The Nuclear Force and Limitations to the Lorentz Electrostatic Force Equation

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Abstract

If we need to find the total electric field vector from multiple charges at a point in space we add the vectors to get the resultant vector. When we do this for two electrons near to each other we find that the field vectors oppose in a region between the electrons. This tells us that there is partial field cancellation there. Field cancellation reduces the field energy density leading to attractive forces in this region. With two electrons the net effect of all of space is one of increased energy density and hence net repulsion, so the attractive forces in the region between the electrons is overcome by greater repulsive forces elsewhere. However, with the bounded electric fields of neutrons this attractive region dominates over a limited region leading to first attractive then repulsive forces as the neutrons approach closer and closer to each other. This suggests in turn that the Lorentz Force Equation is limited to particles whose electric fields are not truncated but extend to infinity.

Calculating the potential energy between two electrons from first principles

When there are two sources of electric field such as two electrons, the resultant electric field vector at a point must be calculated by the vector addition of the field from each source at that point:-

$$\mathbf{E}_{\text{resultant}} = \mathbf{E}_1 + \mathbf{E}_2$$

Consider a pair of distributed electric charges q_1 and q_2 whose electric fields extend to infinity, as happens with electrons. Separate them by a distance 'r' and choose the co-ordinates so that one charge is at [-r/2,0,0] and the other at [r/2,0,0] as shown in Figure 1.

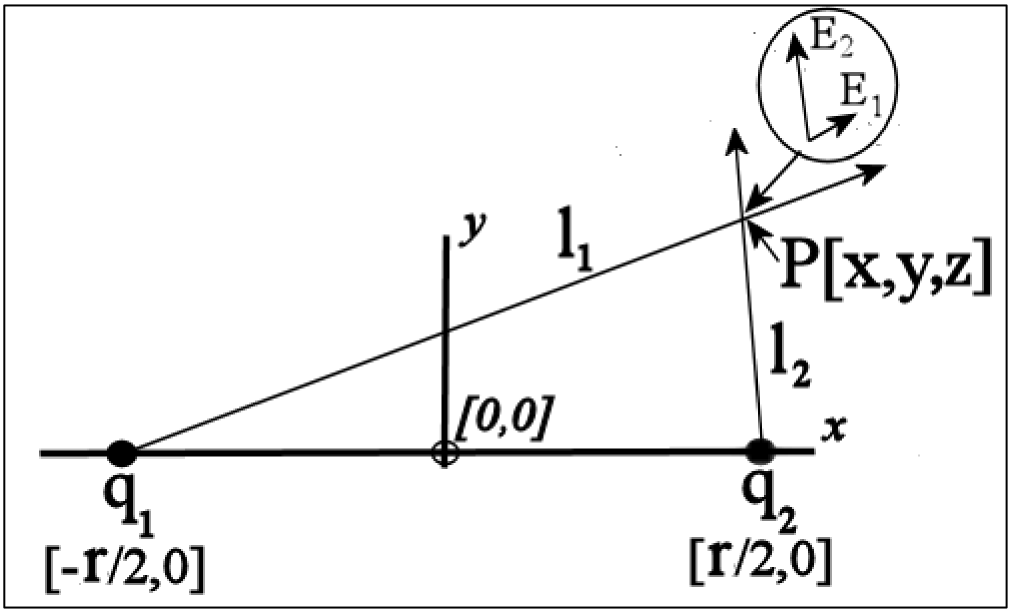

Figure 1

The x-axis is left to right on the sheet. The y-axis is bottom to top, and the z-axis points out of the page. The total energy density at point P is given by:-

$$\frac{\mathrm{dW}}{\mathrm{ds}} = \frac{\varepsilon(|\mathbf{E}_{\mathbf{resultant}}|^2)}{2}$$

$$= \frac{\varepsilon(|\mathbf{E}_1 + \mathbf{E}_2|^2)}{2}$$

$$= \frac{\varepsilon(\mathbf{E}_1 \bullet \mathbf{E}_1 + 2\mathbf{E}_1 \bullet \mathbf{E}_2 + \mathbf{E}_2 \bullet \mathbf{E}_2)}{2}$$

$$= \frac{\varepsilon\mathbf{E}_1 \bullet \mathbf{E}_1}{2} + \boldsymbol{\varepsilon}(\mathbf{E}_1 \bullet \mathbf{E}_2) + \frac{\varepsilon\mathbf{E}_2 \bullet \mathbf{E}_2}{2}$$

