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An Application of Single-Valued Neutrosophic Sets in Medical Diagnosis

Gulfam Shahzadi[1], Muhammad Akram[1] and Arsham Borumand Saeid[2]

1. Department of Mathematics, University of the Punjab, Lahore, Pakistan. E-mail: gulfamshahzadi22@gmail.com, m.akram@pucit.edu.pk

2. Department of Mathematics, Shahid Bahonar university of Kerman Kerman, Iran. E-mail: arsham@mail.uk.ac.ir

Abstract:
In this paper, we present the use of single-valued neutrosophic sets in medical diagnosis by using distance measures and similarity measures. Using interconnection between single-valued neutrosophic sets and symptoms of patient, we determine the type of disease. We define new distance formulas for single valued neutrosophic sets. We develop two new medical diagnosis algorithms under neutrosophic environment. We also solve a numerical example to illustrate the proposed algorithms and finally, we compare the obtained results.

Keywords: Single-valued neutrosophic sets, distance, similarity measures, medical diagnosis.

1 Introduction

The notion of fuzzy set was introduced by Zadeh [1] to deal with ambiguity, vagueness and imprecision. Atanassov [2] popularized the concept of intuitionistic fuzzy set, as a generalization of fuzzy set. Adlassnig [3] employed fuzzy set theory to formalize medical relationships and fuzzy logic to model the diagnostic process and developed a computerized diagnosis system. Important developments and applications of some medical expert systems based on fuzzy set theory were reported in the literature [4-8]. De et al. [9] first proposed an application of intuitionistic fuzzy sets in medical diagnosis. Davvaz and Sadrabadi [10] discussed an application of intuitionistic fuzzy sets in medicine. Several authors [10-15] employed intuitionistic fuzzy sets in medical diagnosis and cited De et al. [9]. However, Hung and Tuan [16] pointed out that the approach studied in [9] contains questionable results that may lead to false diagnosis of patients' symptoms.

It is widely recognized that the information available to the medical practitioners about his/her patient and about medical relationships in general is inherently uncertain. Even information is incomplete as it continually becomes enlarged and gets changed. Heisenberg's Uncertainty Principle [17] reflects that nature possibly is fundamentally indeterministic. It is widely accepted that knowledge may differ according to culture, education, religion, social status, etc., and therefore information derived from different sources may be inconsistent. We may recall Godel's Theorem [18] which clearly reflects that contradictions within a system cannot be eliminated by the system itself. So uncertainty, incomplete and inconsistency should be addressed in medical diagnosis problem which can be dealt with neutrosophic set [19] introduced by Florentin Smarandache. Neutrosophic set [19] consists of three independent objects called truth-membership (μ), indeterminacy-membership (σ) and falsity-membership (ν) whose values are real standard or non-standard subset of unit interval $]0^-, 1^-[$. In 1998, the idea of single-valued neutrosophic set was given by Smarandache [19] and the term "single valued neutrosophic set" was coined in 2010 by Wang et al. [20].

Yang et al. [21] presented the theory of single-valued neutrosophic relation based on single-valued neutrosophic set. In almost every scientific field, the idea of similarity is essentially important. To measure the degree of similarity between fuzzy sets, many methods have been introduced [22-25]. These methods are not suitable to deal with the similarity measures of neutrosophic sets (NSs). Majumdar and Samanta [26] presented several similarity measures of single valued neutrosophic sets based on distances, a matching function, membership grades, and then proposed an entropy measure. Several studies dealt with similarity measures for neutrosophic sets and single-valued neutrosophic sets [27-31]. Salama et. al. [32] defined the neutrosophic correlation coefficients which are another types of similarity measurement. Ye [33] discussed similarity measures on interval neutrosophic set [34] based on Hamming distance and Euclidean distance and showed how these measures can be used in decision making problems. Furthermore, on the domain of neutrosophic sets, Pramanik et al. [35] studied hybrid vector similarity measures for single valued neutrosophic sets as well as interval neutrosophic sets. In medical diagnosis, Ye [36] presented the improved cosine similarity measures of single valued neutrosophic sets as well as interval neutrosophic sets and employed them to medical diagnosis problems. Mondal and Pramanik [37] propose tangent similarity measure and weighted tangent similarity measure for single valued neutrosophic sets and employed them to medical diagnosis.

