Real and Complex Amended Maxwell's Equations for Non-Abelian Gauge Groups

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In this work we analyze, calculate and extend modification of Maxwell's equations in a complex Minkowski metric, M_4 in a $\mathbb{C}_2$ complexified space using the SU_2 gauge, SL(2,c) and other gauge groups, such as SU_n for $n > 2$ expanding the U_1 gauge theories of Weyl. Weyl identified the U_1 gauge group for the standard Maxwell's equations in its nonrelativistic form in M_4 space. We expand the form of the elegant electromagnetic equations and express them in $\mathbb{C}_4$ space for the nonrelativistic formalism and for the relativistic formalism of the equations. The advanced and retarded formalisms are also examined. In the case where Maxwell's equations are solved in $\mathbb{C}_4$ space, or the complex 8-space, we can extend the theory to considerations of other gauge groups such as SL(2,c), SU_n for $n > 2$ and SU_2 expanding the approach beyond U_1 gauge conditions.

1. Introduction – Extended Maxwell's Equations

In addition to our work others have examined complex multidimensional geometries [1-6]. In particular we have examined the complexification of M_4 Minkowski space as an 8D complex $\mathbb{C}_4$ space [5,6]. The complex space is comprised of four real dimensions and 4 imaginary dimensions and this geometry is consistent with Lorentz invariance and analytic continuation.

We have developed an 8D complex Minkowski space, M_4 composed of four real dimensions and four imaginary dimensions which is consistent with Lorentz invariance and analytic continuation in the complex plane [1-6]. The unique feature of this geometry is that it admits of nonlocality consistent with Bell's theorem, (EPR paradox), possibly Young's double slit experiment, the Aharonov-Bohm effect and multi-mirrored interferometric experiment [7].

This work of amending Maxwell's equations yields additional predictions beyond the electroweak unification scheme. Some of these are:

- Modified gauge invariant conditions,
- Short range non-Abelian force terms and Abelian long-range force terms in Maxwell's equations,
- Finite but small rest mass, m_γ of the photon,
- A magnetic monopole like term
- Longitudinal as well as transverse magnetic and electromagnetic field components in a complex Minkowski metric, M_4 in a $\mathbb{C}_4$ space.

Additionally, expressing Maxwell's electromagnetic equations in complex 8-space, leads to some new and interesting predictions in physics, including possible detailed explanation of some of the previously mentioned nonlocality experiments [8-13]. Complexification of Maxwell's equations requires a non-Abelian gauge group which amend the usual theory and which utilizes the usual unimodular Weyl U_1 group. We have examined the modification of gauge conditions using higher symmetry groups such as, SU_n and other groups such as the SL(2,c) double cover group of the rotational group SO(3,1) related to the Ricci curvature tensor [14]. Thus, we are led to new and interesting physics involving extended metrical space constraints. In addition to the usual transverse we also have longitudinal, non-Hertzian electric and magnetic field solutions to Maxwell's equations leading to new communication systems and antennae theory, non-zero solutions to $\nabla \cdot \underline{B}$, and a possible finite but small rest mass of the photon, m_γ.

Comparison of our theoretical approach is made to the work of Vigier et al [15,16] Barrett et al [17] and Harmuth et al [18] on amended Maxwell's theory. We compare our predictions such as our longitudinal field to the $B^{(3)}$ term of Vigier, and our Non-Abelian gauge groups to that of Barrett and Harmuth. We interpret this work as leading to new and interesting physics, including a possible interpretation of nonlocal information transmission properties within the Dirac polarized vacuum.

