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Neutrosophic Refined Sets in Medical Diagnosis

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Abstract. In this paper, a new approach (cosecant similarity measure) is proposed to construct the decision method for medical diagnosis by using neutrosophic refined set. Also, we develop a technique to diagnose which patient is suffering from what disease.

Keywords: Neutrosophic set, Neutrosophic refined set, cosecant similarity measure, medical diagnosis.

AMS Mathematics Subject Classification (2010): 03E72, 92C50, 62P10

1. Introduction

Kumbakonam is a thickly populated town. Although underground drainage system is available here, it is yet to cover all the houses in the town. So, open drainage system continues to be in practice in different places of the town. Further this town is racing fast towards total sanitation in all spheres. As a result, Kumbakonam continues to be a repository of all new kinds of diseases. This created an urge to carry out research in the medical field. By introducing innovative methods in the research, the diseases can be diagnosed instantly and infallibly.

A number of real life problems in engineering, medical sciences, social sciences, economics etc., involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. Such uncertainties are being dealt with the help of topics like probability theory, fuzzy set theory [14], rough set theory [7] etc., Health care industry has been trying to complement the services offered by conventional clinical decision making systems with the integration of fuzzy logic techniques in them. As it is not an easy task for a clinician to derive a fool proof diagnosis it is advantageous to automate few initial steps of diagnosis which would not require intervention from an expert doctor. Neutrosophic set which is a generalized set possesses all attributes necessary to encode medical knowledge base and capture medical inputs.

As medical diagnosis demands large amount of information processing, large portion of which is quantifiable, also intuitive thought process involve rapid unconscious data processing and combines available information by law of average, the whole process offers low intra and inter person consistency. So contradictions, inconsistency, indeterminacy and fuzziness should be accepted as unavoidable as it is integrated in the behavior of biological systems as well as in their characterization. To model an expert doctor it is imperative that it should not disallow uncertainty as it would be then inapt to capture fuzzy or incomplete knowledge that might lead to the danger of fallacies due to misplaced precision.

As medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different truth membership, indeterminate and false membership functions. The unique feature of neutrosophic refined set is that it contains multi truth membership, indeterminate and false membership. By taking one time inspection, there may be error in diagnosis. Hence, multi time inspection, by taking the samples of the same patient at different times gives thebest diagnosis. So, neutrosophic refined sets and their applications play a vital role in medical diagnosis.

In 1965, fuzzy set theory was firstly given by Zadeh which is applied in many real applications to handle uncertainty. Then interval valued fuzzy set, intuitionistic fuzzy set theory and interval valued intuitionistic fuzzy set were introduced by Turksen, Atanassov and Atanassov and Gargov respectively. These theories can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief systems. So, Neutrosophic set (generalization of fuzzy sets, intuitionistic fuzzy sets and so on) defined by Smarandache [1] has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exists in real world from philosophical point of view. Ye [4] proposed the vector similarity measures of simplified neutrosophic sets.

Yager [13] introduced the notion of multisets which is the generalization of the concept of set theory. Sebastian and Ramakrishnan [10] studied a new concept called fuzzy multisets (FMS), which is the generalization of multisets. Since then, Sebastian and Ramakrishnan [11] established more properties on fuzzy multisets. Shinoj and John [12] extended the concept of fuzzy multi sets by introducing intuitionistic fuzzy multisets (IFMS). An element of a multi fuzzy sets can occur more than once with possibly the same or different membership values, whereas an element of a intuitionistic multi fuzzy sets is capable of having repeated occurrences of membership and non-membership values. However, the concepts of FMS and IFMS are not capable of dealing with indeterminacy. In 2013, Smarandache [2] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component T, I, F into respectively, T_1,T_2,..,T_m,I_1,I_2,...,I_p and F_1,F_2,..,F_r. Recently, Deli and Broumi introduced the concept of neutrosophicrefined sets and studied some of their basic properties. The concept of neutrosophic refined sets (NRS) is a generalization of fuzzy multisets and intuitionistic fuzzy multisets. In 2014, Broumi and Smarandache [8] proposed the cosine similarity measure of neutrosophic refined sets. Mondal and Pramanik [5] proposed the cotangent similarity measure of neutrosophic refined sets.

