Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 9, Number 3 (2017), pp. 311-321

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Neutrosophic Q-fuzzy left N-subgroups of a Near-ring

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Abstract

In this paper, the notion of neutrosophic Q- fuzzy left N-subgroups is introduced in a near ring and investigated some related properties. Characterization of neutrosophic Q- fuzzy left N-subgroups with respect to T-norm and S-norm are given. Few homomorphic image and its pre-image on neutrosophic Q- fuzzy are obtained.

Section 1: Introduction

Nagarajan and Manermaran [2013] gave few algebraic properties on M-fuzzy factor group, and obtained some its power fuzzy group, homomorphic image and preimage, union and intersection. Makamba [1992] discussed direct product and isomorphism of fuzzy subgroups. Ajmal [1994] studied on homomorphism on factor fuzzy group. They gave an idea to introduce M-fuzzy group, & its normality in a ring. Das [1981] found some new results on fuzzy group and level subgroups. It is also motivated to find N-fuzzy group in hemi-ring. Fang [1994] studied on fuzzy homomorphism & fuzzy isomorphism, and it leads to N-fuzzy subgroup in near-ring. Kim [1997], Kim & Kim [1994], Mukherjee and Bhattacharya [1984, 1986], and analyzed on some characterizations of fuzzy subgroups, and it give an idea to verify also few characterizations on N-fuzzy group in a near-ring. Kumar et. al. [1992] obtained some new structures on fuzzy normal subgroup and fuzzy quotient group which initiate to make an attempt on N-fuzzy normal subgroup in a near-ring. Kumbhoikar and Bapal

[1991] found correspondence theorem for fuzzy ideals in near-ring. Liu et.al. [2001] and Morsi & Yehia [1994] got few properties on quotient fuzzy group and quotient fuzzy ring induced from fuzzy ideals in near-ring, and same argument will argue quotient N-fuzzy subgroup in near-ring. Further

Section 2: Preliminaries and Definitions

Definition 2.1: A near ring is a non – empty set R with two binary operations + and . satisfying the following axioms: (1). (R, +) is a group; (2). (R, .) is a semigroup; (3). x. (y + z) = x. y + x. z for all x, y, z, \ in R. Then It is called a left near – ring by (3). In this paper, it will use the word near- ring. Here xy denotes x.y; (2). x.0 = 0, and x(-y) = -(xy for x, y in R.

Definition 2.2: A two sided R – subgroup of a near – ring R is a subset H of R such that (1). (H, +) is a subgroup of (R, +); (2). RH $\subset$ H; (3). HR $\subset$ H. If H satisfies (1) and (2), then it is a left R-subgroup of R, while if H satisfies (1) and (3), then it is a right R-subgroup of R.

Definition 2.3: A fuzzy set μ in a set R is a function μ: R $\rightarrow$ [0, 1]. If Q is set, then a Q-fuzzy set λ is a functionλ: R $\times$ Q $\rightarrow$ [0, 1].

Definition 2.4: Let G be any group. A mapping μ: G $\rightarrow$ [0, 1] is a fuzzy group if (FG1). $\mu(xy) \geq \min\{\mu(x),\ \mu(y)\}$ and (FG2). $\mu(x^{-1}) = \mu(x)$, for all $x, y \in G$.

Definition 2.5: Let A be any non-empty set, and X be the universe of discourse. **A neutrosophic Q-fuzzy set A** on X $\times$ Q characterized by a truth membership function $T_A(x, q)$, an indeterminacy function $I_A(x, q)$ and a falsity membership function $F_A(x, q)$ is defined as A = { < (x, q), $T_A(x, q)$, $I_A(x, q)$, $F_A(x, q)$ > : x $\in$ X, q $\in$ Q}, where T_A , I_A ,F_A : X $\times$ Q $\rightarrow$ [0. 1] and $0 \leq T_A(x, q) \leq 1$; $0 \leq I_A(x, q) \leq 1$; $0 \leq F_A(x, q) \leq 1$, for all x $\in$ X, and q $\in$ Q.

