Space and Time, Geometry and Fields: An Historical Essay on the Fundamental and its Physical Manifestation

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We address historical circumstances surrounding the absence of two essential tools - geometric interpretation of Clifford algebra and generalization of impedance quantization - from the particle physicist's tool kit, and present details of the new perspective that follows from their inclusion. The resulting geometric wavefunction model permits one to examine the interface between fundamental and emergent.

Introduction

The topic for the 2018 Foundational Questions Institute's annual essay contest[1] is more than a little recursive, given that 'fundamental' has its origins in the Latin 'found'. Foundational Questions are fundamental questions. The foundation is fundamental. The fundamental is the foundation. A koan for the philosopher physicist.

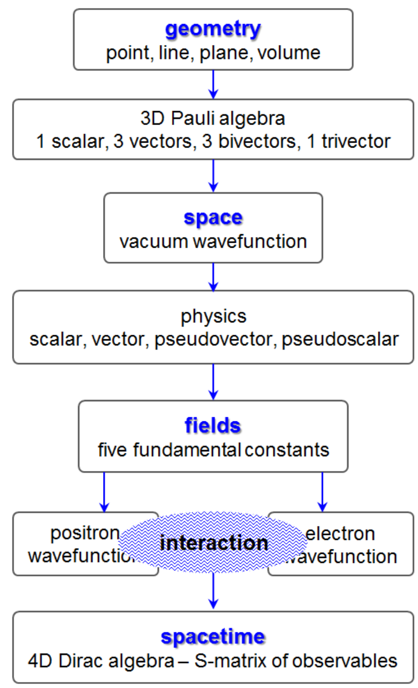

FIG. 1. Fundamentals - the theoretical minimum for philosopher physicists, showing spacetime emerging from wavefunction interactions, manifesting as single measurement observables of the S-matrix[6–9].

Introducing the concept of emergence permits a straightforward delineation of the fundamental. Here we take emergent to mean the whole is greater than the sum of its parts. There is perhaps no better example than the innumerable variety of snowflakes emerging from clusters of but one simple molecular structure - two atoms of hydrogen and one of oxygen.

We take the fundamental to be that which cannot be understood to be emergent in any **observable** sense, where observable is taken to be that which can give information in a single measurement. Phase is relative, not a single measurement observable. This radically diminishes possibilities, puts us at the phase-coherent boundary between classical and quantum, between observables and our models, puts us in the interaction of figure 1.

The figure comprises a partial outline of this essay, with interactions marking the boundary between how we know and what we know, between knowledge and reality, the epistemic and the ontic, the fundamental and the emergent, our models and the world.

It marks the boundary as interactions of that which cannot be observed, the enigmatic wavefunction. Observing the wavefunction changes it. What we see is the change. From many measurements we construct our model. One might argue that the unobservable nature of the wavefunction does not prevent it from being emergent, but then the question arises - emergent from what? From what emerges the unobservable? An easier question - from where comes the universe?

In what follows we don't offer new proposals about some 'fundamental' constituents of the universe, but rather mix old ideas in new ways, starting with Euclid and fundamental geometric objects of physical space, presenting the mix in historical context.

Over two millenia elapsed between discovery of Euclid's fundamental geometric objects[2] and development of their algebra by Grassman and Clifford in the 1800s[3–5]. It was rediscovered by Pauli and Dirac in the 1920s in the much more abstract matrix representation, with the unfortunate consequence that simple intuitive geometric interpretation remains lost in mainstream physics yet today.

Of itself the geometry and its algebra are abstractions. It is only with the possibility of excitation by physical fields that the concept of geometric vacuum wavefunction becomes useful. For interactions to exist requires assignment of quantized electric and magnetic fields to Euclid's fundamental geometric objects, requires the introduction of five fundamental physical constants. One might argue that the constants are not fundamental, but rather emergent. However then the question again arises - emergent from what? Our models that seek to describe the fundamental?

The wavefunction.

