Some hybrid weighted aggregation operators under neutrosophic set environment and their applications to multi criteria decision making

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Abstract Neutrosophic sets (NS) contain the three ranges: truth, indeterminacy, and falsity membership degrees, and are very useful for describing and handling the uncertainties in the real life problem. The aggregation of the neutrosophic sets is one of the important concepts to aggregate the uncertain data. In this paper, some new hybrid aggregation operators based on arithmetic and geometric aggregation operators have been developed to aggregate the information under the single-valued neutrosophic (SVN) set environment. Further, we establish some of its basic properties. Then, we extend these operators to the interval neutrosophic set (INS) environment. Further, an approach based on these operators for multi-criteria decision-making (MCDM) problems is explored under SVN/INS environment. Moreover, to demonstrate its practicality and effectiveness, numerical examples have been presented under both environments. Finally, a comparison analysis has been made with some other existing methods to analyze the superiority of proposed work.

Keywords: Neutrosophic Set; single-valued neutrosophic sets; interval neutrosophic set; aggregation operators.

1 Introduction

During the decision-making process, it is difficult for the person to get the sufficient and accurate data for real decision-making owing to the vagueness on the uncertainties. To address this issue, Zadeh [1] introduced the concept of the fuzzy set (FS) and since then it has been widely used in many real fields. Further, different types of the FSs have been developed and investigated, namely intuitionistic fuzzy set [2], interval-valued intuitionistic fuzzy set [3], Pythagorean fuzzy set [4], neutrosophic set [5]. In the past few decades, under these environments, researchers have gained great attention and been successfully applied to many practical areas such as decision making, pattern recognition, medical diagnosis, and clustering analysis [6, 7, 8, 9, 10, 11, 12].

Among these FSs, neutrosophic set (NS) seems to be more reasonable and acceptable. In NSs, the uncertain information is represented as a tuple of membership, non-membership and indeterminacy degrees, all are independent and are real standard or non-standard subsets of $]0^-, 1^+[$. Thus, it will be difficult to apply in real scientific and engineering areas. Therefore, Wang et al. [13, 14] proposed the concepts of an interval neutrosophic set (INS) and a single-valued neutrosophic (SVN) set (SVNS), which are the classes of the NSs. Under these environments, it is necessary to aggregate the different SVNSs or INSs and hence their corresponding weighted operators and the ordered weighted operators have been applied to it. For instance, Ye [15] developed the single-valued neutrosophic weighted averaging (SVNWA) operator and single-valued neutrosophic weighted geometric (SVNWG) operator. Peng et al. [16] defined the operations of SVNSs and based on it some weighted and ordered weighted averaging and geometric aggregation operators, namely single valued neutrosophic ordered weighted average (SVNOWA) and single valued neutrosophic ordered weighted geometric (SVNOWG) operators have been developed. Liu et al. [17] developed some generalized neutrosophic aggregation operators based

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on Hamacher operations and named as single-valued neutrosophic Hamacher weighted averaging (SVNHWA), single-valued neutrosophic Hamacher ordered weighted averaging (SVNHOWA), single-valued neutrosophic Hamacher weighted geometric(SVNHWG) and single-valued neutrosophic Hamacher ordered weighted geometric (SVNHOWG). Zhang et al. [18] proposed an aggregation operator under the INSs and called as interval neutrosophic number weighted averaging (INNWA) and interval neutrosophic number weighted geometric (INNWG) operators. After that, Aiwu et al. [19] proposed the generalized weighted aggregation order under the INS environment. Recently, Nancy and Garg [20] proposed the weighted averaging and geometric aggregation operator based on the Frank norm operators and called as single-valued neutrosophic Frank weighted averaging and geometric operators denoted by SVNFWA and SVNFWG respectively. Apart from that, some other authors have worked under the NS environment and developed their corresponding aggregation operators [21, 22, 23, 24, 25, 26, 27] and their corresponding references.

Effective aggregation is one of the most important research areas in the field of decision-making. Aggregation, which usually, involves mathematical operators, is not just an average; rather, it represents a more general notion. The results of the aggregation operator are meaningful if the aggregated value given by it is unbiased, that is, it should never tend to one or some number(s)(among to be combined) whose weight is on the higher side and doesn't tend to maximum or the minimum arguments. But, the aggregated value of existing weighted averaging and geometric aggregation operators sometimes tends towards the maximum arguments or the argument with higher importance respectively. Hence, they give unrealistic results in some situations.

So, the main objective of the manuscript is to introduce some new aggregation operators which have characteristics of both the averaging and geometric operators. In order to achieve this objective, we propose the hybrid weighted arithmetic and geometric aggregation operators under the neutrosophic set environment. These aggregation operators are called as hybrid SVN weighted averaging and geometric and hybrid SVN ordered weighted averaging and geometric. Furthermore, this version of operators give us the averaging and geometric operators as special cases which make its formulation robust. Some of its desirable properties have also been investigated. Later on, these aggregation operators have been extended to the interval neutrosophic set(INS) environment. Finally, an illustrative example of the decision-making problem has been given to show the developed method.

The rest of the paper is organized as follows. In section 2, we introduce the neutrosophic set theory along with limitations of weighted averaging and geometric operators. Section 3 proposes the hybrid aggregation operators under SVN environment and also desirable properties of them are investigated. In section 4, we extend these proposed aggregation operators from the SVN to INS domain. Section 5 illustrates the proposed approach with a numerical example and also shows it flexibility by assigning different values to the parameter. Also, the comparative study validates the proposed approach. Section 6 ends up with concluding remarks.

2 Preliminaries

In this section, some basic concepts about the SVNSs and INSs are briefly presented over the universal set X.

2.1 Neutrosophic set theory

Definition 1 [5] A neutrosophic set (NS) β consists of three independent degrees namely truth (ζ_β), indeterminacy (κ_β), and falsity (φ_β) which are defined as

$$\beta = \{\langle x, \zeta_\beta(x), \kappa_\beta(x), \varphi_\beta(x) \mid x \in X\rangle\}$$

where $\zeta_\beta(x)$, $\kappa_\beta(x)$, $\varphi_\beta(x) \in]0^-, 1^+[$ such that $0^- \leq \zeta_\beta(x), \kappa_\beta(x), \varphi_\beta(x) \leq 3^+$.

Definition 2 [14]A single-valued neutrosophic set (SVNS) β in X is defined as

$$\beta = \{\langle x, \zeta_\beta(x), \kappa_\beta(x), \varphi_\beta(x) \mid x \in X\rangle\}$$

where $\zeta_\beta(x)$, $\kappa_\beta(x)$, $\varphi_\beta(x) \in [0, 1]$ such that $0 \leq \zeta_\beta(x), \kappa_\beta(x), \varphi_\beta(x) \leq 3$. A SVNS is an instance of a NS. For convenience, we denote these pairs as $\beta = \langle\zeta_\beta, \kappa_\beta, \varphi_\beta\rangle$, throughout this article, and called as single-valued neutrosophic number (SVNN).

Definition 3 An order relation, based on S and H functions, between two SVNNs β and γ is stated as[14], if $S(\beta) > S(\gamma)$ then $\beta \succ \gamma$ and if $S(\beta) = S(\gamma)$ and $H(\beta) > H(\gamma)$ then $\beta \succ \gamma$, if $H(\beta) = H(\gamma)$ then $\beta = \gamma$, where $S(\beta) = \zeta_\beta - \kappa_\beta - \varphi_\beta$ and $H(\beta) = \zeta_\beta + \kappa_\beta + \varphi_\beta$.

Definition 4 Let $\beta_1 = \langle \zeta_1, \kappa_1, \varphi_1 \rangle$ and $\beta_2 = \langle \zeta_2, \kappa_2, \varphi_2 \rangle$ be two SVNNs. Then the operational laws between them are defined as follows [16]

(i) $\beta_1 \oplus \beta_2 = \langle \zeta_1 + \zeta_2 - \zeta_1\zeta_2, \kappa_1\kappa_2, \varphi_1\varphi_2 \rangle$
(ii) $\beta_1 \otimes \beta_2 = \langle \zeta_1\zeta_2, \kappa_1 + \kappa_2 - \kappa_1\kappa_2, \varphi_1 + \varphi_2 - \varphi_1\varphi_2 \rangle$
(iii) $\lambda\beta_1 = \langle 1 - (1-\zeta_1)^\lambda, {\kappa_1}^\lambda, {\varphi_1}^\lambda \rangle; \lambda > 0$
(iv) ${\beta_1}^\lambda = \langle {\zeta_1}^\lambda, 1 - (1-\kappa_1)^\lambda, 1 - (1-\varphi_1)^\lambda \rangle; \lambda > 0$

In order to aggregate the different SVNNs $\beta_j = \langle \zeta_j, \kappa_j, \varphi_j \rangle$ for $j = 1, 2, \ldots, n$ corresponding to its weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ such that $\omega_j > 0$ and $\sum\limits_{j=1}^{n} \omega_j = 1$, Peng et al. [16] gave the different weighted geometric and averaging aggregation operators which are summarized as follows:

(a) SVNWA operator [16]

$$\text{SVNWA}(\beta_1, \beta_2, \ldots, \beta_n) = \left\langle 1 - \prod_{j=1}^{n} (1-\zeta_j)^{\omega_j}, \prod_{j=1}^{n} (\kappa_j)^{\omega_j}, \prod_{j=1}^{n} (\varphi_j)^{\omega_j} \right\rangle \tag{1}$$

(b) SVNWG operator [16]

$$\text{SVNWG}(\beta_1, \beta_2, \ldots, \beta_n) = \left\langle \prod_{j=1}^{n} (\zeta_j)^{\omega_j}, 1 - \prod_{j=1}^{n} (1-\kappa_j)^{\omega_j}, 1 - \prod_{j=1}^{n} (1-\varphi_j)^{\omega_j} \right\rangle \tag{2}$$

(c) SVNOWA operator [16]

$$\text{SVNOWA}(\beta_1, \beta_2, \ldots, \beta_n) = \left\langle 1 - \prod_{j=1}^{n} (1-\zeta_{\xi(j)})^{\omega_j}, \prod_{j=1}^{n} (\kappa_{\xi(j)})^{\omega_j}, \prod_{j=1}^{n} (\varphi_{\xi(j)})^{\omega_j} \right\rangle \tag{3}$$

(d) SVNOWG operator [16]

$$\text{SVNOWG}(\beta_1, \beta_2, \ldots, \beta_n) = \left\langle \prod_{j=1}^{n} (\zeta_{\xi(j)})^{\omega_j}, 1 - \prod_{j=1}^{n} (1-\kappa_{\xi(j)})^{\omega_j}, 1 - \prod_{j=1}^{n} (1-\varphi_{\xi(j)})^{\omega_j} \right\rangle \tag{4}$$

where ξ is a permutation of $(1, 2, \ldots, n)$ such that $\beta_{\xi(j-1)} \geq \beta_{\xi(j)}$ for $j = 2, \ldots, n$.