Definition 2.6: Let X, Y be two non-empty sets and $f: X \longrightarrow Y$ be a function. (i). If B ={ < (y, q), $T_B(y, q)$, $I_B(y, q)$, $F_B(y, q)$ > : y $\in$ Y, q $\in$ Q} is a neutrosophic Q-fuzzy set in Y, then the **preimage of B under *f***, denoted by $f^{-1}(B)$, is the neutrosophic fuzzy set in X defined by $f^{-1}(B)$ = {<(x, q), $f^{-1}(T_B(x, q))$, $f^{-1}(I_B(x, q))$, $f^{-1}(F_B(x, q))$ > : x $\in$ X, q $\in$ Q} where $f^{-1}(T_B(x, q) = T_B(f(x), q)$; $f^{-1}(I_B(x, q) = I_B(f(x), q)$; $f^{-1}(F_B(x, q) = F_B(f(x), q)$.

(ii). If A = { < (x, q), $T_A(x, q)$, $I_A(x, q)$, $F_A(x, q)$ > : x $\in$ X, q $\in$ Q} is a Neutrosophic fuzzy set in X, then the **image f(A) of A** under f is the neutrosophic fuzzy set in Y defined by f(A) = { A = { < (y, q), $T_{f(A)}(y, q)$, $I_{f(A)}(y, q)$, $F_{f(A)}(y, q)$ > : y $\in$ Y, q $\in$ Q} where

$T_{f(A)}(y, q) = sup_{(x,q) \in f^{-1}(y,q)}(T_A(x, q))\ \ if\ \ f^{-1}(y, q) \neq \{\}$; 0, otherwise.

$I_{f(A)}(y, q) = sup_{(x,q)\in f^{-1}(y,q)}(I_A(x,q))$ if $f^{-1}(y,q) \neq \{\}$; 0, otherwise.

$F_{f(A)}(y, q) = inf_{(x,q)\in f^{-1}(y,q)}F(x,q))$ if $f^{-1}(y,q) \neq \{\}$,where $F_{f(A)}(y, q) = (1\text{-}f(1\text{-}F_A))(y, q)$; 1, otherwise.

Definition 2.7: A neutrosophic left R-fuzzy subgroup of a near ring R is a neutrosophic fuzzy set A = (T_A, I_A, F_A) on X in definition (2.3) such that

(1). (a) $T_A(x\text{-}y) \geq \min \{ T_A(x), T_A(y)\}$; (b). $I_A(x\text{-}y) \geq \min \{ I_A(x), I_A(y)\}$;

(c). $F_A(x\text{-}y) \leq \max \{ F_A(x), F_A(y)\}$;

(2). (a) $T_A(xy) \geq \min \{ T_A(x), T_A(y)\}$; (b). $I_A(xy) \geq \min \{ I_A(x), I_A(y)\}$;

(c). $F_A(xy) \leq \max \{ F_A(x), F_A(y)\}$ for all x, y ∈ R.

Definition 2.8: A = < x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > and B = < x, $T_B(x$, $I_B(x)$, $F_B(x)$ > are two neutrosophic fuzzy sets on X. Then,

(ii). $A \subseteq B$, if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$, for all x ∈ X.

(ii).$A \cup B = < x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min((F_A(x), F_B(x)) >$.

(iii). $A \cap B = < x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max((F_A(x), F_B(x)) >$.

Definition 2.9: The complement A^c of a neutrosophic fuzzy subset A = (T_A, I_A, F_A) on X is defined by $A^c = \{< x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x) > : x \in X \}$ where $T_{A^c}(x) = F_A(x)$, $I_{A^c}(x) = 1 - I_A(x), F_{A^c}(x) = T_A(x)$, for all x ∈ X.

Definition 2.10: Let f be a mapping from a near ring R_1 to a near ring R_2 and A = (T_A, I_A, F_A) be a neutrosophic fuzzy set in R_1. Then f(A) the image of μ is neutrosophic fuzzy set ($T_{f(A)}$, $I_{f(A)}$, $F_{f(A)}$) in R_2 defined by

$T_{f(A)}(y) = \sup (T_A(x): x \in f^{-1}(y) \neq \{\}$; 0, otherwise. for all $y \in R_2$;
$F_{f(A)}(y) = \sup (F_A(x): x \in f^{-1}(y) \neq \{\}$; 0, otherwise. for all $y \in R_2$;
$T_{f(A)}(y) = \inf (I_A(x): x \in f^{-1}(y) \neq \{\}$; 0, otherwise. for all $y \in R_2$;

Definition 2.11: Let A = (T_A, I_A, F_A) be a neutrosophic fuzzy set in a near ring R. For each t ∈ [0, 1], upper cut-set of A is defined as the set $\{x \in R: T_A(x) \geq t;\ F_A(x) \geq t;\ I_A(x) \leq t\}$.

