Does Heisenberg's Uncertainty Principle Predict a Maximum Velocity for Anything with Rest-Mass below the Speed of Light ?

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Abstract

In this paper, we derive a maximum velocity for anything with rest-mass from Heisenberg's uncertainty principle. The maximum velocity formula we get is in line with the maximum velocity formula suggested by Haug in a series of papers. This supports the assertion that Haug's maximum velocity formula is useful in considering the path forward in theoretical physics. In particular, it predicts that the Lorentz symmetry will break down at the Planck scale, and shows how and why this happens. Further, it shows that the maximum velocity for a Planck mass particle is zero. At first this may sound illogical, but it is a remarkable result that gives a new and important insight into this research domain. We also show that the common assumed speed limit of $v < c$ for anything with rest-mass is likely incompatible with the assumption of a minimum length equal to the Planck length. So one either has to eliminate the idea of the Planck length as something special, or one has to modify the speed limit of matter slightly to obtain the formula we get from Heisenberg's uncertainty principle.

Key words: Heisenberg's uncertainty principle, maximum velocity of matter, reduced Compton wavelength

1 Introduction

Haug [3–7] has recently suggested a new maximum velocity for subatomic particles (anything with mass) that is just below the speed of light. The formula is given by

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{1}$$

where $\bar{\lambda}$ is the reduced Compton wavelength of the particle we are trying to accelerate and l_p is the Planck length [8, 9]. This formula can be derived from special relativity by simply assuming that the maximum frequency one can have is the Planck frequency $\frac{c}{l_p}$, or that the shortest wavelength possible is the Planck length. We will also get the same formula if we assume that the ultimate fundamental particle has a spatial dimension equal to l_p and always is traveling at the speed of light; this is a model outlined by [1, 3].

This maximum velocity for anything with rest-mass was first predicted by Haug in 2014 and presented at the Royal Institution in London in October 2015, see [1, 2]. It was first derived from two postulates in atomism. The theory leads to the same mathematical end results as special relativity theory, as long as one uses Einstein-Poincaré synchronized clocks. However, at that time Haug had not yet linked his theory to some of the core concepts of Max Planck. Here the key understanding given in 2014 will lead to the same formula that is described above.[1]

In this paper, we will show that the same formula implicitly agrees with a new result that comes out of Heisenberg's uncertainty principle when combined with an essential insight from Max Planck.

2 Heisenberg's Uncertainty Principle Leads to a Maximum Velocity for Anything with Rest-Mass

The original Heisenberg's uncertainty principle formulation [11] is given by

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[1] Back then I only derived my maximum velocity formula with a link to one-sided relativistic Doppler shift, based on a slightly different clock synchronization procedure and therefore got slightly different results than those given here.

$$\sigma_x \sigma_p \geq \hbar \tag{2}$$

where σ_x is considered to be the uncertainty in the position, σ_p is the uncertainty in the momentum, and $\hbar$ is the reduced Planck constant. Alternatively, one could use the more modern Kennard [24] version of the Heisenberg's uncertainty principle formula as we have done in section 4. The difference is $\hbar$ or $\frac{\hbar}{2}$ in the formulation. Even if this results in a very small difference in maximum velocity, it still leads to exactly the same important main conclusions. The rest-mass of an elementary subatomic particle is given by

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{3}$$

For an electron, for example, we have

$$m = \frac{\hbar}{\bar{\lambda}_e} \frac{1}{c} \approx 9.10938 \times 10^{-31} \text{ kg} \tag{4}$$

This means the mass of an elementary particle can be found by measuring the reduced Compton wavelength of the particle, as has been done experimentally with electrons, see [**?**]. We will assume that the minimum uncertainty in the position of any elementary particle (or any other object) is the Planck length. Setting $\sigma_p = l_p$ and the momentum σ_p to the relativistic momentum we get

$$\begin{aligned}
\sigma_x \sigma_p &\geq \hbar \\
l_p \sigma_p &\geq \hbar \\
l_p \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \hbar \\
l_p \frac{\frac{\hbar}{\bar{\lambda}} \frac{1}{c} v}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \hbar \\
\frac{\frac{1}{c} v}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{l_p} \\
\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{l_p} c
\end{aligned} \tag{5}$$