2.2 Shortcomings of SVNWA, SVNWG, SVNOWA, SVNOWG operators

The existing aggregation operators given in Eqs. (1)-(4) are the basic operators used to aggregate the SVNNs but these operators have some shortcomings which are discussed as: Let $\beta_1 = \langle 0.0001, 0, 0 \rangle$ and $\beta_2 = \langle 1, 0, 0 \rangle$ be two SVNNs and $\omega = (0.9, 0.1)^T$ be the corresponding weight vector. By utilizing the Eqs. (1)-(4), we get $\text{SVNWA}(\beta_1, \beta_2) = \langle 1, 0, 0 \rangle$, $\text{SVNWG}(\beta_1, \beta_2) = \langle 0.002, 0, 0 \rangle$, $\text{SVNOWA}(\beta_1, \beta_2) = \langle 1, 0, 0 \rangle$ and $\text{SVNOWG}(\beta_1, \beta_2) = \langle 0.5012, 0, 0 \rangle$.

On the other hand, if we take $\omega = (0.1, 0.9)$ as weight vector , then by using Eqs. (1)-(4), we get $\text{SVNWA}(\beta_1, \beta_2) = \langle 1, 0, 0 \rangle$, $\text{SVNWG}(\beta_1, \beta_2) = \langle 0.5012, 0, 0 \rangle$, $\text{SVNOWA}(\beta_1, \beta_2) = \langle 1, 0, 0 \rangle$ and $\text{SVNOWG}(\beta_1, \beta_2) = \langle 0.002, 0, 0 \rangle$.

From the above cases, we notice that the aggregated values of SVNWA and SVNOWA operators tend to the maximum argument and the aggregated values of the SVNWG and SVNOWG operator may tend to the value having maximum weight. So, it is concluded from these results that the existing operators may not give the reasonable results and hence there is need to improve the existing aggregation operators so that we can overcome these flaws.

3 Aggregation operators with single-valued neutrosophic information

Let Ω be the collection of all SVNNs. In this section, we have proposed some hybrid aggregation operators namely, Hybrid SVN weighted averaging and geometric (H-SVNWAG) and Hybrid SVN ordered weighted averaging and geometric (H-SVNOWAG) aggregation operators.

3.1 Hybrid single-valued neutrosophic weighted arithmetic and geometric aggregation operator

Definition 5 Let $\beta_j = \langle \zeta_j, \kappa_j, \varphi_j \rangle (j = 1, 2, \ldots, n)$ be SVNNs. A H-SVNWAG operator is a mapping H-SVNWAG : $\Omega^n \to \Omega$ and is defined as

$$\text{H-SVNWAG}(\beta_1, \beta_2, \ldots, \beta_n) = \left(\bigoplus_{j=1}^{n} \omega_j \beta_j \right)^{\lambda} \bigotimes \left(\bigotimes_{j=1}^{n} \beta_j^{\omega_j} \right)^{1-\lambda} \tag{5}$$

where $\lambda \in [0, 1]$ be a real number and ω_j is the standardized weight vector of $\beta_j; (j = 1, 2, \ldots, n)$.

Theorem 1 *The aggregated value by using H-SVNWAG operator for a collection of SVNNs $\beta_j; (j = 1, 2, \ldots, n)$ is given in Eq. (5) and still a SVNN.*

$$\begin{aligned} &\textit{H-SVNWAG}(\beta_1, \beta_2, \ldots, \beta_n) \\ &= \Bigg\langle \left(1 - \prod_{j=1}^{n} (1-\zeta_j)^{\omega_j}\right)^{\lambda} \left(\prod_{j=1}^{n} \zeta_j^{\,\omega_j}\right)^{1-\lambda}, 1 - \left(1 - \prod_{j=1}^{n} \kappa_j^{\,\omega_j}\right)^{\lambda} \left(\prod_{j=1}^{n} (1-\kappa_j)^{\omega_j}\right)^{1-\lambda}, \\ &\quad 1 - \left(1 - \prod_{j=1}^{n} \varphi_j^{\,\omega_j}\right)^{\lambda} \left(\prod_{j=1}^{n} (1-\varphi_j)^{\omega_j}\right)^{1-\lambda} \Bigg\rangle \end{aligned} \tag{6}$$

Proof For SVNNs β_j and a real number $\lambda \in [0, 1]$, we have

$$\begin{aligned} \left(\bigoplus_{j=1}^{n} \omega_j \beta_j \right)^{\lambda} &= \left(\left\langle 1 - \prod_{j=1}^{n} (1-\zeta_j)^{\omega_j}, \prod_{j=1}^{n} (\kappa_j)^{\omega_j}, \prod_{j=1}^{n} (\varphi_j)^{\omega_j} \right\rangle \right)^{\lambda} \\ &= \left\langle \left(1 - \prod_{j=1}^{n} (1-\zeta_j)^{\omega_j}\right)^{\lambda}, 1 - \left(1 - \prod_{j=1}^{n} (\kappa_j)^{\omega_j}\right)^{\lambda}, 1 - \left(1 - \prod_{j=1}^{n} (\varphi_j)^{\omega_j}\right)^{\lambda} \right\rangle \end{aligned} \tag{7}$$

and

$$\begin{aligned} \left(\prod_{j=1}^{n} \beta_j^{\omega_j} \right)^{1-\lambda} &= \left(\left\langle \prod_{j=1}^{n} (\zeta_j)^{\omega_j}, 1 - \prod_{j=1}^{n} (1-\kappa_j)^{\omega_j}, 1 - \prod_{j=1}^{n} (1-\varphi_j)^{\omega_j} \right\rangle \right)^{1-\lambda} \\ &= \left\langle \left(\prod_{j=1}^{n} (\zeta_j)^{\omega_j}\right)^{1-\lambda}, 1 - \left(\prod_{j=1}^{n} (1-\kappa_j)^{\omega_j}\right)^{1-\lambda}, 1 - \left(\prod_{j=1}^{n} (1-\varphi_j)^{\omega_j}\right)^{1-\lambda} \right\rangle \end{aligned} \tag{8}$$

Therefore, based on these and by Definition 5, we have

$$
\begin{aligned}
&\text{H-SVNWAG}(\beta_1, \beta_2, \ldots, \beta_n) \\
&= \left\langle \left(1 - \prod_{j=1}^{n}(1-\zeta_j)^{\omega_j}\right)^{\lambda}, 1 - \left(1 - \prod_{j=1}^{n}(\kappa_j)^{\omega_j}\right)^{\lambda}, 1 - \left(1 - \prod_{j=1}^{n}(\varphi_j)^{\omega_j}\right)^{\lambda} \right\rangle \\
&\quad \bigotimes \left\langle \left(\prod_{j=1}^{n}(\zeta_j)^{\omega_j}\right)^{1-\lambda}, 1 - \left(\prod_{j=1}^{n}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda}, 1 - \left(\prod_{j=1}^{n}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda} \right\rangle \\
&= \left\langle \left(1 - \prod_{j=1}^{n}(1-\zeta_j)^{\omega_j}\right)^{\lambda} \left(\prod_{j=1}^{n}(\zeta_j)^{\omega_j}\right)^{1-\lambda}, 1 - \left(1 - \prod_{j=1}^{n}(\kappa_j)^{\omega_j}\right)^{\lambda} + 1 - \left(\prod_{j=1}^{n}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda} \right. \\
&\quad - \left(1 - \left(1 - \prod_{j=1}^{n}(\kappa_j)^{\omega_j}\right)^{\lambda}\right)\left(1 - \left(\prod_{j=1}^{n}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda}\right), 1 - \left(1 - \prod_{j=1}^{n}(\varphi_j)^{\omega_j}\right)^{\lambda} \\
&\quad \left. +1 - \left(\prod_{j=1}^{n}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda} - \left(1 - \left(1 - \prod_{j=1}^{n}(\varphi_j)^{\omega_j}\right)^{\lambda}\right)\left(1 - \left(\prod_{j=1}^{n}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda}\right) \right\rangle \\
&= \left\langle \left(1 - \prod_{j=1}^{n}(1-\zeta_j)^{\omega_j}\right)^{\lambda} \left(\prod_{j=1}^{n}(\zeta_j)^{\omega_j}\right)^{1-\lambda}, 1 - \left(1 - \prod_{j=1}^{n}(\kappa_j)^{\omega_j}\right)^{\lambda} + 1 - \left(\prod_{j=1}^{n}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda} \right. \\
&\quad - \left(1 - \left(\prod_{j=1}^{n}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda} - \left(1 - \prod_{j=1}^{n}(\kappa_j)^{\omega_j}\right)^{\lambda} + \left(1 - \prod_{j=1}^{n}(\kappa_j)^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda}\right), \\
&\quad 1 - \left(1 - \prod_{j=1}^{n}(\varphi_j)^{\omega_j}\right)^{\lambda} + 1 - \left(\prod_{j=1}^{n}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda} - \left(1 - \left(\prod_{j=1}^{n}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda} - \left(1 - \prod_{j=1}^{n}(\varphi_j)^{\omega_j}\right)^{\lambda}\right. \\
&\quad \left.\left. + \left(1 - \prod_{j=1}^{n}(\varphi_j)^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda}\right) \right\rangle \\
&= \left\langle \left(1 - \prod_{j=1}^{n}(1-\zeta_j)^{\omega_j}\right)^{\lambda} \left(\prod_{j=1}^{n}{\zeta_j}^{\omega_j}\right)^{1-\lambda}, 1 - \left(1 - \prod_{j=1}^{n}{\kappa_j}^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda} \right. \\
&\quad \left. 1 - \left(1 - \prod_{j=1}^{n}{\varphi_j}^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda} \right\rangle
\end{aligned}
$$

Hence, the result.

