《The Color Dimension Theorem》
《色維定理》

English below

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（摘要）

《色維定理》

《色維定理》分為兩個部分：第一部份給出一個關於四色定理的構造性證明，並指出「一維-二色、二維-四色」此關係的存在絕非偶然；第二部分探討空間的維數上限—方法是：假設N維以上的空間存在，則必存在一種幾何體，考慮此幾何體在平面上的投影，會發現此投影圖形產生一個悖論，這意味著N維以上的空間不存在

（內文）

《四色定理的證明》

壹、定義：

①連接線

⑴代表兩區塊有鄰接關係，故鄰接線彼此間必不相交。

⑵為一動點由一定點移動至另一定點之軌跡線。

②基準線

⑴由正交座標平面上，所有的Y軸的零點所構成的曲線或直線。這些Y軸皆上下移動了#距離，其中#可為0。

⑵用於定義下文所稱之Y+與Y-象限：基準線下方為Y-、上方為Y+。

③平行

⑴在某線上的每一點，到另一線的距離皆相等，則此兩線具有平行關係。

④方向的區域

⑴η方向的區域係指零點到η∞之間的區域，其中η為+或-

⑤方向的區域的正負

⑴方向的區域的正負，取決于基準線，基準線為平面上的兩個代表區塊的點的連線，不一定是直的

⑥投影組態

⑴一線段在ρρ座標軸的σσ方向的區域上有投影，則稱其投影組態為ρσρσ—其中ρ為X或Y、σ為+、-或空，若為空則代表不計正負方向

⑦「連接」：若線段C在X(或Y)座標軸上有投影，則稱C在X(或Y)方向上有「連接」

⑧「阻擋」：若平行Y(或X)座標軸的線與線段C有交點，則稱C在Y(或X)方向上有「阻擋」

⑨「鄰阻原理」是一個不證自明的公設(前設)。在本文的證明中，我們僅需用到二維版本的「鄰阻原理」(以下簡稱「2-鄰阻原理」)

「2-鄰阻原理」內容：正交座標平面上，線段C在X方向上的連結④，會造成在線段C的Y方向上的阻擋⑤，反之亦然(X與Y相互調換)

貳、證明

1、四色定理的要求是「任兩區塊相鄰，則這兩區塊的顏色必須不同」

2、為了便於理解，以點代表區塊，點與點之間的連接線代區塊與區塊的鄰接。根據四色定理的要求，該連接線不應該相交

3、因此，我們可以將四色問題等價地改寫成「平面上有N個點，每個點都以一線段和其它N-1個點連結，且線段與其它線段皆不相交，則N最大為多少？」

(若兩點之間沒有連接線，則代表這兩點可以有同一顏色。為了避免某一地圖出現顏色不夠用的情形，所以應使N盡可能地大。)

4、現在的目標在於證明N=4。

5、考慮平面空間上的四個點A、B、C、D：一動點α由A移動至B並留下一軌跡線—連接線，令此線為基準線。現在，A、B已有互相連接的關係。

6、現在，考慮點A、B、C彼此互相連接—一動點β由A移動至C再移動至B，並留下軌跡線—為了避免重合(交點)，該動點β必須繞過基準線佔據的空間，也就是說：該動點β面對兩種選擇—在移動至B之前，先進入Y+或Y-象限與C點相交。

(註：C點必在Y+或Y-象限內，這是來自於基準線的定義的推論。)

7、上述過程將使動點β相對於基準線，向上(或向下)遠離基準線，然後再向下(或向上)靠近基準線—「相對」的意思是：若基準線是一條連接A與B、在正交座標平面的的下凹(或上凸)曲線，則即使β的軌跡線是直的，β在移動的過程中依然是先相對性的向上(或向下)遠離基準線，然後再相對性的向下(或向上)靠近基準線。

(註：這裡所謂的向上(向下)是指往有Y+或Y-方向分量的方向移動。)