In medical diagnosis problem, symptoms and inspecting data of some disease may be changed in different time intervals. It leads to the question that whether only by using a single

period inspection one can conclude for a particular patient with a particular decease or not. Sometimes symptoms of different diseases may appear for a person under treatment. Then, natural question arises, how can we decide a proper diagnosis for the particular patient by using one inspection? To answer this question Ye [38] proposed multi-period medical diagnosis (i.e. dynamic medical diagnosis) strategy based on neutrosophic tangent function. Several medical strategies [39-52] have been reported in the literature in neutrosophic environment including neutrosophic hybrid set environment. Nguyen et al. [53] made a survey of the state-of-the-arts on neutrosophic sets in biomedical diagnoses. The aforementioned strategies [36, 37, 38] employed cosine similarity measure and tangent similarity measure under neutrosophic environment.
The use of single-valued neutrosophic sets in medical diagnosis by using distance measures and similarity measure which have not been addressed in the literature. In this paper, we present two algorithms for medical diagnosis by using distance measures and similarity measures under neutrosophic environment. This study answers the following research questions:

1. Is it possible to formulate a new algorithm for medical diagnosis by using normalized Hamming distance and similarity measure?
2. Is it possible to formulate a new algorithm for medical diagnosis by using normalized Euclidean distance and similarity measure?
3. Is it possible to develop a new algorithm for medical diagnosis by using new distance formula and similarity measure?

The above-mentioned analysis describes the motivation behind proposing two new medical diagnosis algorithms under single valued neutrosophic environment using new distance formulas and similarity measures. This study develops two novel medical diagnosis algorithms under single valued neutrosophic environment. The Objectives of the paper are stated as follows:

1. To define two new neutrosophic distance formulas.
2. To develop two new medical diagnosis algorithms under single valued neutrosophic environment.
3. To show numerical example of medical diagnosis using the proposed algorithms.
4. To compare the obtained results derived from the proposed two algorithms with the algorithms based on normalized Hamming and normalized Euclidean distance.
5. To fill the research gap, we propose two algorithms for medical diagnosis by using distance measures and new similarity measures under neutrosophic environment.

The proposed algorithms can be effective in dealing with medical diagnosis under single valued neutrosophic set environment. It can be extended to interval neutrosophic environment and neutrosophic hybrid environment. The main contributions of this paper are summarized below:

i. We define two new distance formulas for neutrosophic sets.
ii. We develop two new algorithms for medical diagnosis based on new distance formulas and similarity measure.
iii. We present the comparison between the proposed algorithms with the algorithms based on normalized Hamming and normalized Euclidean distance.

The rest of the paper unfolds as follows: In section 2, we describe some basic definitions and operations of single valued neutrosophic sets (SVNSs). In section 3, we present the definition of proposed distance formulas and develop two new algorithms for medical diagnosis and present comparison with numerical example. In section 4, we present conclusion and future scope of the study.

2 Preliminaries

In this section, we review some basic concepts related to neutrosophic sets.

Definition 1. *[19] Let Z be a space of points (objects). A neutrosophic set M in Z is characterized by a truth-membership function $(\mu_M(z))$, an indeterminacy-membership function $(\sigma_M(z))$ and a falsity-membership function $(\nu_M(z))$. The functions $(\mu_M(z))$, $(\sigma_M(z))$, and $(\nu_M(z))$ are real standard or non-standard subsets of $]0^-, 1^+[$, that is, $\mu_M(z) : Z \rightarrow]0^-, 1^+[$, $\sigma_M(z) : Z \rightarrow]0^-, 1^+[$ and $\nu_M(z) : Z \rightarrow]0^-, 1^+[$ and $0^- \leq \mu_M(z) + \sigma_M(z) + \nu_M(z) \leq 3^+$.*
From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]0^-, 1^+[$. In real life applications in scientific and engineering problems, it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]0^-, 1^+[$, , where $0^- = 0 - \epsilon$, $1^+ = 1 + \epsilon$, ϵ is an infinitesimal number > 0. To apply neutrosophic set in real-life problems more conveniently, Smarandache and Wang et al. [20] defined single-valued neutrosophic sets which takes the value from the subset of [0, 1]. Thus, a single-valued neutrosophic set is a special case of neutrosophic set. It has been proposed as a generalization of crisp sets, fuzzy sets, and intuitionistic fuzzy sets in order to deal with incomplete information.