Definition 2.12: (T-norm): A triangular norm is a function T: [0, 1] × [0., 1] → [0, 1] that satisfies the following conditions **(T1):** T(x, 1) = x; **(T2).** T(x, y) = T(y, x);

(T3): $T(x, T(y, z)) = T(T(x, y), z)$; **(T4).** $T(x, y) \leq T(x, z)$ if $y \leq z$ for all x, y, z in [0, 1].

Definition 2.13: Let Q, N be non-empty sets, and . be a map from $N \times R \rightarrow R$. Let A = (T_A, I_A, F_A) be a neutrosophic fuzzy set in a near ring R. Then A is a **neutrosophic Q-fuzzy left N-subgroup** of R if (1a) $T_A(n(x-y),q) \geq T\{T_A(x, q), T_A(y, q)\}$; (1b). $T_A(nx, q) \geq T_A(x, q)$

(2a) $I_A(n(x-y),q) \geq T\{I_A(x, q), I_A(y, q)\}$; (2b). $I_A(nx, q) \geq I_A(x, q)$

(3a) $F_A(n(x-y),q) \leq S\{I_A(x, q), I_A(y, q)\}$; (3b). $I_A(nx, q) \leq T_A(x, q)$

for all x, y in R; q in Q, and n in N.

Section 3: Homomorphic properties of neutrosophic Q-fuzzy left N-subgroups

The following theorem is first started:

Theorem 3.1: If A = (T_A, I_A, F_A) and B = (T_B, I_B, F_B) are neutrosophic Q-fuzzy left N-subgroups of a near ring R′, then their intersection $(A \cap B)$ s a neutrosophic Q-fuzzy left N-subgroup of R.

Proof: Let $x, y \in R$. Let $n \in N$, and $q \in Q$. It gets that

(1a) $T_{(A\cap B)}(n(x-y), q) = \min\{ T_A(n(x-y), q), T_A((n(x-y)), q)$
$\geq \min\{ T\{ T_A(x, q), T_A(y, q)\}, T\{T_B((x, q), T_B(y, q)\}\}$
$= \min\{ T\{ T_A(x, q), T_B(y, q)\}, T\{T_A(x, q), T_B(y, q)\}\}$
$\geq T\{ \min\{ T_A(x, q), T_B(y, q)\}, \min\{T_A(x, q), T_B(y, q)\}\}$
$= T\{T_{(A\cap B)}.(x, q), T_{(A\cap B)}.(x, q)\}$

(2a) $I_{(A\cap B)}(n(x-y), q) = \min\{ I_A(n(x-y), q), I_A((n(x-y)), q)$
$\geq \min\{ T\{ I_A(x, q), I_A(y, q)\}, T\{I_B((x, q), I_B(y, q)\}\}$
$= \min\{ T\{ I_A(x, q), I_B(y, q)\}, T\{I_A(x, q), I_B(y, q)\}\}$
$\geq T\{ \min\{ I_A(x, q), I_B(y, q)\}, \min\{I_A(x, q), I_B(y, q)\}\}$
$= T\{I_{(A\cap B)}.(x, q), I_{(A\cap B)}.(x, q)\}$

(3a) $F_{(A\cap B)}(n(x-y), q) = \min\{ F_A(n(x-y), q), F_A((n(x-y)), q)$
$\leq \min\{ S\{ F_A(x, q), F_A(y, q)\}, S\{F_B((x, q), F_B(y, q)\}\}$
$= \min\{ S\{ F_A(x, q), F_B(y, q)\}, S\{F_A(x, q), F_B(y, q)\}\}$
$\leq S\{ \min\{ F_A(x, q), F_B(y, q)\}, \min\{F_A(x, q), F_B(y, q)\}\}$
$= S\{F_{(A\cap B)}.(x, q), F_{(A\cap B)}.(x, q)\}$

(1b).$T_{(A\cap B)}(nx, q) = \min\{ T_A(nx, q), T_B(nx, q)\} \geq \min\{ T_A(x, q), T_B(x, q)\} = T_{(A\cap B)}(x, q)$.

(2b).$I_{(A\cap B)}(nx, q) = \min\{ I_A(nx, q), I_B(nx, q)\} \geq \min\{ I_A(x, q), I_B(x, q)\} = I_{(A\cap B)}(x, q)$.