Historical Perspective on Geometric Algebra and Geometric Wavefunctions

Over fifty years have passed since the original geometric intent of Clifford algebra was rediscovered by David Hestenes and introduced to physics [10], sixteen years since he was awarded the Oersted Medal by the American Physical Society for "Reformulating the Mathematical Language of Physics"[11]. Figure 2 illustrates an important point - geometric algebra claims to encompass the better part of the particle physicist's mathematical toolkit[11–13]. Geometric interpretation remains unrecognized by mainstream physics, a profound measure of our inertia.

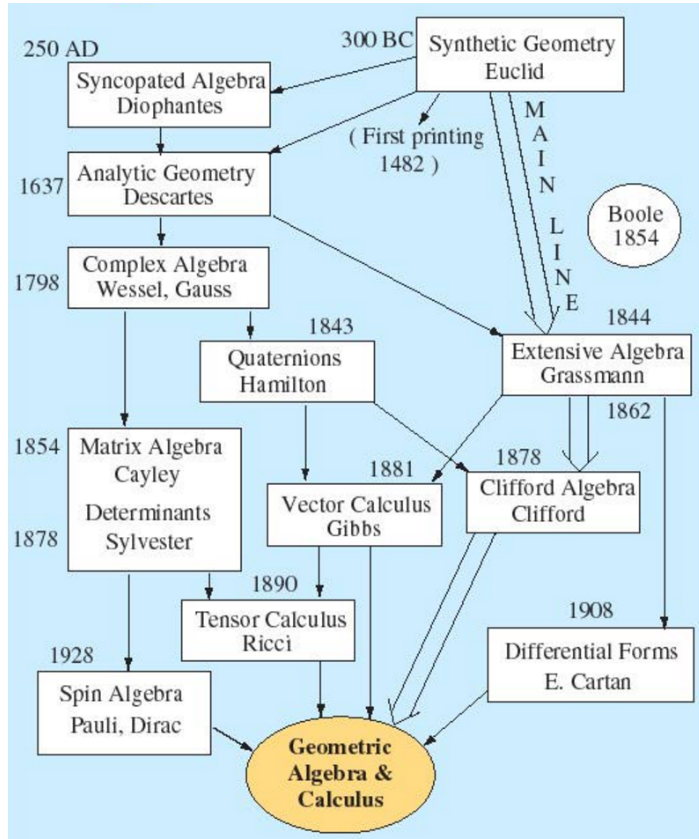

FIG. 2. Geometric algebra (and its extension into geometric calculus) claims to encompass the physicist's mathematical toolkit[16].

The algebra as originally conceived describes interactions of geometric objects. Grassman was "...a pivotal figure in the historical development of a universal geometric calculus for mathematics and physics... He formulated most of the basic ideas and... anticipated later developments. His influence is far more potent and pervasive than generally recognized."[14]

Grassman's work lay fallow until Clifford "...united the inner and outer products into a single *geometric* product. This is associative, like Grassman's product, but has the crucial extra feature of being *invertible*, like Hamilton's quaternion algebra."[15]

While Clifford algebra attracted interest, with his early death in 1879 absence of an advocate to balance the powerful Gibbs led to its eventual neglect. It was "...largely abandoned with the introduction of what people saw as a more straightforward and generally applicable algebra, the *vector algebra* of Gibbs... This was effectively the end of the search for a unifying mathematical language and the beginning of a proliferation of novel algebraic systems..."[13]. Geometric algebra resurfaced, unrecognized, as algebra without geometric meaning in the Pauli and Dirac matrices.

