Gravity and Invariant Cosmic Wavelength Four Volume Action Densities

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Abstract

This paper explores the possibility of a different level of spacetime action symmetry when compared to the usual Lagrangian that Einstein's field equations can be derived from. It relies heavily, and builds on ideas developed in an earlier paper [7] which looked at Standard Model particles built from infinite superpositions, borrowing mass from a Higgs type scalar field, and energy from zero point fields. At cosmic wavelengths, zero point energy densities are infinitesimal. To make available and required zero point energies equal, space expands exponentially with time. This balance occurs at a minimum graviton wavenumber $k_{\min}$ (or maximum wavelength). The density $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$, of $k_{\min}$ gravitons has an invariant $K_{Gk\min}$ in all coordinates, for all spacetime. The value of $k_{\min} \approx R_{Horizon}^{-1}$ decreases with cosmic time T, but increases around mass concentrations, inversely with the clock rate $\sqrt{g_{00}}$ in the local metric. It also increases with peculiar velocities relative to comoving coordinates. This paper proposes that this relates with an "Invariant Four Volume Action Density" at that maximum wavelength $k_{\min}$. Borrowed cosmic wavelength quanta are Planck scale zero point action modes, redshifted from a holographic horizon receding at virtually light velocity. This fits an infinitesimally modified General Relativity. We also extend these arguments to include angular momentum and the Kerr Metri**c**. The earlier paper, for simplicity, included only the vast majority $(\psi \text{Universe} * \psi_m + \psi_m * \psi \text{Universe})$ of $k_{\min}$ gravitons around a mass concentration m, We now include the relatively smaller number $\psi_m * \psi_m$ of $k_{\min}$ gravitons emitted by mass m, adding a dimensionless m^2 / r^2 term to $2m / r$ in the metric, which becomes $g_{00} = 1 - m / r - 1.39 m^2 / r^3 = g_{rr}^{-1}$ in the non rotating case, and is equivalent to ≈ 700 metres extra distance to the centre of the sun for all the planets, with no change in their orbital periods. The effect of m^2 / r^2 is significant close to Black Holes. The radius of a non-rotating Black Hole increases $\approx 27.5\%$ from $r = 2m$ to $r \approx 2.55m$, but maximum spin Black Holes remain at $r = m$. Only the last cycle or so of black hole mergers are significantly affected. The extra acceleration due to m^2 / r^2 could slightly speed up mergers for any total angular momentum and mass. This may allow spins to be aligned with their mutual orbits; as thought more probable in some recent mergers. It also increases their apparent mass slightly. The change in the Riemannian tensor due to m^2 / r^2 is of same form, but opposite sign, when compared with the r_Q^2 / r^2 term in electrically charged, Reissner-Nordstrom and rotating Kerr-Newman, metrics. A negative energy massless particle in the Energy-Momentum tensor can generate this term in the metric, just as massless particles in the electromagnetic field do with diagonal stress tensor terms contracting to zero.

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1 Introduction

The universe we live in is currently described by two models: “The Standard Model of Particle Physics” and “The Standard Model of Cosmology”. While the Standard Model of Particle Physics is remarkably accurate in its predictions, is mathematically elegant and apparently complete in many respects; it is also at the same time incomplete. Supersymmetry, proposed to solve some of its issues, is at this date, not panning out as expected, with some physicists questioning whether supersymmetry is the hoped for answer. Nuetrinoes have a small mass, which the Standard Model does not predict. Gravity is not included and there is no force unification without supersymmetry.

The Standard Model of Cosmology or “The Lambda-CDM Model” requires Dark Energy to explain the accelerating expansion of space, with no good understanding of what causes it. Without initial inflation, there is no good explanation of why space is Euclidean on average, or why regions initially causally separated are so homogeneous; but there is still no widely accepted understanding of what causes this inflation.

In the first paper [7] we attempted to show that the fundamental particles of the Standard Model can be built from infinite superpositions apart from infinitesimal, but important differences. They all had mass which naturally divided into two sets. Spin 2 gravitons, spin 1 photons and gluons, all had infinitesimal mass approximately the inverse (always) of the causally connected horizon radius *of the observable universe* $R_{OU} \approx 46 \times 10^{9}$ light years or $\approx 10^{-33} eV$. (This value is close to some recent proposals **[8]** giving gravitons a mass of $< 10^{-33} eV$ to explain the accelerating expansion of the universe.) The rest had finite masses of micro electron volts upwards. Infinite superpositions are always built in some rest frame in which they had no nett momentum $\mathbf{p}$, but only $\mathbf{p}^2$ terms. In the “infinitesimal” mass set this rest frame can be, and usually is, travelling at virtually light velocity, as seen from our usual (nearly) comoving frame. We also divided the world of all interactions into two sets.

(a) **Primary Interactions** are purely virtual. They build all the fundamental particles in the form of infinite superpositions. We can not see any direct signs of primary interactions.

(b) **Secondary Interactions** are all the others that occur between fundamental particles, both virtual and real. They are the real world of experiments that the Standard Model is all about.

The rules for borrowing energy from zero point fields can be different for both (a) & (b). Primary interactions are between spin zero particles borrowed from a Higgs type scalar field and the zero point fields.

In the 1970’s models were proposed with preons as common building blocks of leptons and quarks [10] [11] [12] [13] In contrast with the spin zero particles in this paper, most of these earlier models used *real spin ½* building blocks. As in these earlier models, this paper also calls the common building blocks preons; *but here the preons are both virtual, and spin zero bosons.* There are only three preons; red, green and blue, all with positive electric charge. There are also three anti preons; antired, antigreen and antiblue, all with negative electric charge. As preons are spin zero there can be no weak charge involved in primary interactions. This is all explained more fully in the first paper. These preons build all spin ½ leptons and quarks, spin 1 gluons, photons, W & Z particles, plus spin 2 gravitons. This is in contrast to only leptons and quarks in earlier preon models. We found that the fundamental forces do not unite at the Planck energy cutoff of superpositions. They relate with each other in a manner that meshes nicely with the Standard Model, but do not relate with versions including supersymmetry. In the final third of this first paper we tried to fit infinite superpositions with General Relativity and The Standard Model of Cosmology. Because these infinite superpositions borrow energy from zero point fields, which have virtually zero density at cosmic wavelengths; it only works if space expands exponentially with time, and if space is flat on average. The equations we derived looked the same for all comoving observers. Regardless of an observer's position in the universe this expansion looked the same apart from the effect of initial quantum fluctuations at the start. This may remove one of the key reasons for inflation. The universe in this proposed scenario should look the same, and be flat on average, for all observers with or without inflation. Even to observers near the horizon or outside it. The properties and equations controlling distant universes should be identical to ours and there would be no metaverses which are a natural endpoint of inflation. We found that all particles have a maximum wavelength that is approximately the same as the size of the causally connected universe at any cosmic time T. At this maximum wavelength there is a minimum wavenumber we called $k_{\min}$. We found that the density of $k_{\min}$ gravitons at this maximum wavelength was always proportional to a universal invariant which we labelled $K_{Gk\min}$. The same invariance applies to action densities $@\, k_{\min}$. We connected this with infinitesimally modified GR equations locally, but significant implications at cosmic scale.

Solutions to $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4}\left[T_{\mu\nu} - T_{\mu\nu}(\text{Background})\right]$ are consistent with $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$
where $K_{Gk\min}$ is invariant in all coordinates through all Spacetime, and $\rho_{Gk\min}$ is the density of maximum wavelength $k_{\min}$ gravitons, but wavenumber $k_{\min}$ depends on local clocks or $\sqrt{g_{00}}$.
These solutions are equivalent to an "Invariant Four Volume $k_{\min}$ Graviton Action Density".

In comoving coordinates $T_{\mu\nu}(\text{Background})$ has just one component $T_{00} = \rho_U$ the average density of the universe, or only a few hydrogen atoms per cubic metre. This modification limits the range of GR to scales smaller than the radius of the universe and guarantees flatness on average regardless of the value of Ω.

The overall exponential expansion of space is controlled by equations balancing the zero point action available at cosmic wavelenths to that borrowed by infinite superpositions. Dark

Energy is not required for this accelerating expansion, but Dark Matter is still required inside galaxies because of centrifugal forces due to their fast rotation. We found a spin 2 massive graviton type infinite superposition as a possible dark matter candidate that won't show up in current weak interaction type searches. Spacetime has to warp in accord with GR around mass concentrations to make available the zero point energy required by their extra cosmic wavelength gravitons. The first paper only looked at long range gravitons emitted by a mass interacting with the rest of the mass in the universe $(\psi \text{Universe} * \psi_m + \psi_m * \psi \text{Universe})$, it ignored the relatively smaller number of long range gravitons emitted by the mass interacting with itself $(\psi_m * \psi_m)$. This paper looks at the $(\psi_m * \psi_m)$ term which is most significant near black holes. It unfortunately messes up a nice agreement with the Schwarzchild solution by adding a dimensionless m^2 / r^2 term to the usual $2m / r$ in the metric. The effect is equivalent to increasing the distance to the centre of the sun by about 700 metres for all the planets, assuming no change in their orbital periods. This may well be measurable in the foreseeable future. But a non rotating black hole radius increases approximately 27.5% from $r = 2m$ to $r \approx 2.55m$. With angular momentum this becomes the modified ergosphere maximum diameter, but the radius of a maximum spin black hole is unchanged at $r = m$.

These changes, mainly close to Black holes, initially appear to introduce a tension with the field equations of General Relativity. However in Section 2.6 we look at the similarities between this m^2 / r^2 term and the dimensionless r_Q^2 / r^2 terms of both the Reissner-Nordstrom and Kerr-Newman charged black hole metrics. Their effects on the Riemannian curvature tensor are of the same form, but opposite sign to that from an m^2 / r^2 term. There are no covariance problems with electromagnetic field massless particles. If we include in the Stress-Energy tensor a massless negative energy particle, covariance is similarly maintained. It is a bit like, but not the same as, including negative gravitational field energy; which Einstein specifically excluded because of covariance problems. This m^2 / r^2 term increases the merging energies for a fixed mass at any radius. The same applies to the gravitational wave radiated energies. It increases the apparent masses of merging black holes above those derived with only an m / r term. It introduces extra radial acceleration, and may speed up final mergers of black holes for any total mass and spin. This may relate with the merger [26], where if General Relativity holds to the horizon, spins were found unlikely to be aligned with their mutual orbits, as current astrophysics theory had expected. The accuracy of these observations will almost certainly increase with time, either confirming this or not.

In the *rest frame* in which the particles are built from infinite superpositions the spin zero preons are born with *zero momentum*. This means they are *born with infinite wavelength* allowing the possibility that they *can borrow zero point energy from an infinite distance*. We proposed that they borrow redshifted Planck energy zero point quanta from a holographic horizon receding at light like velocities relative to comoving coordinates instantaneously on that horizon. This is necessary because at cosmic wavelengths of $\approx R_{OU}$ the density of zero point modes is almost zero, and insufficient to build all the fundamental particles; gravitons in particular. The Riemannian spacetime curvature tensor is controlled by the need to keep both "The Graviton $K_{Gk\min}$, & Action Density@ $k_{\min}$ or, $K(k_{\min}\text{Action})$ " invariant. For the sake of clarity, this paper repeats a heavily revised portion of the final third of that first paper, but now includes cosmic wavelength gravitons emitted by the mass interacting with itself $(\psi_m * \psi_m)$, the effects of angular momentum, and gravitational waves.

Einstein published his General Theory of Relativity [1] 100 years ago. There have been many attempts over the intervening years to modify it with different goals in mind. A dissertation by Germanis [2] discusses some of these modifications [3] [4] [5] [6]. On its initial publication it was criticized for not including gravitational field energy, but over the last century, many physicists have tried unsuccessfully to covariantly do this. The modifications proposed in these papers, are the extra m^2/r^2 term in the metric with its large effect close to black holes, and our equations being consistent with $T_{\mu\nu}$ changing to $T_{\mu\nu} - T_{\mu\nu}(\text{Background})$, which has an infinitesimal effect locally, but significant implications at cosmic scale. The Λ-CDM Model of Cosmology is based on General Relativity as it is currently interpreted. It requires Dark energy to accelerate the expansion, it requires $\Omega \approx 1$, it requires "Inflation" so that regions initially out of causal contact can have (almost) uniform properties, and to produce the observed average flatness. The ideas proposed in these two papers, may well eliminate the need for these requirements. If both $K_{Gk\min}$ & $K(k_{\min}\text{Action})$ are *invariant at all points in spacetime*, the equations controlling the expansion of space and the warping of spacetime around mass concentrations are the same for all observers in this universe and should also be for those far away. There should be no metaverses and no need for anthropic arguments. The original arguments behind the Cosmological Model, of uniformity on average everwhere, should be absolutely true. While the arguments proposed in these two papers are radical, and no doubt contain many errors, the principles behind them may well suggest a possible different path forward. But much tidying up, and putting these ideas on a more rigorous foundation, would be required. It is almost certainly to our evolutionary advantage that what we call established, or collective knowledge, or paradigms particularly in science, change slowly; and only after evidence for change builds to a tipping point. In the end however, science, as it always has in the past, slowly but surely progresses towards the simplest explanations.

Finally, so that these ideas are accessible to the widest possible audience, many more details, and the simplest possible explanations are included, than required by experts in the field.

2 The Expanding Universe and General Relativity

2.1 Zero point energy densities are limited

If fundamental particles can be built from energy borrowed from the spatial component of zero point fields, and this energy source is limited, (particularly at cosmic wavelengths) there must be implications for the maximum possible densities of these particles. In section 2.2.3 in [7] we discussed how preons build massive fundamental particles, and are born from a Higg's type scalar field with zero momentum in the laboratory rest frame. Infinitesimal mass particles such as gravitons borrow their mass from the time component of the same zero point fields. In this frame they have infinite wavelength and can borrow from anywhere in the universe. This suggests there should be little effect on localized densities, but possibly on overall average densities in any of these universes. So which fundamental particle is there likely to be most of? Working in Planck, or natural units with $G=1$ and a graviton coupling between Planck masses of one, there are approximately $M \approx 10^{61}$ Planck masses within the causally connected observable universe. Their average distance apart is approximately the radius R_{OH} of this region. There should be approximately $M^2 \approx 10^{122}$ virtual gravitons with wavelengths of the order of radius R_{OH} within this same volume. No other fundamental particle is likely to approach these values, for example the number of virtual photons *of this extreme wavelength* is much smaller. (Virtual particles emerging from the vacuum are covered in section 2.5.4) If this density of virtual gravitons needs to borrow more energy from the zero point fields than what is available at these extreme wavelengths does this somehow control the maximum possible density of a causally connected universe?

2.1.1 Virtual Particles and Infinite Superpositions

Looking carefully at section 3.3 in [7] we showed there that, for all interactions between fundamental particles represented as infinite superpositions, the actual interaction is between only single wavenumber k superpositions of each particle. *We are going to conjecture that a virtual particle of wavenumber k for example is just such a single wavenumber k member*. Only if we actually measure the properties of real particles do we observe the properties of the full infinite superposition. The full properties do not exist until measurement, just as in so many other examples in quantum mechanics. We will use this conjectured virtual property below when looking at the probability density of virtual gravitons of the maximum possible wavelength. These virtual gravitons would be a superposition of the three modes $n=3,4,5$ of a single wavenumber k, as in Table 4.3.1 in [7]. Time polarized or spherically symmetric versions would be a further equal $(1/\sqrt{5})$ superposition of $m=-2,-1,0,+1,+2$ states of the above $n=3,4,5$ mode superpositions. A spin 2 graviton in an $m=+2$ state is simply a superposition of the three modes $n=3,4,5$ as above but all in an $m=+2$ state. This is explained in the first paper section 3.2.2 page 30 [7].

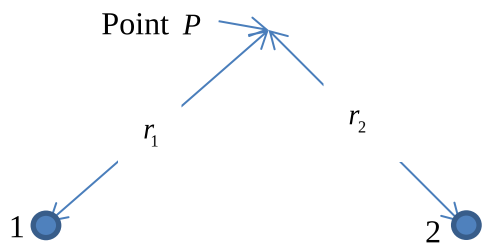
2.1.2 **Virtual graviton density at wavenumber k in a causally connected Universe**

From here on we will work in natural or Planck units where $\hbar = c = G = 1$.

General Relativity predicts nonlinear fields near black holes, but in the low average densities of typical universes *we can assume approximate linearity*. The majority of mass moves slowly relative to comoving coordinates so we can ignore momentum (i.e. $\beta << 1$), provided we limit this analyses initially to comoving coordinates. Spin 2 gravitons transform as the stress tensor in contrast to the 4 current Lorentz transformations of spin 1, but, at low mass velocities the only significant term is the mass density T_{00}. In comoving coordinates the vast majority of virtual gravitons will thus be *time polarized or spherically symmetric* which we will for simplicity call scalar. We should be able to simply apply the equations in sections 3.4 & 3.5 in [7] to spin 2 virtual graviton emissions, as they should apply equally to both spins 1 & 2 at low mass velocities. (This is not necessarily so near black holes.) We will assume spherically symmetric $l = 3$ wavefunctions can emit both spins 1 & 2 scalar virtual bosons, and $l = 3, m = \pm 2$ states can emit both $m = \pm 1$ spin 1 bosons and $m = \pm 2$ spin 2 gravitons. Section 3.4 in [7] derived the electrostatic energy between infinite superpositions. In flat space we looked at the amplitude that each equivalent point charge emits a virtual photon, and then focused on the interaction terms between them. Thus we can use the same scalar wavefunctions Eq's. (3.4.1) in [7] for virtual scalar gravitons as we did for virtual scalar photons. Using $(\psi_1 + \psi_2)*(\psi_1 + \psi_2) = (\psi_1 * \psi_1) + (\psi_1 * \psi_2 + \psi_2 * \psi_1) + (\psi_2 * \psi_2)$ we showed in section 3.4.1 in [7] that the interaction term for virtual photons is

$$\psi_1 * \psi_2 + \psi_2 * \psi_1 = \frac{4k}{4\pi r_1 r_2} e^{-k(r_1 + r_2)} \cos k(r_1 - r_2) \tag{2.1. 1}$$

Figure 2.1. 1



Where r_1 & r_2 are the distances to some point P from two charges or masses 1 & 2, and we are looking at the interaction at point P as in Figure 2.1. 1. Equation (2.1. 1) is strictly true *only in flat space* but it is still approximately true if the curvature is small or when $2m/r <<<< 1$, *which we will assume applies almost everywhere throughout the universe except in the infinitesimal fraction of space close to black holes*. In both sections 3.4 & 3.5 in [7] for simplicity and clarity, we delayed using coupling constants and emission probabilities in the wavefunctions until necessary. We do the same here. There will also be some minimum wavenumber k which we call $k_{\min}$ where for all $k < k_{\min}$ there will be insufficient zero point energy available. We want Eq. (2.1. 1) to still apply at the maximum wavelength where $k_{\min} \approx 1/R_{OU}$ ($=1/R_{ObsevableUniverse}$). In section 6 in [7] we found gravitons have an infinitesimal rest mass m_0 of the same order as this minimum wavenumber $k_{\min}$. At these extreme k values this rest mass must be included in the wavefunction negative exponential term. It is normally irrelevant for infinitesimal masses. Section 6.2 in [7] looks at $N=2$ infinitesimal rest masses finding $\langle K_{k\min} \rangle^2 \approx 1$. Using Equ's. (3.1.11) & (3.2.10) in [7] with $\hbar = c = 1$

$$\langle K_{k\min} \rangle^2 = \frac{s\langle n \rangle^2 k_{\min}^2}{2m_0{}^2} \approx 1 \ \& \ for\ spin\ 2\ gravitons\ \langle K_{k\min} \rangle^2 = \frac{\langle n \rangle^2 k_{\min}^2}{m_0{}^2} \approx 1\ or\ m_0 = \langle n \rangle k_{\min} \qquad (2.1.\ 2)$$

From Table 4.3.1 in [7] we find

$$\text{For } N=2 \text{ spin 2 gravitons } \langle n \rangle \approx 3.33 \text{ so that } m_0 \approx 3.33 k_{\min} \qquad (2.1.\ 3)$$

This virtual mass m_0 increases the ΔE term in $\Delta E \cdot \Delta T \approx \hbar/2$ for the virtual graviton from $\Delta E = k$ to $\Delta E = \sqrt{k^2 + m_0^2}$. This reduces the range $r \approx \Delta T \approx \Delta E^{-1}$ over which it can be found, which is controlled by the exponential decay term e^{-kr} in its wavefunction. This term becomes $e^{-r\sqrt{k^2+m_0^2}}$ as we approach $k_{\min}$. Using Eq. (2.1. 3) we can define a k' such that

$$k' = \sqrt{k^2 + m_0^2} \approx \sqrt{k^2 + 3.33^2 k_{\min}^2} \quad \text{and} \quad k'_{\min} \approx \sqrt{k_{\min}^2 + 11.09 k_{\min}^2} \approx 3.477 k_{\min} \qquad (2.1.\ 4)$$

A normalized virtual massles graviton wavefunction is $\psi = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr+ikr}}{r}$ see Eq. (3.4.1) in [7] and for infinitesimal mass gravitons this becomes using Eq. (2.1. 4)

$$\text{A massless } \psi = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr+ikr}}{r} \quad \text{becomes with infinitesimal mass } \sqrt{\frac{2k'}{4\pi}} \frac{e^{-k'r+ikr}}{r} \qquad (2.1.\ 5)$$

Thus the massless interaction term in Eq. (2.1. **1**) becomes with this infinitesimal mass m_0

$$\psi_1 * \psi_2 + \psi_2 * \psi_1 = \frac{4k'}{4\pi r_1 r_2} e^{-k'(r_1+r_2)} \cos k(r_1 - r_2) \qquad (2.1.\ 6)$$

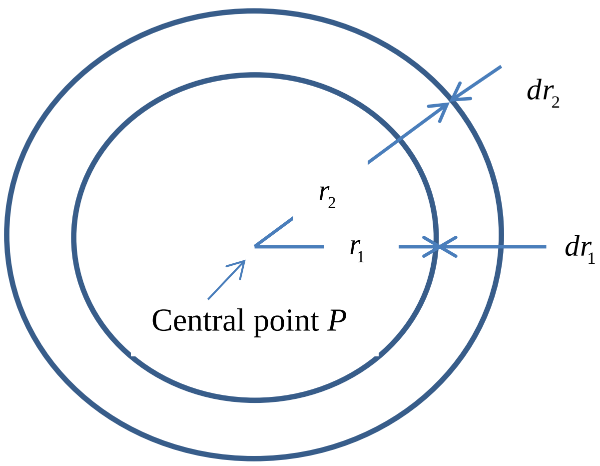


Figure 2.1. 2

Let point P be anywhere in the interior region of a universe as in Figure 2.1. 2. Let the average density be ρ_U (subscript U for homogeneous universe density) Planck masses per unit volume. Consider two spherical shells around the central point P of radii $r_1 \& r_2$ and thicknesses $dr_1 \& dr_2$ with masses $dm_1 = \rho_U dv_1 = 4\pi r_1^2 dr_1 \rho_U$ & $dm_2 = \rho_U dv_2 = 4\pi r_2^2 dr_2 \rho_U$ Now we expect the graviton coupling constant α_G to be $=1$ between Planck masses, but we do not yet really know this for certain, so let us define (we find later that $\alpha_G = 1$ works out)

$$\text{The Secondary graviton coupling constant between Planck masses } = \alpha_G \qquad (2.1.\ 7)$$

In section 3.4.1 in [7] Eq. (3.4.3) used a scalar emission probability $(2\alpha/\pi)(dk/k)$ which becomes $(2\alpha_G/\pi)(dk/k)$ between Planck masses. (We return to this in section 2.4.2) Now quantum interactions are instantaneous over all space but distant galaxies recede at light like and greater velocities. However at the *same cosmic time T in all comoving coordinate systems, clocks tick at the same rate, and a wavenumber k (or frequency) in one comoving coordinate system measures the same in all comoving coordinate systems*. Thus as we integrate over radii $r_1 \& r_2 = 0 \to \infty$ we can still use the same equations as if the distant galaxies are not moving. (The vast majority of mass is moving relatively slowly in these comoving coordinate systems and we return to this important comoving coordinate property in section 2.4.1). Using this proposed coupling probability between Planck masses $(2\alpha_G/\pi)(dk/k)$ we can now integrate over both radii $r_1 \& r_2$; but to avoid counting all pairs of masses $dm_1 \& dm_2$ twice, we must divide the integral by two. The total probability density of virtual gravitons at any point P in the universe at wavenumber k is using Eq. (2.1. 6)

$$\rho_{Gk} = \frac{\rho_U^2}{2}\alpha_G \frac{2}{\pi}\frac{dk}{k}\iint\limits_{0\to\infty} 4\pi r_1^2 dr_1 \cdot 4\pi r_2^2 dr_2 \cdot \frac{4k'}{4\pi r_1 r_2} e^{-k'(r_1+r_2)} \cos k(r_1 - r_2)$$

$$= 16\alpha_G \rho_U^2 \frac{k'}{k} dk \iint\limits_{0\to\infty} r_1 r_2 e^{-k'(r_1+r_2)} \cos k(r_1 - r_2)\cdot dr_1 \cdot dr_2$$

Expanding $\cos k(r_1 - r_2) = \cos kr_1 \cos kr_2 + \sin kr_1 \sin kr_2$, and then using:

$$\int_{r=0}^{r=\infty} rExp(-k'r)\cos(kr)dr = \frac{k'^2 - k^2}{(k'^2 + k^2)^2} \quad \text{and} \quad \int_{r=0}^{r=\infty} rExp(-k'r)\sin(kr)dr = \frac{2k'k}{(k'^2 + k^2)^2}$$

$$\text{finally yields } \rho_{Gk} = 16\alpha_G \rho_U^2 \frac{k'}{k} dk \frac{(k'^2 + k^2)^2}{(k'^2 + k^2)^4} = 16\alpha_G \rho_U^2 \frac{k'}{k} dk \frac{1}{(k'^2 + k^2)^2} \qquad (2.1.\ 8)$$

From Eq.(2.1. 4) $k' = \sqrt{k^2 + m_0^2} \approx \sqrt{k^2 + 11.09k_{\min}^2}$ and we can write Eq.(2.1. 8) as

$$\rho_{Gk} = 16\alpha_G \rho_U^2 \frac{\sqrt{k^2 + 11.09k_{\min}^2}}{k} dk \frac{1}{\left(2k^2 + 11.09k_{\min}^2\right)^2} = 16\alpha_G \frac{\rho_U^2}{k_{\min}^4} dk \frac{\sqrt{x^2 + 11.09}}{x\left(2x^2 + 11.09\right)^2} \quad \text{where } x = \frac{k}{k_{\min}}$$

$$\text{Wavelength } k \text{ Graviton Probability Density } \rho_{Gk} \approx \frac{0.3247\alpha_G \rho_U^2}{k_{\min}^4} dk \left[\frac{49.27}{x} \frac{\sqrt{x^2+11.09}}{\left(2x^2+11.09\right)^2} \right] \quad (2.1.\ 9)$$

$$\text{Maximum wavelength Probability Density } \rho_{Gk\min} \approx \frac{0.3247\alpha_G \rho_U^2}{k_{\min}^4} dk_{\min} \text{ when } \frac{k}{k_{\min}} = x = 1$$

As we think $K_{G\min}$ will prove to be a spacetime invariant, we will write this as follows.

$$\text{Maximum wavelength Probability Density } \rho_{Gk\min} = K_{Gk\min} dk_{\min} \text{ where } K_{Gk\min} \approx \frac{0.3247\alpha_G \rho_U^2}{k_{\min}^4} \quad (2.1.\ 10)$$

2.2 Can we relate this to General Relativity?

The above assumes a homogeneous universe that is essentially flat on average. At any cosmic time T it also assumes there is always some value $k_{\min}$ where the borrowed energy density $E_{Gk\min} = E_{ZP\min}$ the available zero point energy density@ $k_{\min}$. It is also in comoving coordinates. At the same cosmic time T, all comoving observers measure the same probability density $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ as in Eq. (2.1. 10). So what happens if we put a small mass concentration $+m_1$ at some point? The gravitons it emits must surely increase the local density of $k_{\min}$ gravitons upsetting the balance between borrowed energy and that available. However General Relativity tells us that near mass concentrations the metric changes, radial rulers shrink and local observers measure larger radial lengths. This expands locally measured volumes lowering their measurement of the background $\rho_{Gk\min}$. But clocks also slow down, increasing the measured value of $k_{\min}$. Let us look at whether we can relate these changes in rulers and clocks with the $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ of Eq. (2.1. 10).