(3b).$F_{(A\cap B)}(nx, q) = \min\{ F_A(nx, q), F_B(nx, q)\} \leq \min\{ F_A(x, q), F_B(x, q)\} = F_{(A\cap B)}(x, q)$.

Thus $(A \cap B)$ is a neutrosophic Q-fuzzy left N-subgroup of R.

Theorem 3.2: Every imaginable neutrosophic Q-fuzzy left N-subgroup μ = = (T_A, I_A, F_A) on a near ring (R, +, .) is a neutrosophic fuzzy left N-subgroup of R.

Proof: Assume μ is an imaginable Q-fuzzy left N-subgroup of a near ring (S, +,.). Let $x, y \in R$, $q \in Q$, and $n \in N$. Then it observes that

Since 'μ' is imaginable, it finds that

$$\min \{ T_A(x, q), T_A(y, q)\} = T (\min \{ T_A(x, q), T_A(y, q)\}, \min \{ T_A(x, q), T_A(y, q)\} \}$$
$$\geq T \{ T_A(x, q), T_A(y, q)\}$$
$$\geq \min \{ T_A(x, q), T_A(y, q)\} \quad \text{(by property of T-norm).}$$

Thus $T \{ T_A(x, q), T_A(y, q)\} = \min \{ T_A(x, q), T_A(y, q)\}$

It follows that (1a) $T_A(n(x-y),q) \geq T \{T_A(x, q), T_A(y, q)\} = \min \{T_A(x, q), T_A(y, q)\}$.

In addition, (1b). $T_A(nx, q) \geq T_A(x, q)$.

Similarly, it implies that

$$\min \{ I_A(x, q), I_A(y, q)\} = T (\min \{ I_A(x, q), I_A(y, q)\}, \min \{ I_A(x, q), I_A(y, q)\} \}$$
$$\geq T \{ I_A(x, q), I_A(y, q)\}$$
$$\geq \min \{ I_A(x, q), I_A(y, q)\} \quad \text{(by property of T-norm).}$$

Thus $T \{ I_A(x, q), I_A(y, q)\} = \min \{ I_A(x, q), I_A(y, q)\}$

(2a) $I_A(n(x-y),q) \geq T \{I_A(x, q), I_A(y, q)\} = \min \{I_A(x, q), I_A(y, q)\}$.

Also (2b). $I_A(nx, q) \geq I_A(x, q)$.

In addition,

$$\min \{ F_A(x, q), F_A(y, q)\} = S (\max \{ F_A(x, q), F_A(y, q)\}, \max \{ F_A(x, q), F_A(y, q)\} \}$$
$$\leq S \{ F_A(x, q), F_A(y, q)\}$$
$$\leq \max \{ F_A(x, q), F_A(y, q)\} \quad \text{(by property of T-norm).}$$

Thus $S \{ I_A(x, q), I_A(y, q)\} = \max \{ I_A(x, q), I_A(y, q)\}$

(3a) $I_A(n(x-y), q) \leq S \{I_A(x, q), I_A(y, q)\} = \max \{T_A(x, q), T_A(y, q)\}$.

(3b). $I_A(nx, q) \geq I_A(x, q)$.

Hence μ is a neutrosophic Q-fuzzy left N-subgroup of R.

Theorem 3.3: If A = (T_A, I_A, F_A) is a neutrosophic Q-fuzzy left N-subgroup of a near ring R′ and f is an endomorphism (near ring) from $R \to R$, then $A_f = (T_{A_f}, I_{A_f}, F_{A_f})$ is a neutrosophic Q-fuzzy left N-subgroup of R.

Proof: Let $x, y \in R$. Let $n \in N$, and $q \in Q$. It gets that

(1a). $T_{A_f}(n(x\text{-}y), q) = T_A (f (n(x - y)), q) = T_A (f (x, q), f(y, q) \geq T \{ T_A (f (x, q), T_A f(y, q)\}$
$\geq T \{ T_{A_f}(x, q), T_{A_f}(y, q) \}$.

(2a). $I_{A_f}(n(x\text{-}y), q) = I_A (f (n(x - y)), q) = I_A (f (x, q), f(y, q) \geq T \{ I_A (f (x, q), I_A f(y, q)\}$
$\geq T \{ I_{A_f}(x, q), I_{A_f}(y, q) \}$.