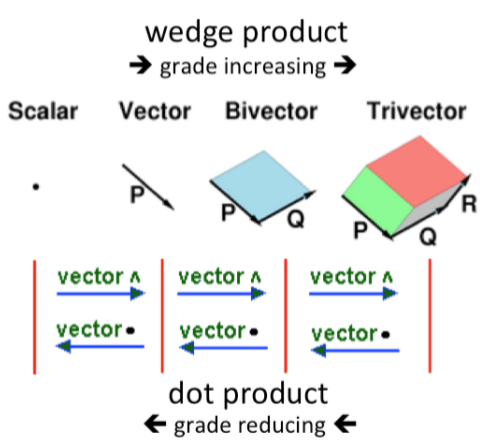

FIG. 3. Geometric algebra components in 3D Pauli algebra of space, showing operation of the grade/dimension raising wedge and lowering dot products.[17]

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Spinors of the Dirac equation wavefunction are comprised of two of the eight fundamental geometric objects of the Pauli algebra - one each scalar and bivector. The full eight component Pauli vacuum wavefunction is comprised of one scalar, three vectors, three bivectors, and one trivector.

Figure 3 illustrates operation of the Pauli algebra of space. The two products (dot and wedge or inner and outer) comprising the geometric product lower and raise the grade. Mixing of grades, of dimensionality, makes geometric algebra unique in the ability to handle geometric concepts in any dimension.

Given vacuum wavefunctions, geometric algebra models interactions by the geometric product, the non-linear process of multiplication, in particular *geometric* multiplication. The vacuum wavefunction is 3D, just geometry. Time emerges from the interactions, from the mixing of grades, dynamic from the static. Interactions of 3D vacuum wavefunctions yield geometry of the 4D spacetime S-matrix shown in figure 4[8–10]. However, this remains an abstraction without fields.

Historical Perspective on Quantized Fields and the resulting Quantized Impedances

The leap from point particle quark and lepton wavefunctions to the full eight component Pauli wavefunction is a big one, and surprisingly easy. What was obscure, intrinsic, populating abstract 'internal' spaces finds representation in our intuitively familiar 3D physical space, benefits from the wisdom of the body.

The fundamental geometric objects we use to model that space introduce geometric quantization. They are individual, discrete, not continuous, clearly defined. One point, three lines, three area elements and one volume element, all orientable. The eight geometric quanta of the Pauli vacuum wavefunction are shown in the detailed example at top and left of figure 4 for electron and positron respectively. Their geometric product generates the Dirac matrix, geometric structure of the impedance representation of the S-matrix[18, 19].

Physical manifestion requires assignment of appropriate fields to the geometric objects of the wavefunction. The simplest possibility is to work with electromagnetic fields only. We know they are quantized - electric charge quantum, magnetic flux quantum, Bohr magneton,... all fundamental constants. To assign topologically appropriate fields to vacuum wavefunction elements of figure 4 requires five fundamental constants - electric charge quantum, electric permittivity of free space, speed of light, Planck's constant, and electron mass to set the scale of space.

