

Title: Expression to get prime numbers and twin prime numbers.

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**Abstract:** This paper develops a modified an old and well-known expression for calculating and obtaining all prime numbers greater than three, composite numbers and all twin prime numbers greater than three. The conditioning (n) will be the key to make the formula work.

**Keywords:** Theory number, Prime numbers, Twin prime numbers, composite numbers.

## Introduction

The study of the prime numbers is wonderful and exciting, in the absence of an expression that involves all of them I have investigated and I have discovered a brilliant expression that contains all the prime numbers greater than 3. This expression comes from investigating first how they are distributed the composite numbers, this allowed me to explore its order and understand its mechanism. The expression of the prime numbers is its result.

The expression to obtain the prime numbers is similar to how we use the sieve of Eratosthenes although all that infinite procedure expressed in symbols in a formula.

From here it was very easy to get to the Twin prime numbers.

## Methods

The prime numbers greater than three can be expressed under the expressions  $(6 * n + 1)$  and  $(6 * n - 1)$ , for some values of (n). This paper demonstrates in Theorem 3 how to obtain the correct (n) values to obtain all the prime numbers.

We can obtain composite numbers that are not divisible by 2 and by 3 under the expressions  $(6 * n + 1)$  and  $(6 * n - 1)$ , for some values of (n). This paper demonstrates how to obtain the correct (n) values to obtain all the Composite numbers in Theorem 5.

Also, Twin prime numbers greater than 3 are expressed under the same expression, so once the values of (n) are obtained for obtaining the prime numbers, combining both expressions we obtain all the twin prime numbers in Theorem 6.

The conditions to which it is subject (n) are represented in tables and calculations that demonstrate it.

**Theorem 1. Sequence of:  $\beta = (6 * n \pm 1)$**

The  $\beta$  sequence will be fundamental since this will allow us to obtain all the composite values and once excluded these will only be the prime numbers.

$$n > 0$$

Sequence numbers $\beta (a)$	Sequence numbers $\beta (b)$
$\beta (a) = (6 * n + 1) = 7, 13, 19, 25, 31, \dots$	$\beta (b) = (6 * n - 1) = 5, 11, 17, 23, 29, 35, \dots$

$$\beta = (6 * n \pm 1) = 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, \dots$$

**Theorem 2. Natural Twin**

The natural twins play a fundamental role in the application of the formula. The natural twins are obtained from the  $\beta$  sequence, they are numbers that take two of difference. One belongs to the form  $\beta (a)$  and the other to the form  $\beta (b)$

$$gn_x = (\beta (b), \beta (a))$$

$gn_x =$  natural twins

$$gn_x = gn_{1=(5,7)}; gn_{2=(11,13)}; gn_{3=(17,19)}; gn_{4=(23,25)}; \dots gn_{\infty=(\infty - 2, \infty)}$$

$$gn_x = gn_{1=1}; gn_{2=2}; gn_{3=3}; gn_{4=4}; \dots gn_{\infty=\infty}$$

**Theorem 3. Formula for calculating and obtaining prime numbers**

This formula allows the obtaining of all the prime numbers greater than three. This formula allows to obtain the correct values of (n), for which we can establish how the prime numbers are located and distributed.

$$P (a, b) = \text{Prime numbers} > 3$$

$$Z = \text{numbers} \geq 0$$

$$P (a, b) = (6 * n \quad n > 0 \quad \pm 1) \\ n \neq \beta \pm gn + \beta * Z$$

**Example and demonstration of the formula is divided into 2 columns.**

In this example I will calculate all the prime numbers greater than 3.

On the left we will calculate the prime numbers located in  $\beta (a)$ , on the right we will calculate the prime numbers located in  $\beta (b)$ .

$P (a) = (6 * n \quad n > 0 \quad + 1) \\ n \neq \beta \mp gn + \beta * Z$	$P (b) = (6 * n \quad n > 0 \quad - 1) \\ n \neq \beta \pm gn + \beta * Z$
$P(a) = \text{Prime numbers in } \beta(a)$	$P(b) = \text{Prime numbers in } \beta(b)$