(3a). $F_{A_f}(n(x\text{-}y), q) = F_A (f (n(x - y)), q) = F_A (f (x, q), f(y, q) \leq S \{ F_A (f (x, q), F_A f(y, q)\}$
$\geq S \{ F_{A_f}(x, q), F_{A_f}(y, q) \}$.

(1b). T_{A_f} (nx), q) = T_A (f (nx, q) = T_A (f (x, q) $\geq$ T_{A_f} (x, q).

(2b). I_{A_f} (nx, q) = I_A (f (nx, q) = I_A (f (x, q) $\geq$ I_{A_f} (x, q).

(3b). F_{A_f} (nx, q) = F_A (f (nx, q) = F_A (f (x, q) $\geq$ F_{A_f} (x, q).

Thus A_f = (T_{A_f}, I_{A_f}, F_{A_f}) is a neutrosophic Q-fuzzy left N-subgroup of R.

Theorem 3.4: Let φ: R → R′ be an onto homomorphism on near rungs. The pre-image of a neutrosophic Q-fuzzy left N-subgroup of near ring R′ under f is a neutrosophic Q-fuzzy left N-subgroup of near ring R.

Proof: Let μ = (T_μ, I_μ, F_μ) be a neutrosophic Q-fuzzy left N-subgroup of a near ring R′. Let λ = (T_λ, I_λ, F_λ) be the pre-image of μ under f. Then it finds that

(1a).T_λ (n(x-y), q) = T_μ (φ (n(x – y)), q) = T_μ (φ (x, q), f(y, q) $\geq$ T { T_μ (φ (x, q), T_μ φ(y, q)}
$\geq$ T { T_λ (x, q), T_λ (y, q) }.

(2a). I_λ (n(x-y), q) = I_μ (φ (n(x – y)), q) = I_μ (φ (x, q), φ(y, q) $\geq$ T { I_μ (φ(x, q), I_μφ(y, q)}
$\geq$ T { I_λ(x, q), I_λ(y, q) }.

(3a). F_λ (n(x-y), q) = F_μ (φ (n(x – y)), q) = F_μ (φ (x, q), φ(y, q)) $\leq$ S { F_μ (φ (x, q), F_μf(y, q)}
$\geq$ S { F_λ (x, q), F_λ (y, q) }.

(1b). T_λ (nx), q) = T_μ (φ (nx, q) = T_μ (φ (x, q) $\geq$ T_λ(x, q).

(2b). I_λ (nx, q) = I_μ (φ (nx, q) = I_μ (φ (x, q) $\geq$ I_λ (x, q).

(3b). F_λ(nx, q) = F_μ (φ (nx, q) = F_μ (φ (x, q) $\geq$ F_λ (x, q).

Thus λ = (T_λ, I_λ, F_λ) is a neutrosophic Q-fuzzy left N-subgroup of R.

Theorem 3.5: Let f: R → R′ be an onto homomorphism on near rungs. The pre-image of a neutrosophic Q-fuzzy left N-subgroup of near ring R′ under f **with supreme property** is a neutrosophic Q-fuzzy left N-subgroup of near ring R.

Proof: Let μ = (T_μ, I_μ, F_μ) be a neutrosophic Q-fuzzy N-subgroup of a near ring R′ **with supreme property**. Let λ = (T_λ, I_λ, F_λ) be a neutrosophic Q-fuzzy N-subset in R′.

Let x^1, y^1 ∈ R′ and $x_0 \in f^{-1}(x^1)$, $y_0 \in f^{-1}(y^1)$ be such that

$$T_\mu(x_0, q) = \sup_{h \in f^{-1}(x^1)} T_\mu(h, q)$$

$T_\mu(y_0, q) = \sup_{h \in f^{-1}(y^1)} T_\mu(h, q)$ respectively

$$I_\mu(x_0, q) = \sup_{h \in f^{-1}(x^1)} I_\mu(h, q)$$

$I_\mu(y_0, q) = \sup_{h \in f^{-1}(y^1)} I_\mu(h, q)$ respectively

$$F_\mu(x_0, q) = \inf_{h \in f^{-1}(x^1)} F_\mu(h, q)$$

$F_\mu(y_0, q) = \inf_{h \in f^{-1}(y^1)} F_\mu(h, q)$ respectively.

