The General Relevance of the Modified Cosmological Model

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May 21, 2018

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The stone the builders rejected has become the cornerstone.
– Psalm 118:22

I Introduction

In lieu of an abstract, each chapter in this book will have a description of its contents. This book is focused on recapping, consolidating, streamlining, and annotating previous work related to gravitation and non-relativistic quantum theory while adding a few new insights when they are modest. Throughout this book, the reader's familiarity with the modified cosmological model is assumed but not strictly required.

The focus of the first section in this chapter is a review of geometry. Section two gives a preliminary overview of an algorithm that will violate conservation of information. In section three we propose to modify Feynman's application of the action principle by replacing the least action complex field trajectories with maximum action hypercomplex field trajectories that still satisfy the action principle.

I.1 An Abstract Psychological Dimension

It is shocking that after this many years of work on the theory of infinite complexity that the associated material calculated and referred to here is not already well known with the entire field of all possible linear nuance being mapped out to the n^{th} degree. It is surprising that there is no Wikipedia article regarding the modified cosmological model (MCM) or the theory of infinite complexity (TOIC) that spells out all of the trivially derived properties. To that end, consider a cube spanned by $\hat{x}$, $\hat{y}$, and $\hat{z}$. The slices of constant z are the subspaces spanned by $\hat{x}$ and $\hat{y}$ at each value of z. Every curve that can be constructed using $\hat{x}$ and $\hat{y}$ will be confined to some slice of z. Any curve leaving the slice would have a component in the $\hat{z}$ direction. Likewise any curve constructed from just $\hat{x}$ and $\hat{y}$ will have its tangent vectors confined to that single slice of constant z. The curve's cotangent space is the first place we could possibly come across vectors with a non-vanishing $\hat{z}$ component. We state these obvious truths because the MCM describes de Sitter (dS) and Anti-de Sitter (AdS) spacetimes as slices of a 5D cube and we want to show the exceptional behavior of our flat universe when it sews together two 5D spaces but is not itself a slice of any 5D space.

Now consider flat empty 5-spaces $\Sigma^{\pm}$ where general relativity in the absence of 5D matter-energy leads to the desired dynamics in the 4D slices through the Kaluza-Klein metric

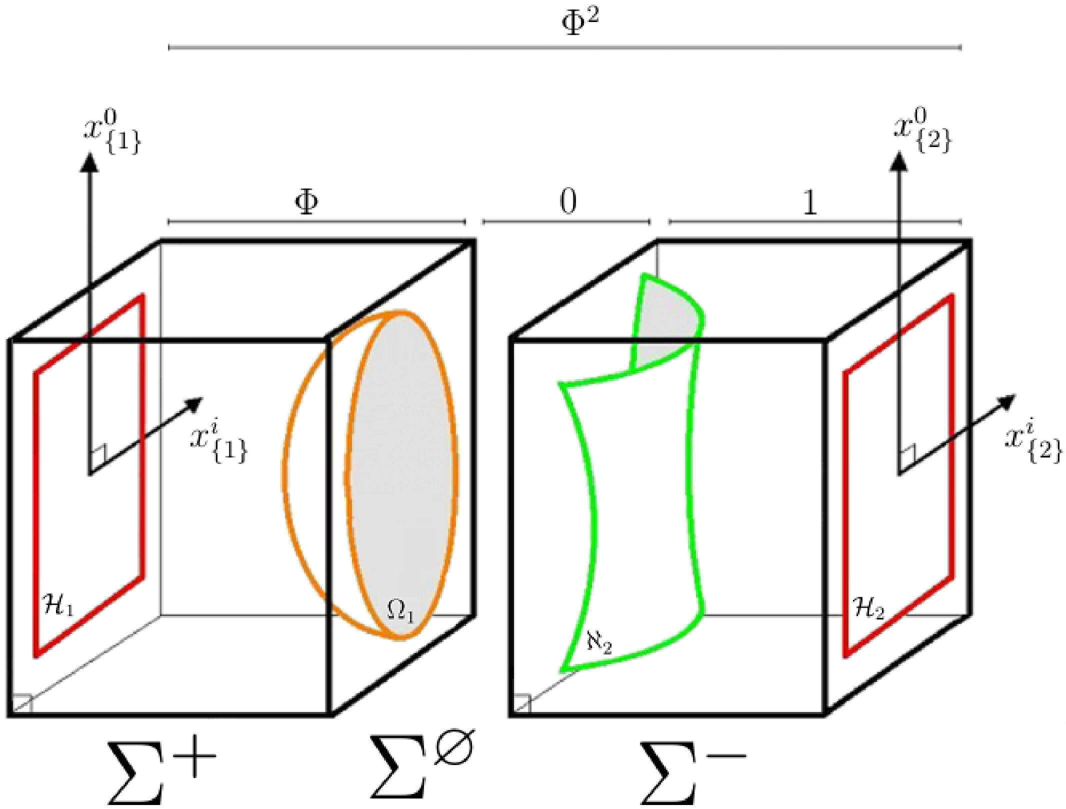


Figure 1: This figure shows the region between two adjacent moments of psychological time: $\mathcal{H}_1$ and $\mathcal{H}_2$. The arrangement immediately suggests a gravitational pilot wave formulation as the path to evolve through the discontinuity of the as-yet-undescribed region inside the null interval between Ω_1 and $\aleph_2$ but we will introduce another simpler formulation in this book. We will introduce new $\chi^A_\varnothing$ coordinates to accommodate this representation wherein $x^\mu \in \mathcal{H}$ are moved away from the center of the MCM unit cell where we have depicted them in previous work. χ^5 is the horizontal direction across this figure. This figure uses the values Φ^2, Φ, and 1 to demonstrate $\Sigma^\varnothing$ but due to the properties of the golden ratio there are many such arrangements.

$$\Sigma_{AB} = \begin{pmatrix} g_{\alpha\beta} + \phi^2 A_\alpha A_\beta & \phi^2 A_\alpha \\ \phi^2_\beta & \phi^2 \end{pmatrix} \quad . \tag{1.1}$$

In this book, we will use the Greek letter χ for the 5D coordinates where we have used ξ previously. Where Latin indices A have previously run from 0 to 4, here they will run from 1 to 5 so $\xi^4 \to \chi^5$. We will add a layer of complexity when we take $\mu, \nu \in \{0,1,2,3\}$ in the usual way but then add a subtle convention for $\alpha, \beta \in \{1,2,3,4\}$. In 5D, we have $A, B \in \{1,2,3,4,5\}$ and $\alpha, \beta \in \{1,2,3,4\}$ or $\dot\alpha, \dot\beta \in \{2,3,4,5\}$. Taking the coordinates of $\Sigma^\pm$ as $\chi^A_\pm$, we will call the bulk metrics $\Sigma^\pm_{AB}$ and they will have the form of equation (1.1). Curves in the flat slices of constant $\chi^5_\pm$ can never have tangent vectors that point to the left or right in the cosmological unit cell. (Figure 1 shows that cell.) The slices $\aleph$ and Ω are flat slices of χ^5 but they appear curved in this figure to demonstrate the curvature associated with the embedded metric of the de Sitter coordinates $x^\mu_\pm \neq \chi^\alpha_\pm$.

The oft-lamented "cylinder condition" that the MCM both embodies and motivates from first principles [1, 2, 3] says that physics in the 4D worldsheets spanned by $\chi^\mu_\pm$ can never depend on the fifth coordinate. This can be accomplished via a generalized disallowance of the appearance of $\chi^5_\pm$ in any equations of motion but we can accomplish the same thing by taking our 4D spacetimes as surfaces of constant $\chi^5_\pm$ in the 5D bulk [3]. The ordinary limitation of the cylinder condition on physics is that position and momentum measured in x^μ can never depend on x^4. However, that doesn't say anything about the abstract coordinates $\{\chi^A_+, \chi^A_\varnothing, \chi^A_-\}$ or vice versa.

Here, we begin to develop the complex behavior that can be derived by modeling our universe of x^μ at the interface of two 5D spaces $\Sigma^\pm$. This is a key point to notice: observables will always be defined on x^μ which can, in principle, depend on all of the χ^A coordinates. This contrasts the normal application of Kaluza–Klein theory which says x^μ cannot depend on x^4. Therefore, even at this early stage, it is apparent that the MCM is very different from the standard cosmological model and other Kaluza–Klein models. One well known issue with standard Kaluza–Klein theory is that the field equations indicate that the electromagnetic field strength tensor must always vanish with respect to 4D general relativity. By adding the 15 **chirological coordinates** $\{\chi^A_+, \chi^A_\varnothing, \chi^A_-\}$ we have a lot of room to develop novel workarounds. For instance, if the Kaluza–Klein requirement for vanishing electromagnetic strength tensors applies to the chirological coordinates then that puts only a loose constraint on what we do with the x^μ, $x^\mu_\varnothing$, and $x^\mu_\pm$ coordinates.

Let $\chi^5_\pm$ be non-relativistic psychological dimensions with identical topological flatness. The identical topological flatness of $\chi^5_\pm$ does not hold for $\chi^5_\varnothing$ which can have an arbitrary non-linear curvature with tangent vectors pointing anywhere because it has no width in the path from $\mathcal{H}_1$ to $\mathcal{H}_2$, as in figure 1. $\Sigma^\varnothing$ exists only to sew $\Sigma^\pm$ together with a single point so we are not concerned with the overall curvature there. There is no constrained object anywhere in the vector bundle of $\Sigma^\varnothing$ so everything about that bundle is introduced as a new MCM degree of freedom. The only constraint on $\Sigma^\varnothing$ is that it has to have at least one point where we can construct a Lorentz frame and then use that point to ensure smooth transport of a Lorentz frame from $\mathcal{H}_1$ to $\mathcal{H}_2$. The 4D slices of flat 5-space are flat but $\aleph$ and Ω, themselves slices, are curved, and what's more: the only flat space we do have, $\mathcal{H}$, isn't even a slice of a 5-space because $\Sigma^\pm$ do not contain their boundary at $\chi^5 = 0$ which specifies the location of $\mathcal{H}$ [3]. $\mathcal{H}$ is the unincluded boundary of two 5D half spaces. How can we get a curved slice out of a flat space? These new degrees of freedom beyond $\mathcal{H}$ will be helpful.

The addition of only one new degree of MCM freedom to go through larger infinity in the **hyperreal number system** $^*\mathbb{R}$ (via $\hat{\Phi}^n \to \hat{\Phi}^{n+1}$) leads to two new degrees of freedom: the two dimensions of $\mathbb{C}$ become hyperreal and **hyperimaginary**. We will name the system that contains hyperreal and hyperimaginary numbers as the **hypercomplex number system**[1] $^\star\mathbb{C}$. We point to hyperimaginarity as the reason for the fourth ontological basis vector $\hat{2}$ which allows us to use $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ as a basis for general relativity (or rather we might choose to call the fourth one $\hat{\imath}$ because it more precisely corresponds to hyperimaginarity.) Our initial desire to add a single degree of freedom in a longitudinal mode along $\hat{\Phi}$ showed that $\{\hat{\pi}, \hat{\Phi}, \hat{\imath}\}$ was insufficient for the intened purposes [4, 5]. Luckily, we found $\hat{2}$ already there

[1] There is already another number system named the hypercomplex numbers, but it is not $^\star\mathbb{C}$. We have previously used the name $\mathbb{C}^3$ in this regard which was also already taken.

in the plane wave solutions whose periodic topology goes like $\psi := e^{2\pi i\theta}$. We assemble all the ontological numbers when the first place we want to look in the phase space is where $\theta = |\hat{\Phi}| = \Phi$ giving $\psi := e^{2\pi i \Phi}$.

The $\hat{\Phi}^{n+1}$-site shall be located at the tip of the $\hat{\Phi}$ vector as pointing from the $\hat{\Phi}^n$-site and this intuitively points to future timelike infinity but we will develop other options in this book. Due to the special MCM boundary condition that χ^5 is always flat, we can retain the non-relativistic notion of a vector connecting two points in a single manifold. $\hat{\Phi}$ points in the direction of χ^5 and all the $\hat{\Phi}^n$ lie end to end in an infinite 1D manifold.[1] All the other structure is foliated from $\hat{\Phi}$. If $\hat{\Phi}$ points to spacelike infinity then $\mathcal{H}_2$ can be another universe beyond the observable horizon of $\mathcal{H}_1$. While there are many different discrete $\hat{\Phi}$ vectors in any significant volume of the cosmological lattice, the string of observations that can be made by one observer is always a straight line in the cosmological lattice. This is what we mean when we say that χ^5 is identically flat.

Feynman considered it a flaw in his own approach [6] that he was forced to choose an arbitrarily short, finite amount of chronological time t so as to avoid divergent integrals within an infinite natural duration of time $t \in [-\infty, \infty]$. In this chapter, we will say a lot about Feynman's framework. Note that when an observer's lifetime is comprised of a finite number of observations we naturally have a way to impose Feynman's mathematical constraint with a more realistic philosophical predicate. Since the theory of infinite complexity regards the observer's ability to test his own theory, and an observer can only make finitely many observations in a lifetime, it is likely that we can impose a constraint based on finite chirological time even when the proper chronological time of the universe has no inherent constraint to finiteness. Feynman's theory works even for artificially finite time so we are able to begin to build the MCM by dividing the real line $\mathbb{R}$ into three non-specific regions [7]

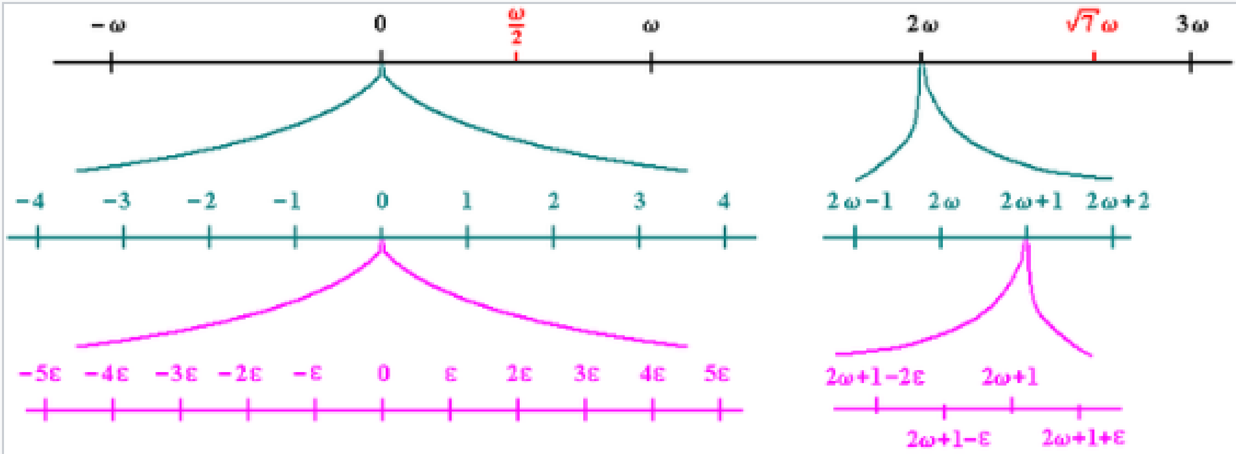
$$\text{Past} \in [t_{\min}, t_0) \quad , \qquad \text{Present} \in [t_0] \quad , \qquad \text{and} \qquad \text{Future} \in (t_0, t_{\max}] \quad . \tag{1.2}$$

Notice that the actual values of $t_{\min}$ and $t_{\max}$ do not enter into consideration. In the MCM we will presume that t is infinite in extent when $t \equiv x^0$ but finite when $t \equiv \chi^5$. χ^5 is built[2] from some stack of $\hat{\Phi}$ vectors that point from one observation to the next. Since an observer can only make finitely many observations in a lifetime, it will never become necessary to consider the implications of an infinitely long χ^5 dimension. This shows what we mean when we say χ^5 is an abstract psychological dimension. Here, Feynman's arbitrary time is replaced with a realistic observation time.

In reference [8], we go into a lot of detail regarding the mathematical analysis of concepts of infinite complexity and we will also do so in chapter four of this book. Here, we introduce ${}^\star\mathbb{C}$ by labeling each tier of infinitude with some unique $\hat{\Phi}^n$ and refer to them as **levels of aleph** [9]. We may efficiently use integers for the logical ordering of tiers of hypercomplex infinitude so we can also use integers to refer to them as levels of $\aleph$. Figure 2 shows three levels of $\aleph$. **Hyperspacetime** will refer to an object constructed from 3D position space by the addition of, first, relativistic chronological time x^0 to make spacetime, and then flat

[1] This manifold is infinite in the sense of an infinite stack of unit cells, each of finite width in χ^5.

[2] Here, χ^5 refers to the construction assembled from $\chi^5_\pm$ and $\chi^5_\varnothing$ across arbitrarily many unit cells.



Infinitesimals (ε) and infinites (ω) on the hyperreal number line (1/ε = ω/1)

Figure 2: From Wikipedia: an example of how hyperreal numbers $^*\mathbb{R}$ work. Note that, in general (not pictured), there is more than one level of infinitude and more than one level of infinitesimality. These levels of infinitesimality and infinitude are what we will call tiers of infinitude when referring to $^*\mathbb{R}$ and levels of $\aleph$ when referring to the exact same principle in the context of $\hat{\Phi}^n$, though we will interchange the terms informally. This figure shows three levels of hyperreal infinitude but the complete hypercomplex number system $^*\mathbb{C}$ goes all the way up and all the way down. The rightward direction in figure 1 is strictly associated with the upward direction in this figure.

chirological time χ^5 to make 5D hyperspacetime. The **hypercosmos** will consist of many hyperspacetimes on many levels of $\aleph$. We will use the **ontological basis** $\{\hat{2},\hat{\pi},\hat{\Phi},\hat{\imath}\}$ as a non-coordinate basis for tensor analysis but we do not yet require that the 5D set $\{\hat{\imath},\hat{1},\hat{\Phi},\hat{2},\hat{\pi}\}$ invented in reference [10] will be the analogue geometric basis in 5D. One way to think about this is to assume that any 5D basis of a geometric manifold is not the ontological basis which is 4D. Rather, the 5D construction $\{\hat{\imath},\hat{1},\hat{\Phi},\hat{2},\hat{\pi}\}$ will refer to an **ontological group** $\aleph^{\Omega}(5,4,3)$ where the arguments five, four, and three represent a standard group theoretical labeling similar to how O(3) distinguishes the group of rotations from the O(1,3) group of Lorentz rotations[1]. However, this book is only about the general relevance of the MCM and we won't develop any group theoretical concepts here.

In previous publications detailing the MCM, $\mathcal{H}$ refers both to the 4D manifold that is flat Minkowski space and also to a Hilbert space $\mathcal{H}'$. Here, we will prime the letter when it refers to a vector space and leave it unprimed when it describes a manifold. $\aleph'$ and Ω' are members of a rigged Hilbert space with $\mathcal{H}'$; $\aleph$ and Ω are anti-de Sitter and de Sitter spaces respectively. We have chosen an arbitrary convention putting AdS in the past since the only thing definite is that $\aleph'$ is a subspace of $\mathcal{H}'$ and that Ω' is the dual of $\aleph'$. We don't yet know for sure which of $\aleph$ and Ω *must* be dS or AdS but by turning the crank on the theoretical constructions presented in this book, it should be possible to determine that one configuration or the other describes time moving in the forward direction.

[1]We remind the reader that O(3,1) is the natural topology of $\{\hat{2},\hat{\pi},\hat{\Phi},\hat{\imath}\}$.

I.2 The Dual Tangent Space

We have previously described a need to expand the phase space from $2N$ dimensions to $3N$ [3] because we had revamped the theory as

$$\mathbb{C} \quad \equiv \quad \{\hat{\imath}, \hat{1}\} \qquad \longrightarrow \qquad \mathbb{C}^3 \quad \equiv \quad \{\hat{\imath}, \hat{\pi}, \hat{\Phi}\} \quad . \tag{1.3}$$

We find that it was wise not to immediately cease exploration in an attempt to calculate in $\mathbb{C}^3$ because by adding $\hat{2}$ we have likely moved to $4N$ dimensional phase space with

$$^{\star}\mathbb{C} \quad \equiv \quad \{\hat{\imath}, \hat{\pi}, \hat{\Phi}, \hat{2}\} \quad . \tag{1.4}$$

When working with $\mathbb{C}^3$, we proposed to find new physics by expanding the 2N-dimensional Hamiltonian phase space to 3N dimensions but now the suggested expansion for new physics on a hypercomplex cosmological lattice is larger still.

In ordinary quantum mechanics, in either of the position or momentum space representations, the other of the observable operators, $\hat{x}$ or $\hat{p}$, is represented as the partial derivative operator. Through the derivative operator, the quantum mechanics is connected to the continuum in the sense that a manifold is connected to its tangent space through the gradient. Classical phase space has $2N=6$ dimensions because it is comprised of three dimensions of space and three of momentum space. The conjugate nature of position and momentum is not quite the same as the duality between the unprimed geometric manifolds $\{\aleph, \mathcal{H}, \Omega\}$ and the abstract vector spaces $\{\aleph', \mathcal{H}', \Omega'\}$ whose states have position and momentum space representations written in the coordinates of different manifolds which are described in the MCM as lattice sites. Multiple simultaneous avenues of complexity in duality and conjugation are the expected source of new complexity in the theory of infinite complexity. Each lattice site has a localized bubble of physically realizable phase space when, for example, the $x^\mu_{\{j\}}$ coordinates are beyond infinity in phase space with respect to the $x^\mu_{\{n<j\}}$ coordinates and infinitesimal with respect to $x^\mu_{\{n>j\}}$. It is at least feasible that new physics lie in phase space when there is an overlap in the momentum space available to universes that are separated by spacelike infinity in position space.

The process of evolution in the lattice is expected to differ from pure Schrödinger evolution as follows: the first derivative operator that appears in quantum mechanics only allows us to go back and forth between $\hat{p} \equiv -i\partial_x$ and $\hat{x} \equiv i\partial_p$ but in $4N$ dimensional phase space we can likely find complex representational loops that achieve novel effects such as arbitrage of information. Perhaps we can use the addition of two new layers of complexity called hyperreality and hyperimaginarity to transfer information from position space into momentum space, then into the dual tangent space, then into the momentum space of a different position space such that $\hat{M}^3$ takes initial conditions in the position space of $\mathcal{H}_1$ and returns the expectation value in the momentum space representation of the $\mathcal{H}_2$ coordinates. Recall the underlying process

$$\hat{M}^3 \;\; : \;\; \mathcal{H}_1 \quad \mapsto \quad \Omega_1 \quad \mapsto \quad \aleph_2 \quad \mapsto \quad \mathcal{H}_2 \;\; , \tag{1.5}$$

and keep in mind that we want to use the dual tangent space to bridge the gap across $\Sigma^\varnothing$ that appears in figure 1. Schrödinger evolution is such that an initial state directly becomes a final state but equation (1.5) says that $\hat{M}^3$ takes an initial state in $\mathcal{H}_1$, sends it into the future Ω_1, then into the past of the next moment $\aleph_2$ before returning the final state in $\mathcal{H}_2$. The coordinates $x^\mu_{\{n\}}$ of each level of $\aleph\ \hat{\Phi}^n$ are separated by infinity but the frequencies associated with the eigenstate basis vectors are separated by finite width in phase space through $\omega_{i+1}=\Phi\omega_i$ [9].[1] Therefore, in momentum space, we can expect tunneling between regions that are separated by infinity in position space. One of the remaining nebulosities in the MCM is how to actually move beyond infinity when transiting the unit cell. Therefore, consider how it will be useful make the Fourier transform between coordinates that can never make it all the way to infinity and wave numbers which don't know anything about "the surface at infinity" and don't have to "reach it" before they can describe what happens behind it.

To begin to lay the groundwork for infinitely complex quantum states that have new degrees of freedom hidden in the dual tangent spaces of their position and momentum space representations, consider that a generic set of non-ontological 4D basis vectors $\hat{e}^\alpha\equiv\hat{e}^\mu$ can be inherited from the generic 5D basis $\hat{e}^A$ exactly via suppression of the component $\hat{e}^5$ that would point in the direction of increasing or decreasing χ^5. The four basis vectors of the observer's lab frame $\hat{x}^\mu$ do not point into the bulk hyperspacetime between each instance of $\mathcal{H}$ because $x^4\notin x^\mu$. When evolving some qubit encoded on a Lorentz frame across the unit cell, we will rely heavily upon the fact that the flat metric of the Lorentz frame is always the same and does not depend on the global topology specified by the $\{+,\varnothing,-\}$ scripting. It is very important to note that it will be possible to take the four unit vectors $\hat{\chi}^\mu_\pm$ of $\Sigma^\pm$ which span slices of $\chi^5_\pm$ as the basis of a local Lorentz frame defined by an observer living in a manifold with **arbitrary global topology** as long as the curvature is mild enough for the Lorentz approximation. We will therefore restrict ourselves to the weak field limit

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}\quad , \tag{1.6}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ is a small perturbation. Specifically we will use $\hat{x}^\mu_+$ for the basis of 4D de Sitter space and $\hat{x}^\mu_-$ as the basis of Anti-de Sitter space. When we consider the unperturbed cosmological solution $h_{\mu\nu}=0$, and let the intergalactic magnetic field go to zero with $B_\mu=0$ in equation (1.1), we have a cosmological Kaluza–Klein metric

$$\Sigma^\pm_{AB}=\begin{pmatrix}\eta_{\mu\nu} & 0\\ 0 & \phi^2(\chi^5_\pm)\end{pmatrix}\quad , \tag{1.7}$$

that will serve as the basis for modification in the sense of the first M in MCM. This metric describes the singularity-free cosmos postulated in reference [7]. The no-singularities condition was initially assumed on the basis of loop quantum cosmology but the condition was independently motivated later and will be discussed in section IV.1. Neither of loop

[1] This feature is reviewed in section IV.6.

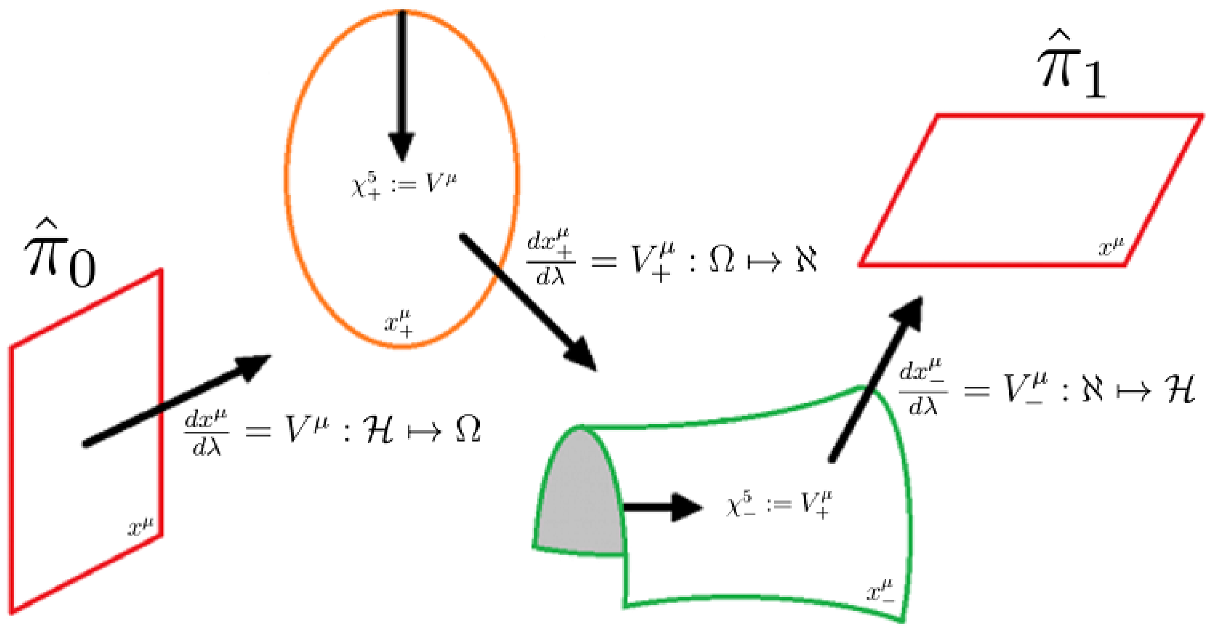


Figure 3: This figure describes the MCM topological configuration. The geodesic that traverses the MCM unit cell and whose tangent vectors include the three vectors $\{V^\mu_+, V^\mu, V^\mu_-\}$ shown here could be akin to a periodic orbit of the Hopf fibration. Note that the connective vectors are never derived from the 5D coordinates χ^A.

quantum cosmology (LQC) or loop quantum gravity (LQG) have any direct relevance to the MCM machinery as it presently exists.

The tangent vectors to the geodesics associated with the perturbation $h_{\mu\nu}$, such as, for example, the local deformation of spacetime due to an ordinary cathode ray tube, while very small, are not strictly confined to the slices of $\chi^5_\pm$. We can say the same thing about the tangent vectors to some set of mega-scale cosmological geodesics in de Sitter space: they point outside of the flat slices of χ^5_+ and, further, the same can be said about anti-de Sitter space and χ^5_-. The tangent vectors point outside of the 4D worldsheet in both cases: vectors associated with some perturbation $h_{\mu\nu}$ and vectors tangent to some cosmological geodesics that show the global topology. A vector in curved space is an object defined at a point and nothing more but χ^5 is not curved so its tangent vectors always point to other points along χ^5. All the Lorentz frame vectors remain in their respective slices. If the slice is Minkowski space then the Lorentz frame defines it identically. The tangent vectors to the cosmological geodesics in Minkowski space remain in the slice but as we have shown there are still other important vectors in the MCM that do point outside of the slices.

How do we know the vectors that point outside of the flat slices are pointing to other slices of $\mathcal{H} \cup \{\Sigma^+, \Sigma^\varnothing, \Sigma^-\}$ and not outside of hyperspacetime altogether? This is where we take advantage of the flat topology that we have assigned to our psychological dimension χ^5. It only has one collinear tangent space. Vectors tangent to χ^5 everywhere point in the direction of $\hat{\chi}^5$; it is a straight line. When we need some vector V^μ to be aligned correctly in the cosmological lattice, as in figure 3, we can begin $\hat{M}^3$ by defining χ^5_+ as pointing in the direction of V^μ which is defined in terms of $x^\mu \in \mathcal{H}$ only. When we need to continue the process on χ^5_-, we can choose a vector V^μ_+ from Ω and use it to the define the direction of

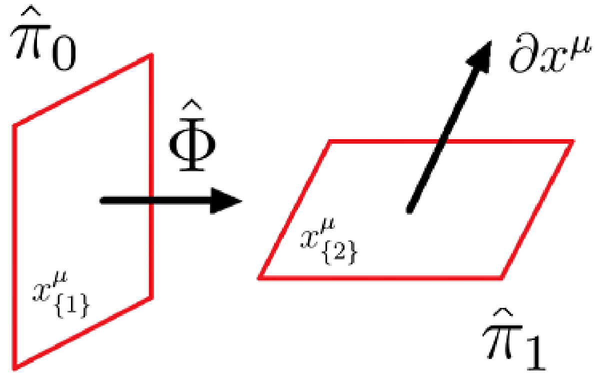


Figure 4: This figure shows our proposal to avoid computing the geodesics in figure 3 through the use of an appropriately defined $\hat{\Phi}$ vector or set of $\hat{\Phi}$ vectors. This figure shows the single instance one 5D $\hat{\Phi}$ vector. In terms of a 4D $\hat{\Phi}$ vector, the connector might be better illustrated as $(\hat{\Phi})^3$ and we begin to see an origin for the $(\Phi\pi)^3$ term in $\alpha_{MCM} = 2\pi + (\Phi\pi)^3$ when each $\hat{\Phi}$ has an accompanying $\hat{\pi}$. We have the option to say that $\hat{\Phi}$ points from one 5D manifold to p in the next or it can act in 4D (more likely) where it points to successive p's in $\{\aleph, \mathcal{H}, \Omega\}$ and where the third 4D p would be the same point as the first 5D p.

$\hat{\chi}^5_-$. If V^μ_+ is the parallel transport of V^μ onto Ω, and V^μ_- is the parallel transport of V^μ_+ onto $\aleph$, then the parallel transport of V^μ_- onto $\mathcal{H}_2$, call it W^μ, will be very nearly the same as V^μ due to the weak field condition. If the Lorentz approximation was perfect then V^μ would be equal to W^μ but in reality there will be small differences between the flat, spherical, and hyperbolic spaces that lead to small "errors" in each step of parallel transport from one Lorentz frame to the next in $\mathcal{H} \mapsto \Omega \mapsto \aleph \mapsto \mathcal{H}$. We expect that the sum of these errors $Q = |V^\mu - W^\mu|$ will look like quantum decoherence.

Here is an important point. Consider an infinite number of 4D de Sitter and anti-de Sitter spaces that have an infinite number of curvature parameters, all unique, that form a continuum of monotonically increasing curvature: curvature parameters in $[-a, 0)$ for AdS and $(0, b]$ for dS. In the worldsheet representation, we can order all the like de Sitter branes by increasing curvature parameter and then abut them to construct the smooth 5-spaces $\Sigma^\pm$. Since each 4D slice is curved individually, the 5D spaces will also be curved and physics in a curved manifold always has tangent vectors pointing outside of the manifold. Compare this to the case mentioned above when we start with flat 5-space and chop it into slices so that it is only the cotangent vectors that point outside of the manifold. The slices are all flat in the mathematical sense of the most natural geometry but in the physical sense of distance we are imposing an embedded metric on the 4D slices that makes them seem curved to an observer within. We can do this because the 4D coordinates x^μ are an independent object from the 5D ones χ^A even if there are certain cases where they are the same. Discerning between either tangent vectors or cotangent vectors leaving the slices of constant coordinate will be a critical distinction used to construct a bridge across $\Sigma^\varnothing$.

If we are relying on the topological disconnection of $\mathcal{H}$ from $\Sigma^\pm$ to motivate quantum weirdness [3, 11] then we need to show how they are still connected in the non-topological sector. If they are not sufficiently connected then the action that steers geodesics through figure 3 will not exist. If they are connected, then the action is guaranteed to exist and the trick (figure 4) will work (if a 5D $\hat{\Phi}$ exists.) It is very easy to demonstrate the non-topological

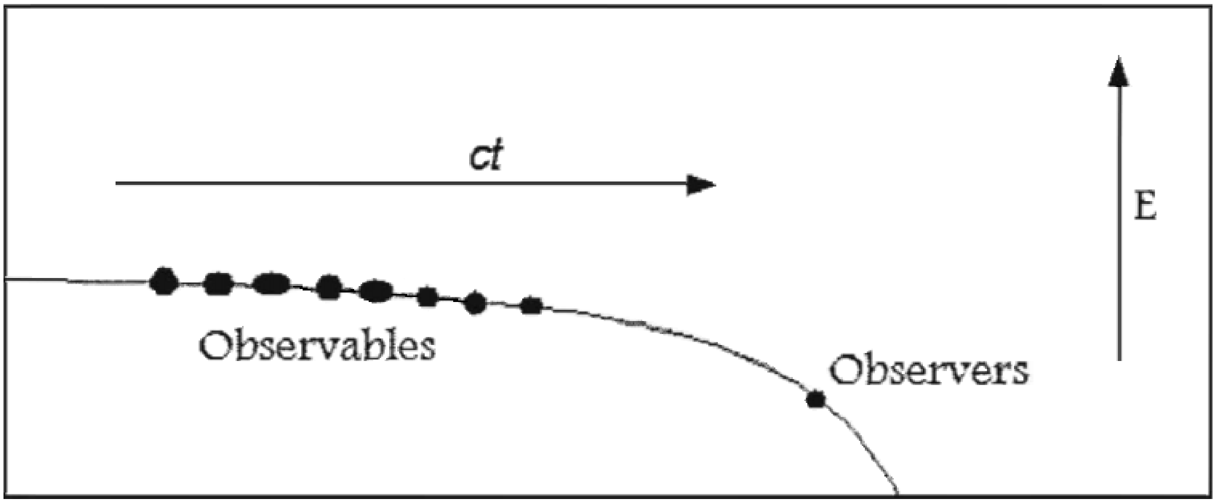


Figure 5: This figure from reference [2] seeks to demonstrate how dark energy is a solution to the Poisson equation inside the cosmological unit cell.

connection. Beginning with an initial state in $\mathcal{H}_1$, parameterize some curve in the Minkowski metric whose tangent vector at a point is collinear with χ^5_+ and its tangent space (which will be accomplished by defining χ^5_+ accordingly.)[1] The statement $\chi^5_+\hat{\Phi} = V^\mu \hat{\pi}$ is a non-topological, purely algebraic statement. Take note that χ^5_+ having the same direction as V^μ is the only constraint on Σ^+ so there is no other constraint that could make it impossible to construct a unit cell around $\mathcal{H}$ when it preexists in a void. The χ^α_+ are attached to χ^5_+ strictly to ensure our ability to transport qubits defined in a local Lorentz frame along χ^5. Above, we introduced the $\hat{x}^\mu$ as a subset of the $\hat{\chi}^A$ but a more physical approach is to begin with observable $\hat{x}^\mu$ and then define $\hat{\chi}^A$ as a superset. Since 5-space has infinitely many 4D subspaces, there are infinitely many χ^A that can be constructed to contain x^μ. Keep in mind that there is no requirement for anything to "slide" along $\chi^5_\pm$. The qubit in $\mathcal{H}_1$ can just as easily be described as tunneling onto Ω, then onto $\aleph$, and then onto $\mathcal{H}_2$. In fact, Kaluza–Klein theory relies on the vanishing 5D Ricci tensor $R_{AB} = 0$ so the qubit *must* tunnel directly from one brane onto the advanced brane in $\mathcal{H} \mapsto \Omega \mapsto \aleph \mapsto \mathcal{H}$. A current of massive particles in the bulk would be the opposite of a vanishing Ricci tensor. Once χ^5_+ establishes a place to put χ^α_+, we may define the coordinates x^μ_+ in the dS metric of the Ω manifold. In $\aleph$ and Ω, it is likely that we can always take $x^\mu_\pm = \chi^\alpha_\pm$ when considering an unperturbed ground state.

Describing figure 3, we parallel transport V^μ onto Ω, call it V^μ_+, and then define χ^5_- so it is pointing in the direction of V^μ_+ along the 1D manifold of stacked $\hat{\Phi}$ vectors, possibly something like $\chi^5_-\hat{\imath} := V^\mu_+\hat{\Phi}$. To avoid conflict with the collinearity of V^μ and χ^5_+, meaning that the topological arrangement should prevent any linear superposition of V^μ and V^μ_-, we say χ^5_- is out of phase with both V^μ and V^μ_+.[2] We can accomplish this in the obvious way with the orthogonality of the ontological basis vectors but there is another avenue available

[1] The method of obtaining this first relativistic 4-vector from some quantum state is suggested in section II.6.
[2] We follow the convention that $\hat{x}$ is out of phase with $\hat{y}$ by $\pi/2$ radians in the Cartesian plane.

when the level of $\aleph$ increases $n \to n+1$ between Ω and $\aleph$. The MCM forbids the interference of vectors on different levels of $\aleph$ [9] and this is already the established behavior for hyperreal quantities on different tiers of infinitude in $^*\mathbb{R}$. With two number lines [10] — chronos x^0 and chiros χ^5 — we can introduce the concept of **double orthogonality**. Consider Hamiltonian systems which only consider the canonical conjugate coordinates wherein any physical system is determined once any two variables are known. There, single orthogonality (regular orthonormalism) can only halfway decouple two elements of a closed system. Double orthogonality will allow complete decoupling of elements.

It is known that every gravitational manifold contains at least one point p at which it is possible to construct a coordinate system wherein the metric is locally the Minkowski metric and its first derivatives all vanish at p. Considering figure 3, let each vector $\{V^\mu, V^\mu_+, V^\mu_-\}$ point to the special point p in the future-adjacent manifold as in $\mathcal{H} \mapsto \Omega \mapsto \aleph \mapsto \mathcal{H}$. This means that V^μ_+ passing through $p \in \Sigma^\varnothing$ is the only constraint on $\Sigma^\varnothing$. Since V^μ_+ was already separated from $\mathcal{H}$ by only appearing in its tangent space, and Σ^- is separated by not having the inverse vector to V^μ_+ anywhere in its vector bundle[1], $V^\mu_- \in \Sigma^-$ is doubly separated from $V^\mu \in \mathcal{H}$, or doubly orthogonal. To begin the three-fold process of observation, calculation, and observation again [12], we start with a vector V^μ defined at a point in Minkowski space $\mathcal{H}$ and say it points to p not in $\mathcal{H}$, and then we construct p's manifolds Σ^+ and Ω around p. Then we do the same thing with V^μ_-: construct χ^5_- from V^μ_+ and then create V^μ_- so that it points to $\mathcal{H}_2$.[2]

We point out that if $\{V^\mu_-, V^\mu, V^\mu_+\}$ are all identically the 4D $\hat{\Phi}$ vector when written in the various coordinates systems on $\{\aleph, \mathcal{H}, \Omega\}$, but $Q = |V^\mu - W^\mu| \neq 0$, then we have an algorithm that violates **conservation of information**.

