QUANTUM HALL EFFECT AND SUPERCONDUCTIVITY

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Quantum Hall Effect and, in particular, fractional quantum Hall Effect have forced theorists to rack their brains over an explanation of these phenomena. However for past decades since the effect was opened theory has remained in an unsatisfactory state.

Existing explanation of a quantum Hall Effect looks less convincing in connection with the fact that it is simultaneously accompanied by even more «strange and inexplicable» phenomenon - superconductivity.

However, the study of these key problems of physics is important not only from the point of view of adequate theory formation, but also as a way to deepen our knowledge about fundamental bases of substance structure.

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1. Preliminary comments

Let's remind that the quantum Hall Effect occurs in semiconductor structures with 2D electron gas (2DEG) at low temperature (about 1K) and high magnetic field (~*10 T*).

The planar structure for such experiments is usually produced on the basis of Si or As-Ga. Thus 2DEG is formed near the semiconductor surface under influence of electric field and actually cannot be perfectly thin, occupying finite thickness about *10* nanometers.

In these conditions the dependence of Hall resistance on magnetic field value gets the unusual step shape.

It is important, that in limits of every «plateau» of this dependence simultaneously arises superconductivity in a longitudinal direction [1] (fig. 1).

As is known, the «normal» Hall Effect is based on the fact that charged particles are deflected from velocity direction under cross magnetic field as a result of Lorentz force action. This drift of charges in transverse direction after a while stops, as the Lorentz force is counterpoised by an incipient cross voltage:

$$I = qn_s v_d (DS), \qquad F = q[v_d \times B] = q\frac{U}{D}. \qquad (1)$$

Here U - potential difference in transverse direction,
D, S - breadth and thickness of the sample cross-section,
q, n_s, v_d - charge, concentration and velocity of current carriers,

I, B - electrical current and magnetic induction values,

F - Lorentz force acting on current carriers.

From (1) follows, that the Hall resistance is directly proportional to magnetic field value and inversely proportional to the current carriers' concentration:

$$R = \frac{U}{I} = \frac{B}{n_s q S}. \qquad (2)$$

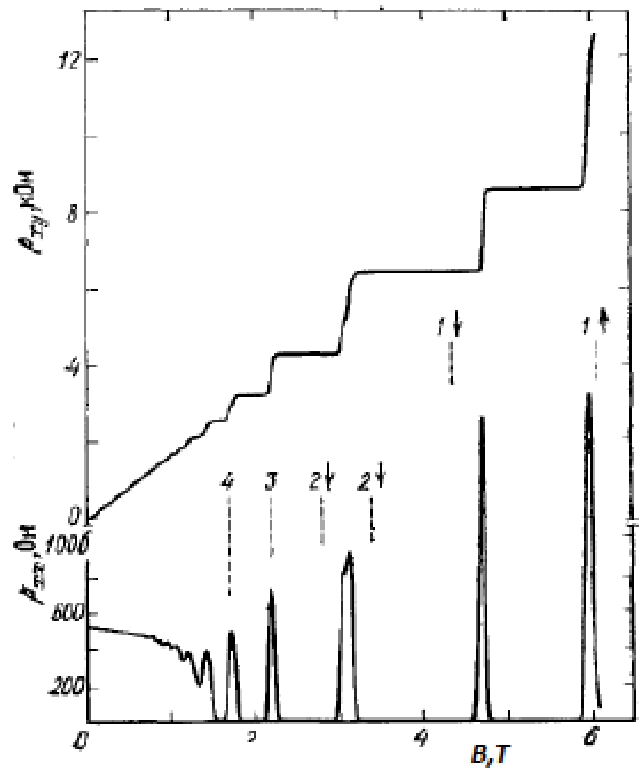

Fig.1. Hall resistance and longitudinal resistance dependences on magnetic induction (*T* = *8mK*) [1].

For a sample with unit thickness size (along a magnetic field) the expression (2) gets the simplest form [2]:

$$\rho_{xy} = \frac{B}{n_s q}. \qquad (3)$$

The quantum Hall Effect, as well as ordinary Hall Effect, is characterized by Hall resistance rising (but stepwise) at magnetic field increasing. The resistance in longitudinal direction in limits of every plateau, as it was mentioned, falls almost to zero (fig. 1).

The existing theory of superconductivity asserts that this macroscopic quantum-mechanical phenomenon is caused by so-called electron - phonon interaction. Figuratively speaking, electrons moving in a superconductor interact with crystal lattice vibrations; therefore the superconducting Cooper pairs are formed. Thus, electrons, being fermions, are transmuted into pairs - bosons and transit in one and the same state, forming Bose condensate of Cooper pairs.

Apparently, in this theory «boosting» action of magnetic field on superconductivity is not predicted. On the contrary, rather small magnetic field generally destroys superconductivity.

It is also known, that the existing theory is not capable to explain high-temperature superconductivity, which cannot be placed in the frameworks of the mechanism developed in the theory of the phenomenon.

As a whole, increasing formalism and shabbiness of the theory cause a sensation of dissatisfaction. The physics is plunged in abstractions and leaves without answer natural questions on physical phenomena essence.

2. Electron closed trajectories motion

In classical representation the electron motion under Lorentz force occurs on circular trajectories with cyclotron frequency in a plane, perpendicular to magnetic field.

In 2DEG conditions electron trajectories do not «overlap» each other, and electrons electrostatic charges do not cause mutual interference, as on the average all charges inside a crystal lattice appear to be neutralized.

Thus, the motion of 2DEG electrons in magnetic field can be represented as a system of circular trajectories on a plane (fig. 2).

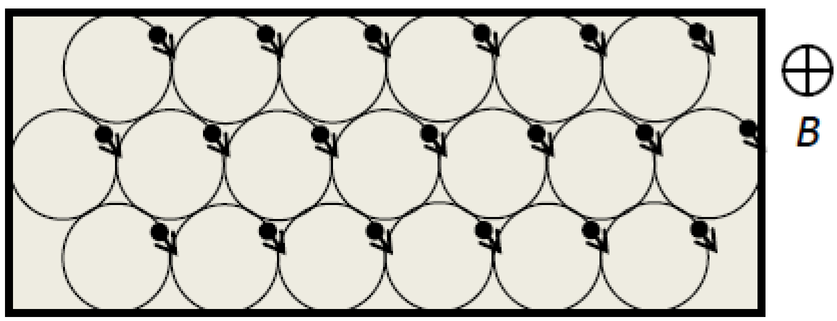

Fig. 2. Trajectories of 2DEG electrons moving in magnetic field.