8、在遠離與靠近的過程中，β產生的軌跡線將阻擋來自該軌跡線的X+方向與X-方向的動點(「阻擋」的意思請參照「定義」)(X+方向與X-方向是平行於基準線的。)，亦即β的軌跡線在Y軸上有投影。由於β跟α都具有同樣的起點與終點，故可知β在α上具有投影，這意味著β產生的軌跡線將會阻擋來自該軌跡線的Y+方向與Y-方向的動點。總而言之，α與β產生的軌跡線，將形成一封閉區域θ。

9、接下來考察點D：顯然，D必位於θ內或θ外。若D在θ外，且一動點γ由A、B、C中的任一點為起點並移動至點D，接著移動至其它兩點中的其中一個，然後沿著它的軌跡線返回D，再移動至最後一個沒抵達過的點，則基於和「8、」相同的理由，將形成一個封閉區域φ，且A、B、C三點中的一點，將落於φ內；若D在θ內，就沒什麼好討論的了。總而言之：無論是哪一種情形，四個點彼此相連，必導致其中一點落於封閉區域內。

10、結果昭然若揭：上述中的、被封閉的點無法與第五點E有連接線而不使連結線們彼此相交，這意味著代表第五個區塊的點E，沒有和該被封閉的點(區塊)相鄰，這意味著第五區塊可以有和被封閉的區塊有相同顏色。因此，只需要四種顏色—四色定理，得證。

參、後記

①

Q：為何會出現「一維—二色」「二維—四色」這種對偶關係？

A：

❶定義

⑴α、β、γ：其為三條在彼此之間與X軸上皆有投影的線段

⑵Ω：一個只能在Y方向上移動的動點

❷證明

⑴若Ω由β移動至α不經過γ、由β移動γ必不經過α，且由α移動至γ，必經過β，則代表Ω由Y+∞或Y-∞遠處移動至β，必經過α或γ—這是由於一個維度只有兩個可移動的方向(+、-)，而α跟γ由於有在彼此與X軸上的投影，故可知α阻擋來Y+∞(或Y-∞)的Ω，γ阻擋來自Y-∞(或Y+∞)的動點，使得Ω從α的Y+(或Y-∞)的方向的區域，與從γ的方向Y-∞(或Y+∞)的區域，移動至β皆與α或γ其中一個有交會處。

⑵將以上敘述中的X和Y互換，敘述依舊成立，因此，一平面上一點透過N條連接線來和其他N個點連接，則N必須等於3，否者將出現連接線彼此相的情況—其它三個點跟自身一個點，總共四個點，四種相異顏色。

《關於N維度以上的空間不存在的證明》

壹、證明

1、一個4-超立方體的每個頂點，連接四個邊

2、從與其中一個頂點相鄰的四個直邊，任意選取兩個邊，皆能組成一個平面

3、因此，這些平面，皆與前述的頂點相連

4、C(4，2)=6，因此，任一4-超立方體的每個頂點，皆與6個平面相連

5、現在，在這些平面上劃一條線(直線或曲線)，每條線皆連接這些邊中的任意兩個邊

6、接著，準備一個獨立於該4-超立方體的平面，它與前述的頂點相接觸，且在整個4-超立方體上，只有該點，與該平面接觸

7、將該點、該線、該平面與劃在平面上的那些線，投影到該獨立平面

8、則該獨立平面上，會產生一個圖形：一個點與四條直線相鄰，且任意兩條直線(與那個點相鄰的)之間，皆有一條線(直線或曲線)將它們連結

9、問題來了：這些用於連結的線，無疑至少兩條會相交①，而相交意味著前述的那些面，彼此之間的夾角並非九十度②。這與4-超立方體的定義相悖

10、若以上推理的前提(公設、前設)與推理的本身沒有瑕疵，則超空間的存在會導致矛盾

貳、後記

①這可以用於證明四色定理，方法是把區塊當成點，區塊間的邊界當成連結點與點的線，作圖之後便會明白了。反之，由於四色定理已經由其它方式證明，因此，四色定理是它的佐證

②可以用正方體作類比—由於三維不會產生矛盾，因此，我們生活在三維空間中

《The Color Dimension Theorem》

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（Summary）

《The Color Dimension Theorem》

The Color Dimension Theorem is divided into two parts: The first part gives a constructive proof of the four-color theorem, and points out that the existence of the "one-dimensional-two-color, two-dimensional-four-color" relationship is not accidental; The second part discusses the upper limit of the dimensionality of the space. The method is: suppose the space above the N dimension exists, there must be a geometry. Considering the projection of this geometry on the plane, we will find that this projection image produces a paradox, which means N The space above the dimension does not exist