Example 1 Let $\beta_1 = \langle 0.9, 0.1, 0.1\rangle$, $\beta_2 = \langle 0.92, 0.1, 0.05\rangle$ and $\beta_3 = \langle 0.7, 0.1, 0.2\rangle$ be three SVNNs and their corresponding weight vector is $\omega = (0.3, 0.5, 0.2)^T$. Without loss of generality, we assume that $\lambda = 0.5$ be a real number. Then, by utilizing the given information, we have

$$
\begin{aligned}
&\text{H-SVNWAG}(\beta_1, \beta_2, \beta_3) \\
&= \left\langle \left(1 - \prod_{j=1}^{3}(1-\zeta_j)^{\omega_j}\right)^{\lambda} \left(\prod_{j=1}^{3}{\zeta_j}^{\omega_j}\right)^{1-\lambda}, 1 - \left(1 - \prod_{j=1}^{3}{\kappa_j}^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{3}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda}, \right. \\
&\quad \left. 1 - \left(1 - \prod_{j=1}^{3}{\varphi_j}^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{3}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda} \right\rangle \\
&= \left\langle \left(1 - (1-0.9)^{0.3} \times (1-0.92)^{0.5} \times (1-0.7)^{0.2}\right)^{0.5} \left((0.9^{0.3}) \times (0.92^{0.5}) \times (0.7^{0.2})\right)^{1-0.5}, \right. \\
&\quad 1 - \left(1 - (0.1)^{0.3} \times (0.1)^{0.5} \times (0.1)^{0.2}\right)^{0.5}\left((1-0.1)^{0.3} \times (1-0.1)^{0.5} \times (1-0.1)^{0.2}\right)^{0.5}, \\
&\quad \left. 1 - \left(1 - (0.1)^{0.3} \times (0.05)^{0.5} \times (0.2)^{0.2}\right)^{0.5}\left((1-0.1)^{0.3} \times (1-0.05)^{0.5} \times (1-0.2)^{0.2}\right)^{0.5} \right\rangle \\
&= \langle 0.8768, 0.1, 0.089\rangle
\end{aligned}
$$

Remark 1 It is evident from the proposed operator that

(i) if $\lambda = 1$, then H-SVNWAG is reduced to SVNWA [16] operator, and
(ii) if $\lambda = 0$ then it is reduced to SVNWG [16] operator.

Hence, the proposed operator is a more generalized as compared to other existing operator.

Furthermore, it has concluded that the proposed H-SVNWAG operator satisfies the properties of idempotency, boundedness and monotonicity for a collection of SVNN $\beta_j = \langle \zeta_j, \kappa_j, \varphi_j \rangle; (j = 1, 2, \ldots, n)$, which can be stated as follows:

(P1) (Idempotency) If $\beta_j = \beta$ for all j, we have

$$\text{H-SVNWAG}(\beta_1, \beta_2, \ldots, \beta_n) = \beta$$

(P2) (Boundedness) For SVNNs $\beta_j; (j = 1, 2, \ldots, n)$, we have

$$\min\{\beta_j\} \leq \text{H-SVNWAG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \max\{\beta_j\}$$

(P3) (Monotonicity) If β_j and γ_j be two collections of SVNNs such that $\beta_j \leq \gamma_j$ for all j, then

$$\text{H-SVNWAG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \text{H-SVNWAG}(\gamma_1, \gamma_2, \ldots, \gamma_n)$$

Since SVNWA and SVNWG operators satisfy these properties so by the definition of H-SVNWAGA operator, it follows directly and hence we omit their proofs.

Theorem 2 *The existing operators SVNWA, SVNWG and the proposed operator H-SVNWAG satisfy the following inequality for a collection of SVNNs $\beta_j (j = 1, 2, \ldots, n)$,*

$$\textit{SVNWG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \textit{H-SVNWAG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \textit{SVNWA}(\beta_1, \beta_2, \ldots, \beta_n) \tag{9}$$

Proof Let $\beta_j = \langle \zeta_j, \kappa_j, \varphi_j \rangle$ be SVNNs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be their normalized weight vector, then we have $\prod\limits_{j=1}^{n} \zeta_j^{\omega_j} \leq 1 - \prod\limits_{j=1}^{n} (1 - \zeta_j)^{\omega_j}$ and $0 \leq \prod\limits_{j=1}^{n} \zeta_j^{\omega_j}, 1 - \prod\limits_{j=1}^{n} (1 - \zeta_j)^{\omega_j} \leq 1$. Therefore, for a real number $\lambda \in [0, 1]$ we have

$$\left(\prod_{j=1}^{n} \zeta_j^{\omega_j} \right)^{1-\lambda} \leq \left(1 - \prod_{j=1}^{n} (1 - \zeta_j)^{\omega_j} \right)^{1-\lambda}$$

which implies that

$$\left(1 - \prod_{j=1}^{n} (1 - \zeta_j)^{\omega_j} \right)^{\lambda} \left(\prod_{j=1}^{n} \zeta_j^{\omega_j} \right)^{1-\lambda} \leq 1 - \prod_{j=1}^{n} (1 - \zeta_j)^{\omega_j} \tag{10}$$

Similarly, for $\prod\limits_{j=1}^{n} \kappa_j^{\omega_j} \geq 1 - \prod\limits_{j=1}^{n} (1 - \kappa_j)^{\omega_j}$ and $\prod\limits_{j=1}^{n} \varphi_j^{\omega_j} \geq 1 - \prod\limits_{j=1}^{n} (1 - \varphi_j)^{\omega_j}$, we have

$$\prod_{j=1}^{n} \kappa_j^{\omega_j} \leq 1 - \left(1 - \prod_{j=1}^{n} \kappa_j^{\omega_j} \right)^{\lambda} \left(\prod_{j=1}^{n} (1 - \kappa_j)^{\omega_j} \right)^{1-\lambda} \tag{11}$$

and

$$\prod_{j=1}^{n} \varphi_j^{\omega_j} \leq 1 - \left(1 - \prod_{j=1}^{n} \varphi_j^{\omega_j} \right)^{\lambda} \left(\prod_{j=1}^{n} (1 - \varphi_j)^{\omega_j} \right)^{1-\lambda} \tag{12}$$

Thus, by using Eqs. (10), (11), (12) and the definition of score function, we get

$$\begin{aligned}
S(\text{SVNWA}(\beta_1, \beta_2, \ldots, \beta_n)) &= 1 - \prod_{j=1}^{n}(1-\zeta_j)^{\omega_j} - \prod_{j=1}^{n}\kappa_j^{\omega_j} - \prod_{j=1}^{n}\varphi_j^{\omega_j} \\
&\geq \left(1 - \prod_{j=1}^{n}(1-\zeta_j)^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{n}\zeta_j^{\omega_j}\right)^{1-\lambda} - \left\{1 - \left(1 - \prod_{j=1}^{n}\kappa_j^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\kappa_j)^{\omega_j}\right)^{1-\lambda}\right\} \\
&\quad - \left\{1 - \left(1 - \prod_{j=1}^{n}\varphi_j^{\omega_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\varphi_j)^{\omega_j}\right)^{1-\lambda}\right\} \\
&= S(\text{H-SVNWAG}(\beta_1, \beta_2, \ldots, \beta_n))
\end{aligned}$$

Hence, H-SVNWAG$(\beta_1, \beta_2, \ldots, \beta_n) \leq$ SVNWA$(\beta_1, \beta_2, \ldots, \beta_n)$. Similarly, we can obtain that H-SVNWAG$(\beta_1, \beta_2, \ldots, \beta_n) \geq$ SVNWG$(\beta_1, \beta_2, \ldots, \beta_n)$. Thus, we get the required proof.

Example 2 Let $\beta_1 = \langle 0.9, 0.1, 0.1\rangle$, $\beta_2 = \langle 0.92, 0.1, 0.05\rangle$ and $\beta_3 = \langle 0.7, 0.1, 0.2\rangle$ be three SVNNs and their corresponding weight vector is $\omega = (0.3, 0.5, 0.2)^T$ as given in Example 1. Then by using Eqs. (1), (2) and (6), we get SVNWA$(\beta_1, \beta_2, \beta_3) = \langle 0.9426, 0.1, 0.08\rangle$, H-SVNWAG$(\beta_1, \beta_2, \beta_3) = \langle 0.8768, 0.1, 0.089\rangle$ and SVNWG$(\beta_1, \beta_2, \beta_3) = \langle 0.865, 0.1, 0.0968\rangle$. Therefore, based on the score functions, we get SVNWG$(\beta_1, \beta_2, \beta_3) \leq$ H-SVNWAG$(\beta_1, \beta_2, \beta_3) \leq$ SVNWA$(\beta_1, \beta_2, \beta_3)$ and it validates the theorem 2.

3.2 Hybrid single-valued neutrosophic ordered weighted arithmetic and geometric aggregation operator

Definition 6 A H-SVNOWAG operator is a mapping H-SVNOWAG$(\beta_1, \beta_2, \ldots, \beta_n) : \Omega^n \to \Omega$ that has an associated positional weight vector $w = (w_1, w_2, \ldots, w_n)$, such that $w_j > 0$ and $\sum\limits_{j=1}^{n} w_j = 1$ and is defined as

$$\text{H-SVNOWAG}(\beta_1, \beta_2, \ldots, \beta_n) = \left(\bigoplus_{j=1}^{n} w_j \beta_{\xi(j)}\right)^{\lambda} \bigotimes \left(\bigotimes_{j=1}^{n} \beta_{\xi(j)}^{w_j}\right)^{1-\lambda} \tag{13}$$

where ξ is a permutation of $(1, 2, \ldots, n)$ such that $\xi(j-1) > \xi(j)$ for $j = 2, 3, \ldots, n$ and λ be any real number in $[0, 1]$.

