Fundamental Theory of Nature

Author: Peter Hickman
20th Nov 2017

Abstract

It is proposed that particles are not fundamental but arise as symmetries of an infinite dimension random field. Space-Time with signature (3,1) and gravitation arises as a double phase transformation of spinors and scalars. The 3 generations of chiral spinors of quarks and leptons and the scalar doublets of the Standard Model are found from 3d random field. The Schwarzschild space-time has no physical singularity.

Introduction

The current description of Nature is the Standard model (SM) of particles and gauge fields and General Relativity (GR). It is well tested; however the reason for the gauge group structure and particle content is unknown. In addition the dark sector i.e. Dark energy and Dark matter are not explained by the SM or GR. The reason for the Space-Time signature (3,1) is also unknown and will not be assumed in this paper. Both the SM and GR suffer from physical singularities which need to be resolved by any proposed fundamental theory of nature.

The approach is to consider Nature at the elementary level does not follow laws of nature but instead particles arise as symmetries of random fields. It will be shown that the spinors and scalar doublets are undetermined up to phase. The particle content and the gauge group structure of the SM are addressed here. The metric and thus gravitation arises from a double phase transformation of a spinor or scalar doublet.

Hypothesis

Nature at the fundamental level is an infinite dimensional, random real scalar field $\boldsymbol{F}$
.

$$\boldsymbol{F} = \{f_i : i \in \mathbb{R}^\infty\}$$

The phenomena of particles arise as endomorphisms of the infinite dimensional scalar field.

Cartan Spinors on 3d space

Consider the 3x3 matrix Φ of random scalars which transforms f_i to f_j

$$f_j = \Phi f_i \qquad (1)$$

Let X be a Hermitean 2x2 matrix representation of an isotropic row vector of the matrix Φ

$$X \in \begin{Bmatrix} \begin{pmatrix} \Phi_{g3} & \Phi_{g1} - i\Phi_{g2} \\ \Phi_{g1} + i\Phi_{g2} & -\Phi_{g3} \end{pmatrix}, \begin{pmatrix} \Phi_{g2} & \Phi_{g1} - i\Phi_{g3} \\ \Phi_{g1} + i\Phi_{g3} & -\Phi_{g2} \end{pmatrix}, \begin{pmatrix} \Phi_{g} & \Phi_{g2} - i\Phi_{g3} \\ \Phi_{g2} + i\Phi_{g3} & -\Phi_{g1} \end{pmatrix}, \\ \begin{pmatrix} \Phi_{g3} & \Phi_{g2} - i\Phi_{g1} \\ \Phi_{g2} + i\Phi_{g1} & -\Phi_{g3} \end{pmatrix}, \begin{pmatrix} \Phi_{g2} & \Phi_{g3} - i\Phi_{g1} \\ \Phi_{g3} + i\Phi_{g1} & -\Phi_{g2} \end{pmatrix}, \begin{pmatrix} \Phi_{g} & \Phi_{g3} - i\Phi_{g2} \\ \Phi_{g3} + i\Phi_{g2} & -\Phi_{g1} \end{pmatrix} \end{Bmatrix} \quad 2$$

The 2-component spinors ζ are a solution to the following equation:

$$X = 2\zeta^{\dagger}\zeta C \quad 3$$

Where $C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $X^2 = 0$ is the isotropic vector condition **[1]**.

Phase transformations of the spinors
Gravitation

Phase transformations of the spinors ζ leaves isotropic vector condition invariant. The 2 component spinors transform under the Lie group $U(1) \times SU(2)$ whose generators are **[2]**:

$$\tau^{\mu} \in u(1) \times su(2), (\tau^0)^2 = 1, \left(\tau^i\right)^2 = -1 \\ i\tau^0\tau^1\tau^2\tau^3 = -1 \quad 4$$

Apply 2 infinitesimal non-commutative phase transformations to the spinor ζ:

$$\zeta' = e\left(i\delta g_{\mu}\frac{d(x\tau)^{\mu}}{dx^0}\right)e\left(-i\delta g_{\nu}\frac{d(x\tau)^{\nu}}{dx^0}\right)\zeta = \left(I + \delta g_{\mu}\delta g_{\nu}\frac{d(x\tau)^{\mu}}{dx^0}\frac{d(x\tau)^{\nu}}{dx^0} + \cdots\right)\zeta \quad 5$$