Solved with respect to v this gives

$$\begin{aligned}
\frac{v^2}{1 - \frac{v^2}{c^2}} &\geq \frac{\bar{\lambda}^2}{l_p^2} c^2 \\
v^2 &\leq \frac{\bar{\lambda}^2}{l_p^2} c^2 \left(1 - \frac{v^2}{c^2}\right) \\
v^2 \left(1 + \frac{\bar{\lambda}^2}{l_p^2}\right) &\leq \frac{\bar{\lambda}^2}{l_p^2} c^2 \\
v^2 &\leq \frac{\frac{\bar{\lambda}^2}{l_p^2} c^2}{\left(1 + \frac{\bar{\lambda}^2}{l_p^2}\right)} \\
v &\leq \frac{c}{\sqrt{1 + \frac{l_p^2}{\bar{\lambda}^2}}}
\end{aligned} \tag{6}$$

We could also have done the derivation below without decomposing the mass in this way; this alternative, which gives exactly the same result, is shown in the Appendix A.

For example, for an electron we have $\bar{\lambda}_e \approx 3.861593 \times 10^{-13}$ m and the Planck length $l_p \approx 1.616229 \times 10^{-35}$ m (2014 NIST CODATA). We then see that the maximum velocity of an electron must be

$$v \leq c \frac{1}{\sqrt{1 + \frac{l_p^2}{\bar{\lambda}_e^2}}} \leq c \times 0.99999999999999999999999999999999999999999999124$$

This is basically identical to the maximum velocity we get from Haug's maximum velocity formula (that corresponds to the one we get from the energy time uncertainty principle)

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}_e^2}} \approx c \times 0.999999999999999999999999999999999999999999999124$$

This is far above velocities one can achieve in today's particle accelerators, but still falls inside Einstein's assumption of $v < c$.

For all observed particles we have $\bar{\lambda} >> l_p$ and then we can use the Taylor series expansion

$$\frac{v}{c} \leq \frac{1}{\sqrt{1 + \frac{l_p^2}{\bar{\lambda}^2}}} \approx 1 - \frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2} + \frac{3}{8}\frac{l_p^4}{\bar{\lambda}^4} - \frac{5}{16}\frac{l_p^6}{\bar{\lambda}^6} + \cdots \tag{7}$$

Further, we have the Taylor series expansion for

$$\frac{v_{max}}{c} = \sqrt{1 - \frac{l_p^2}{\bar{\lambda}_e^2}} \approx 1 - \frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2} + \frac{1}{8}\frac{l_p^4}{\bar{\lambda}^4} - \frac{1}{16}\frac{l_p^6}{\bar{\lambda}^6} + \cdots \tag{8}$$

In most cases, using only the first term of the Taylor series expansion will be more than accurate enough, and then we see that they are the same, which explains why we obtained the same numerical value for an electron. Still there is a structural difference between the two formulas that we will discuss more in the next section.

3 Maximum Velocity from the Heisenberg Energy Time Uncertainty Principle

In 1927, Heisenberg also introduced a corresponding uncertainty principle for energy and time

$$\sigma_E \sigma_t \geq \hbar \tag{9}$$

The energy time uncertainty has not been considered as profound as the momentum position uncertainty principle due to the Pauli objection [13]. However, the Pauli's objection has encountered several counterexamples, criticisms, and discussions; see, for example [14–23]. Haug has also suggested the maximum velocity formula to resolve the Pauli objection [10]. Let us assume that the Heisenberg energy time uncertainty principle holds.

If we also assume that the shortest possible time interval is the Planck time, then we can derive the maximum uncertainty by

$$\begin{aligned}
\sigma_E \sigma_t &\geq \hbar \\
\sigma_E \frac{l_p}{c} &\geq \hbar \\
\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{l_p}{c} &\geq \hbar \\
\frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{l_p}{c} &\geq \hbar \\
\frac{c}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{l_p}{c} &\geq \frac{\bar{\lambda}}{l_p} \\
\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{l_p} \\
\frac{l_p}{\bar{\lambda}} &\geq \sqrt{1 - \frac{v^2}{c^2}} \\
\frac{l_p^2}{\bar{\lambda}^2} &\geq 1 - \frac{v^2}{c^2} \\
v &\leq c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}
\end{aligned} \tag{10}$$

This is the same as the maximum velocity formula given by Haug. We can naturally wonder why we get a structurally different maximum velocity from the momentum position principle and the energy time uncertainty principle. Numerically, for any known particle they give the same value to any practical degree of accuracy, but it is worth evaluating carefully to see if it is fully consistent. In a recent working paper, we have shown how the (old) momentum used by modern quantum mechanics is non-optimal. It is linked to the de Broglie wavelength

