A Short Note From Myself to Myself to Better Understand Hawking

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Abstract

In this short paper we look at the Hawking temperature from a Newtonian perspective as well as a General Relativity perspective. If we are considering the Hawking temperature, and simply replace the gravitational field input in his 1974 formula with that of Newton, we will get a Hawking temperature of half of that of the well-known Hawking temperature formula. This is very similar to the case where Newton's theory predicts half the light bending that GR does. Based on recent theoretical research on Newton's gravitational constant, we also rewrite the Hawking temperature to give a somewhat different perspective without changing the output of the formula, that makes some of the Hawking formulas more intuitive.

Key words: Hawking temperature, Hawking radiation, Newton gravitational field, General relativity, Planck length, gravitational constant.

1 Hawking Temperature in the Newtonian Gravitational Field versus General Relativity Theory

In 1974, Hawking [1] introduced the idea of black-hole radiation and a corresponding temperature at the black hole's surface, better known today as Hawking radiation and Hawking temperature. In this paper, we will look at Hawking temperature from a Newtonian perspective, a general relativity perspective, and a newly-introduced modified Newtonian gravitational field approach.

The Hawking temperature formula is known today as

$$T = \frac{c^3}{8\pi GM}\frac{\hbar}{k_b} \tag{1}$$

where c is the speed of light, G is the Newton gravitational constant, M is the mass$\hbar$ is the reduced Planck constant, and k_b is the Boltzmann constant. The Hawking temperature was originally stated as

> *"one would expect if the black hole was a body with temperature of* $(\kappa/2\pi)(\hbar/2k_b)$*..."*
> – Stephen Hawking 1974

where κ is the surface gravity of the black hole. If we assume, for simplicity's sake, the Newton surface gravity at the Schwarzschild radius, we have

$$\kappa = g = \frac{GM}{r_s^2} \tag{2}$$

Replacing the κ in the Hawking temperature formula with this we get

$$\begin{aligned}
T &= \frac{\frac{\frac{GM}{r_s^2}}{c}}{2\pi}\frac{\hbar}{2k_b} \\
T &= \frac{\frac{\frac{GM}{\left(\frac{2GM}{c^2}\right)^2}}{c}}{2\pi}\frac{\hbar}{2k_b} \\
T &= \frac{\frac{\frac{GM}{\frac{4G^2M^2}{c^4}}}{c}}{2\pi}\frac{\hbar}{2k_b} \\
T &= \frac{c^3}{8\pi GM}\frac{\hbar}{2k_b}
\end{aligned} \tag{3}$$

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this is off by a factor of $\frac{1}{2}$ relative to the Hawking temperature formula. The main reason for the difference is that the Hawking temperature formula is derived from the Schwarzschild metric solution of general relativity theory. The Newtonian gravitational field does not take into account the bending of space-time, which is known to be different in strong gravitational fields than what was described by Newton.

We find it interesting, however, that the Hawking temperature derived from Newton's gravitational field inputs seems to be off by $\frac{1}{2}$ relative to the GR solution. This is very similar to Newton bending of light first derived by Soldner [2, 3], which is also off by a half, relative to GR predictions, which has basically been confirmed experimentally.

2 Hawking Temperature from a Gravitation Composite Constant Perspective

Recently, Haug [4, 5, 6] has given strong evidence for the concept that the Newton gravitational constant is a composite constant of the form

$$G = \frac{l_p^2 c^3}{\hbar} \tag{4}$$

where l_p is the Planck length. The Planck length was introduced by Max Planck in 1899 in the form $l_p = \sqrt{\frac{G\hbar}{c^3}}$. One can solve Max Planck's formula for the Planck length with respect to G and get the formula 4, or one can use dimensional analysis as shown by Haug [4]. In 2014, McCulloch [8] derived basically the same gravitational constant from Heisenberg's uncertainty principle; his formula was

$$G = \frac{\hbar c}{m_p^2} \tag{5}$$

and because the Planck mass can be written as

$$m_p = \frac{\hbar}{l_p}\frac{1}{c} \tag{6}$$

we can see that this is the same as the Haug formula

$$G = \frac{\hbar c}{\frac{\hbar}{l_p}\frac{1}{c}\frac{\hbar}{l_p}\frac{1}{c}} = \frac{l_p^2 c^3}{\hbar} \tag{7}$$

It may seem that we are introducing a circular problem here without a solution. e.g. that to write the Newton gravitaional constant in a composite form, we need to know the Planck length (or Planck mass), and to know the Planck length we need to know the Newton gravitational constant, and that to write the gravitational constant this way is just biting oneself in the tail. Haug [9] has however recently shown that the Planck length (and thereby the Planck mass and Planck time) can be measured experimentally using a Cavendish [10] apparatus totally independent of any knowledge of the Newtonian gravitational constant.