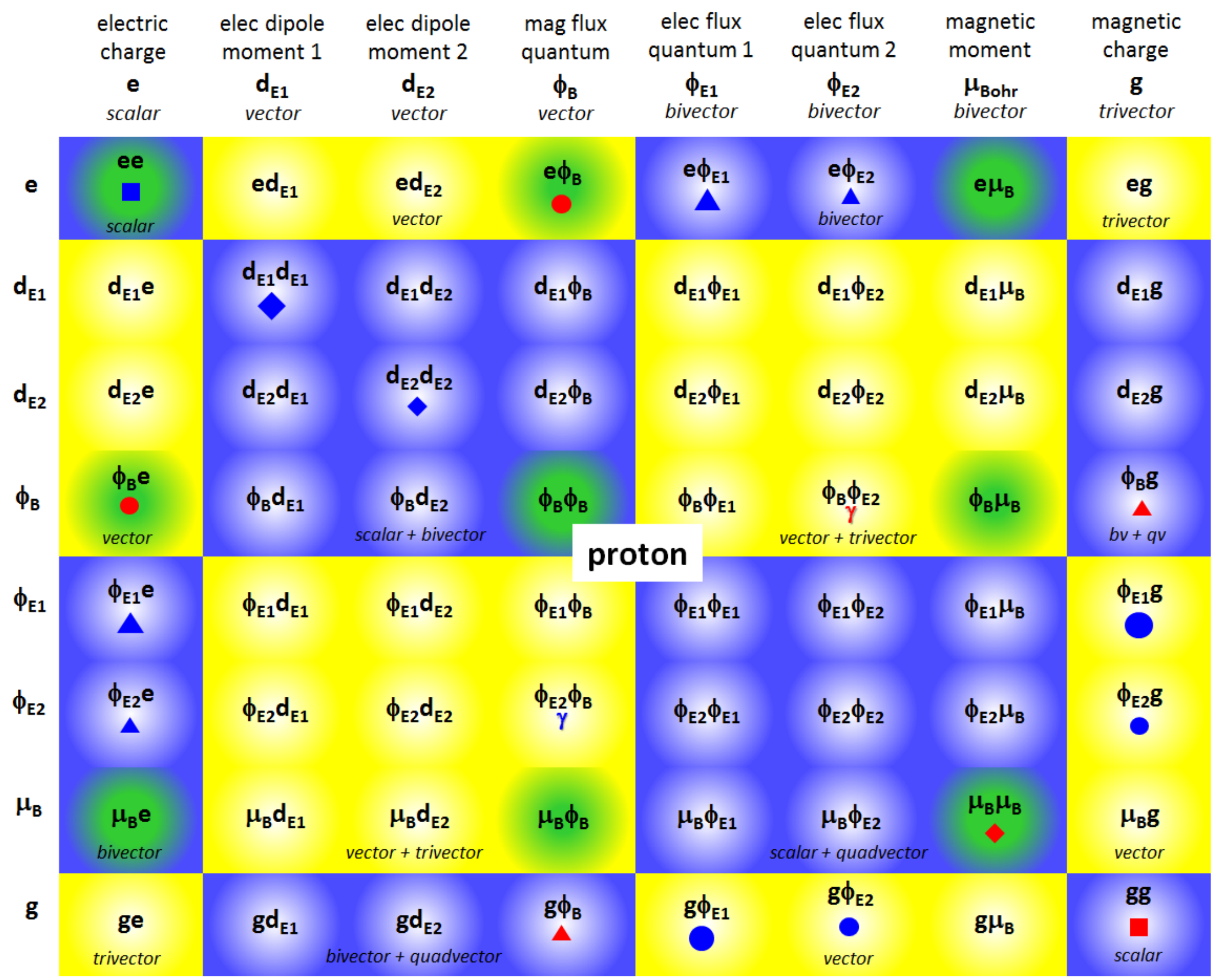

FIG. 4. **Dirac matrix - Impedance Representation of the S-matrix:** As in the Dirac equation, the eight-component wavefunction at figure top can be associated with the electron, and at left with the positron. Their interaction, as described by the geometric product, generates the 4D Dirac algebra of flat Minkowski spacetime, arranged in odd transition modes (yellow) and even eigenmodes (blue) by geometric grade/dimension. Modes indicated by symbols (triangle, square, dot, diamond) have their impedances plotted in figure 8, opening new windows on the unstable particle spectrum. Transition and eigenmodes of the proton are highlighted in green[19, 20].

Impedance may be defined as that which governs the amplitude and phase of the flow of energy. In a model that defines the boundary between fundamental and emergent to be wavefunction interactions, one would like to understand what governs the flow of energy in such interactions. Given that the model is electromagnetic, we know how to calculate electromagnetic impedances, for instance in calculating the 377 ohm vacuum impedance from photon excitation of the virtual Dirac spinor vacuum wavefunction[21]. Extending the analysis to the full Dirac S-matrix is non-trivial, computationally intensive.

However, there is a simple and straightforward shortcut. One can calculate mechanical impedances from Mach's principle[22]. Analysis of the electromechanical oscillator then permits calculating electrical interaction impedances for any element of the matrix, at any length scale. The impedance network of the vacuum is that of the virtual electron, centered upon the lightest stable excitation[23].

