The Simplest Elementary Mathematics Proving Method of Fermat's Last Theorem

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Abstract: In this paper the author gives a simplest elementary mathematics method to solve the famous ***Fermat's Last Theorem*** (FLT), in which let this equation become a one unknown number equation, in order to solve this equation the author invented a method called “Order reducing method for equations” where the second order root compares to one order root and with some necessary techniques the author successfully proved ***Fermat's Last Theorem***.

1. Some Relevant Theorems

There are some theorems for proving or need to be known. ***All symbols in this paper represent positive integers unless stated they are not.***

Theorem 1.1. In the equation of

$$\begin{cases} x^n + y^n = z^n \\ \gcd(x, y, z) = 1 \\ n > 2 \end{cases} \tag{1-1}$$

x, y, z meet $x \neq y$, $x + y > z$ and if $x > y$ then $z > x > y$.

Proof: Let

$x = y$,

we have

$$2x^n = z^n$$

and

$$\sqrt[n]{2}x = z$$

where $\sqrt[n]{2}$ is not an integer and x, z are all positive integers, so $x \neq y$. Since

$$(x + y)^n = x^n + C_n^1 x^{n-1} y + ... + C_n^{n-1} x y^{n-1} + y^n > z^n ,$$

so we get

$x + y > z.$

Since

$$x^n + y^n = z^n ,$$

so we have

$$z^n > x^n , z^n > y^n$$

and get

$$z > x > y$$

when

$$x > y\ .$$

Theorem 1.2. In the equation of (1-1), x, y, z meet

$$\gcd(x, y) = \gcd(y, z) = \gcd(x, z) = 1.$$

Proof: Since $x^n + y^n = z^n$, if $\gcd(x, y) > 1$ then we have $\left(x_1^{\ n} + y^n{}_1\right) \times \left[\gcd(x, y)\right]^n = z^n$ which causes $\gcd(x, y, z) > 1$ since the left side contains the factor of $\left[\gcd(x, y)\right]^n$ then the right side must also contains this factor but contradicts against (1-1) in which $\gcd(x, y, z) = 1$, so we have $\gcd(x, y) = 1$. Using the same way we have $\gcd(x, z) = \gcd(y, z) = 1$.

Theorem 1.3. If there is no positive integer solution for

$$x^p + y^p = z^p$$

when $p > 2$ is a prime number then there is also no positive integer solution for

$$\left(x^p\right)^k + \left(y^p\right)^k = \left(z^p\right)^k .$$

Proof: Since $x^p + y^p = z^p$ has no positive integer solution, so there still no positive integer solution for

$$\left(x^k\right)^p + \left(y^k\right)^p = \left(z^k\right)^p$$

which means there is also no positive integer solution for

$$\left(x^p\right)^k + \left(y^p\right)^k = \left(z^p\right)^k .$$

So we only need to prove there is no positive integer solution for equation (1-1) when n is a prime number.

Theorem 1.4. In the equation of (1-1), x, y, z meet

$$x^{n-i} + y^{n-i} > z^{n-i}$$

where

$$n > i \geq 1.$$

Proof: From equation (1-1), since

$$x^n + y^n = z^n ,$$

let $x > y$, so we have

$$x^{n-i}+y^{n-i}>\left[\left(\frac{x}{z}\right)^i x^{n-i}+\left(\frac{y}{z}\right)^i y^{n-i}=z^{n-i}\right],$$

from **Theorem 1.1** we know $z>x>y$, so we have

$$x^{n-i}+y^{n-i}>z^{n-i}.$$

Theorem 1.5. In **Figure 1-1**, x, y, z of equation (1-1) meet

$$\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}\le 1.$$

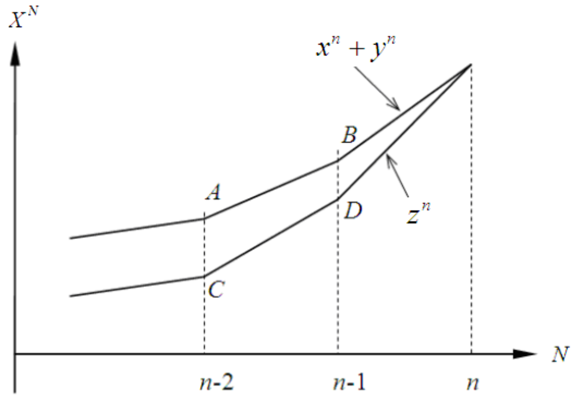

Figure 1-1 *Graph for* $x^n+y^n=z^n$

Proof: Obviously the meaning of $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}\le 1$ is the slope of AB is not greater than that of CD and if $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}=1$ then the slope of AB equals to that of CD.

It is necessary to point out that there is a positive real number R that meets equation

$$\frac{dx^N}{dN}+\frac{dy^N}{dN}=\frac{dz^N}{dN},$$

where

$$x^R\ln x+y^R\ln y=z^R\ln z,$$

Obviously the "Slope" of x^N+y^N equals to that of z^N when $N=R$. There are three cases for R in **Figure 1-1** when $R\le n-2, n-2<R\le n-1$ and $R>n-1$. If $R\le n-2$ then it is very clear that $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}<1$. If $n-2<R\le n-1$ then $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}\le 1$ is

possible and $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}>1$ is also possible. If $R>n-1$ then $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}>1$.

