Minkowski-Einstein spacetime: insight from the Pythagorean theorem

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ABSTRACT

The Pythagorean Theorem, combined with the analytic geometry of a right circular cone, has been used by H. Minkowski and subsequent investigators to establish the 4-dimensional spacetime continuum associated with A. Einstein's Special Theory of Relativity. Although the mathematics appears sound, the incorporation of a hyper-cone into the analytic geometry of a right triangle in Euclidean 3-space is in conflict with the rules of pure mathematics. A metric space of n dimensions is necessarily defined in terms of n independent coordinates, one for each dimension. Any coordinate that is a combination of the others for a given space is not independent so cannot be an independent dimension. Minkowski-Einstein spacetime contains a dimensional coordinate, via the speed of light, that is not independent. Consequently, Minkowski-Einstein spacetime does not exist.

1 Introduction

The radius of a sphere is the hypotenuse of a right triangle (Fig.1).

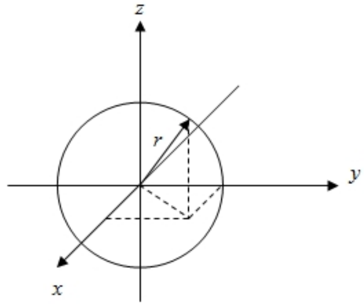

Fig. 1: The sphere and its radius.

The hypotenuse is not an independent dimensional coordinate because it is determined from the three Cartesian coordinates x, y, z. The speed at which a ray of light travels along the radius of a sphere from a light source located at the origin of the (orthogonal) Cartesian system (x, y, z) is determined by the distance light travels along any radial interval (hypotenuse) in a given time. Speed is not an independent dimensional coordinate. The equation for a right circular cone in the Cartesian coordinates (x, y, z), although having the same appearance, is not the equation of a circle. Only a normal cross-section through the axis of the right circular cone produces the equation of a circle. The Minkowski-Einstein 4-dimensional spacetime continuum confounds the hypotenuse of a right triangle in Euclidean 3-space for the equation of a hyper-cone, via the speed of light. It is consequently in conflict with the rules of pure and analytic geometry.

2 Analytic Geometry: Right Triangle and Cone

The equation of a sphere, radius r, centred at the origin of the Cartesian system (x, y, z) is:

$$x^2 + y^2 + z^2 = r^2. \tag{1}$$

The speed of light c is the distance light travels in a given time. The speed of light is not an independent dimensional coordinate. The distance r that light travels along *any radial line* of the sphere in any time t is $r = ct$. Then by eq.(1),

$$x^2 + y^2 + z^2 = c^2t^2, \tag{2}$$

where ct is the length of a hypotenuse (fig.1) in Euclidean 3-space. The quantity ct is not an independent dimensional coordinate because t is a scalar parameter for r. The hypotenuse of a triangle is not orthogonal to x, y, and z; although, in the degenerate case, when it coincides with one axis, it is orthogonal to the other two.

Subtracting c^2t^2 from both sides of eq.(2) gives,

$$x^2 + y^2 + z^2 - c^2t^2 = 0. \tag{3}$$

This is still a statement of the Pythagorean Theorem, hence the equation of a sphere. It is 3-dimensional, not 4-dimensional (fig.1).

The equation for a 45° right circular cone with its axis oriented along the z-axis is [1],

$$x^2 + y^2 = z^2. \tag{4}$$

Its graph is,

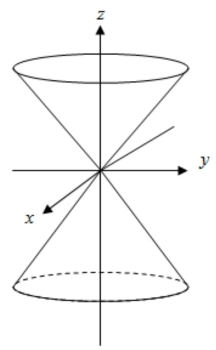

Fig. 2: A right circular cone.