Now the first term is the energy density that the first electric field vector would have at that point if the first charge was alone in the universe. The third term is the energy density the second electric field vector would have at that point if the second charge was alone in the universe. The second term is the change in energy density caused by bringing the two charges together so they interact. When this ‘change’ term is integrated over all of space we get the potential energy difference between the two charges having infinite separation, to their current separation. Hence this “potential energy density” at an arbitrary point P[x,y,z] for a charge separation **r** is...

$$\frac{\mathrm{dU}}{\mathrm{ds}} = \varepsilon(\mathbf{E}_1 \bullet \mathbf{E}_2)$$

Here ‘U’ is the total potential energy, ‘dU/ds’ is the potential energy density at a point, and $\mathbf{E}_1 \bullet \mathbf{E}_2$ is the vector dot product. To find the total potential energy U at a separation **r**, integrate dU/ds over all space.

$$\mathbf{E}_1 = \frac{q_1}{4\pi\varepsilon\mathbf{l}_1^2} = \frac{q_1}{4\pi\varepsilon\left(\left(\mathbf{x} + \frac{r}{2}\right)^2 + \mathbf{y}^2 + \mathbf{z}^2\right)}$$

$$\mathbf{E}_2 = \frac{q_2}{4\pi\varepsilon\mathbf{l}_2^2} = \frac{q_2}{4\pi\varepsilon\left(\left(\mathbf{x} - \frac{r}{2}\right)^2 + \mathbf{y}^2 + \mathbf{z}^2\right)}$$

$$\mathrm{U} = \varepsilon \iiint_{-\infty}^{+\infty} (\mathbf{E}_1 \bullet \mathbf{E}_2)\, \mathrm{dx\, dy\, dz}$$

$$= \frac{q_1 q_2}{16\pi^2\varepsilon} \iiint_{-\infty}^{+\infty} \left(\frac{\left(\left(x + \frac{\mathbf{r}}{2}\right)\left(x - \frac{\mathbf{r}}{2}\right) + y^2 + z^2\right)}{\left(\left(\left(x + \frac{\mathbf{r}}{2}\right)^2 + y^2 + z^2\right)\left(\left(x - \frac{\mathbf{r}}{2}\right)^2 + y^2 + z^2\right)\right)^{3/2}} \right) \mathrm{dz\, dy\, dx}$$

Equation 1

This integration is difficult. There are discontinuities at the centre of each charge and we know that the equation does not apply there or the rest mass of the electron would be infinite (have a search for “The Classical Radius of the Electron” for the argument) so the centre of both electrons cannot be part of the integration; however the electron field goes on to infinity outside the Classical Radius so there are no other limits. It is more straightforward to use Finite Element Analysis which gives us a solution for potential energy numerically equal to...

$$\mathrm{U} = \frac{q_1 q_2}{4\pi\varepsilon\mathbf{r}}$$

... for separations significantly greater than the Classical Radius. Now since Force is equal to the rate of change of energy with distance (i.e. the separation ‘**L**’), to find the force between two charges at a separation ‘L’ we have:-

$$\mathbf{F} = \frac{dU}{d\mathbf{r}} = -\frac{q_1 q_2}{4\pi\varepsilon\mathbf{r}^2}$$

The minus sign indicates that the force opposes any increase in energy as the two charges are brought together. The field from charge q_2 at a distance **r** is...

$$\mathbf{E}_2 = \frac{q_2}{4\pi\varepsilon\mathbf{r}^2}$$

Finally if we assign $\mathbf{E}_2$ to the background field from charge q_2, and q_1 to the 'test point charge' placed at a separation **r** from the centre of q_2, we derive the Lorentz equation...

$$\mathbf{F} = -q_1\mathbf{E}_2$$

As can be seen, it does not mean that one charge has miraculously become a point charge while the other remains a distributed electric field! It is just happenchance that allows this arbitrary assignment. We could of course have used the Lorentz Force Equation to get that result much more simply, but here we are developing an approach that will equally apply to the neutron and as will be seen, the Lorentz Force Equation cannot be applied to the neutron.