Every observable state we could consider has some associated energy density that is unambiguously a perturbation $h_{\mu\nu}$ on the background Minkowski metric $\eta_{\mu\nu}$. $\mathcal{H}$ is globally flat but with the perturbation it is not precisely Minkowski space locally so it has local geodesics with tangent vectors that point outside of the slice, into the bulk. All this allows us to construct the Riemann sphere with poles at successive p and perform the inversion operation that has been described for transporting Hilbert space, along with the geometry, from one moment to the next, as in figure 6 [12].[3] It is likely that the ordinary inversion map between the two coordinate charts on $\mathbb{S}^2$ will be sufficient for the purposes of carrying out what we have called "the inversion operation on the Riemann sphere." When the sphere is situated between two branes $\mathcal{H}_1$ and $\mathcal{H}_2$ (or $\mathcal{H}_1$ and Ω) then it is natural to associate the coordinate chart that covers one of the sphere's poles with the brane that touches that pole, and likewise for the other pole and the other coordinate chart on $\mathbb{S}^2$. Between $\mathcal{H}_1$ and $\mathcal{H}_2$, these would be the $x^\mu_{\{1\}}$ and $x^\mu_{\{2\}}$ coordinates shown in figure 1 or between $\mathcal{H}$ and Ω we would have the x^μ and $\{x^\mu_+, x^\alpha_+\}$ coordinates.

Consider $\mathcal{H}$ and its vector space $\mathcal{H}'$. If we are going to send information across the unit cell from $\mathcal{H}_1$ to $\mathcal{H}_2$ with a vanishing Ricci tensor everywhere in the bulk then we need to use

[1] The inverse vector would go as $1/\infty$ since Σ^- is on a higher level of $\aleph$ than Σ^+.

[2] To put this on a computer, it may be necessary to do this one more time with p whose manifolds are $\varnothing$ and $\Sigma^\varnothing$ being an intermediate location between V^μ_+ and V^μ_-. This could require another object $V^\mu_\varnothing$.

[3] The reader might ask, "If the vector in Hilbert space exists independently of the position space representation, then why transport it all? Surely we can use the same Hilbert space everywhere." If 4D physics is independent of the fifth dimension because the branes are slices of constant $\chi^5_\pm$, and $\chi^5_\pm$ increases with increasing $\mathcal{H}_n$, then we can mitigate the increase of χ^5 with an appropriate linear reduction factor attached to the position space representations of $\mathcal{H}'$ vectors in the successive position spaces $\mathcal{H}_n$. This reduction operation can be inconsequentially included in the inversion operation on the Riemann sphere that occurs between observations [12, 10].

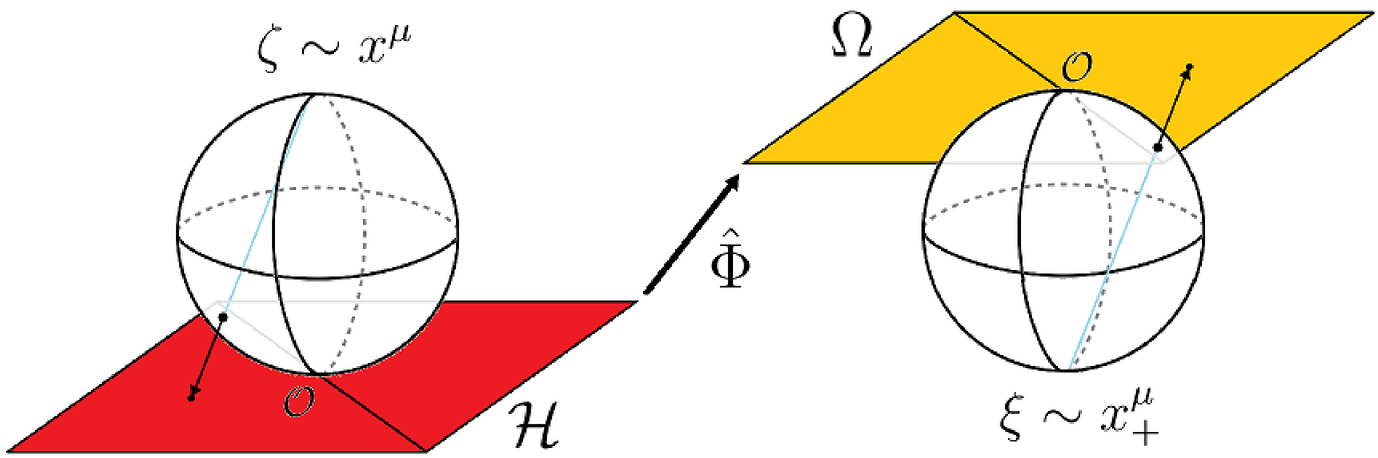


Figure 6: This figure shows the bijection between a sphere and either of two planes: two flat slices of some bulk. $\mathcal{O}$ marks the origin of coordinates on each chart. This figure differs from figure 4 because here the sphere connects $\mathcal{H}$ to Ω but in figure 4 $\hat{\Phi}$ connects $\mathcal{H}_1$ to $\mathcal{H}_2$. Where figure 4 shows what we called the 5D p, the point $\mathcal{O}$ on the right of this figure would be the first 4D p. Since the Gel'fand triple is constructed by first taking a subspace of $\mathcal{H}$, this figure suggests that we should examine a possible reversal of the association of $\aleph'$ and Ω' with dS and AdS ($\aleph$ and Ω.)

an alternate channel. Hilbert space $\mathcal{H}'$ is defined on $\mathbb{C}$ which is topologically equivalent to either of the two charts on $\mathbb{S}^2$. The Riemann sphere is the portion of $\mathbb{S}^2$ that can be covered by a single chart, and the map between $\mathbb{S}^2$'s two chart coverings, call them ξ and ζ, is the inversion map

$$\zeta \quad \equiv \quad \frac{1}{\xi} \quad . \tag{1.8}$$

There are normally two maps between the plane and the Riemann sphere: one where the plane intersects the equator of the sphere and another where the sphere sits on top of the plane and we will use the latter. The two representations completely mirror the choice to put either $\mathcal{H}$ or $\Sigma^{\varnothing}$ in the center of the MCM unit cell. When the sphere rests on the plane, the bijection between points on the sphere and points in the plane is defined by polar ray tracing as in figure 6. After we use the polar ray to trace the plane onto the sphere, we can use the inversion map to invert the information encoded on the sphere and then use the polar ray (which now points always upward instead of always downward) to trace the information back onto a new plane. This new plane can be some other flat slice of 5D bulk with its own unrelated embedded metric that is de Sitter space in some coordinates x_+^μ so that the slice is what we call Ω (or perhaps this should be $\varnothing$.) The inversion map is exactly what is required to go to a higher level of $\aleph$. Points on the sphere very near the pole at the base of the polar ray have planar coordinates that approach infinity so the inversion map between ζ and ξ will give something like $\infty \to 1/\infty$ which is axiomatically zero. However, there is some wiggle room where we can transform the coordinates as $x^\mu \to dx^\mu$ and $\chi^A \to d\chi^A$ which are axiomatically non-zero. In this way, we can build the 1- and 2-forms of general relativity as well as whatever N-forms are required for the differential geometry of the hypercosmos. The reader is referred to reference [8] for further specifications of the Riemann sphere as it relates to the MCM/TOIC.

Here, $\hat{\Phi}$ points at least to Ω but in reference [13] we showed an alternative formulation where $\hat{\Phi}$ points to an intermediate point somewhere inside the bulk of Σ^+ between $\mathcal{H}$ and Ω so the reader should not go in with too much bias regarding $\hat{\Phi}$. We develop the 4D $\hat{\Phi}$ here and we will discuss the very important alternative formulation from reference [13] in chapter four. Here, we will only use the 5D version (figure 4) to demonstrate how $\hat{\Phi}$ has at least the potential to provide a shortcut between an arbitrary initial state and an arbitrary final state. This shortcut regards calculating the transition amplitude which is often the object of interest in quantum theory. The 4D version of $\hat{\Phi}$ is preferred at this time so that the framework is guaranteed to accept input in the classical Lorentz approximation. However, since we know the bulk curves in figure 3 are guaranteed to exist, we may be able to simply define the vector $\hat{\Phi}$ such that it points from $\mathcal{H}_1$ to $\mathcal{H}_2$. All of the determinism can be written in coordinate independent tensors so it could be possible to choose the coordinates where $\hat{\Phi}$ has the correct behavior in 4- or 5D, and then write the tensor equations in those coordinates. To some extent, we are cheating by taking $\chi^5_\pm$ to be in the direction of an arbitrary tangent vector but to a further extent that is exactly what one would expect using $\hat{2}\pi = \hat{\pi} + \hat{\pi} = \hat{\pi} - \varphi\pi\hat{\Phi}$ [10]. Obviously the *copy* assigned to the $\hat{\Phi}$ component will have the same direction as the original component: it is a co-$\hat{\pi}$. They can point in the same direction from the same point and not interfere with each other if we require that $\hat{\pi}$ and $\hat{\Phi}$ are orthogonal and we have already done so. If there are two quantities associated with the single direction indicated by V^μ then that is essentially the system of two number lines described in reference [10]. Double orthogonality means vectors can have the same direction and still be orthogonal due to the appended ontological basis vectors, *e.g.*: $|\psi\rangle\,\hat{\pi}_n$.

We can define the **ontological gauge** to be the one where $\hat{\Phi}^n$ is a vector in $\mathcal{H}_n$ that points to the termination of the worldline on Ω which is then carried forward by the periodic MCM boundary condition to a final termination on $\mathcal{H}_{n+1}$. This holds for any n but not necessarily more than one n at a time. By "termination of the worldline on $\mathcal{H}_{n+1}$" we mean that the observer has computed the mathematical evolution of the initial state in $\mathcal{H}_1$ which correctly describes what is observed when the theory is tested at $\mathcal{H}_2$. In many realistic applications, this will be repeated millions of times iteratively because fine-grained simulation might require millions of time steps, each calculated with $\hat{M}^3$. The ontological gauge is very complicated but the *theory* of the TOIC is that $\hat{\Phi}$ agrees with experiment and there will be no need to actually compute the geodesics whose tangent vectors are shown in figure 3. If we did compute these curves, the idea of the shortcut is that they would give the same answer arrived at with $\hat{\Phi}$. In any case, if we did try to compute the curves in figure 3 we would probably bump into ourselves where we have required that chiros is non-relativistic because the full hypergeometry of the cosmological cell including $\Sigma^\varnothing$ would likely require χ^5 to be non-flat. Such a system has proven too complex even to represent schematically and rightly so if the computation represented by the diagram is technically impossible (without $^\star\mathbb{C}$ tools.) Here, we mean to imply that $\hat{\Phi}$ is not an alternative means by which to compute MCM processes, it will be the only way. Furthermore, if we are somehow accomplishing an "impossible computation" then that gives some loose suggestion of the principle of violation of conservation of information.

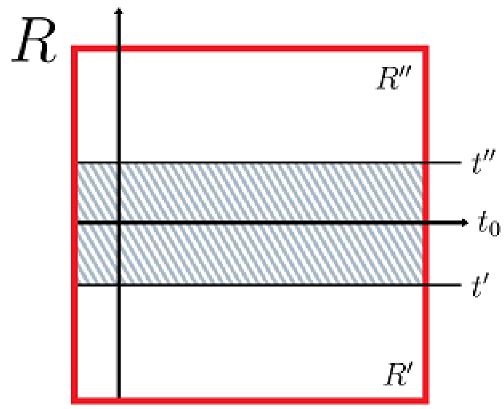


Figure 7: This figure shows the division of the time interval described by Feynman. The red square here is necessarily the same as the one in figure 4 because Feynman's relativistic region R is spanned by x and t. R is Minkowski space.

We have introduced $\{\varnothing, \Sigma^{\varnothing}\}$ as a new degree of topological freedom. Generally, in physics, each degree of freedom is a channel for the flow of information so when we say $\hat{M}^3$ is incomputable, perhaps we can compute

$$\tilde{M}^4 \quad : \quad \mathcal{H}_1 \quad \mapsto \quad \Omega_1 \quad \mapsto \quad \varnothing \quad \mapsto \quad \aleph_2 \quad \mapsto \quad \mathcal{H}_2 \ \ , \tag{1.9}$$

with the additional point $p \in \varnothing \subset \Sigma^{\varnothing}$. $\varnothing$ is a 4D Poincaré section of $\Sigma^{\varnothing}$ where we apply the *ad hoc* operation of discrete translation to the higher level of $\aleph$. We may use the two central steps of $\tilde{M}^4$ to convert to the 1-form basis (description of the manifold in terms of its dual tangent vectors) on the manifold with $\Omega \mapsto \varnothing$ and then convert back to tangent vectors in Σ^- with $\varnothing \mapsto \aleph$. Another possibility that we will discuss in this book has to do with joining Σ_1^+ to Σ_2^- with a twistor representation beyond the limits of the unit cell where $\chi_\pm^5 = \pm\infty$. Yet another possibility, the main one treated in this book, is that $\hat{M}^3$ increases the level of $\aleph$ by two because there is one $\hat{\Phi}$ pointing to $\varnothing$ and another $\hat{\Phi}$ anchored at $\varnothing$.

While we seem to have the correct convention for dS and AdS spaces on Ω and $\aleph$, we should, at some point, consider the reverse ordering of the elements of the Gel'fand triple $\{\aleph', \mathcal{H}', \Omega\}$ with respect to $\{\aleph, \mathcal{H}, \Omega\}$. Since the first step is $\mathcal{H} \mapsto \Omega$, it is likely that the first algebraic step is to select the subspace of Hilbert space which we have been calling $\aleph'$ whereas we have previously assigned the abstract superspace Ω' to Ω. If we do change this convention, we will simply make a revision in the definitions of $\{\aleph', \Omega'\}$ to retain the intuitive object associations through their shared symbols.

I.3 Feynman, Functions, and Functionals

Now we have explained, in principle, how a geodesic can pass from one disconnected space $\mathcal{H}_1$ to another one $\mathcal{H}_2$ as if they were connected. We just have to find the correct algorithm for the computation that makes use of the vectors in the tangent and dual tangent bundles to these cosmological manifolds. As a preliminary for what will follow, insofar as the modified cosmological model is the boundary condition in which we propose to unify general relativity with quantum electrodynamics via the theory of infinite complexity, and by extension, eventually, with the standard model, we will now consider an excerpt[1] from Feynman's seminal paper [6] on the spacetime formulation of quantum theory.[2]

> "We shall see that it is the possibility [*of expressing the action*] $\mathcal{S}$ as a sum, and hence Φ as a product[3], of contributions from successive sections of the path, which leads to the possibility of defining a quantity having the properties of a wavefunction.
>
> "To make this clear, let us imagine that we choose a particular time t_0 and divide the region R [(*figure 7*)] into pieces, future and past relative to t_0. We imagine that R can be split into: (a) a region R', restricted in any way in space, but lying entirely earlier in time than some t', such that $t' < t_0$; (b) a region R'' arbitrarily restricted in space but lying entirely later in time than t'', such that $t'' > t_0$; (c) the region between t' and t'' in which all the values of the x coordinates are unrestricted, i.e., all of space-time between t' and t''. The region (c) is not absolutely necessary. It can be taken as narrow in time as desired. However, it is convenient in letting us consider varying t a little without having to redefine R' and R''. Then $|\varphi(R', R'')|^2$ is the probability that the path occupies R' and R''.[4] Because R' is entirely previous to R'', considering the time t as the present, we can express this as the probability that the path had been in region R' and will be in region R''. If we divide by a factor, the probability that the path is in R', to renormalize the probability we find: $|\varphi(R', R'')|^2$ is the (relative) probability that if the system were in region R' it will be found later in R''.
>
> "This is, of course, the important quantity in predicting the results of many experiments. We prepare the system in a certain way (e.g., it was in region R') and then measure some other property (e.g., will it be found in region R''?)[5] What

[1] We change a few variable names here to enforce consistency between Feynman's notation and the present conventions.

[2] Feynman wrote reference [6], in part, as a response to a paper of Dirac's [14]. At the end of Dirac's paper he points out that his theory still has some problems with it because it returns a complex-valued probability whose only physical utility is to make a hand-waving (but valid!) association between a very small complex number and a very low probability. Large complex numbers were apparently unintelligible. Feynman even dedicates an entire section of his much longer paper [6] to describing the inadequacies of the formulation ge presents. Together, these two war era papers serve to sharply contrast contemporary editorial standards in comparable modern journals. In reference [12], we make a statement similar to Dirac's when we ignore the problems associated with the sum of a vector and a tensor: "Assume an evolution operator that is the sum of a vector part and a tensor part so that $\hat{\Upsilon} \equiv \hat{U} + \hat{M}^3$. We ignore the difficulties associated with adding a vector to a tensor and for now it will suffice to say that $\hat{\Upsilon}$ is a strange mathematical object. The operator ∂ is a unit vector and $\hat{M}^3$ takes on unitary properties in chronos. Using the convention to denote tensor states $|\psi\rangle\hat{\pi}$, we outline a new quantum theory." Then in reference [5] we discovered that $\hat{\Upsilon}$ is the higher rank, complexified analogue of a strange representation of the quaternions $\mathbf{q} = v_0 + \vec{v}$.

Why are papers about partially formed ideas that only kind of work no longer allowed? When such papers appear in the literature, other papers that fill in the gaps often appear in short order. It is only the papers that don't work at all which should be disallowed. Given the current *total standstill* in the pace of discovery in theoretical physics (or recent rather if we *reject the false narrative* about how no one is studying the MCM), one would assume that editors would encourage the kinds of papers that might inspire others to pursue new research directions but, alas, 'tisn't so. The vast bounties of low hanging fruit pointed to by the present research (as yet undescribed even now in 2017) will continue to be ignored (officially) by those who would seemingly rather build Rube Goldbergs on the backs of giraffes in the hopes of randomly grabbing some fruit from the clutches of a bird that might fly by even though no one has seen a bird with any fruit like that for decades.

[3] $\Phi[x(t)]$ is the contribution to the complex phase from the action associated with a given path $x(t)$.

[4] Feynman writes that the region of variation can be taken as thin as desired but that also means that it can be taken as thick as wanted as well. Feynman said that it can even be taken as 0 and when we use the inversion map on $\mathbb{S}^2$ we show that it can even be taken as ∞. The inversion map is what we will use to switch between the description of the MCM unit cell with $\mathcal{H}$ or $\Sigma^{\varnothing}$ in the center, and it will also mark the changing level of $\aleph$ where some yet-to-be-defined transfinite renormalization induces $\infty \to \epsilon$.

[5] Note how Feynman describes the three-fold process of observation, calculation, and then observation again.

does [*equation (1.10)*] say about computing this quantity, or rather the quantity $\varphi(R',\, R'')$ [*which is the square of equation (1.10)*]?

$$\varphi(R) = \lim_{\epsilon \to 0} \int_R \exp\left[\frac{i}{\hbar}\sum_i \mathcal{S}(x_{i+1}, x_i)\right] \ldots \frac{dx_{i+1}}{A}\frac{dx_i}{A} \ldots \tag{1.10}$$

"Let us suppose in [*equation (1.10)*] that the time t corresponds to one particular point k of the subdivision of time into steps ϵ, i.e., assume $t = t_k$, the index k, of course, depending on the subdivision ϵ. Then, the exponential of a sum may be split into a product of two factors

$$\exp\left[\frac{i}{\hbar}\sum_{i=k}^{\infty} \mathcal{S}(x_{i+1}, x_i)\right] \cdot \exp\left[\frac{i}{\hbar}\sum_{i=-\infty}^{k-1} \mathcal{S}(x_{i+1}, x_i)\right] \quad . \tag{1.11}$$

"The first factor contains only coordinates with index k or higher, while the second contains only coordinates with index k or lower. This split is possible because [*the representation of the action as a sum of actions*]

$$\mathcal{S} = \sum_i \mathcal{S}(x_{i+1}, x_i) \quad , \tag{1.12}$$

results essentially from the fact that the Lagrangian is a function only of positions and velocities. First, the integration on all variables x_i for $i > k$ can be performed on the first factor resulting in a function of x_k (times the second factor). Next, the integration on all variables x_i, for $i < k$ can be performed on the second factor also, giving a function of x_k. Finally, the integration on x_k can be performed. That is, $\varphi(R', R'')$ can be written as the integral over x_k of the product of the two factors. We will call these $\vartheta^*(x_k, t)$ and $\psi(x_k, t)$:

$$\varphi(R', R'') = \int \vartheta^*(x_k, t)\psi(x_k, t) dx \quad , \tag{1.13}$$

where

$$\psi(x_k, t) = \lim_{\epsilon \to 0} \int_{R'} \exp\left[\frac{i}{\hbar}\sum_{i=-\infty}^{k-1} \mathcal{S}(x_{i+1}, x_i)\right] \frac{dx_{k-1}}{A}\frac{dx_{k-2}}{A} \ldots \quad , \tag{1.14}$$

and

$$\vartheta^*(x_k, t) = \lim_{\epsilon \to 0} \int_{R''} \exp\left[\frac{i}{\hbar}\sum_{i=k}^{\infty} \mathcal{S}(x_{i+1}, x_i)\right] \frac{dx_{k+1}}{A}\frac{dx_{k+2}}{A} \ldots \quad . \tag{1.15}$$

"The symbol R' is placed on the integral for ψ to indicate that the coordinates are integrated over the region R', and, for t_i between t' and t, over all space. In like manner, the integral for ϑ^* is over R'' and over all space for those coordinates corresponding to times between t and t''. The asterisk on ϑ^* denotes complex conjugate, as it will be found more convenient to define [*equation (1.15)*] as the complex conjugate of some quantity ϑ.

"The quantity ψ depends only upon the region R' previous to t, and is completely denned if that region is known. It does not depend, in any way, upon what will be done to the system after time t. This latter information is contained in ϑ. Thus, with ψ and ϑ we have separated the past history from the future experiences of the system. This permits us to speak of the relation of past and future in the conventional manner[1]. Thus, if a particle has been in a region of space-time R' it may at time t said to be in a certain condition, or state, determined only by its past and described by the so-called wavefunction $\psi(x,t)$. This function contains all that is needed to predict future probabilities. [*sic*]

"Thus, we can say: the probability of a system in state ψ will be found by an experiment whose characteristic state is ϑ (or, more loosely, the chance that a system in state ψ will appear to be in ϑ) is

$$\left| \int \vartheta^*(x,t)\psi(x,t)dx \right|^2 \quad . \tag{1.16}$$

"These results agree, of course, with the principles of ordinary quantum mechanics. They are a consequence of the fact that the Lagrangian is a function of position, velocity, and time only."

When we include the advanced potential, Feynman's nice machinery of the classical formalism will fail because the MCM Lagrangian depends on position, the first derivative of position: the velocity $\dot{x}$, and also at least the third derivative of position $\dddot{x}$ that is almost unique to the theory of advanced (and retarded) potentials. We expect to be able to make the extension in the ontological formalism but emphasize that it will not be a direct extension of the classical action formalism. To achieve direct extension of the classical action formalism we will consider the case when the advanced and retarded potentials $A^\mu_\pm$ vanish and thereby remove the dependence on $\dddot{x}$. $A^\mu_\pm$ represent an esoteric electromagnetic effect whose technical details will be mostly beyond the scope of this book which aims (mostly) to present the modestly technical details of the general relevance of what has been previously reported regarding the MCM. However, we will briefly look at the advanced and retarded potentials $A^\mu_\pm$ in section III.10.

Feynman didn't invent QED with the above quoted paper [6]. QED's greatest success, arguably, was the positive result obtained by Schwinger who wrote the following about *eleven six two* in reference [15].

"The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu = (1/2\pi)e^2/hc = 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium have yielded values that are definitely larger than those to be expected from the directly measured

[1]For example, see definitions (1.2)

nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment."

We eventually need to show how the radiative corrections mentioned by Schwinger are the natural ones expected in the MCM but we will go that way at a later date since it is about twenty years on down the line from the historical period with which the MCM has been preoccupied. However, since Feynman's result from that period is the foundation of the MCM, we will go into lot of detail examining Feynman's method for dividing the time interval. First, note the process we described in reference [12].

"The observer is fixed in the present (at the origin) with the inclusion of $\delta(t)$ and since this function returns an undefined value at $t = 0$ it is impossible to integrate directly from early times to late times. To use an integrand of the form $f(t)\delta(t)$ we must employ the method from complex analysis $f(t)\delta(t) \mapsto g(r, \theta)$. The integral over all times will trace a path through $\aleph$, $\mathcal{H}$, and Ω."

The method is the Cauchy integral formula

$$f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - z_0} dz \quad . \tag{1.17}$$

It is normally possible in physics to ignore boundary terms at infinity because the action principle chooses the minimum of the action that goes from the past to the future along $\aleph \mapsto \mathcal{H} \mapsto \Omega$. That is the process Feynman considered in his famous thought experiment of building a continuum of all possible paths from the limit of infinitely many double slit experiments. We consider here another process when the MCM boundary condition blocks the path of least action with a topological obstruction at the origin so that the two sums in equation (1.11) are joined on $i = \infty$ instead of $i = k$. We expect that the **maximum action** path (which is a perfectly good solution for the equations of motion) is the one that goes around infinity like $\mathcal{H} \mapsto \Omega \mapsto \aleph \mapsto \mathcal{H}$, as in figure 8, where the boundary at infinity cannot be ignored.

We can see one kind of topological obstruction at the origin when we set $z_0 = 0$ in equation (1.17) to obtain

$$f(0) = \frac{1}{2\pi i} \oint \frac{f(z)}{z} d\, z \quad . \tag{1.18}$$

With $f(0)$, clearly, there is no continuous path of integration through the point where $z = 0$. However, by using the path around the outside of the complex plane we can compute the paths like $\lim_{\epsilon \to 0} \int_\epsilon^\infty dz$ knowing that the result is the same as if the central point $z = 0$ had contributed. Instead of the most direct path, we integrate along the path in figure 8 that is exactly of the form

$$\chi^5 \quad \equiv \quad \chi^5_+ \otimes \chi^5_\varnothing \otimes \chi^5_- \quad . \tag{1.19}$$

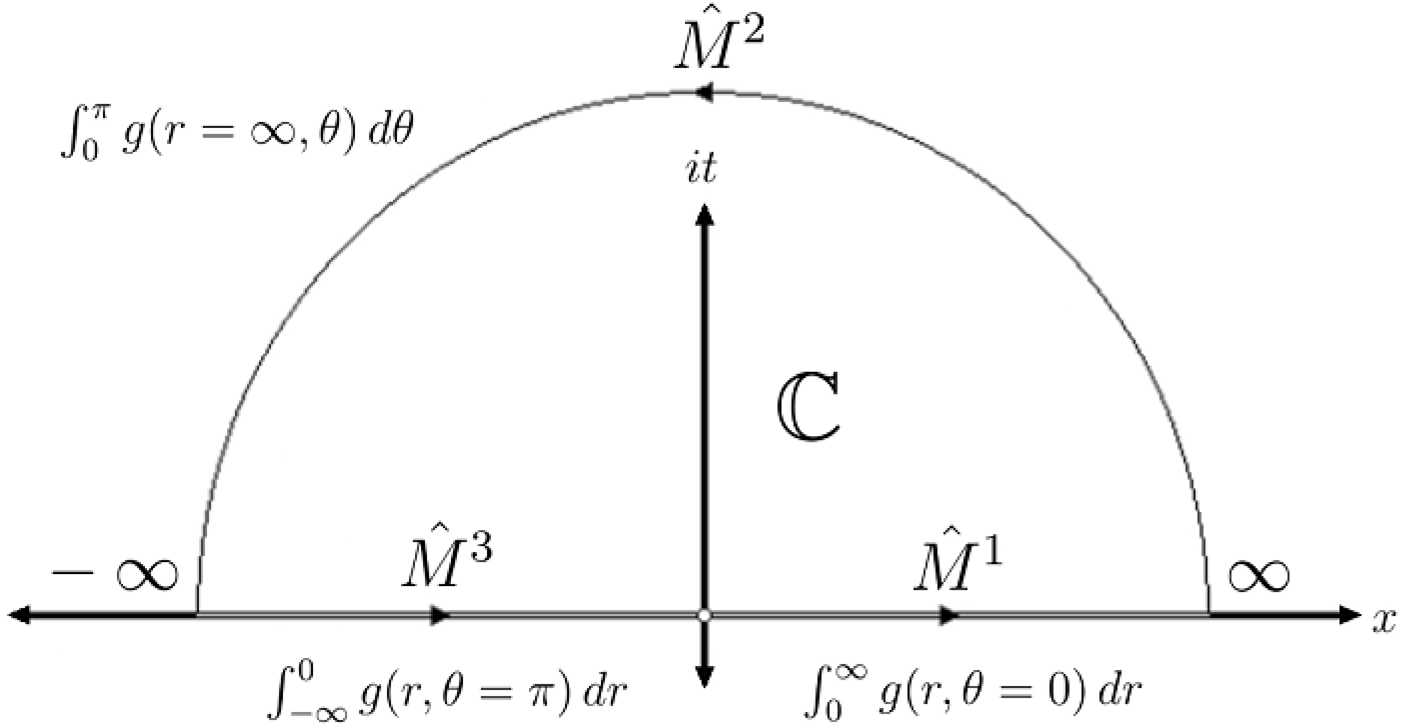


Figure 8: We can understand the path symbolically labeled $\hat{M}^2$ as the path through $\Sigma^{\varnothing}$. The two path segments on the real line labeled x very clearly correspond to χ^5_+ and χ^5_-. We will have to change the orientation of this graph when we want to connect future timelike infinity to past timelike infinity on a higher level of $\aleph$.

The point p in $\Sigma^{\varnothing}$ can be the point z_0 that appears in the Cauchy formula, or any other special point that we might need it to be, such as, perhaps, the location of the observer at the origin. In section II.6, we will examine the twistor representation in which points are deformed such that we might associate $p \in \varnothing$ with the entire outer path labeled $\hat{M}^2$. The important thing is that the Cauchy formula defines three piecewise path lengths around the origin and it is obvious that we have already represented those paths with χ^5_+, χ^5_-, and $\chi^5_{\varnothing}$. The reader should note the excellent qualitative agreement between the specific concept of $\Sigma^{\varnothing}$ and the path at infinity, and in general between $\hat{M}^3$ and the three pieces of the integral over C.

We proposed in reference [12] to use equation (1.17) to write

$$\int_{-\infty}^{\infty} f(t)\delta(t)\,dt = \int_0^{\infty} g\left(r,0\right)\,dr + \int_0^{\alpha^{-1}_{MCM}} g\left(\infty,\theta\right)\,d\theta + \int_{-\infty}^{0} g\left(r,\alpha^{-1}_{MCM}\right)\,dr \quad , \tag{1.20}$$

where the δ function places a singularity (topological obstruction) in the integrand like $1/t$ which blows up at the origin. In more recent iterations, the mechanism that we expected to derive from α^{-1}_{MCM} hyperradians in equation (1.20) has been offloaded onto aspects of hypercomplexity elsewhere and we refer the reader to reference [10] for fuller details on that offloading.

Here is a good place to notice an asymmetry between chronological time x^0 and chirological time χ^5. In chronos, future timelike infinity $t=\infty$ is perfectly balanced with a symmetric value at past timelike infinity $t=-\infty$. However, the hyperreal infinity-valued chirological qubit at $\hat{\Phi}^{\infty}$ is balanced with a hyperreal infinitesimal-valued qubit $\hat{\Phi}^{-\infty}$ at the origin. When $\hat{\Phi}$ points to spacelike infinity but $\hat{\Phi}^{-1}$ points to the origin from a lower level of $\aleph$ there is a manifest asymmetry. Certainly any number of principles can be attached to this asymmetry. The matter/anti-matter imbalance comes to mind because big and little hyperreal infinity are both positive numbers (little hyperreal infinity is like $\lim_{x\to\infty} 1/x>0$) and they contrast the notion of infinity composed from only positive and negative "big infinity." The symmetric concept of plus and minus infinity might dominate the analyses that predict global baryon neutrality whereas the fully transfinite analysis might directly suggest the excess of matter over anti-matter. Also note that in plane polar coordinates there is no such thing as minus infinity because $r\in[0,\infty)$ for $\theta\in(0,2\pi]$.

Originally, we put the rotation in equation (1.20) to be through α_{MCM}^{-1} "hyperradians" to show that the method allows a free parameter when we complexify the geometry of the complex plane. Instead, we have complexified the underlying real analysis. Now that we have a different origin for the free parameter $\alpha_{MCM}^{-1}=2\pi+(\Phi\pi)^3$ [10], the π ordinary radians required for the normal piecewise expansion of C will suffice. Then equation (1.20) becomes

$$\int_{-\infty}^{\infty}\psi(x,t)\delta(t)\,dt=\int_0^{\infty}\psi\left(r,0\right)\,dr+\int_0^{\pi}\psi\left(\infty,\theta\right)\,d\theta+\int_{-\infty}^{0}\psi\left(r,\pi\right)\,dr \quad . \tag{1.21}$$

We will eventually need to show that the proposed representation in equation (1.21) can support a non-exploding integration scheme that solves the problem Feynman encountered when he tried to integrate over all of spacetime: his integrals would invariably explode. It was for the reason of exploding integrals that Feynman imposed his finite time constraint. In developing the associated argument, Feynman used one idea of "all of spacetime" that perhaps relies on an unassumed differentiation between the theory of functions and the theory of functionals. When the probability is a function of x and t

$$P \quad\equiv\quad P(x,t) \quad , \tag{1.22}$$

because the wavefunction is a function of x and t, Feynman's thought experiment does produce in its limit the complete space of all possible paths. However, in functional analysis the probability

$$P \quad\equiv\quad P[\psi] \quad , \tag{1.23}$$

clearly implies that the continuum of all possible paths includes the Cauchy C curve. Unlike equation (1.22), the P in equation (1.23) depends on a complex variable because ψ is complex-valued. Therefore, the ordinary rules of complex analysis must apply and those rules say C needs to be included in the definition of all of spacetime. We will need to decide if we will use the Cauchy integral formula to expand the probability, as examined above, or the probability amplitude which usually depends on purely real variables but there are some subtleties. In general, this relates to the difference between discrete and continuous sets of eigenfunctions that lead to $P[\psi]\in\mathbb{R}$ and $P[\psi]dx\notin\mathbb{R}$ respectively. In the continuous basis, there is an extra integral that is preformed before a real valued probability is obtained for comparison with experiment and we will return to these rudimentary yet critical details in section IV.3. However, the general treatment of the Cauchy formula does not appear in this book.

In equation (1.20), $g(r,\theta)$ is a function of a point in the complex plane and that is to say that g is a function of a complex number. On the other hand, $\psi(x,t)$ appears to be a complex-valued function of two real variables x and t. This would imply $P\equiv P(x,t)$ and that the Cauchy integral formula has no natural application to the integrals Feynman used to build his quantum mechanical wavefunction. However, the reader must recognize that we can force ψ to be a function of a complex variable. How can we force this other possibility? The analytical origin of the minus sign in the line element of Lorentzian spacetime

$$ds^2=-c^2(dt)^2+(dx)^2+(dy)^2+(dz)^2 \quad , \tag{1.24}$$

can come from a definition $x^0=ict$ which means that the region R considered by Feynman is actually the complex plane $\mathbb{C}$ because it is spanned by one real axis and one imaginary axis. The dimensional transposing parameter c is a trivial coefficient that can be ignored so

$$z \quad \equiv \quad x+it\in\mathbb{C} \quad . \tag{1.25}$$

If $\{x,t\}$ is a point in $\mathbb{C}$ then $\psi(x,t)\equiv\psi(z)$ must be a function of a complex variable and the Cauchy formula applies. We will discuss this complex forcing method further in section II.4.

In the famous thought experiment, Feynman builds up his space of all possible paths by adding increasing numbers of screens and slits to a hypothetical double slit experiment. The probability is a function of the wavefunction, which is a complex-valued function, so the path in the Cauchy integral formula is completely contextually correct even when $\psi(x,t)\not\equiv\psi(z)$. In quantum physics, the thing that the observer actually tests at the endpoints of $\hat{M}^3$ is the real valued probability P which, in turn, is a functional of the complex-valued wavefunction. In the Cauchy formula, we have $f(z_0)\equiv P(\psi_0)$ but what is ψ_0? A general question in quantum mechanics is to ask, when given an initial state ψ_i, what is the probability of observing another state later. Call that state ψ_0 so the probability of observing it is $P[\psi_0]$ (or $P'[\psi_0]dx$.)

Feynman has not introduced hypercomplex field variables so he considered the neighborhood around one level of $\aleph$ and the minimum of the action is favored. When all the future levels of $\aleph$ have gravitating objects whose magnitudes (masses) are defined with hyperreal infinities of successively increasing infinitude, the trajectory of a test mass is intuitively pulled out of the Euclidean minimum of action onto the maximum action path that heads off in the direction of $\hat{\Phi}$. Notice the consistency here. An intuitive way to test gravitational equations is to consider the motions of test particles. If we have equations of motion for gravity near the surface of the Earth, they should show that an ordinary object will fall. Now we are proposing to radically modify the entire paradigm of gravitation by adding this new path C and different hypercomplex infinities and infinitesimals, and we expect the reader to believe that it works when we have no significant accompanying calculation similar to Einstein's prediction for the deflection of light from Mercury as it passes deep through the sun's gravitational well.[1] However, we do have many other results including one very much like Schwinger's derivation of the first order correction to the electron's magnetic moment. Schwinger used perturbation theory that depends on an expansion in the magical fine structure constant which is the number we have derived from first principles with help from God. The author expresses his gratitude to God.

What possible reason could there be for a trajectory in which stationary states seem stationary but constantly zoom off to infinity in $\mathcal{H}_1 \mapsto \Omega \mapsto \aleph \mapsto \mathcal{H}_2$ where $\mathcal{H}_2$ lies beyond infinity? The infinitely large hyperreal qubit at the end of $\mathcal{H}_1$'s $\hat{\Phi}$ vector is the most massive thing in the local hypercosmos so when the trajectory always goes straight toward the mass of the universe on the higher level of $\aleph$ (where $m \sim \infty$), it is making a beeline for the most massive object in its local neighborhood. This is exactly what is expected of a theory of gravity so there is a lot of consistency. The main push of this book will be to demonstrate familiar aspects of unification between quantum theory and general relativity but we very much urge a third party undertaking of a rigorous survey of MCM/TOIC objects/methods to check if they are useful in any cutting edge experimental applications or if there is any unnoticed but interesting hypercomplexity hidden somewhere within the infinite complexity. The existence of this complexity has been convincingly demonstrated throughout this research program.

It has been very many decades since Feynman wrote hits paper titled "Space-Time Approach to Non-Relativistic Quantum Mechanics," and some of the problems with that approach had not been resolved until the introduction of the MCM wherein we have proposed to explore the exotic but allowed maximum action equations of motion. Feynman's Ph.D. thesis was about the principle of least action in quantum theory so he discarded the maximum action paths *a priori*, but we will not do so. Since the action principle only allows two solutions, and Feynman has already described one of them, the MCM must rely on the other one.

[1] This work resulted from Einstein's collaborations with Grossmann and Besso so it contrasts greatly with the MCM which did not result from any collaborations.

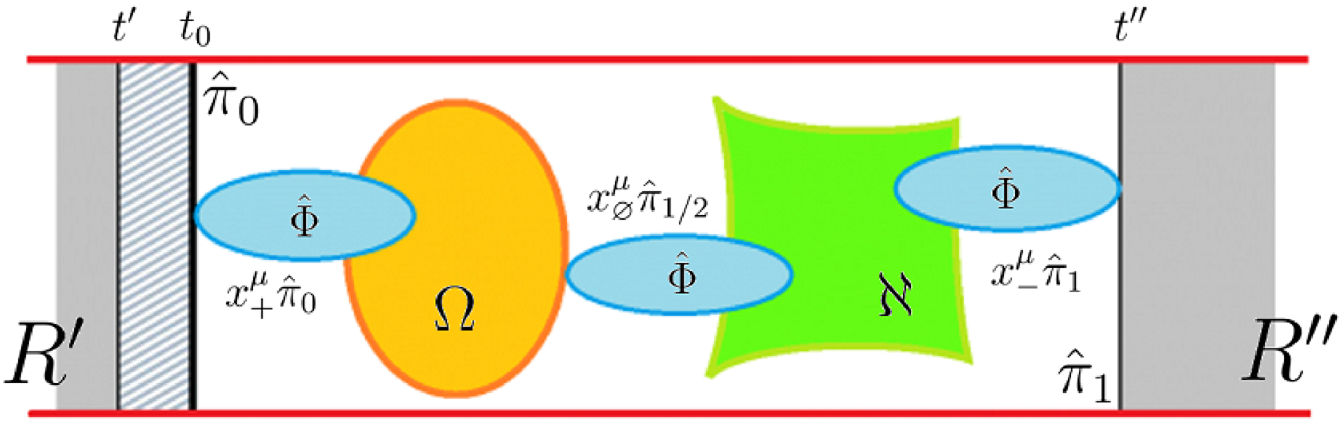


Figure 9: This figure illustrates the region of figure 7 between t_0 and t''. If one considers the third $\hat{\pi}$ component in the middle as $x^\mu_\varnothing \hat{\pi}$ then it is very easy to see where the $(\Phi\pi)^3$ term comes from in α_{MCM}. Simply let the non-unitary component be such that each $\hat{\pi}$ is inflated by Φ from one moment to the next. In a sense we can say $\hat{\pi}_{1/2}$ is out of phase with the other two $\hat{\pi}$s (the normal sense of $\pi/2$ orthogonality) and it is $\hat{\pi}_0$ and $\hat{\pi}_1$ that give, speculatively at least, the 2π for $\alpha_{MCM} = 2\pi + (\Phi\pi)^3$.

