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Algorithms for neutrosophic soft decision making based on EDAS, new similarity measure and level soft set

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Abstract. This paper presents three novel single-valued neutrosophic soft set (SVNSS) methods. First, we initiate a new axiomatic definition of single-valued neutrosophic similarity measure, which is expressed by single-valued neutrosophic number (SVNN) that will reduce the information loss and remain more original information. Then, the objective weights of various parameters are determined via grey system theory. Moreover, we develop the combined weights, which can show both the subjective information and the objective information. Later, we propose three algorithms to solve single-valued neutrosophic soft decision making problem by Evaluation based on Distance from Average Solution (EDAS), similarity measure and level soft set. Finally, the effectiveness and feasibility of approaches are demonstrated by a numerical example.

Keywords: Single-valued neutrosophic soft set, similarity measure, SVNN, Combined weighed , EDAS , level soft set

1. Introduction

Soft set theory, initiated by Molodtsov [1], is free from the inadequacy of the parameterized tools of those theories such as fuzzy set [2], rough set [3], and interval mathematics [4]. The study of hybrid models combing soft sets with other mathematical structures are an important research topic. Maji et al. [5] firstly proposed fuzzy soft sets, a more wide notion combining fuzzy sets and soft sets. Alcantud [6] proposed a novel algorithm for fuzzy soft set based decision making from multiobserver input parameter data set. Feng et al. [7] proposed an adjustable approach to fuzzy soft set based decision making by level soft set. Yang et al. [8] developed the concept of interval-valued fuzzy soft sets. Meanwhile, Peng and Yang applied interval-valued fuzzy soft sets in clustering analysis [9] and decision making [10]. Peng et al. [11] presented Pythagorean fuzzy soft sets, and discussed their operations. Yang et al. [12] proposed the multi-fuzzy soft sets and successfully applied them to decision making, meanwhile they extended the multi-fuzzy soft sets to bipolar multi-fuzzy soft sets [13], which can describe the parameter more precisely. Wang et al. [14] presented the hesitant fuzzy soft sets by aggregating hesitant fuzzy set [15] with soft set, and developed an algorithm to solve decision making problems. Peng and Yang [16] further proposed interval-valued hesitant fuzzy soft set, presented an algorithm, and discussed its calculation complexity with others algorithms.

Smarandache [17] initially presented the concept of a neutrosophic set from a philosophical point of view. A neutrosophic set is characterized by a truth-membership degree, an indeterminacy-membership degree, and a falsity-membership degree. It generalizes the concept of the classic set, fuzzy set [2], interval-valued fuzzy set [4], paraconsistent set [17],

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and tautological set [17]. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Hence, Smarandache [17] firstly introduced the a single valued neutrosophic set (SVNS). Wang et al. [18] provided the set-theoretic operators and various properties of SVNSs. Ye [19, 20] proposed a multi-attribute decision making (MADM) method using the correlation coefficient under single-valued neutrosophic environment. Ye [21, 22] further developed clustering method and decision making methods by similarity measures of SVNS. Meanwhile, Peng and Dai [23] presented new similarity measure of SVNS and applied them to decision making. Biswas et al. [24] extended the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method for multi-attribute single-valued neutrosophic decision-making problem. Sahin and Kucuk [25] defined a subsethood measure for SVNS, and applied to MADM. Yang et al. [26] introduced single valued neutrosophic relations and discussed their properties. Huang [27] developed a new distance measure of SVNSs, and applied them to clustering analysis and MADM. Liu [28] proposed aggregation operators based on Archimedean t-conorm and t-norm for SVNS and also gave an application in MADM. Wu et al. [29] proposed a cross-entropy and prioritized aggregation operator with simplified neutrosophic sets. Single valued neutrosophic graphs [30–32] and bipolar single valued neutrosophic graphs [33, 34] are proposed by Broumi et al. Peng et al. [35, 36] proposed ELECTRE approach and qualitative flexible approach based on likelihood for multi-valued neutrosophic MADM problem, respectively. Ji et al. [37] introduced a projection-based TODIM method for multi-valued neutrosophic MADM problem. Tian et al. [38] presented an improved MULTIMOORA approach for simplified neutrosophic linguistic set. Later, Tian et al. [39] incorporated power aggregation operators and a TOPSIS-based QUALIFLEX method in order to solve green product design selection problems using simplified neutrosophic linguistic information. Tian et al. [40] proposed a MADM based on generalized prioritized aggregation operators under simplified neutrosophic uncertain linguistic environment.

Combining the advantages of the instance of neutrosophic set [17] with soft set [1], Maji [41] proposed the single-valued neutrosophic soft set. Şahin and Küçük [42] presented the definition of similarity and entropy in single-valued neutrosophic soft environment. Mukherjee and Sarkar [43] defined some similarity measures of single-valued neutrosophic soft sets, and applied them to real life problems. Deli and Broumi [44] defined some new operators and soft matrix based on single-valued neutrosophic soft sets. Alkhazaleh [45] introduced time-neutrosophic soft set and studied some of its properties in detail. Al-Quran and Hassan [46] presented the neutosophic vague soft expert set theory and discussed their properties in detail.

Evaluation based on Distance from Average Solution (EDAS), originally proposed by Ghorabaee et al. [47], is a new MADM method for inventory ABC classification. It is very useful when we have some conflicting parameters. In the compromise MADM methods such as TOPSIS and VIKOR [48], the best alternative is got by computing the distance from ideal and nadir solutions. The desirable alternative has lower distance from ideal solution and higher distance from nadir solution in these MADM methods. Ghorabaee et al. [49] extended the EDAS method to supplier selection. As far as we know, however, the study of the MADM problem based on EDAS method have not been reported in the existing academic literature. Hence, it is an interesting research topic to apply the EDAS in MADM to rank and determine the best alternative under single-valued neutrosophic soft environment. Through a comparison analysis of the given methods, their objective evaluation is carried out, and the method which maintains consistency of its results is chosen.

For computing the similarity measure of two SVNSs, we propose a new axiomatic definition of similarity measure, which takes in the form of SVNN. Comparing with the existing literature [21, 22, 42, 43], our similarity measure can remain more original decision information.

By means of level soft sets, Feng et al. [7] presented an adjustable approach to fuzzy soft sets based decision making. By considering different types of thresholds, it can derive different level soft sets from the original fuzzy soft set. In general, the final optimal decisions based on different level soft sets could be different. Thus the newly proposed approach is in fact an adjustable method which captures an important feature for decision making in an imprecise environment: some of these problems are essentially humanistic and thus subjective in nature. As far as we know, however, the study of the single-valued neutrosophic soft MADM problem based on level soft set have not been reported in the existing academic literature.