Theorem 3 *For the collection of SVNNs $\beta_j = \langle \zeta_j, \kappa_j, \varphi_j\rangle$, $(j = 1, 2, \ldots, n)$, the aggregated value by using H-SVNOWAG is still a SVNN and is given by*

$$\begin{aligned}
&\textit{H-SVNOWAG}(\beta_1, \beta_2, \ldots, \beta_n) \\
&= \Bigg\langle \left(1 - \prod_{j=1}^{n}(1-\zeta_{\xi(j)})^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}\zeta_{\xi(j)}^{w_j}\right)^{1-\lambda}, 1 - \left(1 - \prod_{j=1}^{n}\kappa_{\xi(j)}^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda}, \\
&\quad 1 - \left(1 - \prod_{j=1}^{n}\varphi_{\xi(j)}^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda} \Bigg\rangle
\end{aligned} \tag{14}$$

Proof Based on the SVNOWA and SVNOWG operators and the operational laws, we have

$$
\begin{aligned}
\text{H-SVNOWAG}(\beta_1,\beta_2,\ldots,\beta_n) &= \left(\bigoplus_{j=1}^{n} w_j\beta_{\xi(j)}\right)^{\lambda} \bigotimes \left(\bigotimes_{j=1}^{n} \beta_{\xi(j)}^{w_j}\right)^{1-\lambda} \\
&= \left(\left\langle 1-\prod_{j=1}^{n}(1-\zeta_{\xi(j)})^{w_j}, \prod_{j=1}^{n}(\kappa_{\xi(j)})^{w_j}, \prod_{j=1}^{n}(\varphi_{\xi(j)})^{w_j}\right\rangle\right)^{\lambda} \bigotimes \left(\left\langle \prod_{j=1}^{n}(\zeta_{\xi(j)})^{w_j}, 1-\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j},\right.\right. \\
&\quad \left.\left. 1-\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right\rangle\right)^{1-\lambda} \\
&= \left\langle \left(1-\prod_{j=1}^{n}(1-\zeta_{\xi(j)})^{w_j}\right)^{\lambda}, 1-\left(1-\prod_{j=1}^{n}(\kappa_{\xi(j)})^{w_j}\right)^{\lambda}, 1-\left(1-\prod_{j=1}^{n}(\varphi_{\xi(j)})^{w_j}\right)^{\lambda} \right\rangle \\
&\quad \bigotimes \left\langle \left(\prod_{j=1}^{n}(\zeta_{\xi(j)})^{w_j}\right)^{1-\lambda}, 1-\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda}, 1-\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda} \right\rangle \\
&= \left\langle \left(1-\prod_{j=1}^{n}(1-\zeta_{\xi(j)})^{w_j}\right)^{\lambda}, 1-\left(1-\prod_{j=1}^{n}(\kappa_{\xi(j)})^{w_j}\right)^{\lambda}, 1-\left(1-\prod_{j=1}^{n}(\varphi_{\xi(j)})^{w_j}\right)^{\lambda} \right\rangle \\
&\quad \bigotimes \left\langle \left(\prod_{j=1}^{n}(\zeta_{\xi(j)})^{w_j}\right)^{1-\lambda}, 1-\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda}, 1-\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda} \right\rangle
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&= \left\langle \left(1-\prod_{j=1}^{n}(1-\zeta_{\xi(j)})^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(\zeta_{\xi(j)})^{w_j}\right)^{1-\lambda}, 1-\left(1-\prod_{j=1}^{n}(\kappa_{\xi(j)})^{w_j}\right)^{\lambda} + 1-\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda}\right. \\
&\quad -\left(1-\left(1-\prod_{j=1}^{n}(\kappa_{\xi(j)})^{w_j}\right)^{\lambda}\right)\left(1-\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda}\right), 1-\left(1-\prod_{j=1}^{n}(\varphi_{\xi(j)})^{w_j}\right)^{\lambda} \\
&\quad \left. +1-\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda} - \left(1-\left(1-\prod_{j=1}^{n}(\varphi_{\xi(j)})^{w_j}\right)^{\lambda}\right)\left(1-\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda}\right)\right\rangle \\
&= \left\langle \left(1-\prod_{j=1}^{n}(1-\zeta_{\xi(j)})^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(\zeta_{\xi(j)})^{w_j}\right)^{1-\lambda}, 1-\left(1-\prod_{j=1}^{n}(\kappa_{\xi(j)})^{w_j}\right)^{\lambda} + 1-\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda}\right. \\
&\quad -\left(1-\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda} - \left(1-\prod_{j=1}^{n}(\kappa_{\xi(j)})^{w_j}\right)^{\lambda} + \left(1-\prod_{j=1}^{n}(\kappa_{\xi(j)})^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda}\right), \\
&\quad 1-\left(1-\prod_{j=1}^{n}(\varphi_{\xi(j)})^{w_j}\right)^{\lambda} + 1-\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda} - \left(1-\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda}\right. \\
&\quad \left.\left. -\left(1-\prod_{j=1}^{n}(\varphi_{\xi(j)})^{w_j}\right)^{\lambda} + \left(1-\prod_{j=1}^{n}(\varphi_{\xi(j)})^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda}\right)\right\rangle \\
&= \left\langle\left(1-\prod_{j=1}^{n}(1-\zeta_{\xi(j)})^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}\zeta_{\xi(j)}{}^{w_j}\right)^{1-\lambda}, 1-\left(1-\prod_{j=1}^{n}\kappa_{\xi(j)}{}^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda}\right. \\
&\quad \left. 1-\left(1-\prod_{j=1}^{n}\varphi_{\xi(j)}{}^{w_j}\right)^{\lambda}\left(\prod_{j=1}^{n}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda}\right\rangle
\end{aligned}
$$

Hence, the result.

Example 3 Consider $\beta_1 = \langle 0.4, 0.2, 0.6\rangle$, $\beta_2 = \langle 0.3, 0.1, 0.4\rangle$ and $\beta_3 = \langle 0.7, 0.2, 0.1\rangle$ be three SVNNs. Assume that importance of each SVNN is given as $w = (0.6, 0.3, 0.1)^T$ and the $\lambda = 0.5$. Based on the score function of SVNNs, we get $S(\beta_1) = -0.4$, $S(\beta_2) = -0.2$ and $S(\beta_3) = 0.4$. Thus, $S(\beta_3) > S(\beta_2) > S(\beta_3)$

and hence $\beta_{\xi(1)} = \beta_3, \beta_{\xi(2)} = \beta_2$ and $\beta_{\xi(3)} = \beta_1$. Therefore, by using the Eq. (14), we get

$$\begin{aligned}
&\text{H-SVNOWAG}(\beta_1, \beta_2, \beta_3)\\
&= \Bigg\langle \left(1 - \prod_{j=1}^{3}(1-\zeta_{\xi(j)})^{w_j}\right)^{\lambda} \left(\prod_{j=1}^{3} \zeta_{\xi(j)}^{w_j}\right)^{1-\lambda}, 1 - \left(1 - \prod_{j=1}^{3} \kappa_{\xi(j)}^{w_j}\right)^{\lambda} \left(\prod_{j=1}^{3}(1-\kappa_{\xi(j)})^{w_j}\right)^{1-\lambda},\\
&\quad 1 - \left(1 - \prod_{j=1}^{3} \varphi_{\xi(j)}^{w_j}\right)^{\lambda} \left(\prod_{j=1}^{3}(1-\varphi_{\xi(j)})^{w_j}\right)^{1-\lambda} \Bigg\rangle\\
&= \Big\langle \left(1-(1-0.7)^{0.6} \times (1-0.3)^{0.3} \times (1-0.4)^{0.1}\right)^{0.5} \times \left((0.7)^{0.6} \times (0.3)^{0.3} \times (0.4)^{0.5}\right)^{0.5},\\
&\quad 1 - \left(1-(0.2)^{0.6} \times (0.1)^{0.3} \times (0.2)^{0.1}\right)^{0.5} \left((1-0.2)^{0.6} \times (1-0.1)^{0.3} \times (1-0.2)^{0.1}\right),\\
&\quad 1 - \left(1-(0.1)^{0.6} \times (0.4)^{0.3} \times (0.6)^{0.1}\right)^{0.5} \left((1-0.1)^{0.6} \times (1-0.4)^{0.3} \times (1-0.6)^{0.1}\right) \Big\rangle\\
&= \langle 0.5482, 0.1669, 0.2244 \rangle
\end{aligned}$$

According to the properties of the SVNOWA and SVNOWG operators , it is clear that the H-SVNOWAG operator also satisfies the properties of idempotency, boundedness and monotonicity and commutativity.

(P1) (Idempotency) If $\beta_j = \beta$, for all j, then

$$\text{H-SVNOWAG}(\beta_1, \beta_2, \ldots, \beta_n) = \beta$$

(P2) (Boundedness) For a SVNN β_j, we have

$$\min\{\beta_j\} \leq \text{H-SVNOWAG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \max\{\beta_j\}$$

(P3) (Monotonicity) If $\beta_j \leq \gamma_j$ for all j, then

$$\text{H-SVNOWAG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \text{H-SVNOWAG}(\gamma_1, \gamma_2, \ldots, \gamma_n)$$

(P4) (Commutativity)If $(\beta'_1, \beta'_2, \ldots, \beta'_n)$ be any permutation of $(\beta_1, \beta_2, \ldots, \beta_n)$, then

$$\text{H-SVNOWAG}(\beta_1, \beta_2, \ldots, \beta_n) = \text{H-SVNOWAG}({\beta'}_1, {\beta'}_2, \ldots, {\beta'}_n)$$

Theorem 4 *The SVNOWA, SVNOWG and proposed operator H-SVNOWAG satisfies the following inequality*

$$\textit{SVNOWG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \textit{H-SVNOWAG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \textit{SVNOWA}(\beta_1, \beta_2, \ldots, \beta_n) \tag{16}$$

Proof Proof is similar to Theorem 2 so we omit here.

The suitability of aggregated values of the proposed hybrid aggregation operators is discussed in following example.

Example 4 Consider the SVNNs $\beta_1 = \langle 0.0001, 0, 0 \rangle$ and $\beta_2 = \langle 1, 0, 0 \rangle$ as taken in section 2.2. If we consider the weight vector $\omega = w = (0.9, 0.1)^T$, then by using the H-SVNWAG and H-SVNOWAG operators given in Eqs. (6) and (14), we get H-SVNWAG$(\beta_1, \beta_2) = \langle 0.0447, 0, 0 \rangle$ and H-SVNOWAG$(\beta_1, \beta_2) = \langle 0.7079, 0, 0 \rangle$. Clearly, H-SVNWAG lie between SVNWA$(\beta_1, \beta_2) = \langle 1, 0, 0 \rangle$ and SVNWG$(\beta_1, \beta_2) = \langle 0.002, 0, 0 \rangle$ and H-SVNOWAG which has value lie between SVNOWA$(\beta_1, \beta_2) = \langle 1, 0, 0 \rangle$ and SVNOWG$(\beta_1, \beta_2) = \langle 0.5012, 0, 0 \rangle$.

On the other hand, if we utilize the weight vector $\omega = w = (0.1, 0.9)^T$, then from Eqs. (6) and (14) we have, H-SVNWAG$(\beta_1, \beta_2) = \langle 0.7079, 0, 0 \rangle$ which is between SVNWA$(\beta_1, \beta_2) = \langle 1, 0, 0 \rangle$ and SVNWG$(\beta_1, \beta_2) = \langle 0.5012, 0, 0 \rangle$; H-SVNOWAG$(\beta_1, \beta_2) = \langle 0.04447, 0, 0 \rangle$ which is lie between SVNOWA$(\beta_1, \beta_2) = \langle 1, 0, 0 \rangle$ and SVNOWG$(\beta_1, \beta_2) = \langle 0.002, 0, 0 \rangle$.