[25, 26], while a new momentum rooted in the Compton wavelength seems to simplify quantum mechanics and also resolves this inconsistency. It seems to be $v \leq c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}$ that is the exact speed limit and not $v \leq \frac{c}{\sqrt{1+\frac{l_p^2}{\bar{\lambda}^2}}}$, even though they both give the same numerical answer for any observable particle, because when $\bar{\lambda} << l_p$ the first term of the Taylor series expansion is the same $v_{max} \approx c\left(1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}\right)$, see [27]. Only for particles with mass very close to the Planck mass we get significant differences in the numerical output and predictions.

4 The Planck Mass Particle Must Stand Absolutely Still

The rest-mass of the Planck mass particle is given by

$$m_p = \frac{\hbar}{l_p}\frac{1}{c} \approx 2.1765 \times 10^{-8} \text{ kg} \tag{11}$$

That is to say, the reduced Compton wavelength of a Planck mass particle is l_p. Further, we know that the Planck mass particle momentum is $m_p c$. Now let us combine this formula with Heisenberg's uncertainty principle, where we will set $\sigma_x = l_p$ again. In this special case, we think it makes sense to set it only "equal to" rather than "greater than or equal to," because, unlike any other particle, we claim that the Planck mass particle momentum must always be $m_p c$, and then there is no uncertainty per se in the momentum. In other words, we predict that Heisenberg's uncertainty principle breaks down at the Planck scale and becomes a certainty principle. This is not really the topic of this paper and is covered in great detail in a recent Preprint paper [10]. However, the uncertainty principle limit must hold, and this is exactly what we see here

$$\begin{aligned} \sigma_x \sigma_p &\geq \hbar \\ l_p m_p c &= \hbar \\ l_p \frac{\hbar}{l_p}\frac{1}{c} c &= \hbar \\ 1 &= 1 \end{aligned} \tag{12}$$

This can only happen when $v = 0$. That is to say, only a Planck mass particle must have a velocity of zero. This is the same result as given by Haug's maximum velocity formula for anything with rest-mass; in the special case of a Planck mass particle

$$v_{max} = c\sqrt{1-\frac{l_p^2}{l_p^2}} = 0 \tag{13}$$

A velocity of zero (no matter the reference frame from which it is observed) sounds absurd at first. But actually this is not so strange at all. The Planck mass particle, according to Haug, can only last for one Planck second. This is the collision point between two light particles. Recent research has been quite clear on the concept that in a photon–photon collision we likely can create matter, see [12]. What is the speed of a light particle at the very turning point of light? It is zero. This means that light has two invariant "velocities": when it is energy, it always moves at speed c, as measured with Einstein-Poincaré synchronized clocks, no matter what the reference frame may be. And the velocity is zero when the particle is colliding and stands still for one Planck second, also as measured with Einstein-Poincaré synchronized clocks. As we see, at the deepest level the world is likely binary: we only have the Planck mass particle lasting for one Planck second (colliding indivisible particles), and energy (non-colliding indivisible particles).

This could best be interpreted to mean that the Planck mass particle can only have what we will call rest-mass momentum. The rest-mass momentum of the Planck mass particle is zero, as its maximum velocity is zero. It actually has zero momentum, if we define momentum as the momentum for a particle that moves. That is, for a Planck mass particle we have

$$\frac{m v_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = 0 \tag{14}$$

We predict that the Planck mass particle, the Planck length, and the Planck second are invariant and the same as observed from any reference frame. This means that Lorentz symmetry is broken at the Planck scale; this view is consistent with what has been predicted by several scientists in relation to quantum gravity. An important question is then if new physics at the Planck scale could be weakly detected at lower energies; this is discussed by [28–30], for example, and there is clearly room for more investigation here before any final conclusions are made.

5 $\hbar$ or $\frac{\hbar}{2}$?

Heisenberg himself postulated his uncertainty principle as $\sigma_x\sigma_p \geq \hbar$, while Kennard [24] was the first to come up with and partly prove this version of Heisenberg's uncertainty principle $\sigma_x\sigma_p \geq \frac{\hbar}{2}$.