<p><math>gn =</math> natural twins of <math>\beta</math>  <math>Z =</math> numbers <math>\geq 0</math></p> <p><math>\beta = (6 * n \pm 1) = 5,7,11,13,17,19,23,25,29,31,...</math></p> <p><b>First step</b>  <math>\beta - gn</math> Subtraction in the numbers:  <math>\beta (b) = 5,11,17,23,29,35,41,....</math></p> <p><b>Second step</b>  <math>\beta + gn</math> Sum in the numbers:  <math>\beta (a) = 7,13,19,25,31,37,43,.....</math></p>	<p><math>gn =</math> natural twins <math>\beta</math>  <math>Z =</math> numbers <math>\geq 0</math></p> <p><math>\beta = (6 * n \pm 1) = 5,7,11,13,17,19,23,25,29,31,...</math></p> <p><b>First step</b>  <math>\beta + gn</math> Sum in the numbers:  <math>\beta (b) = 5,11,17,23,29,35,41,....</math></p> <p><b>Second step</b>  <math>\beta - gn</math> Subtraction in the numbers:  <math>\beta (a) = 7,13,19,25,31,37,43,.....</math></p>
<p><math>P(a) = (6 * n \quad n &gt; 0 \quad + 1) =</math></p> <p style="text-align: center;"> <math>n \neq 5 - 1 + 5 * Z</math>  <math>n \neq 7 + 1 + 7 * Z</math>  <math>n \neq 11 - 2 + 11 * Z</math>  <math>n \neq 13 + 2 + 13 * Z</math>  <math>n \neq 17 - 3 + 17 * Z</math>  <math>n \neq 19 + 3 + 19 * Z</math> </p> <p style="text-align: center;">↓</p> <p>We can keep adding <b><math>\beta</math> numbers</b> and expanding the formula infinitely. In this way we will obtain the correct values of <math>n</math> that replaced in the formula generate prime numbers.</p> <p><math>P(a) = (6 * n \quad n &gt; 0 \quad + 1) =</math></p> <p style="text-align: center;"> <math>n \neq 4 + 5 * Z</math>  <math>n \neq 8 + 7 * Z</math>  <math>n \neq 9 + 11 * Z</math>  <math>n \neq 15 + 13 * Z</math>  <math>n \neq 14 + 17 * Z</math>  <math>n \neq 22 + 19 * Z</math>  <i>continue infinitely</i> </p> <p><math>n \neq 4, 8, 9, 14, 15, 19, 20, ... ..</math></p> <p><b>Using values correct for:</b>  <math>n = 1, 2, 3, 5, 6, 7, 10, 11, 12, 13, ...</math></p> <p><b>We get the following prime numbers.</b></p> <p><math>P(a) = 7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109, 127, 139, ... ..</math></p>	<p><math>P(b) = (6 * n \quad n &gt; 0 \quad - 1) =</math></p> <p style="text-align: center;"> <math>n \neq 5 + 1 + 5 * Z</math>  <math>n \neq 7 - 1 + 7 * Z</math>  <math>n \neq 11 + 2 + 11 * Z</math>  <math>n \neq 13 - 2 + 13 * Z</math>  <math>n \neq 17 + 3 + 17 * Z</math>  <math>n \neq 17 - 3 + 19 * Z</math> </p> <p style="text-align: center;">↓</p> <p>We can keep adding <b><math>\beta</math> numbers</b> and expanding the formula infinitely. In this way we will obtain the correct values of <math>n</math> that replaced in the formula generate prime numbers.</p> <p><math>P(b) = (6 * n \quad n &gt; 0 \quad - 1) =</math></p> <p style="text-align: center;"> <math>n \neq 6 + 5 * Z</math>  <math>n \neq 6 + 7 * Z</math>  <math>n \neq 13 + 11 * Z</math>  <math>n \neq 11 + 13 * Z</math>  <math>n \neq 20 + 17 * Z</math>  <math>n \neq 16 + 19 * Z</math>  <i>continue infinitely</i> </p> <p><math>n \neq 6, 11, 13, 16, 20, 21, ... ..</math></p> <p><b>Using correct values for</b>  <math>n = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 14, 15, ...</math></p> <p><b>We get the following prime numbers.</b></p> <p><math>P(b) = 5, 11, 17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113, ... ..</math></p>

#### Theorem 4. Graphics tables

These graphs (Table 1 and table 2) show the behavior of  $(n)$  with respect to the prime numbers and compound numbers.

The brown colors are related to the  $\beta$  numbers, where they fall generate compound numbers for  $(n)$

The columns are sorted as appropriate (the addition or subtraction) of the twin natural numbers with  $\beta$ .

The columns are ordered every 7 boxes on one side and the other every 5 boxes.