2.2.1 Approximations with possibly important consequences

Let us refer back to Eq. (3.4.2) in [7] and the steps we took to derive it; but now including $k' = \sqrt{k + m_0} \approx \sqrt{k^2 + 11.09k_{\min}^2}$ as in Eq. (2.1. 4)

$$\psi_1 * \psi_2 + \psi_2 * \psi_1 = \frac{4k'}{4\pi r_1 r_2} e^{-k'(r_1+r_2)} \cos[k(r_1 - r_2)] \quad (2.2.\ 1)$$

And assume that *space has to be approximately flat* with errors $\propto 1-(1-2m/r)^{1/2} \approx m/r$. If we now focus on Figure 2.1. 1, equation (2.2. 1) is the probability that an infinitesimal mass virtual graviton of wavenumber k is at the point P *if all other factors are one*. Let us now put a mass of m_1 Planck masses as in Figure 2.2. 1. Also assume that the *point P is reasonably close to mass* m_1 *(in relation to the horizon radius)* at distance r_1 as in Figure 2.2. 1 and *the* *vast majority of the rest of the mass inside the causally connected or observable horizon* R_{OH} is at various radii r, equal to r_2 of Eq.(2.2. 1) where $r_2 = r >> r_1$ and thus $\cos[k(r_1 - r)] \approx \cos(-kr)$. *Only under these conditions can we approximate Eq.* (2.2. 1) *as*

$$\psi_1 * \psi_2 + \psi_2 * \psi_1 \approx \frac{4k'}{4\pi r_1 r} e^{-k'r} \cos(-kr) \quad (2.2.\ 2)$$

(We will later find that this approximation is consistent with limiting the range of GR to inside the horizon but to vast scales)

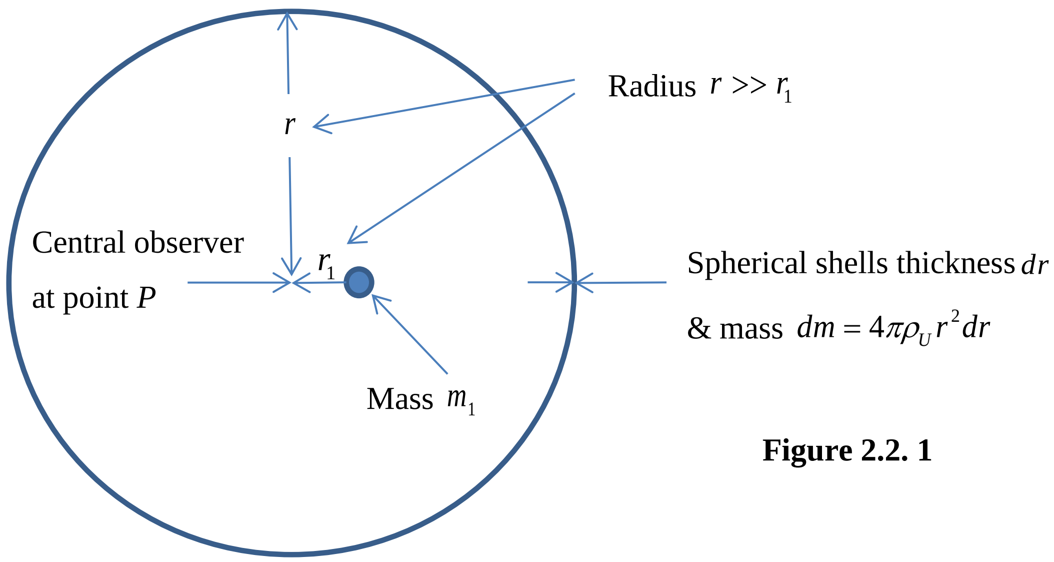


Figure 2.2. 1

As we have assumed average particle velocities are low (relative to comoving coordinates) this is *a time polarized or scalar interaction* and as *there are no directional effects* we can consider simple spherical shells of thickness dr and radius r around a central observer at the point P which have mass $dm = \rho_U 4\pi r^2 dr$. At each radius r the coupling factor $(2\alpha/\pi)(dk/k)$ using Eq. (2.1. 7) again is $(2\alpha_G/\pi)(dk/k)$ between Planck masses *and again we use the fact that all instantaneously connected comoving clocks tick at the same rate*.

$$\text{Coupling factor } = \frac{2\alpha_G m_1}{\pi} dm \frac{dk}{k} = \frac{2\alpha_G m_1}{\pi} \frac{dk}{k} \rho_U 4\pi r^2 dr \quad (2.2.\ 3)$$

Including this coupling factor in Eq. (2.2. 2)

$$(\frac{2\alpha_G m_1}{\pi} \frac{dk}{k} \rho_U 4\pi r^2 dr)(\psi_1 * \psi_2 + \psi_2 * \psi_1) \approx \left(\frac{2\alpha_G m_1}{\pi} \frac{dk}{k} \rho_U 4\pi r^2 dr\right)\left(\frac{4k'}{4\pi r_1 r} e^{-k'r} \cos(-kr)\right) \quad (2.2.\ 4)$$

$$\approx \alpha_G \frac{m_1}{r_1} \frac{8\rho_U}{\pi} \frac{k'dk}{k} r e^{-k'r} \cos(-kr) dr$$

This is virtual graviton density at point P due to each spherical shell. (ignoring the relatively *small number of particularly* $k_{\min}$ gravitons emitted by mass m_1 itself $(\psi_{m_1} * \psi_{m_1})$ see section 2.6.1). Integrating over radius $r = 0 \to \infty$ the virtual graviton density at wavenumber k using Eq's. (2.1. 4 & (2.2. 4)

$$\Delta\rho_G = \alpha_G \frac{m_1}{r_1}\frac{8\rho_U}{\pi}\frac{k'dk}{k}\int_0^\infty re^{-k'r}\cos(-kr)dr$$

$$= \alpha_G \frac{m_1}{r_1}\frac{8\rho_U}{\pi}\frac{k'dk}{k}\left[\frac{(k'^2-k^2)}{(k'^2+k^2)^2}\right] \qquad (2.2.\ 5)$$

Now $k'^2 = k^2 + m_0^2 \approx k^2 + 11.09k_{\min}^2$ and if $k = k_{\min}$ then $k_{\min}'^2 \approx 12.09k_{\min}^2$ & so when $k = k_{\min}$:

$$\Delta\rho_{Gk\min} \approx \alpha_G \frac{m_1}{r_1}\frac{8\rho_U}{\pi}\frac{\sqrt{12.09k_{\min}^2}\,dk_{\min}}{k_{\min}}\left[\frac{(12.09k_{\min}^2 - k_{\min}^2)}{(12.09k_{\min}^2 + k_{\min}^2)^2}\right] \qquad (2.2.\ 6)$$

$$\Delta\rho_{Gk\min} = (\psi_{\text{Universe}} * \psi_{m_1} + \psi_{m_1} * \psi_{\text{Universe}}) \approx \alpha_G \frac{m_1}{r_1} 0.573 \frac{\rho_U}{k_{\min}^2} dk_{\min}$$

Equation (2.1. 10) suggests $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$. In comoving coordinates in a metric far from masses where $g_{\mu\nu} = \eta_{\mu\nu}$, $k_{\min}$ has its lowest value. As we approach any mass $k_{\min}$ increases to $k''_{\min}$ where we use green double primes when $g_{\mu\nu} \neq \eta_{\mu\nu}$ to avoid confusion with the k' & $k'_{\min}$ of Eq. (2.1. 4). At a radius r from mass m the Schwarzchild metric is $(1-2m/r)^{\pm 1/2}$ for the time and radial terms. Radial rulers shrink and clocks slow, measured local volume V & frequency $k_{\min}$ both increase as $\approx 1+\frac{m}{r}$ .Thus using $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$

$$\text{If } r >>> m; \quad 1+\frac{m}{r} \approx \frac{V+\Delta V}{V} = 1+\frac{\Delta V}{V} \approx \frac{k''_{\min}}{k_{\min}} = \frac{dk''_{\min}}{dk_{\min}} = \frac{\rho''_{k\min}}{\rho_{k\min}} \qquad (2.2.\ 7)$$

So in this metric the total number of $k_{\min}$ gravitons is the original $(g_{\mu\nu} = \eta_{\mu\nu})$ $\rho_{Gk\min}$ of Eq. (2.1. 10) plus the extra due to a local mass of Eq. (2.2. 6), but we have to divide this number by the increased volume to get the new density $\rho''_{Gk\min} \approx (1+\frac{m}{r})\rho_{Gk\min}$. Thus using Eq. (2.2. 7)

$$\text{The new} \quad \rho''_{Gk\min} \approx \frac{\rho_{Gk\min} + \Delta\rho_{Gk\min}}{1+\Delta V/V} \approx \frac{\rho_{Gk\min} + \Delta\rho_{Gk\min}}{(1+m/r)} \approx (1+m/r)\rho_{Gk\min}$$

$$\rho_{Gk\min} + \Delta\rho_{Gk\min} = (1+m/r)^2\rho_{Gk\min} \approx (1+2m/r)\rho_{Gk\min} \qquad (\text{if } r >>> m)$$

$$\frac{\rho_{Gk\min}+\Delta\rho_{Gk\min}}{\rho_{Gk\min}}\approx 1+\frac{2m}{r}$$
$$\frac{\Delta\rho_{Gk\min}}{\rho_{Gk\min}}\approx 2\frac{m}{r} \qquad (2.2.\ 8)$$

We can now put Eq's. (2.1. 9), (2.2. 6) into Eq. (2.2. 8), and dropping the now unnecessary subscripts the graviton coupling constant α_G cancels out:

$$\frac{\Delta\rho_{Gk\min}}{\rho_{Gk\min}} \approx \frac{\alpha_G\left[\frac{m}{r}\right]0.573\frac{\rho_U}{k_{\min}^2}dk_{\min}}{\alpha_G 0.3247\frac{\rho_U^2}{k_{\min}^4}dk_{\min}} \approx \left[\frac{m}{r}\right]\left[\frac{1.765k_{\min}^2}{\rho_U}\right] \approx 2\frac{m}{r} \qquad (2.2.\ 9)$$

(Strictly speaking we should be using $dk''_{k\min}$ in the top line of this equation but the error is second order as we are approximating with $r >>> m$. We will do this more accurately below for large masses.) For the above to be consistent with General Relativity this suggests that:

"At all points inside the horizon, and at any cosmic time *T*, the red highlighted part of Eq.(2.2.9) is ≈ 2 in Planck units. This is simply equivalent to putting $G/c^2 = 1 = G = c$".

Thus we can say

$$\text{The average density of the universe } \rho_U \approx (0.8823)k_{\min}^2 \approx 0.8823\frac{\Upsilon^2}{R_{OU}^2} \qquad (2.2.\ 10)$$

Where the parameter $\Upsilon = k_{\min}R_{OH}$ is in radians, and Υ is close to 1.

Putting Eq. (2.2. 10 the average density ρ_U into Eq.(2.1. 10) gives $\rho_{Gk\min}$ & $K_{Gk\min}$.

$$\text{Maximum Wavelength Graviton Probability Density } \rho_{Gk\min} \approx \frac{0.3247\alpha_G\rho_U^2}{k_{\min}^4}dk_{\min}$$

$$\rho_{Gk\min} \approx \frac{0.3247\alpha_G(0.8823k_{\min}^2)^2}{k_{\min}^4}dk \approx 0.253\alpha_G dk_{\min} = K_{Gk\min}dk_{\min}$$

Where we label $K_{Gk\min} \approx 0.253\alpha_G$ as "The $k_{\min}$ Graviton Invariant". (2.2. 11)

If our conjectures are true, this is the number density of maximum wavelength gravitons excluding possible effects of virtual particles emerging from the vacuum. In section 2.5.4 we argue that these do not change the $K_{Gk\min}$ of Eq. (2.2. **11**). However $K_{Gk\min}$ does depend on the graviton coupling constant α_G between Planck masses, but α_G cancels out in Eq.(2.2. 9). It does not affect the allowed universe average density ρ_U in Eq**.** (2.2. 10).

2.2.2 The Schwarzchild metric near large masses

At a radius r from a mass m (dropping the now unnecessary subscripts) the Schwarzchild metric is $(1-2m/r)^{\pm 1/2}$ for the time and radial terms which can be written as

$$\sqrt{g_{rr}} = \frac{1}{\sqrt{1-2m/r}} = \frac{1}{\sqrt{g_{tt}}} = \frac{1}{\sqrt{1-\beta_M^2}} = \gamma_M \tag{2.2. 12}$$

Velocity β_M ($c=1$) is that reached by a small mass falling from infinity and $\gamma_M^{\pm 1}$ is the metric change in clocks and rulers due to mass m. We are using green symbols with the subscript M for metrics $g_{\mu\nu} \neq \eta_{\mu\nu}$ as we did for $k''_{\min}$ above. The symbols $\gamma_M^{\pm 1}$ help clarity in what follows.

$$\beta_M^2 = \frac{2m}{r}$$

$$\gamma_M^2 = \frac{1}{1-2m/r} = g_{rr} = \frac{1}{g_{00}}$$

Using these symbols $k''_{\min} = \gamma_M k_{\min} \rightarrow dk''_{\min} = \gamma_M dk_{\min} \rightarrow \rho''_{Gk\min} = \gamma_M \rho_{Gk\min}$ (2.2. 13)

In sections 2.1.2 & 2.2.2 we approximated in flat space. The wavelength of $k_{\min}$ gravitons span approximately to the horizon. They fill all of space. We can think of the space around even a large black hole as an infinitesimal bubble on the scale of the observable universe. The normalizing constant of a $k_{\min}$ wavefunction emitted from a localized mass is only altered very close to this mass. Over the vast majority of space it is unaltered. Only close to this mass will local observers measure $k''_{\min} = \gamma_M k_{\min}$ due to the change in clocks. There is also a local dilution of the normalizing constant due to the change in radial rulers. We will consider both these changes in two steps to help illustrate our argument. Now repeat the derivation of $\Delta\rho_{Gk\min}$ as in section 2.2.1 but with a large central mass as in Figure 2.2. 1.

At the point P consider Eq.(2.2. 2) $\psi_1 {}^* \psi_2 + \psi_2 {}^* \psi_1 \approx \frac{4k'}{4\pi r_1 r} e^{-k'r} \cos(-kr)$.

The red part is the normalizing factor discussed above where we will *initially ignore the dilution* due to the local increase in volume. The green $k'r$ & kr can be thought of as invariant phase angles. So *if we ignore the dilution factor* this equation is unaltered. However the coupling factor contains all the masses in the universe and the local mass m. But in the Schwarzchild metric this is *the mass dispersed at infinity before it comes together*. At radius r the kinetic energy a mass m acquires on falling from infinity, whether it is still moving, or after stopping, increases this total energy to $\gamma_M m$. The slower local clock rate also increases the measured energy of the rest of the universe by the same factor γ_M. (See section 2.3.5)

We are left with the factor $\frac{2\alpha_G}{\pi}\frac{dk_{\min}}{k_{\min}}$ which is the same as $\frac{2\alpha_G}{\pi}\frac{dk''_{\min}}{k''_{\min}} = \frac{2\alpha_G}{\pi}\frac{\gamma_M dk_{\min}}{\gamma_M k_{\min}}$ in the changed metric. Ignoring the dilution factor, and considering only clock changes Eq.(2.2. 6) becomes, dropping the now uneccesary subscripts

With only time change and no dilution $\Delta\rho_{Gk\min} \approx \alpha_G \gamma_M^2 \frac{m}{r} 0.573 \frac{\rho_U}{k_{\min}^2} dk_{\min}$

But $\frac{\rho_U}{k_{\min}^2} \approx 0.8823$ from Eq. (2.2. 10) so

From Equ's. (2.2. 11) & (2.2. 13) $K_{Gk\min} \approx 0.253\alpha_G$ and $\beta_M^2 = \frac{2m}{r}$ and we finally get

$$\text{Before dilution of the normalization factor } \Delta\rho_{Gk\min} \approx \beta_M^2 \gamma_M^2 K_{Gk\min} dk_{\min} \qquad (2.2.\ 14)$$

So the total $k_{\min}$ graviton density before dilution is the original $\rho_{Gk\min} \approx K_{Gk\min} dk_{\min}$ plus the extra $\Delta\rho_{Gk\min} \approx \beta_M^2 \gamma_M^2 K_{Gk\min} dk_{\min}$. Thus before dilution

$$\rho_{Gk\min}(\text{Total}) = K_{Gk\min} dk_{\min} + \beta_M^2 \gamma_M^2 K_{Gk\min} dk_{\min} = (1+\beta_M^2\gamma_M^2) K_{Gk\min} dk_{\min}$$

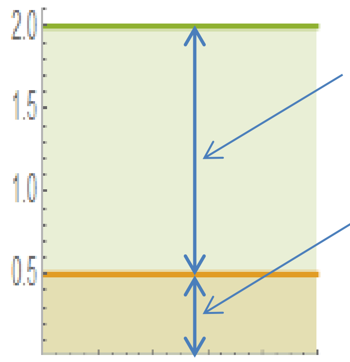
$$\text{But } (1+\beta_M^2\gamma_M^2) = 1 + \frac{\beta_M^2}{1-\beta_M^2} = \gamma_M^2$$

$$\text{So undiluted } \rho_{Gk\min}(\text{Total}) = \gamma_M^2 K_{Gk\min} dk_{\min} \qquad (2.2.\ 15)$$

If we now increase the volume to that in the new metric, the new volume is $\sqrt{g_{rr}} = \gamma_M$ times the original volume. So in the new metric we must divide this value by γ_M .

$$\text{In the new metric } \rho''_{Gk\min} = \frac{\gamma_M^2 K_{Gk\min} dk_{\min}}{\gamma_M} = \gamma_M K_{Gk\min} dk_{\min} = K_{Gk\min} dk''_{\min} \qquad (2.2.\ 16)$$

If for example $\gamma_M = 2$, frequencies are doubled so $k''_{\min} = 2k_{\min}$, the number density of gravitons ($\rho''_{Gk\min} = 2\rho_{Gk\min}$) is doubled, but so is the measurement of a local small volume element, which is now $V = 2$. The above equations tell us that the original $\rho_{Gk\min}$ background gravitons which occupied one unit of volume is now compressed into 1/2 a unit of volume and the remaining 3/2 units of volume is taken up by extra gravitons due to the central mass. Figure 2.2. 2 illustrates this. The metric appears to adjust itself so that $K_{Gk\min}$ *(the maximum wavelength graviton probability constant)* is an invariant number. (See Figure 2.5. 1 also.) What we have done in this section is only true if the increase in measured volume is equal to the increase in measured frequency. In the Schwarzchild metric this is equivalent to saying that $g_{rr} \cdot g_{tt} = 1$. But what happens in the Kerr metric with angular momentum?



Measured local volumes double, & 3/2 units of volume × the increased number density equals the extra maximum wavelength gravitons at that point due to a central mass.

The background $k_{\min}$ gravitons that originally occupied one unit of volume are compressed into 1/2 a unit of volume as number densities are doubled in this new metric.

Figure 2.2. 2 An infinitesimal local volume in a Schwarzchild metric where $\sqrt{g_{rr}} = \gamma_M = 2$.

2.3 Angular Momentum and the Kerr Metric

In the Schwarzchild metric the increase in volume is the same as the frequency increase as $g_{rr} \cdot g_{tt} = 1$ and $g_{\theta\theta} \cdot g_{\phi\phi} = r^4 \sin^2 \theta$ is invariant if there is no angular momentum. With angular momentum both $g_{\theta\theta}$ & $g_{\phi\phi}$ change. The volume ratio of $g_{\mu\nu} \neq \eta_{\mu\nu}$ space, to $g_{\mu\nu} \neq \eta_{\mu\nu}$ space in

any metric at fixed $r \,\&\, \theta$ is
$$\frac{V'}{V} = \sqrt{\frac{(g'_{rr} \cdot g'_{\theta\theta} \cdot g'_{\phi\phi})(g_{\mu\nu} \neq \eta_{\mu\nu})}{(g_{rr} \cdot g_{\theta\theta} \cdot g_{\phi\phi})(g_{\mu\nu} = \eta_{\mu\nu})}} = \sqrt{\frac{(g'_{rr} \cdot g'_{\theta\theta} \cdot g'_{\phi\phi})(g_{\mu\nu} \neq \eta_{\mu\nu})}{r^4 \sin^2 \theta}} \qquad (2.3.\ 1)$$

The Kerr metric can be written in Boyer-Lindquist coordinates as

$$\begin{cases} g_{\theta\theta} = r^2 + \alpha^2 \cos^2 \theta \\ g_{\varphi\varphi} = (r^2 + \alpha^2 + \dfrac{r_S r}{\rho^2} \alpha^2 \sin^2 \theta) \sin^2 \theta \\ g_{t\varphi} = \dfrac{r_S r}{g_{\theta\theta}} \alpha \sin^2 \theta \\ g_{rr} = \dfrac{g_{\theta\theta}}{\Delta} \qquad \& \qquad g_{tt} = 1 - \dfrac{r_S r}{g_{\theta\theta}} \end{cases}$$

Where $\Delta = r^2 + r_S r + \alpha^2$ and $\alpha = \dfrac{J}{mc}$ and $r_S = \dfrac{2Gm}{c} = 2m$ is the Schwarzchild radius in Planck units where $G = c = 1$. Everything is in units of length or $(\text{length})^2$, except g_{rr} & g_{tt} which are dimensionless. Because we want volume ratios as in Eq. (2.3. 1) we can write the above version of the Kerr metric in a dimensionless form, leaving the length squared, and length terms $r^2, r^2 \sin^2 \theta \,\&\, r \sin \theta$ in $r^2 d\theta^2$, $r^2 \sin^2 \theta d\varphi^2$ & $r \sin \theta d\varphi$ etc outside the metric tensor. This effectively gives us the denominator $r^4 \sin^2 \theta$ we want in Eq. (2.3. 1) as we will see. We must also remember that angular momentum parameter α is a length dimension.

Writing the above in dimensionless form as follows, using $-+++$ for the line element ds^2 :

A Dimensionless form of the Kerr Metric where $\Delta = 1 + \frac{\alpha^2}{r^2} - A$ and $A = \frac{2m}{r}$ but we will add an also dimensionless $\frac{m^2}{r^2}$ later. See section 2.6

(We assume silent $G = c^2 = 1$ Planck value constants in $A = \frac{2m}{r} +$ a dimensionless term)

$$\left[\begin{aligned} g_{\varphi\varphi} &= 1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}} \frac{\alpha^2}{r^2} \sin^2\theta \\ g_{\theta\theta} &= 1 + \frac{\alpha^2}{r^2} \cos^2\theta \\ g_{rr} &= \frac{g_{\theta\theta}}{\Delta} \\ g_{t\varphi} &= \frac{A}{g_{\theta\theta}} \frac{\alpha}{r} \sin\theta \\ g_{tt} &= 1 - \frac{A}{g_{\theta\theta}} \end{aligned}\right. \qquad (2.3.\ 2)$$

The space surrounding a rotating mass corotates with it. If we move in this corotating reference frame there is a new metric time component, which after some rearrangement of plus and minus signs for convenience, we can write as: $g'_{tt} = g_{tt} + \frac{g_{t\phi}^2}{g_{\phi\phi}}$.