If $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}>1$ then are three cases have to be considered. The first case (**Case I**) is there is a positive real number $0<r<1$ for $n-r$ between $n-1$ and n whose slope equals to that of AB which means

$$x^{n-1}+y^{n-1}-x^{n-2}-y^{n-2}=\frac{z^{n-r}-z^{n-1}}{1-r}=\frac{\left(z^{1-r}-1\right)z^{n-1}}{1-r}$$

that can be explained by **Figure 1-2** where $AB // DF$.

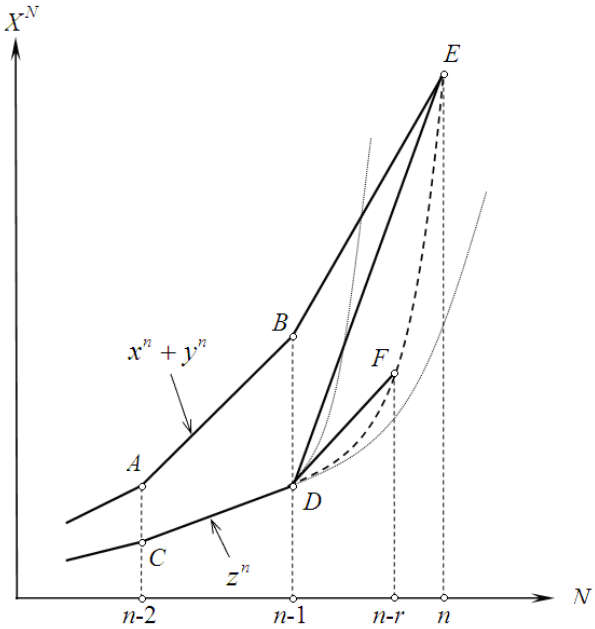

Figure 1-2 *Graph of* $x^n+y^n=z^n$ *when* $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}>1$

and point F *is between* n-1 *and* n *for* ***Case I***

The second case (**Case II**) is there is a positive real number $0<r<1$ for $n-r$ between $n-1$ and $n-2$ whose slope equals to that of AB which means

$$x^{n-1}+y^{n-1}-x^{n-2}-y^{n-2}=\frac{z^{n-1}-z^{n-r-1}}{r}=\frac{\left(1-z^{-r}\right)z^{n-1}}{r},$$

that can be explained by **Figure 1-3** where $AB // DF$.

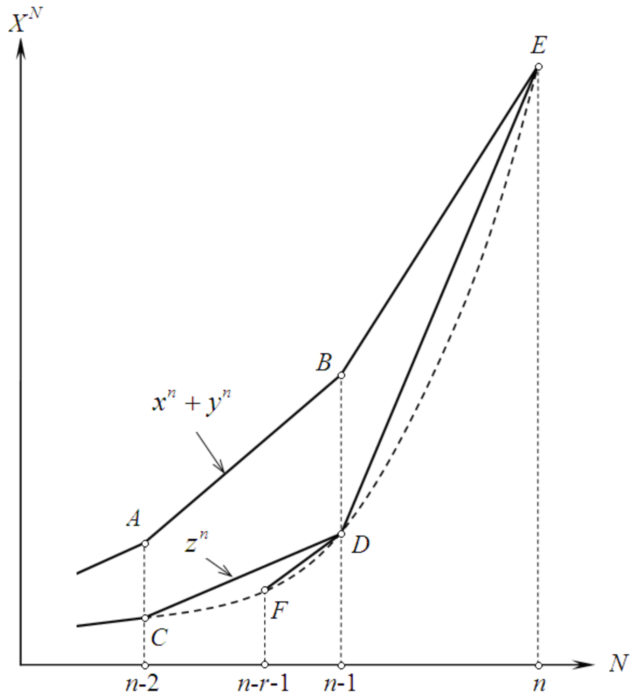

Figure 1-3 *Graph of* $x^n + y^n = z^n$ *when* $\dfrac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}} > 1$

and point F is between n-2 and n-1 for ***Case II***

The third case (**Case III**) is there is a tangent line of curve z^n at D that is $D'DF$ whose slope equals to that of AB which means

$$x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \frac{dz^N}{dN}\Big|_{N=n-1}$$

that can be explained by **Figure 1-4** where $AB \,//\, D'DF$.

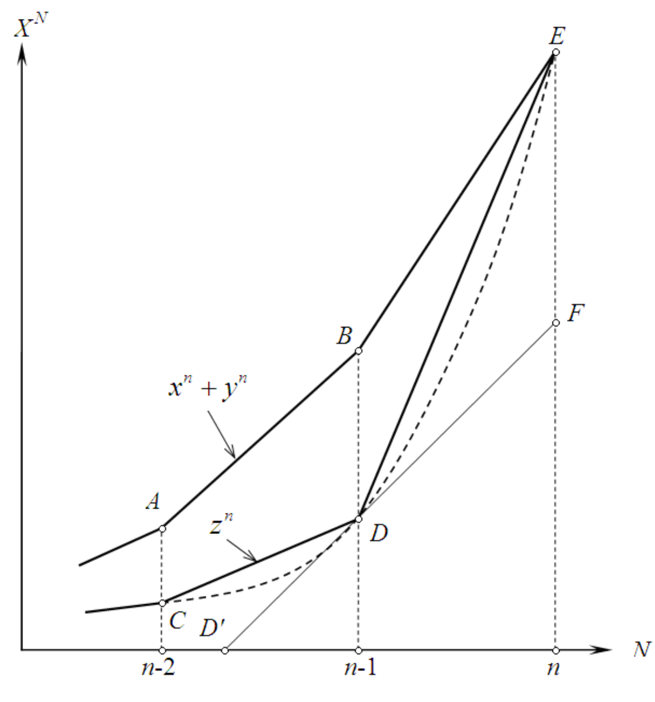

Figure 1-4 *Graph of* $x^n + y^n = z^n$ *when* $\frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} > 1$

and $D'DF$ *is a tangent line of curve* z^n *for **Case III***

Case I : In **Figure 1-2** we have

$$x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \left(\frac{z^{1-r} - 1}{1-r}\right) z^{n-1},$$

and

$$x^{n-1} + y^{n-1} - z^{n-1} - x^{n-2} - y^{n-2} = \left(\frac{z^{1-r} - 1}{1-r}\right) z^{n-1} - z^{n-1} = \left(\frac{z^{1-r} + r - 2}{1-r}\right) z^{n-1}. \qquad (1\text{-}2)$$