Considering that different attribute weights will influence the ranking results of alternatives, we develop a new method to determine the attribute weights by combining the subjective elements with the objective ones. This model is different from the existing methods, which can be divided into two tactics: one is the subjective weighting evaluation methods and the other is the objective weighting determine methods, which can be computed by grey system theory [50]. The subjective weighting methods focus on the preference information of the decision maker [19–21, 27–29], while they ignore the objective information. The objective weighting determine methods do not take the preference of the decision maker into account, that is to say, these methods fail to take the risk attitude of the decision maker into account [22, 24]. The feature of our weighting model can show both the subjective information and the objective information. Hence, combining objective weights with subjective weights, a combined model to obtain attribute weights is proposed.

The remainder of this paper is organized as follows: In Section 2, we review some fundamental conceptions of neutrosophic sets, single-valued neutrosophic sets, soft set and single-valued neutrosophic soft sets. In Section 3, a new axiomatic definition of single-valued neutrosophic similarity measure is presented. In Section 4, three single-valued neutrosophic soft decision making approaches based on EDAS, similarity measure and level soft set are shown. In Section 5, a numerical example is given to illustrate the proposed methods. The paper is concluded in Section 6.

2. Preliminaries

2.1. Neutrosophic set

Neutrosophic set is a portion of neutrosophy, which researches the origin, and domain of neutralities, as well as their interactions with diverse ideational scope [17], and is a convincing general formal framework, which extends the presented sets [2, 4] from philosophical point. Smarandache [17] introduced the definition of neutrosophic set as follows:

Definition 1. [17] Let X be a universe of discourse, with a class of elements in X denoted by x. A neutrosophic set B in X is summarized by a truth-membership function $T_B(x)$, an indeterminacy-membership function $I_B(x)$, and a falsity-membership function $F_B(x)$. The functions $T_B(x)$, $I_B(x)$, and $F_B(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_B(x) : X \to]0^-, 1^+[$, $I_B(x) : X \to]0^-, 1^+[$, and $F_B(x) : X \to]0^-, 1^+[$.

There is restriction on the sum of $T_B(x)$, $I_B(x)$, and $F_B(x)$, so $0^- \leq \sup\ T_B(x) + \sup\ I_B(x) + \sup F_B(x) \leq 3^+$.

As mentioned above, it is hard to apply the neutrosophic set to solve some real problems. Hence, Smarandache [17] presented SVNS, which is a subclass of the neutrosophic set and mentioned the definition as follows:

Definition 2. [17] Let X be a universe of discourse, with a class of elements in X denoted by x. A single-valued neutrosophic set N in X is summarized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. Then a SVNS N can be denoted as follows:

$$N = \{< x, T_N(x), I_N(x), F_N(x) > \mid\ x \in X\}, \quad (1)$$

where $T_N(x), I_N(x), F_N(x) \in [0, 1]$ for $\forall x \in X$. Meanwhile, the sum of $T_N(x)$, $I_N(x)$, and $F_N(x)$ fulfills the condition $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$. For a SVNS N in X, the triplet $(T_N(x), I_N(x), F_N(x))$ is called single-valued neutrosophic number (SVNN). For convenience, we can simply use $x = (T_x, I_x, F_x)$ to represent a SVNN as an element in the SVNS N.

Generally speaking, two special values are taken into consideration, i.e., SVNN 0 and 1. If we think about 0 as the worst value and 1 as the best value, we can set 0 as (0,1,1) and 1 as (1,0,0).

Definition 3. [17, 18] Let $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$ be two SVNNs, then operations can be defined as follows:

(1) $x^c = (F_x, 1 - I_x, T_x)$;
(2) $x \bigcup y = (\max\{T_x, T_y\}, \min\{I_x, I_y\}, \min\{F_x, F_y\})$;
(3) $x \bigcap y = (\min\{T_x, T_y\}, \max\{I_x, I_y\}, \max\{F_x, F_y\})$;
(4) $x \oplus y = (T_x + T_y - T_x * T_y, I_x * I_y, F_x * F_y)$;
(5) $x \otimes y = (T_x * T_y, I_x + I_y - I_x * I_y, F_x + F_y - F_x * F_y)$;
(6) $\lambda x = (1 - (1 - T_x)^\lambda, (I_x)^\lambda, (F_x)^\lambda), \lambda > 0$;
(7) $x^\lambda = ((T_x)^\lambda, 1 - (1 - I_x)^\lambda, 1 - (1 - F_x)^\lambda), \lambda > 0$.

For comparing two SVNNs, Peng et al. [51] introduced a similarity measure method for a SVNN.

Definition 4. [51] Let $x = (T_x, I_x, F_x)$ be a SVNN, then the score function $s(x)$ is defined as follows:

$$s(x) = \frac{2}{3} + \frac{T_x}{3} - \frac{I_x}{3} - \frac{F_x}{3}. \quad (2)$$

It measures the hamming similarity between $x = (T_x, I_x, F_x)$ and the ideal solution (1, 0, 0) for the comparison of SVNNs.

Definition 5. [28] Let $x_j (j = 1, 2, \cdots, n)$ be a series of the SVNNs, and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $x_j (i = 1, 2, \cdots, n)$, then a single-valued neutrosophic weighted averaging (SVNWA) operator is a mapping SVNWA: $X^n \to X$, where

$$\text{SVNWA}(x_1, x_2, \cdots, x_n) = \bigoplus_{j=1}^{n} (w_j x_j)$$
$$= \left(1 - \prod_{j=1}^{n}(1 - T_j)^{w_j}, \prod_{j=1}^{n}(I_j)^{w_j}, \prod_{j=1}^{n}(F_j)^{w_j}\right) \quad (3)$$

Definition 6. [41] A pair $(\widetilde{F}, A)$ is called a single-valued neutrosophic soft set over U, where $\widetilde{F}$ is a mapping given by $\widetilde{F} : A \to \widetilde{P}(U)$.

In other words, the single-valued neutrosophic soft set is not a kind of set, but a parameterized family of subsets of the set U. For any parameter $e \in A$, $\widetilde{F}(e)$ may be considered as the set of e-approximate elements of the single-valued neutrosophic soft set $(\widetilde{F}, A)$. Let $\widetilde{F}(e)(x)$ denote the membership value that object x holds parameter e, then $\widetilde{F}(e)$ can be written as a single-valued neutrosophic set that $\widetilde{F}(e) = \{x/\widetilde{F}(e)(x) \mid x \in U\} = \{x/(T_{\widetilde{F}}(e)(x), I_{\widetilde{F}}(e)(x), F_{\widetilde{F}}(e)(x)) \mid x \in U\}$.