Equation (4) is not the equation of a circle. Subtracting z^2 from both sides of eq.(4) gives,

$$x^2 + y^2 - z^2 = 0. \tag{5}$$

This expression is 3-dimensional. Only in the case of a normal cross-section to the axis of the cone does eq.(4) (or (5)) produce the equation of a circle, for then z has the fixed value $z = z_o$ for a plane parallel to the $x - y$ plane in which the hypotenuse of the triangle is not in the z-direction; it is perpendicular to the z-axis.

Eq.(3) is used to construct a light cone (spacetime diagram) to derive Einstein's length contraction and time dilation, by incorrectly treating ct as an independent dimensional coordinate and hence eq.(3) as that for a hyper-cone separating $t > 0$ from $t < 0$:

> "*The cone* $c^2t^2 - x^2 - y^2 - z^2 = 0$ *with apex 0 consists of two parts, one with values t<0, the other with values t>0.*" [2]

Minkowski's 'cone', the central equation for special relativity and its spacetime continuum, is eq.(3) multiplied by -1. A typical light cone is depicted in fig.3 [2–5], for which light travels upward along the 45° lines.

In treating ct as an independent dimensional coordinate so that the statement of the Pythagorean Theorem, eq.(2), becomes an equation for a hyper-cone, one might just as well treat eq.(4) as the equation of a circle. But in either case the result is achieved only by violation of pure and analytic geometry. If eq.(3) is both the equation of a hyper-cone in a non-Euclidean 4-space and a hypotenuse in Euclidean 3-space, then eq.(5) is both the equation for a cone in Euclidean

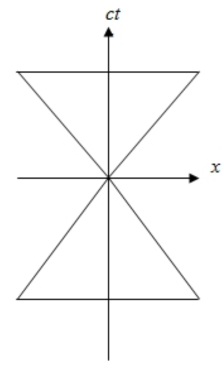

Fig. 3: A typical light cone.

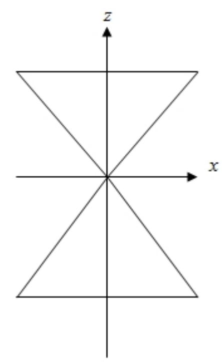

Fig. 4: The cone $x^2 = z^2$.

3-space and the equation for a hypotenuse in Euclidean 2-space. Thus, by *reductio ad absurdum*, eq.(3) is not the equation of a hyper-cone in a non-Euclidean 4-space.

All spacetime diagrams are actually just graphs of the hypotenuse $s = \sqrt{x^2 + y^2 + z^2}$, or a degenerate axial line thereof, plotted against itself, $s = ct$. One can graph anything against itself to obtain a line at 45° to two normal graphical axes. Such a graph does not imply new dimensional coordinates. Setting $y = z = 0$ in eq.(2) gives the degenerate hypotenuse along the x-axis (fig.1):

$$x^2 = c^2t^2, \tag{6}$$

where x is the distance light travels along the x-axis and ct is the same distance light travels along the x-axis. The graph of eq.(6) is fig.3; the graph of the distance x against itself, positive and negative (x and ct are actually collinear (fig.1)).

Setting $y = 0$ in eq.(4) gives:

$$x^2 = z^2. \tag{7}$$

Although eq.(7) rendered in fig. 4 has the same form as fig. 3, it is not the degenerate line of a hypotenuse: x and z are *not* collinear.

3 Vector Forms

Using spherical coordinates the vector equation for a sphere expanding at speed c can be written as,

$$\mathbf{r} = ct\cos\theta\sin\varphi\,\mathbf{i} + ct\sin\theta\sin\varphi\,\mathbf{j} + ct\cos\varphi\,\mathbf{k}, \tag{8}$$

where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors for the mutually orthogonal x, y and z axes respectively, and t is a scalar parameter for time [6].