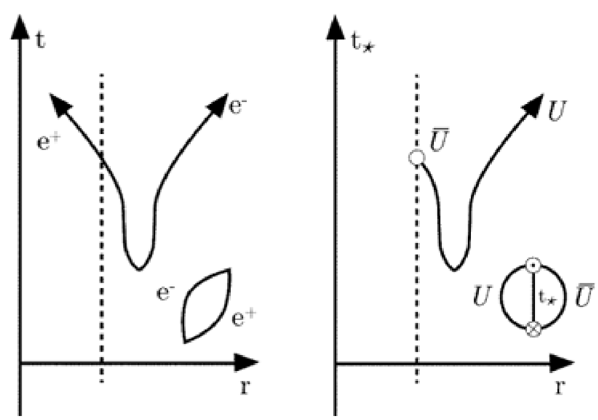


Figure 10: This is the the mechanism from reference [7] through which we claimed to have unified gravity and electromagnetism, and here we point out that that reference would have been better titled *Dark Energy in the Modified Cosmological Model with Ancillary Takeaways*. We use the "in" $\odot$ and "out" $\otimes$ diagrammatic notation of introductory electromagnetism to show the places where the piecewise bulk geodesics are joined across different tiers of infinitude (levels of $\aleph$) in $^*\mathbb{C}$. The original caption for the diagram was, "The Feynman diagrams of gauge theory generate surfaces which represent interacting strings [16]. On the left: electromagnetic pair creation near the horizon. On the right: polarized gravitational pair creation." This figure is adapted from reference [17] and it went unfortunately uncited when we used it in reference [7].

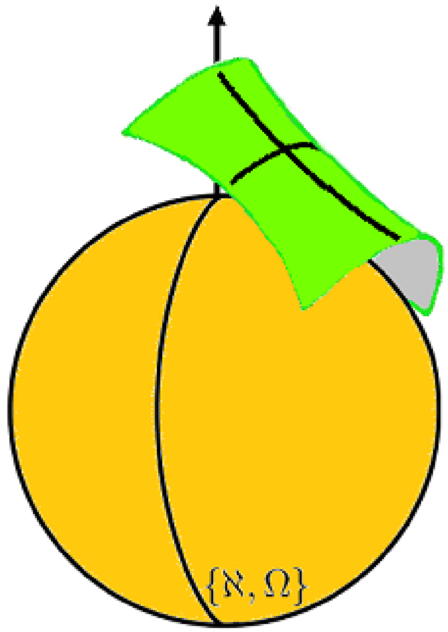


Figure 11: There are a lot of ways that $\aleph$ and Ω could be connected.

II General Relevance with Emphasis on Gravitation

The first section of this chapter is an elementary review. Section two is dedicated to general relativity. In section three, we make a nice comment about the Bekenstein–Hawking formula for the entropy of black holes. Section four describes how coordinates in spacetime can be defined so that the corresponding metric becomes either Lorentzian or Euclidean while maintaining a valid general relativity. This method can force the condition that causes the wavefunction to be a function of a complex variable. Section five describes a method by which we may insert qubits onto a geometric manifold and emphasize the integration of quantum mechanics into MCM quantum gravity/quantum cosmology. Here, we provide some modest but original insights into the structure of perturbation theory with a new definition for $\hat{\varphi}$. Sections six and seven deal with twistors, spinors, dyads, and quaternions. The final section in this chapter is about the MCM mechanism of unification between quantization and gravitation.

Whereas the bulk of the research conducted in this program has focused on the quantum sector, here we also investigate the gravitational sector. This chapter applies general relativity to derive properties of the MCM and the reader should keep in mind that general relativity is not called "general" because it is a high-ranking theory but rather because it is generic. Special relativity to the contrary is not generic. This book is more comprehensive than previous work and is therefore more rigorous but it is also very reiterative. The tools of both hyperreal and complex analyses provide synergy for new tools of **hypercomplex analysis** in the theory of infinite complexity.

The entire history of physics shows that the rigorous mathematical connection of modes is sufficient to demonstrate inter-modal energy transfers in representative experiments and research regarding the MCM has uncovered a novel new mathematical connection. There is no guarantee that a "correct" mathematical demonstration of a principle will show the principle in the lab, but if it does not then that would be the first time. The emphasis, of course, is on correctness but, due to the simplicity of the TOIC principles, it is totally obvious that they are correct. Any defect cited in this research program is of at most genus errata.[1] As Jesus said in John 18:23, "If I said something wrong, testify as to what is wrong." Jesus immediately goes on to ask, "But if I spoke the truth, why did you strike me?," which can be taken, very much so, in the context of this banned research as well.

II.1 Relevant Aspects of Classical Physics

General relativity has a Newtonian limit whose structure is the same as the classical electric force. Those solutions are commonly written as

$$\vec{F}_g = -\frac{GMm}{r^2}\hat{r} \quad , \qquad \text{and} \qquad \vec{F}_e = -\frac{e\,Qq}{r^2}\hat{r} \quad , \tag{2.1}$$

where $\vec{F}_g$ and $\vec{F}_e$ are Newton's law for gravity and the Lorentz force law with no moving charges, a.k.a. Coulomb's law. If quantum electrodynamics (QED) is one kind of multiplectic expansion of the Poisson equation then all we have to do to unify gravity with electromagnetism is to show that general relativity is another multiplectic expansion of the Poisson equation using the same set of objects for each in a common structure. However, the Poisson equation, as in equations (2.1), is not a prominent topic in this book.

A common representation of the classical force law is

$$\vec{F}_{jk} = \frac{\beta}{r_{jk}^2}\left(\frac{\vec{r}_{jk}}{|\vec{r}_{jk}|}\right) \quad , \tag{2.2}$$

where β is an electric or gravitational coupling constant. The multiplectic expansion from Newton's vector gravity to Einstein's tensor theory brings in tensor indices to track curvature, it brings in the metric to define the distance along $\vec{r}_{jk}$, and it throws out the idea of a vector such as $\vec{r}_{jk}$ connecting two points in a curved manifold. To accommodate dynamical spacetime geometry, it is required to use metrical and other tensors, and non-tensorial connection coefficients to fully describe gravity, but even those tensor equations have to reduce in the Newtonian limit to representations of the form of equations (2.1-2.2). These formulae are derivable from the Poisson equation for gravity

$$\nabla^2\phi = 4\pi G\rho \quad , \tag{2.3}$$

when the gravitational potential ϕ at a distance r away from mass M is

$$\phi(r) = -\frac{GM}{r} \quad . \tag{2.4}$$

[1] During the preparation of this book we discovered and corrected an important erratum in reference [10].

Classical gravitation uses the scalar potential ϕ to determine the equations of motion but general relativity has no gravitational potential and instead uses the non-tensorial connection $\Gamma^{\mu}_{\rho\sigma}$ to compute geodesics with the geodesic equation

$$\frac{d^2x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\rho\sigma}\frac{dx^{\rho}}{d\tau}\frac{dx^{\sigma}}{d\tau} = 0 \quad . \tag{2.5}$$

Geodesics are the paths followed by unaccelerated objects in spacetime. The connection, also called the Christoffel symbol, the Christoffel connection, or the affine connection, is determined from the metric via

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(\partial_{\mu}g_{\nu\sigma} + \partial_{\nu}g_{\sigma\mu} - \partial_{\sigma}g_{\mu\nu}\right) \quad . \tag{2.6}$$

The inverse metric $g^{\lambda\sigma}$ appears in equation (2.6) so when we convert to 5D we will make use of the inverse Kaluza–Klein metric Σ^{AB} which has, thankfully, already been calculated and posted on Wikipedia. We can simply plug it into the formula for the 5D connection coefficients where $g^{\lambda\sigma}$ appears in equation (2.6). Other important objects include the Riemann tensor which is (in 4D)

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} \quad , \tag{2.7}$$

and one of its contractions: the Ricci tensor

$$R_{\mu\nu} \quad \equiv \quad R^{\lambda}_{\mu\lambda\nu} \quad . \tag{2.8}$$

Equations (2.7-2.8) generalize to 5D by changing the Greek indices to Latin. General relativity can become a very complex theory when the Ricci tensor and the metric are not linearly dependent on each other and we will discuss related nuance throughout chapter three. To date, we have only worked with Einstein's equation in the form

$$8\pi T_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}\Lambda \quad , \tag{2.9}$$

but the avenue toward greatest complexity is shown when the Einstein tensor

$$G_{\mu\nu} \quad \equiv \quad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \quad , \tag{2.10}$$

is decomposed to give Einstein's equation as

$$8\pi T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + g_{\mu\nu}\Lambda \quad , \tag{2.11}$$

with the Ricci scalar R being

$$R \quad \equiv \quad g^{\mu\nu}R_{\mu\nu} \quad . \tag{2.12}$$

The main result of the MCM has been to develop a new connection between gravitation and quanta. When the ontological basis was considered to have three elements[1] $\{\hat{\pi}, \hat{\Phi}, \hat{\imath}\}$, we showed a one-to-one unification

$$f^3\big|\psi;\hat{\pi}\big\rangle \quad \mapsto \quad T_{\mu\nu} \tag{2.13}$$

$$\big|\psi;\hat{\Phi}\big\rangle \quad \mapsto \quad G_{\mu\nu} \tag{2.14}$$

$$i\big|\psi;\hat{\imath}\big\rangle \quad \mapsto \quad g_{\mu\nu}\Lambda \quad , \tag{2.15}$$

but now the ontological basis is $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$. This is better suited to Einstein's equation as it appears in equation (2.11) but there are several choices for that which $\hat{2}$ should map to. This freedom to choose compounds the original arbitrariness imposed when we chose mappings for $\{\hat{\pi}, \hat{\Phi}, \hat{\imath}\}$. When there were only three objects in Einstein's equation, we chose one particular set of maps to demonstrate that the maps exist. Now that there is a fourth object, we will not make a guess about how the four tensors in Einstein's equation are connected to the four objects in the ontological basis. Instead, we simply point out that since there are four of each, there exist some other one-to-one mappings even if we don't pick one right now. The important thing isn't the form of the specific maps. The most important feature is that we have obtained via the MCM the correct dimensionless coefficient of proportionality: 8π [12, 4], as in equation (2.11). We will discuss the origin of 8π in the MCM in chapter three.

The Newtonian gravitational potential, by construction, does not include the gravitational self-force of a particle when it deforms the spacetime of its own inertial frame. Likewise with the electromagnetic potential: when we probe the field of a strong magnet with a weak test charge e_0, the equations of motion, though completely valid, are only an excellent approximation to the equations of motion that include the small deformation of magnetic field lines around e_0 as it moves through the magnetic field. The geodesics of general relativity also do not account for the gravitational self-force but they do allow other generalized energy densities to change the course of particles in a way that the classical gravitational potential cannot. It is for this reason, among others, that we say Einstein's formulation of gravitation is superior to Newton's.

[1] The fourth element $\hat{2}$ first appeared in references [4] and [5].

The electromagnetic self-force is called the Abraham force and it involves the third time derivative of position as

$$\vec{F}_{\text{A}} := q^2 \dddot{\vec{x}} \quad . \tag{2.16}$$

The electromagnetic force is often taken as the Lorentz force

$$\vec{F}_{\text{L}} = m\ddot{\vec{x}} = q\big(\vec{E} + \dot{\vec{x}} \times \vec{B}\big) \quad , \tag{2.17}$$

alone but the force equation becomes third order in ∂_t as the Lorentz force is taken in superposition with the Abraham force. In general, the approximation to the real electric force with the second order Lorentz force is valid but there are extreme regimes such as plasma physics where the third order equations

$$\vec{F}_{\text{AL}} = m\ddot{\vec{x}} = q\big(\vec{E} + \dot{\vec{x}} \times \vec{B}\big) + q^2 \dddot{\vec{x}} \quad , \tag{2.18}$$

must be considered if a realistic answer is to be obtained. As discussed in the previous chapter, equations of this form are not directly compatible with Feynman's formalism. The Lagrangian that gives the Abraham–Lorentz force is not solely a function of position and velocity.

Consider Carroll's words from reference [17].

> "The primary usefulness of geodesics in general relativity is that they are the paths followed by unaccelerated test particles. A **test particle** is a body that does not itself influence the geometry through which it moves[1] – never perfectly true, but often an excellent approximation. This concept allows us to explore, for example, the properties of the gravitational field around the Sun, without worrying about the field of the planet whose motion we are considering. The geodesic equation can be thought of as the generalization of Newton's law $\mathbf{f} = m\mathbf{a}$, for the case of $\mathbf{f} = 0$, to curved spacetime. It is also possible to introduce forces by adding terms to the right-hand side [*of equation (2.5)*],
>
> $$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = \frac{q}{m}F^\mu{}_\nu\frac{dx^\nu}{d\tau} \quad ." \tag{2.19}$$

Equation (2.19) is a second order force law because, as stated, it does not account for the self-acceleration of test particles. The full relativistic force law is more complicated and Dirac is credited with working out those details in 1938. The familiar object $F^\mu{}_\nu = g_{\rho\nu}F^{\mu\rho}$

[1]This is the gravitational analogue of ignoring the Abraham force. Its dual is called the gravitational self-force or sometimes the gravitational backreaction.

in equation (2.19) is the electromagnetic field strength tensor. It is the tensor that most directly influences observables. It is defined via

$$\partial_\mu F^{\mu\nu} = J^\nu \quad , \qquad \text{or} \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \tag{2.20}$$

where the 4-vector potential A^μ defines the current through

$$J^\mu = \frac{1}{4\pi}\eta^{\mu\nu}\partial_\nu\partial_\rho A^\rho \quad . \tag{2.21}$$

Note how equation (2.21) (the tensor Poisson equation) has a dimensionless coefficient 4π that is also the coefficient of the leading term of the ontological resolution of the identity

$$\hat{1} = \frac{1}{4\pi}\,\hat{\pi} - \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{\imath} \quad . \tag{2.22}$$

It is really striking that the leading coefficient of a logical ordering of the the ontological resolution of the identity is $1/4\pi$. The dimensionless constants that effortlessly fall out of the MCM include: exactly the electromagnetic coefficient 4π, exactly the gravitational coefficient 8π, and the fine structure constant to within 0.4%. On the last, it is totally certain that we can reformulate quantum theory so that the inverse fine structure constant is exactly $2\pi+(\Phi\pi)^3$ and then the 0.4% disagreement will be shuffled into some other quantum fuzziness elsewhere. Quantum electrodynamical perturbation theory works because α_{QED} is a very small number and another number that differs by 0.4% is also very small.

The special behavior of the ontological basis hails from the MCM notation which specifies a manifold and a vector space for state vectors

$$\big|\psi;\hat{\imath}\big\rangle \quad \equiv \quad \psi(x^\mu_-) \tag{2.23}$$

$$\big|\psi;\hat{\pi}\big\rangle \quad \equiv \quad \psi(x^\mu) \tag{2.24}$$

$$\big|\psi;\hat{\Phi}\big\rangle \quad \equiv \quad \psi(x^\mu_+) \quad , \tag{2.25}$$

that would otherwise all live in $\mathcal{H}'$ and have position space representations in the x^μ coordinates of Minkowski space $\mathcal{H}$ due to the implicit assumptions of quantum mechanics. Equation (2.23) says ψ lives in $\aleph'$ which relies on coordinates x^μ_- and equation (2.25) says ψ lives in Ω' relying on coordinates x^μ_+. x^μ are the coordinates of Minkowski space $\mathcal{H}$. $\mathcal{H}$ is where the theory is required to match observations and $\hat{\pi}$ is the element of the ontological basis whose coefficient is the electromagnetic coupling constant. We expect to extend equations (2.23-2.25) to include the complete ontological basis with

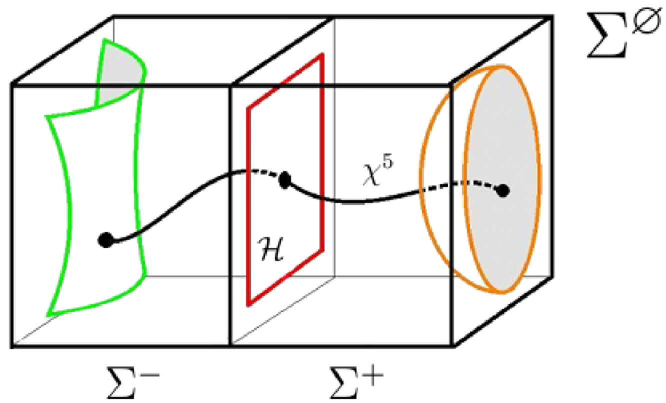


Figure 12: $\mathcal{H}$ is not at the point $p \in \Sigma^{\varnothing}$. $\Sigma^{\varnothing}$ lies all around the outside of this MCM representation.

$$\big|\psi;\hat{2}\big\rangle \quad \equiv \quad \psi(x^\mu_\varnothing) \quad . \tag{2.26}$$

This equation says that $|\psi;\hat{2}\rangle$ lives in the same Hilbert space $\mathcal{H}'$ that $|\psi;\hat{\pi}\rangle$ lives in but the position space representation is in the $x^\mu_\varnothing$ coordinates that do not belong to $\mathcal{H}$. The original three maps were associated with the three vector spaces of a Gel'fand triple and this fourth map should be associated with the dual space to Hilbert space $\mathcal{H}'$ which is also $\mathcal{H}'$. Since $\mathcal{H}'$ is self-dual, the fourth map for $\hat{2}$ is easily accommodated in the Gel'fand triple.

A fourth map for $|\psi;\hat{2}\rangle$ implies a new topological component associated with $\{\aleph,\mathcal{H},\Omega\}$ which we will call simply $\varnothing$ and say that it is a 4D section of the 5D manifold $\Sigma^{\varnothing}$. $\Sigma^{\varnothing}$ lies around the outside of the old unit cell (figure 12) and is centered in the most recent depiction: figure 1.[1] The description of the MCM unit cell has always been as a *partial* unit cell because we had never even begun to examine all the diagrammatic subtleties that would fully include $\Sigma^{\varnothing}$. In figure 1, we put $\Sigma^{\varnothing}$ in the center to emphasize that it is the main unknown at this point but the figure still only shows a partial cosmological unit cell because it does not show how $\Sigma^{\pm}$ are actually connected, which is vital. We have been able to get away with not stating the relationship between $\hat{2}$, $\varnothing$, and $\Sigma^{\varnothing}$ because, as indicated by the "$\varnothing$" notation, it does not directly contribute to observables. However, before the set of MCM observables that would be of interest to experimentalists can be derived, the indirect dependence on $\Sigma^{\varnothing}$ must be clarified and we have mentioned $\tilde{M}^4$ as a next step in that direction.

Regarding $\varnothing$, note that the path over $d\theta$ on the right hand side of

$$\int_{-\infty}^{\infty}\psi(x,t)\delta(t)dt = \int_0^{\infty}\psi\left(r,0\right)dr + \int_0^{\pi}\psi\left(\infty,\theta\right)d\theta + \int_{-\infty}^{0}\psi\left(r,\pi\right)dr \quad , \tag{2.27}$$

is zero because $\psi(\infty)\!=\!0$ and, similarly, the path over $\chi^5_\varnothing$, which lies entirely at infinity, has zero length in our "affine" parameter χ^5. We say $\psi(\infty)\!=\!0$ in compliance with the ordinary

[1] We use a loose definition of manifold because it is possible to leave the MCM "manifolds" along the χ^5 direction.

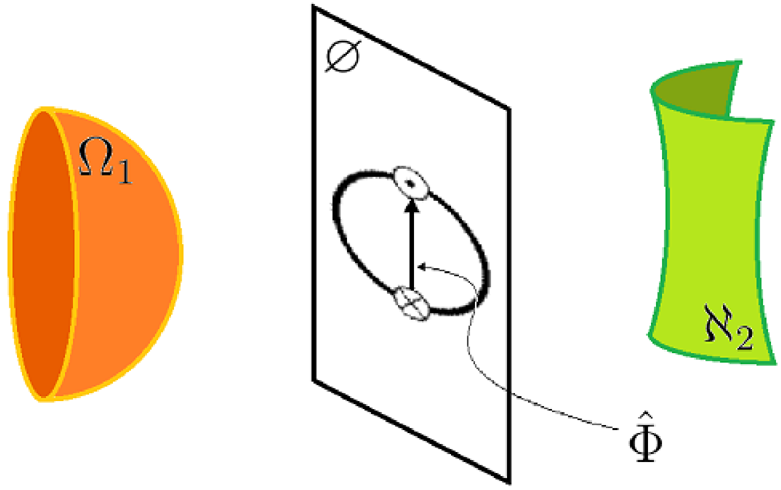


Figure 13: This figure makes reference to figure 10 and describes where the fourth step in $\tilde{M}^4$ could be added to the existing process $\hat{M}^3$. While we will not develop the method in this section, the general idea is to use $\hat{2}$ as the first step to create a copy of $\mathcal{H}$ and then put it in between $\aleph$ and Ω where the level of $\aleph$ increases. If in some sense we can "leave $\hat{\Phi}$ behind" when evolving through $\varnothing$, possibly by folding it into the coefficient of a Fourier transform, then we have a good clue about how to restore unitarity after the non-unitary MCM operations: use modified Fourier coefficients.

square integrability condition in quantum mechanics.[1] If the particle is at some finite $\vec{r}$ at some time t then the probability of finding it at $\vec{r}=\infty$ finitely later should be zero. The entire manifold $\Sigma^{\varnothing}$ exists only to guarantee the existence of an exceptional point where the level of $\aleph$ can increase during $\Omega\mapsto\aleph$, as in figure 13. For now, we simply say that there is some manifold $\Sigma^{\varnothing}$ which has at least one point that can sew together Σ^{+} and Σ^{-}, and then we introduce a non-trivial parameter

$$\chi^5 \quad \equiv \quad \chi^5_+ \otimes \chi^5_\varnothing \otimes \chi^5_- \quad , \tag{2.28}$$

called chiros. We can use this definition to draw diagrams that show a smooth affine parameter χ^5 even when the manifolds it passes through have been disconnected by $\mathcal{H}$- and $\varnothing$-branes.

Why is being able to represent the non-smooth topology on a smooth affine parameter important? It is important because affine parameters are the best way to convert a diagram's lines into equations. No matter how complex a given curve is, be it geodesic or field line, once the curve is known, it is possible to parameterize the curve in a way that makes it trivial. There are other parameterizations which are more complex but the simplest one uses an affine parameter. A good definition for an affine parameter λ on a geodesic is that it is a linear rescaling of the proper time $\tau\rightarrow\lambda=a\tau+b$ that would be experienced by an observer traveling along the path. Once we have constructed χ^5 as in equation (2.28), we can put an affine parameter on that construction and retain the label χ^5. χ^5 is the parameter that allows us to represent the MCM unit cell with a smoothly connected diagram.

[1]The MCM condition necessarily gives $\psi(0)=0$ because the observer will never find the particle at his own exact location.

Why are diagrams important? As we pointed out in reference [7], "Diagrams are used in physics to transmit information with a clarity that is not present in excessively quantified arguments." Diagrams are important because they allow us to visualize the degrees of freedom of a system in a way that lets us easily write down the kinetic and potential energy functions, which then let us formulate the Lagrangian or Hamiltonian. Thereafter, known methods are applied to derive a solution to either the Euler-Lagrange equation or the action principle and the equations of motion result. For instance, while we have spent a lot of time developing the diagrammatic representation which led to the principle of most action, other researchers may have neglected the diagrammatic (geometric) representation in favor of some algebraic descriptions of their models and then become bogged down trying to squeeze the theory onto a software module that only computes the path of least action.

It is quite popular in the contemporary physics scene to assume a Hamiltonian without first sketching out a system's diagram. We completely shy away from this on philosophical grounds. This is why we have never made any statements like, "Let there be a diagonal Hamiltonian matrix operator," and it is why we have not considered arbitrarily specific objects like the Einstein–Cartan–Kibble–Schiama action or similar. The TOIC represents only one single point in parameter space where the normal analysis is to consider gauge theories as entire equivalence classes, or broad hypersurfaces in parameter space. By guessing random Hamiltonians, the idea is that the guesses should intersect these planar equivalence classes. However, the TOIC, with its infinitely precise irrational coefficients, is pointlike and singular in parameter space so it is highly unlikely be intersected in this way. It must be developed from the diagram. Furthermore, many classical methods for probing theoretical parameter space only use one kind of time and therefore could never find the parameters of a two time model.

In the course of doing physics, only when we have developed the diagrams enough to extract one intuitive, unambiguous energy function (a Hamiltonian or a Lagrangian) should we begin to compute the equations of motion. This diagrammatic aspect is the infamous "underlying conceptual component" [7] whose illusory nature is currently the main bottleneck preventing progress in theoretical physics. Diagrams have been among the most important contributions of the TOIC and we have been lucky enough to uncover several irrefutable, non-diagrammatic, quantitative results as well. However, we still have not written the MCM Hamiltonian so that will be a worthwhile task to undertake in the future Ω. The purpose of this research has been to develop a method by which one could calculate new equations of motion that have remained elusive to very many other researchers that are looking for them. While Mathematica or Matlab are happy to give the equations of motion for a given Hamiltonian, we philosophers of Nature, we physicists, should only be happy with equations of motion that come from a diagram so guessing random Hamiltonians can never lead to true happiness.

Reference [18] lists five general categories for theories of gravitation. Mann writes, "By setting various conditions on [*the manifold*] M and choosing an appropriate Lagrangian $\mathcal{L}$ we can construct a variety of theories of gravity." They are: general relativity, torsion theories, Kaluza–Klein theories, supergravity theories, and algebraically extended theories. Mann defines the last as follows [18].

> "[*Algebraically extended theories are*] theories in which all geometric objects take their values in an algebra $\mathcal{A}$, instead of the real numbers $\mathbb{R}$. [*sic*] The spacetime

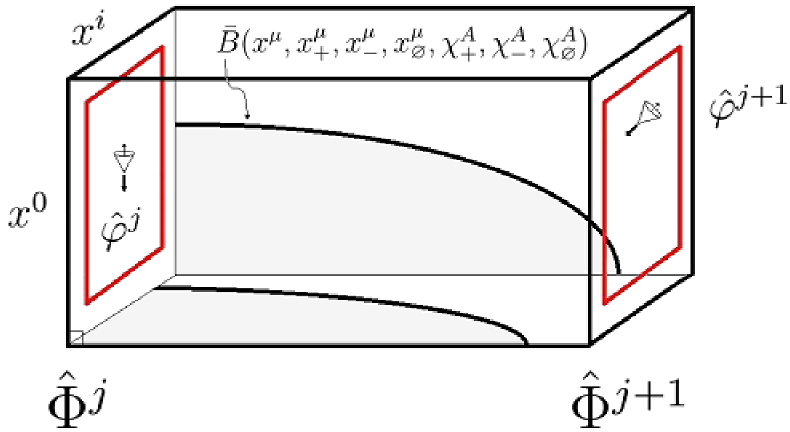


Figure 14: This figure shows a hypothetical hypercomplex gravitational potential $\bar{B}$ that replicates the motion along hypercomplex geodesics

> manifold M is still real; by requiring the reality of physically measurable quantities on M it has been shown that the only allowable algebras which may be introduced are the real numbers (general relativity), the complex and hypercomplex numbers and the quaternions and hyperquaternions."[1]

The MCM combines aspects of general relativity, torsion theories, Kaluza–Klein theories, and algebraically extended theories. Infinite complexity is too much complexity to consider and luckily there is a simplifying constraint built into Kaluza–Klein theory: the 5D Ricci tensor R_{AB} has to vanish at all times. This severely restricts the complexity of the bulk hyperspacetime in $\Sigma^\pm$ but it still allows rather complex solutions to the 5D metric because gravitational radiation is a non-trivial solution to $R_{AB}=0$. We will therefore not be constrained to say that the bulk hyperspacetime is a void; we are only constrained to say that the 5D Ricci tensor R_{AB} always vanishes and the hyperspacetime bulk is void of any points where the wavefunction collapses. (By definition, collapse inducing measurements happen in $\mathcal{H}$ and never in the bulk.) The infinite number of $\mathcal{H}$-branes in the hypercosmos can imply a physical multiverse, or there is a more philosophically untouchable interpretation when the hypercosmos is only a mathematical potential. It can be the observer's non-physical tool for making abstract calculations to derive predictions for observables in spacetime.

Figure 14 shows the expected gravitational potential energy curve between adjacent levels of $\aleph$ $\hat{\Phi}^n$ and $\hat{\Phi}^m$. As levels of $\aleph$ increase in the future, the arrow of time points strictly in the direction of increasing magnitude in $^\star\mathbb{C}$, which is the normal downhill energy condition seen in figure 14. Downhill leads toward the infinitely greater mass-energy that exists in the future under the assumptions of the MCM. We can say that the gravitational potential energy landscape in the time direction is completely determined by the object in the future because the gravitational amplitude of the object in the past, being on a lower level of $\aleph$, is on the order of an infinitesimal with respect to the future object. However, since the future is out there at future infinity, it will be difficult to make the direct extension of the Newtonian

[1] Mann goes on to make an aside about how the theory based on hypercomplex numbers is testable in the solar system, but those hypercomplex numbers, as mentioned in an earlier footnote, are likely not $^\star\mathbb{C}$.

potential from spatial dependence to temporal as

$$\phi(r) = -\frac{GM}{r} \qquad \longrightarrow \qquad \bar{B}(\chi^5) = -\frac{GM}{\chi^5} \quad . \tag{2.29}$$

If an object is infinitely far away, or at least approximately infinitely far away, then $\lim_{\chi^5\to\infty} \bar{B}(\chi^5) = 0$ but if we use hypercomplex numbers to get $M := \infty$ then there exists a likely workaround involving infinite mass divided by infinite distance. For instance, if we assume a level of $\aleph$ j and write the infinite distance as $\chi^5 = r\hat{\Phi}^{j+1}$, and the mass of the universe on the higher level of $\aleph$ as $M\hat{\Phi}^{j+1}$, then

$$\bar{B}(r) = -\frac{GM\hat{\Phi}^{j+1}}{r\hat{\Phi}^{j+1}} = -\frac{GM}{r} \quad . \tag{2.30}$$

Equation (2.30) is completely trivial but, if it is even possible to formulate the dynamics in this way, when we include the non-trivial properties of χ^5 such that the resultant $\bar{B}$ has all the dependences in figure 14 then the equation might be more interesting.

We know the Newtonian gravitational potential describes a limit of general relativity but there is no guarantee that a new scalar potential function $\bar{B}$ can represent hypercomplex gravitation. On the other hand we would be remiss not to at least look for the case where it is possible to encode the entire thing on a Newtonian potential which could then later be shown to be a limit of the the MCM, but here we will work in the post-Newtonian tensor language. Regarding an overall MCM potential energy function, the notion will be to assemble a periodic array of δ-valued energy wells corresponding to allowed regions $\mathcal{H}_n$ between unallowed bulk regions where $V = \infty$ such that the infinite depths of the successive energy wells increase as tiers of infinitude in $^\star\mathbb{C}$ from one $\hat{\pi}$-site to the next. Hypothetically, we would take the limit of infinitely many discrete wells to construct the hypercomplex version of the continuous $1/r$ gravitational potential well.

Now we will examine the metric in the bulk of the MCM unit cell. In reference [19], Bailin and Love report the Kaluza–Klein metric as follows and once again we change certain variable names for global consistency.

> “We adopt coordinates $\tilde{\chi}^A$, with $A = 1, ..., 5$ with
>
> $$\tilde{\chi}^\mu = \chi^\mu \qquad \mu = 0, 1, 2, 3 \tag{2.31}$$
>
> being coordinates for ordinary four-dimensional spacetime, and
>
> $$\tilde{\chi}^5 = \theta \tag{2.32}$$
>
> being an angle to parameterize the compact dimension with the geometry of a

circle.[1] The ground-state metric after compactification is

$$\tilde{\Sigma}_{AB} = \text{diag}\{\eta_{\mu\nu}, -\tilde{\Sigma}_{55}\} \tag{2.33}$$

where

$$\eta_{\mu\nu} = (1, -1, -1, -1) \tag{2.34}$$

is the metric of Minkowski space, M_4, and

$$\tilde{\Sigma}_{55} = \tilde{R}^2 \tag{2.35}$$

is the metric of the compact manifold $\mathbb{S}^1$, where $\tilde{R}$ is the radius of the circle.[2]

"The identification of the gauge field arises from an expansion of the metric about the ground state. Quite generally, we may parameterize the metric in the form

$$\tilde{\Sigma}(\chi, \theta) = \begin{pmatrix} g_{\mu\nu}(\chi,\theta) - A_\mu(\chi,\theta)A_\nu(\chi,\theta)\phi(\chi,\theta) & A_\mu(\chi,\theta)\phi(\chi,\theta) \\ A_\nu(\chi,\theta)\phi(\chi,\theta) & -\phi(\chi,\theta) \end{pmatrix} \quad . \tag{2.36}$$

To extract the graviton and the Abelian gauge field theories it proves sufficient to replace $\phi(\chi, \theta)$ by its ground-state value $\tilde{\Sigma}_{55}$, and to use the ansatz without θ dependence:

$$\tilde{\Sigma}(\chi) = \begin{pmatrix} g_{\mu\nu}(\chi) - A_\mu(\chi)A_\nu(\chi)\tilde{\Sigma}_{55} & A_\mu(\chi)\tilde{\Sigma}_{55} \\ A_\nu(\chi)\tilde{\Sigma}_{55} & -\tilde{\Sigma}_{55} \end{pmatrix} \quad . \tag{2.37}$$

We write

$$B_\mu(x) = \xi A_\mu(\chi) \tag{2.38}$$

where ξ is a scale factor we shall choose later so that $A_\mu(\chi)$ is a conventionally normalized gauge field."

We see unification between classical gravity and classical electromagnetism beginning to occur when the electromagnetic potential appears in the Kaluza–Klein metric. However, we know Kaluza–Klein theory doesn't work by itself (without MCM modifications [3].) Furthermore, the Newtonian gravitational potential is a scalar field whereas the the electromagnetic potential transforms as a 4-vector so there is some fundamental imbalance of the formalism that remains to be balanced through unification.

[1] This is what we have suggested by mapping Minkowski space onto a cylinder [2]. While we will refer to our 5D metric as the Kaluza–Klein metric, the Kaluza–Klein model is the one with the compactified circular topology of the fifth dimension. In the MCM, the fifth dimension has flat Euclidean topology so it is not the ordinary Kaluza–Klein model. Because the Kaluza–Klein metric is consistent with the MCM/TOIC we will still speak of the Kaluza–Klein metric. Rather than a fifth dimension compactified on a tiny circle or spiral, the MCM compactifies the 4D gravitational manifold $\mathcal{H}$ as a hypercomplex infinitesimal on an unending, monotonic march of infinitude in $^\star\mathbb{C}$ where some hypercomplex normalization restores finite unitarity at the end of each unit cell.

[2] Dolce has extensively documented how the dynamical topological radius can be used to scale gauge theories. We we refer the reader to reference [20].

Where general relativity has geodesics, electromagnetism has field lines and we need to unify these approaches. To that end, consider what Zangwill[1] says about chaotic magnetic field lines in reference [21].

> "The complexity of a large class of magnetic field line configurations can be appreciated using a field constructed from a constant B_0 and an arbitrary scalar function $f(\mathbf{r})$:
>
> $$\mathbf{B}(\mathbf{r}) = B_0\hat{z} + \hat{z} \times \nabla f(\mathbf{r}) \quad . \tag{2.39}$$
>
> The field satisfies $\nabla \cdot \mathbf{B} = 0$ by construction. [*sic*] The equation for the field lines is
>
> $$\frac{B_x}{dx} = \frac{B_y}{dy} = \frac{B_z}{dz} = \lambda \tag{2.40}$$
>
> or
>
> $$\frac{dx}{dz} = -\frac{1}{B_0}\frac{\partial f}{\partial y} \qquad \text{and} \qquad \frac{dy}{dz} = +\frac{1}{B_0}\frac{\partial f}{\partial x} \quad . \tag{2.41}$$
>
> Now, change the variables in [*equation (2.41)*] so $x = q$, $y = p$, and $z = t$. If, in addition, we let $f = -B_0 H$, the two equations above are exactly Hamilton's equations of classical mechanics,
>
> $$\dot{q} = \frac{\partial H}{\partial p} \qquad \text{and} \qquad \dot{p} = -\frac{\partial H}{\partial q} \tag{2.42}$$
>
> Therefore the magnetic field lines are the 'time'-dependent trajectories in (q, p) phase space of a 'particle' with Hamiltonian $H = -f/B_0$. Since most Hamiltonians are non-integrable and produce chaotic trajectories,[2] the magnetic field line configuration will be very complex indeed."

In general relativity, we can derive a a set of differential equations in Hamiltonian form whose solutions are geodesics. Zangwill has shown how field lines can be derived in the same way and yet, at some level, the two approaches to physics, geodesics and field lines, are not compatible. Electromagnetic field lines can be derived as a limit of a quantum theory but geodesics cannot. The main push of the MCM is to identify and resolve these outstanding discrepancies. Luckily, this research program has had no aim to calculate any field lines or geodesics, to calculate an expectation value, or anything like that. Where we have seen other researchers stuck with conceptual difficulties, the main contribution of this research program to the total body of human knowledge has been to suggest to those other researchers, "Try doing it this other way." The difficult problem was not to make the other calculation, the difficult part was to *suggest* the other calculation which we have now identified as computing the path of maximum action. Before we can do that we must

[1] Andrew Zangwill is credited, and in fact lauded, as the person who first explained the Abraham–Lorentz force to this writer along with its anomalous reliance on the ∂_t^3 operator.

[2] We expect the simplest MCM Hamiltonian to be of the non-integrable variety but, after the hypercomplex formalism is fully developed, it may be simpler than expected.

first develop the field of hypercomplex analysis in ${}^\star\mathbb{C}$, and a survey of algebra is also in order. If someone wants to see a calculation that does not appear in this research, they should make it themselves or ask someone to do it for them. Detractors can continue to harrumph that they would have used a computer to solve some equations and then copied their software's output into LaTeX, and then sent their paper to a publisher other than viXra, but waiting for this writer to abandon his research in fundamental physics in favor of some problem in applied physics will not prove fruitful. We do not mean to disparage applied physics in any way. Rather our aim is to, however unlikely, get one or more detractors to finally clear the conceptual hurdle where they begin to understand that this writer's research program is his own and not his detractors', that it is already experimentally verified, and that said detractors have received the new theory stupidly. How can a body of research be both "experimentally verified," as this research is, and also "pending peer-review" as this research is not? The widespread desire to ignore or obfuscate this issue pending a change in this writer's research direction will be a dire mark of shame on the history of the professional conduct of science by supposedly reasonable men.

To move forward with the present considerations in classical physics, consider Carroll's further words from reference [17].