If we treat r as constant then $f(z) = \frac{z^{1-r} + r - 2}{1-r}$ is a "Monotonically increasing function"; if we treat z as constant then $f(r) = \frac{z^{1-r} + r - 2}{1-r}$ is a "Monotonically decreasing function" that can be explained by **Figure 1-5**.

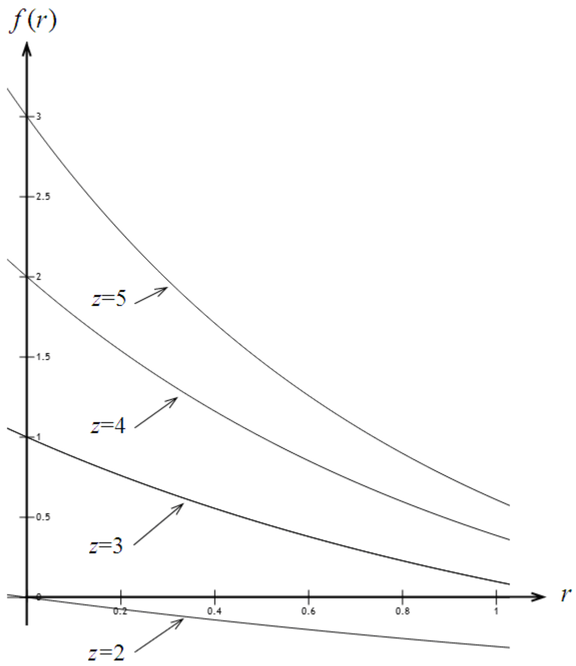

Figure 1-5 *Graph of* $f(r) = \frac{z^{1-r} + r - 2}{1-r}$ *when* $z = 2,3,4,5$

The reason why $f(r) = \frac{z^{1-r} + r - 2}{1-r}$ is a "Monotonically decreasing function" is because:

$$f'(r)=\frac{d\left(\dfrac{z^{1-r}+r-2}{1-r}\right)}{dr}=\frac{\left(-z^{1-r}\ln z+1\right)(1-r)+z^{1-r}+r-2}{(1-r)^2}$$

$$=\frac{-z^{1-r}\ln z(1-r)+z^{1-r}-1}{(1-r)^2}=\frac{\left[(r-1)\ln z+1\right]z^{1-r}-1}{(1-r)^2}.$$

For function

$$g(z)=\frac{\left[(r-1)\ln z+1\right]z^{1-r}-1}{(1-r)^2},$$

it is a "Monotonically decreasing function" since

$$g'(z)=\frac{d\left\{\dfrac{\left[(r-1)\ln z+1\right]z^{1-r}-1}{(1-r)^2}\right\}}{dz}=\frac{\dfrac{(r-1)}{z}z^{1-r}+(1-r)z^{-r}\left[(r-1)\ln z+1\right]}{(1-r)^2}$$

$$=-z^{-r}\ln z<0.$$

For function

$$g(r)=\frac{\left[(r-1)\ln z+1\right]z^{1-r}-1}{(1-r)^2},$$

we give the plot of it in **Figure 1-6**, in which it shows that $g(r)\neq 0, g(r)<0$ that is because

$$\lim_{r\to\infty}\left\{g(r)=\frac{\left[(r-1)\ln z+1\right]z^{1-r}-1}{(1-r)^2}\right\}=\lim_{r\to\infty}\frac{\left[(r-1)\ln z+1\right]z}{(1-r)^2z^r}$$

where

$$\lim_{r\to\infty}(1-r)^2z^r=\infty$$

$$\lim_{r\to\infty}\left[(r-1)\ln z+1\right]z=\infty,$$

and

$$\lim_{r\to\infty}\frac{\left[(r-1)\ln z+1\right]z}{(1-r)^2z^r}=\lim_{r\to\infty}\frac{\dfrac{d\left[(r-1)\ln z+1\right]z}{dr}}{\dfrac{(1-r)^2z^r}{dr}}=\lim_{r\to\infty}\frac{z\ln z}{\left[(1-r)\ln z-2\right](1-r)z^r}=0,$$

which means $g(r)$ has no finite value to intersect axis r and $g(r)\neq 0, g(r)<0$, since when $0<r<1$ the value of $g(r)$ is less than 0 and $g(z)$ is a "Monotonically decreasing function" , so $f(r)$ is a "Monotonically decreasing function" when $0<r<1$(*we have to say because we can not solve "Exponent equation" where the "Exponent" is the unknown number, so the solutions have to be found in numerical way, which is just "Function plot" does*).