Table 1																
gn	7	6	5	4	3	2	1	P=6n+1	1	2	3	4	5	6	7	gn
$\beta(a)$	43	37	31	25	19	13	7	(n)	5	11	17	23	29	35	41	$\beta(b)$
								1								
								2								
								3								
								4								
								5	1							
								6	2							
								7	3							
								8	4							
								9	5							
							1	10	1	1						
							2	11	2	2						
							3	12	3	3						
							4	13	4	4						
							5	14	5	5						
							6	15	6	6						
							7	16	7	7						
							1	17	8	8						
							2	18	9	9						
							3	19	10	10						
							4	20	11	11						
							5	21	12	12						
							6	22	13	13						
							7	23	14	14						
							1	24	15	15						
							2	25	16	16						
							3	26	17	17						
							4	27	18	18						
							5	28	19	19						
							6	29	20	20						
							7	30	21	21						
							1	31	22	22						
							2	32	23	23						
							3	33	24	24						
							4	34	25	25						
							5	35	26	26						
							6	36	27	27						
							7	37	28	28						
							1	38	29	29						
							2	39	30	30						
							3	40	31	31						
							4	41	32	32						
							5	42	33	33						
							6	43	34	34						
							7	44	35	35						
							1	45	36	36						
							2	46	37	37						
							3	47	38	38						
							4	48	39	39						
							5	49	40	40						
							6	50	41	41						
							7	51	42	42						
							1	52	43	43						
							2	53	44	44						
							3	54	45	45						
							4	55	46	46						
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Table 3

In yellow we have the prime numbers in red the composite numbers.

$$\beta(a) = (6 * n + 1) \quad \beta(b) = (6 * n - 1)$$

Prime Numbers					
$\beta(a)$			$\beta(b)$		
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120
121	122	123	124	125	126
127	128	129	130	131	132
133	134	135	136	137	138
139	140	141	142	143	144
145	146	147	148	149	150

### Theorem 5. Get composite numbers of columns A and B

These two formulas allow to obtain all the composite numbers that are divisible by prime numbers greater than 3. The variable is in the sign of  $\pm 1$

#### Integrated formula

$Nc = (6 * n_{=\beta-gn+\beta*z}) \pm 1$ $Z \geq 0$ $Nc = \text{compound numbers}$ <p>+1)= Sum in the numbers:  <math>\beta(b) = 5,11,17,23,29,35,.....</math></p>	$Nc = (6 * n_{=gn+\beta*z}) \mp 1$ $Z > 0$ $Nc = \text{compound numbers}$ <p>-1)= Subtraction in the numbers:  <math>\beta(b) = 5,11,17,23,29,35,.....</math></p>
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<p><math>-1)=</math> Subtraction in the numbers:  <math>\beta (a) = 7,13,19,25,31,37,\dots\dots\dots</math></p> <p><math>Nc = (6 * n_{=5-1+5*z})+1</math>  <math>n=7-1+7*z) -1</math>  <math>n=11-2+11*z) +1</math>  <math>n=13-2+13*z) -1</math>  <math>n=17-3+17*z)+1</math>  <math>n=19-3+19*z)-1</math></p> <p>the signs are interspersed.          We can keep adding more numbers</p>	<p><math>+1)=</math> Sum in the numbers:  <math>\beta (a) = 7,13,19,25,31,37,\dots\dots\dots</math></p> <p><math>Nc =(6 * n_{=1+5*z})-1</math>  <math>n=1+7*z)+1</math>  <math>n=2+11*z)-1</math>  <math>n=2+13*z)+1</math>  <math>n=3+17*z)-1</math>  <math>n=3+19*z)+1</math></p> <p>the signs are interspersed.          We can keep adding more numbers</p>
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### Individual formula

<p><math>Nc =(6 * (\beta - gn + \beta * n) \pm 1</math></p> <p><math>n \geq 0</math>  <math>+1)=</math> Sum in the numbers:  <math>\beta (b) = 5,11,17,23,29,35,\dots\dots</math></p> <p><math>-1)=</math> Subtraction in the numbers:  <math>\beta (a) = 7,13,19,25,31,37,\dots\dots\dots</math></p>	<p><math>Nc =(6 * (gn + \beta * n) \mp 1)</math></p> <p><math>n &gt; 0</math>  <math>-1)=</math> Subtraction in the numbers:  <math>\beta (b) = 5,11,17,23,29,35,\dots\dots</math></p> <p><math>+1)=</math> Sum in the numbers:  <math>\beta (a) = 7,13,19,25,31,37,\dots\dots\dots</math></p>
$Nc 5 (a)=(6 * (4 + 5 * n) + 1)=$	$Nc 5(b) = (6 * (1 + 5 * n) - 1) =$
$Nc 7 (b)=(6 * (6 + 7 * n) - 1)=$	$Nc 7 (a)=(6 * (1 + 7 * n) + 1)=$
$Nc 11 (a)=(6 * (9 + 11 * n) + 1)=$	$Nc 11 (b)=(6 * (2 + 11 * n) - 1)=$
$Nc 13 (b)=(6 * (11 + 13 * n) - 1)=$	$Nc 13 (a)=(6 * (2 + 13 * n) + 1)=$
$Nc 17 (a)=(6 * (14 + 17 * n) + 1)=$	$Nc 17 (b)=(6 * (3 + 17 * n) - 1)=$