However for the analysis of a quantum Hall Effect it is necessary to use quantum-mechanical representations, which bring substantial changes to the classical description.

So we shall engage in view of this question, all the more recently the concept of quantum-mechanical electron motion in substance has undergone cardinal changes. The point is that in papers [3 - 5] on the basis of repeatedly tested and well known experimental data the inconsistency of the existing atom theory was shown.

In particular, it was shown that many-electron shells of atoms are resonant formations, in which electrons settle down on equipotential spherical surfaces (forming a common resonance) and, thus, make collectives - Bose condensates.

Besides because of principal microcosm nonlinearity [6] electron shell resonances in atoms occur not only on basic frequencies, but also on harmonic components. As a result main quantum numbers in atoms can be fractional numbers. For example, hydrogen-like atoms with one outer shell electron in an unexcited basic state are characterized by fractional main quantum numbers (tab. 1) [4].

Табл. 1

Atom	3 (Li)	11 (Na)	19 (K)	55 (Cs)	37 (Rb)	47 (Ag)	78 (Pt)	81 (Tl)
$n \approx N_1/N_2$	8/5	5/3	7/4	15/8	9/5	4/3	5/4	3/2

It is essentially important, that the nearest to a nucleus electron shell (excepting hydrogen atom), consisting of two electrons, makes a Cooper electron pair. Thus electrons are located on two opposite sides apart of a kern in maximums of a «ring» resonance with main quantum number *n = 1* (fig. 3).

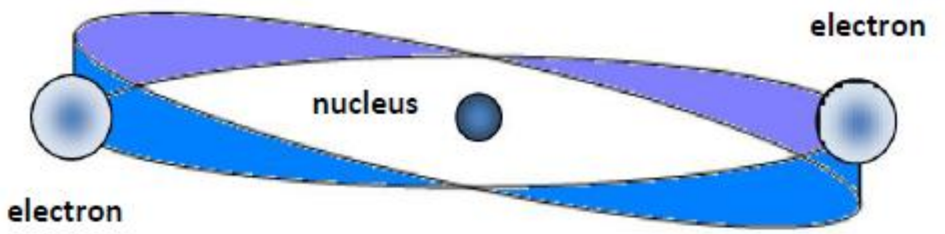

Fig. 3. Nearest to the atomic nucleus electron shell forms a Cooper pair.

Apparently, electrons «motion» in atoms resembles superconductivity phenomenon. The basic property of this state is the resonant interaction of fixed electrons, which actually only «simulate» motion by means of exchanging energy.

Therefore good superconductors, as a rule, are the substances with rigidly bound electrons. For example, there are high-temperature superconductors made of ceramics, which at usual temperatures do not conduct electric current. At the same time, the best conductors - copper, gold, silver never transit in a superconducting state.

It is also known, that distance between electrons in a Cooper electron pair has an order of de Broglie wavelength of an electron having velocity, corresponding to the energy of a Fermi level [2]:

$$\delta = \frac{hu_F}{kT_c} \sim \frac{h}{p} = \lambda. \tag{4}$$

Here δ - distance between electrons in Cooper pair,
u_F, p - electron velocity and impulse corresponding to Fermi level,
k - Boltzmann constant,
$h=2\pi\hbar$ - Planck's constant,
T_c - critical temperature of transition in a superconducting state,
λ - de Broglie wavelength.

As electrons have wave property, quite expected is the fact, that closed trajectories of moving electrons have features of resonances. This property brightly shows itself not only in electron shells of atoms, but also in superconductors.

The vivid example of such resonances is superconductivity in a ring superconductor. An integer number of wavelengths of a wave function ($\sim A \cdot e^{i\theta}$) go in a ring length, and quantization of magnetic flux through superconducting ring is also simultaneously observed, [7]:

$$\Phi = n\Phi_0, \qquad \Phi_0 = \frac{h}{2e}. \tag{5}$$

Similar resonances arise when electron moves along a ring trajectory in a plane, perpendicular to the exterior magnetic field (fig. 2).

Really, the circumference of electron trajectory is multiple to de Broglie wavelength:

$$2\pi r = n\lambda = n\frac{h}{p}. \tag{6}$$

On the other hand, equating the Lorentz force to centrifugal force we obtain formulae for mechanical impulse and cyclotron frequency:

$$\frac{mv^2}{r} = evB, \qquad p = mv = eBr, \qquad \omega = \frac{v}{r} = \frac{eB}{m}. \tag{7}$$

Substituting expression for mechanical impulse (7) in formula (6), we get magnetic flux value covered with electron trajectory:

$$r^2 = n\frac{h}{2\pi eB}, \qquad \Phi = \pi r^2 B = n\frac{h}{2e} = n\Phi_0. \tag{8}$$

As expected, it coincides with (5).

The resonances on closed electron trajectories, apparently, have even numbers of wave antinodes that cause Cooper pairs' formation.

Abrupt change of electrons properties in semiconductors under magnetic field was known long before quantum Hall Effect opening. So, in L.S. Stilbans's book «Physics of semiconductors», issued in 1967 by publishing house «The Soviet radio» [8], it is mentioned:

« … if we shall gradually increase magnetic field (and, hence, ω) so that through a Fermi level there will transit consistently crowdings and exhaustions of states density, then all properties of electron gas, caused by states density on Fermi surface, will oscillate. And all other kinetic coefficients will oscillate too.

Many of these phenomena were named after scientists discovered it. So, magnetizability oscillations were named as de Haas-van Alphen effect, conductance oscillations – Shubnikov-De Haas effect.

