Field Theory of Quintessence: Derivation of equation of motion of such scalar Fields

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Abstract

The variant of the quintessence theory is proposed in order to get an accelerated expansion of the Friedmannian Universe in the frameworks of relativistic theory of gravitation.The substance of quintessence is built up the scalar field of dark energy. It is shown, that function V (Φ) , which factorising scalar field Lagrangian (is a scalar field) has no influence on the evolution of the Universe. Some relations, allowing to find explicit dependence Φ on time, were found, provided given function . V (Φ)

Introduction

On the edge between 20th and 21st centuries an outstanding discovery had been made: it was established that expansion of the Universe presently was going with acceleration. To explain this phenomenon, it is sufficient to assume, that in the homogeneous and isotropic Universe (Friedmann Universe) there is a matter having an unusual equation of state:

$P_{\mathrm{Q}} = -(1-v)r_{\mathrm{Q}}$

where is the isotropic pressure, P_{Q}, ρ_{Q} is the mass density, and the following restriction] for parameter ν is a consequence of general statements of relativistic theory of gravitation (RTG)

$0 < v < 2/3$

A substance having such an equation of state has been called the "quintessence". It is supposed, that the quintessence is a real scalar field. No wonder, that modeling of the perfect fluid with negative preassure is provided with a rather strange scalar field Lagrangian.

Let us designate the scalar field of quintessence as $\Phi(x^{\mathrm{a}})$and postulate, that its Lagrangian density is given as follows

$$L = -\sqrt{-g}\, V(\Phi)\, (I^2)^q \qquad \ldots\ldots\ldots\ldots (1)$$

where g = det $g_{\mu\nu}$, $g_{\mu\nu}$ is a metric tensor of the effective Riemannian space , q is a number, $V(\Phi)>0$, is some function of the field Φ, and

$$I = g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi = g^{\alpha\beta}\nabla_\alpha\Phi\nabla_\beta\Phi$$

where ,[?] is a covariant derivative with respect to metric $g^{\alpha\beta}$

The Lagrangian densities of the form

$$L = -\sqrt{-g}V(F)F(I)$$

where F(I) is an arbitrary function of I , have been considered earlier

Our choice follows from intention to get equation of state in the frameworks of the field theory of quintessence, which in their turn can explain the accelerated expansion of the Friedmannian Universe in Relativistic Gravitation theory.

The Equation Of motion for the field $F(x^a)$

According to the variational principle, the equation of motion for the field Φ which has the Lagrangian) can be obtained from the Euler-Lagrange formula

$$\frac{\partial L}{\partial\Phi} - \partial_\mu \frac{\partial L}{\partial(\partial_\mu\Phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

After substitution of Eq. (1) into Eq. (3) we obtain

$$(4q-1)\frac{d\ln V(\Phi)}{d(\Phi)}I^2 - 8q\,(2q-1)\,g^{\alpha\mu}\partial_\alpha\Phi\left(\Gamma^\lambda_{\mu\tau}g^{\tau\beta}\partial_\lambda\partial_\beta\Phi - g^{\lambda\beta}\right)\partial_\beta\Phi\partial_\lambda\Phi - 4q\left(g^{\mu\tau}\Gamma^\alpha_{\mu\tau}\partial_\alpha\Phi - g^{\lambda\mu}\partial_\lambda\partial_\mu\Phi\right)I = 0$$

Here we have

$$\Gamma^\alpha_{\mu\tau} = \frac{1}{2}g^{\alpha\beta}\left(\partial_\mu g_{\beta\tau} + \partial_\tau g_{\beta\mu} - \partial_\beta g_{\mu\tau}\right)$$

In terms of covariant derivatives [?]α , Eq. (4) has the following form

$$(4q-1)\frac{d\ln V(\Phi)}{d\Phi}I^2 + 8q(2q-1)g^{\alpha\mu}g^{\lambda\beta}\nabla_\alpha\Phi\nabla_\beta\Phi\nabla_\lambda\nabla_\mu\Phi +$$

$$+4qg^{\lambda\mu}\nabla_\lambda\nabla_\mu\Phi I = 0.$$

$\sqrt{-g} = a^{6}_{\max}$From the langrangian desity we can derive the symmetric energy momentum tensor density of the field $\phi(x^a)$. according to Hilbert,

