A Speculative Relationship between the Proton Mass, the Proton Radius, and the Fine Structure Constant and between the Fine Structure Constant and the Hagedorn Temperature

Espen Gaarder Haug*
Norwegian University of Life Sciences

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Abstract

In this short note we present a possible connection between the proton radius and the proton mass using the fine structure constant. The Hagedorn temperature is related to the energy levels assumed to be required to free the quarks from the proton, where hadronic matter is unstable. We also speculate that there could be a connection between the Hagedorn temperature and the Planck temperature through the fine structure constant. Whether there is something to this, or it is purely a coincidence, we will leave to others and future research to explore. However, we think these possible relationships are worth further investigation.

Key words: proton radius, fine structure constant, proton mass, reduced Compton wavelength.

1 The Proton Radius, the Proton Mass, and the Fine Structure Constant

The proton radius is given by NIST CODATA as 0.8751×10^{-15} meters, with a standard uncertainty of 0.0061×10^{-15}. There has been considerable discussion about the proton radius in the recent years; see [1, 2]. The proton mass is given by CODATA as $1.672621898 \times 10^{-27}$. Arnold Sommerfeld [3] introduced the fine structure constant in relation to spectral lines. The fine structure constant, $\alpha \approx 0.0072973525664$ (2014 CODATA recommended values), plays an important role in modern physics. There is no consensus on exactly why the fine structure constant has the "magical" value it has, but there are a number of interesting, speculative ideas about what it could represent; see [4, 5, 6, 7], for example.

Here we speculate that the proton radius could be connected to the proton mass, m_P, through the fine structure constant by the following formula

$$r_p = \frac{\hbar}{2m_P c}\sqrt{\frac{1}{2\alpha - \alpha^2}} \approx 0.872015 \times 10^{-15} \text{ m} \tag{1}$$

This is well inside the uncertainty in the proton radius given by CODATA. However, it does not necessarily mean there is anything to this relationship. Many readers will recognize that $\frac{\hbar}{m_P c}$ is the reduced Compton wavelength formula, so we could alternatively have written the formula simply as

$$r_p = \frac{\bar{\lambda}_P}{2}\sqrt{\frac{1}{2\alpha - \alpha^2}} \approx 0.872015 \times 10^{-15} \text{ m} \tag{2}$$

where $\bar{\lambda}_P$ is the reduced Compton wavelength of the proton. Naturally, there is uncertainty here, just as there is some uncertainty in the proton mass and even, to some degree, in the fine structure constant.

Another interesting point is that if we had a hypothetical particle with a reduced Compton wavelength equal to half the proton radius given by the formula above, then if this particle travelled at a velocity equal to $v = c - \alpha c = c(1-\alpha)$, it would have a relativistic mass exactly equal to the proton mass.

It is well-known that any elementary mass can be described as

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{3}$$

*e-mail espenhaug@mac.com. Thanks to Victoria Terces for helping me edit this manuscript.

Assume now we have $\bar{\lambda} = \frac{1}{2}r_p$, then we get a relativistic mass of

$$\frac{m}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m}{\sqrt{1-\frac{(c(1-\alpha))^2}{c^2}}} = \frac{\frac{\hbar}{\frac{1}{2}r_p}\frac{1}{c}}{\sqrt{1-(1-\alpha)^2}} = \frac{\frac{\hbar}{\frac{1}{2}r_p}\frac{1}{c}}{\sqrt{2\alpha-\alpha^2}} = m_P \approx 1.6726 \times 10^{-27} \text{ kg} \tag{4}$$

where m_P is the proton mass. That would be rather "fun" ? a particle traveling with speed $c(1-\alpha)$ and relativistic mass equal to the proton surrounded by a particle (electron) traveling with speed $c\alpha$.