The theory for such so-called «quantizing fields» should also be considerably reconsidered, as the Boltzmann kinetic equation in this case is inapplicable. Really, the kinetic equation (more exactly, its left side) means continuous particles drift in phase space, but it is impossible when energy levels are discrete and trajectories in ordinary space are restricted by circles.

Further in footnote the author explains: « Actually discreteness of energy is a consequence of trajectory finiteness. The stability requirement for any self-sustained orbit demands, that on its length the integer number of wavelength should be placed: $2\pi r = n\lambda$, so discrete values of momentum and energy from here arise».

Following this logic, let us determine energy states spectrum of an electron in 2DEG under magnetic field. The kinetic energy subject to (7) and (8) makes:

$$W_k = \frac{p^2}{2m} = n\frac{eB\hbar}{2m} = \frac{1}{2}n\hbar\omega. \qquad (9)$$

Simultaneously electron moving on trajectory creates magnetic moment and consequently has the second energy component - potential energy.

If to assume, that all antinodes of a resonance on a circular orbit are occupied by electrons (total charge $q = 2ne$), then the magnetic moment, produced by them, subject to (7) will be multiple of Bohr magneton

$$P_n = I\cdot\pi r^2 = 2ne\frac{v}{2\pi r}\cdot\pi\frac{n\hbar}{eB} = ne\frac{eBr}{\pi rm}\cdot\pi\frac{n\hbar}{eB} = 2n^2\frac{e\hbar}{2m} = 2n^2P_B. \qquad (10)$$

Accordingly energy of this moment under magnetic field is:

$$W_n = P_nB = 2n^2\frac{eB\hbar}{2m} = n^2\hbar\omega. \qquad (11)$$

The potential energy (without energy of intrinsic magnetic moment of an electron) per one electron is equal to its kinetic energy (C (21))

$$W_p = \frac{W_n}{2n} = \frac{1}{2}n\hbar\omega. \qquad (12)$$

These results are confirmed by measurement data of activation energy [1] (fig. 4).

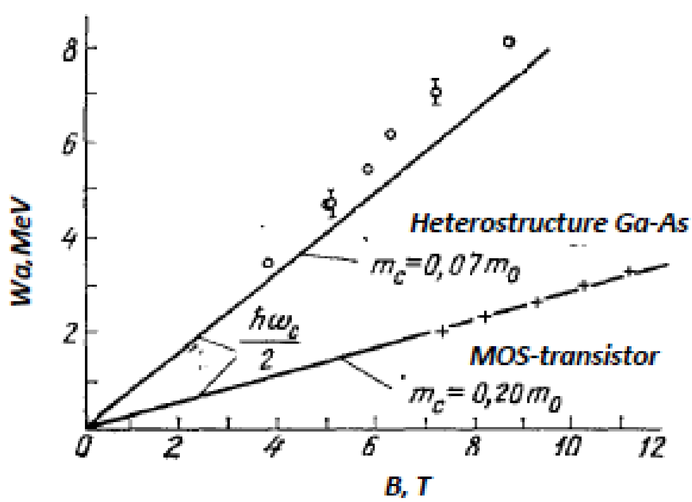

Fig. 4. Dependence of activation energy on a magnetic field for the factors of filling *i=2* (heterostructure on the basis of Ga-As) and *i=4* (silicon MOS-transistor). Experimental data are compared with $0{,}5\hbar\omega_c$ [1].

Apparently, the energy distribution of electrons will be quantized so long as a medial thermal energy will be less, than distance between discrete energy levels $\hbar\omega_c$, determined by cyclotron frequency. It determines the temperature requirement:

$$kT << \hbar\omega_c. \qquad (13)$$

At a reverse inequation the thermal motion spreads energy levels, and the electron spectrum becomes the same, as it was at the absence of external magnetic field.

3. «Superconducting» and ordinary current carriers

It is safe to say, that the macroscopic quantum-mechanical effects «demonstrate» microcosm processes. And especially visually wave and resonance microcosm properties are shown

The level-by-level electrons Bose condensation in atomic electron shells represents «a tiny model» of superconductivity. These resonant structures «help» to understand the superconductivity essence.

Turning back to the analysis of quantum Hall Effect, we shall note, that resonances antinodes occupancy, apparently, depends on current carriers' concentration. Accordingly fig. 2 could be revised and show not only one electron on each circular orbit but some electrons (the maximum number depends on antinodes quantity). The more electrons are already on a closed orbit, the more intensive is resonance, so the greater is a force pulling the subsequent electron in not yet filled antinode.

Similarly the electron shell vacancy pulls in a free electron more actively the greater is the number of electrons being already in the shell.

This apparently reasonable principle corresponds with the formalized quantum-mechanical representation of Bose condensate properties.

The resonant nature of superconductivity appears also in the quantum Hall Effect.

In ordinary superconductor electrons interact with periodic structure of a crystal lattice, but in quantum Hall Effect the magnetic field itself creates «periodic structure», in which resonance becomes possible (fig. 5).

Thus, «superconducting» electrons differ from «ordinary» electrons only by the matter that they are involved in common resonance - are in antinodes of a resonance and have the corresponding energy (wave length).

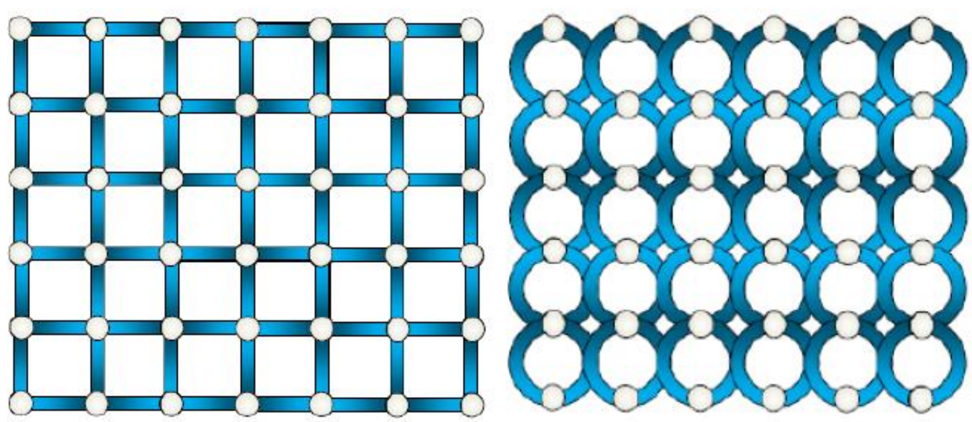

Fig. 5. Resonant structures of superconductivity in crystalline substance (at the left) and in «periodic structure» formed by magnetic field in experiments on a quantum Hall Effect.