> "As we shall see, the metric tensor contains all the information we need to describe the curvature of the manifold (at least in what is called Riemannian geometry [*sic*]). In Minkowski space we can choose coordinates in which the components of the metric are constant; but it should be clear that the existence of curvature is more subtle than having the metric depend on the coordinates [*sic*]. Later, we shall see that the constancy of the metric components is sufficient for a space to be flat, and in fact there always exists a coordinate system on any flat space in which the metric is constant. But we might not know how to find such a coordinate system, and there are many ways for a space to deviate from flatness; we will therefore want a more precise characterization of the curvature [*sic*].
>
> "A useful characterization of the metric is obtained by putting $g_{\mu\nu}$ into its **canonical form**. In this form the metric components become
>
> $$g_{\mu\nu} = \text{diag}(-1,-1,...,-1,+1,+1,...,+1,0,0,...,0) \quad , \tag{2.43}$$
>
> where [*equation (2.43)*] means a diagonal matrix with the given elements. The **signature** of the metric is the number of both positive and negative eigenvalues; we speak of 'a metric with signature minus-plus-plus-plus' for Minkowski space, for example. If any of the eigenvalues are zero, the metric is 'degenerate,' and the inverse metric will not exist; if the metric is continuous and nondegenerate, its signature will be the same at every point. We will always deal with continuous nondegenerate metrics[1]. If all of the signs are positive, the metric is called **Euclidean** or **Riemannian** (or just positive definite), while if there is a single minus sign it is called **Lorentzian** or **pseudo-Riemannian**, and any metric with some $+1$'s and some -1's is called indefinite. [*sic*] The spacetimes of interest in general relativity typically have Lorentzian metrics.
>
> "We have not yet demonstrated how it is always possible to put the metric in canonical form. In fact it is always possible to do so at some point $p \in M$, but in

[1]We, however, cannot always do this because the MCM has piecewise discontinuous and degenerate metrics.

general it will only be possible at that single point, not in any neighborhood of p. Actually we can do slightly better than this; it turns out that at any point p there exists a coordinate system $x^{\hat{\mu}}$ in which $g_{\hat{\mu}\hat{\nu}}$ takes its canonical form and the first derivatives $\partial_{\hat{\sigma}} g_{\hat{\mu}\hat{\nu}}$ all vanish (while the second derivatives $\partial_{\hat{\rho}}\partial_{\hat{\sigma}} g_{\hat{\mu}\hat{\nu}}$ cannot be made to all vanish):

$$g_{\hat{\mu}\hat{\nu}}(p) = \eta_{\hat{\mu}\hat{\nu}} \quad , \qquad \partial_{\hat{\sigma}} g_{\hat{\mu}\hat{\nu}}(p) = 0 \quad . \tag{2.44}$$

“Such coordinates are known as **locally inertial coordinates**, and the associated basis vectors constitute a **local Lorentz frame**; we often put hats on the indices when we are in these special coordinates. Notice that in locally inertial coordinates the metric at p looks like that of flat space to first order. This is the rigorous notion of the idea that ‘small enough regions of spacetime look like flat (Minkowski) space.’ Also, there is no difficulty in simultaneously constructing sets of *basis vectors* at every point in M such that the metric takes its canonical form, the problem is that in general there will not be a *coordinate system* from which this basis can be derived. [*sic*]

“The idea is to consider the transformation law for the metric

$$g_{\hat{\mu}\hat{\nu}} = \frac{\partial x^{\mu}}{\partial x^{\hat{\mu}}} \frac{\partial x^{\nu}}{\partial x^{\hat{\nu}}} g_{\mu\nu} \quad , \tag{2.45}$$

and expand both sides in Taylor series in the sought after coordinates $x^{\hat{\mu}}$. The expansion of the old coordinates x^{μ} looks like

$$x^{\mu} = \left(\frac{\partial x^{\mu}}{\partial x^{\hat{\nu}}}\right)_p x_1^{\mu} + \frac{1}{2}\left(\frac{\partial^2 x^{\mu}}{\partial x^{\hat{\mu}_1}\,\partial x^{\hat{\mu}_2}}\right)_p x^{\mu_1} x^{\mu_2} + \frac{1}{6}\left(\frac{\partial^3 x^{\mu}}{\partial x^{\hat{\mu}_1}\,\partial x^{\hat{\mu}_2}\,\partial x^{\hat{\mu}_3}}\right)_p x^{\mu_1} x^{\mu_2} x^{\mu_3} \ldots \quad , \tag{2.46}$$

with the other expansions proceeding along the same lines. For simplicity we have set $x^{\mu}(p) = x^{\hat{\mu}}(p) = 0$. Then, using some extremely schematic notation, the expansion of [*equation (2.45)*] to second order is

$$\begin{aligned}(\hat{g})_p + (\hat{\partial}\hat{g})_p \hat{x} + (\hat{\partial}\hat{\partial}\hat{g})_p \hat{x}\hat{x} &= \left(\frac{\partial x}{\partial \hat{x}}\frac{\partial x}{\partial \hat{x}} g\right)_p + \left(\frac{\partial x}{\partial \hat{x}}\frac{\partial^2 x}{\partial \hat{x}\,\partial \hat{x}} g + \frac{\partial x}{\partial \hat{x}}\frac{\partial x}{\partial \hat{x}}\hat{\partial} g\right)_p \hat{x} \\ &+ \left(\frac{\partial x}{\partial \hat{x}}\frac{\partial^3 x}{\partial \hat{x}\,\partial \hat{x}\,\partial \hat{x}} g + \frac{\partial^2 x}{\partial \hat{x}\,\partial \hat{x}}\frac{\partial^2 x}{\partial \hat{x}\,\partial \hat{x}} g + \frac{\partial x}{\partial \hat{x}}\frac{\partial^2 x}{\partial \hat{x}\,\partial \hat{x}}\hat{\partial} g + \frac{\partial x}{\partial \hat{x}}\frac{\partial x}{\partial \hat{x}}\hat{\partial}\hat{\partial} g\right)_p \hat{x}\hat{x} \quad .\end{aligned} \tag{2.47}$$

“We can set terms of equal order in $\hat{x}$ on each side equal to each other. Therefore, the components $g_{\hat{\mu}\hat{\nu}}(p)$, 10 numbers in all (to describe a two-index tensor[1]), are determined by the matrix $(\partial x^{\mu}/\partial x^{\hat{\mu}})_p$. This is a 4×4 matrix with no constraints; thus, we are free to choose 16 numbers. Clearly this is enough freedom to put the 10 numbers of $g_{\hat{\mu}\hat{\nu}}(p)$ into canonical form, at least as far as having enough degrees of freedom is concerned. (In fact there are some limitations–if you go through the procedure carefully, you find for example that you cannot change the signature.)

[1] This is true for a symmetric two-index tensor.

> The six remaining degrees of freedom can be interpreted as exactly the six parameters of the Lorentz group; we know that these leave the canonical form unchanged. At first order we have the derivatives $\partial_{\hat{\sigma}} g_{\hat{\mu}\hat{\nu}}(p)$, four derivatives of ten components for a total of 40 numbers. But looking at the right hand side of [*equation (2.47)*] we see that we now have additional freedom to choose $(\partial^2 x^\mu / \partial x^{\hat{\mu}_1} \partial x^{\hat{\mu}_2})_p$. In this set of numbers there are ten independent choices of the indices $\hat{\mu}_1$ and $\hat{\mu}_2$ (it's symmetric since partial derivatives commute) and four choices of μ for a total of 40 degrees of freedom. This is precisely the number of choices we need to determine all of the first derivatives of the metric, which we can therefore set to zero. At second order, however, we are concerned with $\partial_{\hat{\rho}} \partial_{\hat{\sigma}} g_{\hat{\mu}\hat{\nu}}(p)$; this is symmetric in $\hat{\rho}$ and $\hat{\sigma}$ as well as $\hat{\mu}$ and $\hat{\nu}$, for a total of 10×10 numbers. Our ability to make additional choices is contained in $(\partial^3 x^\mu / \partial x^{\hat{\mu}_1} \partial x^{\hat{\mu}_2} \partial x^{\hat{\mu}_3})_p$ [***!!!***] This is symmetric in the three lower indices, which gives 20 possibilities, times four for the upper index gives us 80 degrees of freedom–20 fewer than we require to set the second derivatives of the metric to zero. So in fact we cannot make the second derivative vanish; the deviation from flatness must therefore be measured by the 20 degrees of freedom representing the second derivatives of the metric tensor field. We will see later how this comes about, when we characterize the curvature using the Riemann tensor which will turn out to have 20 independent components in four dimensions.
>
> "Locally inertial coordinates are unbelievably useful. Best of all, their usefulness does not generally require that we do the work of constructing such coordinates [*sic*], but simply that we know they do exist."[1]

Wow! Rather than using tensor language for all of that we can use commas to separate all the items needed to make a calculation, put them in curly brackets, and call it a multiplex. Is the best way to keep track of the 20 curvature components of the Riemann tensor really in an object that transforms like a tensor? Why not use a 4×5 matrix, or a 20×1 matrix that more directly represents the idea of a multiplex? Quantum mechanics does fine with matrices instead of tensors so we know that, in some sense, there exists an analytical channel for physics other than tensor analysis on manifolds. There are already hundreds of multiplectic positions needed just to cover tensor analysis in Lorentzian 4-space so two copies of the 14D MCM system (the requirement for two copies of the system appears in reference [10]) will need many thousands of positions to describe the general relativity of one MCM qubit. Due to the sphere theorem and well-known methods of Ricci flow, the whole thing boils down to large systems of partial differential equations but report format is not the correct venue in which to describe possibly hundreds of thousands or millions of constraint equations that would rigorously demonstrate how the curvature is related to the quantized electric current. (Whoever has the facility to make those calculations should do so and upload his result to a public mirror such as Wikipedia.) Instead of specific constraint equations in the MCM, we have a powerful argument that goes something like, "You see how these two things connect? There is only one way that we can pull an answer out of these objects." As an example, consider the MCM operator $\hat{M}^3 := \partial^3 / \partial \chi^5_+ \, \partial \chi^5_\varnothing \, \partial \chi^5_-$ [10] and how, among all the objects in general relativity, it can only connect to the $\partial^3 / \partial x^{\hat{\mu}_1} \, \partial x^{\hat{\mu}_2} \, \partial x^{\hat{\mu}_3}$ operator that appears

[1] It is this line of reasoning that leads us to believe it will be possible to make a vastly simplified calculation of trajectories across the MCM unit cell whose various coordinate systems might be very difficult to construct... and unnecessary.

in equation (2.46). This is the object Carroll identifies as containing "our ability to make additional choices."

To avoid confusion with very many other hats, we will not use hats to specify locally inertial coordinates. It will be the convention that $\{x_-^\mu, x^\mu, x_+^\mu\}$ are always locally inertial coordinates on $\{\aleph, \mathcal{H}, \Omega\}$ and we have a lot of freedom to use $x_\varnothing^\mu$ as required. The reader must take extreme care to note that Carroll's setting $\partial_\sigma g_{\mu\nu} = 0$ at the origin to reduce the complexity of the equations complements what we have referred to as **the MCM condition**. The MCM condition states that the observer is always at the origin $x^\mu = 0$ and any coordinate system that places him elsewhere will make the MCM contraption falter. Setting the first derivative to zero might be eventually be shown to be a required by the full MCM condition rather than the option as which Carroll has presented it. When evolution moves the observer from $t_i = 0$ to $t_{i+1} \neq 0$, that is simply $t'_{i+1} = 0$ in the coordinates of the time advanced manifold.

II.2 General Relativity

We have firmly established the requirement for four ontological vectors $\{\hat{2}, \hat{\pi}, \hat{\Phi}, \hat{\imath}\}$ [4, 5, 10] as opposed to the three-fold subset $\{\hat{\pi}, \hat{\Phi}, \hat{\imath}\}$ proposed in reference [12]. Therefore, we shall consider Einstein's equation resolved into four components

$$8\pi T_{\mu\nu} \;=\; R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} \quad , \tag{2.48}$$

and how they connect to $\{\mathcal{H}, \aleph, \varnothing, \Omega\}$ via something along the lines of

$$8\pi f^3 \big|\psi; \hat{\pi}\big\rangle \;=\; \frac{\Phi}{4} \big|\psi; \hat{2}\big\rangle - \frac{1}{2} \big|\psi; \hat{\Phi}\big\rangle + i \big|\psi; \hat{\imath}\big\rangle \quad . \tag{2.49}$$

The reader should note that we have chosen this arbitrary position for $\hat{2}$ only to show matching between equations (2.48-2.49) on the 1/2 coefficient and the 8π coefficient, and also that we have used hand waving for some signs and i's. This begins to show that we might assign the geometry $R^\lambda_{\mu\sigma\nu}$ to the $\hat{\imath}$ component, and then use the fundamental incompatibility between irrational π and maximally irrational Φ to establish the computational topology of the topological incompatibility between dS and AdS. When we apply the operator that replaces $\pi/2 \approx 1.57$ with $\Phi \approx 1.62$ (section IV.1), all of the U(1) periodicity in 2π vanishes because

$$\sin(\theta - \pi/2) = \cos(\theta) \quad , \qquad \text{but} \qquad \sin(\theta - \Phi) \neq \cos(\theta) \quad . \tag{2.50}$$

Therefore, this will be a symmetry breaking operation.

When the trigonometric functions are represented as distinct infinite series, they are assumed to all have an even or odd infinite number of terms. It is the fundamental property of sine that it is an even function and the fundamental property of cosine that it is an odd function so we have good reason to consider the case when the topology of the MCM unit cell must be such that it can accommodate a remainder term after operations pairing two unlike infinite numbers of terms. Whatever the origin of the remainder, in this way, we may represent two infinities in $\Sigma^{\pm}$ and then use the remainder to define something in $\mathcal{H}$ or $\varnothing$. For example, consider the inner product of two orthogonal wavefunctions $\psi^{\pm}$ written in some discrete bases $\{\psi_j^+,\psi_k^-\}$ in $\Sigma^{\pm}$

$$\big\langle\psi_-\big|\psi_+\big\rangle=\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}c_k^*c_j\delta_{jk}\int\psi_k^{-\,*}\psi_j^+\,dx=0\quad . \tag{2.51}$$

If we revise the concept of infinity such that one of the sums starts with iterator 0 and the other with iterator 1, possibly motivated by the inclusion of a boundary in hyperbolic AdS when none exists in spherical dS, then we will have an equation like

$$\big\langle\psi_-\big|\psi_+\big\rangle=\sum_{j=0}^{\infty}\sum_{k=1}^{\infty}c_k^*c_j\delta_{jk}\int\psi_k^{-\,*}\psi_j^+\,dx\neq0\quad . \tag{2.52}$$

Therefore the inner product must be such that

$$\sum_{j=0}^{\infty}\sum_{k=1}^{\infty}c_k^*c_j\delta'_{jk}\int\psi_k^{-\,*}\psi_j^+\,dx:=c_0\psi_0^+\quad , \tag{2.53}$$

where δ'_{jk} has special behavior such that

$$\sum_{j=0}^{\infty}\sum_{k=1}^{\infty}c_k^*c_j\delta'_{jk}\int\psi_k^{-\,*}\psi_j^+\,dx=c_0\int\psi_0^+\,d\hat{\gamma}\quad . \tag{2.54}$$

Then it only remains to define $\hat{\gamma}$ such that

$$\int d\hat{\gamma}=V_0\left(\frac{1}{4\pi}\,\hat{\pi}-\frac{\varphi}{4}\,\hat{\Phi}+\frac{1}{8}\,\hat{2}-\frac{1}{4}\,\hat{\imath}\right)\quad . \tag{2.55}$$

where the implication is that the inner product of the extra term in the odd infinite series is taken with the identity operator $\hat{1}$. We will return to these details in chapter four. When we say "the topology must be able to accommodate a remainder term" we have implied at least the existence of a special kind of inner product and equation (2.55) shows how those remainders can be objects in general relativity.

Previous work developing the TOIC has drifted in the direction of quantum theory from its cosmological roots as the MCM and here we swing back in the other direction toward the physics of the continuum described (with random assignments) by

$$f^3\big|\psi;\hat{\pi}\big\rangle\quad\mapsto\quad T_{\mu\nu} \tag{2.56}$$

$$\big|\psi;\hat{\Phi}\big\rangle \quad\mapsto\quad G_{\mu\nu} \tag{2.57}$$

$$i\big|\psi;\hat{i}\big\rangle \quad\mapsto\quad g_{\mu\nu}\Lambda \quad . \tag{2.58}$$

Various iterations of these maps are derived in references [22, 12, 13, 4, 10]. In this section, we will examine what needs to be done to split the Einstein tensor $G_{\mu\nu}$, and therefore equation (2.57), into the components that appear in equation (2.48). Throughout this book, we will ignore the signs of terms when they seem irrelevant and as justification we refer the reader to the as-yet-undiscussed channel for $\sqrt{i}$ (or $\sqrt[n]{i}$) which can be added later to generate an arbitrary sign convention. Note that we continue to choose arbitrary matching for objects rather than represent them in the most general fashion. For instance, the $\hat{2}$ term in equation (2.49) could be the one that connects to $T_{\mu\nu}$, or any of the tensors in Einstein's equation, but we choose a particular (and reasonable) ansatz purely for convenience in maps (2.56-2.58). Even the selection of the spherical and hyperbolic spaces Ω and $\aleph$ as belonging to the future or the past was arbitrary. Likewise, the assignment of the members of our Gel'fand triple $\aleph'$ and Ω' to either dS or AdS manifolds was arbitrary. Such ansatzes are fine and if something about maps (2.56-2.58) is found to be backward in the final analysis then later we can switch it or perhaps we will complement the work of Benjamin Franklin and end up with an electric current vector that does not point in the opposite direction to the motion of electrons. It may even be that the other form of Einstein's equation $R_{\mu\nu}{=}T_{\mu\nu}-1/2\,T g_{\mu\nu}$ is the one that shows the most natural mapping of the objects in maps (2.56-2.58).

Throughout this research program, we have made many arbitrary choices when they make it easier to analyze an idea than would be possible with a fully rigorous but overly general case. Reasonable choices have consistently pointed the way to better choices such as the replacement of the original basis vector $\hat{\varphi}$ [7] with $\hat{\Phi}$.[1] By choosing three ontological vectors, we discovered that three are not enough to do all of the heavy lifting and now there are four objects in the ontological basis. Since $\hat{2}$ now appears along side $\{\hat{\pi},\hat{\Phi},\hat{i}\}$, we should examine what role it might play in relating the quantum and gravitational sectors as in equations (2.56-2.58). To that end, consider the Einstein tensor

$$G_{\mu\nu} \quad\equiv\quad R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu} \quad , \tag{2.59}$$

where we modify map (2.57) with $1{=}1/2+1/2$ and $\hat{\Phi}{=}\Phi/2\;\hat{2}$ as

$$\frac{1}{2}\big|\psi;\hat{\Phi}\big\rangle+\frac{\Phi}{4}\big|\psi;\hat{2}\big\rangle \quad\mapsto\quad R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu} \quad . \tag{2.60}$$

There are multiple ways to assign the components of $G_{\mu\nu}$ because the expanded map is from two objects to two objects (and even this rests upon the assumption that $\hat{\Phi}$ is the piece that

[1] In this book, we use a different $\hat{\varphi}$ and the reader should understand that it is a completely new object unrelated to the original notation for what we now call $\hat{\Phi}$.

should connect to the Einstein tensor.) The first obvious one-to-one possibility of non-mixed linear dependence is

$$\frac{1}{2}\big|\psi;\hat{\Phi}\big\rangle \quad\mapsto\quad R_{\mu\nu} \quad, \qquad\qquad \text{and} \qquad\qquad \frac{\Phi}{4}\big|\psi;\hat{2}\big\rangle \quad\mapsto\quad -\frac{1}{2}Rg_{\mu\nu} \quad, \tag{2.61}$$

which gives $R=\Phi/2$ and the second is

$$\frac{1}{2}\big|\psi;\hat{\Phi}\big\rangle \quad\mapsto\quad -\frac{1}{2}Rg_{\mu\nu} \quad, \qquad\qquad \text{and} \qquad\qquad \frac{\Phi}{4}\big|\psi;\hat{2}\big\rangle \quad\mapsto\quad R_{\mu\nu} \quad, \tag{2.62}$$

which gives $R{=}1$. Either way we obtain a nice constraint on the Ricci tensor $R_{\mu\nu}$.

In reference [3], we connected the curvature of the cosmos described by the Ricci tensor to the golden ratio by defining embedded hyperboloids $\{\aleph,\Omega\}$ in 5D hyperspacetime with curvature parameters equal to

$$\Phi=\frac{1+\sqrt{5}}{2}\approx 1.62 \quad, \qquad\qquad \text{and} \qquad\qquad \varphi=\frac{1-\sqrt{5}}{2}\approx -0.62 \quad. \tag{2.63}$$

The reader should be aware of this odd notation that φ is a negative number, or what Gauss would have called an inverse number. We initially defined hyperboloids directly in the 5D bulk [3] but now we take flat slices of bulk in the χ^A coordinates and then impose the hyperboloidal geometry for the $x^\mu_\pm$ coordinates with a 4D embedded metric $g^\pm_{\mu\nu}(\chi^5_\pm)$ on each flat slice. The curved embedded metric is assigned by the MCM boundary condition so it is not inherited from the 5D metric. The slices stay flat in the 4D metric $\Sigma^\pm_{\alpha\beta}(\chi^5_\pm)$ that is inherited from the 5D metric $\Sigma^\pm_{AB}$.

We already have two places where the Riemann tensor $R^\sigma_{\mu\lambda\nu}$ and the golden ratio Φ connect: the first is in the embedded hyperboloid constraints

$$\Phi^2=-\big(x^0_+\big)^2+\sum_{i=1}^{3}\big(x^i_+\big)^2 \quad, \qquad\qquad \text{and} \qquad\qquad -\varphi^2=-\big(x^0_-\big)^2+\sum_{i=1}^{3}\big(x^i_-\big)^2 \quad, \tag{2.64}$$

for the $x^\mu_\pm$ coordinates of dS and AdS which were given in reference [3]. These two equations were derived with an ansatz that the hyperboloid parameter on each slice should equal to the square of χ^5 at that slice. The new, second connection is derived from equations (2.61-2.62) as a constraint on the Ricci scalar

$$R\in\left\{1,\frac{\Phi}{2}\right\} \quad, \tag{2.65}$$

and we should check to see if they are compatible. Equations (2.64) were proposed in reference [3] as a way to show how the golden ratio could be forced into the metric and the derivations in reference [3] did not depend on any specific choice of hyperboloid parameter. To check the validity, note that in O(N,1) dS with $N=3$ and hyperboloid parameter $\alpha^2=\Phi^2$ the formula for the Ricci scalar is

$$R=\frac{N\big(N-1\big)}{\alpha^2}=\frac{6}{\Phi^2} \quad . \tag{2.66}$$

Clearly these details do not all come together here because $6/\Phi^2\notin\{1,\Phi/2\}$. Therefore, we should give precedence to the new matching condition shown in equation (2.65) and then use equation (2.66) to derive the corresponding hyperboloid parameters $\alpha^2_\pm$ in a revision to the $\{\Phi^2,-\varphi^2\}$ ansatz that appears in equations (2.64). There is no requirement to use χ^5 identically as the hyperboloid parameter as in reference [3]; we can create a the same smoothly varying hyperboloid parameter as some other function of the pseudo-affine parameter χ^5 as needed. In reference [3], the ansatz was that this function should be $\alpha^2_\pm=\pm\Phi^{\pm1}$ now we have a requirement for a different function.

Consider the case when $R=\Phi/2$, as per equation (2.65). Moving past the not very relevant hyperboloid parameters that will describe the exact curvature on $\aleph$ and Ω (it is highly relevant that they are curved, not what the curvature is), note that R becomes the coefficient in the tensor transformation law

$$x^\mu_\varnothing=\frac{\Phi}{2}\,x^\mu_+ \quad , \tag{2.67}$$

associated with a change of coordinates from the $\hat{\Phi}$-site to the $\hat{2}$-site (via $\hat{\Phi}=\Phi/2\;\hat{2}$). The form $R=\Phi/2$ is very natural looking and it seems reasonable but we have another nice possibility as well. We may set the Ricci scalar R to unity and let that be a scale factor that enforces unitarity in lieu of the *ad hoc* normalization that is usually applied. Both choices can be interpreted as an obvious (non-contrived) scale factor so we have good evidence of the *reasonableness* of sticking $\hat{2}$ into the maps that were already doing fine without it.

If we are going to analyze the MCM and TOIC with the methods of general relativity then its objects better satisfy the tensor transformation law. Mirroring equation (2.67), the tensor transformation associated with a change of coordinates from x^μ to x^μ_+ for some vector A^μ is

$$A^\mu=\frac{\partial x^\mu}{\partial x^\nu_+}\,A^\nu_+ \quad . \tag{2.68}$$

To consider a change of coordinates from x^μ associated with $\hat{\pi}$ to x^μ_+ associated with $\hat{\Phi}$, let $A^\mu\equiv A\hat{\pi}$ so that

$$A^{\mu} = \begin{pmatrix} A \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad , \qquad \text{and} \qquad \hat{\Phi} : A^{\mu} \quad \mapsto \quad A_{+}^{\mu} \quad . \tag{2.69}$$

By construction we have $\hat{\pi} = -\varphi\pi\hat{\Phi}$ so

$$x^{\mu}\,\hat{\pi} = -\varphi\pi x^{\mu}\,\hat{\Phi} \qquad \Longrightarrow \qquad x_{+}^{\mu} = -\varphi\pi x^{\mu} \quad . \tag{2.70}$$

Therefore

$$\frac{\partial x_{+}^{\mu}}{\partial x^{\mu}} = -\varphi\pi \quad , \tag{2.71}$$

and evidently the tensor transformation law is

$$A_{+}^{\mu} = -\varphi\pi A^{\mu} \quad . \tag{2.72}$$

It is good that the x^{μ} coordinates of $\mathcal{H}$ have a different scale than the x_{+}^{μ} coordinates of Ω. The change of linear scale should have a direct association with the MCM's expected non-unitarity. Furthermore, we should explore in future work what it means for the quantity $-\varphi\pi$ to be inverse (prefaced with a negative sign) but also positive with $-\varphi\pi > 0$.

The boundary condition given in reference [3] for determining $\mathcal{H}$'s metric $\eta_{\mu\nu}$ (or similarly $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$) was

$$\lim_{\chi_{+}^{5} \to 0^{+}} \Sigma_{AB}^{+} + \lim_{\chi_{-}^{5} \to 0^{-}} \Sigma_{AB}^{-} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \quad . \tag{2.73}$$

The RHS of equation (2.73) shows the 5D analogue of the 4D metric in $\mathcal{H}$. It has a vanishing determinant and only one sign different than the other four so it is degenerate definite and the metrics $\Sigma_{AB}^{\pm}$ are nondegenerate because $\Sigma_{55}^{\pm} = \chi^5$ is never equal to zero in $\Sigma^{\pm}$. Among $\Sigma_{AB}^{\pm}$, one of them is a Lorentzian metric and one is an indefinite metric because χ^5 being timelike in one of $\Sigma^{\pm}$ implies that it is spacelike in the other. Whichever of $\Sigma^{\pm}$ has two timelike dimensions has an indefinite metric and the other is Lorentzian.

We chose the definition in equation (2.73) to show that the present needs to be considered a superposition of the past and future which would not have been specified from a simpler boundary condition such as

$$\lim_{\chi_{+}^{5} \to 0^{+}} \Sigma_{AB}^{+} = \lim_{\chi_{-}^{5} \to 0^{-}} \Sigma_{AB}^{-}. \tag{2.74}$$

Equation (2.74) does not show that the metric in $\mathcal{H}$ is *always* the flat Minkowski metric $\eta_{\mu\nu}$. Equation (2.73) does show that requirement. However, if we suppose to define the metric in $\mathcal{H}$ with

$$\eta_{AB} \quad \equiv \quad \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \quad , \tag{2.75}$$

then no inverse metric will exist (η_{AB} is degenerate) and it will be impossible to compute the connection coefficients according to the ordinary Christoffel prescription. That formula relies on the inverse metric as in

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \big(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} \big) \quad . \tag{2.76}$$

To get the object η_{AB} in equation (2.73) into invertible form, we can consider

$$\lim_{\chi^5_+ \to 0^+} g^+_{\mu\nu} + \lim_{\chi^5_- \to 0^-} g^-_{\mu\nu} = \eta_{\mu\nu} \quad , \tag{2.77}$$

where $g^{\pm}_{\mu\nu}$ is the non-flat embedded metric on each 4D slice of constant χ^5. We could even start to build complexity using the $\dot{\alpha}$ and tensor indices to write

$$\lim_{\chi^5_+ \to 0^+} \Sigma^+_{\alpha\beta} + \lim_{\chi^5_- \to 0^-} \Sigma^-_{\dot{\alpha}\dot{\beta}} = \eta_{\mu\nu} \quad . \tag{2.78}$$

Equation (2.77) shows that the "size," or linear scale, of $g^{\pm}_{\mu\nu}$ is not equal to $\eta_{\mu\nu}$ and this is exactly what we would expect from the tensor transformations shown in the previous section. This non-uniformity of scale must be associated with the non-unitary component of the MCM. If the scale of different sectors was the same then we would have

$$\frac{1}{2} \lim_{\chi^5_+ \to 0^+} g^+_{\mu\nu} + \frac{1}{2} \lim_{\chi^5_- \to 0^-} g^-_{\mu\nu} = \eta_{\mu\nu} \quad , \tag{2.79}$$

but this is not the stated relationship.

The inherited metric on each slice of constant χ^5 is $\Sigma^{\pm}_{\alpha\beta}$ and the embedded metric of the chronological coordinates on each slice is $g^{\pm}_{\mu\nu}$. The MCM is such that $\Sigma^{\pm}_{55} \equiv \pm\chi^5_{\pm}$ so it is easy to see that the embedded metrics are defined by setting the parameter of curvature in dS and AdS equal to some tailored function of the values of χ^5_+ and χ^5_- on each respective slice of χ^5. Therefore, the embedded metric on each slice of χ^5 has subtle curvature near $\chi^5 = 0$ and greater curvature where the absolute value of χ^5 is larger. When we take the limit $\chi^5 \to 0$, it means the curvature approaches zero on each side of $\mathcal{H}$. The difference of the two metrics will be very small because AdS with infinitesimal negative curvature is approximately indistinguishable from dS with infinitesimal positive curvature. Note the

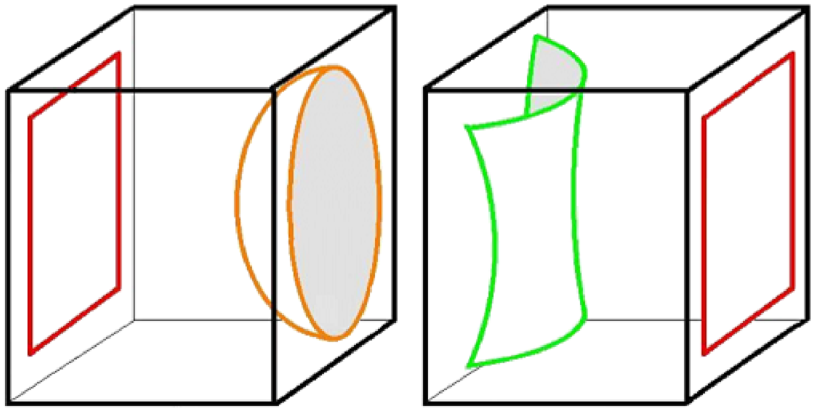


Figure 15: This iteration of the MCM unit cell emphasizes that $\mathcal{H}$ does not connect to Σ^+ and Σ^- simultaneously.

critical distinction that dS with positive curvature and AdS with negative curvature are topologically irreconcilable but all the flat slices of χ^5 share the same flat topology.

Equation (2.77) shows how the slices of $\Sigma^\pm$ approach a matching condition at $\mathcal{H}$ but we can just as easily put the matching condition at $\Sigma^\varnothing$, as in figure 15. Then equation (2.77) becomes

$$g^{\varnothing}_{\alpha\beta} = g^{\Omega}_{\mu\nu} + g^{\aleph}_{\mu\nu} \quad , \tag{2.80}$$

where $g^{\Omega}_{\mu\nu} = g^{+}_{\mu\nu}(\Phi)$ and $g^{\aleph}_{\mu\nu} = g^{-}_{\mu\nu}(\varphi)$. Here, we place $\aleph$ at $\chi^5_- = \varphi$ but in this book we will sometimes place it at $\chi^5_- = -1$. One piece of possibly relevant nuance is that we can introduce a non-commutative concept in which the distance from $\mathcal{H}$ to $\aleph$ is $|\varphi|$ but the distance from $\aleph$ to $\mathcal{H}$ is one. Equation (2.80) shows that $\Sigma^\pm$ do contain their boundary at $\Sigma^\varnothing$ to contrast the definition that $\Sigma^\pm$ are infinite half spaces which do not contain their boundary at $\chi^5_\pm = 0$. If they did not contain the other boundary, which is Ω or $\aleph$ in Σ^+ or Σ^- respectively, we would write

$$g^{\varnothing}_{\mu\nu} = \lim_{\chi^5_+ \to \Phi^-} g^{+}_{\mu\nu} + \lim_{\chi^5_- \to \varphi^+} g^{-}_{\mu\nu} \quad . \tag{2.81}$$

When we include the boundary, we can define $g^{\aleph}_{\mu\nu}$ and $g^{\Omega}_{\mu\nu}$ directly and we can even take another definition

$$g^{\varnothing}_{\mu\nu} = g^{+}_{\mu\nu}(\infty) + g^{-}_{\mu\nu}(\infty) \quad . \tag{2.82}$$

This definition looks pretty good because dS and AdS with infinite curvature are singularities. Mild negative curvature at $\chi^5_- = \varphi$ isn't exactly balanced out with larger positive curvature at $\chi^5_+ = \Phi$ so equation (2.80) does not give the kind of neutral value expected for the $\varnothing$

component. With equation (2.82), we get something along the lines of $\infty+(-\infty)=0$ which is a good neutral value for the point $p\in\Sigma^{\varnothing}$ where the level of $\aleph$ increases. The case of infinite curvature could be a unique case that will allow us to smoothly connect a hyperbolic topology with a spherical one.

In the arrangement described by figure 15, we see that $\mathcal{H}$ touches only one of $\Sigma^{\pm}$ at a time so simultaneous matching on both sides of $\mathcal{H}$ is not implied in any way. We are joining $\Sigma^{\pm}$ with $\varnothing$ instead of $\mathcal{H}$ so the new matching condition in equation (2.80) or (2.82) is very much implied. The original condition about the smoothness of the bulk hyperspacetime across $\mathcal{H}$ has now been dropped. It had to be smooth across $\mathcal{H}$ in the picture where $\mathcal{H}$ was in the center of the unit cell but it does not have to be smooth across $\mathcal{H}$ in figure 15 because there is no "across $\mathcal{H}$" in the that representation of the MCM unit cell. Normally, the notion of something going across $\mathcal{H}$ is required for physics but the MCM considers the regions beyond timelike and spacelike infinity to construct this alternate representation where it is not *strictly* required to pass through $\mathcal{H}$. Hyperspacetime does not need to be smooth across $\mathcal{H}$ because the non-trivial parameter χ^5 is only defined between two adjacent instances of $\mathcal{H}$ that are doubly piecewise disconnected through $\Sigma^{\pm}$ and $\Sigma^{\varnothing}$. We might fill in the piecewise disconnection between Ω and $\aleph$ with a smooth interface but it might not be necessary to do that to compute $\psi\in\mathcal{H}_{j+1}$ given $\psi\in\mathcal{H}_j$. Therefore, we can define the MCM bulk-boundary correspondence [3] to be the one in equation (2.82) rather than equation (2.77). The notation without limits implies that the boundary is included, and the limit notation implies that the boundary is not included. The $\eta_{\mu\nu}$ in equation (2.77) has been replaced with $g^{\varnothing}_{\mu\nu}$ because $\Sigma^{\varnothing}$ is not defined *a priori* to be flat but we might be able to use $g^{\varnothing}_{\mu\nu}=\eta_{\mu\nu}$. In any case, we will make use of the fact that every manifold has at least one point where we can make the metric be the Minkowski metric with vanishing first derivatives to ensure a smooth connection across $\varnothing$. The reader will recall from the earlier excerpt that Carroll[1] has associated the second derivatives with the curvature and we expect there to be some discontinuity in the manifolds as the topology changes between O(1,4) and O(2,3) in $\Sigma^{\pm}$.

We still have not exactly specified the how the size of $g^{+}_{\mu\nu}$ relates to $g^{-}_{\mu\nu}$ but it is likely related to the numbers in the ontological basis. Regarding the other two g metrics: $\hat{2}g_{\mu\nu}\equiv g^{\varnothing}_{\mu\nu}$ a natural assumption. The convention in classical EM is to say that the potential in the present is 1/2 times the advanced potential in superposition with 1/2 times the retarded potential (via $1/2+1/2=1$) but the MCM more likely relies on the non-trivial case $\Phi+\varphi=1$. When we have these two different metrics $g^{\pm}_{\mu\nu}$, they might provide a channel through which to apply complexity directly to the weak field limit of GR

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}\quad, \tag{2.83}$$

so that equation (2.73) becomes

$$\lim_{\chi^5_+\to0^+}\Sigma^{+}_{AB}+\lim_{\chi^5_-\to0^-}\Sigma^{-}_{AB}=\begin{pmatrix}\eta_{\mu\nu}+h_{\mu\nu} & 0\\ 0 & 0\end{pmatrix}\quad. \tag{2.84}$$

[1]Carroll's general relativity textbook [17] has an entire appendix dedicated to analysis of embedded metrics.

This gives us a good place to make a distinction between $\hat{M}^3$ and $\tilde{M}^4$. The former might be impossible to compute because it is impossible for the different topologies of O(1,4) and O(2,3) to smoothly merge in the way that would be required for them to both perfectly and simultaneously approach an observable state that is exactly the Minkowski metric $\eta_{\mu\nu}$. Perhaps the two limits are guaranteed never to sum perfectly to the degenerate 5D extended Minkowski metric η_{AB}. Perhaps due to the numerical approximations that are required (those now assigned to the extra step of $\tilde{M}^4$) the two limits in equations (2.73) and (2.84) can never sum to η_{AB} but instead sum to

$$g_{AB} \quad \equiv \quad \begin{pmatrix} \eta_{\mu\nu} + h_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \quad . \tag{2.85}$$

We may take the small perturbation that arises naturally in the attempt to approximate spherical and hyperbolic manifolds as equal objects (by taking the Lorentz approximation in $\aleph$ and Ω) to be the difference of two large perturbations as

$$h_{\mu\nu} \quad \equiv \quad H^+_{\alpha\beta} - H^-_{\alpha\beta} \quad , \tag{2.86}$$

where $H^\pm_{\mu\nu}$ are perturbations in $\Sigma^\pm$. Similarly to equation (2.86), there exists another interpretation wherein we may write the real metric including perturbations $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ as

$$g_{\mu\nu} \quad \equiv \quad H^+_{\alpha\beta} - H^-_{\alpha\beta} \quad . \tag{2.87}$$

This idea probably can be applied to the hierarchy problem but we are not there yet.[1] Wikipedia says, "In theoretical physics, the hierarchy problem is the large discrepancy between aspects of the weak force and gravity," and, in general, it has to do with the large relative scale of many of the empirical parameters of the standard model. In the MCM, we get large relative scale by taking nested resolutions of the identity to generate arbitrarily small numbers such as those that appear in the standard model, and also as the amplitudes of arbitrarily unlikely quantum processes. Perhaps the irreconcilable topological element, represent it with some arbitrarily large perturbation $H^\varnothing_{\alpha\beta}$, can be decomposed into two perturbative modes that live in $\Sigma^\pm$

$$H^\varnothing_{\alpha\beta} \quad \equiv \quad H^+_{\mu\nu}\hat{\Phi}^1 - H^-_{\mu\nu}\hat{\Phi}^2 \quad . \tag{2.88}$$

For this, we must note that Σ^+ and Σ^- exist on two adjacent levels of $\aleph$ when the unit cell is centered on $\varnothing$ but they exist on the same level of $\aleph$ in the representation centered on $\mathcal{H}$. Mirroring what we have done with

[1] Some particle physics applications of the MCM unit cell, along with its most specific experimental prediction are found in reference [11] and in section II.8.

$$\eta_{AB} \equiv \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \ , \tag{2.89}$$

we can embed the 4D perturbation in a larger perturbation $H^{\varnothing}_{AB}$ and then use the nine extra matrix positions to be the numerical representation of the remainder of the topological incompatibility between $\Sigma^{\pm}$. In a practical sense, it may be possible to reverse engineer what these numerical coefficients must be and thereby complete the “impossible calculation” without knowing them beforehand. This will be another stark MCM departure from the almost two dozen experimentally determined parameters that must be manually inserted to prop up the standard model of particle physics. In chapter four, we will call some specific attention to the idea that

$$\tilde{M}^4 \equiv \hat{\Upsilon} = \hat{U} + \hat{M}^3 \ , \tag{2.90}$$

and also that $\hat{M}^3$ was never meant to be the complete MCM operation [22]. We have added $\hat{2}$ as an accommodation for a fourth component, and even the first MCM representation of fine structure in reference [22] used two in the ratio $\Phi D = 2L$ were D and L were the lengths of the sides of Minkowski space taken as a 2D quantum box.

We have been modestly specific about what all of the MCM coordinates are and now we will list all of the metrics. The real metric $g_{\mu\nu}$ in $\mathcal{H}$ given by equation (2.83) is the idealized Minkowski metric $\eta_{\mu\nu}$ plus a small perturbation $h_{\mu\nu}$. $\Sigma^{\pm}_{AB}$ are the metrics of the 5D manifolds $\Sigma^{\pm}$. The slices of $\chi^5_{\pm}$ in $\Sigma^{\pm}$ are flat in the χ^{α} coordinates associated to the inherited metric $\Sigma^{\pm}_{\alpha\beta} \equiv \eta_{\mu\nu}$. The embedded metric of the $x^{\mu}_{\pm}$ coordinates on any slice is $g^{\pm}_{\mu\nu}(\chi^5_{\pm})$. $g^{\pm}_{\mu\nu}(\chi^5_{\pm})$ is the dS or AdS metric when $\chi^5_{\pm}$ is positive or negative. The metrics on $\aleph$ and Ω are $g^{-}_{\mu\nu}(\varphi) \equiv g^{\aleph}_{\mu\nu}$ and $g^{+}_{\mu\nu}(\Phi) \equiv g^{\Omega}_{\mu\nu}$ respectively. When it comes to the metric for $x^{\mu}_{\varnothing}$, we will probably rely on the mechanism described by Carroll when he wrote, “Best of all, their usefulness does not generally require that we do the work of constructing such coordinates [*sic*], but simply that we know they do exist.”