Definition 2. *Let $Z = \{z_1, z_2, ..., z_n\}$ be a discrete confined set. Consider M, N, O be three neutrosophic sets in Z. For all $z_i \in Z$ we have:*

$$d_H(M,N)=H(M,N)=\max\{|\mu_M(z_i)-\mu_N(z_i)|, |\sigma_M(z_i)-\sigma_N(z_i)|, |\nu_M(z_i)-\nu_N(z_i)|\}.$$

where $d_H(M,N)=H(M,N)$ denotes the extended Hausdroff distance between between two neutrosophic sets M and N.
The above defined distance $d_H(M,N)$ between neutrosophic sets

M and N satisfies the following properties:
(D1) $d_H(M,N) \geq 0$,
(D2) $d_H(M,N) = 0$ if and only if $M = N$; for all $M, N \in NS$,
(D3) $d_H(M,N) = d_H(N,M)$,
(D4) If $M \subseteq N \subseteq O$ for all $M, N, O \in NS$, then $d_H(M,O) \geq d_H(M,N)$ and $d_H(M,O) \geq d_H(N,O)$.
then d is called the distance measure between two neutrosophic sets.

Definition 3. *A mapping $S : NS(Z) \times NS(Z) \longrightarrow [0,1]$, $NS(Z)$ denotes the set of all NS in $Z = \{z_1, z_2, ..., z_n\}$, $S(M,N)$ is said to be the degree of similarity between $M \in NS$ and $N \in NS$, if $S(M,N)$ satisfies the properties of conditions (S1-S4):*
(S1) $S(M,N)$=$S(N,M)$,
(S2) $S(M,N)$=(1,0,0). If $M = N$ for all $M, N \in NS$,
(S3) $S_\mu(M,N) \geq 0$, $S_\sigma(M,N) \geq 0$, $S_\nu(M,N) \geq 0$,
(S4) If $M \subseteq N \subseteq O$ for all $M, N, O \in NS$, then $S(M,N) \geq S(M,O)$and $S(N,O) \geq S(M,O)$.

Definition 4. *The normalized Hamming distance between two neutrosophic sets M and N is defined by*

$$d_3(M,N) = \frac{1}{2n}\sum_{j=1}^{n}(|\mu_M(z_j) - \mu_N(z_j)| + |\sigma_M(z_j) - \sigma_N(z_j)| + |\nu_M(z_j) - \nu_N(z_j)|).$$

Definition 5. *The normalized Euclidean distance between two neutrosophic sets M and N is defined by*

$$d_4(M,N) = \left\{ \frac{1}{2n}\sum_{j=1}^{n}((\mu_M(z_j) - \mu_N(z_j))^2 + (\sigma_M(z_j) - \sigma_N(z_j))^2 + (\nu_M(z_j) - \nu_N(z_j))^2) \right\}^{\frac{1}{2}}.$$

3 Neutrosophic Sets in Medical Diagnosis

We first correct the formulas for the Definitions 4 and 5, where in both of them the we should put "$\frac{1}{3n}$" instead of "$\frac{1}{2n}$" in order for the Hamming distance and respectively Euclidean distance to be "normalized". These formulas are extended from intuitionistic fuzzy sets, where indeed one uses "$\frac{1}{2n}$ " since there are only two intuitionistic fuzzy sets memberships (membership and non-membership). But, we have three components in neutrosophic sets.
For example, if we compute the Hamming distance between the neutrosophic numbers: $(1,1,1)$ and $(0,0,0)$, we get $\frac{1}{2}\{|1-0| + |1-0| + |1-0|\} = \frac{3}{2} = 1.5 > 1$. Therefore, it is not normalized since the result is not in $[0,1]$. Similarly for the Euclidean formula, where we get for the same neutrosophic numbers: $\sqrt{\frac{1}{2}\{|1-0| + |1-0| + |1-0|\}} = \sqrt{\frac{3}{2}} > 1$.

We write normalized formulae for two neutrosophic sets as follows.