2. Complexified Electromagnetic Fields in Minkowski Space and Nonlocality

We expand the usual 4D line element metric $ds^2 = g_{\mu\nu} dx^\nu dx^\mu$ in the following manner. We consider a complex 8D space, $\hat{M}_4$ as $\mathbb{C}_4$ so that $Z^\mu = X^\mu_{\mathrm{Re}} + iX^\mu_{\mathrm{Im}}$ and likewise for Z^ν where the indices ν and μ run 1 to 4 yielding (1, 1, 1, -1). Hence, we now have a new complex 8-space metric as $ds^2 = \eta_{\nu\mu} dZ^\nu dZ^{*\mu}$. We have developed this space and other extended complex spaces and examined their relationship with the twister algebras and asymptotic twister space and the spinor calculus and other implications of the theory [6]. The Penrose twistor SU(2,2) or U_4 is constructed from 4D-spacetime, $U_2 \otimes \widetilde{U}_2$ where U_2 is the real part of the space and $\widetilde{U}_2$ is the imaginary part of the space, this metric appears to be a fruitful area to explore.

The twistor, $\mathbb{Z}$ can be a pair of spinors, U^A and π_A which Penrose created to represent a twistor. The condition for these representations is:

- The null infinity condition for a zero spin field is $\mathbb{Z}^\mu \overline{\mathbb{Z}}_\mu = 0$
- Conformal invariance
- Independence of the origin.

The twistor is derived from the imaginary part of the spinor field. The underlying concept of twister theory is that of conformally invariance fields occupy a fundamental role in physics and may yield some new approaches to physics. Since the twister algebra falls naturally out complex space.

Other researchers have examined complex dimensional Minkowski spaces. In [2], Newman demonstrates that M_4 space does not generate any major "weird physics" or anomalous physics predictions and is consistent with an expanded or amended special and general relativity. In fact the Kerr metric falls naturally out of this formalism as demonstrated by Newman [4] and Rauscher [5,6,19,20].

As we know twistors and spinors are related by the general Lorentz conditions in such a manner that all signals are luminal in the usual 4*n* Minkowski space but this does not preclude super or transluminal signals in spaces where *n* > 4. Stapp, for example, has interpreted Bell's theorem experimental results in terms of transluminal signals to address the nonlocality issue of the Clauser, and Aspect experiments [8]. Newman et al demonstrate the role of nonlocal fields in complex 8-space [2,3].

We believe that there are some very interesting properties of the complex M_4 space which include the nonlocality properties of the metric applicable in the non-Abelian algebras which are related to the quantum theory and the conformal invariance in relativity as well as new properties of Maxwell's equations. In addition, complexification of Maxwell's equations in $\mathbb{C}_4$ space yields interesting predictions, yet we find the usual conditions on the manifold hold [19,20]. Some of these new predictions come out of the complexification of 4-space and appear to relate to the work of Vigier, Barrett, Harmuth and others [15,17.18]. Also, we find that the twistor algebra of the complex 8D $M_4 + \mathbb{C}_4$ space is mapable 1 to 1 with the twistor algebra of the Kaluza-Klein 5D electromagnetic - gravitational metric [21-23].

Some of the predictions of the complexified form of Maxwell's equations are:

- A finite but small rest mass of the photon, m_γ
- A possible magnetic monopole, $\nabla \cdot \underline{B} \neq 0$
- Transverse as well as longitudinal $B^{(3)}$ like components of $\underline{E}$ and $\underline{B}$,
- New extended gauge invariance conditions to include non-Abelian algebras
- An inherent fundamental nonlocality property on the manifold. Evans and Vigier also explore longitudinal $\underline{E}$ and $\underline{B}$ components in detail and finite rest mass of the photon, m_γ [16].

We consider both the electric and magnetic fields to be complexified as $\underline{E} = E_{\text{Re}} + iE_{\text{Im}}$ and $\underline{B} = \underline{B}_{\text{Re}} + iB_{\text{Im}}$ for $E_{\text{Re}}, E_{\text{Im}}, B_{\text{Re}}$ and B_{Im} are real quantities. Then substitution of these two equations into the complex form of Maxwell's equations above yields, upon separation of real and imaginary parts, two sets of Maxwell-like equations. The first set is

$$\begin{aligned} &\nabla \cdot \underline{E}_{\text{Re}} = 4\pi\rho_e \qquad && \nabla \times E_{\text{Re}} = -\frac{1}{c}\frac{\partial \underline{B}_{\text{Re}}}{\partial t} \\ &\nabla \cdot \underline{B}_{\text{Re}} = 0 && \nabla \times \underline{B}_{\text{Re}} = \frac{1}{c}\frac{\partial \underline{E}_{\text{Re}}}{\partial t} = \underline{J}_e\,; \end{aligned} \tag{1}$$