Let's assume that all antinodes are filled with electrons, and thus the electric charge on each circular trajectory is equal *2ne*.

The number of circular orbits N per unit area of sample is determined by formula (8) that is by the total of magnetic flux quanta piercing through one orbit

$$N = \frac{B}{nh/2e} = \frac{2eB}{nh}. \tag{14}$$

Hence, surface charge density (on N circular orbits)

$$\sigma = 2ne \cdot N = \frac{4e^2 B}{h}. \tag{15}$$

Further, similarly to the derivation of the formula for ordinary Hall resistance, we shall write down expressions for electric current and cross voltage, that is, Hall voltage (1):

$$I = \sigma v_d D, \qquad q[v_d \times B] = q\frac{U}{D}. \tag{16}$$

Finally we are sure, that the quantum Hall resistance does not depend on sample sizes and considering (15) equals

$$R = \frac{U}{I} = \frac{h}{4e^2}. \tag{17}$$

This magnitude can be considered as a «basic» one because it meets the situation, when the quantity of current carriers is sufficient for filling all antinodes of resonances on circular orbits.

The point is that along with this basic situation the quantity of carriers can be insufficient for filling all resonance antinodes. At such carriers' deficit are observed diversions of Hall resistance from the formula (17).

The carriers' deficit can be aggravated by occurrence of resonances on harmonic components, just as it occurs in electron shells of atoms (tab. 1).

On the other hand, at greater magnetic fields the circular electron orbit is so small, that the de Broglie wavelength of an electron does not find enough «room» in it.

Under such conditions arises so-called fractional quantum Hall Effect. In this case superconducting electron pair disposes more than one circle orbit, that is perceived as fractional occupation number i (*1/3, 2/5, 3/7* etc.). Generally (17) looks like:

$$R = \frac{U}{I} = \frac{h}{ie^2}. \tag{18}$$

What role act electrons which are not involved in resonance?

Others «not superconducting» carriers remain practically imperceptible against the background of «superconducting» electrons, as the electron Bose condensate provides the most effective of all known electric conductors - superconductor.

4. What experiments say about?

Most evidently resonant nature of superconductivity is revealed by the experimental dependence shown in fig. 6 [1]. Voltage change in the centre of a sample (in comparison with total Hall voltage) under magnetic field here is given.

Singularity and nontriviality of processes underlying this experimental regularity deserve the most attentive analysis.

What causes repetitive voltage oscillations in the centre of a sample?

First, from formulas (4, 6, 7) follows, that radius of an electron orbit (and the number of wavelengths going into orbit) multiplied by magnetic induction make some magnitude, which, in turn, depends on momentum p_F (Fermi level W_F) and Bohr magneton P_B

$$eBr = p_F, \quad B(2\pi r) = B(n\lambda) = \frac{2\pi p_F}{e}, \qquad B \cdot n = W_F \frac{2m}{e\hbar} = \frac{W_F}{P_B}. \qquad (19)$$

The experimental dependence shown in fig. 6 completely confirms formula (19): voltage maximums in the centre of a sample correspond to integer numbers n. The biggest maximum on the right meets $n = 1$, at the centre there is resonance $n = 2$, and to the left we see maximums with numbers successively increasing one after another.

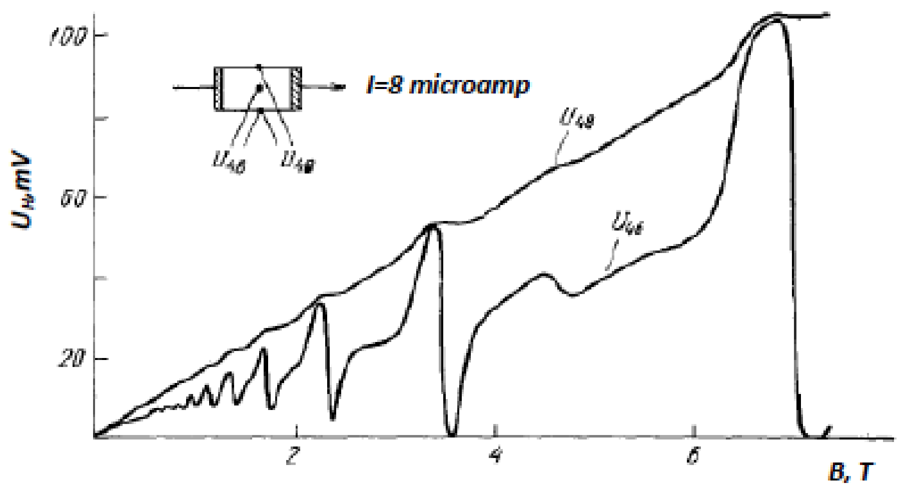

Fig. 6. Dependence of the measured potential distribution in Ga-As heterostructure on magnetic field [1].

The second feature of this plot is a very wide voltage swing in sample centre. On the one hand it testifies that the resonances per se are intensive and sharp. But on the other hand, it seems rather strange, that while total Hall voltage is a monotonic function, the voltage in sample centre is nonmonotonic: now it comes nearer to zero, now reaches the maximal value near to the total voltage.

The explanation for this effect can also be found using formula (19). It is necessary to bear in mind, that Fermi level changes under electric potential.

The matter is that Hall voltage provides potential gradient along sample breadth and thus affects Fermi level. So, the right member of equation (19) depends on actual potential magnitude in a concrete place. That is why resonance conditions vary along sample breadth, and the superconducting channel (resembling a resonant strip) occupies not all breadth, but only its part (fig. 7).