### Five examples of composite numbers.

- 1)  $Nc 5 (a)=(6 * (4 + 5 * n) + 1)= Nc (a)=25,55,85,115,145,175,205,235,265,\dots\dots\dots$   
 $Nc 5 (b)=(6 * (1 + 5 * n) - 1)= Nc(b)= 35,65,95,125,155,185,215,245, \dots\dots\dots$   
 The distance between each number is  $\beta *6= 5*6=30$   
 In  $Nc (a)$  begins in  $\beta *5= 5*5=25$   
 In  $Nc (b)$  begins in  $\beta *7= 5*7=35$
- 2)  $Nc 7 (a)=(6 * (1 + 7 * n) + 1)= Nc(a)= 49,91,133,175,217,259,301,343, \dots\dots\dots$   
 $Nc 7 (b)=(6 * (6 + 7 * n) - 1)= Nc (b)=35,77,119,161,203,245,287,329,371,\dots\dots\dots$   
 The distance between each number is  $\beta *6= 7*6=42$   
 In  $Nc (a)$  begins in  $\beta *7= 7*7=49$   
 In  $Nc (b)$  begins in  $\beta *5= 7*5=35$

$$3) \quad \text{Nc } 11 (a) = (6 * (9 + 11 * n) + 1) = \text{Nc } (a) = 55, 121, 187, 253, 319, \dots$$

$$\text{Nc } 11 (b) = (6 * (2 + 11 * n) - 1) = \text{Nc } (b) = 77, 143, 209, 275, 341, \dots$$

The distance between each number is  $\beta * 6 = 11 * 6 = 66$

In Nc (a) begins in  $\beta * 5 = 11 * 5 = 55$

In Nc (b) begins in  $\beta * 7 = 11 * 7 = 77$

$$4) \quad \text{Nc } 13 (a) = (6 * (2 + 13 * n) + 1) = \text{Nc } (a) = 91, 169, 247, \dots$$

$$\text{Nc } 13 (b) = (6 * (11 + 13 * n) - 1) = \text{Nc } (b) = 65, 143, 221, 299, \dots$$

The distance between each number is  $\beta * 6 = 13 * 6 = 78$

In Nc (a) begins in  $\beta * 7 = 13 * 7 = 91$

In Nc (b) begins in  $\beta * 5 = 13 * 5 = 65$

$$5) \quad \text{Nc } 17 (a) = (6 * (14 + 17 * n) + 1) = \text{Nc } (a) = 85, 187, 289, \dots$$

$$\text{Nc } 17 (b) = (6 * (3 + 17 * n) - 1) = \text{Nc } (b) = 119, 221, 323, \dots$$

The distance between each number is  $\beta * 6 = 17 * 6 = 102$

In Nc (a) begins in  $\beta * 5 = 17 * 5 = 85$

In Nc (b) begins in  $\beta * 7 = 17 * 7 = 119$

We can keep adding more numbers and expanding the formula infinitely.

### **Theorem 6. Formula for obtaining twin prime numbers**

This formula allows the obtaining of all the Twin prime numbers  $>3$ . This formula is armed with the combination of the two main variables of prime numbers. Both formulas come together to condition (n) and obtain the correct values for (n).

This formula does not pretend to demonstrate the infinity of twin prime numbers, it only shows and proves how to obtain them.

$$Tp = (6 * n \begin{matrix} n > 0 \\ n \neq \beta \mp g_n + \beta * Z \end{matrix} \pm 1)$$

$$\beta = (6 * n \pm 1) = 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, \dots$$

$$Z \geq 0$$

Tp = twin prime numbers  $>3$

Example, application and demonstration of the formula.

In this example I will calculate all the twin prime numbers greater than 3.