We will go into more detail about $\aleph$ and Ω in the next chapter but, for now, note that since equation (2.77) suggests a scale factor of 2 when we take $g^{\pm}_{\mu\nu}$ to be equal in size, we are compelled to show that

$$\varphi\pi \approx 1.94 \ , \qquad \text{and} \qquad 2 = 2 \ , \qquad \text{give} \qquad \Delta\aleph \approx 3\% \ , \tag{2.91}$$

with the intention of reminding the reader that

$$\frac{\pi}{2} \approx 1.57 \ , \qquad \text{and} \qquad \Phi \approx 1.62 \ , \qquad \text{give} \qquad \Delta\aleph \approx 3\% \ , \tag{2.92}$$

and even pointing out that

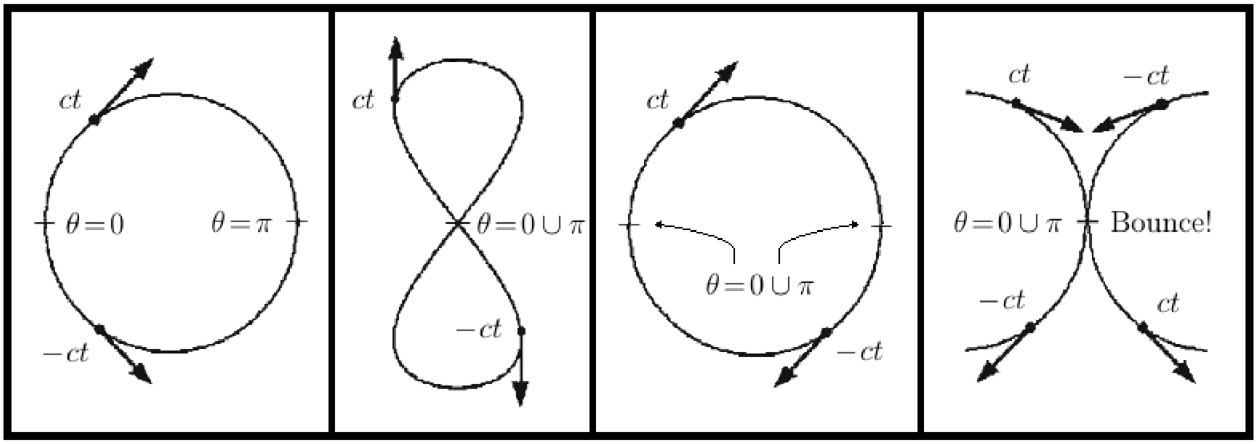


Figure 16: This figure shows the twisting mechanism for modularizing co-$\hat{\pi}$s and creating paths that connect various sites in the cosmological lattice.

$$\Phi^2 \approx 2.62 \quad , \qquad \text{and} \qquad e \approx 2.72 \quad , \qquad \text{give} \qquad \Delta\aleph \approx 3\% \quad . \tag{2.93}$$

If the scale of $\{g_{\mu\nu}, g^+_{\mu\nu}, g^-_{\mu\nu}\}$ was uniform then we would have to multiply the LHS of equation (2.77) by 1/2 as in equation (2.79). This is how we come to say that the simplest scale factor is two, but we also need to consider scales relative to $1 = \Phi + \varphi$, and similar.

Here, we clearly have a mechanism that can lead to fuzzy geometry when the topology of the boundary conditions on the plane waves in the cosmological lattice changes from O(1,4) to O(2,3) across $\Sigma^\varnothing$ in the MCM unit cell. Perhaps what is e on one side of $\mathcal{H}$ becomes Φ^2 on the other and we can use the extra freedom of $H^\varnothing_{AB}$ to make those transformations. χ^5_+ has the opposite sign to χ^5_- so, therefore, $\Sigma^\pm$ have different numbers of spacelike and timelike dimensions. We aim to pass this all off with a topological twist on $\chi^5 \in \{0, \infty\}$ as in figure 16. When we use the $\hat{2}$ operator to generate two topological spaces from one, the operation that separates the co-$\hat{\pi}$s can scale the ratio of their lengths from 1 to Φ such that they can no longer be combined to form equal halves of a circle. They can still be combined into a circle but one co-$\hat{\pi}$ will have more than π radians of arc length. This is nothing but a conformal transformation, but it is an additional operation beyond twisting that breaks the symmetry retained throughout figure 16. We will go into a lot of detail in this regard in section

Lorentzian metrics only have one timelike dimension. This means that when we write the metric as a diagonal matrix, all the eigenvalues are positive except one, or the whole thing is shifted and they are all negative except one: $\{\pm\mp\mp\mp\}$. A new problem that will eventually need to be tackled is to compute 5D geodesics of the form

$$\frac{d^2\chi^C}{(d\chi^5)^2} + \Gamma^C_{AB}\frac{d\chi^A}{d\chi^5}\frac{d\chi^B}{d\chi^5} = 0 \quad , \tag{2.94}$$

when the background topology of piecewise χ^5 is O(1,4) on one side of $\chi^5_\varnothing$ and O(2,3) on the other side. We will begin to treat that problem in section III.8 but it is likely the these geodesics are in the impossible regime of $\hat{M}^3$ and will be replaced by some new technique in $\tilde{M}^4$.

II.3 An Entropic Application

Here, we state a new facet of the model. Clearly $|\varphi\pi| = 1.94$ so we can use $x^\mu_+ = \varphi\pi x^\mu$ (equation (2.70)) to write

$$x^\mu_+ = |\varphi\pi x^\mu| \approx 2x^\mu \quad , \qquad \text{and} \qquad x^\mu_\varnothing = 2x^\mu \quad . \tag{2.95}$$

This is a new definition for $x^\mu_\varnothing$: it is twice as big as x^μ so it is just bigger than x^μ_+. We have already defined the tensor transformation law between x^μ and x^μ_+ that gives $|\varphi\pi| \approx 1.94 \approx 2$ and now we define the $x^\mu_\varnothing$ coordinates as *exactly* twice as big as the observable x^μ coordinates. This is similar to what we have done making chirological orthogonality on $\Phi \approx 1.62$ just larger than the ordinary orthogonality on $\pi/2 \approx 1.57$.

When we want to write a Lorentz frame on $\varnothing$ so that we can sew $\Sigma^\pm$ together, we find

$$x^0_\varnothing = 2x^0 \quad , \qquad \text{and} \qquad x^i_\varnothing = 2x^i \quad . \tag{2.96}$$

This leads to a Minkowski diagram in $\varnothing$ that has four times the area of the same diagram written in $\mathcal{H}$. Since $\hat{M}^3$ is expected to preserve analytical qubits in the ground state along geodesics, the point density of the fabric of the diagram is rarefied by a factor of four, as in figure 17. We could say that entropy increases in physics because there is some boundary condition misalignment between how the present connects to the past and future respectively, possibly on the order of

$$2 - 1.94 \approx 0.06 << 1 \quad . \tag{2.97}$$

The value 0.06 is about ten times larger than

$$\alpha_{QED} \approx \alpha_{MCM} \approx \frac{1}{137} = 0.00729927007299270072992700729... \quad , \tag{2.98}$$

so have good reason for this mode to dominate as decoherence does over determinism in the quantum sector. Here, we imply that α_{MCM} is a deterministic structure constant but 0.06 is an "error" term. Perhaps this mismatch being constantly inflated from one moment to the next results in the second law of thermodynamics: $dS > 0$ where S is the entropy. Note that the area A and the number 4 are the two main components of the Bekenstein–Hawking formula for the entropy of a black hole. In units where $c = G = \hbar = 1$, that formula is

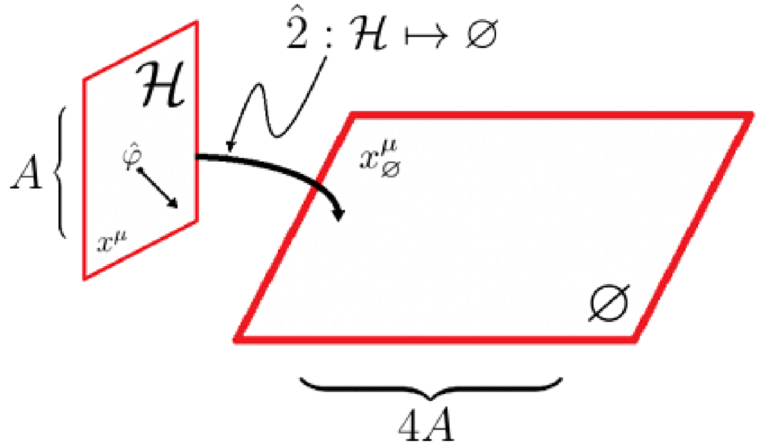


Figure 17: The non-unitary properties of the $\hat{M}^3$ operator require that the Minkowski diagram is stretched. In this map, the area increases exactly by a factor of 4 but in the map $\hat{\Phi} : \mathcal{H} \mapsto \Omega$ the area of Ω only increases by about $(\varphi\pi)^2 \simeq 1.94^2 \simeq 3.7$.

$$S_{BH} = \frac{A}{4} \quad . \tag{2.99}$$

This could be irrelevant but since we have criticized Hawking's own analysis of dynamics near black holes as missing a single point [5], and $\Sigma^\varnothing$ has been created to supply a single point, there could be some profound connection.

II.4 Complex Coordinates

It is quite common to write the metric as a diagonal matrix and here we will examine a few of the ordinary definitions used for tensor analysis in gravitational manifolds. For the Minkowski metric with $c \neq 1$, we must choose from the two allowed Lorentzian signatures $\{\pm \mp \mp \mp\}$ that are out of phase by $e^{i\pi}$. The signature that implies distance in the x^0 direction is imaginary but gives real-valued spacelike distance is

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad . \tag{2.100}$$

The eigenvalues of the matrix define the line element in the manifold

$$ds^2 = -c^2\big(dx^0\big)^2 + \big(dx^1\big)^2 + \big(dx^2\big)^2 + \big(dx^3\big)^2 \quad , \tag{2.101}$$

and all of this can be condensed by writing the line element as

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \quad . \tag{2.102}$$

Of course, the proper time τ is given by

$$d\tau^2 = -\eta_{\mu\nu} dx^\mu dx^\nu = -ds^2 \quad . \tag{2.103}$$

Since 3-velocity is a measure of relative time, and proper time always passes at the same rate, we see the reason for the surprisingly ordinary normalization of the 4-velocity U^μ of an object in motion in spacetime

$$g_{\mu\nu} U^\mu U^\nu = -c^2 \quad . \tag{2.104}$$

The 4-velocity is always normalized as in equation (2.104) without regard for the metric. The passage of time recorded by non-inertial clocks is universally relative to this normalization convention. The 3-velocity $\vec{v}$ always has to be measured with respect to some external clock since the thing that is moving with $\vec{v}$ is stationary in its own inertial frame. Therefore the object of relativity is the external clock. It has to slow down if observers are going to agree about distances due to the surprising property of the universe that the velocity and the time always combine as in equation (2.104).

This is the origin of time dilation: if the 3-velocity increases, then the 1-velocity must decrease in proportion to maintain the normalization of the 4-velocity. Within the normalization of the 4-velocity, the freedom to move around phase space among a range of momenta p^i, and to move throughout space x^i at relativistic speeds, is counterbalanced by time dilation in x^0. Since the 4-velocity includes the real velocity $\vec{v}$ and the relativistic kinetic energy E, we are able to show that

$$p^\mu = mU^\mu \qquad \Longrightarrow \qquad p^\mu p_\mu = -\frac{E^2}{c^2} + \big|\vec{p}\big|^2 = -\big(mc\big)^2 \quad , \tag{2.105}$$

where m is a particle's proper mass. Hypersurfaces in phase space satisfying equation (2.105) lead to the concepts of "on shell" real particles, and "off shell" virtual particles. When QFT shows a particle with energy and momentum not related as in equation (2.105), then we say the particle is "off shell" in some kind of bulk of virtual states. This is why the shell in the on or off shell condition is called the mass shell. The implication of equation (2.105) is exactly a hyperboloid condition in the manner of

$$f_+(\chi^5_+) = -\big(x^0_+\big)^2 + \sum_{i=1}^{3} \big(x^i_+\big)^2 \quad , \qquad \text{and} \qquad f_-(\chi^5_-) = -\big(x^0_-\big)^2 + \sum_{i=1}^{3} \big(x^i_-\big)^2 \quad , \tag{2.106}$$

that define $\aleph$ and Ω as hyperboloids in $\Sigma^{\pm}$.

Given the coordinates x^μ and the Minkowski metric $\eta_{\mu\nu}$, we have assembled all the pieces of the flat 4D line element

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \quad , \tag{2.107}$$

which is a good starting point for making calculations in general relativity. We have used $x^\mu : \{ct, x, y, z\}$ with $\eta_{\mu\nu} = \text{diag}(-+++)$ but it is equally valid to build Lorentzian spacetime with $x^0 \equiv ict$ which requires a different metric to preserve equation (2.107). In that case we have a 4D Euclidean metric

$$\eta'_{\mu\nu} = \begin{pmatrix} c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , \tag{2.108}$$

which leads to a significantly different formulation of the theory. The spacetime is still Lorentzian — that is an immutable aspect of Nature — but now the metric is Euclidean because we have made the change $ct \to ict$. With complex coordinates that use a Euclidean metric, there will be no distinction between raised and lowered tensor indices and any eventual calculations will be simplified.

The most general form of the 4D line element for a given diagonal metric $\bar{Z}_{\mu\nu}$ is

$$ds^2 = \bar{Z}_{00}\big(dZ^0\big)^2 + \bar{Z}_{11}\big(dZ^1\big)^2 + \bar{Z}_{22}\big(dZ^2\big)^2 + \bar{Z}_{33}\big(dz^3\big)^2 \quad , \tag{2.109}$$

where Z^μ is some set of coordinates. If we set

$$Z^\mu \quad \equiv \quad x^\mu : \begin{cases} x^0 & \equiv & ct \\ x^1 & \equiv & x \\ x^2 & \equiv & y \\ x^3 & \equiv & z \end{cases} \quad , \tag{2.110}$$

and we want to make $\bar{Z}_{\mu\nu}$ give the Minkowski line element

$$ds^2 = -c^2\, dt^2 + dx^2 + dy^2 + dz^2 \quad , \tag{2.111}$$

then evidently the Lorentzian metric $\text{diag}(-1, 1, 1, 1)$ is required. *However!* We can alter the Z^μ coordinates as

$$Z^{\mu} \quad \equiv \quad y^{\mu} : \begin{cases} y^0 & \equiv \quad ict \\ y^1 & \equiv \quad x \\ y^2 & \equiv \quad y \\ y^3 & \equiv \quad z \end{cases} \quad , \tag{2.112}$$

and now

$$dZ^0 \quad \equiv \quad ic\,dt \qquad \Longrightarrow \qquad \big(dZ^0\big)^2 = -c^2\big(dt\big)^2 \quad . \tag{2.113}$$

Therefore, if we want to write the line element in the form of equation (2.111), we have to use the Euclidean metric. Here, we have the convenient option to finally make a definition for $\hat{i}$ with

$$\hat{Z}^0 \quad = \quad ct\,\hat{i} \qquad \Longrightarrow \qquad \big(d\hat{Z}^0\big)^2 = c^2\big(dt\big)^2\,\hat{i}^2 = -c^2\big(dt\big)^2 \quad . \tag{2.114}$$

This looks very much like $\hat{i}$ should be the arrow of time and there is another option for using $\hat{i}$, one that nicely demonstrates the principle of complexity. We can use equation (2.114) with the Euclidean metric when we take

$$\hat{Z}^{\mu} \quad \equiv \quad z^{\mu}\,\hat{i} \;, \qquad \text{with} \qquad z^{\mu} : \begin{cases} z^0 & \equiv \quad ct \\ z^1 & \equiv \quad ix \\ z^2 & \equiv \quad iy \\ z^3 & \equiv \quad iz \end{cases} \quad , \tag{2.115}$$

Furthermore, definition (2.115) is the one that naturally eliminates the need for Wick rotation of the time component $z^0 \mapsto -iz^0$ during the computation of QFT action integrals as in reference [23]. Wick rotation will be unnecessary if we replace the differential volume element of four real dimensions dx^{μ} the volume element of one real dimension and three imaginary. When we integrate over dz^{μ} as in equation (2.115), we will recover three factors of i giving $i^3 = -i$ which is exactly the term inserted manually during analytic continuation via Wick rotation. Another configuration, one that seeks to demonstrate the concept of $\hat{\pi}$ as an arrow of space is

$$\hat{Z}^{\mu} \quad \equiv \quad \hat{z}^{\mu} \;, \qquad \text{with} \qquad \hat{z}^{\mu} : \begin{cases} z^0 & \equiv \quad ct\,\hat{i} \\ z^1 & \equiv \quad ix\,\hat{\pi} \\ z^2 & \equiv \quad iy\,\hat{\pi} \\ z^3 & \equiv \quad iz\,\hat{\pi} \end{cases} \quad . \tag{2.116}$$

Due to infinite complexity, there are very many schemes that can be explored.

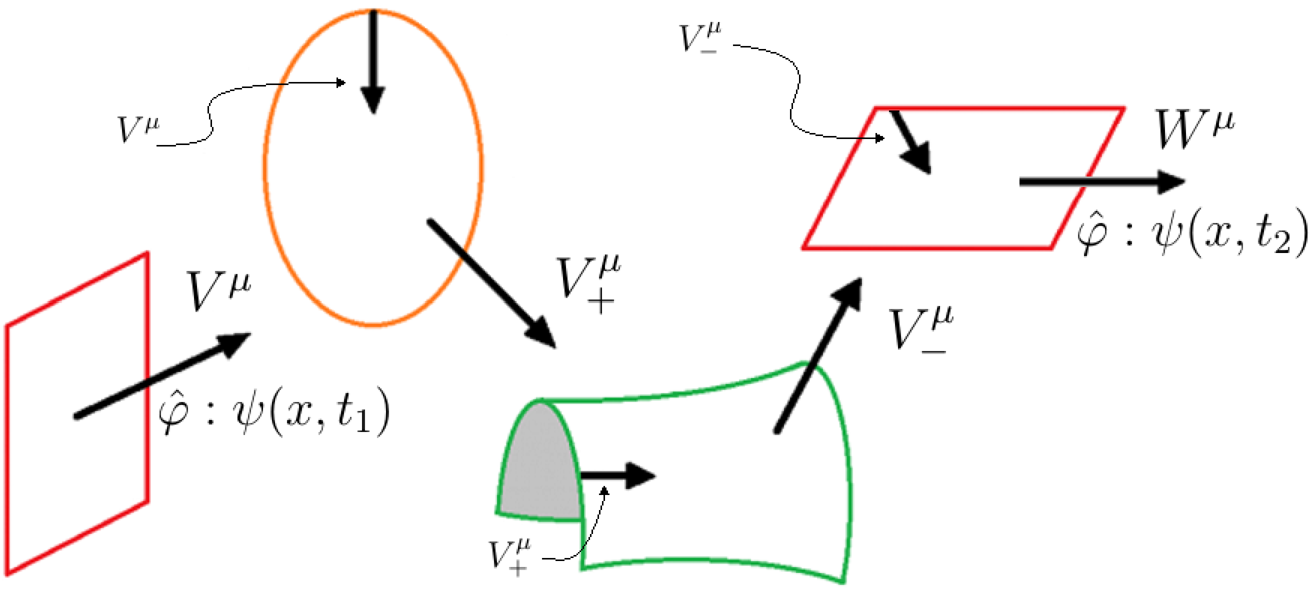


Figure 18: The new object $\hat{\varphi}$ encodes the quantum sector in the MCM unit cell along side the ontological basis $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ which should be used, among other things, as a non-coordinate basis for general relativity. This system needs to be able to support at least two distinct phases: decoherence and stability. The non-decoherence mode of repeated measurement is such that all the V are linearly dependent on $\hat{\Phi}$. Then $Q = |V^\mu - W^\mu| = |\hat{\Phi} - \hat{\Phi}| = 0$.

II.5 What is $\hat{\varphi}$?

In section I.2, we introduced the vector V^μ that will be transported through $\mathcal{H} \mapsto \Omega \mapsto \aleph \mapsto \mathcal{H}$ and figure 18 gives the general idea. The object $\hat{\varphi}$ will be the initial V^μ written in ontological notation. The $\hat{\varphi}$ component shall complement the perturbation tensor $h_{\mu\nu}$ as the computational source for what happens across a given MCM unit cell. So far, we have only considered the ontological sector with $h_{\mu\nu} = 0$ so, in context, $\hat{\varphi}$ is the complete computational source. We described how $\hat{M}^3$ pushes a vector V^μ through the MCM unit cell, and now we begin to clarify those preliminary definitions as

$$V^\mu \quad \equiv \quad \psi\big(x; t_{\text{initial}}\big)\, \hat{\varphi} \quad , \qquad \text{and} \qquad W^\mu \quad \equiv \quad \psi\big(x; t_{\text{final}}\big)\, \hat{\varphi} \quad . \tag{2.117}$$

We can imagine a method in which $\hat{M}^3$ (or $\tilde{M}^4$) takes $\psi(x; t_{\text{initial}})\, \hat{\varphi}$ and returns a qubit in the unitary sector like $\psi(x; t_{\text{final}})\hat{1}$ which then becomes the seed term for evolution across the next level of $\aleph$ when we make the change $\psi(x; t_{\text{final}})\, \hat{1} \to \psi(x; t_{\text{initial}})\, \hat{\varphi}$. We will implement a transfinite normalization $\hat{\Phi}^2 \to \hat{\Phi}^1$ as part of the ontological rescaling at the boundary of each unit cell so we can see how we might also come across $\hat{\Phi}^0 \equiv \hat{1}$ becoming $\hat{\Phi}^{-1} \equiv \hat{\varphi}$. As the level of $\aleph$ increases, the magnitude of infinitude must be decreased to maintain finite normalization. Since $\hat{\varphi}$ is new, we will not be too specific about it other than to mention that it should exist in some form. If $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ are going to be a basis for 4D spacetime then we need some other informatic channel for the quantum component of our quantum gravity. We propose to send that channel on $\hat{\varphi}$. It will be the object that holds the quantum initial condition which will be operated upon with $\hat{M}^3 : \mathcal{H}_1 \mapsto \mathcal{H}_2$. Since the TOIC is a theory of quantum gravity and not just gravity, we need to evolve the dynamical geometry of $\mathcal{H}$ across the unit cell and also whatever quantum information from $\mathcal{H}'$ was encoded on $\mathcal{H}$ through its representations in the coordinates x^μ or momenta p^μ.

The utility of $\hat{\varphi}$, without being specific about one exact mathematical definition or another, will be as follows. The true history is the string of earlier $\hat{\Phi}$ vectors that precede an observer on a given level of $\aleph$ but $\hat{\varphi}$ encodes the record of that history in the present $\mathcal{H}$. For this reason, we will attach the new object to $\mathcal{H}$. The qubit on $\hat{\varphi}$ is $\psi \in \mathcal{H}'$. It defines the entire quantum mechanical sector because states ψ are complete boundary conditions in quantum theory under the Schrödinger equation. We can also take $\hat{\varphi}$ as the source of torsion as in reference [5]. Recall that the source of torsion is a vector and not a scalar like gravitational or electric sources. The torsion can be encoded with a potential so if we use the wavefunction as the potential then we see how the quantum gravity will act by sending qubits into the geometry through the torsion. We would like to show that the probability density on $\mathcal{H}_2$ exhibits decoherence with respect to that on $\mathcal{H}_1$ because of the Ricci flow, or some other geometric flow, acting on the quantum information during its geometric representation in the hyperspacetime between adjacent $\mathcal{H}$-branes. When we discussed torsion in reference [5], we set $\mathcal{H}$ as a torsion free boundary surface in the MCM unit cell. We have also defined $\mathcal{H}$ as the place where measurements of quantum states happen, and that corresponds to what is called the collapse of the wavefunction. It is an intuitive arrangement when the wavefunction is encoded in the torsion and then it collapses on the torsion-free surfaces $\mathcal{H}$.

One of quantum mechanics' many odd empirical results is that a state will not exhibit decoherence if the state is measured sufficiently often. If the geometric flow is always present between successive measurements then we will always see decoherence which is not the desired behavior. Instead, we will need to introduce some parameter[1] that sets a transition between laminar flow across the unit cell and then some supercritical non-laminar phase. In the case of laminar flow, decoherence will not be observed because the laminar flow corresponds to each of $\{V^\mu, V_+^\mu, V_-^\mu\}$ being $\hat{\Phi}$ giving $Q = |V^\mu - W^\mu| = 0$. When the parameter, which might be the length of chronological time x^0 between successive measurements, becomes too large, then there must be a phase change. In the non-laminar phase, the vectors $\{V^\mu, V_+^\mu, V_-^\mu\}$ will all have to be defined individually so that $Q > 0$.

In section I.2, we described how hyperspacetime can be constructed from spacetime by taking some vector in $\mathcal{H}$ and then defining $\{V^\mu, V_+^\mu, V_-^\mu\}$ accordingly. This is the $Q > 0$ decoherence mode where the V are defined by some local qubits instead of the ontological qubit $\hat{\Phi}$. Additionally, we might explore a situation in which decoherence does always occur but the qubit remains in some local minimum that is preserved through brief geometric flow during the period between repeated measurements. If we associate one qubit with one local minimum in the flow then the eigenstate $|\psi; \hat{\pi}_0\rangle$ at the beginning of the flow will be the same one at the end of the flow $|\psi; \hat{\pi}_1\rangle$ as long as the valley of stability representing the eigenstate is preserved. If the duration of the flow is very short then we expect the local minimum will not merge with some other basin of attraction where the quantum state changes from an eigenstate to a superposition of eigenstates. This notion might form a long sought after quantum determinism.

Dirac was able to predict anti-matter on the basis of sign reversal symmetry in quantum

[1] This parameter might describe some experimental configuration and not Nature itself fundamentally.

mechanics. If we extend that principle into another sector in QED then we would expect that the perturbation theory that agrees with Nature would be a Laurent series expansion in the fine structure constant, but instead the one that describes Nature is only a Taylor series. QED uses series whose terms are proportional to α_{QED}^n but a continuation of the symmetry argument made by Dirac might lead one to conclude that terms of order $\alpha_{QED}^{\pm n}$ should contribute as in a Laurent series. Why are the terms associated with α_{QED}^{-n} not present in Nature? Of course, those terms would make the probability interpretation of the theory explode but we could add some non-trivial non-unitary component to the $\sqrt{i}$ channel which would allow the Laurent terms and not just the Taylor terms. If we put the observer on the $\hat{\Phi}^0$ level of $\aleph$ then it is easy to associate all qubits from the past levels $\hat{\Phi}^{\{n<0\}}$ with the Taylor series terms and all the future qubits on levels $\hat{\Phi}^{\{n>0\}}$ with the terms that would add to the Taylor terms to make a complete Laurent series. The critical distinction to note in this regard is that the past terms get encoded on $\hat{\varphi}$ but the future terms do not. To examine the full Laurent series representation, we could take a Laurent series and split it in half. Put the positively and negatively exponentiated terms respectively into the chronological and chirological sectors. It is natural to say that the terms in the chronological sector are QED's Taylor series terms α_{QED}^n and the other terms α_{QED}^{-n} are not encoded in the $\hat{\varphi}$ sector because chronological initial conditions like $\psi_{\text{initial}}\hat{\varphi}$ describe the past only. For now, we leave the future terms as a logical remainder but perhaps the continuation of Dirac's symmetry argument will yield observable consequences of the Laurent representation as advanced effects dual to the retarded effects on the Taylor terms. We may be able to use the transfinite analytical tools of hypercomplexity to write the causality violating advanced potential effects from the future levels of $\aleph$ directly into the well known Taylor series representation of the past levels. Furthermore, we have struggled with a reason to invert α_{MCM} so that it becomes $1/137$ instead of just 137, but now that matters less as we have freedom to choose positive or negative exponents in each sector respectively.[1] Terms in the Laurent exclusive sector are assigned to all future levels of $\aleph$, one term to each level, so they are much more complicated than the Taylor terms that exist together on a single level of $\aleph$. This distinction between a single level of $\aleph$ or many levels could be a clue regarding how the chronological sector could be so well understood for so many decades while the other parallel sector has remained mysterious.

We might motivate the Taylor/Laurent asymmetry in perturbation theory by noting that we have previously expected to define the past of some moment $\hat{\Phi}^n$ on the $\hat{\Phi}^{n-m}$-sites with $m \geq 1$ but now we see that $\hat{\varphi}$ is a better place to define the past. The analytical expression of the past has to exist in the present with the observer so we may encode the information from the past as $\hat{\varphi}$ on $\mathcal{H}_1$. Instead of a series of past terms in the hypercosmos like $\{\hat{\Phi}^{n-1}, \hat{\Phi}^{n-2}, ...\}$, we have a series of terms in perturbation theory like $\{\alpha_{MCM}, \alpha_{MCM}^2, ...\} \approx \{137^{-1}, 137^{-2}, ...\}$ which all coexist on the same level of $\aleph$ with $\hat{\varphi}$.

To say a little more about α_{MCM}, note that we still have not decided which of the ontological basis is the arrow of time. The vector $\hat{\pi}$ is a good candidate, but it does not have to be the arrow of time and could be the "arrow of space" when $\hat{i}$, or another one, is the arrow of time, as in equation (2.116). The motivation in proposing an arrow of space is the factor of π^3 that appears in $\alpha_{MCM} = 2\pi + (\Phi\pi)^3$. While we have made some good definitions for the numerical origin of α_{MCM} in reference [10], if there is a $\hat{\pi}$ pointing in each of the three

[1]In reference [10], we did finally come up with a good way to get 137 into the denominator.

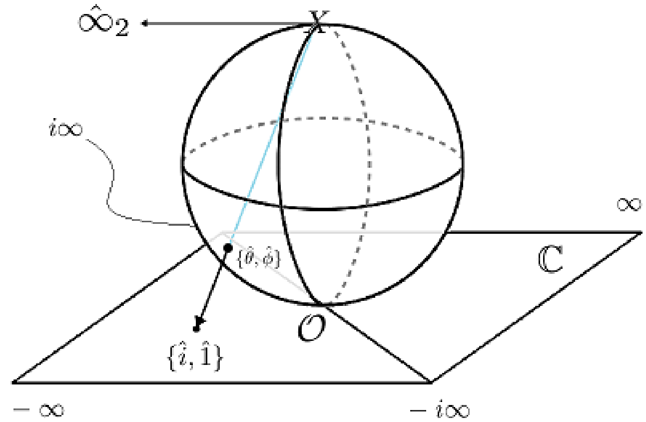


Figure 19: The "volume integral" over the fundamental element of the abstract psychological space is the fine structure constant α_{MCM} when the volume of the ball is $(\Phi\pi)^3$ and the two vectors each contribute π as non-trivial embedded objects corresponding the two $\hat{\pi}$-sites at the beginning and end of every MCM unit cell, as in reference [10]. We might obtain the volume element $(\Phi\pi)^3$ through the inflation of 3-space where the arrow of space is $\hat{\pi}$. Recall that inflation of 3-space is the default scenario in ΛCDM cosmology where the Hubble parameter increases with time. This figure first appeared in reference [8] where we argued against the Riemann hypothesis. Reference [8] is a good introduction to the idea of infinite complexity.

dimensions of space where the $\hat{\varphi}$ object encodes an input for $\hat{M}^3$, and then space is inflated by Φ when $\hat{\Phi}$ acts on $\hat{\varphi}$ (perhaps generating the unitary sector $\hat{1}$), then that could be another natural origin for $(\Phi\pi)^3$. If that 3-space is on the interior of the 2-sphere in figure 19, and the two vectors on its surface are delta functions that return π when integrated over, likely pointing to $\hat{\pi}_0$ and $\hat{\pi}_1$ on two different levels of $\aleph$, $\hat{\Phi}^1$ and $\hat{\Phi}^2$, then the volume integral over the object in the figure will be

$$\int dV_{MCM} \quad \equiv \quad \alpha^{-1}_{MCM} = 2\times\left(\frac{\pi}{2}+\frac{\pi}{2}\right)+\big(\Phi\pi\big)^3 \quad . \tag{2.118}$$

In reference [24], it is shown that topology change is usually represented by a boundary condition on Hilbert space $\mathcal{H}'$ so we will make an entirely rigorous definition for the modular topology change $\mathcal{H}\mapsto\Omega\mapsto\aleph\mapsto\mathcal{H}$. The topological boundary condition that has proven difficult to depict in 2D is such that we take two zenith coordinates on $\mathbb{S}^2$, put them in a bispinor with two more zenith degrees of freedom whose orthogonal endpoints lie in $\theta\in(-\pi/2,\pi/2)$ and then include those 1D intervals in the volume of the physical Hamiltonian or Lagrangian phase space. Perhaps we might generate exotic effects, such as the decoherence of classical reality into quantum weirdness, by scaling only position or momentum space with non-unitary operations based on irrational numbers, but not both, because $\psi\in\mathcal{H}$ are represented solely with position space representations in the MCM maps to general relativity. Therefore, let the changing level of $\aleph$ be such that $\hat{\Phi}:V\to\Phi^3V$ when V is the volume of the O(3) sector of an O(3,1) manifold. The completion of the volume of "all of space," including the ring(s) at infinity, is defined when the rings contribute as an infinitesimal volume on the lower level of $\aleph$ but their portion gets chirologically inflated to finiteness in transit through hyperspacetime from one moment of psychological time to the next. Then the changing level of $\aleph$ for the finite objects that represent conformal infinity is such that the zero volume of four 1D rings contribute on the higher level as $2\times(\pi/2+\pi/2)$. Each zenith coordinate domain on $\mathbb{S}^2$ has length π because $\theta\in(\pi/2,\pi/2)$ along each ring, and we remove the point $\theta=0$ due to the MCM condition that the observer can never observe anything at his own location. The four intervals $\theta\in[0,\pi/2)$ can each have a conformal infinity at $\pi/2$ which is then effectively an orthogonal topological element in the observer's local $\hat{M}^3$ theories. Thus we have defined rigorously the MCM model of topology change as

$$V\hat{\Phi}^0=\pi^3 \quad , \qquad \text{with} \qquad V\hat{\Phi}^1=(\Phi\pi)^3+2\pi \quad . \tag{2.119}$$

Here, we also note that the changing level of $\aleph$ could be such that $\hat{\Phi}^j$ goes to $\hat{\Phi}^{j+2}$ because we have to select the $\hat{\Phi}$ component before we can operate with $\hat{\Phi}$ to increase the level of $\aleph$. Therefore, in section IV.1, we will revisit the argument from reference [13] about whether the level of $\aleph$ should go up by one or two. If we use two then we have a natural disconnection between two chirological degrees of freedom χ^5_+ and χ^5_-.

II.6 Twistors and Spinors

The mystery of unification in physics is to find a scheme by which quantum particles dynamically warp spacetime, meaning that the spacetime is not just a background. In the previous section, we suggested to send qubits into the geometry by encoding them in the torsion. In this section, we show that another option leads to an object in twistor theory.

One obvious way to define a reciprocal mechanism for sending perturbations between QM and GR will be through

$$\psi \quad \longleftrightarrow \quad h_{\mu\nu} \quad , \tag{2.120}$$

where ψ is a disturbance on the vacuum and $h_{\mu\nu}$ is the perturbation on the Minkowski metric. In reference [3], we showed how an electromagnetic boundary condition on $\mathcal{H}$ could induce gravitation in the bulk hyperspacetime and vice versa. This reciprocity between the electric and gravitational sectors implies the existence of more than one configuration for perturbative information and we are drawn to consider many possible definitions for perturbations such as those in equations (2.86-2.88). Each method for writing qubits as perturbative tensors, through the torsion earlier and now directly with $\psi\rightarrow h_{\mu\nu}$, are simply ansatzes taken for the purposes of discussion. To that end, one is inclined to consider perturbations of the form

$$h_{\mu\nu} = \psi \begin{pmatrix} -\dfrac{i}{4} & 0 & 0 & 0 \\ 0 & -\dfrac{\varphi}{4} & 0 & 0 \\ 0 & 0 & \dfrac{1}{8} & 0 \\ 0 & 0 & 0 & \dfrac{1}{4\pi} \end{pmatrix} \quad , \tag{2.121}$$

where the reader should recognize the ontological coefficients. We may associate them with $\int d\hat{\gamma}$ and/or the $\{+-+-\}$ non-Lorentzian metric signature representation of the inherently O(3,1) topology of Minkowski space found in the linear space spanned by $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$. In equation (2.121), the state causes a perturbation in ontological form having two timelike dimensions and two spacelike. Such objects have been treated as a problem in string theoretical twistor theory by Witten. He writes the following in reference [25].

> "[*W*]e will review some kinematics in four dimensions. We start out in signature $+---$, but we sometimes generalize to other signatures. Indeed, [*in perturbation theory*] the signature is largely irrelevant as the scattering amplitudes are holomorphic functions of the kinematic variables. Some things will be simpler with other signatures or for complex momenta with no signature specified.
>
> "First we recall that the Lorentz group in four dimensions, upon **complexification** [*emphasis added*], is locally isomorphic to $SL(2) \times SL(2)$, and thus the finite-dimensional representations are classified as (p, q), where p and q are integers or half-integers. The negative and positive chirality spinors transform in the $(1/2, 0)$ and $(0, 1/2)$ representations, respectively. We write generally λ_a, $a = 1, 2$, for a spinor transforming as $(1/2, 0)$, and $\widetilde{\lambda}_{\dot{a}}$, $\dot{a} = 1, 2$, for a spinor transforming as $(0, 1/2)$.
>
> "Spinor indices of type $(1/2, 0)$ are raised and lowered with the antisymmetric tensor ϵ_{ab} and its inverse ϵ^{ab} (obeying $\epsilon^{ab}\epsilon_{ab} = -\delta^a_c$): $\lambda_a = \epsilon_{ab}\lambda^b$, $\lambda^b = \epsilon^{bc}\lambda_c$. Given two spinors λ_1, λ_2 both of positive chirality, we can form the Lorentz invariant $\langle\lambda_1, \lambda_2\rangle = \epsilon_{ab}\lambda_1^a\lambda_2^b$. From the definitions, it follows that $\langle\lambda_1, \lambda_2\rangle = -\langle\lambda_2, \lambda_1\rangle = -\epsilon^{ab}\lambda_{1a}\lambda_{2b}$.
>
> "Similarly, we raise and lower indices of type $(0, 1/2)$ with the antisymmetric tensor $\epsilon_{\dot{a}\dot{b}}$ and its inverse $\epsilon^{\dot{a}\dot{b}}$, again imposing $\epsilon^{\dot{a}\dot{b}}\epsilon_{\dot{b}\dot{c}} = \delta^{\dot{a}}_{\dot{b}}$. For two spinors $\widetilde{\lambda}_1, \widetilde{\lambda}_2$, both of negative chirality, we define $[\widetilde{\lambda}_1, \widetilde{\lambda}_2] = \epsilon_{\dot{a}\dot{b}}\widetilde{\lambda}_1^{\dot{a}}\widetilde{\lambda}_2^{\dot{b}}$.
>
> "The vector representation of $SO(3, 1)$ is the $(1/2, 1/2)$ representation. Thus, a momentum vector p_μ, $\mu = 0, ..., 3$, can be represented as a 'bi-spinor' $p_{a\dot{a}}$ with one spinor index a or $\dot{a}$ of each chirality. The explicit mapping from p_μ to $p_{a\dot{a}}$ can be made using the chiral part of the Dirac matrices. With signature $+---$, one can take the Dirac matrices to be
>
> $$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad , \tag{2.122}$$

where we take $\sigma^\mu = (1, \vec{\sigma})$, $\bar{\sigma}^\mu = (-1, \vec{\sigma})$, with $\vec{\sigma}$ being the 2×2 Pauli spin matrices. In particular, the upper right hand block of γ^μ is a 2×2 matrix $\sigma^\mu_{a\dot{a}}$ that maps spinors of one chirality to the other. For any spinor p_μ, define

$$p_{a\dot{a}} = \sigma^\mu_{a\dot{a}} p_\mu \quad . \tag{2.123}$$

Thus, with the above representation of σ^μ, we have $p_{a\dot{a}} = p_0 + \vec{\sigma} \cdot \vec{p}$ (where p_0 and $\vec{p}$ are the "time" and "space" parts of p^μ), from which it follows that

$$p_\mu p^\mu = \det(p_{a\dot{a}}) \quad . \tag{2.124}$$

Thus a vector p^μ is lightlike if and only if the corresponding matrix $p_{a\dot{a}}$ has determinant zero.

"Any 2×2 matrix $p_{a\dot{a}}$ has rank at most two, so it can be written $p_{a\dot{a}} = \lambda_a \widetilde{\lambda}_{\dot{a}} + \mu_a \widetilde{\mu}_{\dot{a}}$ for some spinors λ, μ, and $\widetilde{\lambda}, \widetilde{\mu}$. The rank of a 2×2 matrix is less than two if and only if its determinant vanishes. So the lightlike vectors p^μ are precisely those for which

$$p_{a\dot{a}} = \lambda_a \widetilde{\lambda}_{\dot{a}}, \tag{2.125}$$

for some spinors λ_a and $\widetilde{\lambda}_{\dot{a}}$.