Definition 6. *The normalized Hamming distance between two neutrosophic sets M and N is defined by*

$$d_3(M,N) = \frac{1}{3n}\sum_{j=1}^{n}(|\mu_M(z_j) - \mu_N(z_j)| + |\sigma_M(z_j) - \sigma_N(z_j)| + |\nu_M(z_j) - \nu_N(z_j)|).$$

Definition 7. *The normalized Euclidean distance between two neutrosophic sets M and N is defined by*

$$d_4(M,N) = \left\{ \frac{1}{3n}\sum_{j=1}^{n}((\mu_M(z_j) - \mu_N(z_j))^2 + (\sigma_M(z_j) - \sigma_N(z_j))^2 + (\nu_M(z_j) - \nu_N(z_j))^2) \right\}^{\frac{1}{2}}.$$

In this section, we give new concepts for medical diagnosis via distances between neutrosophic sets. In fact our purpose is to find an accurate diagnosis for each patient $p_i, i = 1, 2, 3$. The relation between neutrosophic sets for all the symptoms of the i-th patient from the k-th diagnosis is as follows:

$$d_1(p_i,d_k) = \frac{1}{n}\sum_{j=1}^{n}\left[\frac{1}{6}\big[|\mu_{p_i}(z_j) - \mu_{d_k}(z_j)| + |\sigma_{p_i}(z_j) - \sigma_{d_k}(z_j)| + |\nu_{p_i}(z_j) - \nu_{d_k}(z_j)|\big] + \frac{1}{3}\big[\max(|\mu_{p_i}(z_j) - \mu_{d_k}(z_j)|, |\sigma_{p_i}(z_j) - \sigma_{d_k}(z_j)|, |\nu_{p_i}(z_j) - \nu_{d_k}(z_j)|)\big]\right]. \quad (1)$$

We take $n = 5$.
We consider there are three patients: Ali, Hamza, Imran and symptoms of patient are Temperature, Insulin, Blood pressure, Blood plates, Cough and finally we get diagnosis as Diabates, Dengue, Tuberculosis.
In Table 1, the data are explained by three parameters: membership function (μ), non-membership function (ν) and indeterminacy function (σ). In Table 2, the symptoms are described by (μ, σ, ν). For example, Diabates temperature is low $(\mu = 0.2, \sigma = 0.0, \nu = 0.8)$, while Dengue temperature is high $(\mu = 0.9, \sigma = 0.0, \nu = 0.1)$.

Table 1. Membership function μ, Indeterminacy function σ and non-membership function ν.

I_1	Ali	Hamza	Imran
Temperature	(0.8,0.1,0.1)	(0.6,0.2,0.2)	(0.4,0.2,0.4)
Insulin	(0.2,0.2,0.6)	(0.9,0.0,0.1)	(0.2,0.1,0.7)
Blood pressure	(0.4,0.2,0.4)	(0.1,0.1,0.8)	(0.1,0.2,0.7)
Blood plates	(0.8,0.1,0.1)	(0.2,0.1,0.7)	(0.3,0.1,0.6)
Cough	(0.3,0.3,0.4)	(0.5,0.1,0.4)	(0.8,0.0,0.2)

Table 2. Symptoms

I_2	Temperature	Insulin	Blood pressure	Blood plates	Cough
Diabates	(0.2,0.0,0.8)	(0.9,0.0,0.1)	(0.1,0.1,0.8)	(0.1,0.1,0.8)	(0.1,0.1,0.8)
Dengue	(0.9,0.0,0.1)	(0.0,0.2,0.8)	(0.8,0.1,0.1)	(0.9,0.0,0.1)	(0.1,0.1,0.8)
Tuberculosis	(0.6,0.2,0.2)	(0.0,0.1,0.9)	(0.4,0.2,0.4)	(0.0,0.2,0.8)	(0.9,0.0,0.1)

By using formula (1), for $n = 5$, we obtain Table 3.

Table 3. Using formula (1), for $n = 5$.

I	Ali	Hamza	Imran
Diabates	0.38	0.14	0.27
Dengue	0.15	0.40	0.34
Tuberculosis	0.25	0.25	0.14

The best medical diagnosis in each column is identified by the lowest difference. Therefore, in the first column, Ali suffers from Dengue, in the second column, Hamza suffers from Diabates, in the third column, Imran suffers from Tuberculosis. Now we define another relation for the best medical diagnosis:

$$d_2(p_i, d_k) = \frac{1}{3}\sqrt[r]{n}\Big\{\sum_{j=1}^{n}(|\mu_{p_i}(z_j) - \mu_{d_k}(z_j)| + |\sigma_{p_i}(z_j) - \sigma_{d_k}(z_j)| + |\nu_{p_i}(z_j) - \nu_{d_k}(z_j)|)^r\Big\}^{\frac{1}{r}}. \quad (2)$$

and r is a positive number. We take $n = 5$. We examine the above relation for $r = 1, 2, ..., 10$. First, for $r = 1$ we calculate Table 4.