From the above, we conclude that the proposed hybrid operators are unbiased as they are not tending towards maximum argument shown by SVNWA and SVOWA operators and also not tending towards the maximum weight value as shown by SVNWG and SVNOWG operators. Hence, the proposed operators gives the aggregated values which are much more informative.

4 Extensions to the Interval neutrosophic information

Wang et al. [13] have introduced the interval neutrosophic set (INS), which is generalization of the SVNS in which the degree of the membership functions corresponding to the set are represented in the form of the intervals rather than exact numbers.

Definition 7 An interval neutrosophic set β is defined over X is given as [13]

$$\beta = \{\langle x, \tilde{\zeta}_\beta(x), \tilde{\kappa}_\beta(x), \tilde{\varphi}_\beta(x)\rangle \mid x \in X\}$$

where $\tilde{\zeta}_\beta(x), \tilde{\kappa}_\beta(x), \tilde{\varphi}_\beta(x) \subset [0,1]$ are interval numbers such that $0 \leq \sup(\tilde{\zeta}_\beta(x)) + \sup(\tilde{\kappa}_\beta(x)) + \sup(\tilde{\varphi}_\beta(x)) \leq 1$ for all $x \in X$. For convenience, let $\tilde{\zeta}_\beta(x) = [a,b]$, $\tilde{\kappa}_\beta(x) = [c,d]$ and $\tilde{\varphi}_\beta(x) = [e,f]$ then this pair is often denoted by $\beta = \langle [a,b],[c,d],[e,f]\rangle$ and called an interval-valued neutrosophic number (INN).

For any INN $\beta = \langle [a,b],[c,d],[e,f]\rangle$, a score function S is defined as $S(\beta) = (a+b-c-d-e-f)/2$, $S(\beta) \in [-1,1]$ and an accuracy degree H is defined as $H(\beta) = (a+b+c+d+e+f)/2$, $H(\beta) \in [0,1]$. An order relation between these two INNs is defined as : if $S(\beta) < S(\gamma)$ then β is smaller than γ and if $S(\beta) = S(\gamma)$ then, if $H(\beta) < H(\gamma)$ then β is smaller than γ and if $H(\beta) = H(\gamma)$ then β and γ represent the same information denoted by $\beta = \gamma$.

Furthermore, in order to aggregate the different INNs $\beta_j = \langle [a_j,b_j],[c_j,d_j],[e_j,f_j]\rangle$, $(j = 1,2,\ldots,n)$, the following aggregation operators have been defined as follows

Definition 8 Let $\beta_j = \langle [a_j,b_j],[c_j,d_j],[e_j,f_j]\rangle$, $(j = 1,2,\ldots,n)$ be n INNs then

(i) an interval neutrosophic weighted average (INWA) operator is defined as [28]

$$\begin{aligned}\text{INWA}(\beta_1,\beta_2,\ldots,\beta_n) = \Big\langle &\Big[1-\prod_{j=1}^{n}(1-a_j)^{\omega_j}, 1-\prod_{j=1}^{n}(1-b_j)^{\omega_j}\Big], \Big[\prod_{j=1}^{n}(c_j)^{\omega_j}, \prod_{j=1}^{n}(d_j)^{\omega_j}\Big] \\ &\Big[\prod_{j=1}^{n}(e_j)^{\omega_j}, \prod_{j=1}^{n}(f_j)^{\omega_j}\Big]\Big\rangle\end{aligned}$$

(ii) an interval neutrosophic weighted geometric (INWG) operator is defined as [28]

$$\begin{aligned}\text{INWG}(\beta_1,\beta_2,\ldots,\beta_n) = \Big\langle &\Big[\prod_{j=1}^{n}(a_j)^{\omega_j}, \prod_{j=1}^{n}(b_j)^{\omega_j}\Big], \Big[1-\prod_{j=1}^{n}(1-c_j)^{\omega_j}, 1-\prod_{j=1}^{n}(1-d_j)^{\omega_j}\Big], \\ &\Big[1-\prod_{j=1}^{n}(1-e_j)^{\omega_j}, 1-\prod_{j=1}^{n}(1-f_j)^{\omega_j}\Big]\Big\rangle\end{aligned}$$

where ω_j be the weight vector of it such that $\omega_j > 0$ and $\sum_{j=1}^{n}\omega_j = 1$.

In the following, we have proposed some new interval neutrosophic weighted aggregation operators namely, hybrid interval neutrosophic weighted and geometric average (H-INWAG) operator and hybrid interval neutrosophic ordered weighted and geometric average (H-INOWAG) operator for a collection of the INNs, denoted by Γ, whose weight vector is $\omega = (\omega_1,\omega_2,\ldots,\omega_n)^T$, $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$.

Definition 9 For any real number $\lambda \in [0,1]$ and for the collection of INNs $\beta_j = \langle [a_j,b_j],[c_j,d_j],[e_j,f_j]\rangle$, $(j = 1,2,\ldots,n)$, a H-INWAG operator is a mapping H-INWAG : $\Gamma^n \rightarrow \Gamma$ and is defined as

$$
\begin{aligned}
&\text{H-INWAG}(\beta_1, \beta_2, \ldots, \beta_n) \\
&= \Big(\bigoplus_{j=1}^{n} \omega_j \beta_j\Big)^{\lambda} \bigotimes \Big(\bigotimes_{j=1}^{n} \beta_j^{\omega_j}\Big)^{1-\lambda} \\
&= \Bigg\langle \Bigg[\Big(1 - \prod_{j=1}^{n}(1-a_j)^{\omega_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n} a_j{}^{\omega_j}\Big)^{1-\lambda}, \Big(1 - \prod_{j=1}^{n}(1-b_j)^{\omega_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n} b_j{}^{\omega_j}\Big)^{1-\lambda}\Bigg] \\
&\quad \Bigg[1 - \Big(1 - \prod_{j=1}^{n} c_j{}^{\omega_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-c_j)^{\omega_j}\Big)^{1-\lambda}, 1 - \Big(1 - \prod_{j=1}^{n} d_j{}^{\omega_j}\Big)^{\lambda} \\
&\quad \Big(\prod_{j=1}^{n}(1-d_j)^{\omega_j}\Big)^{1-\lambda}\Bigg], \Bigg[1 - \Big(1 - \prod_{j=1}^{n} e_j{}^{\omega_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-e_j)^{\omega_j}\Big)^{1-\lambda}, \\
&\quad 1 - \Big(1 - \prod_{j=1}^{n} f_j{}^{\omega_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-f_j)^{\omega_j}\Big)^{1-\lambda}\Bigg]\Bigg\rangle
\end{aligned}
\tag{17}
$$

where ω_j is the weight of β_j; $(j = 1, 2, \ldots, n)$.

Theorem 5 *The INOWA, INOWG and proposed operator H-INOWAG satisfies the following inequality*

$$\textit{INWG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \textit{H-INWAG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \textit{INWA}(\beta_1, \beta_2, \ldots, \beta_n) \tag{18}$$

Proof Proof is similar to Theorem 2, so we omit here.

Definition 10 Let $\beta_j = \langle [a_j, b_j], [c_j, d_j], [e_j, f_j] \rangle$, $j = 1, 2, \ldots, n$ be a collection of INNs. An H-INOWAG operator of dimension n is a mapping H-INOWA : $\Gamma^n \to \Gamma$, that has an associated positional weight vector $w = (w_1, w_2, \ldots, w_n)^T$, such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$, Furthermore,

$$
\begin{aligned}
&\text{H-INOWAG}(\beta_1, \beta_2, \ldots, \beta_n) \\
&= \Big(\bigoplus_{j=1}^{n} w_j \beta_{\xi(j)}\Big)^{\lambda} \bigotimes \Big(\bigotimes_{j=1}^{n} \beta_{\xi(j)}^{w_j}\Big)^{1-\lambda} \\
&= \Bigg\langle \Bigg[\Big(1 - \prod_{j=1}^{n}(1-a_{\xi(j)})^{w_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n} a_{\xi(j)}{}^{w_j}\Big)^{1-\lambda}, \Big(1 - \prod_{j=1}^{n}(1-b_{\xi(j)})^{w_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n} b_{\xi(j)}{}^{w_j}\Big)^{1-\lambda}\Bigg], \\
&\quad \Bigg[1 - \Big(1 - \prod_{j=1}^{n} c_{\xi(j)}{}^{w_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-c_{\xi(j)})^{w_j}\Big)^{1-\lambda}, 1 - \Big(1 - \prod_{j=1}^{n} d_{\xi(j)}{}^{w_j}\Big)^{\lambda} \\
&\quad \Big(\prod_{j=1}^{n}(1-d_{\xi(j)})^{w_j}\Big)^{1-\lambda}\Bigg], \Bigg[1 - \Big(1 - \prod_{j=1}^{n} e_{\xi(j)}{}^{w_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-e_{\xi(j)})^{w_j}\Big)^{1-\lambda}, \\
&\quad 1 - \Big(1 - \prod_{j=1}^{n} f_{\xi(j)}{}^{w_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-f_{\xi(j)})^{w_j}\Big)^{1-\lambda}\Bigg]\Bigg\rangle
\end{aligned}
\tag{19}
$$

where $(\xi(1), \xi(2), \ldots, \xi(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $\beta_{\xi(j-1)} \geq \beta_{\xi(j)}$ for $j = 2, 3, \ldots, n$.

Theorem 6 *The INOWA, INOWG and proposed operator H-INOWAG satisfies the following inequality*

$$\textit{INOWG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \textit{H-INOWAG}(\beta_1, \beta_2, \ldots, \beta_n) \leq \textit{INOWA}(\beta_1, \beta_2, \ldots, \beta_n) \tag{20}$$

Proof Proof is similar to Theorem 2 so we omit here.

5 Proposed operators based decision-making approach

In this section, we have presented a decision-making method for solving MCDM problem by using proposed aggregation operators. A practical example from a field of decision-making has been taken for an illustrative and finally the proposed result has been compared with the existing approaches results under neutrosophic environment for demonstrating the validity of the proposed approach.