Then it can deduce that

(1a). $T_\mu{}^f(n(x^1 - y^1), q) = \sup_{z \in f^{-1}(n(x^1 - y^1), q)} T_\mu(z, q)$

$\geq T\{T_\mu(x_0, q), T_\mu(y_0, q)\}$

$\geq T\left\{\sup_{h \in f^{-1}(x_0)} T_\mu(x_0, q), \sup_{(h,q) \in f^{-1}(y)} T_\mu(y_0, q)\right\}$

$= T\{T_\mu{}^f(x, q), T_\mu{}^f(y, q)\}$.

(2a). $I_\mu{}^f(n(x^1 - y^1), q) = \sup_{z \in f^{-1}(n(x^1 - y^1), q)} I_\mu(z, q)$

$\geq T\{I_\mu(x_0, q), I_\mu(y_0, q)\}$

$\geq T\left\{\sup_{h \in f^{-1}(x_0)} I_\mu(x_0, q), \sup_{(h,q) \in f^{-1}(y)} I_\mu(y_0, q)\right\}$

$= T\{I_\mu{}^f(x, q), I_\mu{}^f(y, q)\}$.

(3a). $F_\mu{}^f(n(x^1 - y^1), q) = \inf_{z \in f^{-1}(n(x^1 - y^1), q)} F_\mu(z, q)$

$\leq S\{F_\mu(x_0, q), F_\mu(y_0, q)\}$

$\geq S\left\{\inf_{h \in f^{-1}(x_0)} F_\mu(x_0, q), \inf_{(h,q) \in f^{-1}(y)} F_\mu(y_0, q)\right\}$

$= T\{F_\mu{}^f(x, q), F_\mu{}^f(y, q)\}$.

(1b). $T_\mu{}^f(nx, q) = \sup_{(z,q) \in f^{-1}(nx, q)} T_\mu(z, q)$

$\geq T_\mu(y_0, q)$

$= \sup_{(h,q) \in f^{-1}(y, q)} T_\mu(x_0, q)$

$= \mu^t(x, q)$.

(2b). $I_\mu{}^f(nx, q) = \sup_{(z,q) \in f^{-1}(nx, q)} I_\mu(z, q)$

$\geq I_\mu(y_0, q)$

$= \sup_{(h,q) \in f^{-1}(y, q)} I_\mu(x_0, q)$

$= I_\mu{}^f(x, q)$.

(3b). $F_\mu{}^f(nx, q) = \inf_{(z,q) \in f^{-1}(nx, q)} F_\mu(z, q)$

$\geq F_\mu(y_0, q)$

$= \inf_{(h,q) \in f^{-1}(y, q)} F_\mu(x_0, q)$

$= F_\mu{}^f(x, q)$.

Hence λ is a neutrosophic Q-fuzzy left N-subset of R.

Section 4: Other properties of neutrosophic Q-fuzzy left N-subgroups

Theorem 4.1: Let f: R → R′ be an onto homomorphism on a near rings. If λ = = (T_λ, I_λ, F_λ) be a neutrosophic Q-fuzzy N-subset in R. Then its image λ^f = (T_λ^f, I_λ^f, F_λ^f) is a neutrosophic Q-fuzzy left N-subgroup of R′.

Proof: Here f(R) = R′. Let y_1, y_2 ∈ R′, and q ∈ Q. Assume that $A_1 = f^{-1}(y_1, q)$; $A_2 = f^{-1}(y_2, q)$; and $A_{12} = f^{-1}(n(y_1- y_2), q))$.

Consider $A_1 – A_2$ = {x ∈ R: (x, q) = (a_1, q) – (a_2, q) for some (a_1, q) ∈ A_1, and (a_2, q) ∈ A_2.

If (x, q) ∈ $A_1 – A_2$, then (x, q) = (a_1, q) – (a_2, q) for some (a_1, q) ∈ A_1, and (a_2, q) ∈ A_2. implies that f(x, q) = f(x_1, q) – f(x_2, q) = $y_1 – y_2$.

Thus (x, q) ∈ f^{-1}((y_1, q) – (y_2, q)) = f^{-1}(n($y_1 - y_2$), q)) = A_{12}.

This gives that $A_1 – A_2 \subseteq A_{12}$.