Note that the minus sign that appears in this equation is because the force opposes higher-energy configurations so causes like charges to be forced apart. This minus sign is often by convention dropped in the Lorentz Force Equation and a positive charge is then taken to be impelled in line with the field vector, which gives the same result – like charges are forced apart.

The Structure of the Electrostatic Interaction between two Electrons

Let us look in more detail at the energy interaction between two electrons. Consider the numerator from Equation 1:-

$$\left(\left(x + \frac{\mathbf{r}}{2}\right)\left(x - \frac{\mathbf{r}}{2}\right) + y^2 + z^2\right)$$

This expands to:-

$$\left(x^2 + y^2 + z^2 - \frac{r^2}{4}\right)$$

Inspection shows that the numerator is zero on the circumference of a sphere whose diameter is the line joining the two charges. The sign of the function changes as we move from the inside of the sphere to the outside. This implies that between two electrons there is a region inside this sphere where there are attractive forces, even though these forces are overcome by the stronger repulsive forces outside the sphere to create a net repulsion. Likewise between two opposite charges there is a region of repulsion inside this sphere even though this repulsion is overcome by the stronger attractive forces outside the sphere. For two like charges we may term this the "sphere of attraction".

To get an idea of the structure of attraction and repulsion in the interaction between two like charges, consider Figure 2, which shows how electric field lines from each charge are orthogonal to each other on the sphere whose diameter joins the centres of the two charges.

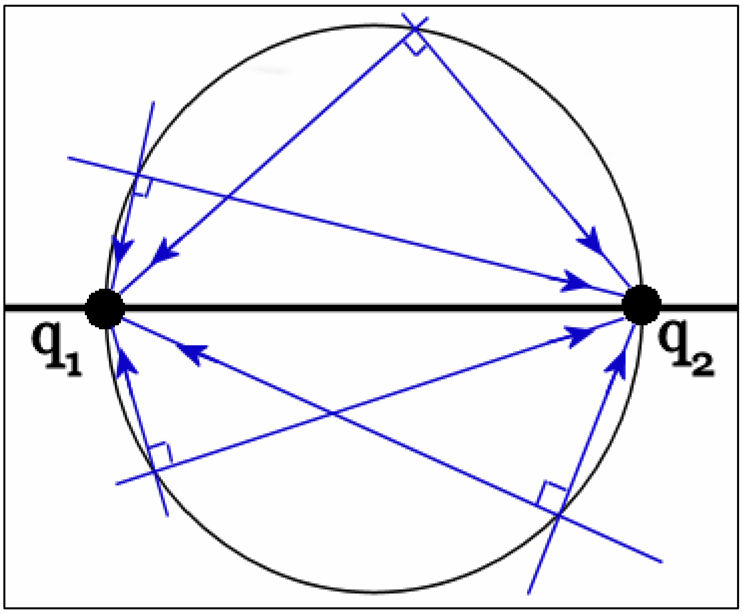

Figure 2

Being orthogonal to each other, there is no interaction on the surface of this sphere between the charges. Inside the circle, however, the field lines intersect at between 90 to 270 degrees, so the fields oppose each other and hence tend to cancel each other, even if only partially. This reduces the energy density inside the sphere, leading to attractive forces in this region. Outside the sphere, however, the field vector intersection is between -90 to +90 degrees and the fields augment each other so that the energy density increases and repulsive forces are generated. The net effect is one of repulsion between electrons, as the sum total of the attractive forces inside the sphere is a little over one fifth of the repulsive forces outside the sphere. This applies at all separations as the electron's fields extend to infinity.

Figure 3 shows the classical field pattern which results from this field cancellation within the sphere of attraction...