#### **Twin Prime numbers in $in \beta (a)$**

$$Tp (a) = (6 * n \begin{matrix} n > 0 \\ n \neq \beta \mp g_n + \beta * Z \end{matrix} + 1)$$

*First step*  $\beta - g_n = \text{Subtract in the numbers } \beta = 5, 7, 11, 13, 17, 19, 23, 25 \dots$

*Second step*  $\beta + g_n = \text{Sum in the numbers } \beta = 5, 7, 11, 13, 17, 19, 23, 25 \dots$

This process is repeated for each number of  $\beta$ .

$$Tp(a) = (6 * n \quad n > 0 \quad + 1) = (6 * n \quad n > 0 \quad + 1) =$$

$n \neq 5-1+5*Z$ $n \neq 5+1+5*Z$ $n \neq 7-1+7*Z$ $n \neq 7+1+7*Z$ $n \neq 11-2+11*Z$ $n \neq 11+2+11*Z$ $n \neq 13-2+13*Z$ $n \neq 13+2+13*Z$ $n \neq 17-3+17*Z$ $n \neq 17+3+17*Z$ $n \neq 19-3+19*Z$ $n \neq 19+3+19*Z$ <i>continue infinitely</i>	$n \neq 4+5*Z$ $n \neq 6+5*Z$ $n \neq 6+7*Z$ $n \neq 8+7*Z$ $n \neq 9+11*Z$ $n \neq 13+11*Z$ $n \neq 11+13*Z$ $n \neq 15+13*Z$ $n \neq 14+17*Z$ $n \neq 20+17*Z$ $n \neq 16+19*Z$ $n \neq 22+19*Z$ <i>continue infinitely</i>
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$n \neq 4, 6, 8, 9, 11, 13, 14, 15, 16, 19, 20, \dots \dots \dots$

Using values for  $n = 1, 2, 3, 5, 7, 10, 12, 17, 18, 23, 25 \dots$

We get the following Prime Numbers

$Tp(a) = 7, 13, 19, 31, 43, 61, 73, 103, 109, \dots \dots \dots$

### Twin Prime numbers in $in \beta(b)$ .

$$Tp(b) = (6 * n \quad n > 0 \quad - 1) = (6 * n \quad n > 0 \quad - 1) =$$

$$Tp(b) = (6 * n \quad n > 0 \quad - 1) = (6 * n \quad n > 0 \quad - 1) =$$

$n \neq 5-1+5*Z$ $n \neq 5+1+5*Z$ $n \neq 7-1+7*Z$ $n \neq 7+1+7*Z$ $n \neq 11-2+11*Z$ $n \neq 11+2+11*Z$ $n \neq 13-2+13*Z$ $n \neq 13+2+13*Z$ $n \neq 17-3+17*Z$ $n \neq 17+3+17*Z$ $n \neq 19-3+19*Z$ $n \neq 19+3+19*Z$ <i>continue infinitely</i>	$n \neq 4+5*Z$ $n \neq 6+5*Z$ $n \neq 6+7*Z$ $n \neq 8+7*Z$ $n \neq 9+11*Z$ $n \neq 13+11*Z$ $n \neq 11+13*Z$ $n \neq 15+13*Z$ $n \neq 14+17*Z$ $n \neq 20+17*Z$ $n \neq 16+19*Z$ $n \neq 22+19*Z$ <i>continue infinitely</i>
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$n \neq 4, 6, 8, 9, 11, 13, 14, 15, 16, 19, 20, \dots \dots \dots$

Using correct values for:  $n = 1, 2, 3, 5, 7, 10, 12, 17, 18, 23, 25 \dots$

We get the following Prime Numbers

$Tp(b) = 5, 11, 17, 29, 41, 59, 71, 101, 107, \dots \dots \dots$

**In the two variables of the formula we obtain the same values for (n)**

### Theorem 7. Twin prime numbers: $Tp=(Tp(b),Tp(a))$

From the formula of Theorem 6 we obtain the correct prime numbers to form the pairs of twin prime numbers.

$Tp$  = Twin prime numbers

$Tp(a)$  = Twin prime numbers in  $\beta(a)$        $Tp(a) = 7,13,19,31,43,61,73,103,109, \dots \dots$

$Tp(b)$  = Twin prime numbers in  $\beta(b)$        $Tp(b) = 5,11,17,29,41,59,71,101,107, \dots \dots$

$Tp = (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73), (101, 103), \dots \dots$

### Conclusion

The order of the prime numbers and the prime twin numbers is done by combining the  $\beta$  numbers and the natural twins.

These wonderful formulas under their conditioning generate what has been sought throughout history. For the first time we can find an expression that generates absolutely all the Prime numbers greater than three, compound numbers and also all the twin prime numbers greater than three. These formulas are simple and easy although extensive, and infinity. Understanding the behavior of  $(n)$  is equivalent to understanding how prime numbers, twin prime numbers and composite numbers are distributed.

This is the expression that is combined with the mechanism of the sieve Eratosthenes.

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