If the mathematical relationship above holds (is more than a coincidence), then we can also find the fine structure constant from the proton radius and the reduced Compton wavelength of the proton

$$\begin{aligned} \frac{2r_p}{\bar{\lambda}_P} &= \sqrt{\frac{1}{2\alpha-\alpha^2}} \\ \frac{4r_p^2}{\bar{\lambda}_P^2} &= \frac{1}{2\alpha-\alpha^2} \\ \alpha^2 - 2\alpha + \frac{\bar{\lambda}_P^2}{4r_p^2} &= 0 \end{aligned} \tag{5}$$

This is a standard quadratic equation with the following solution

$$\alpha = \frac{2 \pm \sqrt{2^2 - 4\frac{\bar{\lambda}_P^2}{4r_p^2}}}{2} = \frac{2 \pm \sqrt{4 - \frac{\bar{\lambda}_P^2}{r_p^2}}}{2} = 1 \pm \sqrt{1 - \frac{\bar{\lambda}_P^2}{4r_p^2}} \tag{6}$$

and only the negative solution seems to make sense, so this would mean

$$\alpha = 1 - \sqrt{1 - \frac{\bar{\lambda}_P^2}{4r_p^2}} \tag{7}$$

Again, this solution is based on the idea that there may be a fundamental relationship here; granted this is a speculative suggestion on a possible relationship with the fine structure constant. We leave it up to future inquiry to see whether this will lead to anything useful or not.

2 The Fine-Structure Constant and the Hagedorn Temperature

The Hagedorn temperature [8] is a theoretical temperature where quarks can be extracted (evaporated) from a proton. It is essentially the temperature where hadronic matter is unstable. The Hagedorn temperature is extremely high; according to traditional grand-unified string models, it is said to be about 10^{30} kelvin, see [9]. This is about two orders of magnitude smaller than the Planck temperature, which is $\frac{m_p c^2}{k_b} \approx 1.42 \times 10^{32}$ kelvin. If the Hagedorn temperature should be $\frac{m_p c^2 \alpha}{k_b} \approx 1.03391 \times 10^{30}$ kelvin and we divide this temperature by the Planck temperature [10, 11], we would get the fine structure constant. Actually, there is considerable uncertainty, even regarding the size of the Planck mass. However, we can indirectly measure the Planck mass from a Cavendish experiment without any knowledge of the Newton gravitational constant; see [12]. Further, the uncertainty in the Planck mass is half the uncertainty of the Newton gravitational constant.

It is important to note that the Hagedorn temperature has never been measured and if its true temperature is different than the speculation here, then there is no such connection.

3 Conclusion

In this paper we have suggested a possible interesting relationship between the proton mass and the proton radius "through" the fine structure constant. We have also suggested a possible connection between the Hagedorn temperature, the Planck temperature, and the fine structure constant. We do not know if there are truly such connections or not, and more observational and theoretical work is needed.

We will end this note with a quote from Feynman about the fine structure constant:

> *It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.* – Richard Feynman

References

[1] R. Pohl, A. Antognini, et al. The size of the proton. *Nature*, 466.

[2] A. Beyer, L. Maisenbacher, P. R. Matveev, A. K. Khabarova, A. Grinin, T. Lamour, D. Yost, T. Hnsch, N. Kolachevsky, and T. Udem. The Rydberg constant and proton size from atomic hydrogen. *Science*, 358.

[3] A. Sommerfeld. On the quantum theory of spectral lines. *Annals of Physics*, 51, 1916.

[4] M. A. Sherbon. Wolfgang Pauli and the fine-structure constant. *Journal of Science*, 2(3):148–154, 2012.

[5] R. Heyrovska. Golden ratio based fine structure constant and Rydberg constant for hydrogen spectra. *International Journal of Sciences*, 2, 2013.

[6] J. P. Koshy. Fine structure constant: A mystery resolved. *vixra.org. 1701.0310*, 2017.

[7] E. G. Haug. The mystery behind the fine structure constant. *viXra:1706.0094*, 2017.

[8] T. Ericson and J. Rafelski. The tale of the Hagedorn temperature. *CERN Courier*, 2003.

[9] J. J. Atick and E. Witten. The Hagedorn transition and the number of degrees of freedom of string theory. *Nuclear Physics B*, 310(2), 1988.

[10] M. Planck. *Naturlische Masseinheiten*. Der Königlich Preussischen Akademie Der Wissenschaften, p . 479., 1899.

[11] M. Planck. *Vorlesungen über die Theorie der Wärmestrahlung*. Leipzig: J.A. Barth, p. 163, see also the English translation "The Theory of Radiation" (1959) Dover, 1906.

[12] E. G. Haug. Can the Planck length be found independent of big G ? *Applied Physics Research*, 9(6), 2017.