Alternate Proof for z_n is Irrational

Timothy W. Jones

December 28, 2017

Introduction

$+1/4$							
$+1/9$	$+1/4$	$+1/4$	$+1/4$	$+1/4$	$\dots$	$+1/4$	
$\notin D_4$	$+1/9$	$+1/9$	$+1/9$	$+1/9$	$\dots$	$+1/9$	
	$\notin D_9$	$+1/16$	$+1/16$	$+1/16$		$\vdots$	
		$\notin D_{16}$	$+1/25$	$+1/25$		$\vdots$	
			$\notin D_{25}$	$+1/36$		$\vdots$	
				$\notin D_{36}$			
						$+1/(k-1)^2$	
						$+1/k^2$	
						$\notin D_{k^2}$	
							$\ddots$

Table 1: A list of all rational numbers between 0 and 1 modified to exclude them all via partial sums of z_2.

Table 1 gives a result from an article that claims to show $\zeta(n)$, $n > 1$ is irrational [1]. Here we give a proof that builds on this result and may be more convincing. It is based on Sondow's geometric proof of the irrationality of e [2].

Nested intervals

By the convergence of z_2, for every ϵ there is an N_ϵ such that $s_2^n - \epsilon < z_2 < s_2^n + \epsilon$ for all $n > N_\epsilon$, where

$$s_2^n = \sum_{k=2}^{n} \frac{1}{k^2}.$$

Using our visualization circles, we have corresponding radii that demark the upper and lower bounds of z_2 cast as angles. We also have by way of Table 1 knowledge that radii for s_2^n never have area values in D_{k^2}, where $2 \leq k \leq n$; that is, they never are single decimals in base k^2, where $2 \leq k \leq n$.

Next, we imagine all the radii associated with all the rational values generated by a partial and determine a lower, l, and upper bound, u, of rationals in D_{k^2} sets that are in $[s_2^n - \epsilon, s_2^n + \epsilon]$. We build intervals using ϵ values of $1/n$ (or any decreasing sequence converging to 0). We will use these values to define intervals $[l_n, u_n]$. These are nested intervals and must intersect at the point of convergence and yet the endpoints can't be the convergent point as no end point is in the intersection of all such intervals. The set of endpoints comprise the closest points in D_{k^2} sets and other points can be rejected. As the union of these D_{k^2} sets are all rational numbers, z_2 must be irrational.

References

[1] T.W. Jones, Visualizing $\zeta(n > 1)$ and Proving its Irrationality (2017), available at `available at http://vixra.org/abs/1710.0145`

[2] J. Sondow, A geometric proof that e is irrational and a new measure of its irrationality, *Amer. Math. Monthly*, **113**, (2007), 637–641.