Example 1. Let $U = \{x_1, x_2, x_3\}$ and $A = \{e_1, e_2, e_3\}$. Let $(\widetilde{F}, A)$ be a single-valued neutrosophic soft set over U, defined as follows:

$$\widetilde{F}(e_1) = \{x_1/(0.3, 0.4, 0.7), x_2/(0.3, 0.5, 0.6), x_3/(0.4, 0.4, 0.6)\},$$
$$\widetilde{F}(e_2) = \{x_1/(0.7, 0.4, 0.8), x_2/(0.6, 0.4, 0.6), x_3/(0.3, 0.5, 0.7)\},$$
$$\widetilde{F}(e_3) = \{x_1/(0.5, 0.4, 0.6), x_2/(0.3, 0.6, 0.7), x_3/(0.3, 0.4, 0.8)\}.$$

Then, $(\widetilde{F}, A)$ is described by the following Table 1.

Table 1
The single-valued neutrosophic soft set $(\widetilde{F}, A)$

	e_1	e_2	e_3
x_1	(0.3,0.4,0.7)	(0.7,0.4,0.8)	(0.5,0.4,0.6)
x_2	(0.3,0.5,0.6)	(0.6,0.4,0.6)	(0.3,0.6,0.7)
x_3	(0.4,0.4,0.6)	(0.3,0.5,0.7)	(0.3,0.4,0.8)

Definition 7. [41] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two single-valued neutrosophic soft sets over the common universe U. $(\widetilde{F}, A)$ is said to be single-valued neutrosophic soft subset of $(\widetilde{G}, B)$ if $A \subset B$, $T_{\widetilde{F}}(e)(x) \leq T_{\widetilde{G}}(e)(x)$, $I_{\widetilde{F}}(e)(x) \leq I_{\widetilde{G}}(e)(x)$, $F_{\widetilde{F}}(e)(x) \geq F_{\widetilde{G}}(e)(x)$, $\forall e \in A, x \in U$. We denote it by $(\widetilde{F}, A) \subseteq (\widetilde{G}, B)$.

3. A new single-valued neutrosophic similarity measure

For the existing single-valued neutrosophic similarity measures [21, 22, 42, 43], its similarity result is a fuzzy number, it may lost some original information. In the following, we will propose a new single-valued neutrosophic similarity measure, its similarity result is still a single-valued neutrosophic number, hence it can be remain more original decision information in some extent.

Definition 8. Let A_1, A_2 and A_3 be three SVNSs on X. A similarity measure $S^{\Delta}(A_1, A_2)$ is a mapping S^{Δ} : SVNS$(X) \times$ SVNS$(X) \to$ SVNN, possessing the following properties:

(1) $S^{\Delta}(A_1, A_2)$ is a SVNN;
(2) $S^{\Delta}(A_1, A_2) = (1, 0, 0)$, iff $A_1 = A_2$;
(3) $S^{\Delta}(A_1, A_2) = S^{\Delta}(A_2, A_1)$;
(4) If $A_1 \subseteq A_2 \subseteq A_3$, then $S^{\Delta}(A_1, A_2) \supseteq S^{\Delta}(A_1, A_3)$ and $S^{\Delta}(A_2, A_3) \supseteq S^{\Delta}(A_1, A_3)$.

Theorem 1. *Let A_i and A_k be two SVNSs, then $S^{\Delta}(A_i, A_k)$ is a similarity measure.*

$$S^{\Delta}(A_i, A_k) = \Big(\min\{L(A_i, A_k), M(A_i, A_k), R(A_i, A_k)\}, \min\{L(A_i, A_k), W(A_i, A_k), R(A_i, A_k)\}, 1 - \max\{L(A_i, A_k), M(A_i, A_k), R(A_i, A_k)\}\Big), \quad (4)$$

where $L(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{T_{ij}, T_{kj}\}}{\sum_{j=1}^{n} w_j \max\{T_{ij}, T_{kj}\}}$,

$$M(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij}, 1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij}, 1-I_{kj}\}},$$

$$W(A_i, A_k) = 1 - \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij}, 1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij}, 1-I_{kj}\}},$$

$$R(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-F_{ij}, 1-F_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-F_{ij}, 1-F_{kj}\}}$$ and w_j is the weight of jth SVNN.

Proof. In order for $S^{\Delta}(A_i, A_k)$ to be qualified as a sensible similarity measure for SVNSs, it must satisfy the (1)-(4) of axiomatic requirements.

(1) Since $0 \leq L(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{T_{ij}, T_{kj}\}}{\sum_{j=1}^{n} w_j \max\{T_{ij}, T_{kj}\}} \leq 1,$

$$0 \leq M(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij}, 1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij}, 1-I_{kj}\}} \leq 1,$$

$$0 \leq W(A_i, A_k) = 1 - \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij}, 1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij}, 1-I_{kj}\}} \leq 1,$$

$$0 \leq R(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-F_{ij}, 1-F_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-F_{ij}, 1-F_{kj}\}} \leq 1,$$

therefore,
$0 \leq \min\{L(A_i, A_k), M(A_i, A_k), R(A_i, A_k)\} \leq 1,$
$0 \leq \min\{L(A_i, A_k), W(A_i, A_k), R(A_i, A_k)\} \leq 1,$
$0 \leq 1 - \max\{L(A_i, A_k), M(A_i, A_k), R(A_i, A_k)\} \leq 1.$

Furthermore,

$$\begin{aligned} 0 \leq \; & \min\{L(A_i, A_k), M(A_i, A_k), R(A_i, A_k)\} \\ & + \min\{L(A_i, A_k), W(A_i, A_k), R(A_i, A_k)\} + 1 \\ & - \max\{L(A_i, A_k), M(A_i, A_k), R(A_i, A_k)\} \leq 3. \end{aligned}$$

Consequently, $S^{\Delta}(A_i, A_k)$ is a SVNN.

(2) ① Necessity:
Since $S^{\Delta}(A_i, A_k) = (1, 0, 0)$, we have
$\min\{L(A_i, A_k), M(A_i, A_k), R(A_i, A_k)\} = 1,$
$\min\{L(A_i, A_k), W(A_i, A_k), R(A_i, A_k)\} = 0,$
$\max\{L(A_i, A_k), M(A_i, A_k), R(A_i, A_k)\} = 1.$

It means that $L(A_i, A_k) = M(A_i, A_k) = R(A_i, A_k) = 1$, $W(A_i, A_k) = 0$.

Furthermore,

$$L(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{T_{ij}, T_{kj}\}}{\sum_{j=1}^{n} w_j \max\{T_{ij}, T_{kj}\}} = 1,$$

$$M(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij}, 1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij}, 1-I_{kj}\}} = 1,$$

$$W(A_i, A_k) = 1 - \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij}, 1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij}, 1-I_{kj}\}} = 0,$$

$$R(A_i, A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-F_{ij}, 1-F_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-F_{ij}, 1-F_{kj}\}} = 1.$$

Based on the randomicity of w_j, we can have $T_{ij} = T_{kj}$, $I_{ij} = I_{kj}$, $F_{ij} = F_{kj}$, i.e., $A_i = A_k$.