Table 4. Using formula (2), for $r = 1$.

I	Ali	Hamza	Imran
Diabates	0.39	0.15	0.26
Dengue	0.16	0.4	0.36
Tuberculosis	0.25	0.25	0.15

Now, for $r = 2$ we get Table 5.

Table 5. Using formula (2), for $r = 2$.

I	Ali	Hamza	Imran
Diabates	0.4	0.22	0.32
Dengue	0.19	0.43	0.38
Tuberculosis	0.32	0.32	0.15

The result for $r = 3$ is given in Table 6.

Table 6. Using formula (2), for $r = 3$.

I	Ali	Hamza	Imran
Diabates	0.41	0.25	0.35
Dengue	0.2	0.45	0.39
Tuberculosis	0.35	0.37	0.16

For $r = 4$, we obtain Table 7.

Table 7. Using formula (2), for $r = 4$.

I	Ali	Hamza	Imran
Diabates	0.42	0.28	0.37
Dengue	0.21	0.47	0.41
Tuberculosis	0.39	0.41	0.17

For $r = 5$, we get Table 8.

Table 8. Using formula (2), for $r = 5$.

I	Ali	Hamza	Imran
Diabates	0.43	0.3	0.39
Dengue	0.22	0.48	0.41
Tuberculosis	0.41	0.44	0.17

By calculation for $r = 6$, we find Table 9.

Table 9. Using formula (2), for $r = 6$.

I	Ali	Hamza	Imran
Diabates	0.43	0.31	0.4
Dengue	0.23	0.49	0.41
Tuberculosis	0.42	0.46	0.17

For $r = 7$, we find Table 10.

Table 10. Using formula (2), for $r = 7$.

I	Ali	Hamza	Imran
Diabates	0.43	0.32	0.41
Dengue	0.23	0.5	0.42
Tuberculosis	0.43	0.48	0.18

For $r = 8$, we get Table 11.

Table 11. Using formula (2), for $r = 8$.

I	Ali	Hamza	Imran
Diabates	0.44	0.33	0.41
Dengue	0.24	0.51	0.43
Tuberculosis	0.44	0.49	0.18

For $r = 9$, we get Table 12.

Table 12. Using formula (2), for $r = 9$.

I	Ali	Hamza	Imran
Diabates	0.44	0.33	0.42
Dengue	0.24	0.51	0.43
Tuberculosis	0.45	0.5	0.18

For $r = 10$, we obtain Table 13.

Table 13. Using formula (2), for $r = 10$.

I	Ali	Hamza	Imran
Diabates	0.45	0.34	0.43
Dengue	0.24	0.52	0.43
Tuberculosis	0.45	0.51	0.18

As r becoming larger, the difference between the data in tables become inferior, that is, the data approaches to the real amount. In Tables 4-13, the results are same. In fact in all tables, in the first column, the lowest difference is related to Ali and Dengue, so Ali suffers from Dengue, also in the second column Hamza suffers from Diabates, in the third column Imran suffers from Tuberculosis.
The normalized Hamming distance for all the symptoms of the i-th patient from the k-th diagnosis [**?**] is

$$d_3(p_i,d_k) = \frac{1}{3n}\sum_{j=1}^{n}(|\mu_{p_i}(z_j)-\mu_{d_k}(z_j)| + |\sigma_{p_i}(z_j)-\sigma_{d_k}(z_j)| + |\nu_{p_i}(z_j)-\nu_{d_k}(z_j)|). \quad (3)$$

and the normalized Euclidean distance [**?**] is

$$d_4(p_i,d_k) = \left\{\frac{1}{3n}\sum_{j=1}^{n}((\mu_{p_i}(z_j)-\mu_{d_k}(z_j))^2 + (\sigma_{p_i}(z_j)-\sigma_{d_k}(z_j))^2 + (\nu_{p_i}(z_j)-\nu_{d_k}(z_j))^2)\right\}^{\frac{1}{2}}. \quad (4)$$

We set $n=5$.
By formulas (3), (4) respectively, the results are given in Tables 14 and 15.

Table 14. Using formula (3).

I	Ali	Hamza	Imran
Diabates	0.39	0.15	0.26
Dengue	0.16	0.4	0.36
Tuberculosis	0.25	0.25	0.15

Table 15. Using formula (4).

I	Ali	Hamza	Imran
Diabates	0.46	0.24	0.37
Dengue	0.20	0.49	0.43
Tuberculosis	0.35	0.37	0.18

Thus, we studied results that have been obtained from formulas (3), (4) are same with relations (1), (2). Another idea for medical diagnosis is

$$d(M,N) = \max(|\mu_M(z_i)-\mu_N(z_i)|, |\sigma_M(z_i)-\sigma_N(z_i)|, |\nu_M(z_i)-\nu_N(z_i)|). \quad (5)$$

Table 16. Medical diagnosis.

I	Ali	Hamza	Imran
Diabates	0.7	0.6	0.7
Dengue	0.4	0.9	0.7
Tuberculosis	0.8	0.9	0.3

The similarity measures between two neutrosophic sets M and N is defined as follows :

$$S_1(M,N) = \frac{1}{n}\sum_{i=1}^{n}\Big[\big[\min(\mu_M(z_i),\mu_N(z_i)) + \min(\sigma_M(z_i),\sigma_N(z_i)) + \min(\nu_M(z_i),\nu_N(z_i))\big] \div \big[\max(\mu_M(z_i),\mu_N(z_i)) + \max(\sigma_M(z_i),\sigma_N(z_i)) + \max(\nu_M(z_i),\nu_N(z_i))\big]\Big]. \quad (6)$$

We set $n=5$ (Table 17).

Table 17. Using formula (6), for $n=5$.

I	Ali	Hamza	Imran
Diabates	0.28	0.70	0.45
Dengue	0.63	0.27	0.32
Tuberculosis	0.51	0.52	0.65

$$S_2(M,N) = \frac{1}{n}\Big[\sum_{i=1}^{n}(1-\frac{1}{3}(|\mu_M(z_i)-\mu_N(z_i)| + |\sigma_M(z_i)-\sigma_N(z_i)+|\nu_M(z_i)-\nu_N(z_i))\Big]. \quad (7)$$

We set $n=5$ (Table 18).

Table 18. Using formula (7), for $n=5$.

I	Ali	Hamza	Imran
Diabates	0.69	0.45	0.72
Dengue	0.84	0.4	0.66
Tuberculosis	0.55	0.55	0.85

$$S_3(M,N) = \sum_{i=1}^{n}\big[\min(\mu_M(z_i),\mu_N(z_i)) + \min(\sigma_M(z_i),\sigma_N(z_i)) + \min(\nu_M(z_i),\nu_N(z_i))\big] \div \sum_{i=1}^{n}\big[\max(\mu_M(z_i),\mu_N(z_i)) + \max(\sigma_M(z_i),\sigma_N(z_i)) + \max(\nu_M(z_i),\nu_N(z_i))\big]. \quad (8)$$

We set $n=5$ (Table 19).

Table 19. Using formula (8), for $n=5$.

I	Ali	Hamza	Imran
Diabates	0.27	0.64	0.41
Dengue	0.61	0.25	0.31
Tuberculosis	0.45	0.45	0.64

$$S_4(M,N) = 1-\frac{1}{3}(\max_i(|\mu_M(z_i)-\mu_N(z_i)|) + \max_i(|\sigma_M(z_i)-\sigma_N(z_i)|) + \max_i(|\nu_M(z_i)-\nu_N(z_i)|)). \quad (9)$$

We set $n=5$ (Table 20).

Table 20. Using formula (9), for $n = 5$.

I	Ali	Hamza	Imran
Diabates	0.47	0.6	0.5
Dengue	0.5	0.1	0.25
Tuberculosis	0.67	0.4	0.73

$$S_5(M,N) = 1 - \Bigg[\sum_{i=1}^{n}\big[|\mu_M(z_i) - \mu_N(z_i)| + |\sigma_M(z_i) - \sigma_N(z_i)| + |\nu_M(z_i) - \nu_N(z_i)|\big] \div \sum_{i=1}^{n}\big[|\mu_M(z_i) + \mu_N(z_i)| + |\sigma_M(z_i) + \sigma_N(z_i)| + |\nu_M(z_i) + \nu_N(z_i)|\big]\Bigg]. \quad (10)$$

We set $n = 5$ (Table 21).

Table 21. Using formula (10), for $n = 5$.

I	Ali	Hamza	Imran
Diabates	0.42	0.78	0.58
Dengue	0.76	0.4	0.46
Tuberculosis	0.62	0.62	0.78

We can see that the results obtained by using the relations S_1, S_2, S_3, S_4, S_5 are different from relations $1 - 5$. Therefore, these similarity measures are not applicable.

The new similarity measures between neutrosophic sets M and N are defined as follows. The first one is

$$S_{new1} = \frac{1}{1-\exp(-n)}\Bigg[1 - \exp\Big(-\frac{1}{3}\sum_{i=1}^{n}(|\mu_M(z_i) - \mu_N(z_i)| + |\sigma_M(z_i) - \sigma_N(z_i)| + |\nu_M(z_i) - \nu_N(z_i)|)\Big)\Bigg]. \quad (11)$$

We set $n = 5$ (Table 22).

Table 22. Using formula (11), for $n = 5$.

I	Ali	Hamza	Imran
Diabates	0.86	0.52	0.75
Dengue	0.55	0.88	0.84
Tuberculosis	0.73	0.73	0.52

The second one is

$$S_{new2} = \frac{1}{1-\exp(-n)}\Bigg[1 - \exp\Big(-\frac{1}{3}\sum_{i=1}^{n}(|\sqrt{\mu_M(z_i)} - \sqrt{\mu_N(z_i)}| + |\sqrt{\sigma_M(z_i)} - \sqrt{\sigma_N(z_i)}| + |\sqrt{\nu_M(z_i)} - \sqrt{\nu_N(z_i)}|)\Big)\Bigg]. \quad (12)$$

We set $n = 5$ (Table 23).

Table 23. Using formula (12), for $n = 5$.

I	Ali	Hamza	Imran
Diabates	0.83	0.50	0.75
Dengue	0.60	0.86	0.84
Tuberculosis	0.74	0.75	0.55

The obtained relations from $S_{new1}(M,N), S_{new2}(M,N)$ are closely same with relations $1 - 5$. Consequently, the obtained results from the relations between neutrosophic sets (1), (2), (5), (11), (12) are equivalent to the results of formula (3), (4). By using the distance and similarity measures formulas between neutrosophic sets, we establish the most applicable medical diagnosis that in all tables are related to the lowest difference in each column. Finally, we conclude that the methods which have the results equivalent to normalized hamming and normalized Euclidean formulas are best to determine the diseases of a patient. Now we present our first method in the following algorithm 1.

Algorithm 1:

Step 1. Input the truth membership, indeterminacy and non-membership values of patients and diagnosis.

Step 2. Compute the diseases by different distance measures given in steps $3 - 7$.

Step 3.

$$d_1(p_i,d_k) = \frac{1}{n}\sum_{j=1}^{n}\Bigg[\frac{1}{6}\big[|\mu_{p_i}(z_j) - \mu_{d_k}(z_j)| + |\sigma_{p_i}(z_j) - \sigma_{d_k}(z_j)| + |\nu_{p_i}(z_j) - \nu_{d_k}(z_j)|\big] + \frac{1}{3}\big[\max(|\mu_{p_i}(z_j) - \mu_{d_k}(z_j)|, |\sigma_{p_i}(z_j) - \sigma_{d_k}(z_j)|, |\nu_{p_i}(z_j) - \nu_{d_k}(z_j)|)\big]\Bigg].$$

Step 4.

$$d_2(p_i,d_k) = \frac{1}{3}\sqrt[r]{n}\Bigg\{\sum_{j=1}^{n}(|\mu_{p_i}(z_j) - \mu_{d_k}(z_j)| + |\sigma_{p_i}(z_j) - \sigma_{d_k}(z_j)| + |\nu_{p_i}(z_j) - \nu_{d_k}(z_j)|)^r\Bigg\}^{\frac{1}{r}}.$$

Step 5.

$$d_3(p_i,d_k) = \frac{1}{3n}\sum_{j=1}^{n}(|\mu_{p_i}(z_j) - \mu_{d_k}(z_j)| + |\sigma_{p_i}(z_j) - \sigma_{d_k}(z_j)| + |\nu_{p_i}(z_j) - \nu_{d_k}(z_j)|).$$

Step 6.

$$d_4(p_i,d_k) = \Bigg\{\frac{1}{3n}\sum_{j=1}^{n}((\mu_{p_i}(z_j) - \mu_{d_k}(z_j))^2 + (\sigma_{p_i}(z_j) - \sigma_{d_k}(z_j))^2 + (\nu_{p_i}(z_j) - \nu_{d_k}(z_j))^2)\Bigg\}^{\frac{1}{2}}.$$

Step 7.

$$d(M,N) = \max(|\mu_M(z_i) - \mu_N(z_i)|, |\sigma_M(z_i) - \sigma_N(z_i)|, |\nu_M(z_i) - \nu_N(z_i)|).$$

W present our second method in the following algorithm 2.