"If we wish $p_{a\dot{a}}$ to be real with Lorentz signature, we must take $\widetilde{\lambda} = \pm\bar{\lambda}$ (where $\bar{\lambda}$ is the complex conjugate of λ). The sign determines whether p^μ has positive energy or negative energy.

"It will also be convenient to consider other signatures. In **signature** $+ + - -$ [*emphasis added*], λ and $\widetilde{\lambda}$ are independent, real, two-component objects. Indeed, with signature $+ + - -$, the Lorentz group $SO(2, 2)$ is, without any complexification, locally isomorphic to $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$, so the spinor representation is real. With Euclidean signature $+ + + +$, the Lorentz group is locally isomorphic to $SU(2) \times SU(2)$; the spinor representations are pseudoreal. A lightlike vector cannot be real with Euclidean signature.

"Obviously, if λ and $\widetilde{\lambda}$ are given, a corresponding lightlike vector p is determined, via (2.125). It is equally clear that if a lightlike vector p is given, this does not suffice to determine λ and $\widetilde{\lambda}$. They can be determined only modulo the scaling

$$\lambda \to u\lambda, \; \widetilde{\lambda} \to u^{-1}\widetilde{\lambda} \tag{2.126}$$

for $u \in \mathbb{C}^*$, that is, u is a nonzero complex number. (In signature $+ - - -$, if p is real, we can restrict to $|u| = 1$. In signature $+ + - -$, if λ and $\tilde{\lambda}$ are real, we can restrict to real u.) Not only is there no natural way to determine λ as a function of p; there is in fact no continuous way to do so, as there is a topological obstruction to this. Consider, for example, massless particles of unit energy; the energy-momentum of such a particle is specified by the momentum three-vector $\vec{p}$, a unit vector which determines a point [*on a sphere*] $\mathbb{S}^2$. Once $\vec{p}$ is given, the space of possible λ's is a non-trivial complex line bundle over $\mathbb{S}^2$ that is known as the Hopf line bundle [(*figure 20*)]; non-triviality of this bundle means that one cannot pick λ as a continuously varying function of $\vec{p}$.

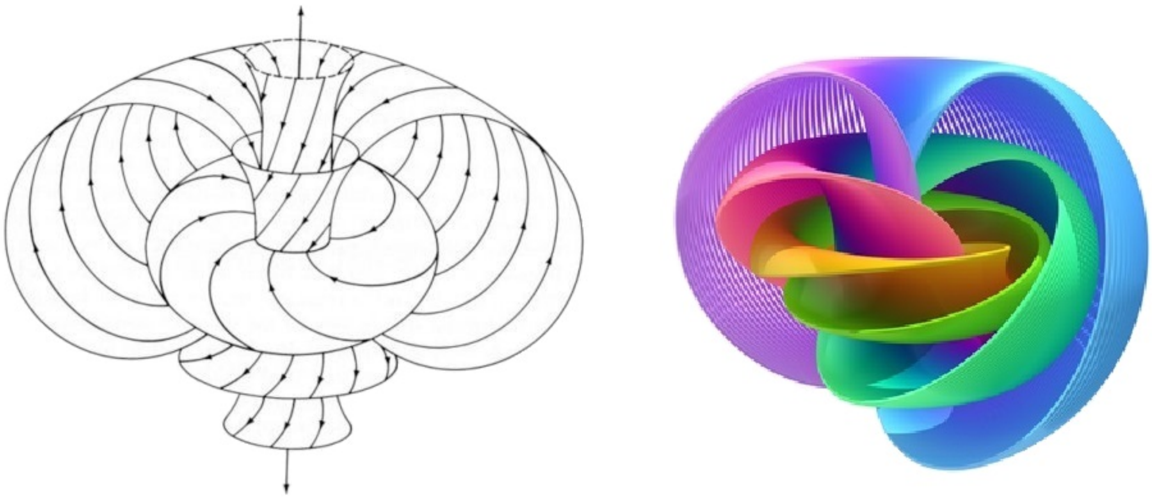


Figure 20: The Hopf fiber bundle describes a 4D shape drawn with circles so it offers an alternative viewpoint on hyperdimensional geometry from rectangular representations such as the tesseract or the MCM unit cell. The bundle looks like the stereographic projection of the 3-sphere onto parallels, meridians, and hypermeridians, as in figure 21. In reference [7], we assigned the topologies flat, hyperbolic, and spherical to those projections and later they were associated to $\{\aleph, \mathcal{H}, \Omega\}$. Now, we are likely proposing to better understand the Hopf fibration by modeling it as the complete cosmological unit cell including $\Sigma^{\varnothing}$. On the left, the arrows at the top and bottom can be thought of as the input and output of $\hat{M}^3$. The Wikipedia caption of the image to the right is, "The Hopf fibration can be visualized using a stereographic projection of $\mathbb{S}^3$ to $\mathbb{R}^3$ and then compressing $\mathbb{R}^3$ to a ball. This image shows points on $\mathbb{S}^2$ and their corresponding fibers with the same color."

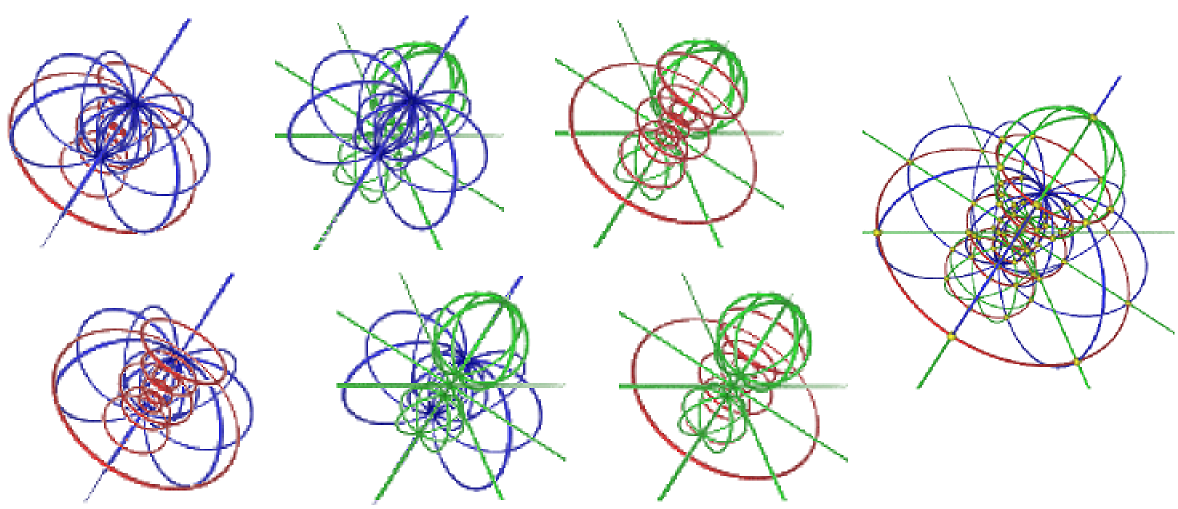


Figure 21: In the stereographic projection of the 3-sphere onto parallels, meridians, and hypermeridians, every parallel, meridian, and hypermeridian is a circle. To get straight lines, we simply consider a circles of infinite radius.

"Once p is given, the additional information that is involved in specifying λ (and hence $\widetilde{\lambda}$) is equivalent to a choice of wavefunction for a spin one-half particle with momentum vector p. In fact, the chiral Dirac equation for a spinor ψ^a is

$$i\sigma^{\mu}_{a\dot{a}}\frac{\partial\psi^a}{\partial x^{\mu}}=0\quad.\text{"}\tag{2.127}$$

Equation (2.127) is exactly the kind of mechanism wherein some hypercosmological vector V^{μ} is derived from the qubit ψ in the lab frame. Witten writes, "The explicit mapping from p_{μ} to $p_{a\dot{a}}$ can be made using the chiral part of the Dirac matrices," and that calls attention to the fact that we have still not defined an explicit $\hat{M}^3$. We have a preliminary definition that $\hat{M}\equiv\partial_t$ but the Dirac operator

$$\widehat{\mathcal{D}}\quad\equiv\quad i\sigma^{\mu}_{a\dot{a}}\partial_{\mu}\quad,\tag{2.128}$$

suggests that we might add complexity to $\hat{M}^3$ with matrix-valued or other multiplectic coefficients $\mathbb{M}$ to each $\hat{M}$. Just as the Pauli matrices define nonrelativistic spinors, the Dirac matrices $\sigma^{\mu}_{a\dot{a}}$ define Dirac spinors (what are often called Dirac vectors are rigorously Dirac bispinors) so if we add some matrix coefficients to the derivatives in $\hat{M}^3$ then that will also be a way to probe the MCM for multiplectic spinor analogues. For instance, we might write

$$\hat{M}^3=\big(\mathbb{M}_1\partial_+\big)\big(\mathbb{M}_2\partial_{\varnothing}\big)\big(\mathbb{M}_3\partial_-\big)\quad,\tag{2.129}$$

where the roots of the algebra of $\mathbb{M}_i$ are representative of physical systems. The matrices $\mathbb{M}_i$ do not disrupt the mechanism of $\partial^3_{\chi}\equiv(C_1\partial_+)(C_2\partial_{\varnothing})(C_3\partial_-)$ demonstrated in reference [10]; they complement the scalars C_i with multiplectic structure.

We will not be going too much into twistor theory and it will suffice to say that twistors satisfy the twistor equation. Wikipedia says, "For Minkowski space [*sic*] the solutions to the twistor equation are of the form

$$\Omega^a(x)=\omega^a-ix^{a\dot{a}}\pi_{\dot{a}}\quad,\tag{2.130}$$

where ω^a and $\pi_{\dot{a}}$ are two constant Weyl spinors and $x^{a\dot{a}}=\sigma^{a\dot{a}}_{\mu}x^{\mu}$ is a point in Minkowski space." Among the primary utilities of twistor theory is to change the number of indices of objects ($x^{\mu}\mapsto x^{a\dot{a}}$ or $p_{\mu}\mapsto p_{a\dot{a}}$) which is exactly what is needed for maps like

$$f^3\big|\psi;\hat{\pi}\big\rangle\quad\mapsto\quad T_{\mu\nu}\tag{2.131}$$

$$\big|\psi;\hat{\Phi}\big\rangle\quad\mapsto\quad Rg_{\mu\nu}\tag{2.132}$$

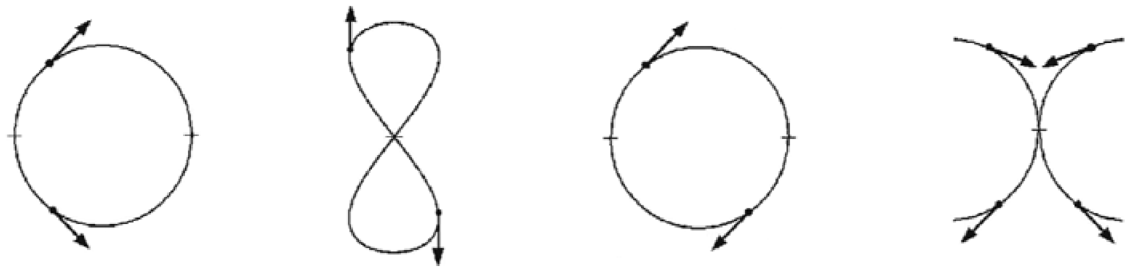


Figure 22: If the figure on the left represents a condition of symmetry then the output of the operation to the right will represent a condition of antisymmetry. Bosons have symmetric wavefunctions and fermions have antisymmetric wavefunctions.

$$\frac{\Phi}{4}\big|\psi;\hat{2}\big\rangle \quad\mapsto\quad R_{\mu\nu} \tag{2.133}$$

$$i\big|\psi;\hat{i}\big\rangle \quad\mapsto\quad g_{\mu\nu}\Lambda \quad , \tag{2.134}$$

wherein rank one state vectors become rank two tensors in general relativity. It follows that we can rewrite

$$\psi \quad\longleftrightarrow\quad h_{\mu\nu} \quad , \qquad\quad \text{as} \qquad\quad \big|\psi;\hat{\varphi}\big\rangle \quad\mapsto\quad h_{\mu\nu} \quad . \tag{2.135}$$

A substantial issue, however, remains unresolved with this method for adding an index: spinors do not transform as tensors. Under ordinary circumstances, the map that adds an index is the covariant derivative

$$\nabla_\lambda \quad : \quad \psi^\mu \quad\mapsto\quad \nabla_\lambda\psi^\mu \quad , \tag{2.136}$$

but shortly we will describe some nuance indicating that spinor objects do have a natural place in general relativity.

We have not used spinors at all in the MCM because we have not carried out any quantum mechanical analysis requiring the specification of a qubit as a fermion. Even when the two universes U and $\bar{U}$ together have fermionic properties attributed to the antisymmetry of the twisting in figure 22, we have not yet considered both universes simultaneously other than to show that a second universe must exist. The MCM unit cell describes U but not simultaneously $\bar{U}$. The MCM proposes to wrap the ct axis of the Minkowski diagram around a cylinder. The initial formulation [2] of the MCM includes a big bang which begrudgingly violates causality — even with cyclic bouncing we have the causal problem of turtles stacked on the backs of turtles — but we do not concede the violation of conservation of momentum

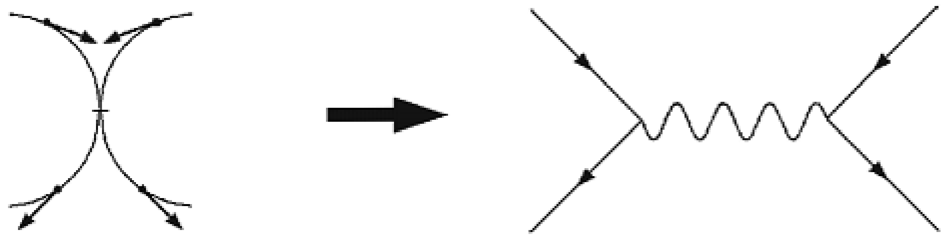


Figure 23: The MCM vertex is exactly the Feynman diagram for electron-electron scattering via the exchange of an off shell photon.

which is a shared property of most big bang theories. We say that if a 4-momentum vector came into existence for some reason at the beginning of the universe then there must also have come into existence another 4-momentum vector pointing in the other direction. Thus, figure 22 shows two arrows of time: the arrows of time for two universe that left the event of the big bang at $\theta = 0$ moving in opposite directions through time. The non-trivial parameter $\chi^5 \equiv \chi^5_+ \otimes \chi^5_\varnothing \otimes \chi^5_-$ is specially designed to accommodate all the twisting in the diagrams with overlaps at $\theta = 0$ and $\theta = \pi$. In general, we can replace the concept of the origin of the universe at an eccentric bounce whose apex represents the spontaneous creation of the wavenumber 4-vector k^μ at t_0 with a permanent non-equilibrium condition of turbulence between a source and sink of information attached to the legs of the diagram, as per Feynman. String theory is known to have the Feynman theory encoded as a limit and we see the MCM is exactly the Feynman digram for two electrons interacting via the exchange of a virtual photon, as in figure 23.

A spinor is just a multiplex introduced to get the right eigenvalue algebra needed to describe anomalous angular momentum in quantum mechanics. The angular momentum vector of a spin-1/2 particle does not transform as a vector under rotations and, therefore, that fermionic angular momentum cannot be represented with vectors; we use spinors instead. For example, if one wishes to know the component in the $\hat{z}$-direction of the angular momentum of a spin-1/2 state then one must operate on the state with the σ_z Pauli matrix. That matrix is 2×2 so the state has to be an array with one column and two rows. We often say that any array with one column is a vector but the rigorous definition has to do with whether or not the array transforms as a vector. The spin-1/2 state does not transform as a vector: it transforms as a spinor and here we are led to point out another general relativistic artifact related to quantum theory. By making the revision $ct \to ict$, as in section II.4, we have shifted x^0 by $\pi/2$ radians in the complex plane but the corresponding revision to the metric $\eta_{00} = 1 \to \eta_{00} = -1$ is a rotation through a full π of arc in the complex plane. This is the same behavior seen in rotations of the half-integer spin vectors (spinors) in relation to the locally inertial lab frame: rotating one makes the other one rotate twice (half) as much. Furthermore, since vectors are the sources of torsion, we might later use spinors as the sources "hypertorsion" which might be useful in some advanced application.

A long-standing problem throughout the history of quantum theory has been the question of how to represent the fermionic eigenfunctions of the spin-1/2 operator without spinors. In a very fundamental way, if we turn a system upside down then we expect that the momentum of the system will be inverted but this is not the case for spin-1/2. There is no classical description with objects that transform as vectors such as the objects associated with orbital

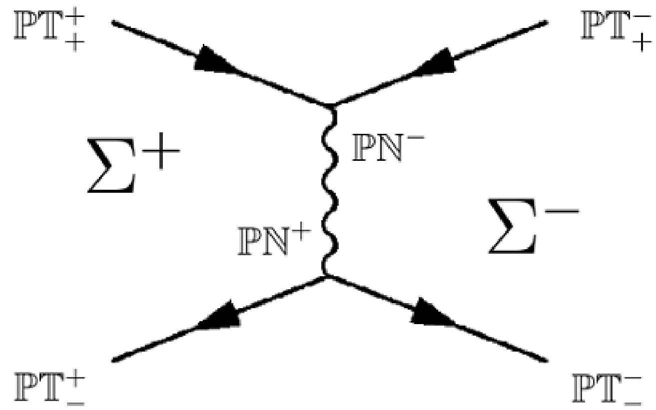


Figure 24: $\mathbb{PN}$ is the projective subspace of null chirality and $\mathbb{PT}_\pm$ are the projective subspaces of a twistor space with either left or right chirality. This figure is well isomorphic to the end of figure 22 when we stretch the bounce point across an entire null surface.

angular momentum, *e.g.*: if a spinning ball is turned over then its angular momentum vector will also turn over in kind, and, *i.e.*: for a boson we can say that $\psi(x) = f(x)$ so that the state is a $1{\times}1$ array but it is always impossible to write the wavefunction of a fermion as an ordinary function. Fermionic wavefunctions always require a multiplectic component as in

$$|\uparrow\rangle \quad \equiv \quad \big|\psi;\tfrac{1}{2}\big\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad , \qquad \text{and} \qquad |\downarrow\rangle \quad \equiv \quad \big|\psi;-\tfrac{1}{2}\big\rangle = \begin{pmatrix}0\\1\end{pmatrix} \quad , \tag{2.137}$$

with

$$\hat{S}_z = \frac{1}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \quad . \tag{2.138}$$

It is easy to see that $\hat{S}_z$ operating on $|\uparrow\rangle$ or $|\downarrow\rangle$ will give the appropriate value: $\pm 1/2$.

We propose that by shifting certain complexity into the topology it will be possible to define spin algebras without spinors. In lieu of spinors, we want to use the topological twisting mechanism from figure 22 which should be amenable to description with spinors if that formalism is desired. Where a wavefunction on one arc of π radians (in figure 22) would be symmetric, we can create antisymmetric fermionic wavefunctions by twisting conformally deformed semicircles around the bounce point which is the origin of coordinates. Where the final pane of figure 22 shows sinusoidal ct and $-ct$, if we invert only the semicircle on one side, or simply select the complete upper or lower path, then the resultant sinusoidal or piecewise sinusoidal wavefunction that propagates on either ct or $-ct$ will be antisymmetric. Witten proposes to use a "bispinor" and that sounds like exactly the kind of thing one might need to twist the topology. However, where Witten builds on the spinor, we aim to never introduce the spinor except as a limit of some ontological considerations. In figure 24, we can see the general outline of how the chirality of twistor space relates the interface of $\Sigma^{\pm}$. The are joined on the projective null space but have chiral legs extending outward. We can directly associate the positive and negative projective twistor subspaces $\mathbb{PT}_{\pm}$ with components of a spinor but we would have to make some special accommodation for $\mathbb{PN}$, possibly as the superposition of two distinct spinor states $\mathbb{PT}_{\pm}$.

The general idea to construct MCM spinors is to take

$$\hat{\pi} \quad \equiv \quad \begin{pmatrix} \pi \\ 0 \end{pmatrix} \qquad \text{and} \qquad \hat{i} \quad \equiv \quad \begin{pmatrix} 0 \\ i \end{pmatrix} \quad , \tag{2.139}$$

as the eigenbasis of a non-relativistic spinor algebra. In the relativistic limit, spinors become Dirac bispinors. We will describe the two degrees of freedom of relativistic quantum theory, hypercharge and isospin, as in figures 25 and 26, with

$$\hat{2} \quad \equiv \quad c_2 \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad , \qquad \text{and} \qquad \hat{\Phi} \quad \equiv \quad c_\Phi \begin{pmatrix} 0 \\ \Phi^0 \end{pmatrix} = c_\Phi \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad . \tag{2.140}$$

We can construct a bispinor from two spinors built on π and i, and 2 and Φ. In terms of the MCM, what Witten says about the Lorentz group being isomorphic to $SL(2)\times SL(2)$ must be in reference to the idea that the ontological gauge has a $\{+-+-\}$ topology in

$$\hat{1} = \frac{1}{4\pi}\,\hat{\pi} - \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{i} \quad , \tag{2.141}$$

but the numbers themselves $\{i, \Phi, 2, \pi\}$ which don't rely on $\varphi < 0$ have the O(3,1) Lorentzian topology of three real dimensions and one imaginary, or vice versa. This topology is usually associated with the Lorentzian signature $\{-+++\}$. Furthermore, the topology of spacetime is notable in general for the conformal properties of spacelike, timelike, and lightlike vectors, and in the MCM we want to add a fourth kind vector that describes a longitudinal direction. Associate to this $\hat{\Phi}$ which points out of spacetime $\mathcal{H}$ to the future Ω. It might be useful to define $\hat{\Phi}$ as a **chiroslike vector** leaving an association for timelike, spacelike, and null vectors as **chronoslike vectors** $\{\hat{2}, \hat{\pi}, \hat{i}\}$. We can associate complexity with $\hat{\Phi}$ when we replace $(0\ 1)^T$ in equation (2.140) with $(0\ \hat{1})^T$ and decompose onto nested ontological resolution of $\hat{1}$.

The Pauli matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \qquad \text{and} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \tag{2.142}$$

and they are related to the Dirac matrices, as in equation (2.122), which gives a good picture of bispinor structure in matrix algebra. If we consider in the Pauli matrices a topology of space associated with $\hat{i}$ and time with $\hat{2}$ then we can associate the imaginary Pauli matrix with space and the other two matrices with time so that there are two kinds of spacetime: space

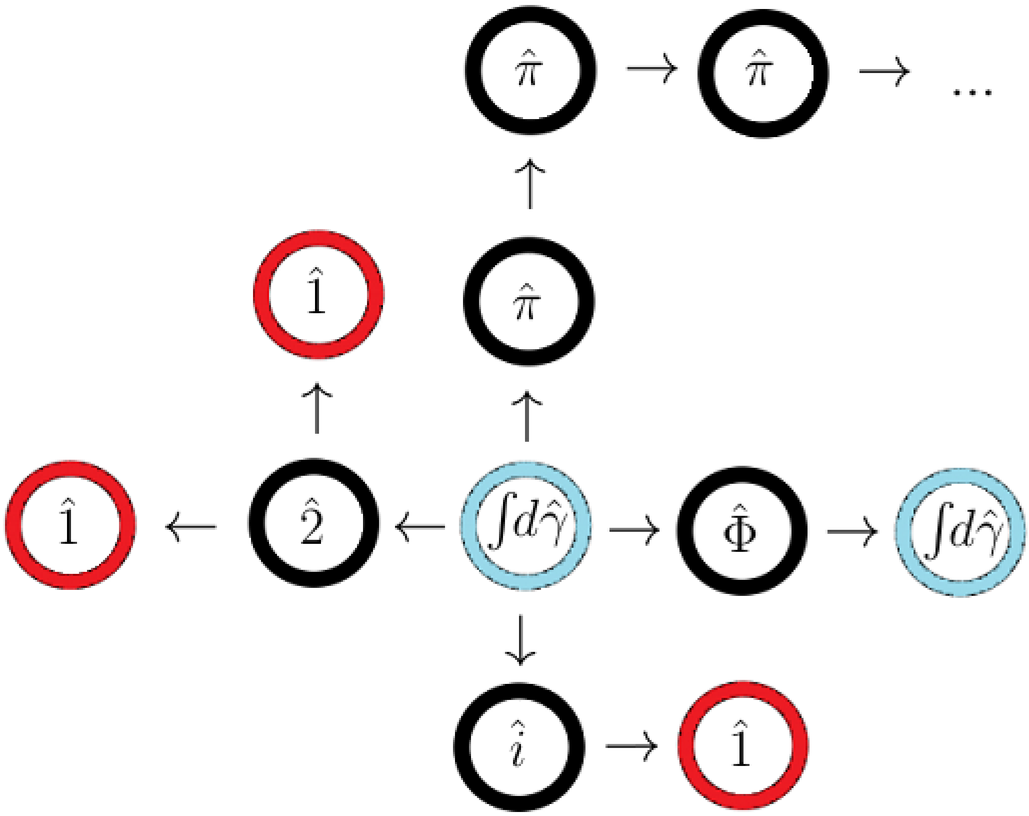


Figure 25: The logical network is similar in structure to common representations of hypercharge and isospin.

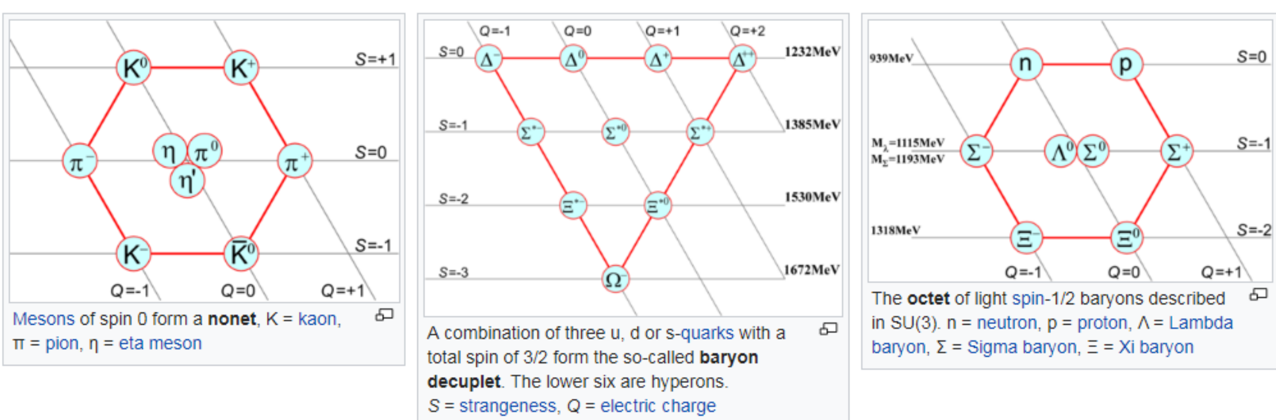


Figure 26: Figures taken from Wikipedia describe the quantization of hypercharge and isospin.

with chronos and space with chiros. In reference [11], we showed that these distinct classes of spacetime, represented here as matrices acting on spinors, when considered altogether, form the basic structure of the standard model of particle physics.

Among $\{\hat{2},\hat{\pi},\hat{i}\}$, $\hat{\pi}$ should probably be the null interval because the coefficient of $\hat{\pi}$ is the coupling constant of the theory of the propagation of photons at the speed of light on $ds^2=0$. If we let $\hat{2}$ describe the two light cones whose symmetry axis is the arrow of time then $\hat{2}$ should be the arrow of time and this leaves spacelike vectors associated with $\hat{i}$. There is an odd spatial property in quantum mechanics that, even with linear quantum mechanical multiplexes, there never exists a simultaneous eigenbasis for $\{\hat{S}_x,\hat{S}_y,\hat{S}_z\}$. The most information that we can extract from the wavefunction is by choosing one direction in an experimental device, calling that the $\hat{z}$-direction, and then measuring the component of the spin angular momentum along that direction. This is likely a profound connection so we will repeat the specification of a novel arrangement. The chirological vector is $\hat{\Phi}$ and χ^5 is 1D, timelike vectors have the topology of $\hat{2}$ giving chronos and chiros, conformally invariant null vectors are like $\hat{\pi}$, and all three dimensions of space are topologically like 1D $\hat{i}$. All of the multiplectic structure in quantum theory is built on the small unitary basis $\{\hat{1},\hat{i}\}$ and the fundamental weirdness of quantum mechanics is that we cannot extract information from the wavefunction about the components of the half integer spin along all three spatial axes.

When we replace $\{\hat{1},\hat{i}\}$ with $\{\hat{\pi},\hat{i}\}$ to construct an "ontological" spinor algebra, the analogues to equations (2.137-2.138) are

$$\big|\uparrow\big\rangle \quad\equiv\quad \big|\psi;\tfrac{1}{2}\big\rangle=\begin{pmatrix}\pi\\0\end{pmatrix}\quad,\qquad\text{and}\qquad \big|\downarrow\big\rangle \quad\equiv\quad \big|\psi;-\tfrac{1}{2}\big\rangle=\begin{pmatrix}0\\i\pi\end{pmatrix}\quad. \tag{2.143}$$

with

$$\hat{S}'_z=\frac{1}{2\pi}\begin{pmatrix}1&0\\0&-i\end{pmatrix}\quad. \tag{2.144}$$

This redefinition of σ_z will require a revision to σ_x and σ_y to preserve the commutation relations $[\sigma_a,\sigma_b]=2i\varepsilon_{abc}\sigma_c$ but that can surely be accomplished because the Pauli matrices are not a unique representation. Furthermore, in dimensionalized form ($\hbar\neq1$) the $\hat{S}'_z$ operator is

$$\hat{S}'_z=\frac{\hbar}{2\pi}\begin{pmatrix}1&0\\0&-i\end{pmatrix}=\big(2\pi\big)^{-2}h\begin{pmatrix}1&0\\0&-i\end{pmatrix}\quad. \tag{2.145}$$

Therefore, perhaps we have redundantly included a factor of 2π in these preliminary definitions. Perhaps the 2π contribution to α_{MCM} on the higher level of $\aleph$ contributes as 0 on the level of $\aleph$ where equation (2.145) is relevant, as in

$$V\hat{\Phi}^0=\pi^3\quad,\qquad\text{with}\qquad V\hat{\Phi}^1=(\Phi\pi)^3+2\pi\quad. \tag{2.146}$$

If these matrices are encoded on $\hat{\varphi}$ then they will be on the same level of $\aleph$ as $\mathcal{H}$ where the quantum theory is canonically defined. We will not develop all these objects here but the origin of spinors in the MCM has been suggested concisely.

An exemplary feature of twistor theory is the mapping of the null interval $ds^2 = 0$ in Minkowski space to its corresponding object in twistor space. In twistor theory, the light ray is a point in twistor space[1] and points in spacetime become Riemann spheres in twistor space. Using these definitions, we can further clarify the inversion operation on the Riemann sphere that we have referred to very many times [12]. This operation takes a Riemann sphere tangentially situated between two branes and inverts it so that the origin of coordinates moves to the null point of $\mathbb{S}^2$ that is not included in the Riemann sphere. In the non-spherical planar representation, this means that the 0D pointlike origin becomes a 1D ring at infinity. If the map swaps the Riemann sphere's pole with its null point at the other pole then that is a map between the entire Riemann sphere and a point, *i.e.*, it is the inverse of the map to twistor space. Furthermore, if the line of $ds^2 = 0$ is a point in twistor space then, recalling that the wavy line on the right side of figure 23 has $ds^2 = 0$, we can say that the diagram on the left side is a twistor representation of the right side that shrinks the wavy line to a point.

Witten's spinors $\lambda, \widetilde{\lambda}$ are only determined up to an inverse scale relationship between u and u^{-1} exactly like the one satisfied by Φ and φ, and the two canonical coordinate charts on $\mathbb{S}^2$. Perhaps it is specifically when we fix this as the golden ratio with $\{u, u^{-1}\} \equiv \{\Phi, -\varphi\}$ that we are able to set the new boundary condition that lets us solve for new physics. Regarding that special kind of physics, the inverse scale relationship is exactly the relationship between the two charts that cover $\mathbb{S}^2$; they are related by the canonical inversion map. Where Witten is not able to uniquely determine his spinors λ and $\widetilde{\lambda}$, in our argument against the Riemann hypothesis [8] we used a single chart on $\mathbb{S}^2$ but were unable to uniquely determine which of the two possible charts it was. Furthermore, since any point on $\mathbb{S}^2$ can be the origin of a chart, we were actually unable to determine which of the infinite number of possible charts we were working with. Our argument against the Riemann hypothesis relied heavily on the relationship between a point and the Riemann sphere, and if a spacetime point is the Riemann sphere in twistor space then there likely exists an important connection between that argument, the Riemann ζ function, and twistor theory. However, we will not be going in that direction in this book because such nuance lies far beyond the general relevance of the MCM.

We previously described how the complete geometry of the bulk, including the fiber bundle of all the computed geodesics, was very complicated. Witten suggests that this is the complexity of the famous Hopf fibration so we will have another new result if the ontological basis provides a new understanding of Hopf's famous mathematical representation of geometric complexity. In reference [13], we described the topology of $\hat{M}^3$ as being isomorphic to that of the Kerr–Newman geometry surrounding a charged, rotating black hole. The topology of the singularity in the Kerr–Newman black hole is $\mathbb{S}^1$, not the normal pointlike singularity, which is actually only one half of $\mathbb{S}^0$ (which is two points.) In the natural universe, it is unlikely that there exist any uncharged, non-rotating black holes, and, therefore, there will be no pointlike singularities in Nature. One might speculate that, at a very funda-

[1] See video reference [26]. In this talk Penrose explains that his main interest in twistor theory is to discover a spacetime calculus specific to 3+1 dimensions, and perhaps we have done that with $\{\Phi, 2, \pi\} \in {}^*\mathbb{R}$ and $\{i\} \in {}^*\mathbb{C}$. He states that 4D twistor theory is the best variety of twistor theory and he casts erudite dispersions on the arrangement shown in figure 24 which shows the abutment of two 5D twistor spaces.

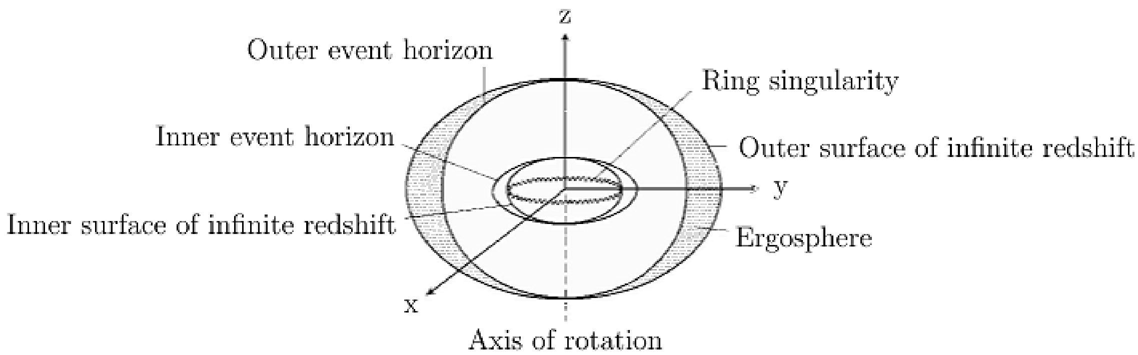


Figure 27: This figure shows the multifaceted region surrounding a Kerr–Newman ring singularity.

mental level, the problems with Hawking's derivation of radiation near a static Schwarzschild black hole described in reference [5] are related to the oversimplification of the multifaceted Kerr–Newman event horizon as a simple mathematical surface. The Kerr–Newman horizon shown in figure 27 is multilayered and very much looks like the Hopf fibration whereas the Schwarzschild event horizon is a simple surface.

To close this section in which we only lightly touch on either of twistors or spinors, and then only with the tip of a long polearm, consider what Misner, Thorne, and Wheeler write about Penrose in reference [27]. Penrose's name appeared in the title of the first paper [2] this writer submitted to arXiv in 2009. His name also appeared in the abstract of the second such paper [7] submitted in 2011 wherein one finds a statement, "Following the program of Penrose, geometry rather than differential equations will be the mathematical tool."

> "Roger Penrose started out as an algebraic geometer. [*sic*] Because if his pure mathematical background, **his approach to the subject was different** [*emphasis added*] from those which had been adopted hitherto. He was particularly interested in the global light-cone structure of spacetime and in the equations of zero rest-mass fields both of which are preserved under conformal transformations. He exploited this conformal invariance to give an elegant and powerful treatment of gravitational radiation in terms of a null surface $\mathscr{I}^+$ at infinity. More recently this interest has led him to develop the theory of twistors, which are the spinors corresponding to the conformal group of Minkowski space."

II.7 Dyads and Quaternions

Noting that the three Pauli matrices together with the identity matrix are isomorphic to the quaternions (whose Lie group is $\mathbb{S}^3$), we have considered a great many things and now we will generate complexity by considering pairs of ontological basis vectors. There is an intuitive picture wherein one needs to compute arbitrarily long strings of such vectors to define the addresses of arbitrarily many unique lattice sites in the hypercosmos. It is with these long addresses that we propose to generate the small numbers needed to solve the hierarchy problem. A first step in that direction is to consider pairs exhibiting the behavior proposed in reference [5]

$$\hat{\pi}\hat{\pi} \equiv \pi^2 \tag{2.147}$$

$$\hat{\Phi}\hat{\Phi} \equiv \Phi + 1 \tag{2.148}$$

$$\hat{2}\hat{2} \equiv 1+1+1+1 \tag{2.149}$$

$$\hat{i}\hat{i} \equiv -1 \quad . \tag{2.150}$$

A dual vector is a map from vectors to real numbers so these four definitions mean that the ontological basis vectors are their own dual vectors. The ontological numbers need to have these properties if they are to retain their everyday numerical properties. However, there are several available product operations to explore.

Hilbert space vectors are their own dual vectors but that does not mean the ontological state vectors live in ordinary Hilbert space. The definition for vectors in $\mathcal{H}' \equiv \mathcal{L}^2$ is

$$\psi(x) \in \mathcal{L}^2 \qquad \Longleftrightarrow \qquad \int_{-\infty}^{\infty} \psi^*(x)\psi(x)\,dx < \infty \quad , \tag{2.151}$$

and that they have an inner product

$$\langle \vartheta | \psi \rangle \equiv \int \vartheta^* \psi \, dx \quad , \tag{2.152}$$

which is symmetric under complex conjugation as

$$\langle \vartheta | \psi \rangle^* = \langle \psi | \vartheta \rangle \quad . \tag{2.153}$$

There are some exceptions for eccentric and/or contrived wavefunctions where $\psi(\infty) \neq 0$ but equation (2.151) is given generally as motivation for $\psi(\infty) = 0$ along the Cauchy C curve around infinity. For any value less than ∞, the wavefunction can be normalized to unity but (without a transfinite framework in place) it is not possible to normalize ψ if the integral in equation (2.151) is equal to ∞.