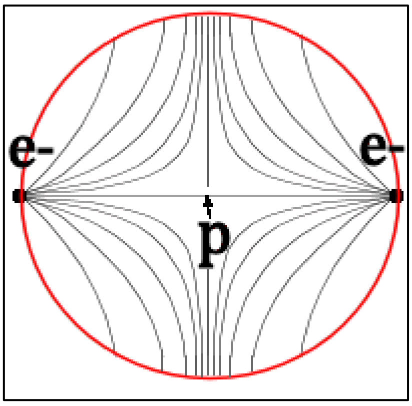

Figure 3

This pattern looks rather like a repulsion pattern between the two electrons but nothing could be farther from the truth. At point ‘p’ in Figure 3, midway between the electrons, the fields from the electrons are equal and opposite, so the energy density is zero. Field regions vertically up and down from this point have their horizontal component cancelled leaving only their vertical component; the central vertical flux lines runs from the periphery, dropping in strength all the way to point ‘p’ where they meet and are zero. At point ‘p’ two new horizontal flux lines start and run each way, one to the centre of each electron, gaining in strength all the way. Other flux lines adjacent to these are continuous, running from the periphery to the centre of one or other electron, but the nearer they are to point ‘p’ the greater the cancellation and the weaker the field strength.

It can be seen that the distributed forces between two charges make a rather beautiful composite field pattern in space. More complex arrangements of charges create an art form of their own. No matter how complex the charge distribution, providing all the charges have fields that extend to infinity the gross forces are perfectly described by the Lorentz Force Equation and its “point charge in a distributed field” approach works well.

However the neutron’s field does *not* extend to infinity? What are the consequences?

The Electrostatic Forces between two Neutrons

Now consider what happens if the charges do not extend to infinity but are suddenly truncated at some finite radius. Equation 1 above is equally valid, but because the limits of the interaction are truncated at a finite radius the limits of the integration changes and that result is no longer the Lorentz one. The neutron is such a particle, known to have a positive electric field truncated at a radius of about 10^{-15} . Consider Figure 4, showing the interaction between two neutrons at three different separations. At each separation the black circles show the limits of the electric field of the two neutrons and the red circle indicates the region of the potential sphere of attraction. Assume that the field strength of the neutron follows the classic $1/r^2$ profile inside its charge radius.

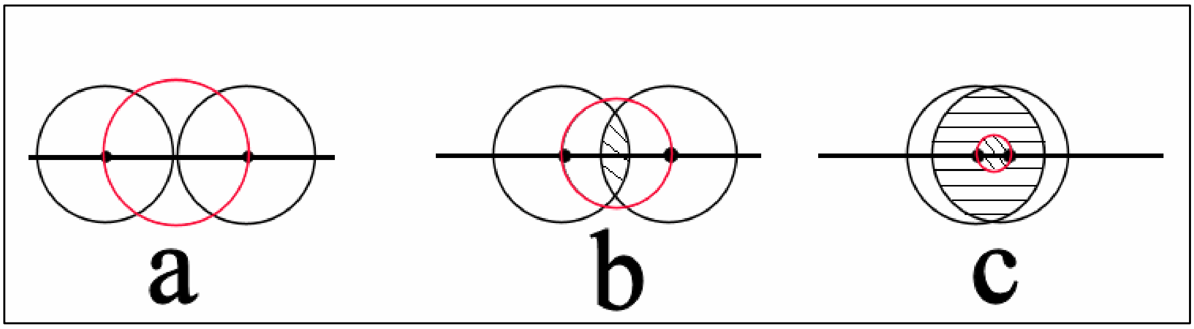

Figure 4

In Figure 4a the neutron fields do not overlap, so at all separations greater than twice the truncated charge radius there is no interaction. In Figure 4b there is a partial overlap of the neutrons' fields, and so there is an interaction; however, this interaction is entirely contained within the "sphere of attraction" region and so at this separation the forces are entirely attractive. In Figure 4c the "sphere of attraction" region is small compared with the total overlap, and bearing in mind that most of the forces are generated inside a radius of about five times the separation it can be seen that the region of repulsion dominates and is tending to the classic $1/r^2$ force profile, where 'r' is separation between the centres of the neutrons. The awkward limits make direct mathematical integration impractical but Finite Element Analysis gives the force profile shown in Figure 5 (here 't' represents the radius of the neutron's truncated field limit). To derive the force we calculate the total energy over all space at one separation then change the separation very slightly and recalculate, to give us the change in energy over the change in separation. This Figure compares the force/separation curve of two neutrons of radius 'r' with the $1/r^2$ curve that would apply if the neutron's electric field extended to infinite radius as it does with the electron.