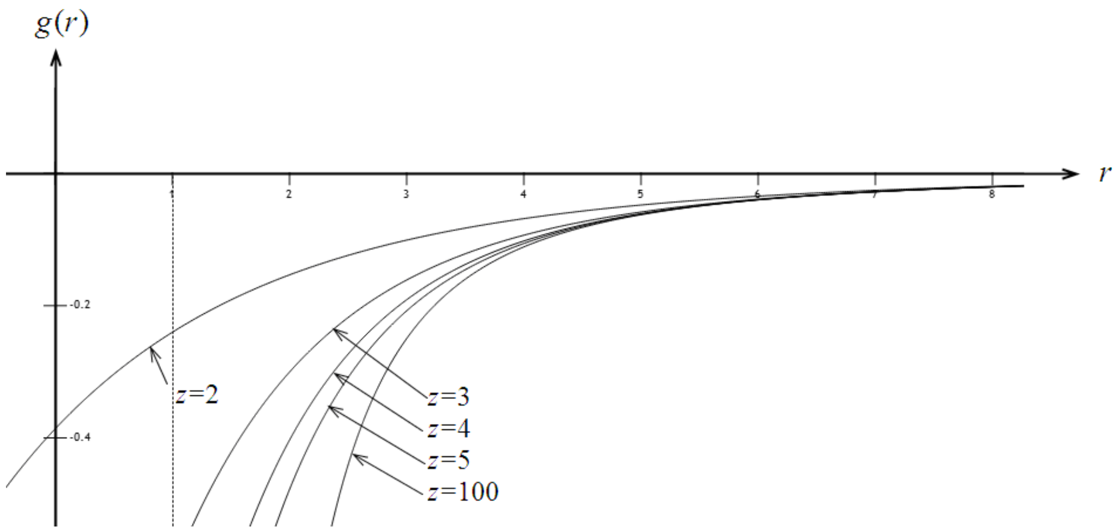

Figure 1-6 *Graph of* $g(r)=\dfrac{[(r-1)\ln z+1]z^{1-r}-1}{(1-r)^2}$ *when* $z=2,3,4,5,100$

From (1-2) we know if z (a positive real number) increases then the left side decreases and the right side also decreases. The minimum value for the right side is

$$\lim_{r\to 1}\left(\frac{z^{1-r}+r-2}{1-r}\right)z^{k-1}=\lim_{r\to 1}\left[\frac{\dfrac{d\left(z^{1-r}+r-2\right)}{dr}}{\dfrac{d(1-r)}{dr}}\right]z^{k-1}=\lim_{r\to 1}\left(\frac{-z^{1-r}\ln z+1}{-1}\right)z^{k-1},$$

$$=\lim_{r\to 1}\left(z^{1-r}\ln z-1\right)z^{k-1}=(\ln z-1)z^{k-1}$$

since

$$\begin{cases}\lim\limits_{r\to 1}\left(z^{1-r}+r-2\right)=0\\ \lim\limits_{r\to 1}(1-r)=0\end{cases}.$$

From **Theorem 1.8** we know $z\geq 4$, so we get

$$\left[\lim_{r\to 1}\left(\frac{z^{1-r}+r-2}{1-r}\right)z^{n-1}=(\ln z-1)z^{n-1}\right]>(\ln 4-1)\times 4^2>9.$$

From (1-2) we have

$$\left(x^{n-1}+y^{n-1}-z^{n-1}\right)-\left(x^{n-2}+y^{n-2}-z^{n-2}\right)=\left(\frac{z^{1-r}+r-2}{1-r}\right)z^{n-1}+z^{n-2},$$

where both sides plus z^{n-2}, in **Figure 1-2** we know

$$x^{n-1}+y^{n-1}-z^{n-1}=BD,$$

$$x^{n-2}+y^{n-2}-z^{n-2}=AC,$$

there must exist a situation in **Figure 1-2** when we increase z (a positive real number) that causes

$$BD \to AC, BD > AC, r < 1,$$

so the left side is almost 0 but the right side is bigger than $9 + z^{n-2} \geq (9 + 4 = 13)$, that is a contradiction which means there are no positive integer solutions of equation (1-1) at **Case I**.

Case II : In **Figure 1-3** we have

$$x^{n-1} + y^{n-1} - x^{n-2} - y^{n-2} = \frac{(1 - z^{-r})z^{n-1}}{r},$$

and

$$x^{n-1} + y^{n-1} - z^{n-1} - x^{n-2} - y^{n-2} = \left(\frac{1 - z^{-r}}{r}\right)z^{n-1} - z^{n-1} = \left(\frac{1 - z^{-r} - r}{r}\right)z^{n-1}. \tag{1-3}$$

If we treat r as constant then $f(z) = \dfrac{1 - z^{-r} - r}{r}$ is a “Monotonically increasing function”; if we treat z as constant then $f(r) = \dfrac{1 - z^{-r} - r}{r}$ is a “Monotonically decreasing function” that can be explained by **Figure 1-7**.