$T_{\mu\nu} = 2\frac{\delta L}{\delta g^{\mu\nu}}$

as L does not depend on derivatives of the metric tensor $g_{\alpha\beta}$ we obtain,

$$T_{\mu\nu} = 2\ \frac{\delta L}{\delta g^{\mu\nu}} = \sqrt{-g}V\left(\phi\right)\left(I^2\right)^{q-1}\left[g_{\mu\nu}I^2 - 4qI\vartheta_\mu\phi\vartheta_\nu\phi\right] \quad ....4$$

Equation of motion for field (ϕ) and energy -momentum tensor in the homogeneous and isotropic universe

For a homogeneous and isotropic universe metric tensors $g^{\alpha\beta},\ g_{\alpha\beta}$ in cartesian coordinates are given as follows:

$$g_{00} = 1;\ g_{11} = g_{22} = g_{33} = -a^4_{\max}a^2,\ g_{\alpha\beta} = 0,\ \alpha \text{ and } \beta \text{ are not equal} \ ....5$$

and

$$g^{00} = 1\ ;\ g^{11} = g^{22} = g^{33} = \frac{-1}{a^4_{\max}a^2}\ ,\ g^{\alpha\beta} = 0 \quad .... \ 5"$$

here $a\left(\tau\right)$ is the scalar factor and $a_{\max} < \frac{1}{0}$ is maximal value . the square root $\sqrt{-g}$ is equal to

$$\sqrt{\sqrt{\sqrt{-g}}} = a^6_{\max}a^3 \quad .... \ 6$$

to provide that field (ϕ) does not break homogeneous and isotropy of the universe , it is neccasasry to assume , that $\phi\left(x^a\right)$ depands on τ only .then , according to

$$I = \left(\partial_0\phi\right)^2$$

and for $T_{\mu\nu}$ taken from equation 4 in the view of eq. 5 we obtain,

$$\left[T_{00} = (1-4q)\ a^6_{\max}a^3V\left(\phi\right)\left[\partial_0\phi\right)^2\right]^{2q} \ .... \ 6$$

$$T_{11} = T_{22} = T_{33} = -a^{10}_{\max}a^5V\left(\phi\right)\left[\left(\partial_0\phi\right)^2\right]^{2q} \ .... \ 7$$

$T_{\mu\nu} = 0,\ \mu\nu$ are not equal

it is standardly assumed that in the friedman universe the matter can be described by the energy -momentum tensor density of some perfect fluid. This tensor is written as follows

$$T_{\mu\nu} = \sqrt{-g}\left[\left(\rho+p\right)U_\mu U_\nu - \ g_{\mu\nu}p\right]..... \ .... \ 8$$

As for the homogeneous and isotropic universe 4-velocity U_μ has the form:

$U_0 = 1,\ U_k = 0$

we find from equation 8 in view of eq. 5,

$T_{00} = a_{\max}^6 a^3 \rho$ 9

$T_{11} = T_{22} = T_{33} = a_{\max}^{10} a^5 p$, $T_{\mu\nu} = 0$ for μ is not equal to ν 10

if we extend this hypothesis of an opportunity to represent the energy momentum tensor density of matter by ideal fluid energy momentum tensor density, onto the quintessence field , then after comparison equation 6 and equation 7 with equation 9 and 10 accordingly , we get

$\rho_Q = (1 - 4q)\ V(\phi) \left[(\partial_0 \phi)^2\right]^{2q}$, ... 11

$P_Q = -V(\Phi) \left[(\partial_0 \Phi)^2\right]^{2q}$ 12

Hence we have

$$P_Q = -\frac{1}{(1-4q)} \rho_Q \ \ldots\ldots\ldots\ldots \ 13$$

This is required equation for quintessence

by putting

$$1 - 4q = \frac{1}{1-\nu} \ \ldots\ldots\ldots\ldots \ 14$$

we see that equation 13 transform into equation of state . It gives a ground to treat field Φ with langrangian as quintessence field if only

$$q = -\frac{v}{4(1-v)} \ \ldots\ldots\ldots\ldots \ 15$$

considering restriction for a parameter v , we obtain from eq. 15

$-\frac{1}{2} < q < 0$

thus the langrangian density for a quintessence has the following fo0rm

$$L = -\sqrt{-g} V(\Phi) \left(I^2\right)^{-\frac{v}{4(1-v)}} \ \ldots\ldots\ldots\ldots 16$$

Result:

I considered first of all a arbitrary function which can be kinetic term in langrangian of quintessence and here you can see that quintessence must have negative pressure which leads to understand that it has some anti-gravity properties which accelerates the friedman universe ,this is my result .

Referances:

introduction to quintessence

https://arxiv.org/abs/gr-qc/0407023

https://arxiv.org/abs/gr-qc/0001051