② Sufficiency:
Since $A_i = A_k$, we have $T_{ij} = T_{kj}$, $I_{ij} = I_{kj}$, $F_{ij} = F_{kj}$.

Furthermore,

$$\sum_{j=1}^{n} w_j \min\{T_{ij}, T_{kj}\} = \sum_{j=1}^{n} w_j \max\{T_{ij}, T_{kj}\},$$

$$\sum_{j=1}^{n} w_j \min\{1 - I_{ij}, 1 - I_{kj}\} = \sum_{j=1}^{n} w_j \max\{1 - I_{ij}, 1 - I_{kj}\},$$

$$\sum_{j=1}^{n} w_j \min\{1 - F_{ij}, 1 - F_{kj}\} = \sum_{j=1}^{n} w_j \max\{1 - F_{ij}, 1 - F_{kj}\}.$$

Consequently, $S^{\Delta}(A_i, A_k) = (1, 0, 0)$.

(3) It is obvious.

(4) If $A_1 \subseteq A_2 \subseteq A_3$, then $\forall j, T_{1j} \leq T_{2j} \leq T_{3j}$, $I_{1j} \geq I_{2j} \geq I_{3j}$ and $F_{1j} \geq F_{2j} \geq F_{3j}$.

Hence,

$$L(A_1, A_3) = \frac{\sum_{j=1}^{n} w_j \min\{T_{1j}, T_{3j}\}}{\sum_{j=1}^{n} w_j \max\{T_{1j}, T_{3j}\}}$$

$$= \frac{\sum_{j=1}^{n} w_j T_{1j}}{\sum_{j=1}^{n} w_j T_{3j}} \le \frac{\sum_{j=1}^{n} w_j T_{1j}}{\sum_{j=1}^{n} w_j T_{2j}}$$

$$= \frac{\sum_{j=1}^{n} w_j \min\{T_{1j}, T_{2j}\}}{\sum_{j=1}^{n} w_j \max\{T_{1j}, T_{2j}\}} = L(A_1, A_2),$$

$$M(A_1, A_3) = \frac{\sum_{j=1}^{n} w_j \min\{1 - I_{1j}, 1 - I_{3j}\}}{\sum_{j=1}^{n} w_j \max\{1 - I_{1j}, 1 - I_{3j}\}}$$

$$= \frac{\sum_{j=1}^{n} w_j (1 - I_{1j})}{\sum_{j=1}^{n} w_j (1 - I_{3j})} \le \frac{\sum_{j=1}^{n} w_j (1 - I_{1j})}{\sum_{j=1}^{n} w_j (1 - I_{2j})}$$

$$= \frac{\sum_{j=1}^{n} w_j \min\{1 - I_{1j}, 1 - I_{2j}\}}{\sum_{j=1}^{n} w_j \max\{1 - I_{1j}, 1 - I_{2j}\}}$$

$$= M(A_1, A_2),$$

$$W(A_1, A_3) = 1 - \frac{\sum_{j=1}^{n} w_j \min\{1 - I_{1j}, 1 - I_{3j}\}}{\sum_{j=1}^{n} w_j \max\{1 - I_{1j}, 1 - I_{3j}\}}$$

$$= 1 - \frac{\sum_{j=1}^{n} w_j (1 - I_{1j})}{\sum_{j=1}^{n} w_j (1 - I_{3j})}$$

$$\le 1 - \frac{\sum_{j=1}^{n} w_j (1 - I_{1j})}{\sum_{j=1}^{n} w_j (1 - I_{2j})}$$

$$= 1 - \frac{\sum_{j=1}^{n} w_j \min\{1 - I_{1j}, 1 - I_{2j}\}}{\sum_{j=1}^{n} w_j \max\{1 - I_{1j}, 1 - I_{2j}\}}$$

$$= W(A_1, A_2),$$

$$R(A_1, A_3) = \frac{\sum_{j=1}^{n} w_j \min\{1 - F_{1j}, 1 - F_{3j}\}}{\sum_{j=1}^{n} w_j \max\{1 - F_{1j}, 1 - F_{3j}\}}$$

$$= \frac{\sum_{j=1}^{n} w_j (1 - F_{1j})}{\sum_{j=1}^{n} w_j (1 - F_{3j})} \le \frac{\sum_{j=1}^{n} w_j (1 - F_{1j})}{\sum_{j=1}^{n} w_j (1 - F_{2j})}$$

$$= \frac{\sum_{j=1}^{n} w_j \min\{1 - F_{1j}, 1 - F_{2j}\}}{\sum_{j=1}^{n} w_j \max\{1 - F_{1j}, 1 - F_{2j}\}}$$

$$= R(A_1, A_2).$$

Furthermore,

$$\min\{L(A_1, A_2), M(A_1, A_2), R(A_1, A_2)\}$$
$$\ge \min\{L(A_1, A_3), M(A_1, A_3), R(A_1, A_3)\},$$
$$\min\{L(A_1, A_2), W(A_1, A_2), R(A_1, A_2)\}$$
$$\ge \min\{L(A_1, A_3), W(A_1, A_3), R(A_1, A_3)\},$$
$$1 - \max\{L(A_1, A_2), M(A_1, A_2), R(A_1, A_2)\}$$
$$\le 1 - \max\{L(A_1, A_2), M(A_1, A_2), R(A_1, A_2)\}.$$

Consequently, $S^{\Delta}(A_1, A_2) \supseteq S^{\Delta}(A_1, A_3)$.

Similarly, $S^{\Delta}(A_2, A_3) \supseteq S^{\Delta}(A_1, A_3)$. This completes the proof.

Example 2. [21] Assume that we have the following three SVNSs in a universe of discourse $X = \{x_1, x_2\}$, $A \subseteq B \subseteq C$, $w = \{0.5, 0.5\}$:

$$A = \{< x_1, 0.1, 0.5, 0.6 >, < x_2, 0.2, 0.5, 0.7 >\},$$

$$B = \{< x_1, 0.3, 0.4, 0.5 >, < x_2, 0.5, 0.3, 0.4 >\},$$

$$C = \{< x_1, 0.6, 0.1, 0.2 >, < x_2, 0.8, 0.1, 0.3 >\}.$$

By applying the proposed similarity measure,

$$S^{\Delta}(A, B) = (0.3750, 0.2308, 0.2308),$$

$$S^{\Delta}(B, C) = (0.5714, 0.2778, 0.2778),$$

$$S^{\Delta}(A, C) = (0.2143, 0.2143, 0.4444).$$

Thus, $S^{\Delta}(A, C) < S^{\Delta}(A, B)$ and $S^{\Delta}(A, C) < S^{\Delta}(B, C)$.