5.1 Proposed approach

Assume that there is a set of m alternatives $A = \{A_1, A_2, \ldots, A_m\}$ which are evaluated with respect to the set of n criteria $C = \{C_1, C_2, \ldots, C_n\}$. Consider that decision maker gives his/her preference values corresponding to each alternative A_i;$(i = 1, 2, \ldots, m)$ with respect to criteria C_j;$(j = 1, 2, \ldots, n)$ in terms of neutrosophic numbers denoted by β_{ij} . The collected information has been summarized in terms of neutrosophic decision matrix D and is represented as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} \beta_{11} & \beta_{12} & \ldots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \ldots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1} & \beta_{m2} & \ldots & \beta_{mn} \end{bmatrix} \end{array}$$

Approach I:

Assume that the decision maker/expert gave their preferences towards the alternative in the form of SVNNs $\beta_{ij} = \langle \zeta_{ij}, \kappa_{ij}, \varphi_{ij} \rangle$, $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$ where ζ_{ij}, φ_{ij} represent the membership and non membership degrees of alternative A_i corresponding to the attribute C_j respectively and κ_{ij} represent the degree of indeterminacy. Then, in the following, we develop an approach, based on the proposed operators to find the best alternative(s) which involves the following steps.

Step 1. Obtain the collective information in terms of the decision matrix $D = (\beta_{ij})_{m\times n}$.

Step 2. Utilize appropriately the H-SVNWAG operator:

$$\begin{aligned} r_i &= \text{H-SVNWAG}(\beta_{i1}, \beta_{i2}, \ldots, \beta_{in}) \\ &= \Bigg\langle \Big(1 - \prod_{j=1}^{n}(1-\zeta_{ij})^{\omega_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}\zeta_{ij}{}^{\omega_j}\Big)^{1-\lambda}, 1 - \Big(1 - \prod_{j=1}^{n}\kappa_{ij}{}^{\omega_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-\kappa_{ij})^{\omega_j}\Big)^{1-\lambda}, \\ &\qquad 1 - \Big(1 - \prod_{j=1}^{n}\varphi_{ij}{}^{\omega_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-\varphi_{ij})^{\omega_j}\Big)^{1-\lambda} \Bigg\rangle \end{aligned}$$

or the H-SVNOWAG operator

$$\begin{aligned} r_i &= \text{H-SVNOWAG}(\beta_{i1}, \beta_{i2}, \ldots, \beta_{in}) \\ &= \Bigg\langle \Big(1 - \prod_{j=1}^{n}(1-\zeta_{i\xi(j)})^{w_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}\zeta_{i\xi(j)}^{w_j}\Big)^{1-\lambda}, 1 - \Big(1 - \prod_{j=1}^{n}\kappa_{i\xi(j)}^{w_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-\kappa_{i\xi(j)})^{w_j}\Big)^{1-\lambda}, \\ &\qquad 1 - \Big(1 - \prod_{j=1}^{n}\varphi_{i\xi(j)}^{w_j}\Big)^{\lambda}\Big(\prod_{j=1}^{n}(1-\varphi_{i\xi(j)})^{w_j}\Big)^{1-\lambda} \Bigg\rangle \end{aligned}$$

to aggregate all the individual SVNNs $\beta_{ij}(j = 1, 2, \ldots, n)$ into the collective SVNN $r_i(i = 1, 2, \ldots, m)$.

Step 3. Compute the score value of the aggregated SVNNs $r_i, (i = 1, 2, \ldots, m)$ and arrange their values in the descending order.

Step 4. Rank the alternative according to the score value of final aggregated numbers and hence choose the best one(s).

Approach II:

If the decision-makers' are given their preferences towards each alternative $A_i(i = 1, 2, \ldots, m)$ in the form of INNs $\beta_{ij} = \langle [a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}] \rangle$, $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$ rather than the crisp numbers $\langle \zeta_{ij}, \kappa_{ij}, \varphi_{ij} \rangle$ for $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$ then in the following, we develop an approach based on interval operators which involves the following steps.

Step 1. Obtain the collective information in terms of the interval-valued decision matrix $D = (\beta_{ij})_{m\times n}$.

Step 2. Utilize appropriately H-INWAG operator

$$
\begin{aligned}
r_i &= \text{H-INWAG}(\beta_{i1}, \beta_{i2}, \ldots, \beta_{in}) \\
&= \Bigg\langle \Bigg[\Big(1 - \prod_{j=1}^{n} (1 - a_{ij})^{\omega_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} a_{ij}{}^{\omega_j}\Big)^{1-\lambda}, \Big(1 - \prod_{j=1}^{n} (1 - b_{ij})^{\omega_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} b_{ij}{}^{\omega_j}\Big)^{1-\lambda} \Bigg] \\
&\qquad \Bigg[1 - \Big(1 - \prod_{j=1}^{n} c_{ij}{}^{\omega_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} (1 - c_{ij})^{\omega_j}\Big)^{1-\lambda}, 1 - \Big(1 - \prod_{j=1}^{n} d_{ij}{}^{\omega_j}\Big)^{\lambda} \\
&\qquad \Big(\prod_{j=1}^{n} (1 - d_{ij})^{\omega_j}\Big)^{1-\lambda}\Bigg], \Bigg[1 - \Big(1 - \prod_{j=1}^{n} e_{ij}{}^{\omega_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} (1 - e_{ij})^{\omega_j}\Big)^{1-\lambda}, \\
&\qquad 1 - \Big(1 - \prod_{j=1}^{n} f_{ij}{}^{\omega_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} (1 - f_{ij})^{\omega_j}\Big)^{1-\lambda}\Bigg] \Bigg\rangle
\end{aligned}
$$

or the H-INOWAG operator

$$
\begin{aligned}
r_i &= \text{H-INOWAG}(\beta_{i1}, \beta_{i2}, \ldots, \beta_{in}) \\
&= \Bigg\langle \Bigg[\Big(1 - \prod_{j=1}^{n} (1 - a_{i\xi(j)})^{w_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} a_{i\xi(j)}{}^{w_j}\Big)^{1-\lambda}, \Big(1 - \prod_{j=1}^{n} (1 - b_{i\xi(j)})^{w_j}\Big)^{\lambda} \\
&\qquad \Big(\prod_{j=1}^{n} b_{i\xi(j)}{}^{w_j}\Big)^{1-\lambda}\Bigg], \Bigg[1 - \Big(1 - \prod_{j=1}^{n} c_{i\xi(j)}{}^{w_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} (1 - c_{i\xi(j)})^{w_j}\Big)^{1-\lambda}, \\
&\qquad 1 - \Big(1 - \prod_{j=1}^{n} d_{i\xi(j)}{}^{w_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} (1 - d_{i\xi(j)})^{w_j}\Big)^{1-\lambda}\Bigg], \Bigg[1 - \Big(1 - \prod_{j=1}^{n} e_{i\xi(j)}{}^{w_j}\Big)^{\lambda} \\
&\qquad \Big(\prod_{j=1}^{n} (1 - e_{i\xi(j)})^{w_j}\Big)^{1-\lambda}, 1 - \Big(1 - \prod_{j=1}^{n} f_{i\xi(j)}{}^{w_j}\Big)^{\lambda} \Big(\prod_{j=1}^{n} (1 - f_{i\xi(j)})^{w_j}\Big)^{1-\lambda}\Bigg] \Bigg\rangle
\end{aligned}
$$

to aggregate the preference values into the collected INN $r_i (i = 1, 2, \ldots, m)$.

Step 3. Compute the score value of the aggregated INNs $r_i, (i = 1, 2, \ldots, m)$ and arrange their values in the descending order.

Step 4. Rank the alternative according to the score value of final aggregated numbers and hence choose the best one(s).

5.2 Illustrative example

The above proposed operators have been illustrated with a numerical example from the field of decision-making.

Demonetization is the withdrawal of a particular form of currency from circulation. On 8 November 2016, Government of India announced in a broadcast to the nation that Rs. 500 and Rs. 1,000 currency notes would no longer be recognized legally as currency. This step was taken to crack down the use of illicit and counterfeit cash to fund illegal activity and terrorism. As Government knew that demonetization will effect the Indian economy and may also drop country's Gross domestic product (GDP) growth. So before the announcement of the demonetization, Government wanted to conduct the survey of effect of this bold move on various sectors of Indian economy to take the further decisions. For doing this, Indian government hired a economist or decision maker who is able to handle this kind of situation and able to crack that which sector (alternative) of Indian economy will be effected by the demonetization. For this, decision maker assume the five important sectors on which our Indian economy depends and were given as: A_1(Agriculture Sector), A_2(Real-Estate Sector), A_3(Information Technology Sector), A_4(Educational Sector), A_5(Industrial Sector). For evaluation, decision maker considered criterion in the terms 'how much effect of demonetization on particular sector in linguistic terms' which are summarized as: C_1(Very low effect), C_2(Low effect), C_3(Regular effect), C_4(High effect) and C_5(Very high effect). The importance of each criteria $C_j; (i = 1, 2, 3, 4, 5)$ is taken in the form of weight vector as $\omega = (0.2, 0.25, 0.15, 0.3, 0.1)^T$ in the decision-making problem and in order to make the decision more pessimistic for future goals then manipulate the aggregation by using the OWA weighted vector $w =$

$(0.1, 0.2, 0.2, 0.2, 0.3)^T$. The procedure to get the most desirable alternative(s) by using the proposed operator is discussed as follows:

Step 1. The evaluation of these strategies are taken using the neutrosophic numbers by the decision makers under the above five general characteristics and hence construct the decision matrix as given in Table 1.

Insert Table 1 here.

Step 2. By utilizing the proposed arithmetic and geometric aggregation operator to aggregate these decision information, their corresponding results are summarized in Table 2 which include the SVNWA, SVNWG, SVNOWA, SVNOWG, SVNFWA, SVNFWG, SVNHWA, SVNHWG, H-SVNWAG and H-SVNOWAG operators.

Insert Table 2 here.

Step 3. The score values of aggregated SVNNs, shown in Table 2, are summarized in Table 3.

Step 4. According to score function of the aggregated values, the ordering of the alternatives is shown in Table 4 in which $\succ$ means "preferred to". From these results, it has been seen that the best alternative is A_2 by all the operators while the different aggregation operators have different ranking strategies which is slightly different. Thus, based on the decision makers preference in terms of their aggregation operators used, the results may leads to the different decisions.

On the other hand, if the given five strategies $A_i (i = 1, 2, \ldots, 5)$ are to be evaluated using the INNs by the decision makers under the above five characteristics, then the following steps have been executed to find the most desirable alternative(s).

Step 1: The rating values of each alternative is measured in the form of INNs and are summarized in Table 5.