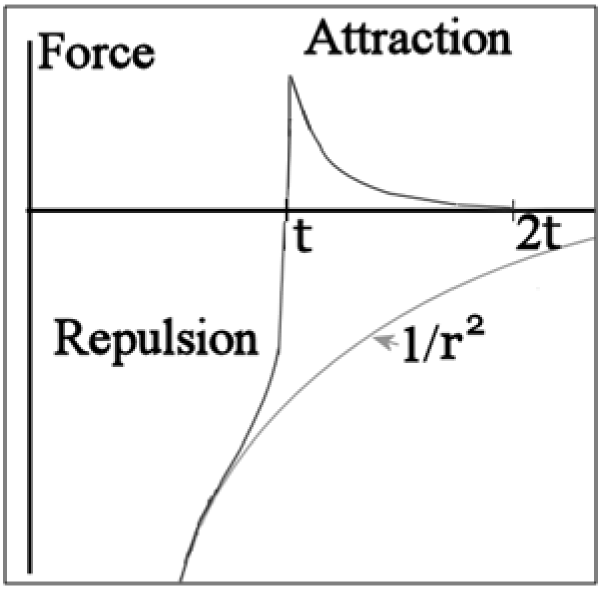

Figure 5

There is no interaction at a separation of more than twice the neutron's charge radius 't' as the fields do not overlap. Below this, as the neutrons come together, the attractive force rapidly climbs to a peak at a separation of 1.02 neutron radii, then collapses very suddenly, dropping through zero into repulsive forces at a separation of 1.0 neutron radii and continuing falling to converge with the $1/r^2$ curve. Two neutrons would therefore come to rest at a centre-to-centre separation of one neutron radius.

Measurements of how the neutron-neutron force changes with separation generally show a softer curve than that shown in Figure 5, having a soft peak to right of the theoretical peak. There are two possibilities for this difference. The first possibility is that nuclear kinetics limits the accuracy of measurement, softening the curve as if viewed through a low-pass filter and essentially smoothing out the sharp peak and instead showing just the broad outline of the curve. The second possibility is that the softer curve is precise which would mean that the outer edge of the neutron's electric field does not abruptly drop to zero at the limiting radius but fades to zero over a small shift in that radius (our calculation in Figure 5 assumed a hard sudden truncation of the electric field).

The Force/Separation Profile of the Force holding the nucleus together

Existing studies of the force between two neutrons agree with the right hand side of the plot in Figure 5 but details of the force as it drops below zero do not seem to exist, especially in the region of convergence to a $1/r^2$ curve at separations significantly less than the charge radius 't'. This information would be a real test of this approach. However with the information we already have there seems to be a good match to the attraction/repulsion pattern between neutrons.

It also confirms the inapplicability of the Lorentz Force Equation in this situation. For example in Figure 4b Lorentz predicts that there is no force because the ***centre*** of one neutron is outside the distributed field of the other so is in a region of zero external field strength. Yet there is clearly an interaction from the energy density integration. Hence the Lorentz Force Equation disagrees with the Principle of Conservation of Energy, reporting no forces where the energy density integration clearly predicts forces in Figure 4b. Clearly the Lorentz Force Equation works only for those particles whose electric fields extend to infinity.

The field strengths of neutrons, and therefore the forces involved in neutron-neutron interactions, are orders stronger than the forces between electrons.

A Model for the Proton

So how is the proton held in the nucleus of the atom? With the electron we know that the electric field continues inwards to a fixed radius at which it is truncated, because if it continued to zero radius the rest mass/energy of the electron would be infinite. So we could perhaps model the proton electromagnetically as a sort of super-electron where the inner radius of the field continues to a smaller radius than that of the electron. However, it is impossible to make such an object hold position inside the atomic nucleus – it would be expelled immediately – and such a model will not work.