Increasing magnetic field magnitude causes augmentation of the left side in equation (19) (for each fixed resonance number). Accordingly area of resonance will move in direction, where the right side of the equation also arises (where Fermi level is greater).

Thus, under rising magnetic field the superconducting «strip» moves crosswise the sample to the part where potential is greater (in figure is indicated by dotted lines and shading) and then, at the moment of crossing the middle of a sample causes sharp transformation of characteristics measured in sample center.

Really, when there is no resonance (superconductivity), the voltage at sample centre is approximately equal to the half of total Hall voltage and changes accordingly (fig. 6). When there is a resonance, and the superconducting area (below in fig. 7) «involves» all electric current, the voltage in the middle of a sample becomes almost equal to a total Hall voltage (fig. 6). At further magnetic field increase the superconducting area passes the middle of a sample (above in fig. 7), and the voltage in sample centre sharply decreases, coming near to zero (fig. 6).

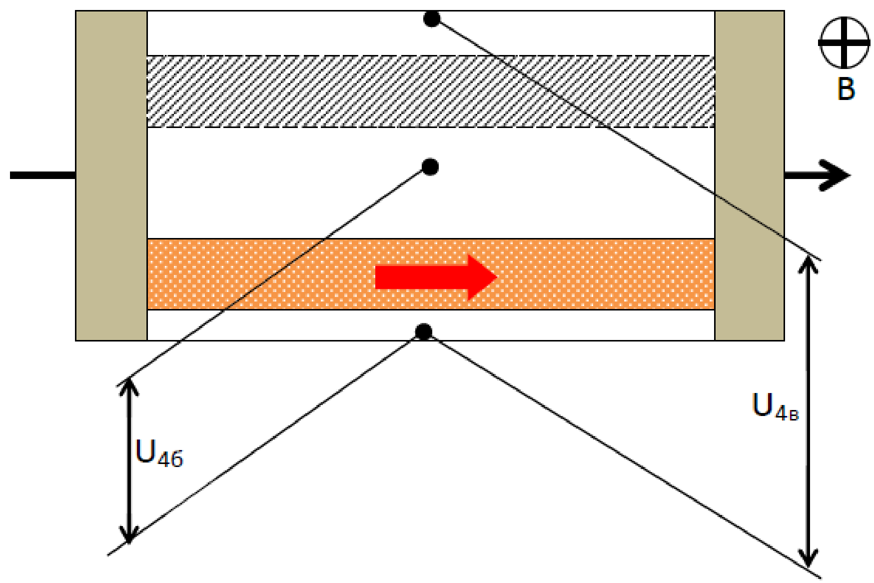

Fig. 7. The superconducting area drifts along the sample breadth under increasing magnetic field.

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But a real «surprise» is a discovery of one more aspect in a fig. 6, which the theorists prefer «not to note». Therefore fig. 6 is reproduced once more and is supplied with the necessary explanations (fig. 8).

Rate of curves for total Hall voltage and voltage in the middle of a sample definitely indicates the presence of one more resonance with $n = 3/2$ between $n = 1$ and $n = 2$.

This resonance, though not brightly expressed, has a common nature with quantum states of atoms given in tab. 1 and specified by fractional main quantum numbers.

In both cases there are nonlinear effects - resonances on harmonic components.

It is necessary to note, that the nature of fractional quantum Hall Effect with $n < 1$ ($1/3$, $2/5$, $3/7$ etc.) is entirely other and is caused by the fact that under greater magnetic field de Broglie wavelength «does not find enough room» in a circular orbit of an electron.

In fig. 9 the results of one more experiment are represented which show the dependence of basic characteristics of the quantum Hall Effect on gate voltage [1].

Preparatory to analyze these results, let us remind, that the change of a Fermi level under the influence of electric potential is also observed in other experiments. For example, at contact of two superconductors under different potentials arises Fermi levels difference which results in Josephson Effect [7].

It is considered, that the gate adjusts carriers concentration and thickness of 2DEG in near-surface layer of semiconductor. However gate voltage «adjusts» also Fermi level.

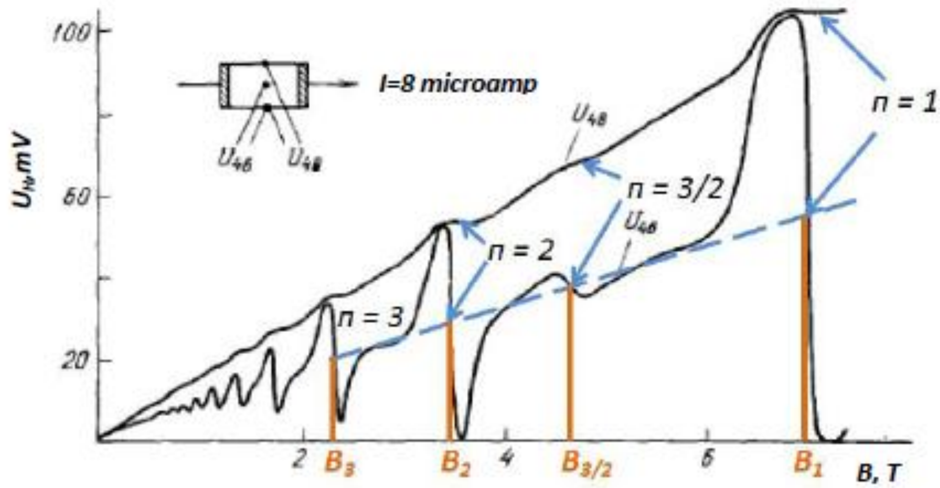

Fig. 8. The resonance *n = 3/2* is similar to resonances on harmonic components in atomic electron shells (tab. 1).

Therefore, similarly to the above-mentioned description of processes (fig. 6, 7), the resonance area (superconductivity) also can move along the sample breadth, neutralizing Fermi level change when gate voltage is adjusted.