4. Three algorithms for single-valued neutrosophic soft decision making

4.1. Problem description

Let $U = \{x_1, x_2, \cdots, x_m\}$ be a finite set of m alternatives, $E = \{e_1, e_2, \cdots, e_n\}$ be a set of n

Table 2
Tabular representation of $(\widetilde{F}, E)$

	e_1	e_2	$\cdots$	e_n
x_1	$\widetilde{F}(e_1)(x_1)$	$\widetilde{F}(e_2)(x_1)$	$\cdots$	$\widetilde{F}(e_n)(x_1)$
x_2	$\widetilde{F}(e_1)(x_2)$	$\widetilde{F}(e_2)(x_2)$	$\cdots$	$\widetilde{F}(e_n)(x_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_m	$\widetilde{F}(e_1)(x_m)$	$\widetilde{F}(e_2)(x_m)$	$\cdots$	$\widetilde{F}(e_n)(x_m)$

parameters, and the weight of parameter e_i is w_i, $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$. $(\widetilde{F}, E)$ is single-valued neutrosophic soft set which can be expressed in Table 2. $\widetilde{F}(e_j)(x_i) = (T_{\widetilde{F}}(e_j)(x_i), I_{\widetilde{F}}(e_j)(x_i), F_{\widetilde{F}}(e_j)(x_i))$ represents the possible SVNN of the ith alternative x_i satisfying the jth parameter e_j which is given by the decision maker.

In the following, we will apply the EDAS, similarity measure and level soft set methods to SVNSS.

4.2. The method of computing the combined weights

We develop a novel method to obtain the weights by combining the subjective elements with the objective ones. This model is different from the existing methods, which can be divided into two tactics: one is the subjective weighting determine methods and the other is the objective weighting methods, which can be computed by grey system theory [50]. The feature of our weighting model can show both the subjective information and the objective information. Hence, combining subjective weights with objective weights, we provide a combined model to determine attribute weights.

4.2.1. Determining the objective weights: The grey system method

The grey system theory [50] is an excellent tool to deal with small sample and poor information. For a decision maker problem, the alternatives and attributes are generally small which accord with the condition of grey system theory, so we take the method of grey system into consideration.

Theoretically, if an attribute information with respect to other attribute more matches in the average information of the attribute, the attribute contains more information for decision making, the greater the weight. Based on this idea, we propose a grey relational analysis method to determine the attribute weights.

Definition 9. Suppose $R = (r_{ij})_{m\times n}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ be a single-valued neutrosophic matrix. And $S = (s_{ij})_{m\times n}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ is the score function s (Equation (2)) of R. Let $\bar{s}_i = \frac{1}{n}\sum_{j=1}^{n} s_{ij}$, then the attribute weight ω_j is defined as follows:

$$\omega_j = \frac{1 - \frac{1}{m}\left(\sum_{i=1}^{m}(o_{ij}^{q})\right)^{\frac{1}{q}}}{n - \frac{1}{m}\sum_{j=1}^{n}\left(\sum_{i=1}^{m}(o_{ij}^{q})\right)^{\frac{1}{q}}}, \tag{5}$$

where $o_{ij} = \frac{\min_i |s_{ij}-\bar{s}_i| + \xi \max_i |s_{ij}-\bar{s}_i|}{|s_{ij}-\bar{s}_i| + \xi \max_i |s_{ij}-\bar{s}_i|}$ is grey mean relational degree, in general, we set $\xi = 0.5$.

In order to improve the effective resolution, this paper uses Euclidean distance instead of Hamming distance, i.e., $q = 2$.

4.2.2. Determining the combined weights: The non-linear weighted comprehensive method

Suppose that the vector of the subjective weight, given by the decision makers directly, is $w = \{w_1, w_2, \cdots, w_n\}$, where $\sum_{j=1}^{n} w_j = 1, 0 \le w_j \le 1$. The vector of the objective weight, computed by Equation (11) directly, is $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$, where $\sum_{j=1}^{n} \omega_j = 1, 0 \le \omega_j \le 1$.

Therefore, the vector of the combined weight $\varpi = \{\varpi_1, \varpi_2, \cdots, \varpi_n\}$ can be defined as follows:

$$\varpi_j = \frac{w_j * \omega_j}{\sum_{j=1}^{n} w_j * \omega_j}, \tag{6}$$

where $\sum_{j=1}^{n} \varpi_j = 1, 0 \le \varpi_j \le 1$.

The objective weight and subjective weight are aggregated by non-linear weighted comprehensive method. According to the multiplier effect, the larger the value of the subjective weight and objective weight are, the larger the combined weight is, or vice versa. At the same time, we can obtain that the Equation (6) overcomes the limitation of only considering either subjective or objective factor influence. The advantage of Equation (6) is that the attribute weights and rankings of alternatives can show both the subjective information and the objective information.

4.3. The method of EDAS

In this section, an extended version of the EDAS method is proposed to deal with decision making problems in the single-valued neutrosophic soft environment. Therefore, the concepts and arithmetic operations of the SVNN are utilized for extending the EDAS method.

Algorithm 1: EDAS

Step 1. Identify the alternatives and parameters, and obtain the single-valued neutrosophic soft set $(\widetilde{F}, E)$ which is shown in Table 2.

Step 2. Normalize the single-valued neutrosophic soft set $(\widetilde{F}, E)$ into $(\widetilde{F}', E)$ by Equation (7).

$$\widetilde{F}'(e_j)(x_i) = \begin{cases} \left(T_{\widetilde{F}}(e_j)(x_i), I_{\widetilde{F}}(e_j)(x_i), F_{\widetilde{F}}(e_j)(x_i)\right), & e_j \in B, \\ \left(F_{\widetilde{F}}(e_j)(x_i), 1 - I_{\widetilde{F}}(e_j)(x_i), T_{\widetilde{F}}(e_j)(x_i)\right), & e_j \in C, \end{cases} \tag{7}$$

where B is benefit parameter set and C is cost parameter set.

Step 3. Compute the combined weights by Equation (6).

