Liar Paradox and Completeness of Theory

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Let us define the completeness of the complex system as a state where all individual components composing the system are in equilibrium. Mathematically it can be expressed as the case when the sum of systems constituents is equal to zero. Any deviation from the completeness state would be consequently defined as the incompleteness of the system. Here we postulate that the incompleteness of the system whether it is the formulation of any logical conclusion, the technical construction or physical law leads to paradoxes or instabilities of the system itself.

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Introduction

The idea of completeness of the system, for instance, the system of logical statements can be shown by the Liar paradox. The ancient Greek priest and seer Epimenides (born VII century BC in Crete) had said: "All Cretans lie". It is also mentioned in the New Testament, Paul's book of Titus 1:12. As Epimenides was Cretan, then, according to his own statement, he was lying. However, if he was lying, then the statement “All Cretans lie” itself was a lie. Therefore one can continue with that the opposite is true, that is, all Cretans do not lie. If Cretans did not lie Epimenides told the truth, and so on. The statement denies himself.

Throughout history, the Liar paradox has resulted in many variants, for example, the king ordered that if the offender in his last name lied then he would be hanged; if he told the truth then his head would be cut down. The offender said: "I will be hanged." The judges were not able to fulfill the king's orders. Nowadays the Liar paradox is reduced to a simple statement, "I lie."

The incompleteness of Liar paradox and solution

The key to the solution of the Liar paradox is to recognize the complete set of logical entities, which in this case would be "a lie" and its opposite "the truth". Since the original Epimenides statement “All Cretans lie” includes only one of both logical entities, it is incomplete and therefore yields to a paradoxical state or logical instability. On the other hand, if one uses the both entities, for example: "I lie or tell the truth", the logical paradox is not present anymore. To express this situation mathematically, one could assign a positive quantity to “the truth”, for example, “+1”. In this case “a lie" would be a negative quantity of the same magnitude. Using the above example, it will be "-1". The sum of both entities is zero. It is a criterion of entities completeness.

Liar paradox in techniques

In Boolean algebra the statement "the truth" is designated by "1" and the opposite value "a lie" or "false" by "0". From the logical point of view, the opposite statement must be marked by "-1". Therefore the sum is zero and both statements are complete. From the technical point of view, the Boolean statements are well-founded because electronic logic gates have the statements "high voltage" and "low voltage". This corresponds with the Boolean statements - “the truth” and “false”. Therefore the statements of logic gates are incomplete. In some cases it leads to the instability of a logical circuit or, in other words, to a paradox.

For example, the logical gate NON has only one function - inversion of an input signal. If the input statement is "1" then the output statement must be "0" and vice versa. By connecting the input with the output (Fig. 1a) one gets an analog of the Liar paradox realized by a logical gate. The circuit at the best is unstable, but in the worst case can burn out.

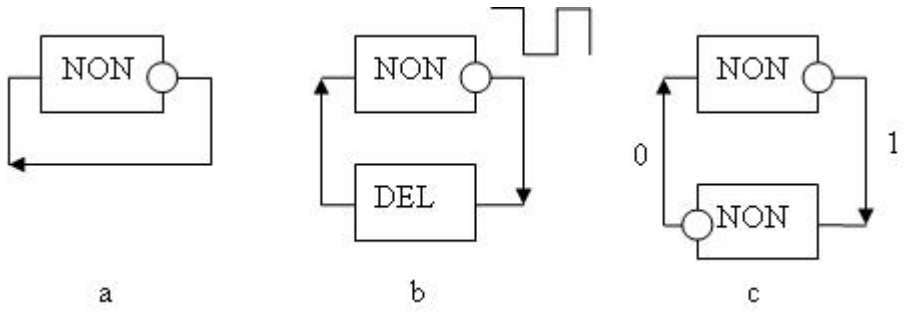

Fig. 1. Circuits of NON gate

The instability of a circuit is widely used to generate pulses. In this case (Fig 1b) the delay element DEL must be added in the feedback. The pulse duration is equal to the delay time. The stable circuit (Fig 1c) must be created by feedback with another NON

gate. In this case the circuit has a complete set of statements: "1" and the opposite "-1" marked by "0". It is a trigger which is widely used as a basic memory element.

Another example of paradox in the technical system is a three phase alternating current (AC) supply. Three phase voltages can be represented as vectors A, B and C mutually shifted by 120 degrees (Fig. 2). In the steady state all vectors have the same length and the sum is zero (Fig. 2a). The starting point of vectors has a ground potential G.