Returning to the Hawking temperature formula, the formula itself gives very little intuition, except we can see that the temperature is inversely related to the mass. Interestingly, if we rewrite it in the view that Newton's gravitational formula is a composite, we get

$$\begin{aligned} T &= \frac{c^3}{8\pi GM}\frac{\hbar}{k_b} \\ T &= \frac{c^3}{8\pi \frac{l_p^2 c^3}{\hbar} N m_p}\frac{\hbar}{k_b} \\ T &= \frac{c^3}{8\pi \frac{l_p^2 c^3}{\hbar} N \frac{\hbar}{l_p}\frac{1}{c}}\frac{\hbar}{k_b} = \frac{1}{N8\pi}\frac{\frac{\hbar}{l_p}c}{k_b} = \frac{1}{N8\pi}\frac{m_p c^2}{k_b} \end{aligned} \tag{8}$$

where N is the number of Planck masses in the mass. The part $\frac{m_p c^2}{k_b}$ is easily recognizable as the Planck temperature. So, the Hawking temperature is basically a factor $\frac{1}{N8\pi}$ multiplied by the Planck temperature (known from before, but not derived in this way). We can also clearly see from the formula that the Hawking temperature falls with the number of Planck masses in the gravitational mass. This way of writing the Hawking temperature is more intuitive than its original form and we can see that after it is rewritten in this form we no longer have c^3, but rather c^2, which is simply is connected to the well-known energy mass relationship of Einstein [7].

Also related to Hawking radiation is the Hawking luminosity for a black hole; this is given by

$$P = \frac{\hbar c^6}{15360\pi G^2 M^2} \tag{9}$$

Again, we can rewrite this based on the view the gravitational constant is a composite constant, and we get

$$\begin{aligned}
P &= \frac{\hbar c^6}{15360\pi \left(\frac{l_p^2 c^3}{\hbar} N \frac{\hbar}{l_p}\frac{1}{c}\right)^2} \\
P &= \frac{\hbar c^6}{15360\pi l_p^2 c^4 N^2} \\
P &= \frac{1}{N^2 15360}\frac{\hbar c^2}{l_p^2}
\end{aligned} \tag{10}$$

According to Hawking, the time it takes for a black hole to dissipate is

$$t_{ev} = \frac{15360\pi G^2 M^3}{\hbar c^4}$$

Rewritten based on the view that the gravitational constant is a composite constant, we get

$$\begin{aligned}
t_{ev} &= \frac{15360\pi \left(\frac{l_p^2 c^3}{\hbar}\right)^2 \left(N \frac{\hbar}{l_p}\frac{1}{c}\right)^3}{\hbar c^4} \\
t_{ev} &= \frac{15360\pi \frac{l_p^4 c^6}{\hbar^2} N^3 \frac{\hbar^3}{l_p^3}\frac{1}{c^3}}{\hbar c^4} \\
t_{ev} &= \frac{15360\pi l_p c^3 N^3 \hbar}{\hbar c^4} \\
t_{ev} &= 15360\pi \frac{l_p}{c} = 15360\pi t_p
\end{aligned} \tag{11}$$

that is the time it takes for a black hole to dissipate is the constant 15360π times the Planck time. This formula is already known and one can write the Hawking dissipate time as

$$t_{ev} = 15360\pi \sqrt{\frac{G\hbar}{c^5}} \tag{12}$$

where $\sqrt{\frac{G\hbar}{c^5}}$ is the well-known Planck time as first described by Planck in 1899. In Table 1 we show the traditional form of writing the Hawking dissipation time and luminosity, as well as the deeper form shown here. This is presented from a somewhat different perspective, as we do not need to know G to measure the Max Planck natural units, all we need is a Cavendish apparatus, according to recent work by Haug.

	Original form	**Deeper form**
Hawking temperature	$\frac{c^3}{8\pi GM}\frac{\hbar}{k_b}$	$T = \frac{1}{N8\pi}\frac{m_p c^2}{k_b}$
Hawking dissipation time	$t_{ev} = \frac{15360\pi G^2 M^3}{\hbar c^4}$	$T = 15360\pi \frac{l_p}{c} = 15360\pi t_p$
Bekenstein?Hawking luminosity	$P = \frac{\hbar c^6}{15360\pi G^2 M^2}$	$P = \frac{1}{N^2 15360}\frac{\hbar c^2}{l_p^2}$

Table 1: The table of a series of measurements that actually can be observed/measured in relation to gravity, and also the gravitational force that we cannot observe and measure.

3 Conclusion

In this short paper, we have shown that by simply replacing the gravitational field in the Hawking temperature with that of Newton we get a Hawking temperature of just half of that predicted by Hawking. This seems natural as the gravitational field in GR and Newton are different. By viewing the gravitational constant as a composite, we can rewrite the Hawking temperature without changing its output; this may be a more intuitive way of studying this temperature. This alternative way of writing these formulas has existed for some time, but it was assumed that we needed to know G in order to calculate the Planck mass, the Planck length, and the Planck time. Based on new work, we find that this is not the case, since this calculation can be done with no knowledge of big G, by using results from a Cavendish apparatus, as recently shown by Haug.

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