We could also have derived the limit on the velocity based on the Kennard version of the Heisenberg principle, but here we will on purpose use the energy time version (the momentum version is shown in appendix B); when assuming the minimum uncertainty in the position is the Planck length this gives

$$\begin{aligned}
\sigma_t\sigma_E &\geq \frac{\hbar}{2} \\
\frac{l_p}{c}\sigma_E &\geq \frac{\hbar}{2} \\
\frac{l_p}{c}\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\hbar}{2} \\
\frac{l_p}{c}\frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\hbar}{2} \\
\frac{l_p}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{1}{2}
\end{aligned} \tag{15}$$

Solved with respect to v this gives

$$v \leq c\sqrt{1-4\frac{l_p^2}{\bar{\lambda}^2}} \tag{16}$$

For example, for an electron this would mean a speed limit of

$$v \leq c\sqrt{1-4\frac{l_p^2}{\bar{\lambda}_e^2}} \approx c \times 0.99999999999999999999999999999999999999999999649$$

Interestingly, when $\bar{\lambda} >> l_p$, as it is for all observed particles, numerically this is almost identical to the maximum velocity limit that one would get from the maximum velocity formula given by Haug in 2014. Haug's 2014 formula shows that when we set the diameter of the indivisible particle to l_p we get

$$v_{max} = c\left(\frac{\bar{\lambda}^2 - l_p^2}{\bar{\lambda}^2 + l_p^2}\right) \tag{17}$$

For an electron, we have

$$v_{max} = c\left(\frac{\bar{\lambda}_e^2 - l_p^2}{\bar{\lambda}_e^2 + l_p^2}\right) \approx c \times 0.99999999999999999999999999999999999999999999649$$

In addition, if we take a Taylor series expansion of formula 16 we get

$$\frac{v}{c} \leq \sqrt{1+4\frac{l_p^2}{\bar{\lambda}^2}} \approx 1 - 2\frac{l_p^2}{\bar{\lambda}^2} - 2\frac{l_p^4}{\bar{\lambda}^4} - 4\frac{l_p^6}{\bar{\lambda}^6}\cdots \tag{18}$$

Using only the first term of the series expansion and we have $v \leq c\left(1 - 2\frac{l_p^2}{\bar{\lambda}^2}\right)$ and also a series expansion of $\frac{1-x^2}{1+x^2} \approx 1 - 2x^2 + 2x^4 - 2x^6\cdots$. This means when we have $\bar{\lambda} >> l_p$, we can write Haug's maximum formula as

$$v_{max} = c\left(\frac{\bar{\lambda}^2 - l_p^2}{\bar{\lambda}^2 + l_p^2}\right) = c\left(\frac{1 - \frac{l_p^2}{\bar{\lambda}^2}}{1 + \frac{l_p^2}{\bar{\lambda}^2}}\right) \approx c\left(1 - 2\frac{l_p^2}{\bar{\lambda}^2}\right) \tag{19}$$

Yet an alternative is to indicate that half the Planck length is the minimum possible uncertainty in position, instead of the full Planck length. This would lead to the same formula for the maximum limit on velocity when calculated using the Kennard version of the Heisenberg principle formula, just as we calculated using the Planck length from the original Heisenberg principle formula, that is formula 6.

We could introduce further discussion on the Heisenberg principle, including the linkage to atomism, why both versions of the Heisenberg principle make sense, and how they are closely related. However, we will leave the more philosophical discussions for a separate paper.

The fact that two postulates in atomism lead to all of the known equations in special relativity theory, plus the result we have derived from combining the theory from Heisenberg and Max Planck indicate that this theory should be considered as an viable alternative. Atomism is quantized from the very beginning by returning to a spatial dimension for one unique particle that makes up all other masses and energy.

6 Conclusion

We have shown that Heisenberg's uncertainty principle can predict an exact maximum velocity that is below the speed of light for anything with rest-mass. For any practical purpose, this seems to be the same limit as given by Haug's earlier suggested maximum velocity formula.

This could have major implications for how we look at light particles at the very collision point with other light particles. This also indicates that Lorentz symmetry breaks down at the Planck scale. The Planck mass particle stands absolutely still and is invariant and the same as observed across different reference frames.

Below we show some possible choices of assumptions, where the theory presented above seems to be compatible with the Planck length being a minimum length. The idea that the velocity of a mass has to be below c, but can come as close to c as we want, is actually not compatible with accepting the Planck length as a minimum length.