the second set is

$$\begin{aligned} &\nabla \cdot \left(i\underline{B}_{\text{Im}}\right) = 4\pi i\rho_m \qquad && \nabla \times \left(i\underline{B}_{\text{Im}}\right) = \frac{1}{c}\frac{\partial\left(i\underline{E}_{\text{Im}}\right)}{\partial t} \\ &\nabla \cdot \left(i\underline{E}_{\text{Im}}\right) = 0 && \nabla \times \left(i\underline{E}\right) = \frac{1}{c}\frac{\partial\left(i\underline{B}_{\text{Im}}\right)}{\partial t} = i\underline{J}m\,. \end{aligned} \tag{2}$$

The real part of the electric and magnetic fields yield the usual Maxwell's equations and complex parts generate "mirror" equations. For example, the divergence of the real component of the magnetic field is zero, but the divergence of the imaginary part of the electric field is also zero, and so forth. The structure of the real and imaginary parts of the fields is parallel with the electric real components being substituted by the imaginary part of the magnetic fields and the real part of the magnetic field being substituted by the imaginary part of the electric field. In the second set of equations, (2), the i's, "go out" so that the quantities in the equations are real, hence $\nabla \cdot \underline{B}_{\mathrm{Im}} = 4\pi\rho_m$, and not zero, yielding a term that may be associated with some classes of monopole theories [20].

We express the charge density and current density as complex quantities based on the separation of Maxwell's equations above. Then, in generalized form $\rho = \rho_e = i\rho_m$ and $\underline{J} = \underline{J}_e + i\underline{J}_m$ where it may be possible to associate the imaginary complex charge with the magnetic monopole and conversely the electric current has an associated imaginary magnetic current. Using the invariance of the line element $s^2 = x^2 - c^2t^2$ for $r = ct = \sqrt{x}^{\,2}$ and for $x^2 = x^2 + y^2 + z^2$ for the distance from an electron charge, we can write the relation,

$$\frac{1}{c}\frac{\partial\left(i\underline{B}_{im}\right)}{\partial t} = i\underline{J}m \qquad \text{or} \qquad \frac{1}{c}\frac{\partial B_{im}}{\partial t} = \underline{J}_m$$

$$\nabla\times\left(i\underline{E}_{\mathrm{Im}}\right) = 0 \ \text{ for } \ \underline{E}_{\mathrm{Im}} = 0 \quad \text{or} \quad \frac{1}{c}\frac{\partial\left(i\underline{B}_{\mathrm{Im}}\right)}{\partial t} = i\underline{J}m \tag{3}$$

3. The General Concept of Gauge Symmetry in Current Physics

Gauge symmetry is the basic concept required in field theory to describe a field for which the equations describing the field do not change when an operation applied to all particles and fields everywhere in space is globally invariant. It is also possible to have local gauge symmetry where the operation is applied to some particular region of space. Fields with gauge symmetry are, for example, gravity, electromagnetism and QED. The gauge symmetry approach was a key development in the theory of weak, and electroweak interactions and QCD. The quantum field is restored to symmetry by its Yang-Mills gauge field. Thus, the origin of the concept of broken symmetry in gauge theory which led to the development of the electroweak theory in 1967 by Weinberg and Salam. This was a key benchmark in developing a grand unified theory (GUT).

