# Gravitational Force and Conservation of Momentum

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In the history of physics, momentum has been represented by two expressions. One from Issac Newton, the other from Special Relativity. Both expressions are expected to describe a physical system that demands conservation of momentum. By examining the gravitational force between two identical particles in two different inertial reference frames, the momentum expression from Issac Newton is found to obey conservation of momentum while the momentum expression from Special Relativity is found to violate conservation of momentum.

## I. INTRODUCTION

In 17th century, Issac Newton proposed a definition of force, $F = m * a$. From this definition, both kinetic energy and momentum can be derived.

In 20th century, Special Relativity[1] proposed a new definition of kinetic energy. This results in new definitions of both momentum and force.

However, the physics law, conservation of momentum, remains intact. Any definition of kinetic energy is expected to generate a force that results in the conservation of momentum.

This paper examines both expressions of momentum in an isolated gravitational system. The total momentum is calculated for this physical system in two different inertial reference frame. The total momentum from each expression of momentum is expected to obey conservation of momentum in both reference frames.

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativistic momentum do not share the same relativistic mass. The momentum of a particle is represented by either $\gamma(v) * m(0) * v$ or $m(v) * v$. Both representations are equivalent to each other mathematically. In this paper, $\gamma(v) * m * v$ is chosen to emphasize Lorentz Factor, $\gamma(v)$, from Lorentz Transformation.

$$\frac{dm}{dv} = \frac{dm(0)}{dv} = 0 \tag{1}$$

## II. PROOF

Consider two-dimensional motion.

### A. Kinetic Energy and Momentum

In Newtonian Mechanics, force F is defined as multiplication of mass and acceleration.

$$F = m * a \tag{2}$$

In Special Relativity, kinetic energy K is defined as

$$K = (\gamma(v) - 1) * m * C^2 \tag{3}$$

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{4}$$

These two definitions generate different expressions for momentum. However, the derivation of momentum from kinetic energy has not been changed. Kinetic Energy K is defined as integration of force over distance.

$$K = \int F\, dx \tag{5}$$

Momentum P is defined as integration of force over time.

$$P = \int F\, dt \tag{6}$$

$$\frac{dP}{dt} = F \tag{7}$$

In Newtonian Mechanics,

$$K = \frac{1}{2} * m * v^2 \tag{8}$$

$$P = m * v \tag{9}$$

In Special Relativity,

$$K = (\gamma(v) - 1) * m * C^2 \tag{10}$$

$$P = \gamma(v) * m * v \tag{11}$$

The difference in two expressions indicates that only one expression of momentum can be correct By applying conservation of memntum to both expressions of momentum in a physical system such as gravitational attraction, the correct expression can be distinguished.

### B. Gravitational Force

Two identical particles move along y axis under a gravitational force between them along y axis. The single force on this isolated physical system demands that total momentum P should remain constant.

$$\frac{dP}{dt} = 0 \tag{12}$$

### C. Reference Frame

Let the center of mass be stationary in a reference frame $F_1$. Both particles move at the same speed u' but in opposite direction along y axis in $F_1$.

$$\frac{du'}{dt'} > 0 \tag{13}$$

TABLE I. Velocity and Momentum in $F_1$

| Particle | Value |
|---:|---|
| Velocity of particle 1 $O_1$ | is $(0, u')$ |
| Velocity of particle 2 $O_2$ | is $(0, -u')$ |
| Newtonian momentum of $O_1$ | is $m * (0, u')$ |
| Newtonian momentum of $O_2$ | is $m * (0, -u')$ |
| Relativistic momentum of $O_1$ | is $\gamma(u') * m * (0, u')$ |
| Relativistic momentum of $O_2$ | is $\gamma(-u') * m * (0, -u')$ |

Let another reference frame $F_2$ move at a constant velocity of -V relatively to $F_1$ along x axis. In $F_2$, both particles acquire a new velocity V in x direction.

$$\frac{dV}{dt} = 0 \tag{14}$$

Their velocity in y direction become u and -u. The gravitational force accelerates both particles toward each other. u increases with time.

$$\frac{du}{dt} > 0 \tag{15}$$

$O_1$ moves at the speed $v_1$ in $F_2$.

$$v_1 = \sqrt{V^2 + u^2} \tag{16}$$

$O_2$ moves at the speed $v_2$ in $F_2$.

$$v_2 = \sqrt{V^2 + (-u)^2} = v_1 \tag{17}$$

TABLE II. Velocity and Momentum in $F_2$

| Particle | Value |
|---:|---|
| Velocity of $O_1$ | is $(V, u)$ |
| Velocity of $O_2$ | is $(V, -u)$ |
| Newtonian momentum of $O_1$ | is $m * (V, u)$ |
| Newtonian momentum of $O_2$ | is $m * (V, -u)$ |
| Relativistic momentum of $O_1$ | is $\gamma(v_1) * m * (V, u)$ |
| Relativistic momentum of $O_2$ | is $\gamma(v_2) * m * (V, -u)$ |

### D. Conservation of Momentum

The single force in this isolated gravitational system demands conservation of momentum in both $F_1$ and $F_2$.

In Newtonian Mechanics, total momentum in $F_2$ is $P_n$. $P_n$ remains constant.

$$P_n = m * (V, u) + m * (V, -u) = 2 * m * (V, 0) \tag{18}$$

$$\frac{dP_n}{dt} = 2 * m * (\frac{dV}{dt}, 0) = (0, 0) \tag{19}$$

In Special Relativity, total momentum in $F_2$ is $P_r$. $P_r$ varies with time.

$$P_r = \gamma(v_1) * m * (V, u) + \gamma(v_2) * m * (V, -u) \tag{20}$$

$$= 2 * \gamma(v_1) * m * (V, 0) \tag{21}$$

$$\frac{dP_r}{dt} = 2 * m * (V, 0) * \frac{d\gamma(v_1)}{dt} \tag{22}$$

$$= 2 * m * (V, 0) * \gamma(v_1)^3 * \frac{v_1}{C^2} * \frac{dv_1}{dt} \tag{23}$$

$$= 2 * m * (V, 0) * \gamma(v_1)^3 * \frac{u}{C^2} * \frac{du}{dt} \tag{24}$$

Total momentum remains constant in Newtonian Mechanics but not in Special Relativity.

## III. CONCLUSION

Special Relativity violates conservation of momentum in an isolated gravational system.

Conservation of momentum fails to hold if momentum is defined as $\gamma(v) * m * v$. The failure of this physics law is due to the introduction of Lorentz factor, $\gamma(v)$, from Lorentz Transformation[8][11].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] and Conservation of Momentum[10] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity.

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