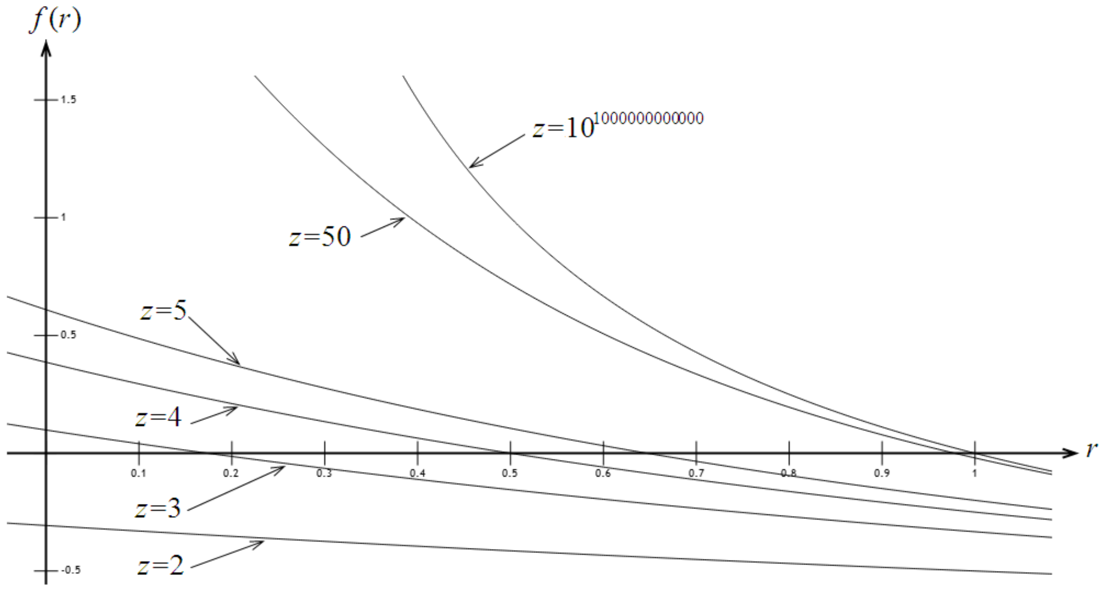

Figure 1-7 *Graph of* $f(r) = \dfrac{1 - z^{-r} - r}{r}$ *when* $z = 2,3,4,5,50,10^{1000000000000}$

The reason why $f(r) = \dfrac{1 - z^{-r} - r}{r}$ is a “Monotonically decreasing function” is because:

$$f'(r)=\frac{d\left(\frac{1-z^{-r}-r}{r}\right)}{dr}=\frac{rz^{-r}\ln z-r-\left(1-z^{-r}-r\right)}{r^2}=\frac{\left(r\ln z+1\right)z^{-r}-1}{r^2}.$$

For function

$$g(z)=\frac{\left(r\ln z+1\right)z^{-r}-1}{r^2},$$

it is a "Monotonically decreasing function" since

$$g'(z)=\frac{d\left[\frac{\left(r\ln z+1\right)z^{-r}-1}{r^2}\right]}{dz}=\frac{\left[\frac{r}{z}-r\left(r\ln z+1\right)\right]z^{-r}}{r^2}<0,$$

in which from **Theorem 1.8** we know $z\geq 4$, so we have $\frac{r}{z}-r\left(r\ln z+1\right)<0$ where $\frac{r}{z}<r$ and $r^2\ln z>0$.

For function $g(r)=\frac{\left(r\ln z+1\right)z^{-r}-1}{r^2}$, we plot the graph of it in **Figure 1-8**, in which it shows that $g(r)\neq 0$ and $g(r)<0$ that is because:

$$\lim_{r\to\infty}\left[g(r)=\frac{\left(r\ln z+1\right)z^{-r}-1}{r^2}\right]=\lim_{r\to\infty}\frac{\left(r\ln z+1\right)}{r^2z^r}$$

where

$$\lim_{r\to\infty}\left(r\ln z+1\right)=\infty$$

$$\lim_{r\to\infty}r^2z^r=\infty,$$

and

$$\lim_{r\to\infty}\frac{\left(r\ln z+1\right)}{r^2z^r}=\lim_{r\to\infty}\frac{\frac{d\left(r\ln z+1\right)}{dr}}{\frac{r^2z^r}{dr}}=\lim_{r\to\infty}\frac{\ln z}{2rz^r+r^2z^r\ln z}=0$$

which means $g(r)$ has no finite value to intersect axis r and $g(r)\neq 0, g(r)<0$, since when $0<r<1$ the value of $g(r)$ is less than 0 and $g(z)$ is a "Monotonically decreasing function", so $f(r)$ is a "Monotonically decreasing function" when $0<r<1$.

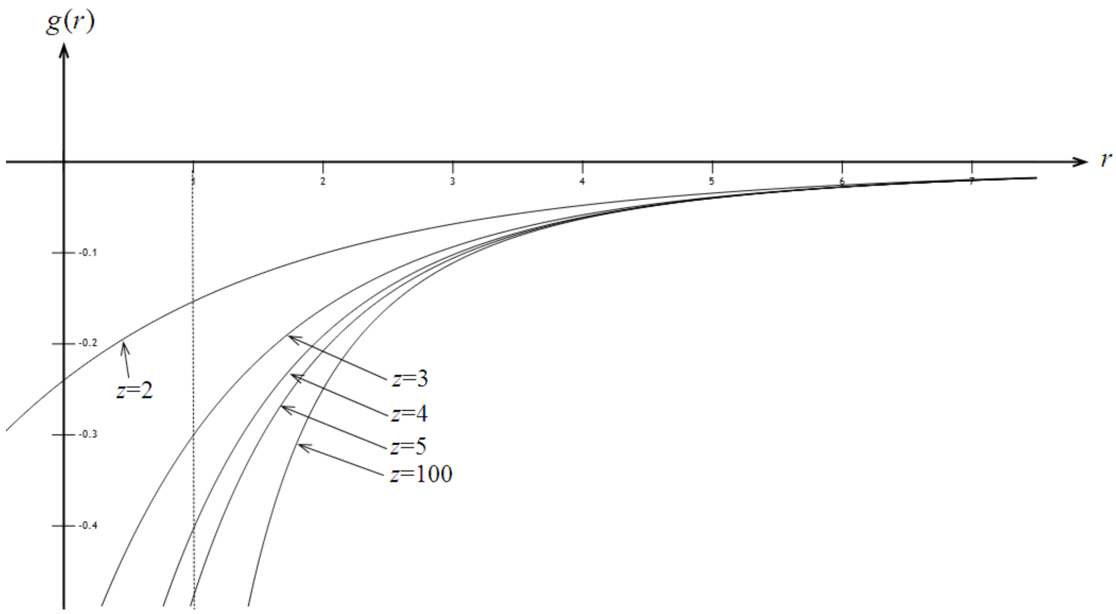

Figure 1-8 *Graph of* $g(r)=\dfrac{(r\ln z+1)z^{-r}-1}{r^2}$ *when* $z=2,3,4,5,100$