Where the notation $e \equiv exp$. Note that $\delta g_{\mu}\frac{d(x\tau)^{\mu}}{dx^0}$ is free of parameters.

$$\zeta' = \zeta + \delta g_{\mu}\delta g_{\nu}\frac{d(x\tau)^{\mu}}{dx^0}\frac{d(x\tau)^{\nu}}{dx^0}\zeta \quad 6$$

For orthogonal frames, and noting that the components $\zeta_a \neq 0$.

$$\delta\zeta_a = \zeta'_a - \zeta_a \quad 7$$

$$\frac{\delta\zeta_a}{\zeta_a} = \delta g_{\mu}\delta g_{\nu}\frac{d(x\tau)^{\mu}}{dx^0}\frac{d(x\tau)^{\nu}}{dx^0} \quad 8$$

$$\zeta_a = Ae\left(g_{\mu\nu}\frac{d(x\tau)^{\mu}}{dx^0}\frac{d(x\tau)^{\nu}}{dx^0}\right) = Ae\left(\left(\frac{d\tau}{dt}\right)^2\right) \quad 9$$

$$g_{\mu\nu} = \iint dg_{\mu}dg_{\nu} = \iint dg_{\nu}dg_{\mu} \quad 10$$

The metric $g_{\mu\nu}$ is symmetric since (10) is independent of the order of integration.

Gravitation arises as 2 phase transformations of a spinor and space-time has signature (3,1). Transform the random scalars into a probability density function by multiplication by $A^{\dagger}A$:

$$A^{\dagger}A\zeta^{\dagger}\zeta = \frac{1}{2}C^{-1}\,A^{\dagger}AX \qquad 11$$

Normalisation of the probability density function is given by the 3 volume integral:

$$\int A^{\dagger}A\zeta^{\dagger}\zeta\,dV = \int \frac{1}{2}C^{-1}\,A^{\dagger}AX\,dV = 1 \qquad 12$$

The spinor ψ is $\psi \equiv A\zeta$ with the normalisation condition:

$$\int \psi^{\dagger}\psi\,dV = 1 \qquad 13$$

Phase transformations of the spinors
Chiral Dirac Equation

Apply an infinitesimal phase transformation to the spinor ψ

$$\psi' = e\left(i\delta g_{\mu}\frac{\delta(x\tau)^{\mu}}{\delta x^{0}}\right)\psi \qquad 14$$

$$\psi' - \psi = i\delta g_{\mu}\frac{\delta(x\tau)^{\mu}}{\delta x^{0}}\psi \qquad 15$$

$$\frac{\delta\psi}{\delta x^{\mu}} = i\frac{\delta(g\tau)^{\mu}}{\delta x^{0}}\,\psi \qquad 16$$

$$\frac{\partial\psi}{\partial x^{\mu}} = i\frac{d(g\tau)^{\mu}}{dx^{0}}\,\psi \qquad 17$$

$$\tau^{\mu}\frac{\partial\psi}{\partial x^{\mu}} = i\dot{g}\psi \qquad 18$$

(18) is the equation of motion for the chiral spinor ψ where $\dot{g} = m$

Particles of Standard Model
Spin ½ Quarks

Each row vector of the transformation matrix Φ forms 3d isotropic vectors with spinors ζ_{agf} where a is the spin component, g is the matrix row.

For linear independent matrix columns

$$\zeta_{agf} : f \in \{(1,2),(2,1),(1,3),(3,1),(2,3),(3,2)\} \qquad 19$$

$$\zeta_{1g1} = \pm\sqrt{\frac{\Phi_{g1} + i\Phi_{g2}}{2}} \quad \zeta_{2g1} = \pm\sqrt{\frac{-\Phi_{g1} - i\Phi_{g2}}{2}} \tag{20}$$

The subsets

$$\begin{gathered}\{\zeta_{1gc}\}: \; c \in \{(1,2),(1,3),(2,3)\} \\ \{\zeta_{2gc}\}: c \in \{(2,1),(3,1),(3,2)\}\end{gathered} \tag{21}$$

The spinors ζ_{1gc} and ζ_{2gc} are 3 component spinors
The doublets transform under emergent symmetries

$$SU_L(2) \times U_Y(1) \quad \text{and } SU(3) \tag{22}$$

Weak-Hypercharge $Y_W = 2(Q - T_3)$ **[3]** is generalised to Y'_W

$$Y'_W = 2(Q' - \sum_c T_{3c}) \tag{23}$$

$$Y'_W = 1, c = 1,2,3 \qquad Q = Q'/3,\ Q = \binom{-1/3}{2/3} \tag{24}$$

Where Q is the electric charge quantum number.
Since $g = 1,2,3$ it follows there are 3 generations of quark doublets. $Y'_W = -1$ results in 3 generations of anti-quark doublets.