Insert Table 5 here.

Step 2: By using the information provided by the decision maker related to these strategies and the weight vector, we compute, the aggregated values corresponding to it by using INWA, INWG, INOWA, INOWG, INFWA, INFWG, INHWA, INHWG, H-INWAG and H-INOWAG operators. The results corresponding to these are summarized in Table 6.

Step 3: Calculate the score of aggregated values by using Definition 7 and summarized in Table 7.

Step 4: Based on these score values, we summarized their preference orders in Table 8 and it has been concluded that the best region for investment is A_2.

5.3 Sensitivity Analysis

In order to access the impact of the parameter λ on the score values and ranking of the alternative, an investigation has been done by making a change in values of the parameter λ from 0 to 1. The overall score values of the alternative based on the different $\lambda' s$ are summarized in the Table 9. From this table, it has been seen that when $\lambda = 0$, then the H-SVNWAG/H-INWAG reduces to SVNWG/INWG and H-SVNOWAG/H-INOWAG reduces to SVNOWG/INOWG. Also, it has been highlighted that if $\lambda = 1$, then the H-SVNWAG/H-INWAG reduces to SVNWA/INWA and H-SVNOWAG/H-INOWAG reduces to SVNOWA/INOWA. Further, the overall score values of different alternates are increasing as the increase of λ. Also, the ranking order of H-SVNWAG/H-INWAG and H-SVNOWAG/H-INOWAG for different values of λ are not identical but the final decision given by them is identical. The decision maker can select the desired value λ according to his preference or practical demand. Thus, the proposed operators are more flexible than the existing operators.

Insert Table 9 here

6 Conclusion

The main objective of this paper is to present new aggregation operators which are capable to fuse the behavior of the existing averaging and geometric operators. From these existing operators, it has been observed that the aggregated values given by them are either tends towards the maximum arguments values or towards the maximum weight values in some cases and hence they don't give the unbiased aggregated values. For handling their limitations, the new hybrid neutrosophic weighted average and geometric aggregation operators have been developed which can give the moderate values in the aggregation process. We have also studied some properties of these operators such as idempotency, boundedness, and monotonicity. Moreover, the parameter λ makes the proposed operator more flexible and general than existing operators such as SVNWA/INWA, SVNOWA/INOWA, SVNWG/INWG and SVNOWG/IVNOWA. Further, from the results and their corresponding comparative studies, it has been observed that the decision-making approaches defined in this paper are more stable for solving the MCDM problems. In future, we will extend the proposed work to other complex fields.

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Table 1 Neutrosophic decision making matrix

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.5, 0.3, 0.4\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.2, 0.2, 0.6\rangle$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.3, 0.3, 0.4\rangle$
A_2	$\langle 0.7, 0.1, 0.3\rangle$	$\langle 0.7, 0.2, 0.3\rangle$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.6, 0.4, 0.2\rangle$	$\langle 0.7, 0.1, 0.2\rangle$
A_3	$\langle 0.5, 0.3, 0.4\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.6, 0.4, 0.3\rangle$
A_4	$\langle 0.7, 0.3, 0.2\rangle$	$\langle 0.7, 0.2, 0.2\rangle$	$\langle 0.4, 0.5, 0.2\rangle$	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.4, 0.5, 0.4\rangle$
A_5	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.4, 0.1, 0.5\rangle$	$\langle 0.4, 0.3, 0.6\rangle$	$\langle 0.3, 0.2, 0.4\rangle$

Table 2 Neutrosophic aggregated results by using aggregated operators($\lambda = 0.5$)

	SVNWA [16]	SVNWG [16]	SVNHWA [17]		SVNOWA [16]
			$\gamma = 2$	$\gamma = 3$	
A_1	$\langle 0.3862, 0.2259, 0.3956\rangle$	$\langle 0.3553, 0.2314, 0.4132\rangle$	$\langle 0.3812, 0.2264, 0.3980\rangle$	$\langle 0.3782, 0.2266, 0.3992\rangle$	$\langle 0.3413, 0.2352, 0.4389\rangle$
A_2	$\langle 0.6585, 0.2125, 0.2400\rangle$	$\langle 0.6531, 0.2548, 0.2467\rangle$	$\langle 0.6578, 0.2165, 0.2407\rangle$	$\langle 0.6575, 0.2181, 0.2410\rangle$	$\langle 0.6536, 0.2169, 0.2352\rangle$
A_3	$\langle 0.5528, 0.1702, 0.3213\rangle$	$\langle 0.5477, 0.2020, 0.3337\rangle$	$\langle 0.5520, 0.1726, 0.3230\rangle$	$\langle 0.5516, 0.1735, 0.3237\rangle$	$\langle 0.5528, 0.2107, 0.3326\rangle$
A_4	$\langle 0.5842, 0.2727, 0.2144\rangle$	$\langle 0.5502, 0.3074, 0.2227\rangle$	$\langle 0.5790, 0.2765, 0.2151\rangle$	$\langle 0.5766, 0.2781, 0.2154\rangle$	$\langle 0.5301, 0.3429, 0.2462\rangle$
A_5	$\langle 0.4178, 0.1490, 0.3708\rangle$	$\langle 0.4110, 0.1751, 0.4271\rangle$	$\langle 0.4167, 0.1508, 0.3788\rangle$	$\langle 0.4160, 0.1515, 0.3825\rangle$	$\langle 0.4193, 0.1261, 0.3455\rangle$
	SVNOWG [16]	SVNFWA[20]	SVNFWG[20]	H-SVNWAG	H-SVNOWAG
A_1	$\langle 0.3096, 0.2416, 0.4605\rangle$	$\langle 0.3840, 0.2262, 0.3968\rangle$	$\langle 0.3576, 0.2311, 0.4117\rangle$	$\langle 0.3704, 0.2286, 0.4045\rangle$	$\langle 0.3251, 0.2384, 0.4498\rangle$
A_2	$\langle 0.6481, 0.2598, 0.2416\rangle$	$\langle 0.6581, 0.2149, 0.2404\rangle$	$\langle 0.6535, 0.2525, 0.2463\rangle$	$\langle 0.6558, 0.2340, 0.2434\rangle$	$\langle 0.6508, 0.2386, 0.2384\rangle$
A_3	$\langle 0.5477, 0.2483, 0.3432\rangle$	$\langle 0.5524, 0.1716, 0.3222\rangle$	$\langle 0.5481, 0.2002, 0.3329\rangle$	$\langle 0.5502, 0.1862, 0.3275\rangle$	$\langle 0.5502, 0.2297, 0.3380\rangle$
A_4	$\langle 0.4947, 0.3842, 0.2661\rangle$	$\langle 0.5815, 0.2748, 0.2148\rangle$	$\langle 0.5528, 0.3047, 0.2220\rangle$	$\langle 0.5669, 0.2903, 0.2185\rangle$	$\langle 0.5121, 0.3639, 0.2563\rangle$
A_5	$\langle 0.3862, 0.1848, 0.4563\rangle$	$\langle 0.4173, 0.1501, 0.3751\rangle$	$\langle 0.4115, 0.1738, 0.4230\rangle$	$\langle 0.4144, 0.1622, 0.3996\rangle$	$\langle 0.3893, 0.1723, 0.4365\rangle$

Table 3 Score values of aggregated SVNNs

	SVNWA [16]	SVNWG [16]	SVNOWA[16]	SVNOWG [16]	SVNFWA [20]
A_1	-0.2353	-0.2894	-0.3328	-0.3924	-0.2390
A_2	0.2060	0.1516	0.2015	0.1467	0.2028
A_3	0.0613	0.0120	0.0094	-0.0438	0.0886
A_4	0.0971	0.0201	-0.0591	-0.1556	0.0919
A_5	-0.1020	-0.1913	-0.1834	-0.2549	-0.1079
	SVNFWG [20]	SVNHWA[17]		Proposed Operators	
		$\gamma = 2$	$\gamma = 3$	H-SVNWAG	H-SVNOWAG
A_1	-0.2852	-0.2432	-0.2476	-0.2627	-0.3631
A_2	0.1547	0.2006	0.1984	0.1785	0.1738
A_3	0.0150	0.0564	0.0544	0.0365	-0.0174
A_4	0.0261	0.0874	0.0831	0.0581	-0.1081
A_5	-0.1853	-0.1129	-0.1180	-0.1474	-0.2196

Table 4 Ordering of the alternatives

Existing operators	Ordering	Proposed operators	Ordering
SVNWA [16]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$	H-SVNWAG	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$
SVNOWA [16]	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	H-SVNOWAG	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$
SVNWG [16]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$		
SVNOWG [16]	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$		
SVNHWA [17]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$		
SVNFWA [20]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$		
SVNFWG [20]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$		

Table 5 Information about each alternative under five characteristics in form of INNs

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle[0.4,0.6],[0.2,0.3],[0.3,0.5]\rangle$	$\langle[0.45,0.55],[0.1,0.2],[0.3,0.4]\rangle$	$\langle[0.2,0.3],[0.1,0.2],[0.6,0.7]\rangle$	$\langle[0.2,0.4],[0.2,0.3],[0.3,0.4]\rangle$	$\langle[0.2,0.4],[0.3,0.4],[0.4,0.6]\rangle$
A_2	$\langle[0.7,0.8],[0.1,0.2],[0.3,0.5]\rangle$	$\langle[0.65,0.80],[0.2,0.3],[0.3,0.5]\rangle$	$\langle[0.5,0.7],[0.2,0.3],[0.2,0.3]\rangle$	$\langle[0.6,0.7],[0.4,0.5],[0.2,0.3]\rangle$	$\langle[0.6,0.7],[0.1,0.3],[0.2,0.3]\rangle$
A_3	$\langle[0.5,0.6],[0.3,0.4],[0.4,0.5]\rangle$	$\langle[0.60,0.65],[0.2,0.3],[0.3,0.4]\rangle$	$\langle[0.4,0.6],[0.1,0.2],[0.2,0.3]\rangle$	$\langle[0.4,0.5],[0.1,0.3],[0.3,0.4]\rangle$	$\langle[0.6,0.7],[0.4,0.5],[0.3,0.4]\rangle$
A_4	$\langle[0.6,0.7],[0.3,0.5],[0.2,0.4]\rangle$	$\langle[0.70,0.80],[0.1,0.2],[0.1,0.2]\rangle$	$\langle[0.4,0.5],[0.5,0.6],[0.1,0.2]\rangle$	$\langle[0.3,0.5],[0.1,0.2],[0.2,0.3]\rangle$	$\langle[0.3,0.4],[0.5,0.6],[0.4,0.5]\rangle$
A_5	$\langle[0.4,0.5],[0.1,0.2],[0.3,0.4]\rangle$	$\langle[0.50,0.60],[0.1,0.2],[0.2,0.3]\rangle$	$\langle[0.3,0.4],[0.1,0.2],[0.5,0.6]\rangle$	$\langle[0.3,0.4],[0.2,0.3],[0.5,0.6]\rangle$	$\langle[0.2,0.3],[0.2,0.4],[0.3,0.4]\rangle$