From **Figure 1-3** we know if z (a positive real number) increases then r also increases. From (1-3) we have

$$\left(x^{n-1}+y^{n-1}-z^{n-1}\right)-\left(x^{n-2}+y^{n-2}-z^{n-2}\right)=\left(\frac{1-z^{-r}-r}{r}\right)z^{n-1}+z^{n-2},$$

where both sides plus z^{n-2}, in **Figure 1-3** we know

$$x^{n-1}+y^{n-1}-z^{n-1}=BD,$$

$$x^{n-2}+y^{n-2}-z^{n-2}=AC,$$

there must exist a situation when we increase z (a positive real number) that causes

$$BD\to AC, BD>AC, r\to 1, r<1,$$

so the left side is

$$\left(x^{n-1}+y^{n-1}-z^{n-1}\right)-\left(x^{n-2}+y^{n-2}-z^{n-2}\right)=0_{+}>0,$$

but the right side is

$$\lim_{\substack{r\to 1\\ r<1}}\left[\left(\frac{1-z^{-r}-r}{r}\right)z^{n-1}+z^{n-2}\right]=\lim_{\substack{r\to 1\\ r<1}}\left(-z^{n-1-r}+z^{n-2}\right)=0_{-}\le 0,$$

which means the right side is less than the left side, so this is a contradiction which means there are no positive integer solutions of equation (1-1) at **Case II**.

Case III : In **Figure 1-4** we have

$$x^{n-1}+y^{n-1}-x^{n-2}-y^{n-2}=\frac{dz^N}{dN}\Big|_{N=n-1}=z^{n-1}\ln z\,,$$

and

$$x^{n-1}+y^{n-1}-z^{n-1}=z^{n-1}\ln z-z^{n-1}+x^{n-2}+y^{n-2}=(\ln z-1)z^{n-1}+x^{n-2}+y^{n-2}\,,$$

that is impossible since for any positive integer solutions of equation (1-1) when z increases then the left side is becoming smaller but the right side is becoming bigger(*since from* ***Theorem 1.8*** *we know* $z\geq 4$, *so* $(\ln z-1)>0$) which is a contradiction, so there are no positive integer solutions of equation (1-1) at **Case III**.

So from **Case I**, **Case II** and **Case III** we have the conclusion of $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}>1$ is impossible and $\frac{x^{n-1}+y^{n-1}-z^{n-1}}{x^{n-2}+y^{n-2}-z^{n-2}}\leq 1$.

Using the same way we can prove

$$\frac{x^{n-j}+y^{n-j}-z^{n-j}}{x^{n-j-1}+y^{n-j-1}-z^{n-j-1}}\leq 1,$$

where j is a positive integer that can be explained by **Figure 1-9** in which $w=n-j$, $w>2,\ AC\geq BE,\ EF\geq GH$.

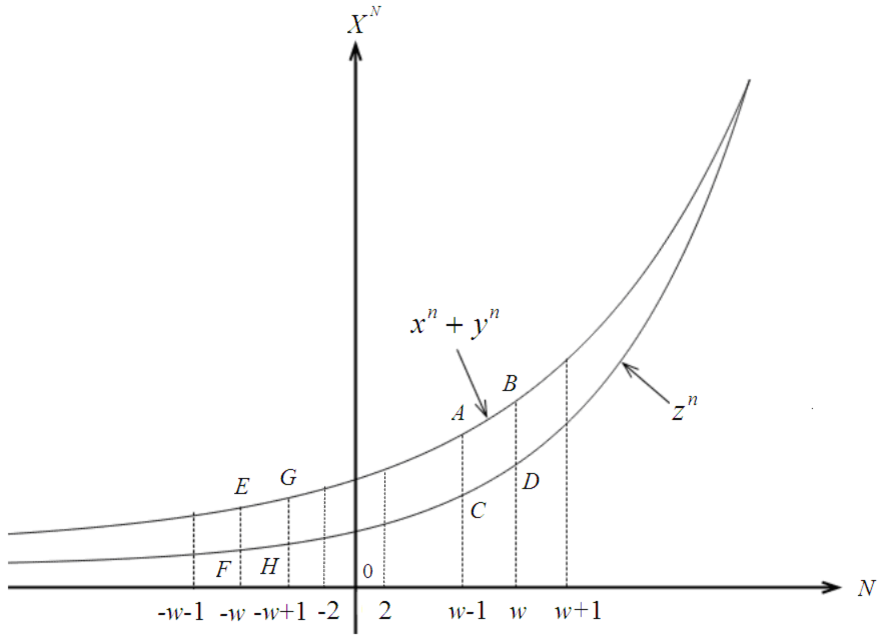

Figure 1-9 *Graph of* $x^n+y^n=z^n$ *when* $\frac{x^{n-j}+y^{n-j}-z^{n-j}}{x^{n-j-1}+y^{n-j-1}-z^{n-j-1}}\leq 1$ *where* j *is a*