Table 1 The Color Octet of Gluon Gauge

$(r\bar{b} + b\bar{r})/\sqrt{2}$	$-i(r\bar{b} - b\bar{r})/\sqrt{2}$
$(r\bar{g} + g\bar{r})/\sqrt{2}$	$-i(r\bar{g} - g\bar{r})/\sqrt{2}$
$(b\bar{g} + g\bar{b})/\sqrt{2}$	$-i(b\bar{g} - g\bar{b})/\sqrt{2}$
$(r\bar{r} - b\bar{b})/\sqrt{2}$	$(r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$

In field theory a gauge group corresponds mathematically to a fiber symmetry group, whereas gauge theory corresponds mathematically to the principle fiber bundle. The gauge group for the electromagnetic photon is a U_1 gauge group. The gauge group for the strong force, SU_3 is mediated by eight independent gluons binding the three quarks. See Table 1. The electroweak force, SU_2 x U_1 corresponds to the quantum gauge groups of quantum chromodynamics (QCD); the electroweak gauge force is mediated by $W^{\pm}, Z^0$ which are termed intermediate vector Bosons. Gauge Bosons couple to conserved currents.

4. New Gauge Conditions, Complex Minkowski Space and New Implications for Physics

In a series of papers, Barrett, Harmuth and Rauscher have examined the modification of gauge conditions in modified or amended Maxwell theory. The Rauscher approach, as briefly explained in the preceding section is to write complexified Maxwell's equation in consistent form to complex Minkowski space [20].

The Barrett amended Maxwell theory utilizes non-Abelian algebras and leads to some very interesting predictions. He utilizes the non commutitative SU_2 gauge symmetry rather than the U_1 symmetry. Although the Glashow electroweak theory utilizes U_1 and SU_2, but in a different manner, but his theory does not lead to the interesting and unique predictions of the Barrett theory. Barrett, in his amended Maxwell theory, predicts that the velocity of the propagation of signals is not the velocity of light. See Chap. 12. He presents the magnetic monopole concept resulting from the amended Maxwell picture. His motive goes beyond the standard Maxwell formalism and generates new physics utilizing a non-Abelian gauge theory [17].

The SU_2 group gives us symmetry breaking to the U_1 group which can act to create a mass splitting symmetry that yield a photon of finite (but necessarily small) rest mass which may be created as self energy produced by the existence of the vacuum. This finite rest mass photon can constitute a propagation signal carrier less than the velocity of light. We can construct the generators of the SU_2 algebra in terms of the fields $\underline{E}$, $\underline{B}$, and $\underline{A}$. The usual potentials, A_μ are expressed as the important 4-vector quality, $A_\mu = \left(\underline{A}, \phi\right)$ where the index runs 1 to 4. One of the major purposes of introducing the vector and scalar potentials is to subscribe to their non-physicality because of the desire by physicists to avoid the issue of action at a distance. In fact in gauge theories, A_μ is all there is! Yet it appears that in fact these potentials yield a basis for a fundamental nonlocality and have real physical consequences!

Let us address the specific case of the SU_2 group and consider the elements of a non-Abelian algebra such as the fields with SU_2 (or even SU_n) symmetry then we have the commutation relations where $XY-YX \neq 0$ or $[X,Y] \neq 0$. This is reminiscent of the Heisenberg uncertainty principle non-Abelian gauge. Barrett explains that SU_2 fields can be transformed into U_1 fields by symmetry breaking. For the SU_2 gauge amended Maxwell theory additional terms appear in term of operations such $\underline{A} \cdot \underline{E}, \underline{A} \cdot \underline{B}$ and $\underline{A} \times \underline{B}$ and their non-Abelian cases. For example $\nabla \cdot \underline{B}$ no longer equals zero but is given as $\nabla \cdot \underline{B} = -jg\left(\underline{A} \cdot \underline{B} - \underline{B} \cdot \underline{A}\right) \neq 0$ where $[A,B] \neq 0$ for the dot product of $\underline{A}$ and $\underline{B}$ and hence we have a magnetic monopole term and j is the current and g is a constant. Also, Barrett gives references to the Dirac, Schwinger and t'Hooft monopole work. Further commentary on the SU_2 gauge conjecture of Harmuth [18] that under symmetry breaking, electric charge is considered but magnetic charges are not. Barrett further states that the symmetry breaking conditions chosen are to be determined by the physics of the problem. These non-Abelian algebras have consistence to quantum theory.