In this paper, by using the notion of neutrosophic refined set,it was provided an exemplary for medical diagnosis. In order to make this, a novel method was executed.
Rest of the article was structured as follows. In Section 2, the basic definitions were briefly presented. Section 3 deals with proposed definitions and some of its properties. Sections 4, 5, 6 contain methodology, algorithm and case study related to medical diagnosis respectively. Conclusion was given in Section 7.

1. Preliminaries

Definition 2.1. [9] Let X be a Universe of discourse, with a generic element in X denoted by x, the neutrosophic set (NS) A is an object having the form
$A=\{\langle x: T_A(x), I_A(x), F_A(x)\rangle, x\in X\}$ where the functions define
$T, I, F: X \rightarrow]^-0,1^+[$ respectively the degree of membership (or Truth), the degree of indeterminacy and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition $^-0 \le T_A(x)+I_A(x)+F_A(x) \le 3^+$

Definition 2.2. [9] Let X be a Universe, a neutrosophic refined set on X can be defined as follows:
$A=\{\langle x,(T_A^1(x),T_A^2(x),\dots,T_A^p(x)),(I_A^1(x),I_A^2(x),\dots,I_A^p(x)),(F_A^1(x),F_A^2(x),\dots,F_A^p(x))\rangle : x\in X\}$
Where $T_A^1(x),T_A^2(x),\dots,T_A^p(x): X\rightarrow[0,1]$ $I_A^1(x),I_A^2(x),\dots,I_A^p(x): X\rightarrow[0,1]$ and $F_A^1(x),F_A^2(x),\dots,F_A^p(x): X\rightarrow[0,1]$ such that $0\le T_A^j(x)+I_A^j(x)+F_A^j(x)\le 3$ for $j=1,2,\dots,p$ for any $x\in X$, $(T_A^1(x),T_A^2(x),\dots,T_A^p(x))$, $(I_A^1(x),I_A^2(x),\dots,I_A^p(x))$ & $(F_A^1(x),F_A^2(x),\dots,F_A^p(x))$ is the truth-membership sequence, indeterminate-membership sequence &falsity-membership sequence of the element x, respectively. Also, p is called the dimension of neutrosophic refined set(NRS) A.

Definition 2.3. [3] Let
$A=\{\langle x,(T_A^1(x),T_A^2(x),\dots,T_A^p(x)),(I_A^1(x),I_A^2(x),\dots,I_A^p(x)),(F_A^1(x),F_A^2(x),\dots,F_A^p(x))\rangle : x\in X\}$ and
$B=\{\langle x,(T_B^1(x),T_B^2(x),\dots,T_B^p(x)),(I_B^1(x),I_B^2(x),\dots,I_B^p(x)),(F_B^1(x),F_B^2(x),\dots,F_B^p(x))\rangle : x\in X\}$
be two neutrosophic refined sets then
$A\subseteq B \Rightarrow T_A^j(x)\le T_B^j(x), I_A^j(x)\ge I_B^j(x) \,\&\, F_A^j(x)\ge F_B^j(x)$ (1) for all $x\in X$

3. Proposed definition

Definition 3.1. Let
$A=\{\langle x,(T_A^1(x),T_A^2(x),\dots,T_A^p(x)),(I_A^1(x),I_A^2(x),\dots,I_A^p(x)),(F_A^1(x),F_A^2(x),\dots,F_A^p(x))\rangle : x\in X\}$ and
$B=\{\langle x,(T_B^1(x),T_B^2(x),\dots,T_B^p(x)),(I_B^1(x),I_B^2(x),\dots,I_B^p(x)),(F_B^1(x),F_B^2(x),\dots,F_B^p(x))\rangle : x\in X\}$
be two neutrosophic refined sets then the cosecant similarity measure is defined as

$$COSEC_{NRS}(A,B)=\frac{1}{6np}\sum_{j=1}^{p}\left[\sum_{i=1}^{n}\operatorname{cosec}\left\langle\frac{\pi\left(1+|T_A^j(x_i)-T_B^j(x_i)|+|I_A^j(x_i)-I_B^j(x_i)|+|F_A^j(x_i)-F_B^j(x_i)|\right)}{4n}\right\rangle\right] \quad (2)$$

Proposition 3.1.
(i) $COSEC_{NRS}(A,B)>0$
(ii) $COSEC_{NRS}(A,B)=COSEC_{NRS}(B,A)$
(iii) If $A\subseteq B\subseteq C$ then
$COSEC_{NRS}(A,C)\le COSEC_{NRS}(A,B)$ & $COSEC_{NRS}(A,C)\le COSEC_{NRS}(B,C)$