Similarly the vector equation for a 45° right circular cone ($\varphi = \pi/4$ and $\varphi = 3\pi/4$: upper and lower nappes) is,

$$\mathbf{r} = ct\frac{\cos\theta}{\sqrt{2}}\,\mathbf{i} + ct\frac{\sin\theta}{\sqrt{2}}\,\mathbf{j} \pm ct\frac{1}{\sqrt{2}}\,\mathbf{k}. \tag{9}$$

Taking dot products for eq.(8),

$$\mathbf{r}\cdot\mathbf{r} = c^2t^2 = x^2 + y^2 + z^2, \tag{10}$$

and eq.(9),

$$\mathbf{r}\cdot\mathbf{r} = c^2t^2 = x^2 + y^2 + z^2. \tag{11}$$

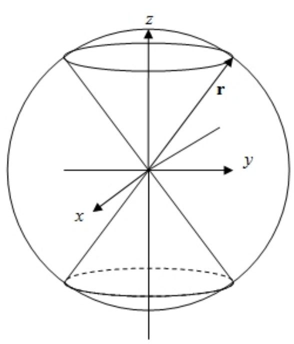

Fig. 5: A right circular cone enclosed by a sphere.

Equations (10) and (11) are the same because the cone is circumscribed by a spherical surface, shown in fig.5. The intersection of the spherical surface and the cone is a circle of radius $z = z_0$, in the plane $z = z_0$ parallel to the $x - y$ plane. Equations (10) and (11) do not however describe the same figure, as their derivations attest: the former a sphere the latter a cone. But they both describe the very same hypotenuse of a 45° isosceles triangle since eq.(10) from (8) is indifferent to radial direction. It is the hypotenuse that is the common element. The difference between eqs.(10) and (11) is that in the latter, from eq.(9),

$$x^2 + y^2 = \frac{c^2t^2}{2},\ z^2 = \frac{c^2t^2}{2},\ \Rightarrow x^2 + y^2 = z^2. \tag{12}$$

Subtracting c^2t^2 from both sides of eqs.(10) and (11) gives,

$$x^2 + y^2 + z^2 - c^2t^2 = 0. \tag{13}$$

Equation (13) is the equation for a hypotenuse and yields either a spherical surface or a 45° right circular cone, in Euclidean 3-space. If a cone, as by relations (12), it cannot be the equation of a cone in Euclidean 3-space and the equation of a hyper-cone in a non-Euclidean 4-space. Thus, it is not the equation of a hyper-cone in a non-Euclidean 4-space. The act of subtracting c^2t^2 from both sides of eqs.(10) and (11) does create a new dimensional coordinate.

4 Discussion and Conclusions

Minkowski-Einstein spacetime is represented by an indefinite Riemannian metric 4-space. A Riemannian metric space of n dimensions must have n independent coordinates.

> "*Any n independent variables x^i, where i takes values 1 to n, may be thought of as the coördinates of an n-dimensional space V_n in the sense that each set of values of the variables defines a point of V_n.*" [7]

Minkowski-Einstein 4-dimensional spacetime has only three independent dimensional coordinates because the hypotenuse of its triangle is neither independent of nor orthogonal to the Cartesian coordinates x, y, z. The quantity ct, is no more an independent dimensional coordinate perpendicular to x, y, and z than the quantities xy/z, y^2/x, or $\left(x^2 + y^2 + z^2\right)/y$, all of which have the units of length. Neither is the speed of light c independent of x, y, z. The equation for the hypotenuse of a triangle in Euclidean 3-space is not the equation of a hyper-cone in a non-Euclidean 4-space, just as the equation for the hypotenuse of a triangle in Euclidean 2-space is not the equation of a right circular cone in Euclidean 3-space. Minkowski-Einstein spacetime confounds the equation for a hypotenuse in three dimensions as an equation for a hyper-cone in four dimensions, thereby violating pure and analytic geometry. The 'spacetime' metric,

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2,$$

'timelike' for $ds^2 > 0$, 'spacelike' for $ds^2 < 0$, 'lightlike' for $ds^2 = 0$, is meaningless except for $ds^2 = 0$. As a result, either physically or as a mathematical construct, Minkowski-Einstein spacetime does not exist.

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