Step 4. Determine the average solution according to all parameters, shown as follows:

$$AV = (AV_j)_{1\times n}, \tag{8}$$

where

$$\begin{aligned} AV_j &= \frac{1}{m}\bigoplus_{i=1}^{m}\widetilde{F}(e_j)(x_i) \\ &= \frac{1}{m}\Bigg(1 - \prod_{i=1}^{m}(1 - T_{\widetilde{F}}(e_j)(x_i)), \\ &\qquad \prod_{i=1}^{m} I_{\widetilde{F}}(e_j)(x_i), \prod_{i=1}^{m} F_{\widetilde{F}}(e_j)(x_i)\Bigg) \\ &= \Bigg(1 - \left(\prod_{i=1}^{m}(1 - T_{\widetilde{F}}(e_j)(x_i))\right)^{\frac{1}{m}}, \\ &\qquad \left(\prod_{i=1}^{m} I_{\widetilde{F}}(e_j)(x_i)\right)^{\frac{1}{m}}, (\prod_{i=1}^{m} F_{\widetilde{F}}(e_j)(x_i))^{\frac{1}{m}}\Bigg). \end{aligned} \tag{9}$$

Step 5. Calculate the positive distance from average (PDA) with $PDA = (P_{ij})_{m\times n}$ and the negative distance from average (NDA) with $NDA = (N_{ij})_{m\times n}$ matrixes according to the type of parameters, shown as follows:

$$P_{ij} = \begin{cases} \frac{\max\{0, s(\widetilde{F}(e_j)(x_i)) - s(AV_j)\}}{s(AV_j)}, & e_j \in B, \\ \frac{\max\{0, s(AV_j) - s(\widetilde{F}(e_j)(x_i))\}}{s(AV_j)}, & e_j \in C, \end{cases} \tag{10}$$

$$N_{ij} = \begin{cases} \frac{\max\{0, s(AV_j) - s(\widetilde{F}(e_j)(x_i))\}}{s(AV_j)}, & e_j \in B, \\ \frac{\max\{0, s(\widetilde{F}(e_j)(x_i)) - s(AV_j)\}}{s(AV_j)}, & e_j \in C, \end{cases} \tag{11}$$

where $s(AV_j)$ and $s(\widetilde{F}(e_j)(x_i))$ are score function of AV_j and $\widetilde{F}(e_j)(x_i)$, respectively.

Step 6. Determine the weighted sum of PDA and NDA for all alternatives, shown as follows:

$$SP_i = \sum_{j=1}^{n} w_j P_{ij}, \tag{12}$$

$$SN_i = \sum_{j=1}^{n} w_j N_{ij}. \tag{13}$$

Step 7. Normalize the values of SP_i and SN_i for all alternatives, shown as follows:

$$NSP_i = \frac{SP_i}{\max_i\{SP_i\}} \tag{14}$$

$$NSN_i = 1 - \frac{SN_i}{\max_i\{SN_i\}} \tag{15}$$

Step 8. Calculate the appraisal score $AS_i (i = 1, 2, \cdots, m)$ for all alternatives, shown as follows:

$$AS_i = \frac{1}{2}(NSP_i + NSN_i), \tag{16}$$

where $0 \le AS_i \le 1$.

Step 9. Rank the alternatives by means of the decreasing values of AS_i. The alternative with the highest AS_i is the best choice among the candidate alternatives.

4.4. The method of similarity measure

In this section, we introduce a method for the decision making problem by the proposed similarity measure between SVNSs. The concept of ideal point has been applied to help determine the best alternative in the decision process. Although the ideal alternative does not exist in practical problems, it does offer a useful theoretical construct against which to appraise alternatives. Therefore, we define the ideal alternative x^* as the SVNN $x_j^* = (T^*, I^*, F^*) = (1, 0, 0)$ for $\forall j$.

Hence, by applying Equation (3), the proposed similarity measure S^{Δ} between an alternative x_i and the ideal alternative x^* represented by the SVNSs is defined by

$$\begin{aligned} &S^{\Delta}(x_i, x^*)\\ &= \Big(\min\{L(x_i, x^*), M(x_i, x^*), R(x_i, x^*)\},\\ &\min\{L(x_i, x^*), W(x_i, x^*), R(x_i, x^*)\},\\ &1 - \max\{L(x_i, x^*), M(x_i, x^*), R(x_i, x^*)\}\Big), \quad (17)\end{aligned}$$

where

$$L(x_i, x^*) = \frac{\sum_{j=1}^{n} \varpi_j \min\{T_{\widetilde{F}}(e_j)(x_i), 1\}}{\sum_{j=1}^{n} \varpi_j \max\{T_{\widetilde{F}}(e_j)(x_i), 1\}} = \sum_{j=1}^{n} \varpi_j T_{\widetilde{F}}(e_j)(x_i),$$

$$M(x_i, x^*) = \frac{\sum_{j=1}^{n} \varpi_j \min\{1 - I_{\widetilde{F}}(e_j)(x_i), 1\}}{\sum_{j=1}^{n} \varpi_j \max\{1 - I_{\widetilde{F}}(e_j)(x_i), 1\}} = 1 - \sum_{j=1}^{n} \varpi_j I_{\widetilde{F}}(e_j)(x_i),$$

$$W(x_i, x^*) = 1 - \frac{\sum_{j=1}^{n} \varpi_j \min\{1 - I_{\widetilde{F}}(e_j)(x_i), 1\}}{\sum_{j=1}^{n} \varpi_j \max\{1 - I_{\widetilde{F}}(e_j)(x_i), 1\}} = \sum_{j=1}^{n} \varpi_j I_{\widetilde{F}}(e_j)(x_i),$$

$$R(x_i, x^*) = \frac{\sum_{j=1}^{n} \varpi_j \min\{1 - F_{\widetilde{F}}(e_j)(x_i), 1\}}{\sum_{j=1}^{n} \varpi_j \max\{1 - F_{\widetilde{F}}(e_j)(x_i), 1\}} = 1 - \sum_{j=1}^{n} \varpi_j F_{\widetilde{F}}(e_j)(x_i).$$

Algorithm 2: Similarity measure

Steps 1-3. Similarly to Steps 1-3 in Algorithm 1.

Step 4. Calculate the similarity measure $S(x_i, x^*)$ $(i = 1, 2, \cdots, m)$ by Equation (17).

Step 5. Compute the each alternative of score function $s(S(x_i, x^*))$ by Equation (2).

Step 6. Rank the alternatives by $s(S(x_i, x^*))\,(i = 1, 2, \cdots, m)$. The most desired alternative is the one with the biggest value of x_i.

4.5. The method of level soft set

We present an adjustable approach to single-valued neutrosophic soft set based decision making problems. This proposal is based on the following novel concept called level soft sets.

Definition 10. Let $\Gamma = (\widetilde{F}, E)$ be a SVNSS over U. Let $\lambda : E \to [0, 1]$ be a function, i.e. $\forall e_j \in E$, $\lambda(e_j) = (r(e_j), k(e_j), t(e_j))$, and $r(e_j), k(e_j), t(e_j) \in [0, 1]$. The level soft set of Γ with respect to λ is a crisp soft set $L(\Gamma; \lambda) =< \widetilde{F}_{\lambda}, E >$ defined by $\widetilde{F}_{\lambda}(e_j) = L(F(e_j); \lambda(e_j)) = \{x_i \in U | T_{\widetilde{F}}(e_j)(x_i) \geq r(e_j), I_{\widetilde{F}}(e_j)(x_i) \geq k(e_j), F_{\widetilde{F}}(e_j)(x_i) \leq t(e_j)\}$, for all $e_j \in E (j = 1, 2, \cdots, n)$, $x_i \in U (i = 1, 2, \cdots, m)$.