Proof:
(i) The proof is straightforward
(ii) It was given that,

$$\left|T_A^j(x_i)-T_B^j(x_i)\right|=\left|T_B^j(x_i)-T_A^j(x_i)\right|$$
$$\left|I_A^j(x_i)-I_B^j(x_i)\right|=\left|I_B^j(x_i)-I_A^j(x_i)\right|$$
$$\left|F_A^j(x_i)-F_B^j(x_i)\right|=\left|F_B^j(x_i)-F_A^j(x_i)\right|$$

$$COSEC_{NRS}(A,B)=\frac{1}{6np}\sum_{j=1}^{p}\left[\sum_{i=1}^{n}\operatorname{cosec}\left\langle\frac{\pi\left(1+\left|T_A^j(x_i)-T_B^j(x_i)\right|+\left|I_A^j(x_i)-I_B^j(x_i)\right|+\left|F_A^j(x_i)-F_B^j(x_i)\right|\right)}{4n}\right\rangle\right]$$
$$=\frac{1}{6np}\sum_{j=1}^{p}\left[\sum_{i=1}^{n}\operatorname{cosec}\left\langle\frac{\pi\left(1+\left|T_B^j(x_i)-T_A^j(x_i)\right|+\left|I_B^j(x_i)-I_A^j(x_i)\right|+\left|F_B^j(x_i)-F_A^j(x_i)\right|\right)}{4n}\right\rangle\right]$$
$$=COSEC_{NRS}(B,A)$$

(iii) By (1),

$$T_A^j(x_i)\le T_B^j(x_i)\le T_C^j(x_i)$$
$$I_A^j(x_i)\ge I_B^j(x_i)\ge I_C^j(x_i)$$
$$F_A^j(x_i)\ge F_B^j(x_i)\ge F_C^j(x_i)$$
$$(\because A\subseteq B\subseteq C)$$

Hence,

$$\left|T_A^j(x_i)-T_B^j(x_i)\right|\le\left|T_A^j(x_i)-T_C^j(x_i)\right|$$
$$\left|T_B^j(x_i)-T_C^j(x_i)\right|\le\left|T_A^j(x_i)-T_C^j(x_i)\right|$$
$$\left|I_A^j(x_i)-I_B^j(x_i)\right|\le\left|I_A^j(x_i)-I_C^j(x_i)\right|$$
$$\left|I_B^j(x_i)-I_C^j(x_i)\right|\le\left|I_A^j(x_i)-I_C^j(x_i)\right|$$
$$\left|F_A^j(x_i)-F_B^j(x_i)\right|\le\left|F_A^j(x_i)-F_C^j(x_i)\right|$$
$$\left|F_B^j(x_i)-F_C^j(x_i)\right|\le\left|F_A^j(x_i)-F_C^j(x_i)\right|$$

Here, our cosecant similarity measure is a decreasing function

$\therefore$ $COSEC_{NRS}(A,C)\le COSEC_{NRS}(A,B)$ & $COSEC_{NRS}(A,C)\le COSEC_{NRS}(B,C)$

4. Methodology

In this section, we present an application of neutrosophic refined set in medical diagnosis. In a given pathology, suppose S is a set of symptoms, D is a set of diseases and P is a set of patients and let Q be a neutrosophic refined relation from the set of patients to the symptoms. i.e., $Q(P\to S)$ and R be a neutrosophic relation from the set of symptoms to the diseases i.e., $R(S\to D)$ and then the methodology involves three main jobs:

1. Determination of symptoms
2. Formulation of medical knowledge based on neutrosophic refined sets and neutrosophic sets
3. Determination of diagnosis on the basis of new computation technique of neutrosophic refined sets

4.1. Algorithm

Step 1: The Symptoms of the patients are given to obtain the patient-symptom relation $Q$and are noted in Table 1

Step 2: The medical knowledge relating the symptoms with the set of diseases under consideration are given to obtain the symptom-disease relation R and arenoted in Table 2.

Step3: The computation T of the relation of patients and diseases is found using (2) and are noted in Table 3.

Step 4: Finally, maximum value from Table 3of each row were selected to find the possibility of the patient affected with the respective disease and then it was concluded that the patient $P_k(k = 1,2,3,4)$ was suffering from the disease $D_r(r = 1,2,3,4,5)$.