Thus using Eq. (2.3. 2) $$g'_{tt} = g_{tt} + \frac{g_{t\varphi}^2}{g_{\varphi\varphi}} = (1 - \frac{A}{g_{\theta\theta}}) + \frac{\frac{A^2}{g_{\theta\theta}^2} \frac{\alpha^2}{r^2} \sin^2\theta}{\left[1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}} \frac{\alpha^2}{r^2} \sin^2\theta\right]}$$

$$= (1 - \frac{A}{g_{\theta\theta}}) + \frac{\frac{A^2}{g_{\theta\theta}} \frac{\alpha^2}{r^2} \sin^2\theta}{g_{\theta\theta}\left[1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}} \frac{\alpha^2}{r^2} \sin^2\theta\right]}$$

$$= \frac{g_{\theta\theta}(1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}} \frac{\alpha^2}{r^2} \sin^2\theta) - A(1 + \frac{\alpha^2}{r^2}) - \frac{A^2}{g_{\theta\theta}^2} \frac{\alpha^2}{r^2} \sin^2\theta + \frac{A^2}{g_{\theta\theta}^2} \frac{\alpha^2}{r^2} \sin^2\theta}{g_{\theta\theta}(1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}} \frac{\alpha^2}{r^2} \sin^2\theta)}$$

$$= \frac{-A(1 + \frac{\alpha^2}{r^2} - \frac{\alpha^2}{r^2} \sin^2\theta) + g_{\theta\theta}(1 + \frac{\alpha^2}{r^2})}{g_{\theta\theta} g_{\phi\phi}}$$

$$= \frac{-A(1 + \frac{\alpha^2}{r^2} \cos^2\theta) + g_{\theta\theta}(1 + \frac{\alpha^2}{r^2})}{g_{\theta\theta} g_{\phi\phi}}$$

$$g'_{tt} = g_{\theta\theta} \frac{-A g_{\theta\theta} + g_{\theta\theta}(1 + \frac{\alpha^2}{r^2})}{g_{\phi\phi}} = \frac{g_{\theta\theta}(1 + \frac{\alpha^2}{r^2} - A)}{g_{\theta\theta} g_{\phi\phi}} = \frac{\Delta}{g_{\phi\phi}} \qquad (2.3.\ 3)$$

We have explicitly gone through this to show that if the parameter $A = 2m/r$ is dimensionless, there is potentially freedom to change it without changing Eq. (2.3. **3**).

(See Section 2.6.6 as this is similar to what happens in the Kerr-Newman metric, where instead of a dimensionless m^2/r^2 term, a dimensionless r_Q^2/r^2 or equivalently a dimensionless Q^2/r^2 is included in term A. See for example Table 2.6. 2 and Table 2.6. 3.)

We will work in corotating frames. Space is swirling around the black hole effectively at rest in these frames, simplifying our calculations and equations. (Section 2.9 puts this into a four vector form, invariant in all frames.) If a small mass, at rest at infinity in the same rest frame as the rotating black hole, falls inwards, it will have the same circumferential velocity as the corotating rest frames at all radii. It will be falling radially through these corotating frames. As in section 2.2.2 we call this radial velocity β_M where as in the non-rotating case

$$\frac{1}{\sqrt{1-\beta_M^2}} = \gamma_M \quad \text{but now} \quad \frac{1}{\sqrt{1-\beta_M^2}} = \gamma_M = \frac{1}{\sqrt{g'_{tt}}} \quad \text{the new inverse rate of clocks.}$$

$$\text{In corotating frames} \qquad \frac{1}{\gamma_M^2} = g'_{tt} = \frac{\Delta}{g_{\phi\phi}} \qquad \gamma_M^2 = \frac{g_{\phi\phi}}{\Delta} \qquad (2.3.\ 4)$$

Frequencies measured in corotating frames increase as γ_M. Similarly using Eq's. (2.3. 1) & (2.3. 4) we can get the (three) volume element ratio in this corotating refence frame.

$$\text{The volume element ratio } V = \sqrt{(g_{rr}\cdot g_{\theta\theta}\cdot g_{\phi\phi})} = \sqrt{\frac{g_{\theta\theta}}{\Delta} g_{\theta\theta} g_{\phi\phi}} = g_{\theta\theta}\sqrt{\frac{g_{\phi\phi}}{\Delta}} = g_{\theta\theta}\gamma_M \qquad (2.3.\ 5)$$

With angular momentum we no longer have the same increase in frequency as volume as in the Schwarzchild case. With no angular momentum we found that the probability density of time polarized $k_{\min}$ gravitons Eq. (2.2. 14) $\Delta\rho_{Gk\min} \approx \gamma_M^2 \beta_M^2 K_{Gk\min} dk_{\min} = \gamma_M^2 \dfrac{2m}{r} K_{Gk\min} dk_{\min}$. With rotation we will find a circularly polarized $\cos^2\theta$ type distribution of gravitons around the axis. These will add to the time polarized dimensionless number $\dfrac{2m}{r}$ to get an as yet unknown number we simply label as X where $X > \dfrac{2m}{r}$

$$\text{Let us rewrite Eq.(2.2. 14) as} \quad \Delta\rho_{Gk\min} \approx \gamma_M^2 X K_{Gk\min} dk_{\min} \text{ with rotation} \qquad (2.3.\ 6)$$

Where the factor γ_M^2 is for the same clock rate change effect in the metric as before or see sections 2.2.2 & 2.3.5 and deriving Eq.(2.2. 15). Repeating the derivation of Eq.(2.2. 15)

$$\rho_{Gk\min}(\text{Undiluted Total}) = K_{Gk\min}dk_{\min} + \gamma_M^2 X K_{Gk\min}dk_{\min} = (1+\gamma_M^2 X)K_{Gk\min}dk_{\min}$$

As in Eq.(2.2. 16) we need to divide this undiluted total by the new volume $V = g_{\theta\theta}\gamma_M$ in Eq. (2.3. 5) to get the new $k_{\min}$ graviton density $\rho''_{Gk\min}$.

If our conjectures are correct $\rho''_{Gk\min} = K_{Gk\min}dk''_{\min}$ is always true, and as our measurement of $k_{\min}$ increases to $k''_{\min} = \gamma_M k_{\min}$ in the new metric, $\rho''_{Gk\min} = K_{Gk\min}\gamma_M dk_{\min}$.

So rewriting Eq.(2.2. 16) as follows

$$\rho''_{Gk\min} = \frac{(1+\gamma_M^2 X)K_{Gk\min}dk_{\min}}{V} = \frac{(1+\gamma_M^2 X)K_{Gk\min}dk_{\min}}{g_{\theta\theta}\gamma_M} = \gamma_M K_{Gk\min}dk_{\min} = K_{Gk\min}dk''_{\min}$$

$$(1+\gamma_M^2 X)K_{Gk\min}dk_{\min} = g_{\theta\theta}\gamma_M^2 K_{Gk\min}dk_{\min}$$

$$1+\gamma_M^2 X = g_{\theta\theta}\gamma_M^2$$

$$X = g_{\theta\theta} - \frac{1}{\gamma_M^2} = (1+\frac{\alpha^2}{r^2}\cos^2\theta) - \frac{1}{\gamma_M^2}$$

$$X = (1+\frac{\alpha^2}{r^2}\cos^2\theta) - \frac{\Delta}{g_{\varphi\varphi}}$$ using Eq. (2.3. 4)

$$X = 1+\frac{\alpha^2}{r^2}\cos^2\theta - \frac{1+\frac{\alpha^2}{r^2} - A}{1+\frac{\alpha^2}{r^2}+\frac{A}{g_{\theta\theta}}\frac{\alpha^2}{r^2}\sin^2\theta}$$ using Eq's.(2.3. 2)

We can write this as
$$X = \frac{\alpha^2}{r^2}\cos^2\theta + \frac{\left[1+\frac{\alpha^2}{r^2}+\frac{A}{g_{\theta\theta}}\frac{\alpha^2}{r^2}\sin^2\theta\right] - \left[1+\frac{\alpha^2}{r^2} - A\right]}{1+\frac{\alpha^2}{r^2}+\frac{A}{g_{\theta\theta}}\frac{\alpha^2}{r^2}\sin^2\theta}$$

$$X = \frac{\alpha^2}{r^2}\cos^2\theta + \frac{A\left[1+\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\theta\theta}}\right]}{1+\frac{\alpha^2}{r^2}+\frac{A}{g_{\theta\theta}}\frac{\alpha^2}{r^2}\sin^2\theta}$$

$$X = \frac{\alpha^2}{r^2}\cos^2\theta + \frac{A\left[1+\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\theta\theta}}\right]}{g_{\phi\phi}}$$ using Eq's.(2.3. 2)

$$X = \frac{\alpha^2}{r^2}\cos^2\theta \quad + \quad \frac{A}{g_{\varphi\varphi}} \quad + \quad A\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\phi\varphi}g_{\theta\theta}}$$

We will discuss later why there is no separate term in $A\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\phi\varphi}g_{\theta\theta}}$ so we will write this as

$$X = \frac{\alpha^2}{r^2}\cos^2\theta \quad + \quad \frac{Ag_{\theta\theta}}{g_{\varphi\varphi}g_{\theta\theta}} \quad + \quad A\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\phi\varphi}g_{\theta\theta}}$$

$$X = \frac{\alpha^2}{r^2}\cos^2\theta \; + \; \frac{A(1+\frac{\alpha^2}{r^2}\cos^2\theta)}{g_{\varphi\varphi}g_{\theta\theta}} \; + \; A\frac{\alpha^2}{r^2}\frac{\sin^2\theta}{g_{\phi\varphi}g_{\theta\theta}}$$

Which we finally write as $X = \frac{\alpha^2}{r^2}\cos^2\theta + \frac{A(1+\frac{\alpha^2}{r^2})}{g_{\varphi\varphi}g_{\theta\theta}}$ (2.3. 7)

Putting $A = \frac{2m}{r}$, the *extra* $k_{\min}$ virtual gravitons $\gamma_M^2 X$ (due to a mass m rotating with angular parameter α that has dimensions of length) are the following two polarization groups (The background $k_{\min}$ virtual gravitons have been normalized to one when $\gamma_M = 1$)

Time polarized spin 2 $\gamma_M^2\left[\frac{\frac{2m}{r}(1+\frac{\alpha^2}{r^2})}{g_{\varphi\varphi}g_{\theta\theta}}\right]$

Circularly polarized spin 2: $\gamma_M^2\left[\frac{\alpha^2}{r^2}\cos^2\theta\right]$ by $(m=\pm 2)$

The main thing to notice here is that the circularly polarized $k_{\min}$ gravitons are independent of the central mass, suggesting they are due to the effect of the rotation of space, or frame dragging, on the $k_{\min}$ graviton background. We will discuss this in section 2.3.2. The extra $k_{\min}$ gravitons due to the central mass have a $(1+\alpha^2/r^2)/(g_{\varphi\varphi}g_{\theta\theta})$ factor, distorting them from spherical symmetry. Figure 2.3.1 & Figure 2.3. 2 compare the above with spinning charged spheres in electromagnetism. The electrostatic energy density surrounding a charged sphere however, reduces with radius as r^{-4}, and magnetic energy as r^{-6}, or two more powers of radius. With gravity however we have been looking at the probability density of minimum wavenumber $k_{\min}$ gravitons surrounding a mass. With no angular momentum there are only time polarized $k_{\min}$ gravitons, and their extra probability density drops as r^{-1}, as so far we have only focussed on those $k_{\min}$ gravitons (the vast majority), that interact with the rest of the mass in the universe. If a charged sphere rotates, there is a radial magnetic field of circularly polarized $m=\pm 1$ photons varying in intensity as $\cos^2\theta$ and a transverse magnetic field (of transversely polarized $m=\pm 1$ photons) varying as $\sin^2\theta$ as in Figure 2.3.1.

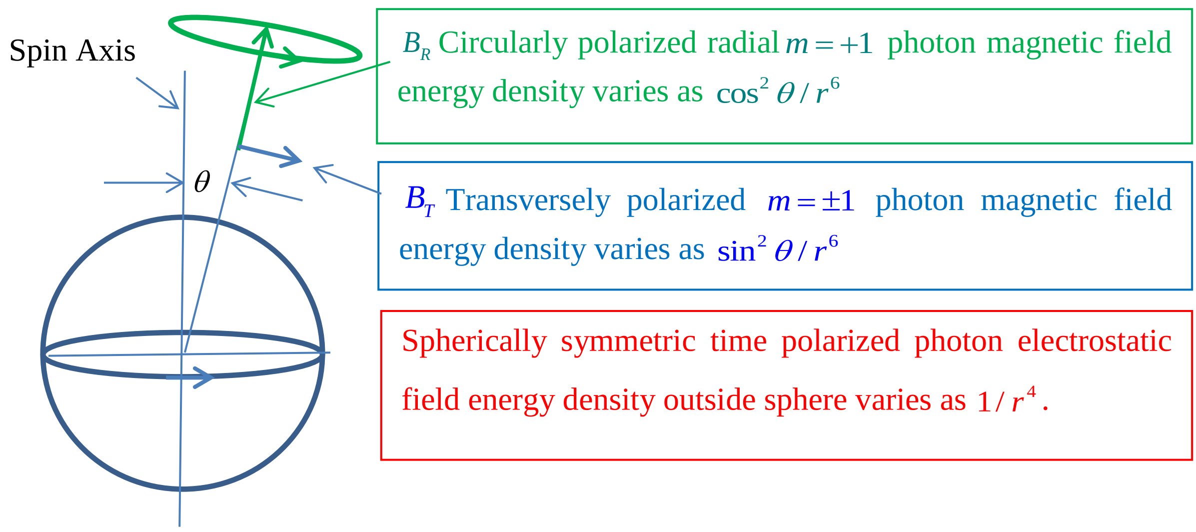


Figure 2.3.1 Spinning electrically charged sphere. At same radius $B_R @(\theta=0) = 2B_T @(\theta=\pi/2)$.

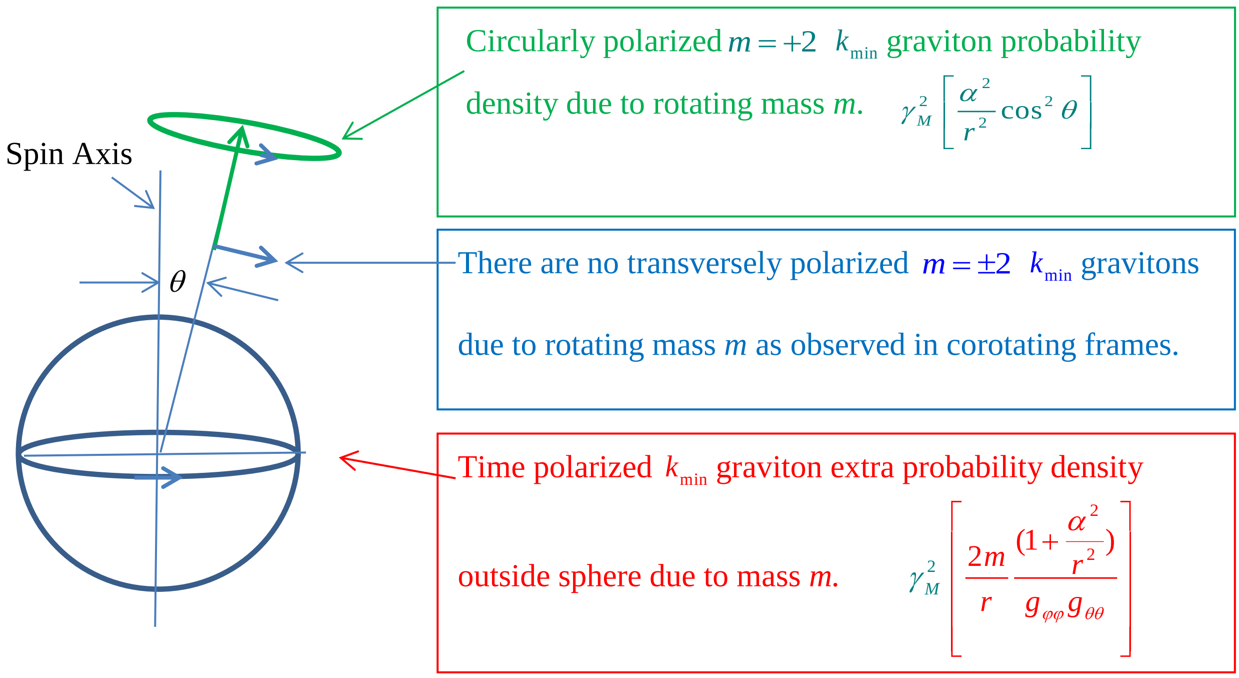


Figure 2.3. 2 Spinning mass m with angular momentum length parameter α as viewed in a corotating frame. There are circularly polarized $(m=\pm2)\times k_{\min}$ gravitons due to the effect of frame dragging on the background time polarized $k_{\min}$ gravitons. There are no transversely polarized $(m=\pm2)\times k_{\min}$ gravitons due to a rotating mass m as seen in a corotating frame. Time polarized $k_{\min}$ gravitons due to mass m are distorted from spherical symmetry as $(1+\alpha^2/r^2)/(g_{\varphi\varphi}g_{\theta\theta})$. For $r >> r_{Sw}$ we can ignore the effects of $g_{\theta\theta}, g_{\varphi\varphi}$ & γ_M^2, as all three rapidly tend to one, with the metric written in dimensionless form as in Equ's.(2.3. 2).

2.3.1 Stress tensor sources for spin 2 gravitons but 4 current sources for spin 1

Spin 1 particles behave like a 4 vector as they come from a 4 current source, transforming with velocity as in the Special Relativity transformations of Minkowski spacetime. Spin 2 gravitons in contrast come from mass/energy density sources. There are two factors in their transformations with velocity. One from the mass increase per source particle, and the second from the increase in particles per unit length due to length contraction. Thus Spin 2 particles transform as a 4x4 rank 2 tensor, which Einstein connected with spacetime curvature.

The rules of quantum mechanics tell us that spherically symmetric spin 2 particles should be equal $1/\sqrt{5}$ superpositions of $m=-2,-1,0,+1,+2$ states. But the shape of gravitational waves behaves like transversely polarized $m=\pm 2$ particles, suggesting the $k_{\min}$ gravitons surrounding mass concentrations may only consist of time polarized, plus $m=\pm 2$ circularly polarized, spin 2 particles. *This suggests that time polarized spin 2 particles could just be equal $1/\sqrt{3}$ superpositions of $m=-2,0,+2$ states and we will assume, at least temporarily, that this is so.* When we looked at non rotating spherical masses it appeared that, even close to black holes, the spherical symmetry of the Schwarzchild metric suggested similarly spherically symmetric, or time polarized, extra $k_{\min}$ gravitons right down to the horizon; with space expanding only radially. Thus before we considered angular momentum we could treat all $k_{\min}$ gravitons as time polarized. (We show in section 2.5.1 that all background $k_{\min}$ gravitons are always only time polarized in flat space at any peculiar velocity.) A stress tensor source with no angular momentum has spherically symmetric spacetime curvature with only time polarized $k_{\min}$ gravitons. But angular momentum in the source produces cylindrically symmetric spacetime curvature. We still have time polarized $k_{\min}$ gravitons (in co-rotating coordinates) due to the central mass, but distorted from spherical symmetry as $(1+\alpha^2/r^2)/(g_{\varphi\varphi}g_{\theta\theta})$ which only affects the close in region, disappearing as $\alpha \to 0$. But there are also circularly polarized $m=\pm 2$ $k_{\min}$ gravitons only related to angular momentum. These circularly polarized gravitons do not have the $2m/r$ factor and must be very different. As we will discuss below it appears that they are generated from the background time polarized $k_{\min}$ gravitons by the swirling velocity of corotating space.

2.3.2 Circularly polarized gravitons from corotating space

The circularly polarized gravitons do not have a $2m/r$ factor. The Kerr metric is an exact solution to Einstein's field equations, which we conjecture (in an infinitesimally modified form as in Eq. (2.5. 6) are consistent with the $k_{\min}$ Graviton constant being invariant at all points in spacetime, or that Eq. (2.2. 11) is always true. If this is so then Eq. (2.3. 7) should be true also. We can perhaps just accept that it must be true, but at the same time we can look at whether it makes sense?

The angular momentum parameter α has dimensions of length, and is defined as $\alpha = \dfrac{J}{mc}$.
Because angular momentum is the cross product of momentum by radius or $m\mathbf{v}\times\mathbf{r}$, we can think of this length parameter as a vector of length α, pointing along the axis of spin, with components $\alpha\cos\theta$ at any polar angle θ to the spin axis. Space corotates around spinning masses with angular velocity $\Omega = \dfrac{g_{t\phi}}{g_{\phi\phi}}$ which in the plane of the equator simplifies to

$$\Omega = \frac{r_S \alpha c}{r^3 + r\alpha^2 + r_S\alpha^2} \approx \frac{r_S \alpha c}{r^3} \text{ when } r >>> r_S \;\&\; \alpha .$$

$$\text{At large radii the corotating velocity} \quad V = \Omega\times\mathbf{r} \approx \frac{r_S \alpha c}{r^2} \tag{2.3. 8}$$

Because $r_S \;\&\; \alpha$ have dimensions of length this equation has dimensions of velocity, and if we divide it by c it is dimensionless. We will call it $\beta_{Coratating} = \beta_C$

$$\text{At large radii} \quad \beta_{Coratating} = \beta_C = \frac{V}{c} = \frac{\Omega\times r}{c} \approx \frac{r_S \alpha}{r^2} \text{ a dimensionless number.} \tag{2.3. 9}$$

If we now think of $\alpha = \dfrac{J}{mc}$ as $\alpha = \dfrac{m}{m}\dfrac{\mathbf{v}\times\mathbf{r}}{c} = \dfrac{\mathbf{v}\times\mathbf{r}}{c}$ we can consider a similar vector along the spin axis consisting of the cross product of the corotating velocity of space $\dfrac{V}{c} \approx \dfrac{r_S\alpha}{r^2}$ by the radius r. The length along the spin axis of this cross product vector $\dfrac{\mathbf{V}\times\mathbf{r}}{c}$ is simply $\dfrac{r_S\alpha}{r}$.

$$\text{At the equator: Length of vector } \frac{\mathbf{V}\times\mathbf{r}}{c} \text{ along the spin axis is } \approx \frac{r_S\alpha}{r} \text{ for } r >>> r_S \tag{2.3. 10}$$

We need this vector length to be a dimensionless number representing the amplitude that a background time polarized $k_{\min}$ graviton generates a circularly polarized $k_{\min}$ graviton around the spin axis. If we divide Eq. (2.3. 10) by the Schwarzchild radius r_S, all rotating black holes with the same percentage of maximum spin look identical, and we get a dimensionless magnitude as required

$$\text{Magnitude of normalized dimensionless vector } \left|\frac{\mathbf{V}\times\mathbf{r}}{r_S c}\right| \approx \frac{r_S\alpha}{r_S r} \approx \frac{\alpha}{r} \tag{2.3. 11}$$

Now the $\Omega = g_{t\phi} / g_{\phi\phi}$ in Eq. (2.3. 8) is measured by the clock rate at infinity, but at rest relative to the centre of the rotating mass. In the corotating frame clocks tick slower and it is measured as the γ_M of Eq. (2.3. 4) times greater than the rate at infinity,

$$\text{In the corotating frame this magnitude becomes } \left|\frac{\gamma_M \mathbf{V}\times\mathbf{r}}{r_S c}\right| \approx \frac{\gamma_M \alpha}{r} \tag{2.3. 12}$$

The whirling velocity of space is a maximum out from the equator, but circularly polarized gravitons generated in this region have to be distributed on this shell around the spin axis as the square of the component of angular momentum. We thus conjecture that the probability of *background* time polarized $k_{\min}$ gravitons, on a corotating thin spherical shell at large radius, generating circularly polarized $k_{\min}$ gravitons around the spin axis on the same shell is (before we expand the volume with the new spatial metric)

$$\text{Probability of } \frac{\text{Extra circularly polarized } m = \pm 2\times k_{\min}\text{gravitons}}{\text{Background time polarized } k_{\min}\text{gravitons}} \approx \frac{\gamma_M^2 \alpha^2 \cos^2\theta}{r^2} \qquad (2.3.\ 13)$$

There is a background density of time polarized $k_{\min}$ gravitons on each corotating spherical shell. The swirling velocity of these $k_{\min}$ gravitons generates extra circularly polarized $k_{\min}$ gravitons around the spin axis with a $\cos^2\theta$ distribution around the spin axis *on the same shell,* in agreement with Figure 2.3. 2. For simple explanatory purposes, we approximated at large radii only. At small radii we must use $\Omega = \frac{g_{t\phi}}{g_{\phi\phi}} = \frac{r_S \alpha c}{r^3 g_{\phi\phi} g_{\theta\theta}}$. On the equator $g_{\theta\theta} = 1$, the co-rotation velocity, as measured in the metric there, is $V = \Omega \times \mathbf{r}\sqrt{g_{\phi\phi}}$ (plus a γ_M term). The circumferential volume generating these circularly polarized gravitons also expands as $\sqrt{g_{\phi\phi}}$. Rederiving Eq's. (2.3. 9) and those following, the effective angular momentum term becomes $\Omega \times \mathbf{r} \times \mathbf{r} \cdot \sqrt{g_{\phi\phi}} \cdot \sqrt{g_{\phi\phi}} = \left[\frac{r_S \alpha c}{r^3 g_{\phi\phi}}\right] \cdot r^2 g_{\phi\phi} = \frac{r_S \alpha c}{r}$ just as before; our derivation applies down to the equatorial horizon. This circular polarization appears to be the result of the swirling or corotating velocity of space as it has no mass term, only angular momentum terms.

2.3.3 **Why there are no transverse polarized gravitons in co-rotating coordinates?**

In Section 2.1 we showed that the majority of $k_{\min}$ gravitons around any non rotating mass *m* is due to the interaction between that mass and the rest of the mass in the universe $\Delta\rho_{Gk\min} \propto (\psi_{\text{Universe}} {*}\psi_m + \psi_m {*}\psi_{\text{Universe}})$; and these were all time polarized $k_{\min}$ gravitons. Let us imagine that a rotating mass emits transversely polarized $k_{\min}$ gravitons, there will only be a small number unless there are also transversely polarized $k_{\min}$ gravitons from the rest of the universe for their amplitudes to interact with. But from what we have just done above there appears to be only circularly polarized $k_{\min}$ gravitons due to the corotation of space. Also if a rotating mass emits its own circularly polarized $k_{\min}$ gravitons, these would interact with the circularly polarized $k_{\min}$ gravitons due to the corotation of space. It thus appears that, when observed in corotating coordinates, a rotating mass does not emit either tranverse or circularly polarized $k_{\min}$ gravitons. This perhaps makes sense, as in corotating frames, we are effectively at rest above the horizon which is rotating in sync with us.