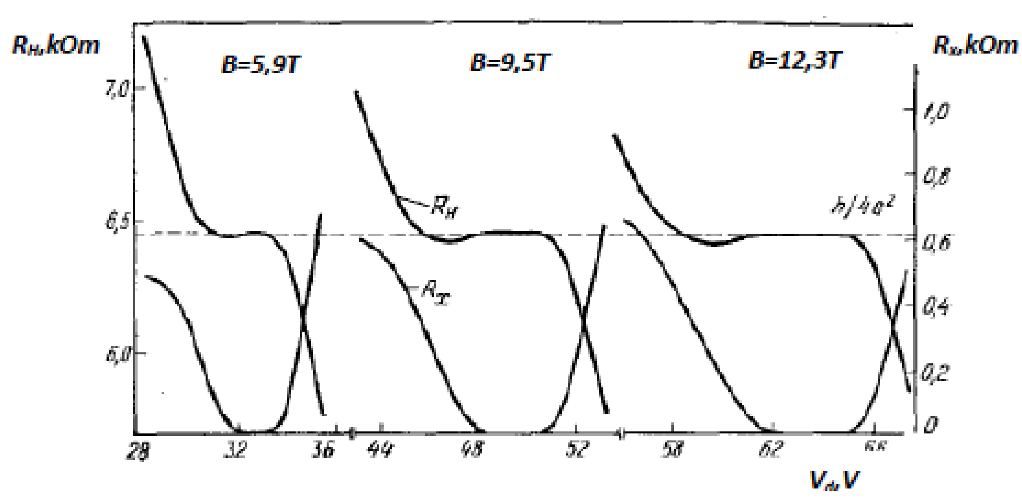

Fig. 9. Hall resistance and longitudinal resistance dependences on gate voltage at different magnetic field values [1].

Greater magnetic field values (fig. 9) correspond with greater gate voltages (Fermi level change provides equation (19)). «Plateaus» in fig. 9 are caused by the resonant superconducting area drift along the sample breadth (as in fig. 7). Thus there is mutual «compensation» of Fermi level changes: on the one hand at the expense of gate voltage change and on the other hand at the expense of superconducting area drift along sample breadth (Hall voltage change).

So, resonances per se, providing superconductivity, are very sharp, and «steps» observed in experiments are explained by resonance area drift along the sample breadth. This superconducting area glide is determined by Hall potential gradient.

5. «Fragile» physical phenomenon

Passing to more detailed discussion of quantum Hall Effect, it is necessary to note, that it may be called as a «fragile» physical phenomenon, because the effect takes place only when a series of rigid conditions are observed.

In addition to low temperature condition maintaining energy levels discreteness (created by «quantizing» magnetic field) there are high requirements to the semiconductor sample properties.

These requirements obviously proceed from the phenomenon nature. In particular, K. von Klitzing noted [1]:

«The Hall resistance will be always quantized, if carriers' density and magnetic field are combined in such a manner that energy levels occupation number

$$i = \frac{n_s}{eB/h}$$

is integer».

Further, there must be more regard for key measuring modes, as voltages and electric currents garble an «ideal» picture of the phenomenon:

« If the Hall field becomes more than approximately $E_H = 60\ V/sm$ under magnetic fields about $5\ T$, so the quantum Hall Effect disappears.

This field value corresponds to classical drift velocity

$v_D = E_H/B \approx 1200\ m/s$.

The restriction on a Hall field, as well as restriction on temperature, is determined by necessary condition of energy levels discreteness.

Really, the size of electron circle orbit in magnetic field, according to formula (8) is about *10 nm*, so the potential difference in this distance at electric field ≈ *60 V/sm* is nearer to ~ $10^{-4}\ V$, that is, energy level spacing ~ $10^{-3}\ eV$ (fig. 4) is subjected to noticeable tailing. Simultaneously there is occupation number i decline.

The next important requirement for Hall Effect observation is the correspondence between a Fermi level of the semiconductor and quantized energy levels of electrons under magnetic field. So, the matter is that such order of a Fermi level in semiconductors can be obtained at the expense of impurities.

Thus, there should be a correspondence between a Fermi level determining a dimensional period of electron waves (de Broglie waves) and a magnetic field force determining (alongside with a Fermi level) a circumference of an electron trajectory, which should be equal to the whole number of electron wavelengths.

It is necessary also to take into account that the gate voltage effect on Fermi level is realized through the change of induced spatial carrier density. Therefore gate voltage change of about tens volt causes Fermi level change of only some unities of millivolt. It is important to take in consideration at the analysis of experimental data (fig. 9).

Let us once again draw attention to the physical mechanism of «plateau» formation in a dependence of Hall resistance on the gate voltage. The fact is that a Fermi level simultaneously depends on gate voltage and on Hall voltage. Thus the compensation occurs «within» Fermi level at the expense of «concerted change» of gate potential and Hall voltage (while drifting along sample breadth), so that the Fermi level is constant:

$$dW_F = \frac{\partial W_F}{\partial U_D} dU_D + \frac{\partial W_F}{\partial U_H} dU_H = 0. \quad (20)$$

The superconductivity area drift along the sample breadth is determined by approximate equality (cross point, fig. 10) of a Fermi level and corresponding discrete electrons energy level W_n, as it would be expected from the equation (19)

$$W_n = P_B B \cdot n = W_F \, . \quad (21)$$

Gate voltage change within «plateau» limits of Hall resistance has practically no effect on electron trajectories because Fermi level and magnetic field remain invariable.

As opposed to this, in limits of every «step» of Hall resistance dependence on magnetic field strength, electron trajectories experience changes. As the magnetic field increases, radius of electron orbits decreases. Accordingly superconducting area drifts along the sample breadth in the direction of greater values of Hall potential and of the Fermi level. Thus de Broglie wavelength of electrons decreases and the same number of wavelengths will fit in a circle orbit of smaller radius.