Table 6 Neutrosophic aggregated results by using aggregated operators($\lambda = 0.50$)

	INWA [28]	INWG [28]
A_1	$\langle[0.3123,0.4731],[0.1578,0.2625],[0.3426,0.4737]\rangle$	$\langle[0.2814,0.4499],[0.1725,0.2729],[0.3662,0.4993]\rangle$
A_2	$\langle[0.6223,0.7500],[0.2000,0.3224],[0.2400,0.3775]\rangle$	$\langle[0.6143,0.7434],[0.2398,0.3501],[0.2467,0.3984]\rangle$
A_3	$\langle[0.4980,0.5981],[0.1702,0.3147],[0.2990,0.4006]\rangle$	$\langle[0.4820,0.5885],[0.2020,0.3304],[0.3075,0.4080]\rangle$
A_4	$\langle[0.5052,0.6344],[0.1863,0.3162],[0.1625,0.2844]\rangle$	$\langle[0.4447,0.5882],[0.2611,0.3876],[0.1852,0.3077]\rangle$
A_5	$\langle[0.3676,0.4691],[0.1320,0.2421],[0.3590,0.4468]\rangle$	$\langle[0.3467,0.4497],[0.1414,0.2532],[0.3985,0.4804]\rangle$
	INOWA [28]	INOWG[28]
A_1	$\langle[0.2725,0.4456],[0.1835,0.2896],[0.3757,0.5283]\rangle$	$\langle[0.2491,0.4228],[0.2038,0.3043],[0.4024,0.5541]\rangle$
A_2	$\langle[0.6043,0.7344],[0.2000,0.3358],[0.2259,0.3497]\rangle$	$\langle[0.5970,0.7286],[0.2398,0.3587],[0.2314,0.3672]\rangle$
A_3	$\langle[0.5170,0.6155],[0.2107,0.3478],[0.3140,0.4156]\rangle$	$\langle[0.5030,0.6063],[0.2483,0.3667],[0.3226,0.4231]\rangle$
A_4	$\langle[0.4424,0.5649],[0.2786,0.4161],[0.2000,0.3280]\rangle$	$\langle[0.3973,0.5242],[0.3621,0.4851],[0.2398,0.3613]\rangle$
A_5	$\langle[0.3169,0.4182],[0.1414,0.2670],[0.4119,0.4571]\rangle$	$\langle[0.2961,0.3995],[0.1515,0.2855],[0.4395,0.4819]\rangle$
	INHWA[17]	
	$\gamma = 2$	$\gamma = 3$
A_1	$\langle[0.3072,0.4693],[0.1589,0.2637],[0.3454,0.4774]\rangle$	$\langle[0.3040,0.4672],[0.1593,0.2641],[0.3468,0.4794]\rangle$
A_2	$\langle[0.6212,0.7493],[0.2035,0.3259],[0.2407,0.3805]\rangle$	$\langle[0.6207,0.7490],[0.2049,0.3275],[0.2410,0.3819]\rangle$
A_3	$\langle[0.4953,0.5967],[0.1726,0.3167],[0.3001,0.4017]\rangle$	$\langle[0.4940,0.5960],[0.1735,0.3175],[0.3005,0.4022]\rangle$
A_4	$\langle[0.4951,0.6277],[0.1923,0.3249],[0.1641,0.2870]\rangle$	$\langle[0.4899,0.6247],[0.1948,0.3290],[0.1647,0.2882]\rangle$
A_5	$\langle[0.3641,0.4658],[0.1325,0.2432],[0.3647,0.4521]\rangle$	$\langle[0.3621,0.4641],[0.1327,0.2437],[0.3672,0.4547]\rangle$
	INFWA [20]	INFWG [20]
A_1	$\langle[0.3101,0.4713],[0.1585,0.2632],[0.3440,0.4755]\rangle$	$\langle[0.2834,0.4518],[0.1719,0.2722],[0.3639,0.4970]\rangle$
A_2	$\langle[0.6217,0.7496],[0.2021,0.3243],[0.2404,0.3791]\rangle$	$\langle[0.6149,0.7437],[0.2374,0.3480],[0.2463,0.3967]\rangle$
A_3	$\langle[0.4967,0.5973],[0.1716,0.3158],[0.2996,0.4012]\rangle$	$\langle[0.4833,0.5893],[0.2002,0.3293],[0.3070,0.4074]\rangle$
A_4	$\langle[0.5003,0.6308],[0.1897,0.3209],[0.1634,0.2859]\rangle$	$\langle[0.4492,0.5914],[0.2562,0.3822],[0.1839,0.3060]\rangle$
A_5	$\langle[0.3661,0.4676],[0.1323,0.2427],[0.3621,0.4494]\rangle$	$\langle[0.3482,0.4513],[0.1410,0.2525],[0.3959,0.4778]\rangle$
	H-INWAG	H-INOWAG
A_1	$\langle[0.2964,0.4613],[0.1652,0.2677],[0.3545,0.4867]\rangle$	$\langle[0.2606,0.4340],[0.1937,0.2970],[0.3892,0.5414]\rangle$
A_2	$\langle[0.6183,0.7467],[0.2201,0.3076],[0.2434,0.3880]\rangle$	$\langle[0.6006,0.7315],[0.2201,0.3184],[0.2286,0.3585]\rangle$
A_3	$\langle[0.4900,0.5933],[0.1862,0.3226],[0.3033,0.4043]\rangle$	$\langle[0.5099,0.6109],[0.2297,0.3573],[0.3184,0.4194]\rangle$
A_4	$\langle[0.4740,0.6109],[0.2246,0.3529],[0.1739,0.2961]\rangle$	$\langle[0.4193,0.5442],[0.3216,0.4517],[0.2201,0.3449]\rangle$
A_5	$\langle[0.3570,0.4593],[0.1367,0.2477],[0.3598,0.4639]\rangle$	$\langle[0.3170,0.4191],[0.1465,0.2670],[0.3863,0.4894]\rangle$

Table 7 Score values of aggregated INNs

	INWA [28]	INWG [28]	INOWA[28]	INOWG [28]	INFWA [20]
A_1	-0.2898	-0.2582	-0.3328	-0.3963	-0.2299
A_2	0.0797	0.1029	0.2015	0.0823	0.1127
A_3	-0.0887	-0.0666	0.0094	-0.1258	-0.0942
A_4	-0.0544	0.0187	-0.0591	-0.2633	0.0856
A_5	-0.2283	-0.1958	-0.1834	-0.3046	-0.1764
	INFWG [20]	INHWA[17]		Proposed operators	
		$\gamma=2$	$\gamma=3$	H-INWAG	H-INOWAG
A_1	-0.2849	-0.3295	-0.3633	-0.2344	-0.2392
A_2	0.0009	0.1245	0.1032	0.1099	0.1072
A_3	-0.08565	-0.0778	-0.1020	-0.04955	-0.0518
A_4	-0.04385	-0.1076	-0.1874	0.0772	0.0689
A_5	-0.23385	-0.2479	-0.2765	-0.1807	-0.1860

Table 8 Ordering of the alternatives

Existing operators	Ordering	Proposed operators	Ordering
INWA [28]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$	H-INWAG	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$
INOWA [28]	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	H-INOWAG	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$
INWG [28]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$		
INOWG [28]	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$		
INHWA [17]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$		
INFWA [20]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$		
INFWG [20]	$A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$		

Table 9 Effect of parameter λ on score values and ranking

	Score Values ($\lambda=0$)				Score Values ($\lambda=0.2$)				Score Values ($\lambda=0.5$)			
	H-SVNWAG	H-SVNOWAG	H-INWAG	H-INOWAG	H-SVNWAG	H-SVNOWAG	H-INWAG	H-INOWAG	H-SVNWAG	H-SVNOWAG	H-INWAG	H-INOWAG
A_1	-0.2894	-0.394	-0.2898	-0.3963	-0.2788	-0.3808	-0.2773	-0.3832	-0.2627	-0.3631	-0.2582	-0.3633
A_2	0.1516	0.1467	0.0797	0.0823	0.1623	0.1574	0.0889	0.0906	0.1785	0.1738	0.1029	0.1032
A_3	0.0120	-0.0438	-0.0887	-0.1258	0.0218	-0.0333	-0.0799	-0.1163	0.0350	-0.0174	-0.0666	-0.1020
A_4	0.0201	-0.01556	-0.0544	-0.2633	0.0352	-0.1368	-0.0255	-0.2334	0.0581	-0.1081	0.0187	-0.1874
A_5	-0.1913	-0.2549	-0.2283	-0.3046	-0.1739	-0.2409	-0.2154	-0.2934	-0.1474	-0.2196	-0.1958	-0.2765
Ranking	(24351)	(24351)	(24351)	(23451)	(24351)	(24351)	(24351)	(23451)	(24351)	(24351)	(24351)	(23451)
	Score Values ($\lambda=0.7$)				Score Values ($\lambda=0.9$)				Score Values ($\lambda=1$)			
A_1	-0.2518	-0.3511	-0.2453	-0.3499	-0.2408	-0.3389	-0.2322	-0.3363	-0.2353	-0.3328	-0.2257	-0.3295
A_2	0.1894	0.1848	0.1124	0.1117	0.2004	0.1959	0.1218	0.1202	0.2060	0.2015	0.1266	0.1245
A_3	0.0464	-0.0068	-0.0577	-0.0924	0.0563	0.0040	-0.0487	-0.0827	0.0613	0.0094	-0.0442	-0.0778
A_4	0.0736	-0.0886	0.0488	-0.1560	0.0892	-0.0690	0.0796	-0.1239	0.0971	-0.0591	0.0952	-0.1076
A_5	-0.1295	-0.2053	-0.1826	-0.2651	-0.1112	-0.1907	-0.1693	-0.2537	-0.1020	-0.1834	-0.1626	-0.2479
Ranking	(24351)	(24351)	(24351)	(23451)	(23451)	(23451)	(23451)	(23451)	(24351)	(24351)	(24351)	(23451)