positive integer and $w = n - j$

In **Figure 1-9** it is obvious to see that the maximum value of $x^{n-j} + y^{n-j} - z^{n-j}$ is at $j = \infty$ since $\dfrac{x^{n-j} + y^{n-j} - z^{n-j}}{x^{n-j-1} + y^{n-j-1} - z^{n-j-1}} \le 1$, and $\lim\limits_{j\to\infty}\left(x^{n-j} + y^{n-j} - z^{n-j}\right) = 0$, but it is also very clear even the value of $x^{n-j} + y^{n-j} - z^{n-j}$ when $j = n+2$ is bigger than that (*since there are two intersections of* $x^n + y^n, z^n$ *although one of them is at* $j = \infty$, *so the curves of* $x^n + y^n, z^n$ *is "Closed" like "Crescent moon" and each* $x^{n-j} + y^{n-j} - z^{n-j}$ *is bigger than 0 when* j *is finite*), so this is a contradiction which means there are no positive integer solutions for equation (1-1) (*this is also the simplest proof that Fermat forgot to write on his narrow margin book and which people have being looking for nearly 400 years*).

The reason why $w > 2$ is because there are three equivalent equations

$$x^2 + y^2 = z^2,$$
$$x + y = z,$$
$$x^{-1} + y^{-1} = z^{-1},$$

for **Case I** or **Case II**, one of their BD, AC is a fixed positive integer or rational number, in order to meet $BD = AC$ (*that means* $BD \to AC, BD > AC$), z must be a positive integer or rational number, but we are not sure about whether there exist a "Positive integer or rational number z" to meet $BD \to AC, BD > AC$ or not, but we do sure about when $w > 2$ there must exist a positive real number z that meets $BD \to AC, BD > AC$ (*for example,* $\pi^{\pi+\sqrt{\pi}}$, *what we are sure of is it is a positive real number, if you tell me it is a positive integer or rational number, then we are not sure about this saying. So it is with the case of a positive real number* z *that meets* $BD \to AC, BD > AC$, *which means there must exist a positive real number* z *that meets* $BD \to AC, BD > AC$, *but we can not say a positive integer or rational number* z *must exist to meet* $BD \to AC, BD > AC$). On the other hand, since we only consider $n > 2$ for equation (1-1), so it is not necessary to consider the situation of $n = 2$. And also for "Order reducing method for equations" that will be explained below, the least number for n is 3.

Theorem 1.6. There are no positive integer solutions for

$$1^n + y^n = z^n.$$

Proof: Since

$$1 = z^n - y^n = (z-y)\left(z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1}\right)$$

where

$$\begin{cases} z - y = 1 \\ \left(z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1}\right) = 1 \end{cases}$$

that causes z, y to be non positive integers, so there are no positive integer solutions for

$$1^n + y^n = z^n .$$

Theorem 1.7. There are no positive integer solutions for

$$2^n + y^n = z^n .$$

Proof: Since

$$2^n = z^n - y^n = (z-y)\left(z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1}\right),$$

if

$$\begin{cases} z - y = 1 \\ z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1} = 2^n \end{cases}$$

then taking the least value for $y = 2, z = 3$, we have

$$3^{n-1} + 2 \times 3^{n-2} + \ldots + 2^{n-1} > 2^n$$

when $n > 2$ that is impossible. If

$$\begin{cases} z - y = 2^i \\ z^{n-1} + z^{n-2}y + \ldots + zy^{n-2} + y^{n-1} = 2^j \\ i + j = n \\ i \geq 1 \end{cases}$$

then $z > 2$ and taking the least value of $y = 2, z = 3$, we get

$$3^{n-1} + 2 \times 3^{n-2} + \ldots + 2^{n-1} > 2^j$$

with $n > 2$ that is also impossible, so there are no positive integer solutions for

$$2^n + y^n = z^n .$$

Theorem 1.8. There are no positive integer solutions for equation (1-1) when $n \to \infty$ and x, y, z in equation (1-1) meet

$z < \sqrt[n]{2}x,$
$x > 2,$
$y > 1,$
$z > 3.$

Proof: Since $x^n + y^n = z^n$, let $x > y$, we get

$$\left(\frac{z}{x}\right)^n - \left(\frac{y}{x}\right)^n = 1,$$

since

$$z > x > y,$$

so we have

$$z < \sqrt[n]{2}x,$$

and

$$\lim_{n\to\infty}\left(\frac{z}{x}\right)^n - \left(\frac{y}{x}\right)^n = \infty > 1$$

which means there are no positive integer solutions for equation (1-1) when $n \to \infty$. And according to **Theorem 1.1, 1.6** we have $x > 2, y > 1, z > 3$.

2. Proving Method

In equation (1-1), let

$$\begin{cases} a = x^{n-2} \\ b = y^{n-2} \\ c = z^{n-2} \end{cases},$$

we have

$$\begin{cases} ax^2 + by^2 = cz^2 \\ a^{\frac{n-1}{n-2}}x + b^{\frac{n-1}{n-2}}y = c^{\frac{n-1}{n-2}}z \end{cases}. \tag{2-1}$$

Since we reduce the order of equation so the method is called “Order reducing method for equations”.