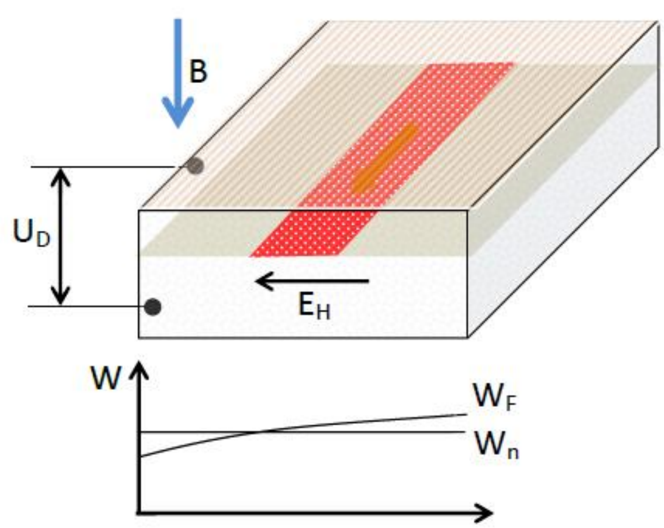

Fig. 10. The gate voltage as well as Hall voltage effect the Fermi level of electrons in 2DEG.

Hence, within «plateau» limits electron trajectories and de Broglie wavelength experience equivalent changes, following simultaneous changes of magnetic field and Fermi level (in fig. 11 such trajectory variation for $n = 3$ is shown).

Gradual «correction» of the resonance configuration without its breaking (or key modification) makes «plateau», in which limits characteristics remain constant.

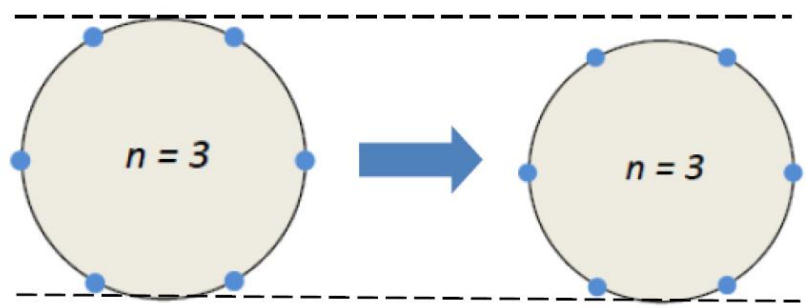

Fig. 11. When the magnetic field increases within «plateau» limits, trajectory radius and de Broglie wavelength of electrons experience «equivalent» changes.

Let's also note that the gate voltage change causes carriers density deviation (besides the above mentioned processes), however it actually has not an effect on the Hall resistance in «plateau» limits according to the experimental data plotted in fig. 9.

It is just that all resonance antinodes are already occupied (about it indicates the Hall resistance value $h/4e^2$ determined by formula (17)), so that the quantity of «ordinary» electrons does not really influence the sample conductance in comparison with contribution of «superconducting» electrons of a resonant Bose condensate.

How electron motion inside semiconductor sample differs from the electron motion in free space?

Electrons in the interior of a sample experience essential restrictions. Besides collisions deforming electrons trajectories, the interaction with surrounding particles of crystal periodic structure causes electron mass «change» which is perceived as some «effective mass».

Moreover, the electrons have utterly different properties depending on whether they are included in a general resonance, or are ordinary «free» particles. The same electron can become «superconducting» or «ordinary» depending on ambient conditions (for example, under external magnetic field).

In a superconducting state, that is, in a state of a general resonance, electrons get surprising properties characteristic for a Bose condensate. Electrons are «grasped» by resonance antinodes and become fixed, so the electrons motion is «simulated» by the most effective transportation agent - by means of electrons interaction. The resonance existing at a level of each circle orbit is extended to all periodic structure consisting of contiguous electron trajectories. Such resonant structure possesses the «teleportation» property, that is, the superconductivity property.

Summary

This article, as well as recent publications devoted to an atom structure [3-5], convincingly shows that the experimental data, accumulated in last decades, call for most careful supplementary analysis.

Relying on experimental data, given in K. von Klitzing «Nobel lecture», the «improper-fractional quantum Hall effect» ($n = 3/2 > 1$) could be discovered, as contrasted to the «proper-fractional quantum Hall effect» characterized by proper fractions (*1/3, 2/5, 3/7…*, that is, $n < 1$).

This «new» effect has other nature and is similar to electron states in atom with fractional main quantum number arising as a result of nonlinear interactions.

As of the existing theory of quantum Hall Effect, there are typical errors of a contemporary physics. Initially analysis of 2DEG properties under magnetic field seems to be logical, but further, more and more there is a feeling of formalism domination.

In particular, the circular movement of electrons is substituted by oscillations in two dimensions; therefore the solution of Schrodinger equation gives the wave function as Hermitian polynomial and relevant discrete energy levels etc.

But in this formalism the physics is lost, and the analysis goes in a false way.

Actually, the electron gas in these experiments, and thereupon the wave functions undergo more rigid physical restrictions.

At first three-dimensional electron gas (3DEG) in the semiconductor-insulator contact is transformed into two-dimensional electron gas (2DEG). But then, as a result of action of strong magnetic field and low temperature, the electron gas actually becomes one-dimensional (1DEG).

Really, the motion of electrons in these conditions is restricted only by linear displacements along the circles periphery.

And what occurs to wave functions of electrons?

They should correspond to substantial physical restrictions.

Therefore electrons, making linear translations along the circles periphery, are characterized by linear parameter - de Broglie wavelength. Accordingly when in perimeter of circle the whole number of wavelengths is fit, the resonance arises.

What occurs in case of a resonance?

The electrons become fixed in antinodes of a resonance (symbolically speaking, in maximums of a real part of a wave function) and form a Bose condensate, that is, transit in a superconducting state.

Speaking about superconductivity nature, let us imagine a visual illustration.

Imagine that some workers carry bricks from one place to another, and thus each worker makes it independently, that is, takes bricks in one place and carries to other place. Such «transportation method» provides a chaotic motion of workers, which quite often hinder each other in their work and even collide.

But now the workers line up and form «strings», which are stretched from a place, where it is necessary to take bricks, up to that place, where they are stacked. Now workers do not go, but interactively and rhythmically transfer bricks down the «chains».

Precisely so in a superconducting state at low temperature Cooper pairs are transferred, when electrons do not move and interacting form the most effective «method of transportation». Such general resonance state is associated with ordered spatial structure of electrons.