2.3.4 Why our time polarized $k_{\min}$ value in co-rotating coordinates makes sense

The inner horizon radius R is defined when $\Delta = 1+\frac{\alpha^2}{r^2} - A = 1 + \frac{\alpha^2}{r^2} - \frac{2m}{r} = 0$ where we initially define the dimensionless number $A = \frac{2m}{r}$. Using $A = \frac{2m}{r}$ the inner horizon is where $r^2 - 2mr + \alpha^2 = 0$. So horizon radius $R = r = m + \sqrt{m^2+\alpha^2} = 2m$ when $\alpha = 0$, and at maximum spin $r = R = m$ when $\alpha = m$. But we will, for generality, revert to the dimensionless A and look at what happens near the horizon for various spins.

Let A_H be the value of A at the horizon where $1 + \frac{\alpha^2}{R^2} - A_H = 0$ is always true

$$\text{So} \quad \frac{\alpha^2}{R^2} = A_H - 1 \quad \text{and} \quad A_H = 1 + \frac{\alpha^2}{R^2} \tag{2.3. 14}$$

$$\text{At the horizon } g_{\theta\theta} g_{\phi\phi} = (1+\frac{\alpha^2}{R^2}\cos^2\theta)(1+\frac{\alpha^2}{R^2} + A_H \frac{\alpha^2}{R^2}\frac{\sin^2\theta}{g_{\theta\theta}})$$

$$= g_{\theta\theta}(A_H + A_H \frac{\alpha^2}{R^2}\frac{\sin^2\theta}{g_{\theta\theta}})$$

$$= g_{\theta\theta} A_H (1 + \frac{\alpha^2}{R^2}\frac{\sin^2\theta}{g_{\theta\theta}})$$

$$= A_H (g_{\theta\theta} + \frac{\alpha^2}{R^2}\sin^2\theta)$$

$$= A_H (1 + \frac{\alpha^2}{R^2}\cos^2\theta + \frac{\alpha^2}{R^2}\sin^2\theta$$

$$= A_H (1 + \frac{\alpha^2}{R^2})$$

And this is true regardless of the degree of spin from zero to maximum as the radius shrinks

$g_{\phi\phi} g_{\theta\theta} = A_H (1+\frac{\alpha^2}{R^2}) = A_H^2$ and is independant of angle θ near the horizon only. (2.3. 15)

Near the black hole horizon the surface area is $4\pi R^2 \sqrt{g_{\phi\phi} g_{\theta\theta}} = 4\pi R^2 A_H$

$$g_{rr} = \frac{g_{\theta\theta}}{\Delta} = \frac{1+\frac{\alpha^2}{r^2}\cos^2\theta}{1+\frac{\alpha^2}{r^2} - \frac{2m}{r}}$$ where we are again reverting to $A = \frac{2m}{r}$

$$g_{rr} = \frac{g_{\theta\theta}}{(1+\frac{\alpha^2}{r^2})(1 - \frac{2m}{r(1+\frac{\alpha^2}{r^2})})} \text{ but } 1 - \frac{2m}{r(1+\frac{\alpha^2}{r^2})} = \frac{r - \frac{2m}{(1+\frac{\alpha^2}{r^2})}}{r} \text{ \& horizon radius is } R = \frac{2m}{(1+\frac{\alpha^2}{R^2})}$$

Near this horizon when $r \approx R$ we can approximate this as $\dfrac{r - \dfrac{2m}{(1+\dfrac{\alpha^2}{R^2})}}{R} \approx \dfrac{r-R}{R} \approx \dfrac{\Delta R}{R}$

$g_{rr} \approx \dfrac{g_{\theta\theta}}{(1+\dfrac{\alpha^2}{R^2})}\dfrac{R}{\Delta R}$ near the horizon and $ds = \sqrt{g_{rr}}dR = \dfrac{\sqrt{g_{\theta\theta}}}{\sqrt{1+\dfrac{\alpha^2}{R^2}}}\sqrt{\dfrac{R}{\Delta R}}\ .\ dR$

Near the horizon the proper distance above it is: $s \approx \dfrac{\sqrt{g_{\theta\theta}R}}{\sqrt{1+\dfrac{\alpha^2}{R^2}}}\int_0^R \dfrac{dR}{\sqrt{\Delta R}} \approx \dfrac{\sqrt{g_{\theta\theta}R}}{\sqrt{1+\dfrac{\alpha^2}{R^2}}}2\sqrt{\Delta R}$

$$\text{Thus}\quad s^2 \approx \frac{4g_{\theta\theta}R\Delta R}{(1+\dfrac{\alpha^2}{R^2})} \quad\text{and}\quad \Delta R \approx \frac{(1+\dfrac{\alpha^2}{R^2})s^2}{4g_{\theta\theta}R} \qquad (2.3.\ 16)$$

Now $\gamma_M^2 = \dfrac{g_{\varphi\varphi}}{\Delta} = \dfrac{g_{\varphi\varphi}}{1+\dfrac{\alpha^2}{r^2}-\dfrac{2m}{r}} = \dfrac{g_{\varphi\varphi}}{(1+\dfrac{\alpha^2}{r^2})(1-\dfrac{2m}{r(1+\dfrac{\alpha^2}{r^2})}} \approx \dfrac{g_{\varphi\varphi}}{(1+\dfrac{\alpha^2}{R^2})}\dfrac{R}{\Delta R}$ near horizon as shown above. But $\Delta R \approx \dfrac{(1+\dfrac{\alpha^2}{R^2})s^2}{4g_{\theta\theta}R}$ so $\gamma_M^2 \approx \dfrac{g_{\varphi\varphi}R}{(1+\dfrac{\alpha^2}{R^2})}\left(\dfrac{1}{\Delta R}\right) \approx \dfrac{g_{\varphi\varphi}R}{(1+\dfrac{\alpha^2}{R^2})}\cdot\dfrac{4g_{\theta\theta}R}{(1+\dfrac{\alpha^2}{R^2})s^2}$

But Eq. (2.3. 15) tells us $(1+\dfrac{\alpha^2}{R^2})(1+\dfrac{\alpha^2}{R^2}) = A_H^2 = g_{\theta\theta}g_{\phi\phi}$, so that near the horizon

$\gamma_M^2 \approx \dfrac{4R^2}{s^2}$ and $\gamma_M \approx \dfrac{2R}{s}$ is always true regardless of the degree of spin, and the value of the dimensionless number A_H, where R is the horizon radius and s the proper distance from it, all measured in co-rotating coordinates. (2.3. 17)

We have shown that in a co-rotating frame the extra time polarized $k_{\min}$ graviton density due to a mass is $\rho_{Gk\min} = \gamma_M^2 A\cdot\dfrac{1+\dfrac{\alpha^2}{r^2}}{g_{\theta\theta}g_{\phi\phi}}K_{Gk\min}dk_{\min}$ where $A = \dfrac{2m}{r}$ so far, and is dimensionless.

Ignoring the factor $K_{Gk\min}dk_{\min}$, and using Eq's. (2.3. 14), (2.3. 15) & ((2.3. 17) above, near the horizon in a corotating frame, this becomes *before volume expansion*:

Extra time polarized $k_{\min}$ graviton density near horizon for all black holes

$$\gamma_M^2 A_H \cdot \frac{1+\frac{\alpha^2}{R^2}}{g_{\theta\theta} g_{\phi\phi}} = \gamma_M^2 \frac{A_H^2}{A_H^2} = \gamma_M^2 = \frac{4R^2}{s^2} \tag{2.3. 18}$$

and is spherically symmetric regardless of spin.

Ignoring the factors γ_M^2 & $K_{Gk\min} dk_{\min}$ the extra time polarized $k_{\min}$ graviton probability density *before expansion is always one in Planck unit*s regardless of spin and is spherically symmetric, but only near the horizon, providing it is observed (somehow) in a corotating reference frame. The region well above the horizon is not spherically symmetric until several Schwarzschild radii away where spherical symmetry is gradually retained as in the non-rotating case. Also near the horizon this density due to a central mass is so great that we can effectively ignore the background value, but the rotation of space still generates circularly polarized $k_{\min}$ gravitons of probability density $\gamma_M^2 \frac{\alpha^2}{r^2}\cos^2\theta$ from this background. The extra time, plus circularly, polarized $k_{\min}$ graviton probability density near the horizon is thus

$$\text{Time polarized } \gamma_M^2 + \text{circularly polarized } \gamma_M^2 \frac{\alpha^2}{R^2}\cos^2\theta = \gamma_M^2 (1+\frac{\alpha^2}{R^2}\cos^2\theta) \tag{2.3. 19}$$

$= \gamma_M^2 g_{\theta\theta}$ as in our original derivation before volume expansion, but ignoring the background, which becomes infinitesimal near the horizon.

As the Kerr metric is an exact solution for rotating black holes we can say that if the extra $k_{\min}$ gravitons due to a rotating mass are consistent with $\gamma_M^2 X$ where X is as in Eq. (2.3. 7) then it is also consistent with keeping the Graviton constant $K_{Gk\min}$ as in Eq. **(2.2. 11)** invariant in the spacetime surrounding it. (We come back to this, and potential changes to the dimensionless term $A = 2m/r$ in section 2.6.) When we looked at non rotating black holes in section 2.2.2 we used simple first principles to show that the warping of spacetime around them is consistent with an invariant Graviton constant $K_{Gk\min}$. With rotating black holes we turned the argument around and assumed this invariance to derive the extra probability densities of time, and circular polarized $k_{\min}$ gravitons, before the density dilution from the expansion of space around the rotating mass. Equations (2.3. **18**) & (2.3. **19**) give us confidence our hypothesis is possibly correct. If it is correct on the horizon, and also far from a rotating black hole, we will conjecture that it is also correct in all regions between, even if it might not initially appear to be so. It is important to remember that the Kerr metric is an exact solution for rotating black holes, not for rotating masses in general. We have only considered here the exact solution. We can thus perhaps summarize section 2.3 as follows:

Spherically symmetric spacetime curvature does not transform time polarized $k_{\min}$ gravitons.

Cylindrically symmetric spacetime curvature, due to angular momentum, generates only time polarized $k_{\min}$ gravitons and circularly polarized $m = \pm 2$ $k_{\min}$ gravitons in corotating coordinates.

2.3.5 Addressing what might appear to be an anomaly in all this

In deriving Eq.(2.2. 14) and the preceeding equations we used the metric factor $\gamma_M = 1/\sqrt{g_{tt}}$ (or $\gamma_M = 1/\sqrt{g'_{tt}}$ in corotating coordinates for the Kerr metric case) twice. (1) The local metric measurement of effective black hole mass & (2) the effective mass of the universe. We have also shown $\gamma_M \approx \frac{2R}{s}$ in Eq.**(2.3. 17**) regardless of spin in corotating frames.

Let us imagine we are (somehow) one Planck unit of proper distance above the horizon of a black hole so that $\gamma_M \approx 2R$ is always true. Or at least imagine some sort of measuring device. Let us start at very small black hole mass and continually add mass so that radius R starts at zero, building linearly with mass, up to its final radius R with an average value $R/2$. The average value of $\gamma_M \approx 2R$ is $\bar{\gamma}_M \approx R$. The average total energy of the black hole, if all the mass were at one Planck unit of proper distance above the horizon is the mass at infinity by this average value of $\bar{\gamma}_M \approx R$ or $E_{\text{Total}} = m\bar{\gamma}_M$ which is half our $m\gamma_M$. In a black hole this energy is virtually all kinetic, in the form of thermal energy with pressure, such that $3P = KE \approx m\bar{\gamma}_M$. And Einstein showed us that this pressure generates curvature as well as energy. In the case of a black hole the three equal pressure terms effectively double the mass/energy, such that $E_{\text{Total}}\ (= m\bar{\gamma}_M) + 3P\ (= m\bar{\gamma}_M)\ = 2m\bar{\gamma}_M = m\gamma_M$) as required, so clearly the pressure terms contribute to gravitons emission amplitudes, as well as mass; just as Einstein found one century ago. It is also easy to show, that for any gravitating mass; the mass, plus energy, plus pressure terms; always total $m\gamma_M$ exactly as our equations require. Our equations also required that the mass of the universe increases as $\gamma_M \approx 2R$ at proper distance $s = 1$. There is no problem with this, as if any part of that mass fell onto this horizon its total energy is also increased by the factor $\gamma_M \approx 2R$. Any energy in starlight from above has this same astronomical frequency increase. We have been looking at scenarios where we are measuring total energies/mass etc in the metric adjacent to that mass. But our equations require it to apply in the metric at any radius, and we conjecture that this must be so. We will later relate all this to an action density $\rho(k_{\min}\text{Action}) = K(k_{\min}\text{Action})dk_{\min}$, where the action constant $K(k_{\min}\text{Action})$ is invariant throughout all spacetime, at maximum wavelength $k_{\min}$. But it only works, if we take the mass at infinity that builds a black hole, and the mass of the universe, times the local value of $\lambda_M^2 = 1/g'_{tt}$ for $k_{\min}$ graviton probability densities?

We have not yet included the relatively small number of $k_{\min}$ gravitons emitted by the mass itself $(\psi_m * \psi_m)$, which has effect close to black holes; but we will first look at the expanding universe. This is a much revised version of section 5.3 in [7] with Figure 2.4. 1 & Equ's.(2.4. 12) helping to make clear why the $k_{\min}$ graviton constant $K_{Gk\min}$ is invariant throughout spacetime. It is the cutoff wavenumber where densities for action available equals action required, by $k_{\min}$ graviton superpositions. The value of $k_{\min}$ reduces with cosmic time T but increases around mass concentrations with the local metric clock rates. See Figure 2.5. 1.

2.4 The Expanding Universe

Section 2.1.1 conjectures that virtual gravitons are single wavenumber k members of superpositions, of width dk. In section 2.3 in [7] wavefunctions ψ_k borrow integral n values of spatially polarized quanta from the zero point fields. From table 4.3.1, also in [7], we see that gravitons are superpositions of integral $n = 3,4,5$ modes with an expection value $\langle n \rangle = 3.33$, thus each $k_{\min}$ graviton borrows $\langle n \rangle = 3.33$ $k_{\min}$ spatially polarized quanta. The same applies to the mass borrowed by these superpositions from the time polarized part of these fields, as the rest mass of each infinitesimal mass $N = 2$ spin 2 superposition obeys $\langle K_k \rangle = \frac{\langle n \rangle k}{m_0} = 1$, but now with $m_0 = \langle n \rangle k_{\min} = 3.33 k_{\min}$ time polarized quanta. Each zero point mode has equal time polarized, and space polarized quanta, so each graviton still only borrows a total of $3.33 k_{\min}$ quanta when near $k_{\min}$. The number density of gravitons at any wavenumber k using Eq.(2.2. 10) for $\rho_U^2 / k_{\min}^4$ & Eq.(2.2. 11) and rewiting Eq. (2.1. 9)

$$\rho_{Gk} \approx \frac{0.3247\alpha_G \rho_U^2}{k_{\min}^4} dk \left[\frac{49.27}{x} \frac{\sqrt{x^2 + 11.09}}{\left(2x^2 + 11.09\right)^2} \right] \approx 0.253\alpha_G dk \left[\frac{49.27}{x} \frac{\sqrt{x^2 + 11.09}}{\left(2x^2 + 11.09\right)^2} \right] \tag{2.4. 1}$$

The blue part of Eq. (2.4. 1) is one when $k / k_{\min} = x = 1$. We simply multiply this by 3.33 to get quanta density required.

$$\rho_{Quanta@k} \approx 3.33 \times 0.253\alpha_G dk \left[\frac{49.27}{x} \frac{\sqrt{x^2 + 11.09}}{\left(2x^2 + 11.09\right)^2} \right]$$

$$\text{When } \frac{k}{k_{\min}} = x = 1 \quad \rho_{Quanta@k\min} = \rho_{Qk\min} \approx 0.84\alpha_G dk_{\min} \tag{2.4. 2}$$

But the density of zero point modes available $@ k_{\min}$ is $k_{\min}^2 dk / \pi^2$ (ignoring some small factors). Even if $\alpha_G << 1$ this is too small by about $k_{\min}^2 \approx 1 / R_{OH}^2$. However the area of the causally connected horizon $4\pi R_{OH}^2$ suggests possible connections with Holographic horizons and the AdS/CFT correspondence [24].

2.4.1 Holographic horizons and red shifted Planck scale zero point modes

Malcadena [24] proposed AntiDesitter or Hyperbolic spacetime where Planck modes on a 2D horizon are (almost) infinitely redshifted at the origin by an (almost) infinite change in the metric. *In contrast we have assumed flat is space on average to the horizon*. In section 2.2.3 in [7] we defined a rest frame, in which preons are born with zero momentum and infinite wavelength, forming infinite superpositions. If there is a spherical horizon with Planck scale zero point modes, that is receding locally at virtually light velocity, these Planck modes can be absorbed by infinite wavelength preons that are born at the centre of this spherical horizon. Special relativity tells us that clocks on that receding horizon will be almost stopped relative to a fixed observer, so these Planck modes can be absorbed from that horizon with wavelengths of the order of the horizon radius. *This potential possibility only exists because zero momentum preons have an infinite wavelength.* If that recession velocity is $\beta = v/c$, where $\beta \to 1$, the frequency/wavenumber reduces as

$$\frac{k_{Observer}}{k_{Source}} = \frac{1}{\gamma} = \sqrt{1-\beta^2}, \text{ where at this Cosmic time } \gamma \approx 10^{61} \qquad (2.4.\ 3)$$

This is not the frequency change of a photon, for example, emitted from that moving horizon. Action, or $\Delta E \cdot \Delta T \approx \hbar$, is invariant in relativity. We are transforming "Large Energy x Small Time" into "Small Energy x Large Time". This as simply special relativity applying locally between, a comoving frame instantaneously at rest on the receding horizon, and a frame on that horizon moving past it at $\beta \approx 1$. There is always some rest frame travelling at nearly light velocity that can transform Planck energy modes into a $k_{\min} \approx 1/R_{OH}$ mode, and also many other frames travelling at various lower velocities that can transform Planck energy modes into any $k > k_{\min}$ mode. In sections 2.1.2 & 2.2.1 we used the fact that clocks in all comoving coordinates, tick at the same rate; so does this effect our logic here? Space between comoving galaxies expands with cosmic or proper time *t*, and called the scale factor $a(t)$. It is normally expressed as $a(t) \propto t^p$ and we will start at time $t = T_0$ with time T now.

$$\text{Thus } \dot{a}(t) \propto pt^{p-1} \text{ and the Hubble parameter } H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{p}{t} \qquad (2.4.\ 4)$$

Writing the present scale factor normalized to one so that $a(T) = 1$ implies $a(t) = t^p / T^P$, we can get the causally connected horizon radius and the horizon velocity *V*. Using Eq. (2.4. 4)

$$\text{The horizon radius } R_{OH} = \int_{T_0}^{T} \frac{dt}{a(t)} = T^p \int_{T_0}^{T} \frac{dt}{t^p} = \frac{T - T_0}{1-p} \approx \frac{T}{1-p} \text{ if } T >> T_0 \text{ \& } p \text{ constant.} \qquad (2.4.\ 5)$$

$$\text{In flat space only, the horizon velocity } \quad V = \frac{dR_{OH}}{dT} = \frac{d}{dT}\left[T^p \int_{T_0}^{T} \frac{dt}{t^p}\right]$$

$$\text{Using } d(uv) = udv + vdu, \text{if } (T >> T_0): \quad \frac{dR_{OH}}{dT} = \frac{T^p}{T^p} + \frac{R_{OH}}{T^p}(pT^{p-1}) = 1 + \frac{p}{T} R_{OH}$$

$$\text{But Hubble parameter } H(t) \ @ \ T \text{ is } \frac{p}{T} \text{ so the horizon velocity } V = 1 + H(T) R_{OH} \qquad (2.4.\ 6)$$

This is always true in flat space regardless of how p behaves.

In flat space at the current time the coordinate, proper and comoving distances are all equal. The *Hubble flow velocity of a comoving galaxy on the horizon* is $V' = H(T)R_{OH}$ and thus from Eq. (2.4. 6) the horizon velocity is always $V = 1 + V'$. In other words the horizon is moving at light velocity relative to comoving coordinates instantaneously on the horizon as measured by a central observer. Now clocks tick at the same rate in all comoving galaxies but clocks moving at almost the horizon light velocity (relative to comoving coordinates instantaneously on the horizon) will tick extremely slowly or as $1/\gamma \approx 10^{-61}$ from Eq.(2.4. 3) as special relativity applies locally in this case. Thus Planck modes on the receding horizon will obey Eq's.(2.4. 3) as seen in all comoving coordinates. Let us now imagine an infinity of frames all travelling at various relativistic velocities relative to comoving coordinates instantaneously on the horizon, and radially as seen by central observers. We can think of these as spherical shells on the horizon *all of one Planck length thickness as measured by observers moving radially with them.* Transverse dimensions do not change for all radially moving observers and the effective surface area of all these shells is $4\pi R_{OH}^2$. The internal volume of all these shells, as measured in rest frames by observers moving radially with them, and each of these observers measures their thickness ΔR as one Planck length; is

$$\text{Rest frame internal shell volume} \quad V = 4\pi R_{OH}^2 \Delta R = 4\pi R_{OH}^2 \tag{2.4. 7}$$

We want the zero point quanta available where *these quanta have near Planck energy* ΔE, *lasting for Planck time* ΔT *such that* $\Delta E \times \Delta T \approx \hbar / 2$. Before frequency changing, a single zero point quanta thus has Planck energy (temporarily using a single primed k' that is not the k' of Eq.(2.1. 4)) where $k' = 1$ before, and k after the frequency change. The density of modes in this shell is $k'^2 dk' / \pi^2$ and is equivalent to

$$\frac{k'^2 dk'}{\pi^2} \text{ quanta, which we will write as mode quanta density } \frac{k'^3}{\pi^2}\frac{dk'}{k'}. \tag{2.4. 8}$$

Now at Planck scale $k' = 1$ and we are transforming to k where from Eq.(2.4. 3) $k = k'/\gamma$ & $dk = dk'/\gamma$. Thus $dk'/k' = dk/k$. As $k = 1$ Eq.(2.4. 8) becomes

$$\text{Planck Scale Quanta Density before frequency changing} = \frac{1^3}{\pi^2}\frac{dk'}{k'} = \frac{1}{\pi^2}\frac{dk}{k} \tag{2.4. 9}$$

Multiplying density by volume ie. Eq's. (2.4. 7) & (2.4. 9) gives the total Planck scale quanta inside the rest frame shell as $4\pi R_{OH}^2 \cdot \frac{1}{\pi^2}\frac{dk}{k}$. The preons that build infinite superpositions are born with zero momentum and infinite wavelength. They can access all the spatial modes regardless of polarization direction. Using Eq. (2.4. 6), after transforming to wavenumber k, these quanta have radius $R' \approx \lambda\!\!\!{}^{-}_C = \frac{1}{k} = \frac{1}{k_{\min}}\frac{k_{\min}}{k} = \frac{R_{OH}}{\Upsilon}\frac{k_{\min}}{k}$ and provided space is flat they occupy spherical volume $V' \approx \frac{4\pi R_{OH}^3}{3\Upsilon^3}\left[\frac{k_{\min}}{k}\right]^3$. As $\Upsilon = k_{\min} R_{OH}$ the quanta density becomes, after dividing by V'

$$\rho_{Quanta@k} \approx \left[4\pi R_{OH}^2 \frac{1}{\pi^2}\frac{dk}{k}\right]\frac{3\Upsilon^3}{4\pi R_{OH}^3}\left[\frac{k}{k_{\min}}\right]^3 \approx \frac{3\Upsilon^2}{\pi^2} dk \left[\frac{k}{k_{\min}}\right]^2 \approx \frac{3\Upsilon^2}{\pi^2} dk \cdot x^2 \text{ where } x = \frac{k}{k_{\min}}$$

$$\text{Density of quanta available after frequency change } \rho_{Quanta@k} \approx \frac{3\Upsilon^2}{\pi^2} x^2 dk \tag{2.4. 10}$$

Now an observer at the centre of all this sees flat space being added inside the horizon at the rate of the horizon velocity $V = 1 + H(T)R_{OH}$ as in Eq. (2.4. 6). We will conjecture that the space added in one unit of Planck time inside the expanding horizon also creates the source of these zero point quanta that we can borrow. Thus Eq. (2.4. 10) becomes

$$\text{Density of } k_{\min} \text{ quanta available } \rho'_k \approx \frac{3\Upsilon^2 V}{\pi^2}\left[\frac{k}{k_{\min}}\right]^2 dk = \frac{3\Upsilon^2(1+H\cdot R_{OH})}{\pi^2}x^2 dk \qquad (2.4.\ 11)$$

2.4.2 Plotting available quanta densities, and required quanta densities

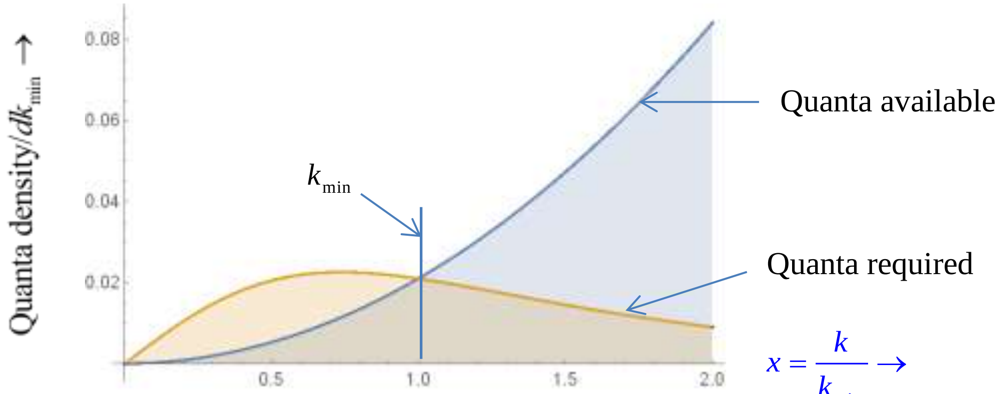


Figure 2.4. 1

Figure 2.4. 1 plots Eq's. (2.4. 2) & (2.4. 11) as a function of $x = k / k_{\min}$. There must be some sort of cutoff in the quanta required below $k = k_{\min}$ probably exponential; but for illustration, we have just used a simple cutoff. We do not yet know the value of the gravitational coupling α_G so we just arrange both curves to cross at $k = k_{\min}$. This is quite valid, as we then equate them in Eq. (2.4. 12). This plot always looks the same, at all cosmic times T and in any metric, only the value of $k_{\min}$ changes. Also, it all only works in a continually expanding flat universe. So when $k = k_{\min}$

$$\text{Quanta available} \approx \frac{3\Upsilon^2 V}{\pi^2}dk_{\min} = \text{Quanta required} \approx 0.84\alpha_G dk_{\min} = K_{Qk\min}dk_{\min}$$

$$\text{Where } K_{Qk\min} = 0.84\alpha_G \text{ is the "Quanta required @ } k_{\min} \text{ Constant" \& } \alpha_G \approx \frac{\Upsilon^2 V}{2.76} \qquad (2.4.\ 12)$$

In Section 2.9 we show this as an invariant "Four Volume Action Density".