（Text）

《The Proof of Four Color Theorem》

壹, definition:

1 connection line

(1) Represents two blocks with adjacency, so adjacent lines must not intersect with each other.

(2) A trajectory where a moving point moves from a certain point to another fixed point.

2 baseline

(1) A curve or straight line formed by the zero point of all Y-axes on the plane of orthogonal coordinates. Both of these Y axes move up and down by #, where # can be 0.

(2) Used to define the Y+ and Y-Quadrants referred to below: Y- and Y+ above the baseline.

3 parallel

(1) At each point on a line, the distance to the other line is equal, then the two lines have a parallel relationship.

4 direction area

(1) The region in the η direction refers to the region between zero and η∞, where η is + or -

Positive and Negative of 5 Directional Area

(1) The positive and negative of the direction of the area depends on the reference line. The reference line is the line connecting the points of the two representative blocks on the plane. It does not have to be straight.

6 Projection configuration

(1) A line segment has a projection on the σσ direction region of the ρρ coordinate axis, then the projection configuration is called ρσρσ—where ρ is X or Y, σ is +, -, or null, and if it is empty, it means that the positive and negative directions are not counted.

7 "Connection": If segment C has a projection on the X (or Y) axis, C is said to have a "connection" in the X (or Y) direction

8 “Blocking”: If the line parallel to the Y (or X) coordinate axis intersects line segment C, then C is said to “block” in the Y (or X) direction.

9 "Principle of Adjacency" is a self-explanatory public design (preceding). In the proof of this paper, we only need to use the two-dimensional version of the "adjacent impedance principle" (hereinafter referred to as the "2-negative resistance principle").

"2-Right Adjacent Principle" content: In the plane of orthogonal coordinates, the connection 4 of the line segment C in the X direction causes blockage 5 in the Y direction of the line segment C, and vice versa (X and Y are exchanged with each other)

贰, proof

1. The requirement of the four-color theorem is "if any two blocks are adjacent, the colors of these two blocks must be different."

2. For ease of understanding, points are represented by blocks, and the connection lines between points are adjacent to blocks. According to the requirements of the four-color theorem, the link should not intersect

3. Therefore, we can rewrite the four-color problem equivalently as “There are N points in the plane. Each point is connected with one line segment and other N-1 points, and the line segment does not intersect with other line segments. Then, N What is the maximum?"

(If there is no connecting line between the two points, it means that these two points can have the same color. In order to avoid the situation that a color map is not enough, you should make N as large as possible.)

4. The goal now is to prove that N=4.

5. Consider the four points A, B, C, D in the plane space: a moving point α moves from A to B and leaves a trajectory-connection line, making this line the reference line. Now, A, B have been connected to each other.

6. Now, consider points A, B, and C to be connected to each other—a moving point β moves from A to C and then to B, and leaves a trajectory—to avoid overlap (intersection), the moving point β must bypass the baseline The occupied space, that is to say: the moving point β faces two choices - before moving to B, the Y+ or Y-quadrant intersects with point C first.

(Note: Point C must be within the Y+ or Y-quadrant, which is a corollary from the definition of the baseline.)

7. The above process will move the moving point β away from the baseline (or down) from the baseline and then down (or up) near the baseline—“relative” means: if the baseline is a connection A With B, the concave (or convex) curve in the orthogonal coordinate plane, even if the trajectory line of β is straight, β is still relatively upward (or downward) away from the

reference line during movement. , and then the relative downward (or upward) close to the baseline.

(Note: The so-called upward (downward) here refers to moving in the direction of Y+ or Y-direction components.)