5. Case study

Let there be four patients P={P_1, P_2, P_3, P_4} and the set of symptoms S={Temperature, Headache, Stomach pain, Cough, Chest pain}. The Neutrosophic Refined Relation $Q(P \to S)$ is given as in Table 1.Let the set of diseases D = {Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}. The Neutrosophic Relation $R(S \to D)$ is given as in Table 2.

Table 1: Using step1

Q	Temperature	Headache	Stomach pain	Cough	Chest pain
P_1	(0.8,0.1,0.1) (0.6,0.3,0.3) (0.6,0.3,0.1)	(0.6,0.1,0.3) (0.5,0.2,0.4) (0.5,0.1,0.2)	(0.2,0.8,0.0) (0.3,0.5,0.2) (0.2,0.3,0.4)	(0.6,0.1,0.3) (0.4,0.4,0.4) (0.4,0.3,0.3)	(0.1,0.6,0.3) (0.3,0.4,0.5) (0.2,0.5,0.4)
P_2	(0.0,0.8,0.2) (0.2,0.6,0.4) (0.1,0.6,0.4)	(0.4,0.4,0.2) (0.5,0.4,0.1) (0.4,0.6,0.3)	(0.6,0.1,0.3) (0.4,0.2,0.5) (0.3,0.2,0.4)	(0.1,0.7,0.2) (0.2,0.7,0.5) (0.3,0.5,0.4)	(0.1,0.8,0.1) (0.3,0.6,0.4) (0.3,0.6,0.3)
P_3	(0.8,0.1,0.1) (0.6,0.4,0.1) (0.5,0.3,0.3)	(0.8,0.1,0.1) (0.6,0.2,0.4) (0.6,0.1,0.3)	(0.0,0.6,0.4) (0.2,0.5,0.5) (0.3,0.4,0.6)	(0.2,0.7,0.1) (0.2,0.5,0.5) (0.1,0.6,0.3)	(0.0,0.5,0.5) (0.2,0.5,0.3) (0.3,0.3,0.4)
P_4	(0.6,0.1,0.3) (0.4,0.3,0.2) (0.5,0.2,0.3)	(0.5,0.4,0.1) (0.4,0.4,0.4) (0.5,0.2,0.4)	(0.3,0.4,0.3) (0.2,0.4,0.5) (0.1,0.5,0.4)	(0.7,0.2,0.1) (0.5,0.2,0.4) (0.6,0.4,0.1)	(0.3,0.4,0.3) (0.4,0.3,0.4) (0.3,0.5,0.5)

Table 2: Using step2

R	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.6,0.3,0.3)	(0.2,0.5,0.3)	(0.2,0.6,0.4)	(0.1,0.6,0.6)	(0.1,0.6,0.4)
Headache	(0.4,0.5,0.3)	(0.2,0.6,0.4)	(0.1,0.5,0.4)	(0.2,0.4,0.6)	(0.1,0.6,0.4)
Stomach pain	(0.1,0.6,0.3)	(0.0,0.6,0.4)	(0.2,0.5,0.5)	(0.8,0.2,0.2)	(0.1,0.7,0.1)
Cough	(0.4,0.4,0.4)	(0.4,0.1,0.5)	(0.2,0.5,0.5)	(0.1,0.7,0.4)	(0.4,0.5,0.4)
Chest pain	(0.1,0.7,0.4)	(0.1,0.6,0.3)	(0.1,0.6,0.4)	(0.1,0.7,0.4)	(0.8,0.2,0.2)

Table 3: Using step 3 and step 4 (Cosecant similarity measure)

T	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.7893**	0.6888	0.6689	0.5909	0.6166
P_2	0.7019	0.6934	**0.7689**	0.7445	0.6693
P_3	**0.7240**	0.6612	0.7169	0.6070	0.5856
P_4	**0.7660**	0.6899	0.6818	0.5784	0.6165

From Table 3, it is obvious that, if the doctor agrees, then P_1 , P_3, P_4 suffers from Viral fever and P_2 suffers from Typhoid.

6. Conclusion

Our propounded techniques are most reliable to handle medical diagnosis problems quiet comfortably. The recommended methods can invade in other areas such as clustering, image processing etc., In future, we will enhance this method to other types of neutrosophic sets.

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