Equation (2.2. 10) the average density of the universe $\rho_U \approx 0.8823\frac{\Upsilon^2}{R_{OH}^2}$ allows us to solve the present value of $\Upsilon = k_{\min}R_{OH}$. Using the 9 year WMAP (March 2013) data for Baryonic and Dark Matter density and radius $R_{OH} \approx 2.7\times10^{61}$ Planck lengths ($\approx 46\times10^9$ light years) puts $\rho_U \times R_{OH}^2 \approx 0.37$ in Planck units. Thus $\rho_U \times R_{OH}^2 \approx 0.8823\Upsilon^2 \approx 0.37$ yields

$$\text{The current value for } \Upsilon = k_{\min}R_{OH} \approx 0.65 \qquad (2.4.\ 13)$$

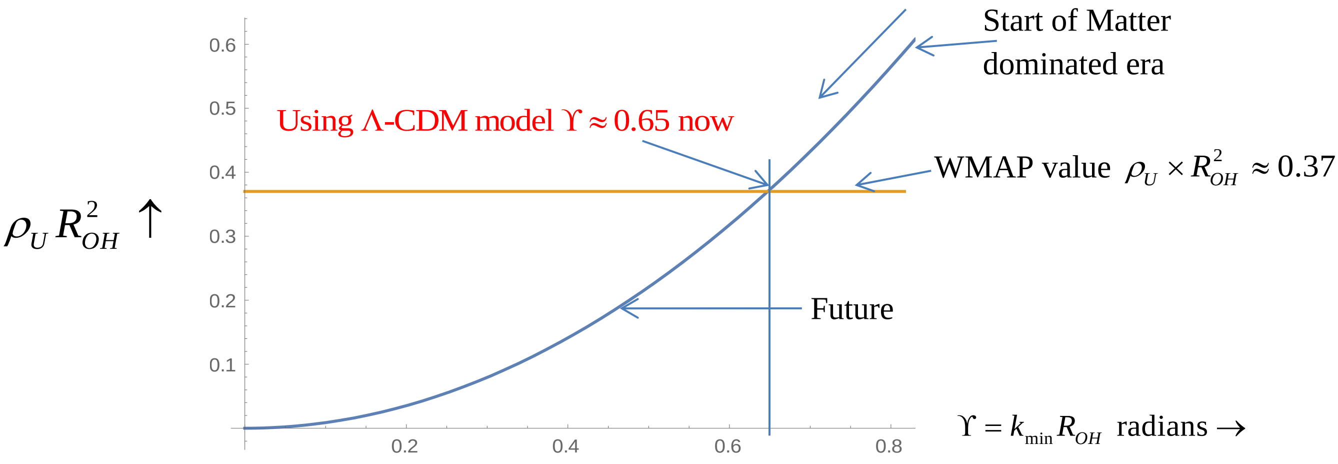


Figure 2.4. 2 plots $\rho_U \times R_{OH}^2 \approx 0.8823\Upsilon^2$

Figure 2.4. 7 plots Eq. (2.4. **20**) $\Upsilon \approx 0.83Exp(-0.24t)$ out to 10 times the current age of the universe showing the exponential decrease with time. The current Horizon Hubble velocity $V = 1 + H(T)R_{OH} \approx 4.37$ and putting this and $\Upsilon \approx 0.65$ into Eq.(2.4. 12) we get a value for the graviton coupling constant α_G. But we can alternatively take $\alpha_G = 1$ as a given, with the different solution $\Upsilon^2 V = 2.76$, as in the two Eq's. (2.4. 14)

$$\text{Using the } \Lambda\text{-CDM model} \qquad \alpha_G \approx \frac{\Upsilon^2 V}{2.76} \approx 0.67$$
$$\text{Alternatively assuming } \alpha_G = 1 \qquad \Upsilon^2 V \approx 2.76 \qquad (2.4.\ 14)$$

2.4.3 **The gravitational coupling constant**

We have made too many assumptions and approximations for these equations to be accurate, however they have simplicity. Also, an $\approx 15\%$ increase in the current value of $\Upsilon \approx 0.65$ to $\Upsilon' \approx 0.75$ which is the same as an $\approx 15\%$ increase in the radius of the universe, and an $\approx 13.5\%$ increase in the horizon velocity gives $\alpha_G \approx \Upsilon^2 V / 2.76 \approx 1$. These increased values are not too far off our proposed exponential expansion in section 2.4.4. This would also be equivalent to $\Upsilon \approx 0.95$ at the start of the matter dominated era in Figure 2.4. 7 with Eq. (2.4. **20**) modified to $\Upsilon \approx 0.95 \times Exp(-0.24t)$. The actual value for α_G is less important than the form of this equation, as provided Eq. (2.2. 10) $\rho_U \times R_{OH}^2 \approx 0.8823\Upsilon^2$ is true (or in other words all comoving observers measure the same *maximum wavelength graviton probability density* $K_{Gk\min}$ *invariant, as in Eq.* (**2.2. 11**) GR is still true locally regardless of graviton coupling α_G. The normal gravitational constant (big) G is directly related to the metric change of GR, and if GR is true locally then G will not change, as most of our equations show it is independent of graviton coupling α_G. However, as we will see, exponential expansion follows quite naturally from either version of Eq's. (2.4. 14). In fact, all that is required for exponential expansion, is for $\Upsilon^2 V$ to be constant.

2.4.4 A possible exponential expansion in a flat matter dominated universe

We have been assuming space is flat on average, and this allows very simple calculations. *Using Euclidean space properties, of equal coordinate, proper and comoving distances at the current time*, let the scale factor be a with density $\rho \propto \frac{1}{a^3}$ in this era. Eq. (2.2. 10) tells us the average density of the universe $\rho_U \approx 0.8823\frac{\Upsilon^2}{R_{OH}^2}$ or $\rho_U = K\frac{\Upsilon^2}{R_{OH}^2} = \frac{1}{a^3}$ where

$$K = 0.8823 \text{ is constant and } a^3 = KR^2\Upsilon^{-2} \rightarrow a = K'R^{2/3}\Upsilon^{-2/3} \text{ where } R = R_{OH} \qquad (2.4.\ 15)$$

The Hubble parameter H is

$$H = \frac{\dot{a}}{a} = \frac{(2/3)K'R^{-1/3}\Upsilon^{-2/3}\, dR/dt}{K'R^{2/3}\Upsilon^{-2/3}} - \frac{(2/3)K'R^{2/3}\Upsilon^{-5/3}\, d\Upsilon/dt}{K'R^{2/3}\Upsilon^{-2/3}} = \frac{2}{3}\left[\frac{1}{R}\frac{dR}{dt} - \frac{1}{\Upsilon}\frac{d\Upsilon}{dt}\right]$$

Thus the Hubble Horizon flow velocity @ R_{OH} is $V' = H\cdot R = \frac{2}{3}\left[\frac{dR}{dt} - \frac{R}{\Upsilon}\frac{d\Upsilon}{dt}\right]$ (2.4. 16)

We can also write either of Eq's.(2.4. 14) as $\Upsilon^2 V$ = a constant K, with $\Upsilon^2 dV + 2\Upsilon d\Upsilon V = 0$.

Thus $\frac{1}{2V}\frac{dV}{dT} = -\frac{1}{\Upsilon}\frac{d\Upsilon}{dT}$ and Eq. (2.4. 6) tells us that the Horizon velocity $V = \frac{dR_{OH}}{dt} = \frac{dR}{dt}$.

Equation (2.4. 6) also tells us that $V' = H\cdot R = V - 1$ so we can write Eq. (2.4. 16) as

$$\left[3(V-1) - 2V = -\frac{2R}{\Upsilon}\frac{d\Upsilon}{dt} = \frac{2R}{2V}\frac{dV}{dt}\right] \rightarrow V - 3 = \frac{R}{V}\frac{dV}{dt} \rightarrow \frac{dV}{dt} = \frac{V}{R}(V-3) \qquad (2.4.\ 17)$$

We will look for an exponential increase of the horizon velocity so $dV/dt > 0$ and $3 \le V \le \infty$. Let us simply try $V = 3Exp(bt)$ with $V > 3$ for all values of b & $t > 0$. Assume a starting time after transition of $t_0 \approx 0$ initially, and only consider times $t >>> t_0$.

If *space is flat* we can simply put $R = \int_{t_0}^{t} Vdt = \int_{t_0}^{t} 3Exp(bt)dt$ & $R \approx \frac{3[Exp(bt)-1]}{b}$ if $(t >> t_0)$

Putting this value for R plus $V = 3Exp(bt)$ & $V - 3 = 3[Exp(bt)-1]$ into Eq. (2.4. 17)

$$\frac{dV}{dt} = \frac{V}{R}(V-3) = 3Exp(bt)\cdot\frac{b}{3[Exp(bt)-1]}\cdot 3[Exp(bt)-1] = 3bExp(bt).$$

But $V = 3Exp(bt)$ and again $\frac{dV}{dt} = \frac{d}{dt}[3Exp(bt)] = 3bExp(bt)$. Thus Eq's. (2.2. 10) & (2.4. 14) are consistent with $V = 3Exp(bt)$ for positive b regardless of the value of graviton coupling α_G

A possible expansion solution is $V = 3Exp(bt)$ & $R = \frac{3[Exp(bt)-1]}{b}$, $b > 0$. (2.4. 18)

But is this consistent with the local special relativity requirement for R_{OH}? In other words does R @ time $T = a(T)\int_0^T \frac{dt}{a(t)} = \frac{3[Exp(bT)-1]}{b}$? Now Eq. (2.4. 15) tells us the scale factor $a^3 = KR^2\Upsilon^{-2} \rightarrow a = K'R^{2/3}\Upsilon^{-2/3}$ but Eq.(2.4. 14) says $\Upsilon^2 \propto 1/V$ so the scale factor $a \propto V^{1/3}R^{2/3}$. From Eq. (2.4. **18**) ignoring the constant factors 3 & b, $V \propto Exp(bt)$ & $R \propto [Exp(bt)-1]$

$$\text{The scale factor } a(t) \propto Exp(bt)^{1/3}[Exp(bt)-1]^{2/3} \text{ and } R = a(T)\int_0^T \frac{dt}{a(t)} \quad (2.4.19)$$

$$= Exp(bT)^{1/3}[Exp(bT)-1]^{2/3}\int_0^T \frac{dt}{Exp(bt)^{1/3}[Exp(bt)-1]^{2/3}}$$

$$= \frac{3[Exp(bT)-1]}{b}$$

And Eq. (2.4. 18) appears to be a consistent exponential expansion for both V and R.

From Eq. (2.4. 14) we showed $\frac{1}{2V}\frac{dV}{dT} = -\frac{1}{\Upsilon}\frac{d\Upsilon}{dT}$. Using Eq. (2.4. 18) $V = 3Exp(bt)$ & $\frac{dV}{dt} = 3bExp(bt)$ implies $\Upsilon = K \cdot Exp(-bt/2)$. The current value of $\Upsilon \approx 0.65$ from Eq.(2.4. 13) and our best guess of $b \approx 0.48$ from Figure 2.4. 3 yields

$$\Upsilon = k_{\min}R_{OH} \approx 0.826Exp(-0.24t) \text{ in radians} \quad (2.4.\ 20)$$

Several of the above formulae only apply in Euclidean on average space.

This simple exponential expansion starts after the transition, and is very different to current Λ-CDM models, which keep the Hubble parameter $H = \dot{a}/a \approx 2/3t$ constant (if $\Omega = 1$) until Dark Energy starts to take effect. This continuous exponential increase could well lead to slightly different values for the radius R_{OH} and also possibly the age $T \approx 13.8\times10^9$ years. Current Λ-CDM models put the Hubble parameter as $H = \dot{a}/a \approx 1/T$ at present (based on $T \approx 13.8\times10^9$ years). In the plots below we put $T \approx 13.8\times10^9$ years $= 1$, with R_{OH} or radius R becoming multiples of $T = 1$. Using Eq. (2.4. 6)) $V = 1 + H(T)R$, Figure 2.4. 3 plots the Hubble parameter x time $(T = 1)$ now, as a function of the exponential time coefficient *b*, showing if $b = 0$ that $H(t)$ always $= 2/(3t)$ *as in current cosmology at critical density with no dark energy*. Also if $H \approx 1/T$ now the best guess is $b \approx 0.48$. This yields $R \approx 3.85T$ or $\approx 15\%$ greater than current cosmology & Figure 2.4. 4 plots horizon velocity which @ $V \approx 4.85$ now is $\approx 11\%$ greater, both not far off giving a graviton coupling of $\alpha_G \approx 1$ as in Eq. (2.4. 14), see section 2.4.3. Figure 2.4. 5 plots the scale factor based on $b \approx 0.48$, but of course the actual value of *b* or rate of change with time must be in agreement with the redshifts currently observed when looking back in time. These could well change *b* and radius *R*. Figure 2.4. 6 plots the transition to positive acceleration of the scale factor showing the effect of changing the value of *b*. Figure 2.4. 7 plots, out to ten times the age of the universe, Eq. (2.4. 20) $\Upsilon = k_{\min}R_{OH} \approx 0.83Exp(-0.24t)$.

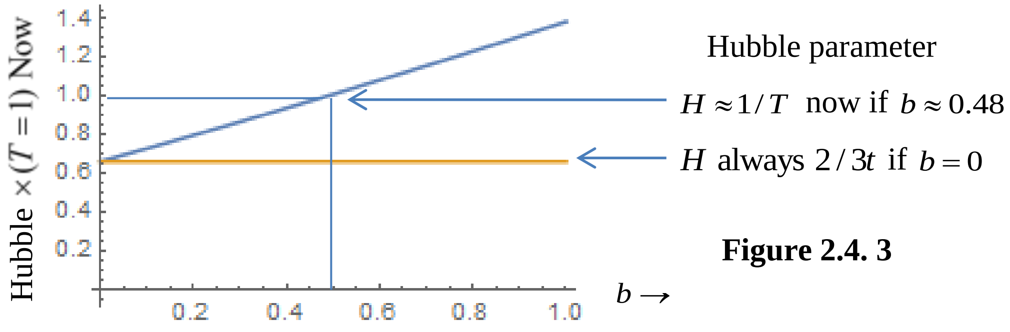


Figure 2.4. 3

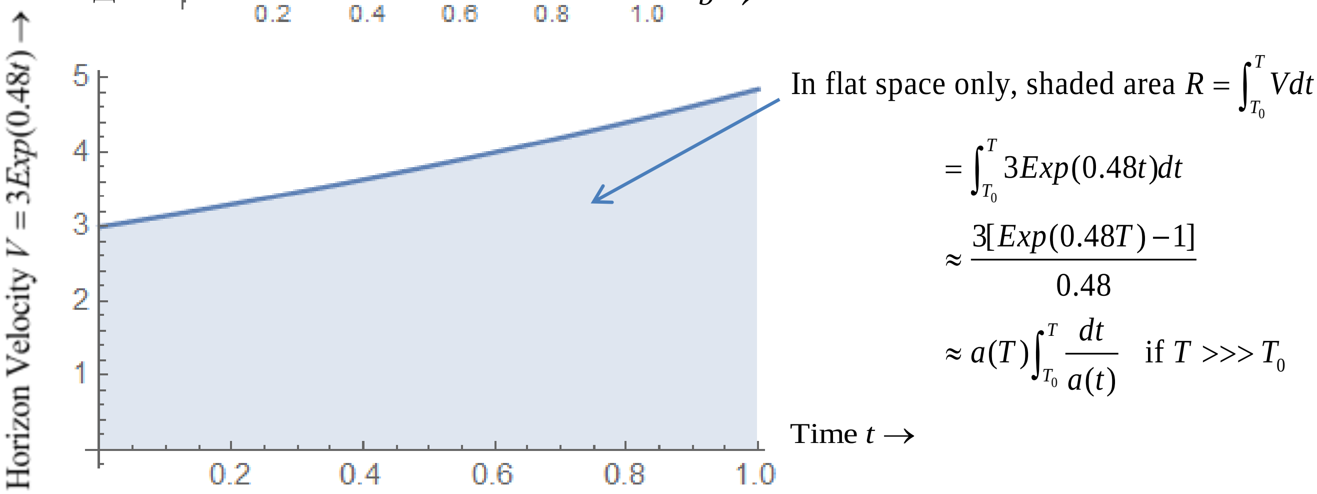


Figure 2.4. 4

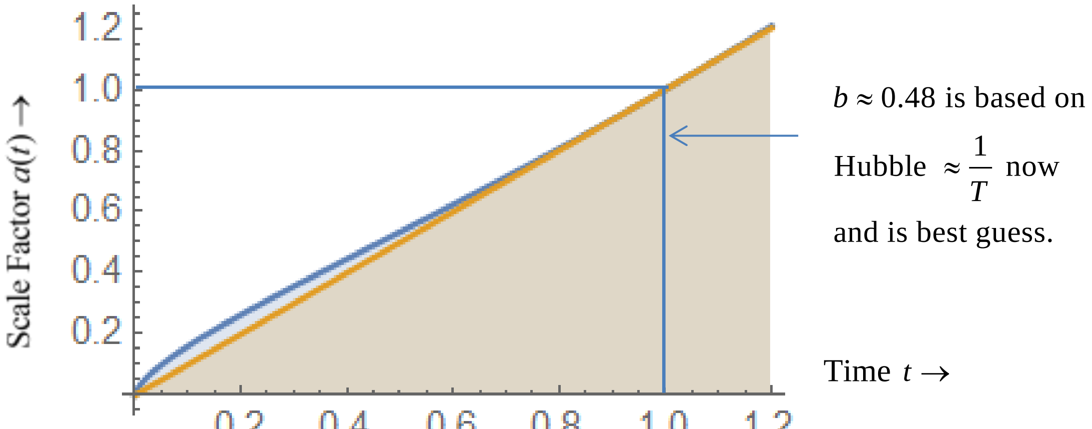


Figure 2.4. 5

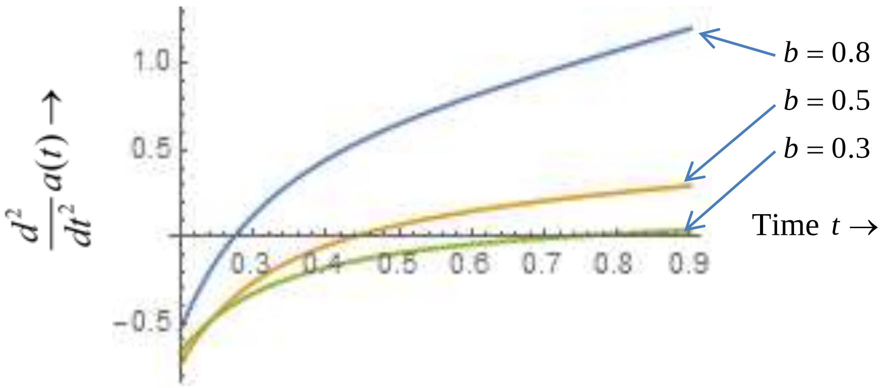


Figure 2.4. 6

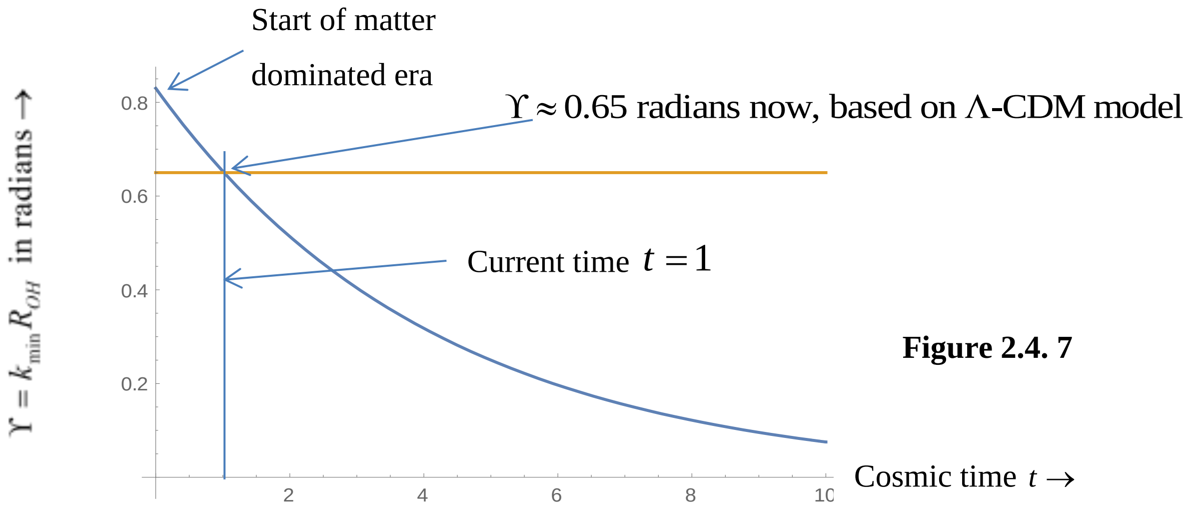


Figure 2.4. 7

2.4.5 The radiation dominated era up to approximately 47,000 years

In the mass dominated era the density $\rho \propto 1/a^3$, and in the preceding radiation dominant era $\rho \propto 1/a^4$. We can repeat section 2.4.4 with horizon velocity $V = 2Exp(ct)$, & horizon radius $R = \int_0^t V dt = \int_0^t 2Exp(ct)dt \;\; = 2[Exp(ct) - 1]/c$. The horizon velocity starts $@V = 2$ and horizon radius $@R = 2t$ with a scale factor $a \propto t^{1/2}$ and is the value used in current models of this era before transition. These models predict results in close agreement with the current measurements of normal matter in the universe. At the start of the matter dominated era, $V = 3$, and one possibility is $V \approx 2Exp(0.4t)$ where time is normalized to one, at the end of this era. This exponential expansion can, with some smooth transition, continue on into the $V = 3Exp(0.48t)$ of the matter era, with the normalization of time then changed, to the current cosmos age. If $\alpha_G = 1$ as in Eq.(2.4. 14) with $V = 2$, Υ has to be ≈ 1.18 initially to get $\Upsilon^2 V \approx 2.76$. When $V = 3$, Υ has to reduce to ≈ 0.96 to keep $\Upsilon^2 V \approx 2.76$. Because the transition time is so small in relation to the $\approx 10^{10}$ year age of the universe, this era has insignificant effect on all our above graphs.

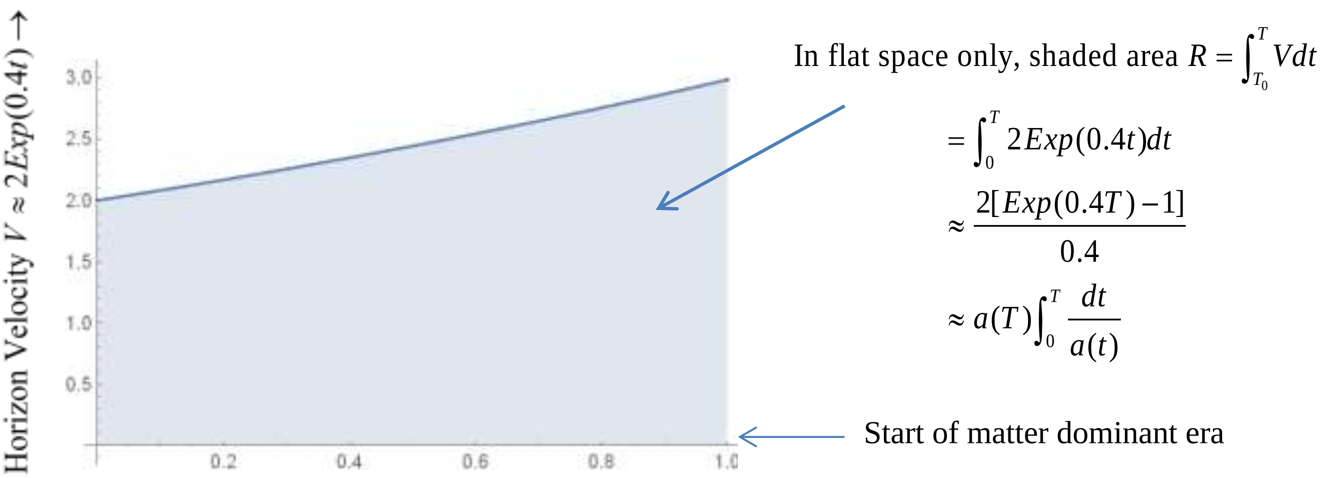


Figure 2.4. 8 An example of a possible radiation dominated exponential exapansion.