Remark 1. In the above definition, the function $\lambda : E \to [0, 1]^3$ is not restricted to be SVNS, it is only a function, $r(e_j) \in [0, 1]$ can be viewed as a given least threshold (the parameter e_j on the degree of truth-membership, $k(e_j) \in [0, 1]$ can be viewed as a given least threshold (the parameter e_j on the degree of indeterminacy-membership), and $t(e_j) \in [0, 1]$ can be viewed as a given greatest threshold (the parameter e_j on the degree of falsity-membership). Maybe some SVNNs may not strictly follow the Definition 10 ($T_{\widetilde{F}}(e_j)(x_i) \geq r(e_j)$, $I_{\widetilde{F}}(e_j)(x_i) \geq k(e_j)$, $F_{\widetilde{F}}(e_j)(x_i) \leq t(e_j)$). For solving this situation, we can continue to compute the two SVNNs by score function defined in Equation (2).

For convenience, we choose mid-level threshold function $\mathrm{mid}_{\Gamma} E \to [0, 1]^3$, i.e.,

$\mathrm{mid}_{\Gamma}(e_j) = (r_{mid_{\Gamma}}(e_j), k_{mid_{\Gamma}}(e_j), t_{mid_{\Gamma}}(e_j))$ for all $e_j \in E$, where

$$r_{mid_{\Gamma}}(e_j) = \frac{1}{m} \sum_{i=1}^{m} T_{\widetilde{F}}(e_j)(x_i),$$

$$k_{mid_{\Gamma}}(e_j) = \frac{1}{m} \sum_{i=1}^{m} I_{\widetilde{F}}(e_j)(x_i),$$

$$t_{mid_{\Gamma}}(e_j) = \frac{1}{m} \sum_{i=1}^{m} F_{\widetilde{F}}(e_j)(x_i).$$

The function mid_Γ is called the mid-threshold of $\Gamma = (\widetilde{F}, E)$, the level soft set w.r.t. mid_Γ, namely $L(\Gamma, \mathrm{mid}_\Gamma)$ is called the mid-level soft set of Γ.

Algorithm 3: Level soft set

Steps 1-3. Similarly to Steps 1-3 in Algorithm 1.

Step 4. Compute the mid-level soft set $L(\Gamma; \mathrm{mid}_\Gamma)$.

Step 5. Present the mid-level soft set $L(\Gamma; \mathrm{mid}_\Gamma)$ in tabular form and compute the weighted choice value c_i of x_i by Equation (18).

$$c_i = \sum_{j=1}^{n} \varpi_i L(e_j)(x_i). \tag{18}$$

Step 6. The optimal decision is to select x_k if $c_k = \max_{i=1}^{m} c_i$.

5. A numerical example

An internet company wants to select a software development project to invest. Assume that there are four software projects: e-commerce development project, game development project, browser development project and web development project. The company selects three parameters to evaluate the four software development projects. Let $U = \{x_1, x_2, x_3, x_4\}$ be the set of software projects, $E = \{e_1, e_2, e_3\}$ be the set of parameters, e_1 stands for economic feasibility, e_2 stands for technological feasibility, e_3 stands for staff feasibility. e_1 and e_2 are benefit parameters, e_3 is cost parameter. The weight vector of the parameters is given as $w = (0.4, 0.1, 0.5)^T$. The decision tabular given by expert is presented in Table 3.

In what follows, we utilize the algorithms proposed above to select software development projects under single-valued neutrosophic soft information.

Algorithm 1: EDAS

Step 1. Identify the alternatives and parameters, and obtain the single-valued neutrosophic soft set $(\widetilde{F}, E)$ which is shown in Table 3.

Table 3
Tabular representation of $(\widetilde{F}, E)$

	e_1	e_2	e_3
x_1	(0.5, 0.4, 0.7)	(0.7, 0.5, 0.1)	(0.6, 0.6, 0.3)
x_2	(0.6, 0.5, 0.6)	(0.6, 0.2, 0.2)	(0.5, 0.4, 0.4)
x_3	(0.7, 0.3, 0.5)	(0.7, 0.2, 0.1)	(0.7, 0.5, 0.4)
x_4	(0.6, 0.4, 0.5)	(0.7, 0.4, 0.2)	(0.5, 0.6, 0.4)

Table 4
The normalized single-valued neutrosophic soft set $(\widetilde{F}', E)$

	e_1	e_2	e_3
x_1	(0.5, 0.4, 0.7)	(0.7, 0.5, 0.1)	(0.3, 0.4, 0.6)
x_2	(0.6, 0.5, 0.6)	(0.6, 0.2, 0.2)	(0.4, 0.6, 0.5)
x_3	(0.7, 0.3, 0.5)	(0.7, 0.2, 0.1)	(0.4, 0.5, 0.7)
x_4	(0.6, 0.4, 0.5)	(0.7, 0.4, 0.2)	(0.4, 0.4, 0.5)

Step 2. Normalize the single-valued neutrosophic soft set $(\widetilde{F}, E)$ into $(\widetilde{F}', E)$ by Equation (7), which is shown in Table 4.

Step 3. Compute the combined weights by Equation (6) as follows:

$$\varpi_1 = 0.4078, \varpi_2 = 0.1017, \varpi_3 = 0.4905.$$

Step 4. Determine the average solution according to all parameters by Equation (9), shown as follows:

$$AV_1 = (0.6064, 0.3936, 0.5692),$$
$$AV_2 = (0.6776, 0.2991, 0.1414),$$
$$AV_3 = (0.5838, 0.5180, 0.3722).$$

Step 5. Calculate the positive distance from average $PDA = (P_{ij})_{4\times3}$ and the negative distance from average $NDA = (N_{ij})_{4\times3}$ matrixes by Equations (10) and (11), shown as follows:

$$PDA = (P_{ij})_{4\times3} \begin{pmatrix} 0.0000 & 0.0000 & 0.0038 \\ 0.0000 & 0.0000 & 0.0038 \\ 0.1560 & 0.0728 & 0.0628 \\ 0.0343 & 0.0000 & 0.0000 \end{pmatrix},$$

$$NDA = (N_{ij})_{4\times3} \begin{pmatrix} 0.1482 & 0.0613 & 0.0000 \\ 0.0873 & 0.0166 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0613 & 0.1143 \end{pmatrix}.$$

Step 6. Determine the weighted sum of PDA and NDA for all alternatives by Equations (12) and (13), respectively, shown as follows:

$$SP_1 = 0.0019, SP_2 = 0.0019,$$
$$SP_3 = 0.1019, SP_4 = 0.0140,$$
$$NP_1 = 0.0667, NP_2 = 0.0373,$$
$$NP_3 = 0.0000, NP_4 = 0.0623.$$

Step 7. Normalize the values of SP_i and SN_i for all alternatives by Equations (14) and (15), respectively, shown as follows:

$NSP_1 = 0.0183$, $NSP_2 = 0.0183$,

$NSP_3 = 1.0000$, $NSP_4 = 0.1375$,

$NSN_1 = -0.0702$, $NSN_2 = 0.4011$,

$NSN_3 = 1.0000$, $NSN_4 = 0.0000$.