It follows that

$$\begin{aligned} T_\lambda^f(n(y_1-y_2), q) &= \sup \{ T_\lambda(x, q) : (x, q) \in f^{-1}(n (y_1-y_2), q) \\ &= \sup \{ T_\lambda(x, q) : (x, q) \in A_{12}\} \\ &\geq \sup \{ T_\lambda(x, q) : (x, q) \in A_1 - A_2\} \\ &\geq \sup \{ T_\lambda(x_1, q) - T_\lambda(x_2, q) : (x_1, q) \in A_1, (x_2, q) \in A_2\} \end{aligned}$$

Since the norm T is continuous, and every $\in > 0$,

$$Sup\,\{T_\lambda(x_1,q)\colon\ (x_1\,,q)\ \in\ A_1\,\}\ -\ (x_1\,*,q)\ \in\ A_2\,\}\ \leq\ \delta \text{ and}$$

$$Sup\{\,T_\lambda(x_2,q)\colon (x_2\,,q)\ \in\ A_2\,\}-\ (x_2\,*,q)\ \in A_2\ \}\ \leq\ \delta$$

implies that

$T\ \{Sup\{\ T_\lambda(x_1\,,q)\ :\ (x_1\,,q)\ \in A_1\,\}, Sup\{T_\lambda(x_2,.\,q)\colon (x_2\,,q)\ \in\ A_2\,\}-\ T\ \{(x_1 * ,q), (x_2 *,q)\}\ \leq \in.$

Choose $(a_1,q) \in\ A_1$ and $(a_2,q) \in\ A_2$ such that $Sup\,\{T_\lambda(x_1,,q)\colon (x_1\,,q) \in A_1\,\} - T_\lambda(a_1,q)\ \leq\ \delta$

implies that

$T\{Sup\ \{\ T_\lambda(x_1,q)\colon (x_1\,,q) \in A_1\,\}, Sup\ \{T_\lambda(x_2,.\,q)\colon (x_2\,,q) \in\ A_2\} - T\{T_\lambda(a_1,q) - T_\lambda(a_2,q)\} \leq \in.$

Consequently it gives that

$$\begin{aligned} &T_\lambda{}^f(n(y_1 - y_2,q) \\ &\quad\geq \sup\{T(T_\lambda(x_1,q) \in A_1 and\ T_\lambda(x_2,q) \in A_2)\colon (x_1,q) \in A_1 and\ (x_2,q) \in A_2\} \\ &\quad\geq T\{sup\{T_\lambda(x_1,q)\colon (x_1,q) \in A_1\}.\sup\{T_\lambda(x_2,q)\colon (x_2,q) \in A_2)\}\} \\ &\quad\geq T\{T_\lambda{}^f(y_1,q), T_\lambda{}^f(y_2,q)\}. \end{aligned}$$

Similarly, it an show $T_\lambda{}^f(nx,q) \geq T_\lambda{}^f[x,q)$.

All these results are hold for $I_\lambda{}^f$ and $F_\lambda{}^f$ by using T-norm , and S-norm respectively.

It concludes that λ^f = (T_λ^f, I_λ^f, F_λ^f) is a neutrosophic Q-fuzzy left N-subgroup of R′.

Theorem 4.2: Let A = (T_A, I_A, F_A) be a neutrosophic Q-fuzzy left N-subgroup of a near ring R. Then < A > is a neutrosophic Q-fuzzy left N-subgroup of R generated by A. In addition, < A > is the smallest neutrosophic Q-fuzzy left N-subgroup containing A.

Proof: Let x, y ∈ R, n ∈ N, and q ∈ Q.

Assume that <A>(x,q) = t_1; <A> (y, q) = t_2, and T_A(n(x-y), q) = t.

Now t = < A > (n(x-y), q)

≤ T { < A > (nx, q), , < A >(ny, q)}

= T { t_1, t_2} = t_1 (say).

Then t_1 = <A>(x, q) = sup {k: (x, q) ∈ <A_k> } ≥ t.

Therefore there exists k_1 with (x, q) ∈ <A_k>.

Also t_2 = < A> (y, q) = sup { k: (y, q) ∈ < A_k >} ≥ t.

Therefore there exist k_2 > t with (y, q) ∈ < A_k >.

Without loss of generality, assume that k_1, k_2 so that < A_{k_1}> ⊆ < A_{k_1}>. Then x, y ∈ < A_k> which is a contradiction since k_2 > t. Thus t ≥ t_1.

Consequently,

A(n(x-y), q) ≥ T { < A > (x, q), < A > (y, q) } …. (1).

Now let, if possible t_3 = { < A> (nx, q)} ≤ {< A > (x, q) } = t_1.