1. $\sigma_x \geq l_p$ and $v \leq c\sqrt{1-\frac{l_p^2}{\lambda^2}}$. What we have derived above.
2. $\sigma_x > 0$ and $v < c$. This is the current theory on velocity, but it is incompatible with idea of Planck length as a minimum. If modern physicists want to hold on to $v < c$, then they must reconsider and likely eliminate the Planck length. This is improbable and probably unwise; instead they should replace $v < c$ with the formula above.
3. $\sigma_x \geq 0$ and $v \leq c$. Impossible for anything with rest-mass, as the relativistic mass could become infinite, so this can easily be excluded.

Choice three can be excluded based on Einstein's analysis that a mass traveling at the speed of light will attain an infinitely large relativistic mass, so it is impossible to have the limit on v for anything with rest-mass. Choice two has the drawback that one must claim the Planck length is nothing special. Choice one is, in our view, an alternative that should be considered seriously by the physics community.

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Appendix A:

This is almost the same derivation as in section 2. However, here we will complete the derivation without initially decomposing the mass:

$$\begin{aligned}
\sigma_x\sigma_p &\geq \hbar \\
l_p\sigma_p &\geq \hbar \\
l_p\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \hbar \\
\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\hbar}{l_p m} \\
\frac{v^2}{\left(1-\frac{v^2}{c^2}\right)} &\geq \frac{\hbar^2}{l_p^2 m^2} \\
v^2 &\leq \frac{\hbar^2}{l_p^2 m^2}\left(1-\frac{v^2}{c^2}\right) \\
v^2 &\leq \frac{\hbar^2}{l_p^2 m^2} - \frac{\hbar^2}{l_p^2 m^2}\frac{v^2}{c^2} \\
v^2 + \frac{\hbar^2}{l_p^2 m^2}\frac{v^2}{c^2} &\leq \frac{\hbar^2}{l_p^2 m^2} \\
v^2\left(1+\frac{\hbar^2}{l_p^2 m^2}\frac{1}{c^2}\right) &\leq \frac{\hbar^2}{l_p^2 m^2} \\
v^2 &\leq \frac{\frac{\hbar^2}{l_p^2 m^2}}{\left(1+\frac{\hbar^2}{l_p^2 m^2}\frac{1}{c^2}\right)} \\
v^2 &\leq \frac{1}{\frac{l_p^2 m^2}{\hbar^2}+\frac{1}{c^2}} \\
v^2 &\leq \frac{c^2}{\frac{l_p^2 c^2 m^2}{\hbar^2}+1} \\
v &\leq \frac{c}{\sqrt{\frac{l_p^2 c^2 m^2}{\hbar^2}+1}} \\
v &\leq \frac{c}{\sqrt{\frac{m^2}{\frac{\hbar^2}{l_p^2 c^2}}+1}}
\end{aligned} \tag{20}$$

and since $\frac{\hbar}{l_p}\frac{1}{c}$ is equal to the Planck mass, m_p, we can also write this as

$$v \leq \frac{c}{\sqrt{1+\frac{m^2}{m_p^2}}} \tag{21}$$

Which is naturally the same formula we derived earlier since

$$\frac{m^2}{m_p^2} = \frac{\left(\frac{\hbar}{\bar{\lambda}}\frac{1}{c}\right)^2}{\left(\frac{\hbar}{l_p}\frac{1}{c}\right)^2} = \frac{l_p^2}{\bar{\lambda}^2} \tag{22}$$

Similarly, if we had derived it this way from the Kennard version of the Heisenberg uncertainty formula we would have gotten

$$v \leq \frac{c}{\sqrt{1+4\frac{m^2}{m_p^2}}} \tag{23}$$

It is also worth pointing out that if we look at Newton's gravitational constant as a composite constant, $G = \frac{l_p^2 c^3}{\hbar}$, as previously suggested by Haug [5, 7, 31], then we can write the maximum velocity as follows, when using the original Heisenberg uncertainty formula

$$\begin{aligned} v &\leq \frac{c}{\sqrt{1+\frac{l_p^2 c^2 m^2}{\hbar^2}}} \\ v &\leq \frac{c}{\sqrt{1+G\frac{mm}{c\hbar}}} \end{aligned} \tag{24}$$

It is worth mentioning that [32] has derived Newton's gravitational formula based on Heisenberg's uncertainty principle; see also [33]. This illustrates that there could be a connection between Heisenberg's uncertainty principle and gravity. One might think that cosmological phenomena have nothing to do with Heisenberg's uncertainty principle, which comes out of quantum physics. However, if the Planck length plays an important role in gravity, then this could actually make sense.