Figure 5 shows a timeline of the impedance concept, with a two century coming-of-age that led to development of the electrical engineer's network analyzers, and importation of the S-matrix formalism from the physicists[6, 7].

Coincident with that was the final crystallization of QED with renormalization procedures that followed from the 1948 Shelter Island conference, and first inquiries into the possibility of impedance quantization.

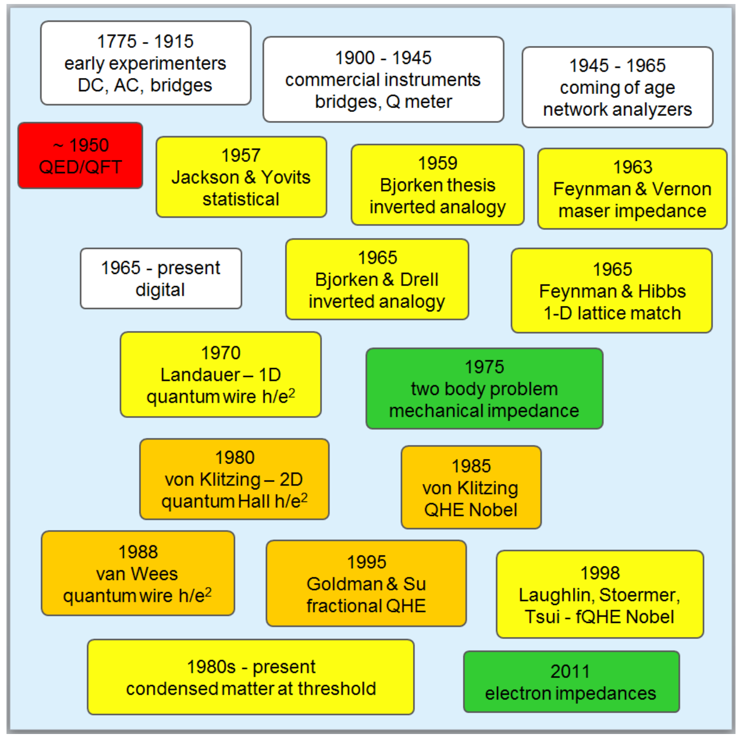

FIG. 5. Development of theory and technology of classical impedances highlighted in white, quantum impedances in yellow (theory) and gold (experiment), generalization of quantum impedances in green, and QED scaffolding in red.[24]

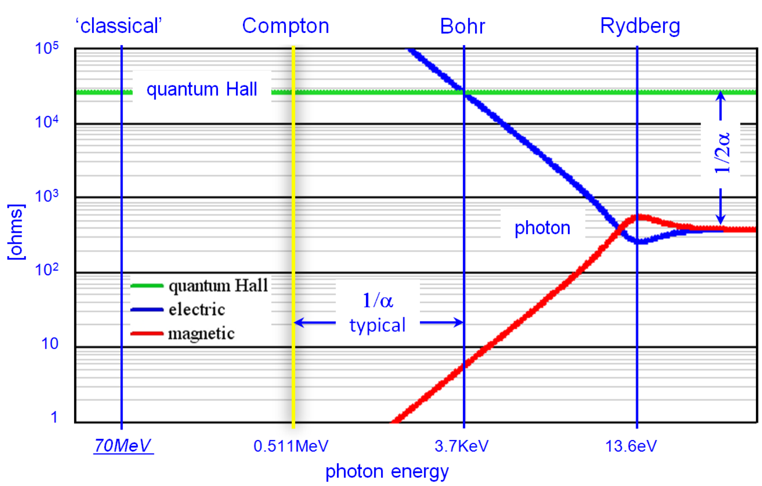

FIG. 6. Scale dependent near-field and invariant 377 ohm far-field impedances of a 13.6eV photon, showing near-field match to quantum Hall impedance at the Bohr radius[25].

The photon-electron interaction is the keystone of QED. What governs energy flow in photon-electron interactions is impedance matching in the transition region between far and near fields, shown in part in figure 6 for the hydrogen atom, and figure 8 for the single free electron. Why is this not common knowledge in physics?