We want to impose a non-commutative component where equation (2.152) holds but equation (2.153) does not strictly hold in all cases. We will not discuss every property of $\mathcal{H}' \equiv \mathcal{L}^2$, but only those we intend to change systematically with modular complexification. Recalling that $\aleph'$ and Ω' are dual spaces in rigged Hilbert space $\{\aleph', \mathcal{H}', \Omega'\}$, we propose to alter equation (2.153) by introducing a new kind of duality along the lines of

$$\left|\psi;\hat{\Phi};\hat{\Phi}^1\right\rangle^* = \left\langle\psi;\hat{\imath};\hat{\Phi}^2\right| \ , \qquad \text{and} \qquad \left|\psi;\hat{\imath};\hat{\Phi}^2\right\rangle^* = \left\langle\psi;\hat{\Phi};\hat{\Phi}^2\right| \ , \tag{2.154}$$

where the second ket position specifies the manifold among $\{\aleph,\varnothing,\Omega;\mathcal{H}\}$ and the third sets the level of $\aleph$. The dual space of either $\aleph'$ or Ω' will be the next forward iteration of the space so that, following a general process $\mathcal{H}_1 \mapsto \Omega_1 \mapsto \aleph_2 \mapsto \mathcal{H}_2$, the dual of an Ω' state is in the same unit cell (centered on $\varnothing$) but the dual of an $\aleph'$ state is in the next unit cell (using the convention that the iterator on the object specifies the level of $\aleph$ to which it belongs). Using these definitions, we can contradict the property of $\mathcal{L}^2$ shown in equation (2.153). The contradictory statement is

$$\left\langle\vartheta;\hat{\imath};\hat{\Phi}^1\middle|\psi;\hat{\Phi};\hat{\Phi}^1\right\rangle^* = \left\langle\psi;\hat{\imath};\hat{\Phi}^2\middle|\vartheta;\hat{\Phi};\hat{\Phi}^1\right\rangle \ , \tag{2.155}$$

but

$$\left\langle\vartheta;\hat{\imath};\hat{\Phi}^1\middle|\psi;\hat{\Phi};\hat{\Phi}^1\right\rangle^* \neq \left\langle\psi;\hat{\imath};\hat{\Phi}^1\middle|\vartheta;\hat{\Phi};\hat{\Phi}^1\right\rangle \ . \tag{2.156}$$

However, we preserve the behavior of equation (2.153) as a special case like

$$\left\langle\vartheta;\hat{\pi};\hat{\Phi}^1\middle|\psi\ \hat{\pi};\hat{\Phi}^1\right\rangle^* = \left\langle\psi;\hat{\pi};\hat{\Phi}^1\middle|\vartheta;\hat{\pi};\hat{\Phi}^1\right\rangle \ . \tag{2.157}$$

Also note that the inner product of $|X_1,Y_1,\hat{\Phi}^1\rangle$ with $\langle X_2,Y_2,\hat{\Phi}^2|$, as in equation (2.155), should give zero when $\hat{\Phi}^2$ is in the ket. This would show that the amplitude for the level of $\aleph$ to decrease is zero. More precisely, it should give zero up to a hypercomplex remainder which is to say that those vectors are nearly orthogonal or, perhaps, nearly Dirac orthogonal. The concept of transfinite infinity requires us to have sectors for three types of numbers: finite, infinite, and infinitesimal. Therefore, when we compute $\langle\psi;\hat{\imath};\hat{\Phi}^1|\vartheta;\hat{\Phi};\hat{\Phi}^1\rangle$ on the level of $\aleph$ denoted $\hat{\Phi}^1$, the answer should be finite but not infinite, and, therefore, we might require the hypercomplex inner product to be carried out on the level of $\aleph$ of the bra rather than the ket. Then in the usual way, the output of the operation will tell us the probability amplitude for a state ψ starting in Ω_1 to be found later in $\aleph_2$. We intuitively expect that this number should be one because time always goes forward but a full analysis of all the requirements should be carried out to determine the complete structure of the inner product including the nuance related to the δ'_{jk} and $\int d\hat{\gamma}$ discussed in section II.2. The point is that inner product must be defined so that the amplitude for a $\hat{\Phi}^{j+1}$ state to end up in a $\hat{\Phi}^j$ state should be zero or very small. Another standard property of vectors in Hilbert space that we aim to alter is

$$\left\langle\vartheta|\psi\right\rangle \in \mathbb{R} \geq 0 \ . \tag{2.158}$$

The $\sqrt{i}$ channel exists specifically to violate this constraint and there is also an inbuilt violation in the ontological basis itself. If the three real ontological basis vectors are such that

$$\big\langle \psi; \{\hat{\Phi}, \hat{2}, \hat{\pi}\} \big| \psi; \{\hat{\Phi}, \hat{2}, \hat{\pi}\} \big\rangle \geq 0 \quad , \tag{2.159}$$

then a direct consequence will be that

$$\big\langle \psi; \hat{i} \big| \psi; \hat{i} \big\rangle \leq 0 \quad . \tag{2.160}$$

We aim to make a lot of modifications to the existing theory but there is no guarantee that everything about $\hat{M}^3$ can be represented in Dirac notation. Therefore all of the objects presented, following the style of this book, are defined tentatively pending a better follow on analysis. We will return to these important issues in section IV.3.

We have not yet begun to address the products defined by arbitrarily long strings of different lattice vectors that are the addresses of different lattice sites. If we were to write the cosmological address of a hypothetical lattice site such as the $\hat{\pi}$-component of the $\hat{i}$-component of the $\hat{2}$-component of the $\hat{\pi}$-component of the $\hat{\pi}$-component of the $\hat{2}$-component of the $\hat{i}$-component of the $\hat{\pi}$-component on $\hat{\Phi}^{11}$ it could look like

$$\prod_1^N \hat{e}^j = \hat{\pi}\hat{i}\hat{2}\hat{\Phi}^7\hat{\pi}\hat{\pi}\hat{\Phi}\hat{2}\hat{\Phi}^3\hat{i}\hat{\pi} \quad . \tag{2.161}$$

We say "could look like" because the order of the eleven $\hat{\Phi}$ in equation (2.161) does not matter. The 1D topological flatness condition on chiros means that no matter how many $\hat{\Phi}^n$'s appear in a string, we will combine them into one hatted object $\hat{\Phi}^N$, and then we will consider the full nested structure of $\{\hat{2}, \hat{\pi}, \hat{i}\}$ on that single level of $\aleph$. By "nested structure" we mean that the order of the non-$\hat{\Phi}$ objects in the string *does* matter when specifying a lattice address. We point this out to demonstrate that $\hat{\Phi}$ is completely different than $\{\hat{2}, \hat{\pi}, \hat{i}\}$. Here, we consider a single observer's timeline but point out that nested $\hat{\Phi}$ structure would imply divergent timelines. By "nested $\hat{\Phi}$ structure" we mean the case when the order of $\hat{\Phi}$ objects in equation (2.161) also matters but we are not yet to that level of complexity. Instead, we use only $\{\hat{2}, \hat{\pi}, \hat{i}\}$ to build an easily understood 3D lattice on one level of $\aleph$ and then we will consider many such levels. In this book, we have proposed to avoid any reliance on the addresses of such sites in the past by encoding the history on $\hat{\varphi}$, but when we want to compute the amplitudes for the evolutionary futures of the qubit on $\hat{\varphi}$, it is likely we will need to consider future cosmological lattice sites identified in a manner roughly like that shown in equation (2.161).

We have considered pairs of like basis vectors acting on each other outside of the Dirac product but we have not yet considered mixed products of different kinds of ontological basis vectors. We will consider a rule to take the outer dyadic product when the pairs of basis vectors are mixed. The cross product is an outer product that only exists in three dimensions but the dyadic product is another outer product valid for any two basis vectors of consistent dimension. The dyadic tensor $\hat{a}\hat{b}$, also called the dyadic tensor product, is defined by its action on vectors. It is a map from one vector to another vector, or a map from two vectors to numbers via

$$\big(\hat{a}\hat{b}\big)\cdot\hat{c}=\hat{a}\big(\hat{b}\cdot\hat{c}\big) \quad , \qquad \text{and} \qquad \hat{d}\cdot\big(\hat{a}\hat{b}\big)\cdot\hat{c}=\big(\hat{d}\cdot\hat{a}\big)\big(\hat{b}\cdot\hat{c}\big) \quad . \tag{2.162}$$

This is an important property because it allows us to construct both types of objects needed to replicate the Dirac formalism with

$$|\psi\rangle\hat{a} \quad \equiv \quad |\psi;\hat{a}\rangle \quad . \tag{2.163}$$

In this notation, we can write

$$\langle\vartheta;\hat{b}|\psi;\hat{a}\rangle \quad \equiv \quad \langle\vartheta|\psi\rangle\,\hat{a}\hat{b}=\hat{a}\hat{b}\,\langle\vartheta|\psi\rangle \quad . \tag{2.164}$$

The dyadic outer product is not commutative. For example, using the totally arbitrary assignments for $\hat{e}^{\mu}$, and going from the 2D spinors in the previous section to the 4D bispinor form, let

$$\hat{\pi}=\begin{pmatrix}\pi\\0\\0\\0\end{pmatrix} \;,\quad \hat{\Phi}=\begin{pmatrix}0\\\Phi\\0\\0\end{pmatrix} \;,\quad \hat{2}=\begin{pmatrix}0\\0\\2\\0\end{pmatrix} \;,\qquad \text{and} \qquad \hat{i}=\begin{pmatrix}0\\0\\0\\i\end{pmatrix} \quad . \tag{2.165}$$

Then we have objects like

$$\hat{\pi}\hat{\Phi}=\begin{pmatrix}0&\pi\Phi&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \qquad \text{and} \qquad \hat{\Phi}\hat{\pi}=\begin{pmatrix}0&0&0&0\\\pi\Phi&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \quad , \tag{2.166}$$

that demonstrate the non-commutativity with $\hat{\pi}\hat{\Phi}\neq\hat{\Phi}\hat{\pi}$.

Equations (2.165-2.166) demonstrate objects natural to quaternion operations since they are 4-vectors and 4×4 matrices. Now that we have discovered hyperquaternions in conference proceedings from 1984 [18], and even they have not proven sufficient to finally develop a unified field theory, perhaps $\hat{2}$ can act on the hyperquaternions to give an “ultraquaternion” solution using the principles outlined in reference [5]. With a fifth component in the quaternion algebra derived from $\hat{2}x \mapsto x + x$, we might design the additional complexity needed to accommodate O(1,4)$\leftrightarrow$O(2,3) on two quaternion algebras. When using the ontological objects as pseudo-quaternions $\mathbb{H}'$ [5] (or ultraquaternions), they do not technically meet the definition of quaternions $\mathbb{H}$ because the coefficient $-i/4$ of $\hat{\imath}$ is not real and the quaternions all have real coefficients. Also, the identity is not in the ontological basis but it is a member of $\mathbb{H}$ as formulated by Hamilton. Therefore, let us solve this problem about the inconsistency of $\hat{\imath}$ with $\mathbb{H}$ by assigning our ontological pseudo-quaternions as $\{\hat{1}, \hat{\Phi}, \hat{\pi}, \hat{\imath}\}$. Then if we add a fifth component $\hat{2}$ to $\mathbb{H}$, and thereby make two copies of a quaternion algebra (on two co-$\hat{\pi}$s), we can take within each instance separately a rigorously perfect set of quaternions $\{\hat{1}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$. The ordinary quaternions $\{\mathbf{1}, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ have the property that $\mathbf{ijk} = -1$ and we can almost certainly replicate this with $\hat{2}\hat{\pi}\hat{\Phi} = 2\pi\Phi$. We have stated that, in each application of $\hat{M}^3$, the phase in the gauge theory is constrained by the ontological gauge to evolve only across a single cycle of 2π radians so we can use the periodicity to get rid of 2π from $2\pi\Phi$ be resetting the final phase $\Delta = 2\pi$ to the initial phase required by the MCM: $\Delta = 0$. We also know that when the level of $\aleph$ increases objects like $\hat{\Phi}^n$ will become like $\hat{\Phi}^{n-1}$. For the specific case of $\hat{\Phi}^1$ this gives $\hat{\Phi}^0 = \hat{1}$. We can attribute the missing negative sign to $\sqrt{i}$. Therefore, up to all of the details which truly need be nothing more than ordinary normalization, we have engineered $2\pi\Phi \to -1$ in a fairly reasonable and direct manner.

Now we have considered pairs of basis vectors. What shall be the general case for the consideration of triples? This is exactly the problem Hamilton was considering when he invented the quaternions. We want to give rigorous definitions to long strings of unit vectors that define each lattice site but the dyadic only involves pairs. It makes a lot of sense to let the self product of two like basis vectors be the inner product and let the mixed product of two dissimilar basis vectors be the outer product but what would be the interpretation for the long product that keeps track of strings like that in equation (2.161)? We have run into a roadblock on the way to complexification after only considering the second order lattice sites. However, it looks like simple pairs will be enough for now; any long string can be constructed with iterative consideration of pairs that say, "from here, go there." If the Dirac matrices can be combined to make exactly 16 independent objects then it is critically important to note that there are exactly sixteen independent objects when we consider all iterations of dyadics like equation (2.166). In this way, perhaps it will be simpler to use the dyadic product exclusively, even for like pairs, and replace equations (2.147-2.150) with

$$\hat{\pi}\hat{\pi} = \begin{pmatrix} \pi^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.167}$$

$$\hat{\Phi}\hat{\Phi} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Phi^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.168}$$

$$\hat{2}\hat{2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.169}$$

$$\hat{i}\hat{i} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad . \tag{2.170}$$

Indeed, the definitions in equations (2.147-2.150) are more like the dot product than the dyadic product. In any case, we left a lot of unanswered questions when we considered the quaternions in reference [5] wherein the main goal was to raise those questions. We won't propose to treat those questions now but will refer to one of the main takeaways reported after examining the quaternions in reference [5]: the 4D structure is a lot like Dirac's theory (and Einstein's theory.) We have now shown another set of 16 objects which is different than, yet exceedingly similar to, the quaternion argument made in reference [5].

After noting how the dyads look like the Clifford algebra, also consider a general Lorentz transformation in matrix form

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \quad . \tag{2.171}$$

Evidently the ontological dyads can build Lorentz transform matrices as well. Every Lorentz transformation can be written as the sum of a rotation and a boost so even the symmetries of the Lorentz group might be motivated by the ontological basis. We know the Lorentz group has six parameters and it will take six dyadic products $\hat{\pi}\hat{\Phi}$, $\hat{\Phi}\hat{\pi}$, $\hat{\pi}\hat{\pi}$, $\hat{\Phi}\hat{\Phi}$, $\hat{2}\hat{2}$, and $\hat{i}\hat{i}$ to build equation (2.171)'s typical example of velocity in one dimension. More complex Lorentz transform matrices will be built with more than six dyads so they are not the Lorentz parameters themselves, but it is good that the small set of dyads we used to demonstrate complexity can be used to demonstrate the Lorentz transform in special relativity.

II.8 Unification

One of the main things that remains to be clarified in the MCM is the precise form of the maps

$$f^3 \big|\psi;\hat{\pi}\big\rangle \quad \mapsto \quad T_{\mu\nu} \tag{2.172}$$

$$\frac{\Phi}{4}\big|\psi;\hat{2}\big\rangle \quad \mapsto \quad R_{\mu\nu} \tag{2.173}$$

$$\big|\psi;\hat{\Phi}\big\rangle \quad \mapsto \quad Rg_{\mu\nu} \tag{2.174}$$

$$i\big|\psi;\hat{i}\big\rangle \quad \mapsto \quad g_{\mu\nu}\Lambda \ \ , \tag{2.175}$$

through which we propose to unify gravitation and quantum theory. This does not appear to be low hanging fruit and, in any case, we have not yet reached for it. Detractors certainly will claim that we have not yet unified the theories because we have not yet defined the maps with some specific definition that would have to be confirmed or denied by experimentalists. This is not an invalid criticism. To date, work on the MCM has only discovered the mechanism of unification. We have not yet demonstrated it by writing the maps as specific functionals of wavefunctions. However, the discovery of a new mechanism for unification, the only one that exists,[1] is independently a great accomplishment. Now it is possible for other researchers to tinker with the mechanism but that was not possible before it was discovered by this writer in 2012 [12]. If no one ever discovers a mechanism of unification then the theories of gravitation and quanta will never be unified. When the theories are eventually unified according to the specifications of detractors, that unification will unequivocally follow the discovery of the mechanism of the unification! Even then, detractors will not desist from their detractions until the experimentalists confirm or deny any specific functionals proposed. Instead of searching for the specific functionals preferred by everyone who was unable to figure out a mechanism for unification on their own, we have wisely pursued the course of research that led to the maximum action result in section I.3 that is among the main new results presented in this book. The absence of a set of specifically defined functions for some problem in applied physics takes away nothing from our discovery of the maximum action path hidden along the Cauchy C curve around infinity. It takes away nothing from very many other, independently valid, results derived in the MCM and yet detractors continue to insist that one incomplete problem means that all of the problems are incomplete. If experimental confirmation of a theory of grand unification was the only praiseworthy achievement in physics then how do praised physicists exist when then that confirmation does not?

Once the mechanism is discovered, it does become possible to unify the theories of gravitation and quanta. This is clearly a two step process and accomplishing either step is a big deal. Thus, the identification of a mechanism of unification, the first one ever, is clearly an important discovery worthy of praise despite the protestations of people who know nothing about the scientific method or who do know but are blinded by their own hubris. If the specific MCM maps are found in the next 90 years or so then history will show that finding the mechanism of unification was the more difficult problem between finding the mechanism and finding the functionals. In the time after unification is achieved according to the semantic definition preferred by detractors and not the one preferred by this writer which means that unification has already been achieved, the thing that will be first demonstrated to the physics students of the future will be the mechanism of the unification. Physics professors will say, "This mechanism unifies the disparate mathematical frameworks of differential geometry and quantum field theory," and the specific forms of the maps will likely be left as a

[1] The reader should be very careful to note that no one has ever proposed a mechanism such as the MCM mechanism. There should be no insistence to treat the MCM as another blip on a radar screen full of many such blips. It is the only blip on the screen, and it is the only blip that has ever appeared on the screen.

homework problem in advanced coursework.

In addition to the mechanism of unification in maps (2.172-2.175), we have demonstrated so many other successful aspects of the theory that any reasonable person should assume that the maps exist and that the two theories will be "properly unified"[1] through the mechanism exposed in this research program. However, the acceptable demonstration of the mechanism of unification, the one that is something other than, "Look at these three dimensionless constants: 4π, 8π, and 137," is likely to be one specific problem in applied physics like Einstein's calculation[2] of the deflection of light from Mercury. From an applied problem such as this, we would only infer the extension to other problems solved in the MCM leaving detractors in the position to say, "Confirmation in one place isn't confirmation everywhere." Similarly in the non-hypothetical case of the extant *and sufficient* MCM demonstrations, detractors can always say, "You didn't do the thing you didn't do yet," and the political agents will maintain the belief that it is a reasonable criticism to accuse this writer of not singlehandedly doing all of physics.

The benefit of dealing with Nature at the fundamental level, and not the applied level, is that we may infer the extension of our fundamental results to all applied problems without having to choose one specifically. If someone can choose an applied problem that shows our extension would fail then they are encouraged to do so, but that work will likely not be stimulated until the original work by this writer is recognized. When the MCM's other myriad, definitive, irrefutable results exist, it is highly irregular that funded researchers have not taken up the search for functionals while we remain focused on more important issues dealing with the fundamentals of infinite complexity (such as those in reference [8].) Thus are the prerogatives of those researchers and if they choose not do it first then we will get to it when we get to it. Since it will be impossible to know if any derived maps are correct without an experimental confirmation, all detractors that currently detract are most likely to continue to do so even if we did propose a specific set of functionals. If the physics community at large has pretended to ignore our other experimental predictions and other mathematical results that are already fully complete then it is likely that specific analytical relationships of the form

$$T_{\mu\nu} = f_T[\psi;\hat{\pi}] \tag{2.176}$$

$$R_{\mu\nu} = f_R[\psi;\hat{2}] \tag{2.177}$$

$$Rg_{\mu\nu} = f_{Rg}[\psi;\hat{\Phi}] \tag{2.178}$$

$$g_{\mu\nu}\Lambda = f_{g\Lambda}[\psi;\hat{i}] \quad , \tag{2.179}$$

[1]Here, we put "properly unified" in quotes to differentiate proper unification from the type of demonstration that would have been made if an acolyte of one of the professional physics societies had somehow found unification inside the box of physics that those societies seem to think is the only allowable, or valid, methodology.

[2]The reader must note that this was not Einstein's calculation but rather was the calculation of Einstein *et al.* and this writer has no *et al.* because he has been shunned rather than embraced by the pool of potential *et al.*s.

would also be ignored.[1] Recall that we have solved very many problems besides the problem of unification, and solving one more is unlikely the change the sentiments of the cretins in the anti-science conspiracy. The primary of example of that to which we refer is the non-publication of the full spin analysis for the Higgslike particle [28]. In reference [11], we derived the MCM's best and most specific experimental prediction for new spin-1 particles. Why has the full spin analysis for the Higgslike particle not been carried out and published if not to spite this writer by denying him the experimental confirmation that detractors will always refer to regardless of how specific our theoretical results eventually become?

The aspects of the theory that provide such strong evidence of its correctness include the production of three numbers which are unambiguously the most important dimensionless constants in physics. The leading coefficient (leading because $\hat{\pi}$ is the primary component) of the ontological resolution of the identity

$$\hat{1} = \frac{1}{4\pi}\,\hat{\pi} - \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{\imath} \quad , \tag{2.180}$$

is the dimensionless electromagnetic coupling constant $1/4\pi$. Electromagnetism is a cornerstone of the physics that we more or less understand, and we have taken $\hat{\pi}$ as the sector of known physics in $\mathcal{H}$ before adding $\{\hat{2}, \hat{\Phi}, \hat{\imath}\}$ to construct an ontological basis for new physics in the hypercosmos.[2] It is a good demonstration of the MCM's consistent, non-contradicting validity that the coefficient associated with $\hat{\pi}$ is the dimensionless electromagnetic coupling constant. We have uncovered consistency in every corner of the MCM but never once have we uncovered any inconsistency, and none has been demonstrated by any detractor. Secondly, we have generated the dimensionless coupling constant from Einstein's equation 8π and we will discuss its origin in the next chapter. Thirdly, we arrive at the first important dimensionless constant identified in the MCM: the fine structure constant $\alpha_{MCM}^{-1} = 2\pi + (\Phi\pi)^3$. This differs from the currently accepted value α_{QED} by about 0.4% and it is totally obvious that quantum theory can be reformulated with α_{MCM} by moving the 0.4% discrepancy into the quantum uncertainty found in some other corner of the theory.

Another very strong indicator for the overwhelming likelihood of the existence of a set of functionals that will clarify and define maps (2.172-2.175) with specific, correct, useful definitions is the MCM's modified model of particle physics (figure 28.) In the standard model of particle physics, there are four fundamental gaugeons plus the spinless scalar Higgs boson. In the MCM scheme, there are no fundamental scalar particles and we predict two spin-1 gaugeons $G^{\pm}$ and ζ^0 where the standard model only predicts the spin-0 scalar particle H^0. At a very fundamental level, it is likely that we have been able to make so much progress through diagrammatic representations alone because the conformal invariance of the MCM gauge theory is not broken by the existence of a fundamental scalar particle. The MCM breaks symmetry in two ways. The first is that "infinitely large" is not balanced with "minus infinitely large." It is balanced instead with infinitesimals at the origin. The

[1] This statement is highly conditional. They would only be ignored if they were correct. If they were demonstrably incorrect, as they likely would be if the endeavors to find them were undertaken by this writer prematurely, then they would likely not be ignored. In that case detractors would probably pounce upon that erroneous result to rhetorically assault the other MCM results which are logically unassailable.

[2] The hypercosmological lattice structure is not unlike the concept of an akashic record. Measurements get made at $\hat{\pi}$-sites and there they remain when the observer makes more observations later, at other $\hat{\pi}$-sites.

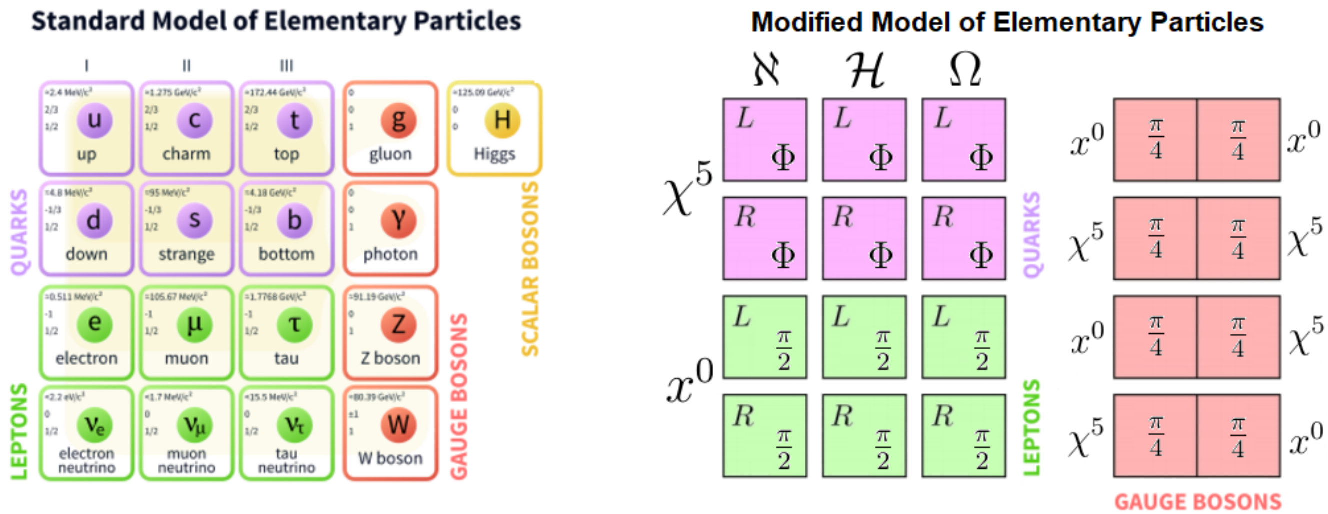


Figure 28: The 4×4 symmetry of the standard model is broken by the spin-0 H scalar boson but the 4×4 symmetry of the modified model is completed with the spin-1 G and ζ gauge bosons. Notice how the labeling, when taken by column, up to an extra Φ in each generation of matter particle, gives $\alpha_{MCM} = (\Phi\pi)^3 + 2\pi$. See reference [11] for details regarding the modified model of elementary particles.

second symmetry breaking mechanism is that chronological time can be computed in any direction but chirological time can only be computed in the forward direction. This is closely related to (if not uniquely derived from) the previous symmetry breaking mechanism about an imbalance in infinitude. All of the pieces of the MCM are conformally invariant in the gauge theory but if we tried to include H^0 then there would be a breakdown in the global conformal invariance, and that would impede the drive to do physics with diagrams.

As an aside about particle physics, we mention the neutrino mass. Neutrino masses violate the predictions of the standard model but they have never been measured directly. The existence of massive neutrinos is inferred from neutrino flavor oscillation. Oscillation means that a neutrino of one flavor at a source can show up in a detector as any of the three flavors: electron, muon, or tau. The argument against the standard model says that if neutrinos were massless then they would move at the speed of light along the null interval in spacetime. When $ds^2 = 0$ for every neutrino trajectory, it means that neutrinos do not experience time and, therefore, they should not be able to change flavors. Change is uniquely a property associated with time; timeless things don't change. However, now that we have introduced a second kind of time into the MCM we should re-evaluate whether or not flavor oscillation strictly implies mass. It might be that massless neutrinos do move on the null interval as predicted by the standard model but there is some oscillation allowed due to the passage of chirological time which is orthogonal to the spacetime of only spacelike, timelike, and null intervals. Massless neutrinos are predicted by the standard model when it says they should have strictly left chirality and perhaps we can associate that with strictly increasing chirological time.

The symmetry of the MCM particle scheme, with respect to the asymmetry of the standard scheme, makes a good case for the overall correctness of the MCM but it still remains for the political agents at the LHC to report the spin of the particle they discovered in 2012. The official source of what is and is not known in particle physics is the Particle Data Group. They publish a yearly volume and the entry for the Higgs boson in the 2016 volume [29] contains the following wherein the reader should note the deliberate uncertainty in the choice of words and how that tone contrasts the ordinary style of concise language one finds in PDG's publications.

> "H^0 refers to the signal that has been discovered in the Higgs searches. Whereas the observed signal is labeled as a spin 0 particle and called a Higgs Boson, the detailed properties of H^0 and its role in the context of electroweak symmetry breaking need to be further clarified."

Indeed they do need to be clarified because no one knows if the particle announced by CERN four years prior to the publication of reference [29] is the spin-0 particle predicted by Higgs and several of his contemporaries or if it is among the pair of spin-1 particles predicted via the MCM. If they report that the particle has spin-1 then that will make an even stronger case for the existence of the specific, physical functionals that will replace the arrows in maps (2.172-2.175). There are very many things that have worked out surprisingly well in the MCM such as the new mechanism for bulk-boundary correspondence [3] and, altogether, there is an irrefutable likelihood that specific physical forms of the maps do exist and can be written down. There have been so many such discoveries in this research program, each making a small (or huge) argument for the overall physical correctness of the MCM, that altogether they do form a convincing argument. However, the fact remains that we still have not identified the specific functionals of wavefunctions or their inverses and that is on the to-do list.

To find the requisite functionals, we should look for a connection between the quantum state vectors and the general relativistic equation of state. Recall that Einstein's equation alone, much like equations (2.176-2.179), does not determine physics. Einstein's equation conserves the energy of a universe but we also need to know the universe's equation of state before we can do physics properly. There are very many common equations of state for a model universe and reference [30] contains a comprehensive survey of the most common and useful ones. One thing that all general relativistic equations of state have in common is a pair of thermodynamics parameters: the pressure p and the energy density ρ. Since any two parameters are defined by their ratio w, we can expect that it will be productive to study ontological equations of state wherein the ratio of these two parameters is the golden ratio. Perhaps we can set them relative to each other as $w=\Phi$ in Σ^+ and $w=\varphi$ in Σ^- to set a non-equilibrium condition across $\mathcal{H}$ or $\varnothing$ which leads to the laws of thermodynamics: entropy increases and time goes forward. Davies has discovered the golden ratio in an independent study of black hole thermodynamics [31]. He published a paper titled "Thermodynamic Phase Transitions of Kerr–Newman Black Holes in de Sitter Space" so here we have the following MCM elements already bundled together in 1989: the golden ratio, Kerr–Newman black holes, phase transitions, and de Sitter space. To repeat what we have stated about the MCM already, we expect that the topological compartmentalization of the MCM unit cell is exactly that of the Kerr–Newman black hole [13], and obviously half of the unit cell is de Sitter space. The MCM adds AdS to the list of things in the title of reference [31], and thereby we make it possible to incorporate AdS/CFT effects beyond the thermodynamics considered by Davies.

Regarding maps (2.172-2.175), we have made them such that they point to both sides of Einstein's equation but we could make the four maps on $|\psi;\{\hat{i},\hat{\Phi},\hat{2},\hat{\pi}\}\rangle$ point to the 16 places in the matrix representation of the Einstein tensor $G_{\mu\nu}$, or the stress energy tensor $T_{\mu\nu}$, or there are very many possible permutations.[1] Since one of $T_{\mu\nu}$ or $G_{\mu\nu}$ determines the other through Einstein's equation (given $g_{\mu\nu}\Lambda$), there is no need for us to send the qubit into both of $T_{\mu\nu}$ and $G_{\mu\nu}$. However, if we do not send the information to both sides of Einstein's equation then we lose the unique connection to the all-important coefficient 8π. Another thing to consider is that a single quantum should not determine the state of the entire universe. In that case, we are drawn to maps of the form

$$f^3|\psi;\hat{\pi}\rangle \quad \mapsto \quad T^{\psi}_{\mu\nu} \ , \qquad \text{where} \qquad T_{\mu\nu}=T^0_{\mu\nu}+T^{\psi}_{\mu\nu} \ , \tag{2.181}$$

or

$$i|\psi;\hat{\Phi}\rangle \quad \mapsto \quad h_{\mu\nu}\big[\psi(x^\mu_+)\big] \ , \qquad \text{where} \qquad G_{\mu\nu}=G^0_{\mu\nu}+h_{\mu\nu}\big[\psi(x^\mu_+)\big] \ . \tag{2.182}$$

By computing only forward chirological time starting from some present moment $\mathcal{H}$, we are forced to compute two separate trajectories in Σ^+ and Σ^-. These are the legs of $\hat{M}^3$ $\mathcal{H}\mapsto\Omega$ and $\aleph\mapsto\mathcal{H}$ respectively and there is also the operation that changes the level of $\aleph$ where $\Omega\mapsto\aleph$. The three pieces of $\hat{M}^3$ are very well matched to the possible polarization states of a spin-1 particle. Classical physics only computes the leg $\aleph\mapsto\mathcal{H}$ and the Higgs boson only has one possible polarization state: no polarization. By adding the additional topological elements associated with $\hat{M}^3$, we can preserve the Higgs mechanism but allow the polarization states $\{1,0,-1\}$ of a spin-1 particle. There is a natural connection between $+1$ polarization in Σ^+, 0 polarization in $\varnothing$, and -1 polarization in Σ^-. Based on figure 28, we have predicted a pair of spin-1 particles [11] but we did not start with the Higgs boson and then generalize it the MCM. We modeled the non-Higgs particles and then saw that there should be two more with spin-1. We have predicted these particles $G^\pm$ and ζ^0 along a line of reasoning independent of what is now called the Englert–Brout–Higgs–Guralnik–Hagen–Kibble mechanism.

In reference [32], we extensively discuss the prediction for spin-1 particles put forward in reference [11]. The main criticism of the prediction for spin-1 is the Landau–Yang theorem which says spin-1 particles cannot decay to a pair of photons but that theorem should be reinterpreted with the tools of hypercomplex analysis. We have shown that Bell's famous theorem has a completely different interpretation in $^\star\mathbb{C}$, and the Landau–Yang theorem may as well. In video reference [33], Arkani-Hamed, who knows about the Landau–Yang theorem

[1] Under ordinary conditions, this writer's fellow researchers would jump at the chance put their names on arbitrarily long catalogs of the implications of finitely many intuitive guesses at specific functionals. The reader is invited to recall that the quantum Hall effect was finally solved by guessing and Laughlin shared the 1998 Nobel Prize in physics for his excellent and well-reasoned guess.

as well as any detractor of the MCM, makes some plain statements that spin-1 is not ruled out.

To finish this chapter, we will compare Higgs' famous result [34] to the MCM maps. The result that earned Higgs the Nobel Prize for Physics 2013, which he shared with Brout, first appeared in reference [34]. On the final (second) page of that paper, one sees the term "70-plet" in the title of the article that follows Higgs', and detractors who detracted from our employment of the word multiplex should take note of the year of publication of that article: 1964. Higgs write the following in reference [34].

> "The Goldstone theorem [*sic*] fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. [*sic*] The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone himself: Two real scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density
>
> $$\mathcal{L} = -\frac{1}{2}\big(\nabla\varphi_1\big)^2 - \frac{1}{2}\big(\nabla\varphi_2\big)^2 - V(\varphi_1^2+\varphi_2^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad , \tag{2.183}$$
>
> where
>
> $$\nabla_\mu\varphi_1 = \partial_\mu\varphi_1 - eA_\mu\varphi_2 \tag{2.184}$$
> $$\nabla_\mu\varphi_2 = \partial_\mu\varphi_2 - eA_\mu\varphi_1 \tag{2.185}$$
> $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\nu \quad , \tag{2.186}$$
>
> e is a dimensionless coupling constant, and the metric is taken as $-+++$. $\mathcal{L}$ is invariant under simultaneous gauge transformations of the first type on $\varphi_1 \pm i\varphi_2$ and of the second kind on A_μ. Let us suppose that $V'(\varphi_0^2)=0$, $V''(\varphi_0^2)>0$; then spontaneous breakdown of U(1) symmetry occurs. Consider the equations [*sic*] governing the propagation of small oscillations about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:
>
> $$\partial^\mu\left\{\partial_\mu\big(\Delta\varphi_1\big) - e\varphi_0A_\mu\right\} = 0 \tag{2.187}$$
> $$\left\{\partial^2 - 4\varphi_0^2V''(\varphi_0^2)\right\}\big(\Delta\varphi_2\big) = 0 \tag{2.188}$$
> $$\partial_\nu F^{\mu\nu} = e\varphi_0\left\{\partial^\mu\big(\Delta\varphi_1\big) - e\varphi_0A_\mu\right\} \tag{2.189}$$
>
> [*Equation (2.188)*] describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; [*equations (2.187) and (2.189)*] may be transformed, by the introduction of new variables
>
> $$B_\mu = A_\mu - \big(e\varphi_0\big)^{-1}\partial_\mu\big(\Delta\varphi\big) \tag{2.190}$$
> $$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad , \tag{2.191}$$

into the form

$$\partial_\mu B^\mu = 0 \quad , \qquad \partial_\nu G^{\mu\nu} + e^2 \varphi_0^2 B^\mu = 0 \quad . \tag{2.192}$$

[*Equation (2.192)*] describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e = 0$) the situation is quite different: [*equations (2.187) and (2.189)*] describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of [*equation (2.189)*] is just the linear approximation to the conserved current: It is linear in the vector potential, gauge invariance being maintained by the presence of the gradient term."

The Goldstone theorem says bosons should appear whenever symmetry is broken and the complexity of fine nuance underlying Higgs' short paper is too much to go into here, and not relevant to the topic of this book. We have included this excerpt to compare the MCM to what Higgs did and was lauded for across decades without any experimental confirmation, or even a calculation to say what the mass of the particle should be. Higgs assumes a Lagrangian and, with sufficient motivation, we have assumed another equation

$$\omega^3 \big|\psi;\hat{\pi}\big\rangle = \pi\Phi^2 \big|\psi;\hat{\pi}\big\rangle \quad . \tag{2.193}$$

From Higgs' Lagrangian, he is able to derive equations (2.187-2.189) and from equation (2.193) we are able to write

$$8\pi^3 f^3 \big|\psi;\hat{\pi}\big\rangle = \pi\Phi\big|\psi;\hat{\pi}\big\rangle + \pi\big|\psi;\hat{\pi}\big\rangle \tag{2.194}$$

$$8\pi f^3 \big|\psi;\hat{\pi}\big\rangle = \big|\psi;\hat{\Phi}\big\rangle - i\big|\psi;\hat{i}\big\rangle \tag{2.195}$$

$$= \frac{\Phi}{4}\big|\psi;\hat{2}\big\rangle + \frac{1}{2}\big|\psi;\hat{\Phi}\big\rangle - i\big|\psi;\hat{i}\big\rangle \quad . \tag{2.196}$$

Higgs' next step following equations (2.187-2.189) is to introduce new variables B_μ and $G_{\mu\nu}$, as in equations (2.190-2.191). We have done the same thing introducing new variables $T_{\mu\nu}$, $R_{\mu\nu}$, $Rg_{\mu\nu}$, and $g_{\mu\nu}\Lambda$ as

$$f^3 \big|\psi;\hat{\pi}\big\rangle \quad \mapsto \quad T_{\mu\nu} \tag{2.197}$$

$$\frac{\Phi}{4}\big|\psi;\hat{2}\big\rangle \quad \mapsto \quad R_{\mu\nu} \tag{2.198}$$

$$-\frac{1}{2}\big|\psi;\hat{\Phi}\big\rangle \quad \mapsto \quad Rg_{\mu\nu} \tag{2.199}$$

$$-i\big|\psi;\hat{i}\big\rangle \quad \mapsto \quad g_{\mu\nu}\Lambda \quad . \tag{2.200}$$

Higgs' derivation concludes by using his new variables to write the equations of vector waves in equation (2.192). We have done the same thing using our new variables to write maps (2.197-2.200) as Einstein's equation

$$8\pi T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + g_{\mu\nu}\Lambda \quad . \tag{2.201}$$

Higgs' result is slightly more formal than ours because he introduces the new variables with equations and we have only introduced them with maps. However, our result is more complicated than Higgs' because he is using Einstein notation throughout his paper but we intermingle Einstein notation, Dirac notation, and also the new MCM notation related to the ontological basis. Since the left hand side of maps (2.197-2.200) do not have tensor indices on them it would be impossible to use equations as Higgs has done. Instead we have used the "maps to" notation $\mapsto$ to call attention to the new channel discovered by this writer and reported in 2012 [12]. Therefore detractors institute a false equivalence when they agree that Higgs' result is praiseworthy but ours is not. Most importantly, where Higgs was not able to derive the mass of the Higgs boson, its coupling constant, we have exactly derived the coupling constant of Einstein's equation. Therefore, when detractors say, "Tooker didn't write equations," one should point out, "Higgs didn't write the coupling constant and no one complained about *that*."

For further reading regarding the Englert–Brout–Higgs–Guralnik–Hagen–Kibble symmetry breaking mechanism, and its clear relationship to the MCM principles such as ∂^3, the reader is referred to reference [35] wherein one finds an exceptionally well written and accessible technical development.

And he shall speak great words against the most High, and shall wear out the saints of the most High, and think to change times and laws: and they shall be given into his hand until a time $[x^0]$ and times $[x^0, \chi^5]$ and the dividing of time $[\chi^5_+, \chi^5_\varnothing, \chi^5_-]$.
– Daniel 7:25

III Maximal Symmetry

The first section in this chapter restates the MCM hypothesis and points out a few obvious criticisms which are mitigated with arguments in later sections. The second section recounts some drama related to the development and publication status of the documents that characterize this research program. Section three contains a few orphaned comments and in section five we make a definition for a multiplex. Sections four and six are dedicated to addressing the criticisms from section one. In section four, we propose to create small perturbations $h_{\mu\nu}$ on the Minkowski metric by taking the difference of large perturbations in $\Sigma^\pm$. Additionally, in section six we strengthen and clarify the loose definition for a 5D ontological wavefunction $\Psi = 0$ that was reported in reference [10]. In section seven, we discuss the properties of eponymous maximally symmetric spacetimes. In section eight, we treat the geodesics of the MCM unit cell but do not calculate them. The main result of this chapter is in section nine where we show how dark energy and expanding space are both expected properties of the piecewise metric used in the MCM. Section ten is a brief summary of relevant aspects of the advanced and retarded electromagnetic potentials.