When using the Kennard version of the Heisenberg uncertainty formula we will have

$$v \leq \frac{c}{\sqrt{1+4G\frac{m^2}{c\hbar}}} \tag{25}$$

Further, many will recognize $G\frac{m^2}{c\hbar}$ as the small gravitational coupling constant, α_g. So, we can also write the maximum limit on velocity simply as

$$v \leq \frac{c}{\sqrt{1+\alpha_g}} \tag{26}$$

These different ways to write the maximum limit on the velocity are essentially the same, except for a small difference that will emerge depending on whether we derive it from the original Heisenberg uncertainty formulation, or from the Kennard formulation.

Solved with respect to big G we get

$$\begin{aligned} v &\leq \frac{c}{\sqrt{1+G\frac{mm}{c\hbar}}} \\ v\sqrt{1+G\frac{mm}{c\hbar}} &\leq c \\ v^2\left(1+G\frac{mm}{c\hbar}\right) &\leq c^2 \\ G\frac{mm}{c\hbar} &\leq \frac{c^2}{v^2}-1 \\ G &\leq \frac{c\hbar}{mm}\left(\frac{c^2}{v^2}-1\right) \end{aligned} \tag{27}$$

The above can only be true when $v = \frac{c}{\sqrt{1+\frac{l_p^2}{\bar{\lambda}^2}}}$; this gives

$$\begin{aligned} G &= \frac{c\hbar}{mm}\left(\frac{c^2}{\frac{c^2}{1+\frac{l_p^2}{\bar{\lambda}^2}}}-1\right) \\ G &= \frac{c\hbar}{mm}\left(\frac{1}{\frac{1}{1+\frac{l_p^2}{\bar{\lambda}^2}}}-1\right) \\ G &= \frac{c\hbar}{mm}\left(\frac{l_p^2}{\bar{\lambda}^2}\right) \\ G &= \frac{c\hbar}{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}\left(\frac{l_p^2}{\bar{\lambda}^2}\right) \\ G &= \frac{l_p^2 c^3}{\hbar} \approx 6.67384 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2} \end{aligned} \tag{28}$$

Naturally, we are simply getting back to what we started with. Still, this mathematical relationship between the maximum velocity from Heisenberg's uncertainty principle combined with the composite view of the gravitational constant indicates that gravity might be related to the potential maximum speed of elementary particles. Alternatively, one might look at this last part, including big G, as merely mathematical "manipulation".

7 Appendix B: Maximum velocity from the Kennard version based on momentum

The Kennard version of the momentum position Heisenberg uncertainty principle; when assuming the minimum uncertainty in the position is the Planck length this gives

$$
\begin{aligned}
\sigma_x \sigma_p &\geq \frac{\hbar}{2} \\
l_p \sigma_p &\geq \frac{\hbar}{2} \\
l_p \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\hbar}{2} \\
l_p \frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\hbar}{2} \\
\frac{\frac{1}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{2l_p} \\
\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{2l_p}c
\end{aligned} \tag{29}
$$

Solved with respect to v this gives

$$
\begin{aligned}
\frac{v^2}{1-\frac{v^2}{c^2}} &\geq \frac{\bar{\lambda}^2}{2^2 l_p^2}c^2 \\
v^2 &\leq \frac{\bar{\lambda}^2}{4l_p^2}c^2\left(1-\frac{v^2}{c^2}\right) \\
v^2\left(1+\frac{\bar{\lambda}^2}{4l_p^2}\right) &\leq \frac{\bar{\lambda}^2}{4l_p^2}c^2 \\
v^2 &\leq \frac{\frac{\bar{\lambda}^2}{4l_p^2}c^2}{\left(1+\frac{\bar{\lambda}^2}{4l_p^2}\right)} \\
v &\leq \frac{c}{\sqrt{1+4\frac{l_p^2}{\bar{\lambda}^2}}}
\end{aligned} \tag{30}
$$

For example, for an electron this would mean a speed limit of

$$v \leq \frac{c}{\sqrt{1+4\frac{l_p^2}{\bar{\lambda}_e^2}}} \approx c \times 0.99999999999999999999999999999999999999999999649$$

Formula 30 is structural different from the speed we get from the Kennard principle when using energy and time, see section 3 and 5.