Missing from formal education of physicists, and essential for reflectionless energy transfer in dissociation of atomic hydrogen, are both photon near-field impedance[25] shown in figure 6 and the corresponding electron dipole impedance[23]. Neither can found in grad school physics texts, the curriculum, or the journals. The same is true of the single free electron quantized impedance network shown in figure 8. What governs the flow of energy in photon-electron interactions is absent from mainstream physics.

The oversight can be attributed to **three primary causes**. The first is historical [26], the second follows from theorists' habit of setting fundamental constants to dimensionless unity, and the third from topological and electromagnetic paradoxes in our systems of units. [23, 27, 28].

First and perhaps foremost, foundations of QED (red in fig.5) were set long before Nobel prize discovery of the scale invariant quantum Hall impedance in 1980 [29]. Prior to that impedance quantization was more implied than explicit in the literature [30, 31, 36–41]. The concept of impedance quantization did not exist, much less *exact* quantization.

The **second** origin of overlooked quantization is setting fundamental constants to dimensionless unity. Doing so with free space impedance made quantization just a little too easy to overlook. And to no useful purpose. What matters are not absolute values but relative, whether impedances are *matched*.

The **third** confusion is seen in an approach [36] summarized [37] as "...an analogy between Feynman diagrams and electrical circuits, with Feynman parameters playing the role of resistance, external momenta as current sources, and coordinate differences as voltage drops. Some of that found its way into section 18.4 of..." the canonical text [38]. As presented there, Feynman parameter units are [sec/kg], units not of resistance, but rather *conductance* [32].

It is not difficult to understand how we lost our way[22, 33–41]. The units of mechanical impedance are [kg/sec]. One would think more [kg/sec] would mean more mass flow. However, the physical reality is more [kg/sec] means more impedance and *less* mass flow. This is one of many interwoven mechanical, electromagnetic, and topological paradoxes [28] found in SI units, which ironically were developed with the intent that they "...would facilitate relating the standard units of mechanics to electromagnetism." [42].

With the confusion that resulted from misinterpreting conductance as impedance and lacking the concept of quantized impedance, the anticipated intuitive advantage [38] of the circuit analogy was lost. Possibility of the jump from well-considered analogy to a quantized photon-electron impedance model was not realized at that time.

Historical Perspective on Geometric Wavefunctions and their Interactions

The geometric wavefunction has been to some extent explored over the years. Early attempts were incomplete, ahead of their time[43, 44]. Seeds of the modern geometric wavefunction were sown by the Pauli and Dirac algebras of Hestenes' short and lucid seminal text[10]. Ensuing geometric approaches have gone astray, one reason being attempts to accomodate two or more fundamental forces[44–47]. The mix of multiple fields and geometric objects remains intractable today. However, proceeding on the assumption of an electromagnetic conceptual synthesis requires nothing more than endowing geometric elements of the Pauli vacuum wavefunction with quantized EM fields.

Parallel evolutions of gauge theory into the Standard Model, S-matrix theory into String Theory, and the synthesis of geometric algebra and impedance quantization into the Geometric Wavefunction Interaction model are shown in figure 7. Lateral boundaries between threads are obviously not as clear as suggested there, with much mixing between threads. Arrows indicate portions of the threads whose experimental and theoretical place in the GWI model is at least partially understood and appears essential.