However, a model that does work well is where the proton has a neutron-like core with a high neutron-like positive field strength, surrounded by an positron-like positive field that extends from the inner core's outer periphery out to infinity. Consider Figure 6, which shows just this layout.

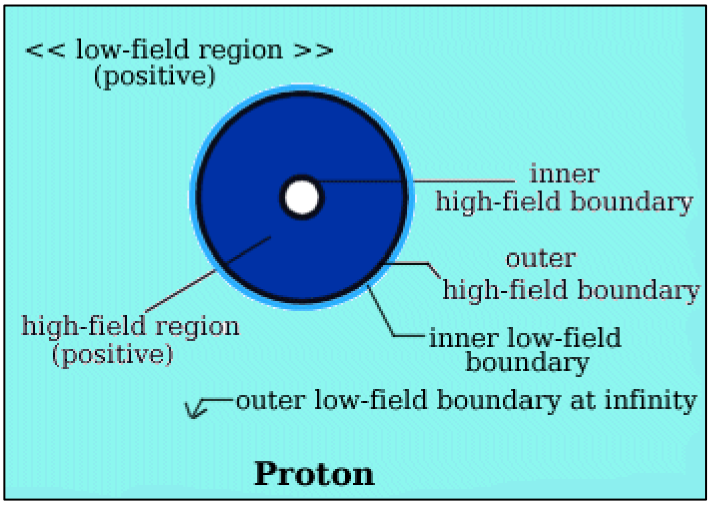

Figure 6

The interaction between a proton and a neutron would be similar to that between two neutrons at close range, but where the neutron and the proton core do not overlap and hence do not interact, but are still within about four times the core radii, Finite Element Analysis (using energy density integration techniques) predicts that there will be a weak interaction between the neutron's field and the proton's outer field that falls off extremely rapidly with increasing separation. This weak interaction has been recorded, confirming the viability of this electromagnetic model of the proton. The reason is demonstrated graphically in Figure 7. Figure 7a shows a proton whose centre is shown at point 'p' and a neutron whose centre is shown at point 'n'. The proton's outer field extends to infinity, but the neutron's field extends only to a finite radius; the inner core of the proton is not delineated but is assumed to lie entirely outside the neutron field and thus takes no part in the interaction. The interaction between the proton's outer field and the neutron occurs over the whole field of the neutron but nowhere else. Some field lines from the proton that intersect with the neutron's field are shown. The separation in Figure 7a is about two neutron radii. Figure 7b is similar except that the separation is much greater. The arc drawn across the neutron field represents the boundary of the sphere of attraction.