Let $x > y$ and

$$\begin{cases} y = x - f \\ z = x + e \end{cases}. \tag{2-2}$$

From (2-1) and (2-2) we have

$$\begin{cases} ax^2 + b(x-f)^2 = c(x+e)^2 \\ a^{\frac{n-1}{n-2}}x + b^{\frac{n-1}{n-2}}(x-f) = c^{\frac{n-1}{n-2}}(x+e) \end{cases}$$

and

$$\begin{cases} (a+b-c)x^2 - 2(bf+ce)x + \left(bf^2 - ce^2\right) = 0 \\ a^{\frac{n-1}{n-2}}x + b^{\frac{n-1}{n-2}}(x-f) - c^{\frac{n-1}{n-2}}(x+e) = 0 \end{cases},$$

the roots are

$$x = \frac{(bf+ce) \pm \sqrt{(bf+ce)^2 - (a+b-c)\left(bf^2 - ce^2\right)}}{x^{n-2} + y^{n-2} - z^{n-2}}, \qquad (2\text{-}3)$$

and

$$x = \frac{c^{\frac{n-1}{n-2}}e + b^{\frac{n-1}{n-2}}f}{a^{\frac{n-1}{n-2}} + b^{\frac{n-1}{n-2}} - c^{\frac{n-1}{n-2}}} = \frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}}. \qquad (2\text{-}4)$$

There are two cases for bf^2, ce^2 when $bf^2 \geq ce^2$ and $bf^2 < ce^2$.

Case A: If $bf^2 \geq ce^2$, from (2-3) when

$$x = \frac{(bf+ce) + \sqrt{(bf+ce)^2 - (a+b-c)\left(bf^2 - ce^2\right)}}{x^{n-2} + y^{n-2} - z^{n-2}},$$

From **Theorem 1.4** we know $a + b - c = x^{n-2} + y^{n-2} - z^{n-2} > 0$, so we have

$$x \leq \frac{2(bf+ce)}{x^{n-2} + y^{n-2} - z^{n-2}},$$

and also from **Theorem 1.4** we have $x^{n-1} + y^{n-1} - z^{n-1} > 0$, compare to (2-4) we get

$$\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} \leq \frac{2(bf+ce)}{x^{n-2} + y^{n-2} - z^{n-2}}.$$

From **Theorem 1.5** we know $\dfrac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} \leq 1$, so we have

$$bfy + cez \leq 2(bf+ce)$$

that is impossible since from **Theorem 1.8** we know $y \geq 2$ and $z > 3$.

When

$$x = \frac{(bf + ce) - \sqrt{(bf + ce)^2 - (a + b - c)(bf^2 - ce^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}.$$

we have

$$x \le \frac{bf + ce}{x^{n-2} + y^{n-2} - z^{n-2}},$$

compare to (2-4) we get

$$\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} \le \frac{bf + ce}{x^{n-2} + y^{n-2} - z^{n-2}}.$$

From **Theorem 1.5** we have

$$bfy + cez \le bf + ce$$

that is impossible since from **Theorem 1.8** we have already known $y \ge 2$ and $z > 3$.

Case B: If $bf^2 < ce^2$, from (2-3) when

$$x = \frac{(bf + ce) + \sqrt{(bf + ce)^2 + (a + b - c)(ce^2 - bf^2)}}{x^{n-2} + y^{n-2} - z^{n-2}},$$

we can prove $(bf + ce)^2 > (a + b - c)(ce^2 - bf^2)$ since if not we have

$$(bf + ce)^2 \le (a + b - c)(ce^2 - bf^2)$$

and

$$[(2b + a) - c]bf^2 + 2bfce + [2c - (a + b)]ce^2 \le 0$$

that is impossible since $a + b - c > 0$ and $c > a, c > b, 2c - (a + b) > 0$. So we have

$$x < \frac{(bf + ce)(1 + \sqrt{2})}{x^{n-2} + y^{n-2} - z^{n-2}}$$

compare to (2-4) we get

$$\frac{bfy + cez}{x^{n-1} + y^{n-1} - z^{n-1}} < \frac{(bf + ce)(1 + \sqrt{2})}{x^{n-2} + y^{n-2} - z^{n-2}}.$$

From **Theorem 1.5** we have

$$bfy + cez < (bf + ce)(1 + \sqrt{2}) < 2.5(bf + ce)$$

and

$$bf(x - f) + ce(x + e) < 2.5(bf + ce)$$

that leads to

$$x < \left[\frac{2.5(bf + ce) + bf^2 - ce^2}{bf + ce} = 2.5 - \frac{ce^2 - bf^2}{bf + ce} \right] < 2.5$$

where possible values for x are 1, 2 but according to **Theorem 1.6**, **1.7** we know there are no positive integer solutions.

When

$$x = \frac{(bf + ce) - \sqrt{(bf + ce)^2 + (a + b - c)(ce^2 - bf^2)}}{x^{n-2} + y^{n-2} - z^{n-2}}$$

is not possible since $x \le 0$.

Now we have completely solved no positive integer solutions for equation (1-1) when $n > 2$ using "Order reducing method for equations".

3. Conclusion

Through the above contents we can see clearly that the proving of *Fermat's Last Theorem* is just a problem of elementary mathematics. "Order reducing method for equations" that the author invented is a very effective method in the proving of *Fermat's Last Theorem* and the author's technique in which let $y = x - f$ and $z = x + e$ is a very important step for solving.

Fermat's Last Theorem is a problem that has lasted for about 380 years. Proving methods are not important but the theorem's correctness is very necessary because many useful inferences can be deduced that are obviously better than "conjectures".

The author has been working on proving of *Fermat's Last Theorem* for quite some times (210 days) without any reference and many methods have been thought about, for example "Method of prime factorization" but not work. So the author has already known that there are no ways to solve except "Solving high order equations" which is also an important aspect in solving other mathematic problems.