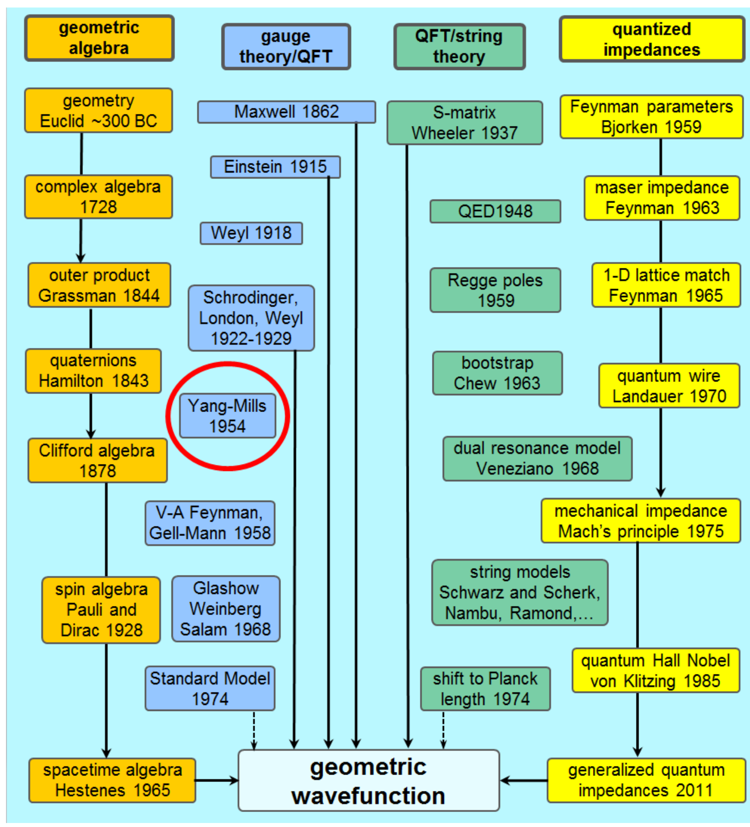

FIG. 7. Historical threads of the geometric wavefunction[48].

Arrangement of the threads in the figure is suggestive of their conceptual relationship. The synthesis of geometric algebra with impedance quantization encompasses the two threads of mainstream particle physics, embraces them, appears to contain them.

Just as geometric structure of the Dirac matrix of figure 4 provides a detailed example of geometric interactions of 3D Pauli vacuum wavefunctions, the impedance networks of figure 8 provides a detailed example of electromagnetic interactions of 3D Pauli electron wavefunctions.