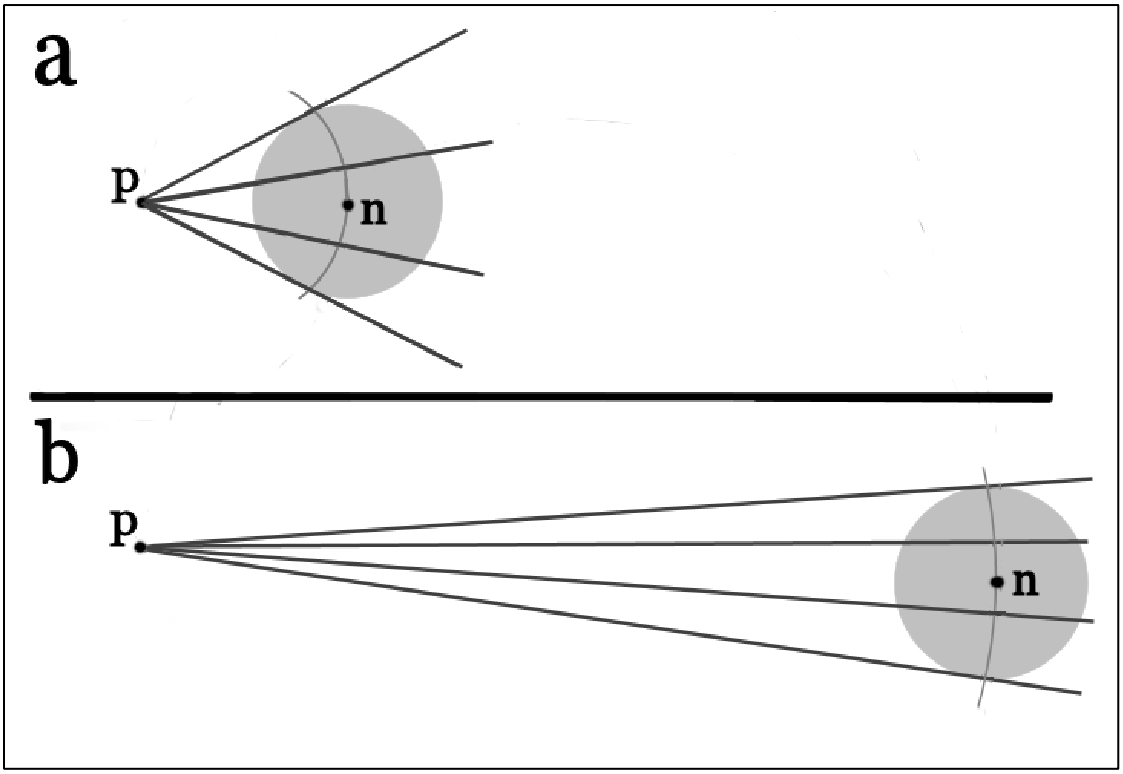

Figure 7

In Figure 7a the field gradient in the region of the proton's outer field that intersects the neutron field is very high, falling off rapidly from left to right in magnitude as the inverse of the square of the separation of the centres of the proton and the neutron. The region inside the sphere of attraction thus dominates, leading to a net attractive force.

In Figure 7b the field strength of the proton's outer field is much lower and the gradient is also much lower, giving almost the same proton field strength on the right of the neutron as on the left. The former makes the force between the particles fall off as the inverse of the square of the separation; the latter causes a second fall-off again as the inverse square. The latter effect is reduced by the fact that the volume of the neutron field is more equally shared between repulsion and attraction in Figure 7b, but the combined effect is still a force that falls of at very roughly the inverse cube of the separation.

So how does the proton structure of a high inner field surrounded by a low outer field hold together? Figure 8 shows the proton inner field (in red) displaced so that its inner field overlaps its outer field (in black).

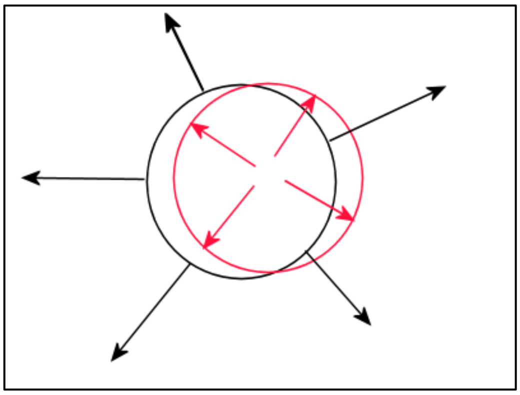

Figure 8

It can be seen that on the left side of Figure 8 the core has moved away from the outer field but there is no change in any energy density there as there never was any overlap between them anyway. On the right, however, the core field now overlaps with the outer field. Where it does so, the fields reinforce each other as they are both positive and this increases the total energy density in the overlap region, giving rise to repulsive forces that tend to restore the inner field to its original position with respect to the outer field. This means that the electromagnetic model indicates that this arrangement is stable. If, however, the core and outer fields had been of different sign any overlap would reduce the energy density, creating attractive forces that would create instability.

Hence a compound model of the proton works well and agrees with the qualitative force/distance interaction between protons and neutrons in an atomic nucleus. Determining the exact values for inner field strength, and the core diameter, needs us to look elsewhere. However, the qualitative behaviour is a good match to measurement. This match requires the use of the energy density integral in analysis rather than the Lorentz Force Equation which gives results known to be at odds with measurement. This demonstrates that the Lorentz Force Equation violates the Principle of Conservation of Energy in this instance.