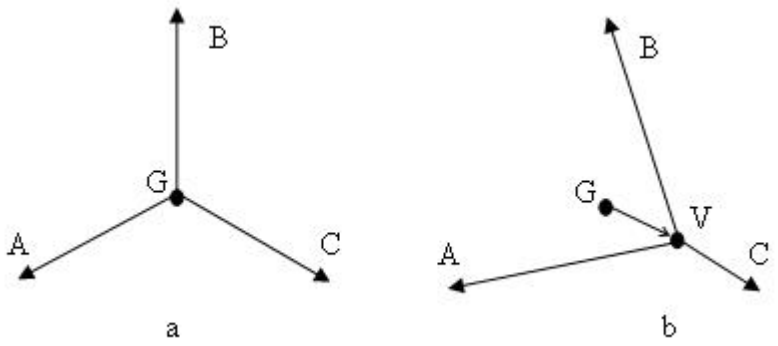

Fig. 2. Vectors A, B, C in the three phase supply.
G – potential of ground, V – voltage on the neutral.

The voltage drops in the case where the phase C is overloaded. This corresponds to the lower length of the vector C (Fig. 2b). The end points of vectors A, B, C are stabilized by a generator. As a result, the starting point of the vectors is shifted to V and voltage VG according to the ground appears in the neutral. From the logical point of view, here is a paradox: the generator gives only three voltages A, B, C, but the fourth V appears in the neutral.

This example shows the importance of balance between all items/entities in a logical system for the prevention of paradox.

Liar paradox and physics

Kirhoff laws and Newton third law are examples of physical laws which use a complete set of items.

Kirchoff current law postulates: At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node (Fig.3). In other words, the algebraic sum of all currents at a node of network is zero. Here it is assumed that the current is a signed (positive or negative) quantity reflecting the direction towards or away from a node.

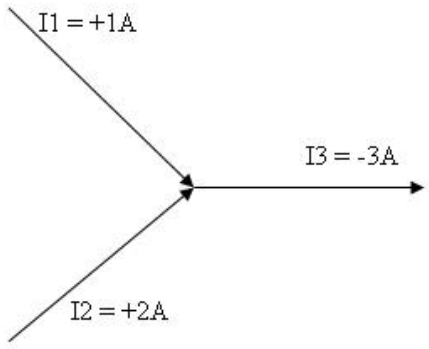

Fig. 3 Currents in the node

Sum of currents I1 + I2 + I3 = 1A+2A-3A = 0

Mathematically this principle can be stated as: $\sum_{j=1}^{n} I_j = 0$ **(1)**

Where: I_j – current in a branch j

n – total number of branches in a node.

The use of an incomplete set of currents leads to a paradox: for example, by omitting the current I2 the output current I3 exceeds the input current I1, but the electrical charge in the node is zero. In this case the law for currents in the node obtains the following kind: The sum of currents in the node is constant.

Kirchoff current law is based on the conservation of charge whereby the charge Q (measured in coulombs) is the product of the current I (in amperes) and the time t (in seconds): $Q = I\,t$

Therefore, the conservation law of charge may be defined as: Sum of all charges in the closed neutral system is zero: $$\sum_{j=1}^{n} Q_j = 0 \qquad \textbf{(2)}$$

Another statement of conservation law: The amount of positive charge minus the amount of negative charge in the Universe is always conserved. This shows that all charges are not taken into account.

Kirchoff voltage law: The directed sum of the electrical potential differences (voltage) around any closed contour of network (Fig. 4) is zero:

$$\sum_{j=1}^{n} V_j = 0 \ . \qquad \textbf{(3)}$$

Where: V_j – voltage on an element j of network

n – total number of elements in contour of network.

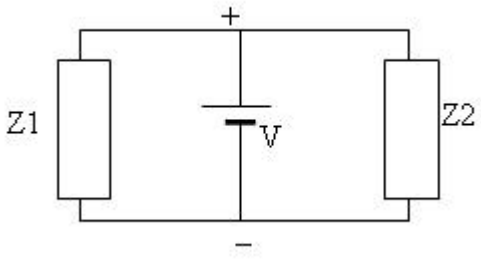

Fig.4. Circuit with three contours (closed loops): Z1 – Z2, Z1 – V and V – Z2.

In all the three loops Kirchoff voltage law is valid independently. This shows that closed systems (loops) can be interconnected.

Kirchoff voltage law is based on the conservation of energy whereby voltage V is defined as energy W per unit charge: $V = W/Q$.

Therefore, the conservation law of energy may be defined as: The sum of all energies in the closed system is zero: $$\sum_{j=1}^{n} W_j = 0 \qquad \textbf{(4)}$$

Here it is assumed that energy is an assigned (positive or negative) quantity reflecting the direction of energy flow towards or away from an object. In other words, the energy that is emitted by one object of a system is equal to the energy that is absorbed by the other objects of a closed system.

