, 1, 1, 1, 1, 1, 1, 0, 1	1, 1, 1, 1, 1, 1, 1, 1, 1		

"... Azumaya algebra is a generalization of ... algebras ... introduced in ... 1951 ...[by]... Goro Azumaya ...[and]... developed further ...[by]... Alexander Grothendieck ..."
Alexander Grothendieck visited North Vietnam in late 1967 teaching mathematics to ... Hoang Xuan Sinh who ... earned her doctorate under Grothendieck's supervision from Paris Diderot University in 1975, with a handwritten thesis ...
on ...[gr-categories that] ...prefigured much of the modern theory of 2-groups ...
[such as Braid Groups]..." (from Wikipedia)

Here are how all 256 Elementary Cellular Automata correspond
to all 256 elements of the Cl(8) Real Clifford Algebra = 16x16 Real Matrices:

Here is how Cl(16) = tensor product Cl(8) x Cl(8) works
and how it was known to the builders of the Giza Pyramids
and how Cl(16) information corresponds to information in 40 micron Microtubules: