A defense of local realism

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Abstract. Bell's inequalities (and the CHSH inequalities) were used in order to rule out certain hidden variable theories (or interpretations of quantum mechanics). In this short note, I will prove that Bell's results represent a strong argument in favor of a deeper, nonlinear underlying reality.

Keywords. Bell's inequality, nonlocality, nonlinearity, chaos, strange attractors.

Introduction.

As Bell emphasized in his article ([2]), when we refer to a hidden variable λ, it stands for any number of variables and the dependencies thereon of entangled photons A and B are unrestricted. These hidden variables could have dynamical significance and laws of motion, our λ could also be thought as initial values of these variables at some suitable instant.

Section 1. The classical derivation of the CHSH inequality ([1]).

We consider experiments involving polarization entangled photons. A version of Bell's inequality requires that we consider experiments involving four different orientations of the two polarization analyzers (PA). We denote these orientations as a, b, c and d. We suppose that the results of measurements made on photon A and photon B are determined by some local hidden variable (or variables) denoted λ. The λ values have a probability density function ρ(λ). We assume that $\int \rho(\lambda)\ d\lambda = 1$ (when not specified, the variable belongs to the hidden variable parameter space).

The expectation values of the results of measurements depend on the particular orientation of the polarizers and the λ value. We write the expectation value for photon A entering PA1 setup with orientation a as A(a, λ). Similarly, the expectation value for photon B entering PA2 setup with orientation b is B(b, λ). The possible result of each measurement is ±1, corresponding to detection in the vertical or horizontal channels respectively. We have the relations:

$$|A(a,\lambda)| \leq 1\ , |B(b,\lambda)| \leq 1 \qquad (1)$$

We assume that the individual results for A depend on a and λ, but are independent of b and vice versa (Einstein separability).

The expectation value for the joint measurement of A and B (averaging over many photon pairs) is :

$$E(a,b) = \int A(a,\lambda)\ \cdot B(b,\lambda)\ \cdot\ \rho(\lambda) d\lambda \qquad (2)$$

We emphasize the statistical nature of the hidden variable approach and the fact that we perform a sufficient number of measurements of photon pairs, so that all possible values of λ are sampled.

We have then:

$$E(a,b) - E(a,d) = \int \big(A(a,\lambda) \cdot B(b,\lambda) - A(a,\lambda) \cdot B(d,\lambda)\big) \cdot \rho(\lambda) d\lambda = \int A(a,\lambda) \cdot \big(B(b,\lambda) - B(d,\lambda)\big) \cdot \rho(\lambda)\, d\lambda \quad (3)$$

Since $|A(a,\lambda)| \leq 1$, we have:

$$|E(a,b) - E(a,d)| \leq \int |B(b,\lambda) - B(d,\lambda)|\, \rho(\lambda)\, d\lambda \quad (4)$$

Similarly, we have:

$$|E(c,b) + E(c,d)| \leq \int |B(b,\lambda) + B(d,\lambda)|\, \rho(\lambda)\, d\lambda \quad (5)$$

From (4) and (5) we have:

$$|E(a,b) - E(a,d)| + |E(c,b) + E(c,d)| \leq \int (|B(b,\lambda) - B(d,\lambda)| + |B(b,\lambda) + B(d,\lambda)|)\, \rho(\lambda)\, d\lambda \quad (6)$$

From (1) we have $|B(b,\lambda) - B(d,\lambda)| + |B(b,\lambda) + B(d,\lambda)| \leq 2$

As a consequence, we have:

$$|E(a,b) - E(a,d)| + |E(c,b) + E(c,d)| \leq 2 \cdot \int \rho(\lambda)\, d\lambda \quad (7)$$

Since $\int \rho(\lambda)\, d\lambda = 1$, we have:

$$|E(a,b) - E(a,d)| + |E(c,b) + E(c,d)| \leq 2 \quad (8)$$

Quantum mechanics predicts the violation of inequality (8) for certain values of a, b, c and d. Experiment validates the prediction of quantum mechanics. As a consequence, this type of hidden variables model cannot explain the experiment. Nonlocality becomes a necessity.

Section 2. Bell's theorem in a nonlinear mathematical framework.

We look at quantum measurement as an irreversible, nonlinear process. Nonlinear processes are characterized by the presence of chaos and strange attractors (see [4]). A path in phase space determines the experimental outcome. The Poincare section of a strange attractor is usually a Cantor dust set. The parameter λ belongs to these types of sets (this is the parameter set), since this parameter will fix the trajectory in phase space, and thus an experimental outcome. In his proof, Bell calculates the expectation value $E(a,b,\lambda)$ for the joint measurement of operators A and B, and is given by the product $A(a,\lambda) \cdot B(b,\lambda)$, where a and b represent the orientation of the apparatus (on both sides A, and B), and λ is the parameter that fixes the experimental outcome. Then he eliminates λ from the expression by averaging the results over many pairs (of entangles particles). Here appears the Lebesgue integral, and this follows if we assume that we can perform measurements over a sufficiently large number of photon (particle) pairs, so that all possible values of λ are sampled.

Here is the argument that fails in the proof of Bell's theorem (in a nonlinear mathematical framework). This averaging is not well defined in our case. Bell assumes that these Lebesgue intergrals exist, and the functions $A(a,\lambda)$, $B(b,\lambda)$ are well behaved, Lebesgue integrable. In the first place, we cannot even use the Lebesgue measure on our fractal set, because the whole Poincare section (of the attractor) is a Cantor dust set, an uncountable set of Lebesgue measure 0. We have to use the Haussdorff measure on Cantor sets (see [3]). Also, on the cross section of the strange attractor we have sensitivity to initial conditions, so the functions $A(a,\lambda)$, $B(b,\lambda)$ are far from continuous, or even integrable, an infinitesimal change in λ can lead to completely different experimental resulta. As a consequence, the averaging process cannot be performed, those integrals are not well defined. We are basically sampling a distribution that might not even have a well defined mean. The fact that experimental results show a violation of Bell's inequalities tells us that those nonlinear processes associated to the act of measurement are real. The standard, linear mathematical framework of quantum mechanics is only an approximation of a deeper, nonlinear underlying reality.

To clarify, I do not raise the issue of loopholes in EPR experiments. I am convinced that larger experimental data sets will show the same strong violations of Bell's inequalities. But the same larger data sets might also show slight deviations from the probabilistic predictions of quantum mechanics.

Local realism is perfectly compatible with experimental evidence, if we consider nonlinear models associated to the act of measurement, This is true, even if there will be strong violations of Bell's inequalities.

We need a model governed by a nonlinear Schrodinger equation, that is consistent with the predictions of usual quantum mechanics (so the no cloning and no FTL signaling theorems are still valid, but note that consistent doesn't necessarily mean identical predictions), for which the wavefunction collapses by the act of amplification. There is no need to invoke the "spooky action at a distance" (as Einstein called it), in order to explain Aspect type experiments, because the proof of Bell's theorem collapses within a nonlinear framework. That means that local realism is preserved. There are at least two models that might be suitable (maybe with small modifications), the pilot wave Bohmian mechanics, and GRW.

Related to experimental methodology, this is a problem of hypothesis testing, or sampling, in a statistical framework. For a confidence level of 0.95, and an error not exceeding 0.005, you would need a sample size of about 40000. For example, for a parametric down conversion source of entangled photons, with an efficiency of a few entangled photons per minute, you would need almost a month of continuous operation, in order to collect the required amount of data, that can be analysed later, based on coincidence timing of detection (including losses). If quantum mechanics is only an approximation of a deeper, nonlinear hidden variable reality, the deviations could involve even smaller margins of error, thus requiring larger samples.

I propose a revision of experimental work (related to EPR type experiments), with much larger experimental data sets, keeping in mind the fact that we could be sampling complex distributions with undefined mean (this is a complex problem in statistics).

The GHZ experiments can also be explained in this framework, if we assume that the acts of measurement performed on entangled quantum systems are associated with trajectories in phase space that start from the same initial conditions.

As a conclusion, local realism is not compromised, Einstein was right, and careful experiments will show the validity of this point of view.

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References :

[1]. Jim Baggott, *“The Meaning of Quantum Theory “*, Oxford University Press, 1992.

[2]. John S. Bell, *“ On the Einstein Podolsky Rosen Paradox“*, Physics, 1, (1964)

[3]. Kenneth Falconer, *“Fractal Geometry “*, John Wiley & Sons Ltd., 2003.

[4]. Steven H. Strogatz , *“Nonlinear Dynamics and Chaos “*, Westview., 1994.

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