Step 8. Calculate the appraisal score $AS_i\,(i = 1, 2, 3, 4)$ for all alternatives by Equation (16), shown as follows:

$AS_1 = -0.0260$, $AS_2 = 0.2097$,

$AS_3 = 1.0000$, $AS_4 = 0.0687$.

Step 9. Rank the software development projects x_i according to the decreasing values of AS_i as follows:

$$x_3 \succ x_2 \succ x_4 \succ x_1.$$

Obviously, amongst them x_3 is the best software development project.

Algorithm 2: Similarity measure

Steps 1-3. Similarly to Steps 1-3 in Algorithm 1.

Step 4. Calculate the similarity measure $S(x_i, x^*)$ $(i = 1, 2, 3, 4)$ by Equation (17), shown as follows:

$S(x_1, x^*) = (0.4223, 0.4102, 0.4101)$,

$S(x_2, x^*) = (0.4815, 0.5019, 0.4897)$,

$S(x_3, x^*) = (0.5529, 0.3879, 0.3879)$,

$S(x_4, x^*) = (0.4695, 0.4000, 0.4000)$.

Step 5. Compute the each alternative of score function $s(S(A_i, A^*))$ by Equation (2), shown as follows:

$s(S(x_1, x^*)) = 0.533993$,

$s(S(x_2, x^*)) = 0.496602$,

$s(S(x_3, x^*)) = 0.592335$,

$s(S(x_4, x^*)) = 0.556487$.

Step 6. Rank the software development projects by $s(S(x_i, x^*))(i = 1, 2, 3, 4)$ as follows:

$$x_3 \succ x_4 \succ x_1 \succ x_2.$$

Obviously, amongst them x_3 is the best software development project.

Algorithm 3: Level soft set

Steps 1-3. Similarly to Steps 1-3 in Algorithm 1.

Step 4. Compute the mid-level soft set $L(\Gamma; \mathrm{mid}_\Gamma)$ by Definition 10. $\mathrm{mid}_\Gamma = \{< e_1, (0.6, 0.4, 0.575) >, < e_2, (0.675, 0.325, 0.15) >, < e_3, (0.375, 0.475, 0.575) >\}$.

Table 5
Tabular representation of the level soft set $L(\Gamma; \mathrm{mid}_\Gamma)$ with weighted choice values

	e_1	e_2	e_3	weighted choice value c_i
x_1	0	1	0	0.1017
x_2	0	0	1	0.4905
x_3	1	1	1	1
x_4	1	0	1	0.8983

Step 5. Present the mid-level soft set $L(\Gamma; \mathrm{mid}_\Gamma)$ in tabular form and compute the weighted choice value c_i of x_i by Equation (18), which is shown in Table 5.

Step 6. The optimal decision is to select software development project x_3.

According to Algorithms 1, 2 and 3, we can conclude that the final decision results are the same, i.e., x_3 is the most desirable investment software development project. Hence, the three approaches proposed above are effective and available.

In the following, some comparisons of Algorithm 1, Algorithm 2 and Algorithm 3 are shown.

(1) Comparison of computational complexity

We know that Algorithm 1 will consume more computational complexity than Algorithm 2 and Algorithm 3, especially in Step 4. So if we take the computational complexity into consideration, the Algorithm 2 and Algorithm 3 are given priority to make decision.

(2) Comparison of discrimination

Comparing the results in Algorithm 1, Algorithm 2 with Algorithm 3, we can find that the results of Algorithm 2 are quite close and vary from 0.496602 to 0.592335. These result of decision values cannot clearly distinguish, in other words, the results obtained from Algorithm 2 are not very convincing (or at least not applicable). That is to say, the Algorithm 1 has a clearly distinguish. So if we take the discrimination into consideration, the Algorithm 1 and Algorithm 3 is given priority to make decision.

In order to further verify the practicability of the proposed SVNSS MADM approaches based on the EDAS, similarity measure and level soft set, a comparison study with some existing methods is now conducted in Table 6.

From the above results shown in Table 6, we can know that the optimal alternative of our proposed three methods is in agreement with the existing methods. That is to say, it is effective and feasible.

Table 6
A comparison study with some existing methods

Method	The final ranking	The optimal alternative
Deli and Broumi [44]	$x_3 \succ x_4 \succ x_1 \succ x_2$	x_3
Maji [53]	$x_3 \succ x_4 \succ x_1 \succ x_2$	x_3
Karaaslan [54]	$x_3 \succ x_4 \succ x_1 \succ x_2$	x_3
Deli and Broumi [55]	$x_3 \succ x_4 \succ x_1 \succ x_2$	x_3
Algorithm 1	$x_3 \succ x_2 \succ x_4 \succ x_1$	x_3
Algorithm 2	$x_3 \succ x_4 \succ x_1 \succ x_2$	x_3
Algorithm 3	$x_3 \succ x_4 \succ x_2 \succ x_1$	x_3

6. Conclusion and remarks

The major contributions in this paper can be summarized as follows:

(1) We construct a new axiomatic definition of single-valued neutrosophic similarity measure and give a similarity formula. Comparing with the existing literature [21, 22, 42, 43], it can reduce the information miss and remain more original information.
(2) A novel single-valued neutrosophic soft decision making approach based on EDAS is explored, which has not been reported in the existing literature. The approach doesn't need to calculate the ideal and the nadir solution.
(3) A novel single-valued neutrosophic soft decision making approach based on similarity measure is proposed, which can reduce the information loss and remain more original information.
(4) A novel single-valued neutrosophic soft decision making approach based on level soft set is proposed.
(5) The subjective weighting methods pay much attention to the preference information of the decision maker [19–21, 27], while they neglect the objective information. The objective weighting methods do not take into account the preference of the decision maker, in particular, these methods fail to take into account the risk attitude of the decision maker [22, 24]. The characteristic of our weighting model can reflect both the subjective considerations of the decision maker and the objective information.

In the future, we shall apply more advanced theories [52] into single-valued neutrosophic soft set and solve more decision making problems.

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