Particles of Standard Model
Spin ½ Leptons

For linear dependent matrix columns:

$$\begin{gathered}\{\zeta_{1gc}\}: \; c \in \{(1,2),(1,3),(2,3)\} \\ \{\zeta_{2gc}\}: c \in \{(2,1),(3,1),(3,2)\}\end{gathered} \tag{25}$$

The 3 doublets transform under emergent symmetries

$$SU_L(2) \times U_Y(1) \tag{26}$$

Weak-Hypercharge $Y_W = 2(Q - T_3)$ is **[3]** generalised to Y'_W

$$Y'_W = 2(Q' - \sum_c T_{3c}) \tag{27}$$

One independent doublet, c=1

$$Y'_W = -1 \qquad Q = Q'/3,\ Q = \binom{0}{-1/3} \tag{28}$$

Summation over 3 spinor doublets gives the electric charge quantum numbers $Q = \binom{0}{-1}$
Since $g = 1,2,3$ it follows there are 3 generations of lepton doublets. $Y'_W = 1$ results in 3 generations of anti-lepton doublets.

Particles of Standard Model
Spin 0, scalars

Each row of the matrix Φ does not form 3d isotropic vectors, then the corresponding Hermitean matrix X where $X^2 \neq 0$, is equivalent to the condition:

$$\sum_f \Phi_{gf}^2 = k \quad k \in \mathbb{R}_{>0} \qquad 29$$

Each row g has 3 complex scalar doublets

$$\begin{gathered}\{\phi_{1gc}\}: \ c \in \{(1,2),(1,3),(2,3)\} \\ \{\phi_{2gc}\}: c \in \{(2,1),(3,1),(3,2)\}\end{gathered} \qquad 30$$

Linear dependent scalar doublets (30) transform under $SU_L(2) \times U_Y(1)$. Linear sum of the 3 scalar doublets gives scalar doublets with electric charge quantum numbers

$$Q \in \left\{\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}0\\-1\end{pmatrix}\right\} \qquad 31$$

The scalar doublets (30) will need to be multiplied by a factor with dimension $L^{-1/2}$
Note that (29) implies that not all scalars can go to zero, hence there is a non zero ground state for scalars.

Schwarzschild space-time
Physical singularity

For the Schwarzschild metric **[4]** the spinor ψ is

$$\psi = Ae\left(-\left(1-\frac{r_s}{r}\right)^{-1}\frac{\dot{r}^2}{c^2} - \frac{r^2}{c^2}\dot{\Omega}^2 + \left(1-\frac{r_s}{r}\right)\right) \qquad 32$$

Where $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$
As $r \to 0$ the amplitude tends to 0 thus the probability of a particle tends to 0 as $r \to 0$ hence there is no physical singularity. The amplitude tends to zero at the event horizon $r = r_s$ and therefore the probability that a particle is located at the event horizon also tends to zero. Thus a particle has to tunnel across the event horizon.

Conclusion

The hypothesis that Nature at the fundamental level is an infinite dimensional random scalar field allows the existence of spinors and scalars with undetermined phases. The case of 3d random scalar field leads to 3 generations of chiral quarks and leptons and to the scalar doublets of the Standard Model**.** Particular phase transformations results in the removal of the Schwarzschild physical singularity and an equation of motion for chiral spinors of the same form as the Dirac equation.

References

[1] Elie Cartan (1981) [1938] The Theory Of Spinors
[2] Quang Ho-Kim, Pham Xuan Yem, [1998] Elementary Particles and Their Interactions p272
[3] Quang Ho-Kim, Pham Xuan Yem, [1998] Elementary Particles and Their Interactions p311
[4] L.D. Landau and E.M. Lifshitz, [2002] The Classical Theory of Fields p323