Algorithm 2:

Step 1. Input the truth membership, indeterminacy and non-membership values of patients and diagnosis.

Step 2. Also compute the diseases by similarity measures given in steps $3-9$.

Step 3.

$$S_1(M,N) = \frac{1}{n}\sum_{i=1}^{n}\Big[\big[\min(\mu_M(z_i),\mu_N(z_i)) + \min(\sigma_M(z_i),\sigma_N(z_i)) + \min(\nu_M(z_i),\nu_N(z_i))\big] \div \big[\max(\mu_M(z_i),\mu_N(z_i)) + \max(\sigma_M(z_i),\sigma_N(z_i)) + \max(\nu_M(z_i),\nu_N(z_i))\big]\Big].$$

Step 4.

$$S_2(M,N) = \frac{1}{n}\Big[\sum_{i=1}^{n}(1 - \frac{1}{3}(|\mu_M(z_i) - \mu_N(z_i)| + |\sigma_M(z_i) - \sigma_N(z_i) + |\nu_M(z_i) - \nu_N(z_i))\Big].$$

Step 5.

$$S_3(M,N) = \sum_{i=1}^{n}\big[\min(\mu_M(z_i),\mu_N(z_i)) + \min(\sigma_M(z_i),\sigma_N(z_i)) + \min(\nu_M(z_i),\nu_N(z_i))\big] \div \sum_{i=1}^{n}\big[\max(\mu_M(z_i),\mu_N(z_i)) + \max(\sigma_M(z_i),\sigma_N(z_i)) + \max(\nu_M(z_i),\nu_N(z_i))\big].$$

Step 6.

$$S_4(M,N) = 1 - \frac{1}{3}(\max_i(|\mu_M(z_i) - \mu_N(z_i)|) + \max_i(|\sigma_M(z_i) - \sigma_N(z_i)|) + \max_i(|\nu_M(z_i) - \nu_N(z_i)|)).$$

Step 7.

$$S_5(M,N) = 1 - \Big[\sum_{i=1}^{n}\big[|\mu_M(z_i) - \mu_N(z_i)| + |\sigma_M(z_i) - \sigma_N(z_i)| + |\nu_M(z_i) - \nu_N(z_i)|\big] \div \sum_{i=1}^{n}\big[|\mu_M(z_i) + \mu_N(z_i)| + |\sigma_M(z_i) + \sigma_N(z_i)| + |\nu_M(z_i) + \nu_N(z_i)|\big]\Big].$$

Step 8.

$$S_{new1} = \frac{1}{1-\exp(-n)}\Big[1 - \exp(-\frac{1}{3}\sum_{i=1}^{n}(|\mu_M(z_i) - \mu_N(z_i)| + |\sigma_M(z_i) - \sigma_N(z_i)| + |\nu_M(z_i) - \nu_N(z_i)|))\Big].$$

Step 9.

$$S_{new2} = \frac{1}{1-\exp(-n)}\Big[1 - \exp(-\frac{1}{3}\sum_{i=1}^{n}(|\sqrt{\mu_M(z_i)} - \sqrt{\mu_N(z_i)}| + |\sqrt{\sigma_M(z_i)} - \sqrt{\sigma_N(z_i)}| + |\sqrt{\nu_M(z_i)} - \sqrt{\nu_N(z_i)}|))\Big].$$

Finally, We compare these methods to normalized hamming and normalized Euclidean formulas and conclude that the methods which have results equivalent to normalized hamming and normalized Euclidean formulas are the best methods to determine the disease of a patient.

4 Conclusion

In this we have developed two new algorithms for medical diagnosis using the proposed distance formula and similarity measures. We have solved a numerical example and compared the obtained results derived from the proposed two algorithms with the algorithms based on normalized Hamming and normalized Euclidean distance. The proposed algorithms can be extended to interval neutrosophic set environment and other neutrosophic hybrid environment for medical diagnosis.

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