In quantum Hall Effect basis lays the superconductivity. It is remarkable, that the resonant periodic structure for Bose condensate formation is created «artificially» (by means of magnetic field) in a form of a set of contiguous circular electrons trajectories.

The superconductivity is observed in limits of every «plateau», when the number of wavelengths *n*, fitting in electron circle trajectories at their «motion» under magnetic field, is integer or common fraction (in case of a fractional quantum Hall effect):

$$W_n = P_B B \cdot n = W_F \, .$$

Fractional numbers *n* turn up, as it was already mentioned, in two cases.

First, it occurs under very strong magnetic field, when de Broglie wavelength appears to be greater than circumference of electrons trajectories in a plane, perpendicular to magnetic field.

Proper-fractional quantum Hall Effect in this case takes place, when one de Broglie wavelength is in keeping with more than one circle trajectory, and the number *n* becomes less than unity (*1/3, 2/5, 3/7* etc.).

Secondly, the fractional number *n* can occur when the resonance takes place on harmonic components as contrasted with resonances on «basic» harmonic (with de Broglie wave length).

Precisely such resonance, when *n* is an improper fraction greater than unity (*3/2*), is seen in the experimental curves of K. von Klitzing paper [1]. Other values *n* in the form of improper fractions, for example, *4/3* are now known. It is logical to term this effect, which was not emphasized by theorists, as an improper-fractional quantum Hall Effect.

Resonances on harmonic components (similar to resonances in external electron shells of hydrogen-like atoms, tab. 1) testify to microcosm nonlinearity. The discovery of resonances on harmonics in electron shells of atoms helps to understand the nature of similar phenomenon apparent at a quantum Hall Effect.

Most informative (in terms of physical processes) are the results of comparative measurements of total Hall voltage and the voltage in the middle of sample breadth subject to magnetic field value - fig. 6 (fig. 8).

First of all, this experiment completely confirms the above mentioned equation featuring a Hall Effect resonance nature.

These dependences also show that Hall voltage «plateaus» are caused by the drift of superconducting area along the sample breadth (it corresponds to descending parts of dependence of voltage in the sample middle on magnetic field).

As a result of superconducting area drift, the Fermi level (the right member of the equation) is «adjusted» by the Hall voltage and «follows» the varying discrete energy level.

Physically it means, that under magnetic field change (in the limits of «plateau») the circular electrons trajectories vary simultaneously with de Broglie wavelength, and consequently the resonance is maintained.

And at last, this experiment has enabled to discover the «improper-fractional quantum Hall effect» with *n = 3/2*.

In this case the resonance occurs on the second harmonic (three wavelengths of the second harmonic are fit in the electron trajectory circumference).

Hall voltage «plateaus» are also observed under the gate voltage change. In this case discrete energy level (left member of equation) does not vary, and the required Fermi level constancy (the right member of equation) is obtained by «coordinated» changes of the gate voltage and Hall voltage (also at the expense of superconducting area drift along the sample breadth).

Let's note furthermore, that in conditions of restricted carriers' density not all resonant antinodes can be filled, so it results in dependence of quantum Hall resistance on occupation number.

In conclusion it is necessary once more to stress, that quality of the theory, in which there is no understanding of phenomena essence, can be improved neither by quantity nor by complexity of mathematical build-ups.

Alongside with formalism domination, the most serious error of the modern theory consists in a priori considering of microcosm as a linear system, but in reality as a matter of fact microcosm is in essence nonlinear. Therefore substantial interactions, for example, in atom occur also on multiple harmonic components.

The authors of the atom quantum-mechanical theory, carried away by formalism, initiated these errors.

For example, they proceeded from the three-dimensional motion of each electron in atom (and accordingly 3D - wave functions).

Actually space around atom nucleus is «ideal» place for electrons Bose condensation, because the thermal motion energy (~ *0,03 eV*) is very small in comparison with depth of this potential well making unity and even hundreds electron-volt.

Recent research [3-5] has completely confirmed these reasons. It was shown that proximate to atom nucleus electron shell (except for hydrogen atom), consisting of two electrons, is a resonant Cooper pair. The electrons are located on the opposite sides from nucleus in antinodes of a ring resonance with a main quantum number equal to unity (one de Broglie wavelength fits in circumference).

It turned out that the many-electron atom shells have the spherical shape. That is, electrons settle down on an imaginary spherical surface, which actually is an equipotential surface; therefore the electrons are involved in a general resonance and form a Bose condensate.

It is safe to say that the common wave function of these electrons is two-dimensional (is specified on a spherical surface), as against to one-dimensional wave function of the first electron shell (figure-of-circle).

And, at last, resonances of outer shell electrons in hydrogen-like atoms occur on harmonic components, therefore its main quantum numbers in the ground state make improper fractions.

The Bose condensation of fixed electrons in atomic electron shells eliminates the known contradiction of the atom theory: if only electrons revolve around the atom nucleus, they should radiate energy.

These facts completely refute the existing atom theory, which regards electrons in atom shells as independent particles moving in some «averaged» field and described by quantum 3D-functions.

In atoms can be used only electron wave functions corresponding to electron Bose condensates: 1D-wave function (first and last shells consisting of 1 or 2 electrons) and 2D-wave function of spherical multi-electron shells.

In conditions of a quantum Hall Effect the resonances in closed circular trajectories in many respects are similar to resonances in atom electron shells.

The quantum Hall Effect has appeared a very «useful» discovery not only from the utilitarian point of view (for calibration of electrical resistances or fundamental constants values improvement), but also as one more fundamental indication of a resonant nature of the superconductivity phenomenon.

As a rule, most people do not doubt theories which are already «inserted» in textbooks. Unfortunately, in the list of such theories there is also the existing quantum-mechanical theory of atom, which was created in far times, when many experimental data contradicting to this theory, still missed. Just the same it is possible to say about the theory of a quantum Hall Effect.

Nevertheless, the gradual comprehension of the falsity of the existing atom theory goes on (especially in chemistry), that creates favorable conditions for reconsideration of out-of-date microcosm representations [9-14].

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