8. In the process of being away from and approaching, the trajectory line generated by β will block the X+ direction and the X-direction moving point from the trajectory (for the meaning of “blocking”, please refer to “Definitions”) (X+ direction and X-direction It is parallel to the baseline.) That is, the trajectory of β is projected on the Y-axis. Since both β and α have the same starting and ending points, it can be seen that β has a projection on α, which means that the trajectory generated by β will block the moving points in the Y+ direction and the Y-direction from the trajectory line. In summary, the trajectory lines generated by α and β will form a closed area θ.

9. Next, check point D: Obviously, D must be within θ or θ. If D is outside θ, and a moving point γ starts from any point in A, B, C and moves to point D, then moves to one of the other two points, then returns to D along its trajectory, and then Moving to the last point that has not arrived, based on the same reason as "8," a closed area φ will be formed, and one of the three points A, B, and C will fall within φ; if D is at θ Inside, there is nothing to discuss. In short: In either case, the four points are connected to each other, and one of them must fall within the enclosed area.

10. The results are clear: The above-mentioned closed points cannot be connected with the fifth point E without making the connecting lines intersect with each other. This means that the point E representing the fifth block is missing and The closed points (blocks) are adjacent, which means that the fifth block can have the same color as the blocked block. Therefore, only four colors are needed—the four-color theorem.

Reference and Postscript

1

Q: Why does the "one-dimensional-two-color" and "two-dimensional-four-color" duality appear?

A:

Definition

(1) α, β, γ: It is a line segment that has projections between each other and the X-axis.

(2) Ω: A moving point that can only move in the Y direction

❷proof

(1) If Ω moves from β to α without passing γ, and from β moving to γ, it does not pass through α, and from α to γ, which must pass through β, it represents that Ω moves from Y+∞ or Y-∞ to β remotely. α or γ - This is because there are only two movable directions (+, -) in a dimension, and α and γ are projected on each other and the X axis, so we know that α blocks Y+∞ (or Y-∞) The Ω, γ block from the moving point of Y-∞ (or Y+∞) such that Ω is from the area of the direction of α+Y (or Y-∞), and the area from the direction of γY-∞ (or Y+∞) Moving to β is one of the intersections with either α or γ.

(2) Interchange X and Y in the above description. The statement is still established. Therefore, if a point on a plane is connected to N points through N connection lines, then N must be equal to 3, otherwise the connection lines will appear in phase. - The other three points are themselves a point, a total of four points, four different colors.

"Proof of Non-existence of Space Above N Dimensions"

壹, proof

1. Each vertex of a 4-hypercube connects four sides

2, from the four straight edges adjacent to one of the vertices, arbitrarily select two sides, can form a plane

3. Therefore, these planes are all connected to the aforementioned vertex

4. C(4,2)=6, so each vertex of any 4-hypercube is connected to 6 planes.

5. Now, draw a line (straight line or curve) on these planes. Each line connects any two of these sides.

6. Next, prepare a plane that is independent of the 4-hypercube, which is in contact with the aforesaid vertex, and on the entire 4-hypercube, only that point, is in contact with the plane.

7. Project the point, the line, the plane, and the lines drawn on the plane to the independent plane

8. On the independent plane, a graph will be generated: one point is adjacent to four straight lines, and between any two straight lines (adjacent to that point), there is a line (straight line or curve) connecting them.

9. The problem has arisen: these lines used to join, no doubt at least two will intersect 1 and the intersection means the aforementioned faces, the angle between them is not nine degrees two. This is contrary to the definition of 4-hypercubes.

10. If the above premise of reasoning (public design, presupposition) and reasoning itself are not flawed, the existence of hyperspace will lead to contradictions.

贰, postscript

1 This can be used to prove the four-color theorem. The method is to treat the block as a point and the boundary between the blocks as the line connecting the point and the point. After the drawing, it will be clear. On the contrary, since the four-color theorem has already been proved by other methods, the four-color theorem is its evidence.

2 You can use cubes for analogy - because 3D does not create contradictions, we live in 3D space