Then t_1 = < A > (x, q) = sup {k: (x, q) ∈ < A_k > } > t_3.

Therefore there exists k with (x, q) ∈ < A_k > and t_1 > k > t_3 implies that (nx, q) ∈ < A_k > ⊆ <A_k > which is a contradiction. Thus t_3 = {<A>(nx, q) ≥ {<A>(x, q) } = t_2 …(2).

Consequently the equations (1) and (2) yield that <A> is a neutrosophic Q-fuzzy left N-subgroup of R.

Finally to show that <A> is the **smallest** neutrosophic Q-fuzzy left N-subgroup containing A.

Let us assume that B is a neutrosophic Q-fuzzy left N-subgroup of R such that A ⊂ B.

Claim that < A > ⊂ B.

Let it possible, t = < A > (x, q) ≥ B (x, q) for some x ∈ R, and q ∈ Q.

Let ε > 0 be given. Then t = A_t = sup {k: (x, q) ∈ < A_k > and t ε ≤ k ≤ t.

So that (x, q) ∈ <A_k> ⊂ < $A_{kt - ε}$ > for all ε > 0.

Now a = a_1x_1 + a_2x2 +…+ a_nx_n where a_i ∈ R, and x_i ∈ $A_{t-ε}$ implies that A(x_i, q) ≥ t - ε.

Then B(x, q) ≥ (t - ε) for all ε > 0. Thus B(x, q) ≥ T { B(x_1, q), B(x_2, q), …, B(x_n, q)} ≥ (t-ε) for all ε > 0. Therefore B(x, q) = t, which is a contradiction to our supposition

Theorem 4.3: Let A = (T_A, I_A, F_A) be a neutrosophic Q-fuzzy left N-subgroup of a near ring R. Define A^+ = (T_A^+, I_A^+, F_A^+) by T_A^+(x,q) = T_A(x,q) + 1 – T_A(0,q), I_A^+(x,q) = I_A(x,q) + 1-T_A(0,q), F_A^+(x, q) = F_A(x, q) + 1 – F_A(0, q), for all x in R, and q in Q,. Then A^+ is a neutrosophic normal Q-fuzzy left N-subgroup of R containing A.

Proof: Let $x, y \in R$, $n \in N$, and $q \in Q$.
It implies that

$$
\begin{aligned}
T_A^+ (n(x-y), q) &= T_A(n(x-y), q) + 1 - T_A(0, q) \\
&\geq T\{T_A(x, q), T_A(y, q)\} + 1 - T_A(0, q) \\
&\geq T\{T_A(x, q) + 1 - T_A(0, q), T_A(y, q)\} + 1 - T_A(0, q)\} \\
&\geq T\{T_A^+(x,q), T_A^+(y,q)\} \\
T_A^+ (nx, q) &= T_A(nx, q) + 1 - T_A(0, q) \\
&\geq T_A(x, q) + 1 - T_A(0, q) \\
&= T_A^+(x, q).
\end{aligned}
$$

Similarly $I_A^+ (n(x-y), q) = I_A(n(x-y), q) + 1 - I_A(0, q)$

$$
\begin{aligned}
&\geq T\{I_A(x, q), I_A(y, q)\} + 1 - I_A(0, q) \\
&\geq T\{I_A(x, q) + 1 - I_A(0, q), I_A(y, q)\} + 1 - I_A(0, q)\} \\
&\geq T\{I_A^+(x,q), I_A^+(y,q)\} \\
I_A^+ (nx, q) &= I_A(nx, q) + 1 - I_A(0, q) \\
&\geq I_A(x, q) + 1 - I_A(0, q) \\
&= I_A^+(x, q).
\end{aligned}
$$

Also $F_A^+ (n(x-y), q) = F_A(n(x-y), q) + 1 - F_A(0, q)$

$$
\begin{aligned}
&\leq S\{F_A(x, q), F_A(y, q)\} + 1 - T_A(0, q) \\
&\leq S\{F_A(x, q) + 1 - F_A(0, q), F_A(y, q)\} + 1 - F_A(0, q)\} \\
&\geq S\{F_A^+(x, q), F_A^+(y,q)\}
\end{aligned}
$$

$$
\begin{aligned}
F_A^+ (nx, q) &= F_A(nx, q) + 1 - F_A(0, q) \\
&\geq F_A(x, q) + 1 - F_A(0, q) \\
&= F_A^+(x, q).
\end{aligned}
$$

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