2.5 An Infinitesimal change to General Relativity at Cosmic Scale

All we have discussed is based on the energy in the zero point fields being limited. We argued that uniform mass density throughout the cosmos has $k_{\min}$ graviton probability density $\rho_{Gk\min}$ as in Eq. (2.2. **11**). At this probability density the zero point quanta density available equals that required. To maintain this required balance as in Figure 2.4. 1 we argued that around any mass concentration the curvature of space expands space locally so as to keep the $k_{\min}$ graviton constant $K_{Gk\min} \approx 0.253\alpha_G$ as in Eq. (2.2. **11**) invariant at all points. In other words *our conjecture only works if the local curvature of space depends on the difference between the local mass density and the uniform background.* Compared to General Relativity this is an infinitesimal change except at cosmic scale. *GR says the curvature of space depends on local mass density* whereas we argue that it depends on *the difference between local mass density and the average background* (only a few hydrogen atoms per cubic metre). This automatically guarantees the universe has to be flat on average. All our aguments start with flat space on average. The equations of GR would look almost identical except the Energy Momentum Tensor $T_{\mu\nu}$ in comoving coordinates requires T_{00} the mass/energy density to change from ρ to $\rho - \rho_U$ where the density of the universe ρ_U is as in Eq.(2.2. 10).

$$\text{In comoving cordinates } T_{00} \text{ changes from } \rho \text{ to } \rho - \rho_U \text{ in the Energy Momentum Tensor } T_{\mu\nu} \tag{2.5. 1}$$

2.5.1 Non comoving coordinates in Minkowski spacetime where $\mathbf{g}_{\mu\nu} = \eta_{\mu\nu}$

To this point everything we have looked at has been in comoving coordinates. Velocities relative to comoving coordinates are usually referred to as peculiar velocities, so, does what we are saying above, still apply in such non comoving coordinates? The average momentum of the universe is zero in comoving coordinates and all background gravitons must be time polarized. If an observer moves at any peculiar velocity β_P, the rest of the universe is moving in the opposite direction relative to this observer at velocity $-\beta_P$. If we repeat section 2.1.2 the spherical shells of Figure 2.1. 2 become ellipsoids, but we can still pair off equal mass densities at equal and opposite radii. Because the average velocity in every large region of the universe is $-\beta_P$, these will create equal and opposite gravitomagnetic vectors, all summing to zero; *resulting in zero transverse, and circularly polarized gravitons*. For the same reasons the magnetic field is zero at the centre of an infinitely long circular conductor with uniform current over its cross-section. Thus the *background, in non rotating Minkowski* $g_{\mu\nu} = \eta_{\mu\nu}$ *spacetime, always contains only time polarized or spherically symmetric* $k_{\min}$ *gravitons, regardless of peculiar velocities.* So let us look again at these background $k_{\min}$ graviton amplitudes and probabilities. In section 2.1.2 we found in Eq. (2.1. 10) the probability density of background $k_{\min}$ virtual gravitons in comoving coordinates.

$$\psi_{\text{Universe}}{}^* \psi_{\text{Universe}} = \rho_{Gk\min} = K_{Gk\min} dk_{\min} \text{ where } K_{Gk\min} \approx \frac{0.3247\alpha_G \rho_U^2}{k_{\min}^4} \text{ in comoving coordinates.}$$

If we move relative to this at peculiar velocity β_P, measured volumes shrink as $\gamma_P^{-1} = (1-\beta_P^2)^{1/2}$ and all comoving mass increases as $\gamma_P = (1-\beta_P^2)^{-1/2}$. (We will use red symbols with the subscript P, and triple primes for wavenumber $k'''_{\min}$ for peculiar velocities, to distinguish them from metric changes where we used green and a double primed $k''_{\min}$.) Thus $\rho'''_U = \gamma_P^2 \rho_U$ .The minimum wavenumber $k_{\min}$ has its lowest value in comoving coordinates (at least far from mass concentrations where $g_{\mu\nu} = \eta_{\mu\nu}$) but at peculiar velocity β_P, $k'''_{\min} = \gamma_P k_{\min}$.

$$\text{Thus } \frac{\rho'''^2_U}{k'''^4_{\min}} = \frac{\gamma_P^4}{\gamma_P^4}\frac{\rho_U^2}{k_{\min}^4} = \frac{\rho_U^2}{k_{\min}^4} \text{ and } K_{Gk\min} \text{ is always invariant.} \qquad (2.5.\ 2)$$

And $\rho_{Gk\min} \approx K_{Gk\min} dk_{\min}$ is always true in non comoving coordinates if $g_{\mu\nu} = \eta_{\mu\nu}$

2.5.2 Non comoving coordinates when $g_{\mu\nu} \neq \eta_{\mu\nu}$

Starting with Eq. (2.1. 10) $\rho_{Gk\min} \approx K_{Gk\min} dk_{\min}$ we have just shown that this equation remains true at any peculiar velocity β_P in flat spacetime. All that happens is that the values of $k_{\min}$, $dk_{\min}$ & $\rho_{Gk\min} \approx K_{Gk\min} dk_{\min}$ all increase as $\gamma_P = (1-\beta_P^2)^{-1/2}$. In other words the probability of finding a $k_{\min}$ graviton is always proportional to whatever the value $k_{\min}$ & $dk_{\min}$ is. Also the *amplitude* to find a $k_{\min}$ graviton is always proportional to either $\sqrt{k_{\min}}$ or $\sqrt{dk_{\min}}$. We have shown in sections 2.1 & 2.2 that around mass concentrations in comoving coordinates, the $k_{\min}$ gravitons are comprised of the background due to the universe plus the interaction between the local mass and the universe as in Figure 2.2. 2 . This background $k_{\min}$ graviton probability, regardless of the local metric, is always proportional to whatever the local value $k_{\min}$ & $dk_{\min}$ is. Amplitudes are also always proportional to local values of $\sqrt{k_{\min}}$ or $\sqrt{dk_{\min}}$.

$$\text{Amplitude } \psi_{Gk\min} \text{ (due to rest of universe) or } \psi_{\text{Universe}} \text{ allways} \propto \sqrt{dk_{\min}} \qquad (2.5.\ 3)$$

As in Figure 2.2. 1 the probability for a small mass m to emit a $k_{\min}$ graviton is $\frac{2}{\pi} m^2 \alpha_G \frac{dk_{\min}}{k_{\min}}$.

The normalized wavefunction $\frac{2k'_{\min}}{4\pi r^2} e^{-2k'_{\min} r} \approx \frac{2k'_{\min}}{4\pi r^2} \approx \frac{2\times 3.477 k_{\min}}{4\pi r^2}$ ($k'_{\min} \approx 3.477 k_{\min}$ & $k'_{\min} r \approx 0$).

$$\text{Amplitude } \psi_{Gk\min} \text{ (due to small mass } m) \approx \sqrt{\frac{2}{\pi} m^2 \alpha_G \frac{dk_{\min}}{k_{\min}} \frac{3.477 k_{\min}}{2\pi r^2}} = \frac{2m}{r}\sqrt{\frac{3.477\alpha_G dk_{\min}}{4\pi^2}}$$

$$\psi_m \text{ is allways } \propto \frac{2m}{r}\sqrt{dk_{\min}} \qquad (2.5.\ 4)$$

The interaction between this small mass and the rest of the universe is

$$\Delta\rho_{Gk\min} \approx \psi_{\text{Universe}}{}^*\psi_m + \psi_m{}^*\psi_{\text{Universe}} \propto \sqrt{dk_{\min}} \times \frac{2m}{r}\sqrt{dk_{\min}} \text{ is allways} \propto \frac{2m}{r} dk_{\min} .$$

We have shown previously that $K_{Gk\min}$ is the proportionality constant.

So regardless of peculiar velocities

$$\Delta\rho_{Gk\min} \approx \psi_{\text{Universe}}{}^*\psi_m + \psi_m{}^*\psi_{\text{Universe}} \text{ is allways } \frac{2m}{r} K_{Gk\min} dk_{\min} \tag{2.5. 5}$$

Thus $\Delta\rho_{Gk\min} \approx \psi_{\text{Universe}}{}^*\psi_m + \psi_m{}^*\psi_{\text{Universe}}$ is always proportional to $dk_{\min}$ and at peculiar velocity β_P ; $k'''_{\min} = \gamma_P k_{\min}$ & $dk'''_{\min} = \gamma_P dk_{\min}$. So both $\rho'''_{Gk\min}$ & $\Delta\rho'''_{Gk\min}$ increase as γ_P and their ratio does not change. The logic of our aguments is not affected by peculiar velocities. The same is true for large masses moving at peculiar velocities. In a metric γ_M as in section 2.2.2 (using four blue primes for combined peculiar velocity and metric changes) $k''''_{\min} = \gamma_P\gamma_M k_{\min}$ and $dk''''_{\min} = \gamma_P\gamma_M dk_{\min}$. Both $\Delta\rho''''_{Gk\min}$ & $\rho''''_{Gk\min}$ increase as $\gamma_P\,\gamma_M$ and again their ratio does not change. All the arguments we used in section 2.2.2 do not change and Equ's. (2.2. 14), (2.2. 15) & (2.2. 16) still apply in non comoving coordinates providing β_M is the velocity reached by a small test mass falling from infinity in the same rest frame as the mass concentration m moving at peculiar velocity β_P . We can think of $K_{Gk\min} \approx 0.253\alpha_G$ as invariant throughout the universe, representing the *Probability Density* of finding a minimum wavenumber $k''''_{\min} = \gamma_P\gamma_M k_{\min}$ virtual graviton at all points in spacetime. Near mass concentrations the metric changes. Local clocks change, also the measurement of $k_{\min}$, but not $K_{Gk\min}$. Locally measured infinitesimal volumes increase to accommodate the extra locally emitted maximum wavelength gravitons, keeping the probability density constant. Section 2.9 expresses this as an "Invariant Four Volume Action Density" we argue is consistent with General Relativity.

If we think of the mass in the universe as a dust of density ρ_U essentially at rest in comoving coordinates we can define a tensor $T_{\mu\upsilon}(\text{Background})$. In comoving coordinates $T_{\mu\upsilon}(\text{Background})$ has only one non zero term $T_{00}(\text{Background}) = \rho_U$. In any other coordinates this same $T_{\mu\upsilon}(\text{Background})$ tensor is transformed by the usual tensor transformations that apply in GR. If these coordinates move at peculiar velocity β_P then $T'''_{00}(\text{Background}) = \gamma_P^2\rho_U$ $= \gamma_P^2 T_{00}(\text{Background})$. This all suggests the infinitesimally modified Einstein field equations

$$G_{\mu\upsilon} \equiv R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = \frac{8\pi G}{c^4}\left[T_{\mu\upsilon} - T_{\mu\upsilon}(\text{Background})\right] \tag{2.5. 6}$$

We argue that Eq. (2.5. 6) is consistent with keeping $K_{Gk\min} \approx 0.253\alpha_G$ invariant throughout all spacetime as in Figure 2.5. 1. This infinitesimal modification is only relevant in the extreme case as $T_{\mu\upsilon}$ approaches $T_{\mu\upsilon}(\text{Background})$. Far from mass concentrations $T_{\mu\upsilon} \le T_{\mu\upsilon}(\text{Background})$. Space curvature, in these remote voids, is in general somewhere between slightly negative and zero; but the causally connected universe is always flat on average regardless of the value of Ω.. If there is no inflation, in comoving coordinates, at the Big Bang or slightly after, $k_{\min}$ starts at just under one and is always close to the inverse of the causally connected horizon radius. It is also close to the inverse of cosmic time T . It is always at its minimum far from mass concentrations, but increases with the slower clock rates in the local metric around mass concentrations as in Figure 2.5. 1

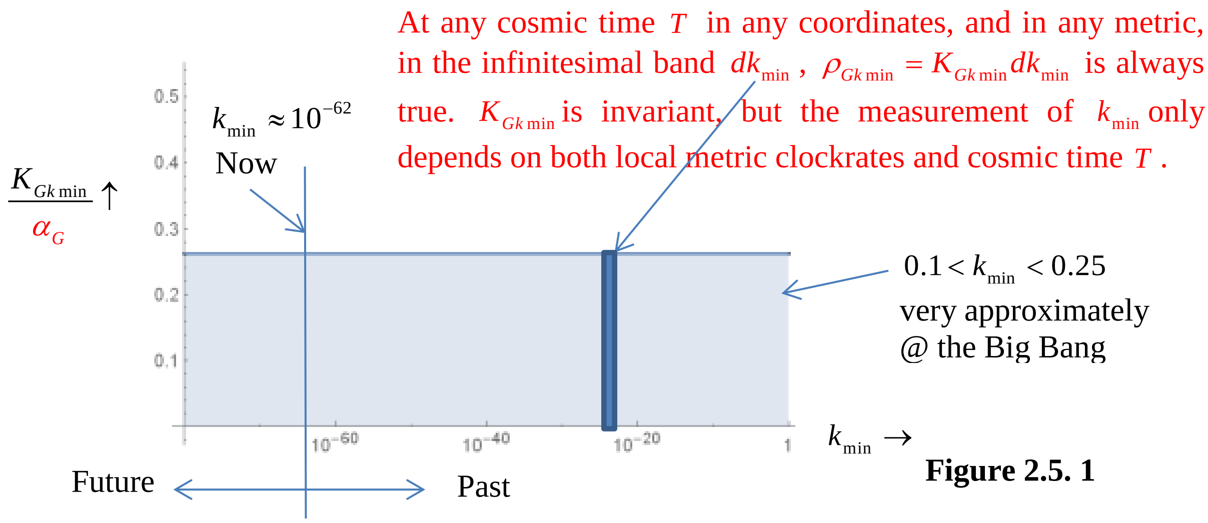


Figure 2.5. 1

2.5.3 Is inflation in this proposed scenario really necessary?

There are two main reasons, usually given, for why inflation is necessary:

(a) The average flatness of space.

(b) The almost uniform temperature of the cosmic microwave background from regions that were initially out of causal contact.

If we put $T_{\mu\upsilon}(\text{Local}) = T_{\mu\upsilon}(\text{Background})$ in Eq. (2.5. 6), the right hand side is identically zero, and $G_{\mu\upsilon} \equiv R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R \equiv 0$ on average throughout all space. The average curvature of all space must be zero and space is compelled to be flat on average. In section 2.4.4 we found that space naturally expands exponentially as in Eq. (2.4. 18) and plotted in Figure 2.4. 4. The value of the constant b in $V = 3Exp(bt)$ has to fit experimental observations. But if it is some fundamental constant, which does not seem unreasonable, it must be the same for all comoving observers. If this is so the physics is identical for all such observers regardless of whether they are in causal contact. Provided we can assume identical starting points everywhere, of say the Planck temperature at cosmic time $T = 0$, then apart from quantum fluctuations, the average background temperature should be some function of cosmic time T for all comoving observers, or at least up to the time the universe became transparent. The physics controlling this should be identical in each comoving frame. Causal contact should not be essential for this. Inflation only guarantees that the starting temperature is uniform everywhere when it stops at approximately $T = 0$. It also has to assume identical physics everywhere from $T \approx 0$ for about the first 375,000 years, or untill the universe is transparent. What we are proposing in this paper should produce results not too different from this.

2.5.4 Why do we think virtual particle pairs do not matter?

For almost a century it has been a puzzle why spacetime is not massively curved by Planck scale zero point energy densities. However space appears to be flat on average regardless of this massive Planck scale zero point energy density so *something must be different and what is it?* In the first paper [7] we conjectured that virtual particles are just single wavenumber k superposition members, whereas real, or observable particles are full infinite superpositions of all wavenumbers k from $k_{\min}$ to k_{Planck}. Only full infinite superpositions have real properties that can be measured (such as measured mass/energy) rather than implied. Because $k_{\min}$ virtual gravitons are such single members they can couple to $k_{\min}$ members of full infinite superpositions. On the other hand virtual particles out of the vacuum are mainly short lived high k single value members that will not couple to $k_{\min}$; provided these virtual particles are single wavenumber k members only, and of a higher k value than $k_{\min}$. The density of $k_{\min}$ virtual pairs from the vacuum is virtually zero as it is based on the Lorentz invariant supply of local zero point fields, not from the receding horizon (sections 2.7.2 & 2.7.3 clarify this).

But this is not the full story. The virtual particles that dress electrons and quarks for example add mass to the real particles. In fact the majority of proton and neutron mass is due to the virtual gluons interacting between quarks. If short lived virtual particles somehow contribute to the mass of full infinite superpositions, then these high k value virtual particles indirectly contribute to the $k_{\min}$ virtual graviton coupling, which is based on the actual mass of the infinite superposition as in Eq. (3.2.3) in [7]. The conservation of energy, or more correctly 4 momentum, says that what we call "real matter or energy" can last for close to the age of the universe. It will have mass and by definition it can be weighed. It can move around, even close to the speed of light, but it is conserved. Gravitons that last this long we have called $k_{\min}$ gravitons and they can only couple to real, or long lasting energy/matter that can be weighed in whatever manner. The rotating dark matter in galaxies we cannot weigh directly, but it contributes to the theoretical weight of a galaxy. We have to allow for this mass when studying galaxy dynamics.

The particle beams in accelerators have real energy which can be temporarily converted into virtual particles. The total energy or 4 momentum is always conserved, but can fluctuate for time $\Delta T \approx 1/\Delta E$. The long term average is what counts. In this sense the mass of short lived virtual particles can contribute to $k_{\min}$ virtual graviton coupling, just as it does in the virtual particle dressing of real charged particles as above. If it can be somehow weighed, it will couple to $k_{\min}$ virtual gravitons. But we cannot weigh the zero point background.

2.6 Messing up what was starting to look promising, or maybe not?

2.6.1 The *k*min virtual gravitons emitted by the mass interacting with itself

In section 2 we started out by finding the average $k_{\min}$ graviton probability density in a uniform universe. We then placed a mass concentration in it, and calculated the extra probability density of $k_{\min}$ gravitons (before the dilution due to local space expansion) due to the amplitude of this mass multiplied by the amplitude of the rest of the mass in the universe. This ended up being proportional to $2m/r$ in Planck units.

$$\Delta\rho_{Gk\min} = (\psi_{\text{Universe}} {}^*\psi_m) + (\psi_m {}^*\psi_{\text{Universe}}) \propto 2m/r \quad \text{as in Eq.(2.2. 6)}$$

And this is true in weak field metrics, except as we start approaching the Schwarzchild radius because of the extra $k_{\min}$ gravitons from the mass interacting with itself: $\psi_m {}^*\psi_m$.

Using Eq. (2.1. **5**) and coupling probability: $\left[\dfrac{2\alpha_G}{\pi} m^2 \dfrac{dk}{k}\right]$

$$\psi_m {}^*\psi_m = \left[\frac{2\alpha_G}{\pi} m^2 \frac{dk}{k}\right] \cdot \left[\frac{2k'}{4\pi} \frac{e^{-2k'r}}{r^2}\right] = \alpha_G \frac{m^2}{r^2} \frac{k' e^{-2k'r}}{\pi^2} \frac{dk}{k}$$

Also using Eq. (2.1. 4) $k' = \sqrt{k^2 + 11.09 k_{\min}^2} \approx 3.477 k_{\min}$ when $k = k_{\min}$

$$\psi_m {}^*\psi_m = \alpha_G \frac{m^2}{r^2} \frac{3.477 k_{\min} e^{-2(3.477 k_{\min} r)}}{\pi^2} \frac{dk_{\min}}{k_{\min}}$$

The exponential term $e^{-2k'r} = e^{-2(3.477 k_{\min} r)} = e^{-6.954 k_{\min} r}$ and we are only interested in radii r that are small in relation to the observable radius of the Universe $R_{OU} \approx k_{\min}^{-1}$, just as in the assumptions we made in section 2.2.1. Thus $k'r \to 0$ and $e^{-2k'r} \approx 1$ in these regions so we can approximate this equation with good accuracy as

$$\psi_m {}^*\psi_m \approx \alpha_G \frac{m^2}{r^2} \frac{3.477}{\pi^2} dk_{\min}$$

$$\Delta\rho_{Gk\min} \text{ due to self emission } \psi_m {}^*\psi_m \approx \alpha_G \frac{m^2}{r^2} 0.352 dk_{\min}$$

$$\approx 1.39 \frac{m^2}{r^2} 0.253 \alpha_G dk_{\min}$$

$$\approx 1.39 \frac{m^2}{r^2} K_{Gk\min} dk_{\min} \quad \text{using Eq.}\mathbf{(2.2.\ 11)}$$

If the local clock rate is $\sqrt{g'_{00}} = \dfrac{1}{\gamma_M}$ as in Eq. (2.2. 13) but with a slightly modified g_{00} (as we will see below), $m' = \gamma_M m$, so before dilution due to the local space expansion:

Before dilution $\Delta\rho_{Gk\min}$ due to $\psi_m * \psi_m \approx 1.39\frac{(\gamma_M m)^2}{r^2} K_{Gk\min} dk_{\min}$ (2.6. 1)

2.6.2 What does this extra term mean for non rotating black holes?

When deriving Eq.(2.2. 14) we found (about two equations previous) that due to interactions with the rest of the Universe $\Delta\rho_{Gk\min} \approx \gamma_M^2 \frac{2m}{r} 0.253\alpha_G dk_{\min} \approx 2\frac{m}{r}\gamma_M^2 K_{Gk\min} dk_{\min}$

$$\text{Thus } \Delta\rho_{Gk\min} \text{ total} \approx \left[2\frac{m}{r} + 1.39\frac{m^2}{r^2}\right]\gamma_M^2 K_{Gk\min} dk_{\min} \text{ in Planck units.} \quad (2.6.\ 2)$$

Staying on our current path appears to contradict General Relativity, but temporarily ignoring this, let us repeat section 2.2.2 which modifies a non rotating black hole metric to

$$g'_{00} = -\frac{1}{g'_{rr}} = 1 - \frac{2m}{r} - 1.39\frac{m^2}{r^2}$$

$$\beta_M^2 = \frac{2m}{r} + 1.39\frac{m^2}{r^2} \quad (2.6.\ 3)$$

$$\gamma_M^2 = \frac{1}{1 - 2m/r - 1.39m^2/r^2}$$

Where β_M is the velocity reached by a small test mass falling in from infinity in the same rest frame. Applying the same proceedures as in section 2.2.2 we can use Equ's. (2.6. 3) to show that $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ in this new metric, and we will discuss how this relates with General Relativity in sections 2.6.5 & 2.6.6. The modified non rotating horizon radius occurs when $r^2 - 2mr - 1.39m^2 = 0$ or the:

Modified non rotating horizon radius $r \approx 2.55m$ (2.6. 4)

or $\approx$ 27.5% larger than the Schwarzchild value.

2.6.3 What does it mean for rotating black holes?

In section 2.3 when we looked at the Kerr Metric we used a dimensionless form of the metric in Equ's.(2.3. 2). We also used a dimensionless parameter A where we initially put $A = 2m/r$. We also showed that we could change A without changing $g'_{tt} = \Delta / g_{\varphi\varphi}$ the time component in the corotating frame, provided there is a modified $\Delta = 1 + \frac{\alpha^2}{r^2} - A$. So again temporarily ignoring potential conflicts with General Relativity let us change $A = \frac{2m}{r}$ to $A = \frac{2m}{r} + 1.39\frac{m^2}{r^2}$ and look at the consequences. Firstly from Equ's. (2.6. 3) we can see that

$A = \beta_M^2$ where β_M is the radial inward velocity, in a corotating rest frame, of a small test mass falling from infinity (in the rest frame of the rotating black hole centre). The inner event horizon is the radius where $g_{rr} \to \infty$ so using Equ's.(2.3. 2) let $g_{rr} = \dfrac{g_{\theta\theta}}{\Delta} \to \infty$ or put

$$\Delta = 1 + \frac{\alpha^2}{r^2} - A = 0$$

$$= 1 + \frac{\alpha^2}{r^2} - \frac{2m}{r} - 1.39\frac{m^2}{r^2} = 0$$

$$\text{or } r^2 + \alpha^2 - 2mr - 1.39m^2 = 0$$

$$r = \frac{2m \pm \sqrt{4m^2 + 5.57m^2 - 4\alpha^2}}{2}$$

Event Horizon radius $\qquad r = \dfrac{2m \pm \sqrt{9.57m^2 - 4\alpha^2}}{2}$ (2.6. 5)

When $\alpha = 0 \quad r = \dfrac{2m \pm \sqrt{9.57m^2}}{2} \approx \dfrac{2m + 3.09m}{2} \approx 2.55m$ as in the non rotating case.

Maximum spin is when $\qquad 4\alpha^2 = 9.57m^2$ or $\alpha_{\max} \approx 1.547m$

At this maximum spin $\qquad r = m$ as in the usual Kerr Metric.

The outer horizon occurs when $g_{tt} = 1 - \dfrac{A}{g_{\theta\theta}} = 0$ or $g_{\theta\theta} - A = 0$ and using Equ's.(2.3. 2)

$$1 + \frac{\alpha^2}{r^2}\cos^2\theta - A = 1 + \frac{\alpha^2}{r^2}\cos^2\theta - \frac{2m}{r} - 1.39\frac{m^2}{r^2} = 0$$

$$r^2 - 2mr - 1.39m^2 + \alpha^2\cos^2\theta = 0$$

$$r = \frac{2m + \sqrt{4m^2 + 5.57m^2 - 4\alpha^2\cos^2\theta}}{2}$$

$$r = \frac{2m + \sqrt{9.57m^2 - 4\alpha^2\cos^2\theta}}{2}$$

Ergosphere radius $r = \dfrac{2m + \sqrt{9.57m^2 - 4\alpha^2}}{2} \quad @\, \theta = 0 \,\&\, \pi$ (2.6. 6)

$$= \frac{2m + \sqrt{9.57m^2}}{2} \approx 2.55m \quad @\, \theta = \frac{\pi}{2}$$

Figure **2.6.1** illustrates these changes from the Kerr Metric. The main effect from changing A is to allow an increase in maximum spin from $\alpha = m$ to $\alpha \approx 1.55m$, and $\approx 27.5\%$ increase in the maximum ergosphere radius from $r = 2m$ to 2.55m. It appears to contradict General Relativity and we will discuss this in sections 2.6.5 & 2.6.6, but provided the extra densities of time polarized, $m = \pm 2$ circular and transverse polarized $k_{\min}$ gravitons are as in Eq.(2.3. 7) with $A = \dfrac{2m}{r} + 1.39\dfrac{m^2}{r^2}$ then $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ is still true in the rotating space outside a black hole.