Newton third law: There is always an opposed equal reaction to every action. The third law states that all forces exist in pairs: if one object A exerts force $\boldsymbol{F}_A$ on a second object B, then B simultaneously exerts force $\boldsymbol{F}_B$ on A, and the two forces are equal and opposite. Therefore the sum of forces is equal to zero or, in other words, the sum of action and reaction is zero.

The gravitation force $\boldsymbol{mg}$ of the pendulum (Fig. 5 a) in the equilibrium position is compensated by the tension $\boldsymbol{T}$ of the cord. The sum of all forces $\boldsymbol{T} + \boldsymbol{mg} = 0$.

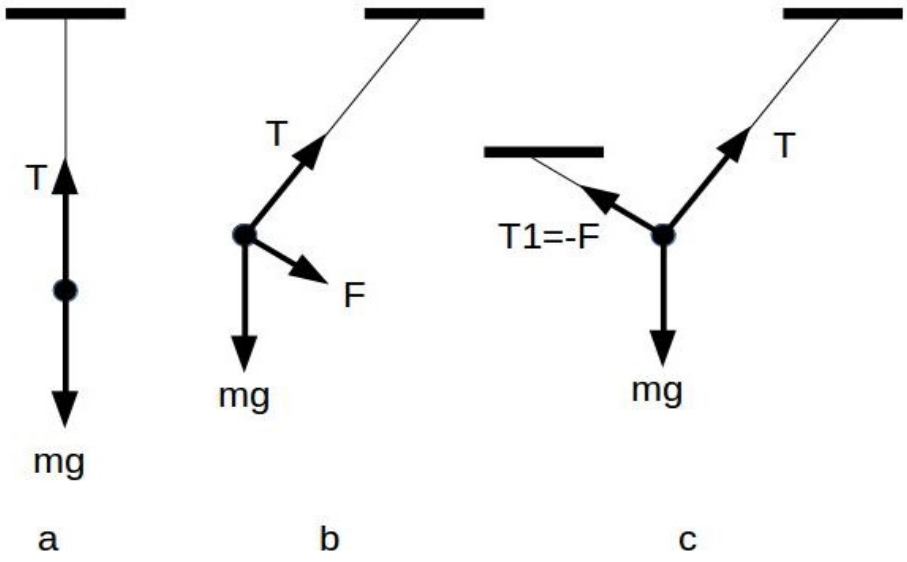

Fig. 5. Forces in the pendulum:
a – in the equilibrium **T** = **mg**, b – restoring force **F**,
c – compensation of restoring force **F** by tension **T1**.

If the pendulum is perturbed away from the equilibrium (Fig. 5 b), the restoring force ***F*** will tend to bring the pendulum back toward the equilibrium. The system is unstable and the pendulum is swinging. The sum of all forces is ***T*** + ***mg*** = ***F***.

If the restoring force ***F*** is compensated (Fig. 5 c) by the tension ***T1*** of another cord, the system is in equilibrium and stable. The sum of forces is equal to zero.

Liar paradox and chemistry

The valence shell of atoms has either one **s** orbital (H and He) or one subshell and three **p** subshells for all other elements. Each subshell can have two electrons with opposite spins. The sum of electron spins in the completed subshell is zero. The closed valence shell contains four subshells with eight electrons altogether. The sum of electron spins

in the closed valence shell is zero. Therefore the closed valence shell has a complete set of items. Such a valence shell has noble gases, which are chemically passive. All other atoms have an incomplete valence shell and they are chemically active.

The activity of atoms significantly decreases when they merge into molecules, where all atoms have a full valence shell. For example, the atomic hydrogen H with one electron in shell is extremely active, but the molecule H_2 or H—H, which has two common electrons forming a complete valence shell, is much more inactive. Like H, the atomic oxygen O with six electrons in the valence shell is extremely active, but molecule O=O with two common electrons forming a complete valence shell with eight electrons for both oxygen atoms, is less active. Molecule of water H-O-H, having complete sets of electrons for all atoms, is inactive.

Conclusions

To avoid paradoxes, all statements in any logical conclusion, technical construction or scientific theory must be in equilibrium. The term “equilibrium” means the use of a logically complete set of items. Mathematically it can be expressed as follows: for logical completeness the sum of all items in any conclusion must be zero. It is proof for the completeness of any theory.

Logical completeness applies only to that part of reality that is covered by a conclusion. For example, in Figure 4 the circuit made from three components has two nodes (at V+ and at V-) and three loops: Z1 – Z2, Z1 – V and V – Z2. Therefore, 5 logically complete sets of items may be set up.

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