In our analysis, using the SU_2 group there is the automatic introduction of short range forces in addition to the long-range force of the U_1 group. U_1 is 1D and Abelian and SU_2 is 2D and is non-Abelian. U_1 is also a subgroup of SU_2. The U_1 group is associated with the long range $1/r^2$ force and SU_2, such as for its application to the weak force yields short range associated fields. Also SU_2 is a subgroup of the useful SL(2,c) group of non compact operations on the manifold. The SL(2,c) group is a semi-simple 4D Lie group and is a spinor group relevant to the relativistic formalism and is isomorphic to the connected Lorentz group associated with the Lorentz transformations. It is a conjugate group to the SU_2 group and contains an inverse. The double cover group of SU_2 is SL(2,c) where SL(2,c) is a complexification of SU_2. Also LS(2,c) is the double cover group of SU_3 related to the set of rotations in 3D space [24]. Topologically, SU_2 is associated with isomorphic to the 3D spherical, O_3^+ (or three-dimensional rotations) and U_1 is associated with the O_2 group of rotations in two dimensions. The ratio of Abelian to non-Abelian components, moving from U_1 to SU_2, gauge is 1 to 2 so that the short-range components are twice as many as the long-range components.

Instead of using the SU_2 gauge condition we use SL (2,c) we have a non-Abelian gauge and hence quantum theory and since this group is a spinor and is the double cover group of the Lorentz group (for spin ½) we have the conditions for a relativistic formalism. The Barrett formalism is non-relativistic. SL (2,c) is the double cover group of SU_2 but utilizing a similar approach using twister algebras yields relativistic physics.

5. Concluding Remarks

It appears that complex geometry can yield a new complementary unification of quantum theory, relativity and allow a domain of action for nonlocality phenomena, such as displayed in the results of the Bell's theorem tests of the EPR paradox [9,25], and in which the principles of the quantum theory hold to be universally. The properties of the nonlocal connections in complex 4-space may be mediated by non -or low dispersive loss solutions. We solved Schrödinger equation in complex Minkowski space [26-29]. See Chaps. 11 and 13. In progress is research involving other extended gauge theory models, with particular interest in the nonlocality properties on the spacetime manifold, quantum properties such as expressed in the EPR paradox and coherent states of matter.

TABLE 2
Comparison of Quantum Theory, Relativistic Maxwell's Equations and Gauge Groups

QUANTUM THEORY	GAUGE THEORY
Physics	Mathematics
Gauge Theory	Principle Fiber Bundle
Gauge Group	Fiber Symmetry Group
Spacetime	Bose Space
Gauge Potential Field, A_μ	Connection 1-Form, U_1
Gauge Field Strength, $F_{\mu\nu}$	Curvature of Connected 2-Form (spin 2)
Gauge Particle (Boson)	Basic Elements of Lie Algebra Symmetry Groups
Matter Field	Spin or Valued Function on the Principle Bundle Basic Elements, Vector Space Acted on by a Symmetry Group

Utilizing Coxeter graphs or Dynkin diagrams, Sirag [24] lays out a comprehensive program in terms of the A_n, D_n and E_6, E_7 and E_8 Lie algebras constructing a hyper dimensional geometry for as a classification scheme for elementary particles. Inherently, this theory utilizes complexified spaces involving twisters and Kaluza-Klein geometries. This space and the complex 8-space incorporate string theory and Grand Unification Theories (GUT) models [30,31]. We display the comparison of relativistic electromagnetic theory, quantum theory and gauge groups in Table 1. Gauge potentials, A^{μ} and gauge field strengths, A^{μ} are compared to U_1 and the Weyl gauge theories and to the Lie algebras of the supersymmetry groups, Su_n.

It appears that utilizing the complexification of Maxwell's equations with the extension of the gauge condition to non-Abelian algebras, yields a possible metrical unification of relativity, electromagnetism and quantum theory. This unique new approach yields a universal nonlocality [32,33]. No radical spurious predictions result from the theory, but some new predictions are made which can be experimentally examined. Also, this unique approach in terms of the twister algebras may lead to a broader understanding of macro and micro nonlocality and possible transverse electromagnetic fields observed as nonlocality in collective plasma state and other media [34]. See Chap. 11.

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