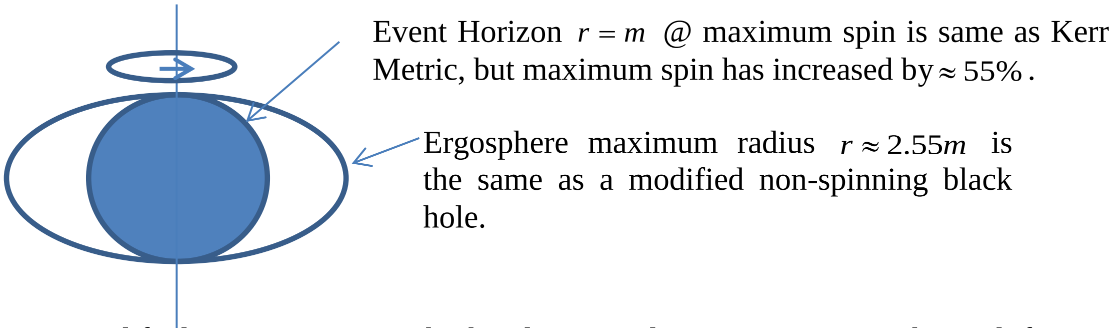


Figure 2.6.1 Modified Kerr Metric with the dimensionless parameter A changed from $A = \frac{2m}{r} \to A = \frac{2m}{r} + 1.39\frac{m^2}{r^2}$. It initially appears to clash with GR near the horizon.

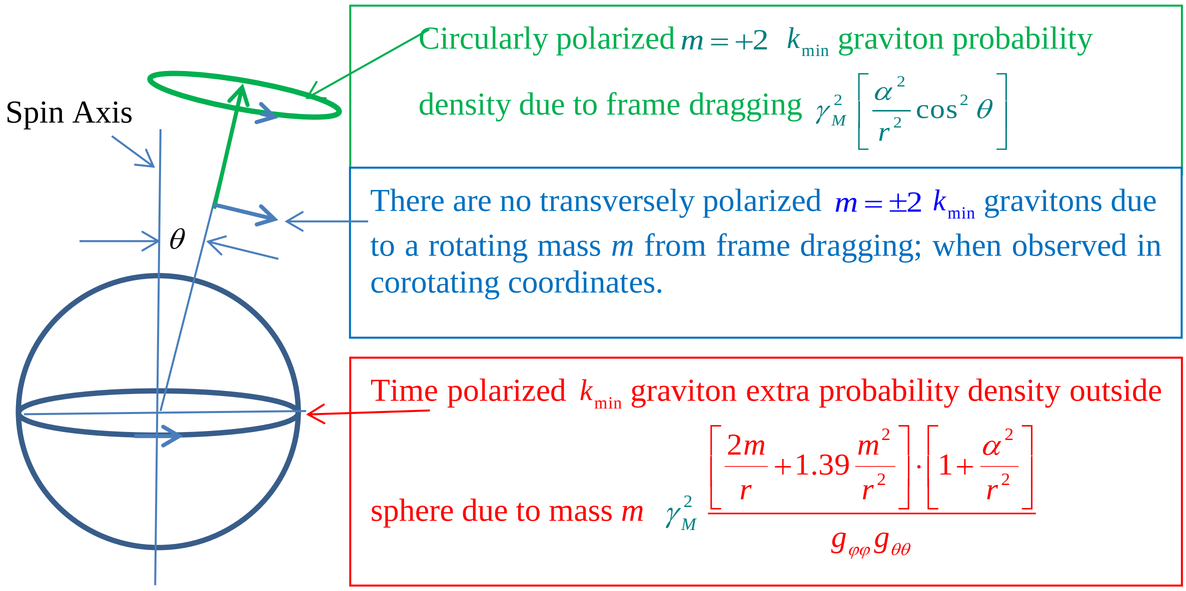


Figure 2.6. 2 Spinning black hole mass m with angular momentum length parameter α, but with the dimensionless parameter A changed from $A = \frac{2m}{r} \to A = \frac{2m}{r} + 1.39\frac{m^2}{r^2}$. The determinant of the metric is independent of A. The denominator terms $g_{\theta\theta}$ & $g_{\phi\phi}$, in dimensionless form as in Equ's. (2.3. 2), rapidly tend to one, as does γ_M^2 for radii $r >> r_{Sw}$, and can then be ignored. It shows the probability densities of time polarized, and circularly polarized $m = \pm 2$ $k_{\min}$ gravitons as in Eq.(2.3. 7) in this modified metric before the expansion of space, which dilutes the probabilies so as to keep the $k_{\min}$ graviton constant $K_{Gk\min}$ invariant outside the black hole. This is as observed in corotating coordinates. A more thorough analysis of everything to here will probably change the m^2/r^2 coefficient 1.39 and the 55% extra maximum spin.

2.6.4 Determinant of the metric and the $k_{\min}$ graviton constant $K_{Gk\min}$

Working in dimensionless form as in Equ's.(2.3. 2), using Eq. (2.3. 3) $g'_{tt}=\dfrac{\Delta}{g_{\phi\phi}}$ and the steps used in its derivation; the determinant of the metric is

$$\left|g_{\mu\nu}\right|=(g_{tt}g_{\phi\phi}-g_{t\phi}^2)g_{\theta\theta}g_{rr}=g'_{tt}g_{\phi\phi}g_{\theta\theta}g_{rr}=\frac{\Delta}{g_{\phi\phi}}g_{\phi\phi}g_{\theta\theta}\frac{g_{\theta\theta}}{\Delta}=g_{\theta\theta}^2=(1+\frac{\alpha^2}{r^2}\cos^2\theta)^2$$

As 4 volumes are invariant in relativity and $\rho_{Gk\min}=K_{Gk\min}dk_{\min}$ is true in corotating frames

If $\left|g_{\mu\nu}\right|=g_{\theta\theta}^2=(1+\dfrac{\alpha^2}{r^2}\cos^2\theta)^2$ then $\rho_{Gk\min}=K_{Gk\min}dk_{\min}$ is true in all frames, and is independent of the dimensionless parameter A. (2.6. 7)

Despite what initially appears to be a conflict with General Relativity (which we discuss below), if the metric determinant Eq. (2.6. 7) is $g_{\theta\theta}^2$ then the $k_{\min}$ graviton probability density is always $\rho_{Gk\min}=K_{Gk\min}dk_{\min}$ in all frames outside the black hole, and this is also true if there is no rotation, regardless of the value of the dimensionless parameter A. (See section 2.9)

2.6.5 The Reissner-Nordstrom Metric and $\mathbf{m}^2/\mathbf{r}^2$ terms

Reissner [27][28] solved the metric surrounding an electrically charged non-rotating mass not long after Schwarzchild had solved the metric around a static mass. He added the electromagnetic stress tensor surrounding a charge to the usual Einstein Energy-momentum tensor, in the region where the mass density term had previously been zero as in the Schwarzchild case. As before we will put $G=c=1$ so we can work in Planck masses. The Schwarzchild radius $r_S=2Gm/c^2$ has length dimension and thus $2Gm/rc^2$ becomes $2m/r$, and both $2m/r$ and m^2/r^2 are effectively dimensionless as, in these units, mass effectively has a length dimension.

Reissner similarly used the characteristic length r_Q where $r_Q^2=\dfrac{Q^2G}{4\pi\varepsilon_0c^4}$

Working in length units of charge with the Coulomb force constant $\dfrac{1}{4\pi\varepsilon_0}=1$ (2.6. 8)

If $G=c=1$ & these units of charge $\dfrac{r_Q^2}{r^2}\equiv\dfrac{Q^2}{r^2}$ are both dimensionless numbers.

Table 2.6. 1. Both parameters, mass m and charge Q, effectively have dimensions of length.

Metric	Schwarzchild	Modified Schwarzchild	Reissner-Nordstrom
$g_{00}=g_{rr}^{-1}$	$1-\dfrac{2m}{r}$	$1-\dfrac{2m}{r}-1.39\dfrac{m^2}{r^2}$	$1-\dfrac{2m}{r}+\dfrac{Q^2}{r^2}$

Using our modified Schwarzchild metric from Eq (2.6. **3**) we can see the similarities to the Reissner-Nordstrom metric for a charged mass, providing we measure charge parameter Q in a similar manner to measuring mass in Planck units. The signs are reversed however.

We can crudely think of this another way as follows: In the units we have been working in, the electrostatic field energy outside any radius r is $Q^2/2r$. This mass/energy must be subtracted from the original central charged mass as work is done bringing these charged particles together to establish the field energy.

So we can very crudely say the original central mass m becomes $m' = m - \dfrac{Q^2}{2r}$ at radius r .

$$\text{Thus } \frac{2m'}{r} = \frac{2m}{r} - \frac{Q^2}{r^2} \text{ and the new } g_{00} = 1 - \frac{2m'}{r} = 1 - \frac{2m}{r} + \frac{Q^2}{r^2}.$$

It is very tempting from this, to think of our modified Schwarzchild metric, as somehow including the negative gravitational field energy; which in Planck units is $-m^2/2r$ outside radius r. Using the same logic as the electrostatic case, but reversing signs, as gravitational field energy is negative, the original central mass m becomes $m' = m + \dfrac{m^2}{2r}$ at radius r .

$$\text{Thus } \quad \frac{2m'}{r} = \frac{2m}{r} + \frac{m^2}{r^2} \qquad \text{and the new} \qquad g_{00} = 1 - \frac{2m'}{r} = 1 - \frac{2m}{r} - \frac{m^2}{r^2}.$$

Of course our coefficient of 1.39 for m^2/r^2 does not fit this scenario, but our analysis is full of approximations and we could have it wrong. Roger Penrose in Chapter 19 of his "Road to Reality" gives a very good discussion on the concerns of many eminent physicists early last century when General Relativity was first published. They worried that gravitational energy was not explicitly included in the stress tensor. But Einstein could not do this and maintain covariance. In the century since, many eminent physicists have tried unsuccessfully to include gravitational energy in a covariant manner. So we must conclude that it is probably not related to gravitational energy; and as we have shown in this section, it is really due to the small number of $k_{\min}$ gravitons (except close in) emitted by the mass itself.

The Maxwell stress tensor tells us in the the electrostatic case, that if the field is in the z direction, there is a tension or negative pressure $P_Z = -E^2/2$ along the z axis and transverse positive pressures $P_X = P_Y = +E^2/2$ such that $P_X + P_Y + P_Z = E^2/2$ and the mass/energy density $\rho = E^2/2$ if they are all in appropriate units. The stress tensor contracts to $\rho - P_X - P_Y - P_Z = 0$ and this is a property of massless particles. Thus the presence of an electromagnetic field does not alter field equation covariance. So if we simply reverse all these signs with a negative mass energy density of $\rho = -1.39m^2/2$ with transverse tensions $P_X = P_Y = -1.39m^2/2$ and in the field direction positive pressure $P_Z = 1.39m^2/2$ such that $P_X + P_Y + P_Z = -1.39m^2/2$ the stress tensor contracts again contracts to $\rho - P_X - P_Y - P_Z = 0$. We can thus include a negative energy massless particle in the stress tensor in the same way as in the positive energy electrostatic case, and similarly maintain covariance.

2.6.6 The Kerr-Newman Metric and $\mathbf{m}^2/\mathbf{r}^2$ terms

In 1965 Newman [29][30] solved the charged version of the axisymmetric rotating black hole solved earlier by Kerr [31] in 1962. In section 2.3 and Equ's .(2.3. 2) we introduced the dimensionless parameter $A = 2m/r$ where as above we have assumed a silent $G = 1$ in the numerator and a silent $c^2 = 1$ in the denominator and in section 2.6 modified this to get a dimensionless $A = \frac{2m}{r} + 1.39\frac{m^2}{r^2}$.We showed in section 2.3 that provided this A is dimensionless it does not change Equ's.(2.3. **3**). If we look carefully at the Kerr-Newman metric we can see that it fits Equ's. (2.3. **2**) provided we put $A = \frac{2m}{r} - \frac{r_Q^2}{r^2}$ which is equivalent to putting $A = \frac{2m}{r} - \frac{Q^2}{r^2}$ where $r_Q^2 = \frac{Q^2 G}{4\pi\varepsilon_0 c^4}$ and we have again measured charge Q in length units as in Equ's. (2.6. **8**).

Thus our modified Kerr metric where $A = \frac{2m}{r} + 1.39\frac{m^2}{r^2}$ is again similar to:

The Kerr-Newman metric where $A = \frac{2m}{r} - \frac{Q^2}{r^2}$ but with opposite signs.

These two metrics are the rotating versions of our modified Schwarzchild metric and the Reissner-Nordstrom metrics. We can perhaps summarize this in the following two tables.

Table 2.6. 2 The non rotating metrics where dimensionless parameter A is as in Eq. (2.3. 2) The modified Schwarzchild and Reissner-Nordrom metrics both have the same form of changes to the Reimannian curvature tensor but of opposite sign.

Schwarzchild	Modified Schwarzchild	Reissner-Nordstrom
$A = \frac{2m}{r}$	$A = \frac{2m}{r} + 1.39\frac{m^2}{r^2}$	$A = \frac{2m}{r} - \frac{Q^2}{r^2}$

Table 2.6. 3 The rotating versions of the above. Again the modified Kerr and Kerr-Newman metrics both have the same form of changes to the Reimannian tensor but of opposite sign.

Kerr	Modified Kerr	Kerr-Newman
$A = \frac{2m}{r}$	$A = \frac{2m}{r} + 1.39\frac{m^2}{r^2}$	$A = \frac{2m}{r} - \frac{Q^2}{r^2}$

Again massless particles in the electromagnetic field apply equally in the Reissner-Nordstrom and Kerr-Newmann metrics. The arguments we used above in the non rotating case using massless negative energy particles in our modified stress tensor apply equally in the rotating case. The small changes in the Riemannian curvature tensor, due to this m^2/r^2 term, are of opposite sign for both our modified Kerr and Schwarzchild metrics, when compared to the Kerr-Newman and Reissner-Nordstrom metrics, but of exactly the same form.

So, provided we include such an appropriate negative energy massless particle in the stress tensor, solutions to $R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = \frac{8\pi G}{c^4}\left[T_{\mu\upsilon} - T_{\mu\upsilon}(\text{Background})\right]$ are consistent with $k_{\min}$ graviton probability density $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$ where $K_{Gk\min}$ is invariant for all observers; whether they are near the horizon of black holes, or if they are at our current cosmic horizon. Also for any observers outside it, who are looking at their own cosmic horizons; and for all cosmic time since the big bang. But wavenumber $k_{\min}$ depends on the local metric and cosmic time. It is approximately the inverse of the causally connected radius at any cosmic time.

Einstein based his remarkable equation on the "Equivalence Principle", or the same physics in all free falling frames as in empty space; with covariant tensor equations that apply in all coordinates throughout all spacetime. He wanted it to be similar to Gauss's law and Poisson's equation $\nabla^2 \varphi = \rho$ ignoring constants, but in curved spacetime. This naturally leads to inverse square force laws with inverse potentials where masses are concerned, but the inclusion of an m^2 / r^2 potential term in the metric due to $\psi_m{}^* \psi_m$ seems to mess all this up. But does it really? Could it be trying to tell us something that we need to know, but did not want to know? Quantum mechanics in the form of QED tells us that, close to the Compton wavelength, the normally simple inverse square force law starts to change, close in shielding makes fundamental electric charge appear to increase, and QED takes over with increadible accuracy. Simple inverse square electric force laws had ruled with remarkable accuracy for over a century before QED arrived on the scene. In fact it was the announcement of the Lamb Shift at the Long Island conference in 1947 that started the big breakthroughs in QED. World War II developments in radar had enabled these remarkably accurate experiments. Is it possible that similar developments today will allow improvements in Gravitational Wave observation accuracy? Developments that may see effects in gravity close to black holes with some parallels to QED changes inside the Compton wavelength of electric charges?

2.6.7 What is the effect of this term in the solar system?

The distance to Mars can be measured very precisely as we have instruments on the surface that can reflect radar from Earth at known locations. On the other hand we don't know the exact diameter of the Sun. If we look at the outer rim it will be deflected outwards by $\approx 1.75/2$ arc seconds (half that of the gravitational bending of starlight because it is coming from the rim). At the distance of the sun, $\approx 150 \times 10^6$ KM this is roughly 640 KM in radius. Even if we optically measure the diameter precisely with no error the actual sun diameter will be about 1275 KM smaller so we only know the true diameter approximately. We also do not know the exact surface of radar reflection. The Astronomical unit is quoted as 149,597,870,700 metres, but this is really based on knowing interplanetary measurements accurately and then using Kepler's laws modified by the Schwarzschild metric to give us this level of accuracy. So, let us do a crude first order approximation of what happens if we include an m^2/r^2 term in the metric. Using low velocity (compared with light) Christoffel symbol approximations and circular orbits for simple comparisons the accelerations are:

$\omega^2 r \approx \frac{1}{2}\frac{d}{dr}g_{00} \approx \frac{1}{2}\frac{d}{dr}(1-\frac{2m}{r}) \approx \frac{m}{r^2}$ in the usual Schwarzschild case if $g_{00} \approx 1$ and

$\omega'^2 r \approx \frac{1}{2}\frac{d}{dr}g'_{00} \approx \frac{1}{2}\frac{d}{dr}(1-\frac{2m}{r}-\frac{1.39m^2}{r^2}) \approx \frac{m}{r^2}+\frac{1.39m^2}{r^3} \approx \frac{m}{r^2}(1+\frac{1.39m}{r})$ in the modified metric case. So $\omega^2 \approx \frac{m}{r^3}$ in the usual Schwarzschild case and $\omega'^2 \approx \frac{m}{r^3}(1+\frac{1.39m}{r})$ in the modified metric case. In weak gravitational field accelerations we can replace mass m with a new effective mass $m' = m(1+\frac{1.39m}{r})$ but orbital periods and angular velocities ω cannot change as we know them very precisely. So we will try the following modification to all planetary radii $\omega^2 = \frac{m}{r^3} = \frac{m}{(r+\Delta r)^3}(1+\frac{1.39m}{r+\Delta r}) = \frac{m}{r^3(1+\frac{\Delta r}{r})^3}(1+\frac{1.39m}{r+\Delta r}) \approx \frac{m}{r^3(1+\frac{3\Delta r}{r})}(1+\frac{1.39m}{r+\Delta r})$

$\omega^2 = \frac{m}{r^3} \approx \frac{m}{r^3}(1-\frac{3\Delta r}{r})(1+\frac{1.39m}{r+\Delta r}) \approx \frac{m}{r^3}(1-\frac{3\Delta r}{r}+\frac{1.39m}{r+\Delta r})$ and if ω is unchanged

$1-\frac{3\Delta r}{r}+\frac{1.39m}{r+\Delta r} \approx 1$ and $\frac{3\Delta r}{r} \approx \frac{1.39m}{r+\Delta r} \approx \frac{1.39m}{r}$ thus $\Delta r \approx \frac{1.39m}{3}$

The Schwarzschild radius $R_{SW} = 2m$ and the extra distance to the sun $\Delta r \approx \frac{1.39m}{3} \approx \frac{1.39R_{SW}}{6}$

The Schwarzschild radius of the Sun is $R_{SW} \approx 3$ km so $\Delta r \approx \frac{1.39R_{SW}}{6} \approx 0.7$ km.

The change Δr for our solar system is about 700 metres. But all interplanetary radial separations do not change. So we can still use the old metric and the astronomical unit unchanged with Kepler's laws to a first approximation, or the new metric and just add 700 metres to all the planetary radii from the sun. Orbital periods are identical to a very high accuracy. The gravitational constant does not change in both cases.

What we have done here is a bit like dipoles with the electrostatic field dropping as $\propto 1/r^2$ and the resultant field as $\propto \Delta r/r^3$ where $\frac{1}{r^2}-\frac{1}{(r+\Delta r)^2} \approx \frac{2\Delta r}{r^3}$. However, in the non-spinning gravity case there is spherical symmetry but not in an electric dipole.

2.6.8 **Can we measure this difference?**

We used circular orbits for a simple crude calculation but the same arguments apply in a slightly more complicated way for eccentric orbits; in a similar manner as Kepler's original arguments with elliptical orbits that sweep out equal angular segment areas with time. The orbit of Mars in particular is highly eccentric and Earth much less so. If the eccentricity of both Earth and Mars orbits were known, to better than say a hundred metres or so between max and min, we should be able to check this difference by measuring the distance (also to around a 100 metre or so accuracy) between Mars and Earth at various points around their orbits. It would seem however that this would be pushing at the very border of current technology, as radar measurements to the sun are inherently a little blurry due to surface variability. We need these to get a very precise value for the eccentricity of Earth's orbit. Even if we can measure Earth-Mars with complete accuracy we have to add in errors due to lack of accuracy in Earth's eccentricity. Also when Mars is on the opposite side of the sun to us, if the distance measuring signal grazes the sun there will a Shapiro type delay that is equivalent to roughly a 15 km error that reduces logarithmically with the minimum radial distance of the signal from the sun. Even if the beam passes through a half Earth-Sun radius there is still a few km error. All these effects introduce possible errors that make it difficult to measure a 700 metre difference in all planetary radii.

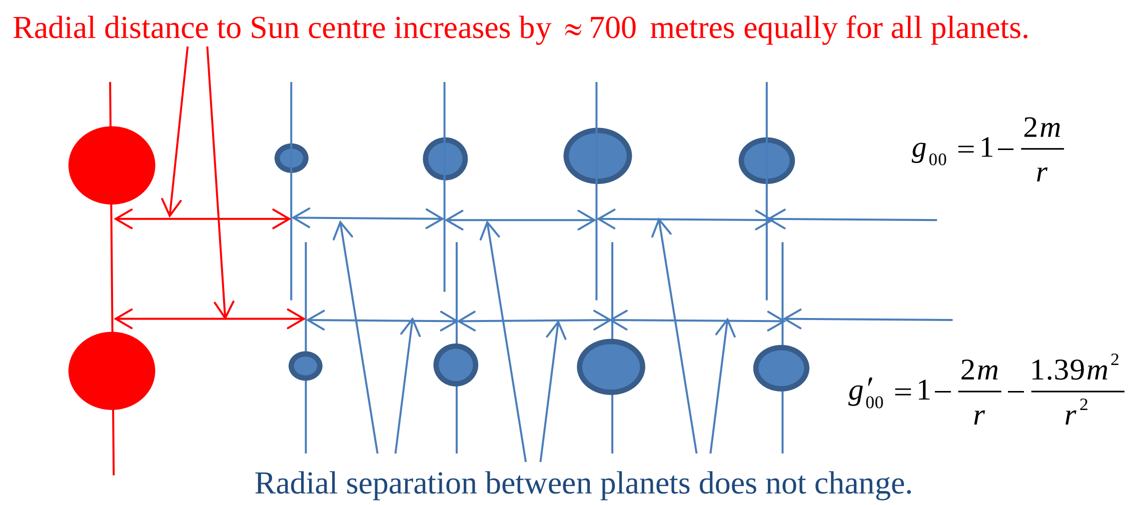


Figure 2.6. 3 Scales are grossly exaggerated for clarity. We have also assumed circular obits here for simplicity, and ignored errors due to the centre of mass of the solar system not being at centre of sun. We have also assumed infinitesimal planet masses so we can simply ignore the effect they have on each other.

2.6.9 What about the Hulse Taylor binary pulsar, can it show this change?

The timing of this pulsar is accurate to 14 significant figures and it would initially seem that this accuracy would show up such differences. However, the semi-major orbit of this binary is $\approx 2\times10^9$ metres, with a decay rate of ≈ 3.5 metres per orbit, or a change $\Delta r \approx 100$ metres over 30 years due to gravitational radiated energy. If we totally ignore this change in the radius, treating it as effectively zero, the accumulated time delay is parabolic; or proportional to elapsed time squared. If we include the small effect of the change in radius, $2m/r + 1.39m^2/r^2$ increases minutely to $2m/(r-\Delta r) + m^2/(r-\Delta r)^2$ adding two minute cubic terms, both proportional to elapsed time cubed, where the m^2/r^2 contribution is about 10^{-7} of that due to the $2m/r$ term. Even the cubic effect of a $\Delta r \approx 100$ metres change in the usual m/r term (which is currently $m/r \approx 5\times 10^{-7}$) on the parabola over 30 years, is miniscule. The chances of measuring either the m/r, or the m^2/r^2 cubic terms are very small in the foreseeable future; let alone distinguish between them. The best chance of measuring any difference will almost certainly turn out to be gravitational wave observations.

2.6.10 Gravitational Wave observations of Black Hole mergers

Some of the mergers observed so far have been suggesting relatively larger Black Hole masses than current astrophysics theory had expected. If we look at our new metric term we can write $g_{00} = 1 - \frac{2m}{r} - 1.39\frac{m^2}{r^2} = g_{00} \approx 1 - \frac{2m}{r}(1 + 0.7\frac{m}{r}) = 1 - \frac{2m'}{r}$ where $m' \approx 1 + 0.7\frac{m}{r}$

For a maximum spin black hole when $r = m$ we can say the effective mass at merger is $m' \approx 1.7m$ or about 70% greater. The addition of an m^2/r^2 term in our modified metrics increases the total merging energy, and hence that in the resulting gravitational waves. Inward radial accelerations would appear to be greater also. However computer simulations with these changed metrics would be required to model all this in detail, but our rough analysis above suggests that the masses of the black holes before merging could well be less than what they have so far seemed. In other words, a pair of smaller black holes merging might create the gravitational waves current theory predicts from the mergers of two, up to maybe 70% larger black holes. Spins had also been expected to be roughly perpendicular to their orbiting plane, but their merging speeds don't tie up with this. Is it possible that this unexpected behaviour is trying to tell us something is different; something different in the metric as we get close to Black Hole Horizons?

Finally in this section, does this extra m^2/r^2 term alter what we said in Eq.(2.3. 18)? We first used $A = 2m/r$, before we introduced the self emission term $1.39m^2/r^2$. Back then we found that the extra time polarized $k_{\min}$ graviton density near the horizon before volume expansion, for all black holes is

$$\gamma_M^2 A_H \cdot \frac{1+\frac{\alpha^2}{R^2}}{g_{\theta\theta} g_{\phi\phi}} = \gamma_M^2 \frac{A_H^2}{A_H^2} = \gamma_M^2 = \frac{4R^2}{s^2}$$ and this is still true, but now $A_H = \frac{2m}{R} + 1.39\frac{m^2}{R^2} = 1 + \frac{\alpha'^2}{R^2}$

Where α' is the increased spin parameter due to the extra $1.39m^2/R^2$ term and we have also reused Eq's.(2.3. 14), (2.3. 15) & (2.3. 17). Everything we did there is not affected by this extra term.