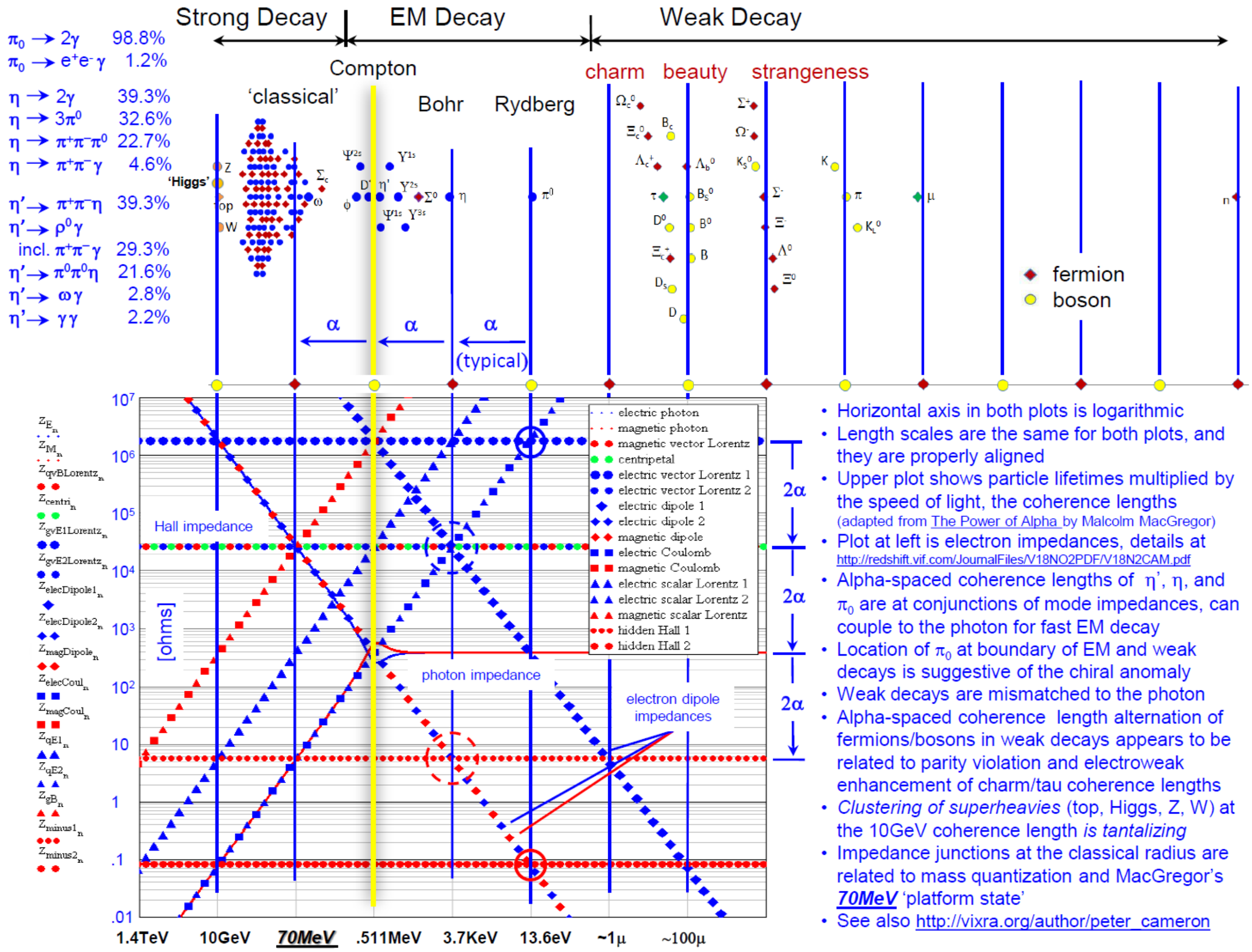

FIG. 8. Correlation of unstable particle lifetimes/coherence lengths with nodes of the electron impedance network comprised of a subset of the S-matrix shown in figure 4 [49], spanning thirteen powers of the electromagnetic fine structure constant α, from neutron to superheavies (top/Higgs/Z/W), with great precision and resolution (a few parts in 10^{27}). Corresponding modes are indicated by like symbols (triangle, square, dot, diamond) in the S-matrix. Strong correlation follows from the requirement that impedances be matched for energy flow between modes during decays.

At the top of figure 8 is yet another long-enduring perspective on the unstable particle spectrum that, in addition to geometric algebra and impedance quantization, has not found favor with the mainstream[50–52]. Unstable particle lifetimes/coherence lengths are clustered in powers of α. Strong correlation of network nodes with coherence lengths (boundary of the light cone) follows from the requirement that impedances be matched for energy flow between modes during decays.

That the GWI model is naturally finite begins to become clear from the figure. IR and UV divergences are cut off by the impedance mismatches as one moves away from the Compton wavelength. A consequence of the numerical values of the five fundamental constants input by hand, the mismatches are in powers of the fine structure constant and correspond to the renormalization coefficients of QED. In a similar manner, as suggested at the upper left of the figure precise π_0, η, and η' branching ratios can be calculated from impedance matches at the nodes indicated by solid and dashed circles in the figure[53].

A welcome corollary to natural finiteness is natural confinement, by reflections from impedance mismatches as one moves away from the quantization scale, the Compton wavelength.

Lifetimes of the superheavies (top, Higgs, Z, W) cluster at the impedance-matched intersection of the ∼10GeV coherence length of the dominant bottomonium decay modes with the 377 ohm photon far-field impedance and the magnetic Coulomb and scalar Lorentz mode impedances, suggesting they are comprised of magnetic resonances.

Summary and Conclusion

We presented a model in which the fundamental is that which cannot be observed, the wavefunction. Detailed examples illustrated how this model might be applied to the elementary particle spectrum, giving new perspective on the boundary between the fundamental and the emergent.

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