2.7 Revisiting some other aspects of the first paper

2.7.1 Infinitesimal rest masses

In section 6.2 in [7] we showed infinitesimal rest mass $N=2$ infinite superpositions have $\langle K_{kmin}\rangle^2 = 1$. From Table 4.3.1 in [7]

$N=2$ infinitesimal rest mass spin 1 superpositions have $\langle n\rangle \approx 3.98$

$N=2$ infinitesimal rest mass spin 2 superpositions have $\langle n\rangle \approx 3.33$

Using Eq's. (3.1.11) in [7] and Eq. (2.2. 10).

$$\langle K_{kmin}\rangle^2 = \frac{\langle n\rangle^2 s}{2} \lambda\!\!\!^{-}{}_C^2 k_{\min}^2 \approx \frac{15.82}{2} \lambda\!\!\!^{-}{}_C^2 k_{\min}^2 = 1 \quad \text{or} \quad \lambda\!\!\!^{-}{}_C \approx 0.355 \frac{R_{OH}}{\Upsilon} \quad \text{for Spin 1}$$

$$\approx \frac{11.09 \times 2}{2} \lambda\!\!\!^{-}{}_C^2 k_{\min}^2 = 1 \quad \text{or} \quad \lambda\!\!\!^{-}{}_C \approx 0.300 \frac{R_{OH}}{\Upsilon} \quad \text{for Spin 2}$$

Using the value for $\Upsilon \approx 0.65$ from Eq. (2.4. 13) based on WMAP data which also puts the horizon radius at $\approx 46\times10^9$ light years and $R_{OH} \approx 2.7\times10^{61}$ in Planck lengths.

Spin	$\langle n\rangle$	Compton Wavelength λ_c	Infinitesimal Rest Mass
1	3.98	$\approx 0.55 R_{OH}$	$\approx 8.3\times10^{-34} eV.$
2	3.33	$\approx 0.46 R_{OH}$	$\approx 9.8\times10^{-34} eV.$

Table 2.6. 4 Infinitesimal rest masses of $N=2$ photons, gluons & gravitons.

These Compton wavelengths and rest masses are the present values, reducing slowly but exponentially with cosmic time *T*. They are based on WMAP data where $\Omega = 1$ and could be slightly different if Ω does not need to be one as we have discussed. They also depend on the actual value of *b* in the exponential expansion $V = 3Exp(bt)$. These infinitesimal rest masses limit the range of virtual photons, gluons and gravitons to approximately the horizon. The graviton rest masses above are close to recent proposals explaining the accelerating expansion of the cosmos [8].

2.7.2 Redshifted zero point energy from the horizon behaves differently to local

Local zero point energies are Lorentz invariant. At high frequencies there is no shortage locally to build the high frequency components of superpositions. If a massive $N=1$ virtual pair emerges from the vacuum its life is short and it places little demand on long range quanta. If there were no redshifted supply from the horizon there would be only a few modes of the local supply of $k_{\min} \approx 1/R_{OU}$ quanta inside the horizon. Because preons are born with zero momentum and infinite wavelength they can however absorb a different supply of redshifted $k_{\min} \approx 1/R_{OU}$ quanta from the receding horizon as we have discussed.

This $k_{\min}$ quanta redshifted supply behaves differently to normal Lorentz invariant zero point local fields. It behaves as $K_{Qk\min} = 0.84\alpha_G$"The Quanta required @ $k_{\min}$Invariant" of Eq. (2.4. 12) Where $K_{Qk\min} \approx 3.33 \times K_{Gk\min}$"The $k_{\min}$Graviton Invariant" of Eq. (2.2. 11).This redshifted supply is only available in a continuously expanding universe to preons born with zero momentum, or infinite wavelength, in the rest frame in which infinite superpositions are built.

2.7.3 Revisiting the building of infinite superpositions

In section 2 of the first paper we developed equations to determine the probability of each mode of a superposition using local zero point fields and when we found the cosmic wavelength supply inadequate we switched to a different redshifted supply for long range quanta. So how do we justify our use of the local zero point fields to determine mode probabilities and behaviours? There is simply a plentifull supply of high frequency local zero point fields. This local supply is adequate for high densities of superpositions for all modes from the Planck energy $k=1$ high energy mode cutoffs to somewhere around $k \approx 10^{-20}$ or near nuclear wavelengths. Thus, until we reach somewhere near nuclear densities, there is a sufficient supply of local high frequency zero point fields to build infinite superpositions. The coupling to local zero point fields in this high frequency region determines the primary coupling behaviour (see page 5) of all the standard model particles. There is however a gradual transition to absorbing quanta from the redshifted horizon supply as the wavelength increases. Because the redshifted supply of $k_{\min}$ quanta behaves as the invariants $K_{Qk\min}$ or $K_{Gk\min}$ above and entirely differently to Lorentz invariant local zero point fields, spacetime has to warp around mass concentrations and the universe has to expand.

2.8 Gravitational Waves

Our hypothesis has been throughout, that the warping of spacetime is directly related to maintaining the maximum wavelength, or $k_{\min}$ graviton density $\rho_{Gk\min} = G_{Gk\min} dk_{\min}$ invariant throughout all spacetime. Around non-rotating (spherically symmetric) mass concentrations this warping decreases inversely with radius (at least well away from black holes) but always in a spherically symmetric manner as the extra $k_{\min}$ gravitons due to this mass are distributed in the same spherical way. Likewise we get cylindrical symmetry for rotating mass concentrations. Both these types of symmetries are the lowest action/energy stable state of the metric. Disturbances to this stable state will travel as waves at the speed of light.

2.8.1 Constant transverse areas in low energy waves

If these mass concentrations accelerate, then just like accelerating electric charges they will radiate gravitational energy in the form of real transversely polarized $m = \pm 2$, gravitons. This energy is a disturbance or oscillation in this lowest energy state $k_{\min}$ graviton background,

but $\rho_{Gk\min} = G_{Gk\min} dk_{\min}$ cannot change during these disturbances, so what is going on?
Let us imagine a region of spacetime far from mass concentrations where the metric $g_{\mu\nu} = \eta_{\mu\nu}$ and using t, x, y, z coordinates let $g_{00} = -1, g_{xx} = +1, g_{yy} = +1, g_{zz} = +1$. Ignoring signs the determinant of the metric $\left|g_{\mu\nu}\right| = 1$. Let a gravitational wave pass through in the z direction with a tranverse wave in the x, y plane. We know that a circular transverse ring of particles will oscillate into, and out of ellipses perpendicular to each other, in such a manner that the enclosed area does not change, or that $g_{xx} \cdot g_{yy} = 1$ during this oscillation. Thus the measured volume of space does not change as the wave passes through and $\rho_{Gk\min} = G_{Gk\min} dk_{\min}$ does not change. The determinant of the metric $\left|g_{\mu\nu}\right| = 1$ also does not change. This is only approximately true as there are extra real transversely polarized $m = \pm 2, k_{\min}$ gravitons passing through due to the energy in the wave, but the error is second order unless the amplitude of the wave is quite large.

2.8.2 What happens in high energy waves?

We can imagine the extra gravitons around a mass concentration and the background gravitons as in section 2.2 (if they are undergoing an acceleration as in binary pairs) generating real transversely polarized $m = \pm 2$, gravitons. This has some parallels to what we found in the Kerr metric, but now with real gravitons. But the intensity, or probability density, of these real gravitons will drop as the inverse radius squared, at least when far away. We can also show from Equ's.(2.1. 9) & (2.2. 5) that most of these gravitons are close to the locally measured value of the $k_{\min}$ wavenumber. About 66% are between $k_{\min}$ & $2k_{\min}$ and about 96% are between $k_{\min}$ & $5k_{\min}$. Thus most of this radiated energy is near $k_{\min}$. Just as measured volumes around mass concentrations had to increase to accommodate extra $k_{\min}$ gravitons, the transverse area of the wave has to increase in relation to the oscillating constant area. Ignoring signs again, if $g_{xx} \cdot g_{yy} = 1 + \varepsilon$ then $g_{00} = (1 - \varepsilon)^{-1}$ to keep the metric determinant $\left|g_{\mu\nu}\right| = 1$. The energy density in the wave increases the local measurement of $k_{\min}$, but $\rho_{Gk\min} = G_{Gk\min} dk_{\min}$ remains invariant as required. Close to orbiting binary black holes or neutron stars this radiated energy intensity is huge and the changes in $g_{xx} \cdot g_{yy}$ & g_{00} become large in relation to the oscillating changes. Transverse areas and hence measured volumes change significantly. This is in complete contrast to what happens at extremely large distances, such as when we observe gravitational waves here on Earth, where the transverse areas are virtually constant during these oscillations.

2.8.3 No connection between wave frequency and radiated quanta energy

The frequency of the radiated wave is twice the orbital frequency of the binary pair source. As most of the energy in the wave is in quanta near $k_{\min}$ there is no connection with the frequency of the radiated wave as in spin 1 photons in electromagnetism. In the recently observed gravitational waves the wave frequency was ≈ 250 cycles per second just before merger with wavelengths ≈ 1200 kilometres or approximately 10^{41} Planck lengths, whereas the wavelength of $k_{\min}$ gravitons is $1/k_{\min} \approx R_{OU} \approx 10^{62}$ Planck lengths. The ratio between them is $\approx 10^{21}$. This ratio is inverse to the binary pair orbital frequency. It could only approach one if the orbital period is approximately twice the age of the universe.

2.9 Invariant Four Volume Action Densities and Four Vectors

2.9.1 Invariants in Relativity

(1)The length of a 4 vector is invariant

(2) An appropriate Scalar x Proper Time, or Action is an invariant

(3) 4 volume $= dtdxdydz$ is invariant. Consider a 4 volume box with n units of $k_{\min}$ action quanta where $\Delta t = \Delta x = \Delta y = \Delta z = 1$ & $\Delta t\Delta x\Delta y\Delta z = \Delta^4 x = 1 = \Delta t'\Delta x'\Delta y'\Delta z'$ & $k_{\min}$ is variable.

$$\text{Then 4 Volume } k_{\min} \text{ Action Density} = \frac{Action}{4volume} = \frac{n_{k\min}}{\Delta t\Delta x\Delta y\Delta z} = \frac{n_{k\min}}{\Delta t'\Delta x'\Delta y'\Delta z'} = n_{k\min}$$

In Flat Space where $g_{\mu\nu} = \eta_{\mu\nu}$ & $\gamma_{\text{Metric}} = \sqrt{1/g_{00}} = 1$. Thus 3 Volume Action Density is

$$\frac{Action}{3volume} = \frac{n_{k\min}}{\Delta^3 x} = n_{k\min}\Delta t \;\propto \rho_{k\min\ \text{Action}} = K_{k\min\ \text{Action}} dk_{\min}$$

In a curved non-rotating spacetime metric with $g_{\mu\nu} \neq \eta_{\mu\nu}$; $\gamma_{\text{Metric}} = \sqrt{1/g_{00}}$

$$\text{The New (3 Volume Action Density)}' = \frac{n_{k\min}}{\Delta^3 x'} = n_{k\min}\Delta t' \;\propto \rho'_{k\min\ \text{Action}} = K_{k\min\ \text{Action}} dk'_{\min}$$

and taking the ratio of these

$$\frac{\left(\text{New (3 Volume Action Density)}' = \dfrac{n_{k\min}}{\Delta^3 x'} = n_{k\min}\Delta t'\right)}{\left(\text{Original 3 Volume Action Density} = \dfrac{n_{k\min}}{\Delta^3 x} = n_{k\min}\Delta t\right)} = \frac{\left(\rho'_{k\min\ \text{Action}} = K_{k\min\ \text{Action}} dk'_{\min}\right)}{\left(\rho_{k\min\ \text{Action}} = K_{k\min\ \text{Action}} dk_{\min}\right)}$$ & changing to

infinitesimals $$\frac{\Delta^3 x}{\Delta^3 x'} = \frac{d^3 x}{d^3 x'} = \frac{\Delta t'}{\Delta t} = \frac{dt'}{dt} = \frac{\rho'_{k\min\ \text{Action}}}{\rho_{k\min\ \text{Action}}} = \frac{dk'_{\min}}{dk_{\min}} = \frac{k'_{\min}}{k_{\min}} = \gamma_{\text{Metric}} = \frac{1}{\sqrt{g_{00}}}$$

In non-rotating metric the ratio of (three volume $k_{\min}$ action densities), equals the ratio of $k_{\min}$ frequencies, and is inverse to the 3 volume ratio. $\dfrac{\rho'_{k\min\ \text{Action}}}{\rho_{k\min\ \text{Action}}} = \dfrac{d^3 x}{d^3 x'} = \dfrac{k'_{\min}}{k_{\min}} = \gamma_{\text{Metric}} = \dfrac{1}{\sqrt{g_{00}}}$

Similarly if we start with a box of $k_{\min}$ gravitons in commoving coordinates (it can be either, as $k_{\min}$ action quanta are $\langle n \rangle = 3.33 \times k_{\min}$ gravitons) , we measure the minimum value of $k_{\min}$, and we measure $k_{\min}$ graviton spatial density $\rho_{Gk\min} \approx K_{Gk\min} dk_{\min}$. If we now move at peculiar velocity β_P our measurement of the time interval dt increases as $\gamma_P = (1-\beta_P^2)^{-1/2}$ such that $dt' = \gamma_P dt$. But this moving observer also measures an increased value of

$k'_{\min} = \gamma_P k_{\min}$. Also their measurement of the new 3 volume $d^3x' = d^3x/\gamma_P$. As their measurement of 3 volume reduces, their measurement of 3 volume action density increases as γ_P, as also does their measurement of both the new time interval $dt' = \gamma_P dt$, and the increased value of $k'_{\min} = \gamma_P k_{\min}$ & $dk'_{\min} = \gamma_P dk_{\min}$ An invariant 4 volume action density, is equivalent to 3 volume action density proportional to a local observer's measurement of maximum wavelength $k'_{\min} = \gamma_P k_{\min}$ for peculiar velocities; or $k'_{\min} = \gamma_M k_{\min}$ in a non-flat metric

$$\frac{\text{Invariant Action}}{\text{Invariant 4 volume}} = \text{another Invariant proportional to what we have called } K_{Gk\min} \text{ or } K_{Qk\min}$$

Invariant 4 volume action density could well rule the cosmos, and the warping of spacetime around mass concentrations, so as to maintain this invariance in the presence of the extra $k_{\min}$ gravitons that mass concentrations emit.

2.9.2 Four volumes in changing metrics

Using our dimensionless form of the metric tensor, the nonrotating space metric determinant has magnitude $|\text{Det } g| = |g| = |g_{tt} g_{rr} g_{\theta\theta} g_{\phi\phi}| = 1$, but we want the square root of this $\sqrt{|g|} = 1$.

However in rotating space this becomes $\sqrt{|g|} = g_{\theta\theta} = 1 + \cos^2 \frac{\alpha^2}{r^2}$ which reverts to $\sqrt{|g|} = g_{\theta\theta} = 1$ when the angular momentum length parameter $\alpha = 0$. At a large radius from any mass concentration let us start with a unit four volume such that $\Delta^4 x = \Delta t \Delta x \Delta y \Delta z = 1$ when $g_{\mu\nu} = \eta_{\mu\nu}$, where for simplicity we use $x, y \,\&\, z$ for the space components. As we approach the central mass in the new metric, this four volume becomes

$$\Delta^4 x' = \sqrt{|g|}\Delta t \Delta x \Delta y \Delta z = g_{\theta\theta} = 1 + \frac{\alpha^2}{r^2}\cos^2\theta = \Delta t' \Delta x' \Delta y' \Delta z'$$

Four volumes at a fixed point in spacetime are invariant as coordinates change, and also as the metric changes if in nonrotating space. In rotating space however it increases as $g_{\theta\theta}$.

$$\frac{\text{Curved spacetime 4 volume}}{\text{Flat spacetime 4 volume}} = \frac{\Delta^4 x'}{\Delta^4 x} = \frac{\Delta t' \Delta x \Delta y' \Delta z'}{\Delta t \Delta x \Delta y \Delta z} = g_{\theta\theta} = 1 \text{ when angular momentim is zero}.$$

We also know that clocks change as $\Delta t' = \sqrt{g_{tt}}\Delta t = \frac{\Delta t}{\gamma_M}$ in curved spacetime so that

$$\frac{\Delta t' \Delta x' \Delta y' \Delta z'}{\Delta t \Delta x \Delta y \Delta z} = \frac{\Delta t \cdot \Delta x' \Delta y' \Delta z'}{\gamma_M \cdot \Delta t \Delta x \Delta y z} = g_{\theta\theta} = \frac{\Delta x' \Delta y' \Delta z'}{\gamma_M \cdot \Delta x \Delta y \Delta z}$$

The expanded spatial volume in the new metric $\Delta^3 x' = \Delta x' \Delta y' \Delta z' = \gamma_M g_{\theta\theta} \Delta x \Delta y \Delta z = \gamma_M g_{\theta\theta} \Delta^3 x$.

Spatial volume in any metric expands as $\frac{V'}{V} = \frac{\Delta^3 x'}{\Delta^3 x} = \frac{d^3 x'}{d^3 x} = \gamma_M g_{\theta\theta} = (1 + \frac{\alpha^2}{r^2}\cos^2\theta)\gamma_M$

Where as above we have defined $\gamma_M = g_{tt}^{-1/2}$ as the local metric clock rate.

2.9.3 **Graviton densities represented as invariant 4 velocities**

Four velocity vectors have the property that $U_0^2 - U_1^2 = 1$ is invariant under local Lorentz transformations; where U_0 is the *time component* of the four velocity, and U_1 the *spatial component.* We will, as previously, use the notation

$$U_0^2 = \gamma_M^2 \quad \text{and} \quad U_1^2 = \gamma_M^2 \beta_M^2 \quad \text{where } \lambda_M^2 = \frac{1}{1-\beta_M^2}$$

We can think of the spatial component U_1 as the four velocity $\gamma_M \beta_M$ of a free falling mass that came from rest at infinity, in the same coordinate frame as the black hole, and pointing radially inwards. We can also write

$$U_0^2 - U_1^2 = 1 \text{ as } 1 + U_1^2 = U_0^2 \text{ or } 1 + \gamma_M^2 \beta_M^2 = \gamma_M^2 .$$

This was what we did for the Schwarzschild metric with the background $k_{\min}$ graviton density normalized to 1 and $\gamma_M^2 \beta_M^2$ the extra $k_{\min}$ graviton density due to a central mass, and γ_M^2 the total; this equation only applies before we have expanded the volume to that in the new metric. Because this is a 4 vector relationship it is true in all coordinates.

We can also add a term $\gamma_M^2 X^2$ to both sides to get $1 + \gamma_M^2 \beta_M^2 + \gamma_M^2 X^2 = \gamma_M^2 + \gamma_M^2 X^2$ and still maintain covariance as $(\gamma_M^2 + \gamma_M^2 X^2) - (\gamma_M^2 \beta_M^2 + \gamma_M^2 X^2) = 1$, and we can put $X^2 = \frac{\alpha^2}{r^2}\cos^2\theta$

so that: $$1 + \gamma_M^2 \beta_M^2 + \gamma_M^2 \frac{\alpha^2}{r^2}\cos^2\theta = \gamma_M^2 + \gamma_M^2 \frac{\alpha^2}{r^2}\cos^2\theta = g_{\theta\theta}\gamma_M^2 .$$

We are not adding another 4 vector here; we are simply adding squared terms, which are equal on each side, so that Lorenz invariance is not affected. This is still an invariant equation in any coordinates. In the above the local metric clock rate is always $1/\gamma_M$. The probability density of circularly polarized $k_{\min}$ gravitons due to rotation before volume expansion always obeys $\gamma_M^2 X^2 = \gamma_M^2 \frac{\alpha^2}{r^2}$ and the remaining $k_{\min}$ graviton probability density, that can be of various polarizations before volume expansion, is $\gamma_M^2 \beta_M^2$; always obeying the above invariant relationships. Just as in QED, aside from the circular polarization due to space rotation, there can be various polarizations in other frames that always obey the 4 vector relationship

$$(\text{Time polarized})^2 \; - \; (\text{Spatially polarized})^2 = \text{Constant}$$

3 Some Loose Ends

3.1.1 Preferred Frames

It would seem that we have been arguing for a preferred frame. But there is really no difference in what we are proposing compared to current views. In commoving frames the cosmic microwave background is isotropic. At peculiar velocity β_P it is no longer isotropic, and the average background temperature increases by γ_P, exactly the same increase as $k_{\min}$ to $k'_{\min} = \gamma_P k_{\min}$, and that is if we could measure it, which is most unlikely. We have frequently throughout this paper talked about local observers measuring $k_{\min}$, but this can only be a thought experiment. There are no other changes in physics in this commoving frame; it is exactly as Einstein originally postulated, and is an important experimentally verified feature of General Relativity (See [33]).

3.1.2 Solar System Constraints and do our proposed changes fit?

See "The Confrontation between General Relativity and Experiment" Clifford M. Will. [33] Probably the most important constraint mentioned in this review is the Cassini Time Delay data that gives a fit with GR of $\approx 10^{-5}$ for signals passing close to the solar horizon, where our extra $1.39m^2/r^2$ term is $\approx 3\times10^{-6}$. So it should be within the Cassini Constraint and also within the light deflection constraint. The remaining changes are discussed in section 2.6.7.

3.1.3 Action Principles and the Einstein Field Equations

The field equations of GR can be derived from an invariant action principle $\delta I = 0$ where

$$I = \frac{1}{16\pi G}\int R\sqrt{-g}\cdot d^4x + I_m(\psi_m, g_{\mu\nu})$$

Where R is the Ricci scalar, and I_m is the matter action, which depends on matter fields ψ_m universally coupled to the metric g. By varying the action with respect to $g_{\mu\nu}$, we obtain the Einstein field equations $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$. This paper suggests however, that an "Invariant 4 volume cosmic wavelength graviton action density" applies to the solutions of infinitesimally modified stress tensor field equations. This thinking is based on our findings summarized very briefly as follows: In commoving coordinates a homogenous flat universe with no mass concentrations has both 3 volume & 4 volume action density invariance for cosmic wavelength gravitons. But with local mass concentrations this is only true for 4 volume densities, which we tried to relate with gravity. This 4 volume invariance is true in all frames. However an infinitesimal change to Einstein's Energy Momentum tensor is required with almost zero local effects, but significant implications at cosmic scale. The extra $\approx 1.39m^2/r^2$ term we had to include in the metric is irrelevant in these invariant properties, but must have some effect on Black hole mergers as we have discussed.

4 Conclusions

If the fundamental particles can be formed from infinite superpositions as outlined in [7], our hypothesis is that the warping of spacetime is consistent with a maximum wavelength, or $k_{\min}$ graviton probability density $\rho_{Gk\min} = K_{Gk\min} dk_{\min}$, or equivalently, an invariant "Four Volume Action Density". But the value of $k_{\min}$ decreases with cosmic time and is roughly inverse to the horizon radius, it also depends on the local clockrate $\sqrt{g_{00}}$

Thinking in a simple way and using the proportionality symbol $\propto$ as follows:
In a universe with no mass concentrations $\rho_{Gk\min} \propto (\psi_{\text{Universe}} * \psi_{\text{Universe}})$. With a concentration of mass m, $\rho'_{Gk\min} \propto (\psi_{\text{Universe}} * \psi_{\text{Universe}}) + (\psi_{\text{Universe}} * \psi_m + \psi_m * \psi_{\text{Universe}}) + (\psi_m * \psi_m)$ but space expands locally to restore $\rho'_{Gk\min}$ back to $\rho_{Gk\min} = G_{Gk\min} dk_{\min}$. The green term $(\psi_{\text{Universe}} * \psi_m + \psi_m * \psi_{\text{Universe}})$ requires $2m/r$ in the metric, and meshes well with an infinitesimally modified General Relativity. This modification changes the T_{00} component from $T_{00} = \rho$, where ρ is the local mass density, to $T_{00} = \rho - \rho_U$, where ρ_U is the average density of the Universe (only a few hyprogen atoms per cubic metre). It matches the Schwarzchild metric, and fits the Kerr metric. In the earlier paper we focused only on this term to illustrate a possible connection with quantum mechanics, provided the fundamental particles can be made from infinite superpositions borrowing action/energy from zero point fields. This paper messes up that nice connection by introducing the troublesome blue term $(\psi_m * \psi_m)$ with its associated m^2/r^2 in the metric. This requires a (possibly problematic?) negative energy massless particle to be added to Einstein's stress tensor, similar to the positive energy massless particles in the electromagnetic field. This m^2/r^2 term is of opposite sign to the r_Q^2/r^2, or equivalently dimensionless Q^2/r^2 term, of both the Reissner-Nordstrom and Kerr-Newman metrics. The effect on the Riemannian curvature tensor is of identical form, but opposite sign to these metrics. It does not, however, alter the event horizon radius of a maximum spin black hole, but allows about 55% more spin. These values as with other findings are only approximate however, as for simplicity, we assumed a square cutoff at $k_{\min}$. An exponential cutoff is most likely and will no doubt change these values, especially the coefficient of the m^2/r^2 term in the metric due to $(\psi_m * \psi_m)$. A more rigorous analysis of everything we have done will almost certainly change the coefficient of m^2/r^2 and the extra maximum spin. But if our conjecture is true, this m^2/r^2 term will not go away. The extra radial acceleration that it introduces could alter the merging rate of black holes. The third merger observed [26] suggests that General Relativity holds to the horizon, but the spins appear not to align with their mutual orbit, possibly challenging conventional astrophysics. If the spins are aligned, as had been thought more probable, the merging rate is slightly too fast. Could we turn this around and say: if the spins are in fact aligned, does General Relativity need modifying near the horizon? This paper requires two such changes, one that is significant, mainly near black holes, and the other most significant at cosmic scale. Testing

these two changes has to await future accuracy improvements in gravitational wave detectors and further refinements in our observations of the accelerating expansion of space.

Finally, supermassive black holes are appearing much earlier in cosmic time than expected. Could this extra m^2 / r^2 term, alter the inflow rate of the surrounding swirling matter; above that of current models? Also the Hubble parameter, predicted by Λ-CDM models based on Cosmic Microwave data, is $\approx 9\%$ less than the recent, or more current, Hubble measurements [32]. The expansion velocity in our exponential expansion model predicts expansion velocities $\approx 11\%$ greater than Λ-CDM models. Is there a connection here? The future refinements expected in these measurements over the coming years, mentioned above, will no doubt clarify this.

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