III.1 The MCM Hypothesis

The first step in building the MCM is to assume a hypothesis that momentum is always conserved and, thereby, pursue a different model than most big bang theorists. Taking a shortcut to something that can be analyzed specifically, we have hypothesized that the third derivative with respect to chronological time should be equal to the third derivative with respect to chirological time.

$$\hat{M}^3\big|\psi;\hat{\pi}\big\rangle = \hat{M}^3\big|\psi;\hat{\pi}\big\rangle \qquad \Longrightarrow \qquad \partial_0^3\big|\psi;\hat{\pi}\big\rangle := \partial_5^3\big|\psi;\hat{\pi}\big\rangle \quad . \tag{3.1}$$

The operator that appeared as ∂_4 in earlier work now appears as ∂_5 because we have changed the counting convention on the Latin indices A. The next step is to choose ψ so that the hypothesis becomes

$$\omega^3\big|\psi;\hat{\pi}\big\rangle = i\pi\Phi^2|\psi;\hat{\pi}\big\rangle \quad , \tag{3.2}$$

and we will say a lot about how to accomplish that in this chapter (after first referring the reader to reference [10] for full details), and we will revisit this important issue in section IV.3. We reduce equation (3.2) which is quadratic in Φ to a linear one with $\Phi^2 = \Phi + 1$ to achieve

$$\omega^3\big|\psi;\hat{\pi}\big\rangle = i\pi\Phi|\psi;\hat{\pi}\big\rangle + i\pi|\psi;\hat{\pi}\big\rangle \quad . \tag{3.3}$$

This gives some general idea about how one might quantize third order equations that are ordinarily taken as unquantizable: we can take an equation of any order in Φ and reduce the order through an operation like $\Phi^3 \to \Phi^2 + \Phi \to 2\Phi + 1$. We have still not identified all the problems we hope to solve through some method of achieving equations in a given order. However, since the theory of infinite complexity uses the advanced potential that is third order in ∂_t, we will almost certainly be required to quantize a third order potential as part of the quantization process for the classical MCM Hamiltonian.

After writing equation (3.3), we use $\omega = 2\pi f$, insert the identities $\hat{\pi} = -\varphi\pi\hat{\Phi}$ and $\hat{\pi} = -i\pi\hat{i}$, and then shuffle the hats to write

$$8\pi^3 f^3\big|\psi;\hat{\pi}\big\rangle = i\pi^2|\psi;\hat{\Phi}\big\rangle + \pi^2|\psi;\hat{i}\big\rangle \quad . \tag{3.4}$$

Then we normalize by π^2 and say that

$$8\pi f^3\big|\psi;\hat{\pi}\big\rangle = i|\psi;\hat{\Phi}\big\rangle + |\psi;\hat{i}\big\rangle \quad , \tag{3.5}$$

is Einstein's equation via

$$f^3\big|\psi;\hat{\pi}\big\rangle \quad \mapsto \quad T_{\mu\nu} := \psi\big(x^\mu\big) \tag{3.6}$$

$$i\big|\psi;\hat{\Phi}\big\rangle \quad \mapsto \quad G_{\mu\nu} := \psi\big(x^\mu_+\big) \tag{3.7}$$

$$\big|\psi;\hat{i}\big\rangle \quad \mapsto \quad g_{\mu\nu}\Lambda := \psi\big(x^\mu_-\big) \quad , \tag{3.8}$$

and we continue to ignore some minus signs. The far right column in maps (3.6-3.8) shows another new trick introduced for this research and, again, this is something we will return to in section IV.3. Even in exotic studies of non-standard quantum theory where the Gel'fand triple is considered, the position space representations of the vectors in $\{\aleph', \mathcal{H}', \Omega'\}$ are taken in the coordinates of $\mathcal{H}$ but in the MCM we take their position space representations in the coordinates of the similarly named manifolds $\{\aleph, \mathcal{H}, \Omega\}$. We have already shown in the previous chapter how to accommodate $\hat{2}$ in the scheme of maps (3.6-3.8). We do it with the decomposed Einstein tensor and the remaining definition to compliment the above will be

$$\big|\psi;\hat{2}\big\rangle \quad \equiv \quad \psi(x^\mu_\varnothing) \quad . \tag{3.9}$$

For the purposes of discussing the logical development of the hypothesis, we will use the original convention given by maps (3.6-3.8) without including $\hat{2}$. There are so many possible ways to insert $\hat{2}$, it is unlikely that we would guess the correct one at this point but we will briefly develop an ansatz.

We say equation (3.5) is Einstein's equation, but what does that mean? Here, we will be more careful with our notation because there are a few things listed below that are not perfect in the sense of the = symbol that appears in equation (3.5). However, that takes away little from the magnitude of the MCM discoveries because we can swap in the := symbol, and it is clear that a previously undocumented mechanism has been documented and now appears in the literature. Rather than a strict equality, we can more freely use the := symbol which means "is defined according to." In section III.5, we will give a formal definition for the **multiplex** although we will also continue to use the word informally. Where the = sign denotes an equality, the := sign shall denote a multiplex.

Before embarking in section III.2 upon an interesting, maximally symmetric aside about the history of the Earth in the years around 2012 A.D., we will list the most obvious criticisms of the hypothesis, and then after discussing the history, we will return to the problems pointing out how the multiplectic formalism is superior, in many ways, to the ordinary ideas of formalism for quantized determinism with arbitrary spinors.

The problems with the hypothesis as defined by equation (3.2) are as follows.

1. Equation (3.2) implies that only one frequency $\omega = \sqrt[3]{i\pi\Phi^2}$ is allowed.

2. When everything is proportional to ψ, as in equations (3.1-3.5), there is no room for complexity-generating double orthogonality in the maps (3.6-3.8). To build double orthogonality we must first start with single orthogonality but ψ is not orthogonal to itself. When we use maps (3.6-3.8) to convert to tensors, all of the tensors will be linearly dependent on each other through ψ. In general, to model an arbitrary cosmos with perturbations, there need to be at least two independent tensors in Einstein's equation.

3. In reference [10], we proposed to make the left and right sides of equation (3.1) independent by switching to a law of the form $\partial_0^3|\psi(x^\mu)\rangle = \partial_5^3|\Psi(\chi^A)\rangle$, but then we have a problem of how a 4D object on one side might be equal to a 5D object on the other side. Recall from reference [9] that not only is Ψ a function of the 5D coordinates, it is a five component vector where ψ is only a four component vector.

4. Even if we solve problem two and accomplish a workaround for the dimensionality issue in problem three, we still have the same problem of linear dependence as in problem two, only to a lesser degree. When we decompose the Einstein tensor into the Ricci tensor and the metric with

$$G_{\mu\nu} \quad \equiv \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad , \tag{3.10}$$

both objects are, by construction, defined by Ψ but we know that fully dynamical spacetime requires $R_{\mu\nu}$ and $g_{\mu\nu}$ to be, at least sometimes, independent.

Since we have introduced the concept of double orthogonality, we could try to solve the problem of the uniform dependence on ψ strictly in the double orthogonal channel for ontological basis vectors (without introducing Ψ as in problem three), but, even then, the Ricci tensor and the metric tensor are neither single nor double orthogonal because of problem four. In equation (3.10), $R_{\mu\nu}$ and $g_{\mu\nu}$ both live inside $i|\Psi;\hat{\Phi}\rangle$ so, even in the second orthogonal channel, they are the same: they are both proportional to Ψ and they both live on $\hat{\Phi}$. This is problematic because there is no infinite complexity; there are just four extra complexes $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ so if we try to get single and double orthogonality from just four numbers we will be in for a burdensome and likely fruitless task (although we develop a novel scheme in this regard throughout chapter four.) Even if we use $\hat{2}$ as in equations (2.61-2.62), which were

$$\frac{1}{2}\big|\Psi;\hat{\Phi}\big\rangle \quad \mapsto \quad R_{\mu\nu} \quad , \qquad \text{and} \qquad \frac{\Phi}{4}\big|\Psi;\hat{2}\big\rangle \quad \mapsto \quad \frac{1}{2} R g_{\mu\nu} \quad , \tag{3.11}$$

and

$$\frac{1}{2}\big|\Psi;\hat{\Phi}\big\rangle \quad \mapsto \quad \frac{1}{2} R g_{\mu\nu} \quad , \qquad \text{and} \qquad \frac{\Phi}{4}\big|\Psi;\hat{2}\big\rangle \quad \mapsto \quad R_{\mu\nu} \quad , \tag{3.12}$$

that will only achieve single orthogonality, albeit in the second orthogonal channel. The ontological basis is hardly infinite; we want to generate complexity by applying the basis to the already infinite dimensional Hilbert space of ψ position eigenstates and, presumably, Ψ as well.

If we wanted to, we could even trace the primary issue regarding linear dependence all the way back to equation (3.1) to say that there is no room for any complexity at all: $\psi = \psi$ and there is only one frequency. However, we will not go that far. We will guarantee some modicum of complexity simply by defining the ∂_0^3 and operators to require a representation of ψ in the chronological and chirological coordinates separately. Recall from chapter one that there is some complex discrepancy between the x^μ and χ^A coordinates due to technical nuance in the dual tangent space.

III.2 Historical Context

In this chapter, we will show how none of problems one through four are actually problems at all. Since we are writing a short book rather than a short paper, with the intention of using these extra pages to review the basics of that which should have been immediately obvious to any subject matter experts with half the entire field committed to memory, we will condense all previous := workarounds into formalism that is rigorously correct using new multiplectic formalism. By "immediately obvious," we mean to derisively imply that the more than seven years it has been since people started passing around MCM literature is far too long and the so-called self-correcting mechanism in science is broken as fuck. This section is dedicated to evidence of the brokenness (non-existence) of the alleged self-correcting mechanism in science and shortly we will return to the problems listed in the previous section.

Reference [2] contains the first written description of the MCM. It was rejected by arXiv in September 2009, and that manuscript is the likely basis for the articles titled "Is the Universe Inside a Black Hole?" that Nikodem Poplawski has been successfully publishing in popular media since 2010. The MCM phrase "inverse radial spaghettification" [7] is a fancy way to say that the universe is inside a black hole and now, in newer research, we have gone on to show that the observer resides on a singularity at the origin of coordinates marking each level of $\aleph$.[1] It is commonly understood that singularities mark the center of black holes so universe-in-a-black-hole is very much a facet of the MCM. We suggest that Poplawski began writing these articles after he was inspired to do so by the original MCM manuscript [2] which he obtained *somehow*.

Similarly at the end of September 2009, Ashtekar, Campiglia, and Henderson published reference [36] wherein the first citation is to the Feynman paper [6] that we have considered in chapter one. This is interesting because Ashtekar had not been citing Feynman's war-era papers from 70 years ago but then he did do so, immediately after this writer distributed reference [2]. Reference [2] begins with a quote taken from one of Feynman's less famous war era papers where he makes a comment about the time ordering of events not being as important as the way they are encoded in his formalism. A main result of reference [2] was an alternative interpretation for the method described by Feynman [6].[2] This is the method we have proposed to modify by the inclusion of the maximum action path. arXiv lists the submission date on Ashtekar *et al.*'s paper, reference [36], as about one or two weeks after an anonymous and/or unscrupulous reviewer at arXiv rejected reference [2].[3] Since we multiply cited LQC[4] within reference [2], a theory whose bottom-liners include Ashtekar,[5] it is likely that the arXiv reviewer, if it was not Ashtekar himself, sent the manuscript to Ashtekar. Additionally, Ashtekar may have obtained the manuscript not

[1] In fact, even by the time arXiv rejected reference [7] in 2011, we had already moved the observer from an arbitrary moment to the moment at the apex of a quantum geometric bounce. Such bounces only occur on the interiors of the event horizons of black holes. When the entire universe bounces *à la* LQC, that is simply occurring inside a super-massive black hole which contains (or is) the entire universe. The MCM goes far beyond LQC when it identifies a dark energy candidate in the mechanism.

[2] This method is the one in the long excerpt from Feynman's reference [6] which appears in section I.3.

[3] Unfortunately, we have no record of the date of the original submission of reference [2] to arXiv, but it was probably around the 15th of September, 2009. After presenting a result to the 2009 meeting of the IceCube Collaboration at Humboldt University in Berlin, this writer enjoyed the gracious patience of the Germans for a few days and then returned to Atlanta to upload the manuscript to arXiv on a Monday or Tuesday expecting it to appear online either Tuesday or Wednesday. At the time of the publication of this book, that was about 3,000 days ago.

[4] LQC and LQG were not cited directly in 2009 but instead we used the terms "bouncing" and "the repulsive force of quantum geometry" which were taken from Ashtekar's 2009 LQC talk at Georgia Tech.

[5] The bottom-liners also include Bojowald who declared LQC "dead" in 2013. See video reference [37].

through arXiv but through another channel. Just weeks before Ashtekar *et al.* published reference [36], this writer had distributed copies of reference [2] in the newly opened Center for Relativistic Astrophysics (CRA) whose founding faculty include two former colleagues of Ashtekar's: Pablo Laguna[1] and Dierdre Shoemaker. The purpose of the email distribution was to advertise that this writer would give a talk on the MCM in the CRA that week. Shoemaker, who had been working side by side with Ashtekar in Pennsylvania just a year earlier, was in attendance but she was most intently on her phone throughout the talk,[2] almost intentionally projecting disinterest, or *disrespect*, and is unlikely to have made any effort to help this writer disseminate his research.

The key point in all of this is that somehow reference [2] was deemed not good enough even to be uploaded to arXiv as a preprint. However, it seems to have been good enough to prompt an immediate response paper [36] from leading names in the field. Usually eliciting a response paper at all is considered a high achievement in theoretical physics and an immediate response from a leader in the field (Ashtekar) is high praise indeed. As a counterexample, consider that most papers passing the "very high," "very meaningful," "critically important" bar of peer-review go on to be completely ignored and accumulate a layer of dust serving as a reminder that the paper did, at one point, pass peer-review meaning that the publishing cartel bestowed a cookie upon the authors who can all add the cookie crumbles to their C.V.s... which mean nothing weighed against the merit of the research that appeared in the publication. The cartel's cookie crumbles have become overly important in the modern era where the merit of the research in question is too often non-existent or not significant.

Despite science's alleged self-correcting mechanism, the exact dynamic from 2009 unfolded again in 2011. Once again, arXiv rejected another manuscript, reference [7], based on some unpublished set of censorship guidelines.[3] It seems that after this newer manuscript made the backchannel rounds, negative frequency resonant radiation was immediately discovered [38] and a team at USC immediately built a working quantum computer [39]. Note that since frequency is inverse time, negative frequency resonant radiation is a negative time mode exactly like the $|t_-\rangle$ state we suggested only months earlier in reference [7]. In reference [22], we suggested to look for correlations with delay and then, just a few months later, the BaBar collaboration announced that they had decided to reanalyze their old data for correlations with delay and that they did affirmatively find them [40].

In 2009, reference [2] was not even good enough to be allowed as a preprint but it garnered a response, which is very high praise. In 2011, the paper [7] still did not meet the bar of arXiv's unpublished censorship criteria and not only did it garner a response paper, it garnered response experiments. This is outrageously high praise because experiments cost time *and* money whereas papers only cost time. It means that the "peers" of this writer have "reviewed" the manuscript and decided to change research direction in favor of the MCM/TOIC. If the results of the experimental response had been negative, then the praise would be lessened only somewhat because it would still be true that we had presented a new

[1]Laguna deserves an honorable mention and thanks for inviting not just Ashtekar to Georgia Tech, but also Penrose, meaning that both of the speakers that inspired the MCM were the invitees of Laguna.

[2]One wonders how Shoemaker could pursue a PhD, make it through the academic grinder into a tenure track position, get a promotion as a founding member of a center for relativistic astrophysics, and then show absolutely no interest when some of the most important astrophysical mysteries of the universe are plainly spelled out before her eyes on a white board. Affirmative action likely explains the whole thing.

[3]If they are unpublished, are they even guidelines? Or does the uncertainty principle mean that they are always whatever the anonymous reviewer wants them to be? What does the uncertainty principle tell us about unpublished guidelines in the national security apparatus?

idea which is the primary function of theorists: to theorize new theories. In this regard, one may compare the MCM/TOIC to other very famous theories that are worse yet still manage to reap all of the theoretical praise. However, unlike the experimental tests of very many respected and praiseworthy theories, the results of the experimental response were all positive. Therefore, although the TOIC has not passed "peer-review," it has been known for an experimental fact, multiple experimental facts actually [38, 39, 40] (*at least!*), that it describes Nature better than any other theory that currently exists. This was known all throughout 2013, 2014, 2015, 2016, and 2017 but there has been no accompanying update to the public understanding of science.

We are essentially accusing Abhay Ashtekar, Nikodem Poplawski, and others of plagiarism but in the technical sense there has been no plagiarism.[1] In the technical sense, the complaints listed here only suggest that the alleged self-correcting mechanism in science "fucked this writer over big time." We pointed out Ashtekar *et al.*'s spurious Feynman citation as evidence of his having viewed reference [2], so consider that, in reference [36], Ashtekar *et al.* wrote that they were being so vague not to avoid writing about the MCM directly, but rather because they would leave "the detailed derivations and discussions to a longer article." Did those derivations exist at the time of the publication of reference [36], or had they been first suggested after someone looked at the manuscript which arXiv rejected [2], but not carried out during the hasty preparation and revision of the rough draft that preceded the preprint cited here as reference [36]? One wonders if the promised detailed derivations ever did appear in the literature. If not, did they ever come into existence? If not, was reference [36] worded so as to mislead readers about the existence of the derivations?

Ashtekar *et al.* write the following in reference [36], and one further wonders how they managed to report a rigorously developed Hamiltonian theory without reporting a rigorous development of anything at all.

> "Because of [*sic*] the Schrödinger equation we can now pass to a sum over histories a la Feynman. [*sic*] We emphasize that the result was derived from a Hamiltonian theory. We did not postulate that [*our equation*] is given by a formal path integral. Rather a rigorously developed Hamiltonian theory guaranteed that [*our equation*] is well-defined."

In reference [2], we did not include a detailed derivation and we did not claim rigor without derivation, which is what Ashtekar *et al.* have done. The diagrams in reference [2] explain an idea much more clearly than Ashtekar *et al.* were able to explain anything with their non-rigorous rigor of math salad in reference [36]. They included neither diagrams nor derivations but, somehow, their paper was good and ours was not just terrible, it was unacceptably terrible. How have Ashtekar *et al.* "rigorously developed" it while leaving the "detailed derivations" to a longer article? Furthermore, the reader should be very careful to note that if the rigor of Ashtekar *et al.*'s result is offloaded elsewhere beyond the paper's pages then references [2] and [36] are very similar indeed. Ashtekar *et al.*'s murky, imprecise, arguably self-contradictory wording starkly contrasts reference [2] where one finds in the abstract a sentence, "No attempt at quantification is made." Instead, we pursue a qualitative

[1] It would be impossible to steal this writer's research because the main intention in carrying it out has been to give it away for free. It is only this writer's accolades that have been stolen and, God willing, much blood will be spilled over this thievery.

analysis of the diagrams that guarantee our framework is well-defined. Again, this sharply contrasts reference [36] when the qualitative discussion of diagrams is practical to a degree that is at least an order of magnitude greater than the practicality of qualitative analysis of quantitative equations that don't, when taken all together, form a rigorous derivation of anything. Generally, quantitative analysis is only superior to qualitative analysis when it is rigorous.

As an example of real quantitative rigor, consider the unassailable truth of the appearance of the coefficient of Einstein's equation 8π in the first intuitive manipulations of the MCM/TOIC once the equally unassailable truth of

$$2\pi + \big(\Phi\pi\big)^3 \approx 137 \quad , \tag{3.13}$$

was established.[1] Somehow, some particular individuals have snuck into the halls of power to convince everyone that Feynman was wrong when he is famously paraphrased as stating that all good physicists have the fine structure constant on the wall in their offices and ask themselves where it comes from, and that no one has a good explanation for it, and that if they did it would "probably be related to π or something." This is paraphrased rather than quoted because the original quote, which this writer had understood to be one of Feynman's greatest quotes of all time, does not appear in any internet search results returned to this writer's computer terminal on February 20, 2017 A.D. Feynman's findable quotes that appear in internet search results at the beginning of the third millennium of *Domini*, include, "We know what kind of a dance to do experimentally to measure [*the fine structure constant*] very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!" The specification of this other dance is a great success of the MCM. Feynman's other quotes include the following.

> "There was no way, without full understanding, that one could have confidence that conditions the next time might not produce erosion three times more severe than the time before. Nevertheless, officials fooled themselves into thinking they had such understanding and confidence, in spite of the peculiar variations from case to case. A mathematical model was made to calculate erosion. This was a model based not on physical understanding but on empirical curve fitting."

Given that Ashtekar *et al.* were able to produce the inferior analysis that became reference [36] *likely within just days* of reading about the MCM, all within the context of their own years or decades long familiarity with the material, it is demonstrated exactly how well-defined the MCM already was in 2009.

Ashtekar *et al.* strongly emphasize that their result was derived from a Hamiltonian theory. They do not say whether or not they were inspired to make that derivation for the first time immediately after viewing the contentious paper that arXiv rejected in 2009: reference [2]. When they write that they did not postulate that their formula is given by a

[1] The reader should note that some further confirmation of the validity of the TOIC is seen when the coefficient $1/4\pi$ of the leading term of the ontological resolution of the identity is also the dimensionless coupling coefficient for the electromagnetic interaction.

formal path integral, is that to distinguish their paper from reference [2] wherein we postulate that the MCM is given by the formal path integral? Their emphasis on the Hamiltonian theory refers to the type of extra mathematical details presented in this book that are not needed to understand the idea. In fact, we are still moving steadily toward that eventual Hamiltonian derivation. In 2011, the purpose of reference [7] was not to make any precise predictions, and this book is only about the theory's general relevance, so we will not derive a new Hamiltonian here. Making precise predictions is clearly the top priority of theoretical physics but it should be clear that there are at least two steps in the process that produces them. Before one makes a prediction, one must define how to predictions are to be made. This illustrates the standard distinction between fundamental science and applied science.

The critical reader will notice that "detailed derivations and discussions" are left out in both references [2] and [36], but only one of them appears on arXiv today. In the acknowledgments section of reference [36], Ashtekar *et al.*'s first thanks are to Jerzy Lewandowski who was the advisor or colleague of Poplawski at the University of Warsaw. In April 2010, around the time Poplawski began publishing his very, very, very many popular science articles about the universe being in a black hole, he also published reference [41]. Note how the title of that paper is evocative of the idea of inverse radial spaghettification:[1] "Radial Motion into an Einstein–Rosen Bridge." Likewise, the title of Lewandowski's October 2009 talk at LSU was evocative: "Spin foams from loop quantum gravity perspective." What was this new perspective that Lewandowski was evangelizing in Louisiana just a month after arXiv rejected reference [2]?

While on the topic of the conduct of science in a manner that is other than ethical, consider the following. At some point in 2011, while preparing a draft of reference [7], this writer encountered a slideshow from another a talk given at LSU. The title was something like "Path Integral Approach to Spin Foams" and the name on the slides was likely Jonathan Engle who was also a speaker in video reference [37]. The slides were dated from the end of 2008, but when this writer checked on the seminar schedule at the host university, LSU, the talk was actually given at the end of 2009 and the date from 2008 appears to have been "a typo." This is notable because the path integral formulation of spin foams was not yet conceived in 2008 and a lesser typo might not have changed the year of initial formulation. Based on the description of a new use for the Feynman path integral in reference [2], and also the fact that Engle was Ashtekar's PhD student, it is likely that the new topic presented and misdated in this talk was inspired by reference [2]. When one views reference [42], which shows the LSU Physics and Astronomy talk schedule archives, one sees all the years 2004–Present except 2009–2012: the window in which Engle presented the misdated slides. If other researchers were already jockeying in 2009 to position themselves to receive credit for a discovery that was not their own, then whose discovery was it? A full forensic accounting of the failure of physics to self-correct in this regard is required.

Finally, we wish to point out that Lewandowski is a coauthor on reference [43] which was published in September 2009 around the same time we were proposing to wrap the Minkowski diagram around a cylinder [2]. Therein, Kamiński *et al.* refer to an unusual cylindrical object

[1] The term "inverse radial spaghettification" did not appear in the literature until 2012 because arXiv did not allow it to be added to the literature in 2011. To understand how the title of Poplawski's 2010 paper is evocative of 2009's reference [2], note that radial motion means 1D motion, and together with "into an Einstein–Rosen bridge," it means motion toward a bridge between two distant regions of the universe along the 1D manifold defined by the motion. The idea presented in reference [2] was that dark energy is an expected feature in pairs of worldsheets in the hypercosmos connected in 1D through a bounce. The connection is 1D because it is along χ^5.

Cyl($\mathcal{A}(\Sigma)$) and one also sees that object in at least one earlier arXiv preprint coauthored by Lewandowski [44]. However, one wonders if perhaps they have contracted with Mossad–Fonseca (or similar) to do a more professional time stamp alteration job than was suggested above when discussing Engle's "Path Integral Formulation of Spin Foams" slides.

For science to self-correct, everyone named in this section will need to consult with the grand inquisitor of the self-correcting mechanism in science.[1]

III.3 A Few Miscellanea

Regarding the technical matters at hand, we restate problem one as

$$\omega^3\big|\psi;\hat{\pi}\big\rangle = i\pi\Phi^2\big|\psi;\hat{\pi}\big\rangle \qquad \Longrightarrow \qquad \omega^3 = i\pi\Phi^2 \quad . \tag{3.14}$$

The implied value $\omega^3\!=\!i\pi\Phi^2$ is a cubic function of ω so the fundamental theorem of algebra says it will have three roots, not one, and two of them might not even be real numbers. However, the problem remains: equation (3.14) does not allow the full frequency spectrum of free particle eigenstates available to unbound quanta gravitating on a continuum.

With quadratic equations, which would define ω^2 rather than ω^3, one has two solutions. An example is the map used to generate the Mandelbrot set $f_c(z)\!=\!z^2+c$, or even $x^2\!=\!x+1$ whose two roots are

$$\Phi = \frac{1+\sqrt{5}}{2} \quad , \qquad \text{and} \qquad \varphi = \frac{1-\sqrt{5}}{2} \quad . \tag{3.15}$$

Quadratic equations $ax^2\!+\!bx\!+\!c\!=\!0$ always have two roots given by "the quadratic equation"

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad , \tag{3.16}$$

but for cubic equations we don't know how to find the roots without guessing because there is no more-complex version of equation (3.16) that we could call "the cubic equation." This is, at least, important to understand before attempting to understand why one "can't quantize" cubic equations of motion despite quadratic second order equations quantizing so famously nicely. About the only thing we do know about cubic equations is that they will never have exactly two real roots.

In the development of the MCM, we have previously vacillated concerning the sign of φ and, independently, the magnitude of $\aleph$'s parameter of curvature because there were many possible choices. Among the many choices, here we have a good reason to choose $\varphi\!<\!0$ with $\varphi\!\approx\!-0.62$, and to take it as the magnitude of the parameter of curvature on $\aleph$. The reader should note that the distance in the χ^5_- direction toward the past from a given $\mathcal{H}_i$ might be

[1] In November 2011, as we were putting reference [7] together with intention to give arXiv a shot at redemption, we discovered all of the above information related to Ashtekar and cohort at Penn State, ranted about it prolifically, and the reader will recall that November 2011 is the month that the FBI swarmed Penn State over "decades old Sandusky allegations."

irrelevant and, instead, the contentious distance may be how far in front of Ω the next $\aleph$ lies in the χ^5_+ direction (or the $\chi^5_\varnothing$ direction.) This follows from the introduction of $\hat{\varphi}$ and the implied separation of the past from the present which contains a record of the past. Another thing we have vacillated on is the length of χ^5 across the complete MCM unit cell. If we say that the length of χ^5_+ between $\mathcal{H}$ and Ω is Φ, that $\chi^5_\varnothing$ has no width between Ω and $\aleph$, and then that the length of χ^5_- between $\aleph$ and $\mathcal{H}$ is φ, then the total chirological distance will be unity via $1=\Phi+0+\varphi$. However, due to the non-unitary component of the MCM, χ^5 across one unit cell could be shorter than χ^5 across the next unit cell. There are very many possible conventions, including the one shown in figure 1 which uses $\chi^5_- = -\varphi\Phi = 1$. This could have a further application to the MCM unitarity constraint that has not been precisely specified other than to say that it exists because we are using quantum theory. However, rather than exploring unitarity in this section, we will remain focused mostly on the general relevance of the existing model without adding too many bells and whistles. When unitarity becomes relevant we will explore it but there are a lot of other things that need to be covered first. For instance, the MCM unit cell considered here does not even begin to make accommodations for the compact unit cell described in reference [13] and it is the opinion of this writer that the ideas presented there are not irrelevant. Reference [13] contains a mechanism through which $\hat{M}^3$ increases the level of $\aleph$ by one where previous work had indicated that the level should increase by two in each application of $\hat{M}^3$. The reader should take note of the condensed mechanism in reference [13], and we will discuss it again in section IV.1.

One of the big struggles in this research has been to show how to unify second order classical mechanics under Newton's laws with first order, or linear, quantum mechanics governed by the Schrödinger equation. To this end, we have chosen the scalar field ϕ in the Kaluza–Klein metric

$$\Sigma_{AB} = \begin{pmatrix} g_{\mu\nu} + A_\mu A_\nu \phi^2(\chi^5) & A_\mu \phi^2(\chi^5) \\ A_\nu \phi^2(\chi^5) & \phi^2(\chi^5) \end{pmatrix} \quad , \tag{3.17}$$

to be linear in χ^5 with

$$\phi^2(\chi^5) \quad \equiv \quad \chi^5 \quad . \tag{3.18}$$

Even when the metric has this linear definition, the double dot operator in $f=m\ddot{x}$ is not linear. We want to make everything linear for two reasons: it is easier to compute linear mathematics and if we can linearize quadratic equations, then we can probably quadratify cubic ones and then quantize them canonically. It would be a big conceptual advance if there was found to be some gauge symmetry associated with the golden ratio that would universally allow such manipulations. Regarding quadratic equations, and not making an argument about gauge symmetry but rather only outlining an idea at a very high level, we need to split the non-linear ∂^2 operator into two parts. They should be unequal so that they can generate complexity but the only obvious way to split it is through

$$\partial^2 = \partial \circ \partial \quad . \tag{3.19}$$

This formulation certainly contains nothing new.

In section II.1, we showed how Hamilton's equations, two first order equations, can give either the geodesics or the field lines of different second order theories but there was no application to quantum theory where the Hamiltonian only appears inside Schrödinger's first order equation. Hamilton's equations, linear or not, are exactly equal to classical mechanics so that formulation gives us little direction regarding a classical description of quantum physics where expectation values conform to classical laws. In reference [10], we discussed the normal decomposition of second order equations into pairs of linear equations that are amenable to analysis with numerical methods. The correlation of numerical x and v to the specific case of Hamiltonian q and p is easy to see with

$$\ddot{x} = \frac{F}{m} \quad \mapsto \quad \begin{cases} \dot{x} &= v \\ \\ \dot{v} &= \dfrac{F}{m} \end{cases} \quad , \tag{3.20}$$

but this is not what we refer to as linearization. The difficult form of linearization we refer to is the one that turns one second order equation into one first order equation. It will likely be easiest to implement the unification of first and second order equations at the level of numerical algorithms, and we will discuss such things in section IV.7. Before moving on to solve problems one through four, presented at the beginning of this chapter, we will discuss one possible analytical feature that could motivate later alterations to existing numerical analysis algorithms. All the physical equations and dynamics that were derived analytically over the years have already been converted into computer ready algorithms so it may be simpler to modify those algorithms and then reverse engineer their analytical underpinnings than to advance solely through the proposition of new underpinnings.

The topological component of double orthogonality uses sums; products should generally be understood as a single object with a single topology. If this avenue of splitting ∂^2 is to pan out, we must learn something about how addition on the right hand side (RHS) of

$$\partial^2 := A + B \quad , \tag{3.21}$$

relates to multiplication on the left hand side (LHS.) In mathematics, multiplication and addition are important operations but they are more important in physics because the physical interpretation assigned to each operation is very well understood. Almost everything in physics depends on some combination of additive and multiplicative operations. Earlier we quoted Feynman's reference [6] as follows.

> "We shall see that it is the possibility [*of expressing the action*] $\mathcal{S}$ as a sum, and hence Φ as a product, of contributions from successive sections of the path

which leads to the possibility of defining a quantity having the properties of a wavefunction."

When Boltzmann was formulating his famous entropy formula $S = k \log \Omega$, he did not randomly pick the logarithm from a very large sea of possible operations. Boltzmann simply wrote down what he knew about the interpretation of addition and multiplication as they relate to entropy, and the logarithm was the only choice that would preserve it. Therefore, pursuing a very high level concept, to obtain some new representation of ∂^2 we should find equation (3.21) when either A or B has to be the partial derivative operator because we must be able to reconstruct ∂^2 from the pieces A and B. For instance if we let $A = \partial$ and $B = \Box^2$ we can reconstruct ∂^2 with

$$\partial^2 = A^2 + 0 \times B \quad , \tag{3.22}$$

but that is not a valid solution because

$$\partial^2 \neq \partial + \Box^2. \tag{3.23}$$

However, we are not totally constrained by $\partial^2 = A + B$ because definition (3.21) is not an equation, and even if it was, it might be irrelevant. (We suggest an idea only vaguely right now.) To get the complexity generating representation of ∂^2, if it exists (and if it is actually useful), we will likely have to solve an equivalence relationship like definition (3.21) and we already know an equation that looks like that. The two roots of

$$x^2 = x + 1 \quad , \tag{3.24}$$

are the two numbers Φ and φ which are both called the golden ratio. If the decomposition of the non-linear operator ∂^2 into a linear form that unifies physics was simple, it would have been done by now. To develop a new law in this regard — for future investigations — we could suggest that ∂^2 has a representation as one full cycle (2π radians) which can be decomposed into co-$\hat{\pi}$s that undergo different types of gauge transformations. Note the similar properties of π and Φ such as

$$\pi + \pi = \left(\sqrt{2\pi}\right)^2 \quad , \qquad \text{and} \qquad \Phi + 1 = \Phi^2 \quad , \tag{3.25}$$

and even the connection between addition and multiplication demonstrated by

$$2 + 2 = 2 \times 2 \quad . \tag{3.26}$$

In the case of $\partial^2 := \pi + \pi$, $A = B$ but it is quadratic in the mysterious coefficient $\sqrt{2\pi}$ from the Fourier transform, and there exists an independent result which proves that $\sqrt{2}$ is an irrational number. Perhaps the ontological basis is best represented as $\{\hat{\imath}, \hat{\sqrt{2}}\ , \hat{\Phi}, \hat{\pi}\}$, and $\hat{\sqrt{2}}$ and $\hat{\imath}$ combine in the $\sqrt{i}$ channel to give the rational real numbers $\mathbb{Q}$. We won't go in this direction now, but we remind the reader of the suggested coupling between "sites" in the continued fraction forms of the irrational number π and the most irrational number Φ

$$\Phi = 1 + \cfrac{1}{1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\ldots}}}} \quad , \qquad \text{and} \qquad \pi = 3 + \cfrac{1}{6 + \frac{3^2}{6+\frac{5^2}{6+\frac{7^2}{6+\ldots}}}} \quad . \tag{3.27}$$

A product defined to combine the piecewise elements of continued fractions might be a good avenue for philosophical inquiry when calculating to assemble 2 from pairs of $\sqrt{2}$. A result of the contraction of information stored in π and Φ respectively might be to combine copies of $\sqrt{2}$ from each. In this way, there will be no rational real numbers in the ontological basis itself, but they are emergent through operations like $i^2 = -1$ or $\sqrt{2}^2 = 2$ which give $i^2 + \sqrt{2}^2 = 1$. Taking these numbers together with addition, we can generate the integers $\mathbb{Z}$ and, when we add multiplication as well, we recover the rationals $\mathbb{Q}$. Then, noting that $\hat{\imath}$ is still in there, we recover complex numbers $\mathbb{C}$, and then through the rules for $\hat{\Phi}$, we recover $^\star\mathbb{C}$. Once we have constructed 2, either from $\hat{\sqrt{2}}$ or $\hat{2}$ directly, then we will have more pieces for building complex cosmological clockwork with

$$\Phi^{n+1} = \Phi^n + \Phi^{n-1} \qquad \text{and} \qquad 2^{n+1} = 2^n + 2^n \quad . \tag{3.28}$$

III.4 Problems One and Two

Problem one references a frequency constraint stated as

$$\omega^3 \big|\psi;\hat{\pi}\big\rangle = i\pi\Phi^2 \big|\psi;\hat{\pi}\big\rangle \qquad \Longrightarrow \qquad \omega^3 = i\pi\Phi^2 \quad . \tag{3.29}$$

The kets on both sides are the same so their coefficients ω^3 and $i\pi\Phi^2$ must also be the same. $i\pi\Phi^2$ is fairly well introduced in reference [4] which revises and improves the original argument [12] for the criticality of that number as it relates to deriving Einstein's equation. All of the MCM structure is encoded on the value $i\pi\Phi^2$ so we can't change it much. Instead of changing the scalar coefficient, we proposed, in reference [10], to change the ket on the right side of the fundamental formula which is recast as

$$\hat{M}^3 \big|\psi;\hat{\pi}\big\rangle = \hat{M}^3 \big|\Psi;\hat{\pi}\big\rangle \qquad \Longrightarrow \qquad \partial_0^3 \big|\psi(x^\mu);\hat{\pi}\big\rangle = \partial_5^3 \big|\Psi(\chi^A);\hat{\pi}\big\rangle \quad . \tag{3.30}$$

Any frequency is allowed in this formulation when the degree of freedom for ω is preserved in the freedom to choose $\Psi \neq \psi$. From equation (3.30), all the steps of equations (3.2-3.5) follow directly, and we have incidentally solved problem two as well. Problem two stated that, when all of the terms in

$$8\pi f^3\big|\psi;\hat{\pi}\big\rangle = i|\psi;\hat{\Phi}\rangle + |\psi;\hat{\imath}\rangle \quad , \tag{3.31}$$

are proportional to ψ, there is no way to achieve double orthogonality. The introduction of Ψ fixes this. With Ψ, when we convert from the quantum language to the gravitational language, the stress energy tensor $T_{\mu\nu}$ on the LHS, defined by ψ, can be different than everything on the other side which will depend on Ψ.

Simply capitalizing ψ on the RHS solves both problems one and two. Ψ is the solution to problem two that we will use moving forward but there is another way to solve problem two that demonstrates complexity, and is worth exposing. We may redefine ψ so that it begins as a third rank tensor

$$\psi \quad \equiv \quad \psi^{\sigma}_{\mu\nu} \quad , \tag{3.32}$$

and then let the ontological vectors act on $\psi^{\sigma}_{\mu\nu}$ like the Kronecker delta per the usual prescription for basis vectors. Consider, for example, only the $\hat{e}^3 \equiv \hat{\imath}$ term that we expect to map to $g_{\mu\nu}\Lambda$. We see how one might use the basis vectors to extract independent tensors via

$$|\psi;\hat{\imath}\rangle \quad \equiv \quad \psi^{\sigma}_{\mu\nu}\hat{e}_3 = \psi^{\sigma}_{\mu\nu}\delta^3_\sigma = \psi^3_{\mu\nu} = g_{\mu\nu}\Lambda \quad . \tag{3.33}$$

The other objects of general relativity, $8\pi T_{\mu\nu}$, $R_{\mu\nu}$, and $1/2\,Rg_{\mu\nu}$, can all be extracted from $\psi^{\sigma}_{\mu\nu}$ with the other $\hat{e}_\mu$ when they are defined accordingly.

Equation (3.33) is reminiscent of the interpretation of the connection coefficients $\Gamma^{\sigma}_{\mu\nu}$ as a set of four 2D matrices: one $\mu\nu$-matrix for each σ. In equation (3.33), $\hat{\imath}$ picks out $g_{\mu\nu}\Lambda$ because that is the definition specified by maps (3.6-3.8). $g_{\mu\nu}\Lambda$ would be just one of the four 2D matrices contained in $\psi^{\sigma}_{\mu\nu}$. This formulation motivates the inclusion of $\hat{2}$ as it will have an object defined for it in the counting over σ, but there would be a leftover element if we tried to do this with only $\hat{e}_i \in \{\hat{\Phi},\hat{\pi},\hat{\imath}\}$. However, we will not presently solve problem two with equation (3.32) because it simply shifts our problem to another sector. If we choose to use $\psi^{\sigma}_{\mu\nu}$, we would have to add a problem to the list about how to use three index tensors as state vectors in quantum mechanics. Between the two proposed solutions, the former is better because it is simpler; we will choose to capitalize ψ rather than adding three Greek indices.

Not only is the three index proposal like the connection, it is also like the torsion. In reference [17] Carroll writes the following.

> "The first thing to notice is that the difference of two connections is a tensor. Imagine we have defined two different kinds of covariant derivative, ∇_μ and $\widehat{\nabla}_\mu$, with associated connection coefficients $\Gamma^{\lambda}_{\mu\nu}$ and $\widehat{\Gamma}^{\lambda}_{\mu\nu}$. Then the difference

$$S^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \widehat{\Gamma}^{\lambda}_{\nu\mu} \quad , \tag{3.34}$$

is a (1,2) tensor. (Notice that we had to choose a convention for the index placement.) We could show this by brute force, plugging in the transformation laws for the connection coefficients, but let's be a little more slick. Given an arbitrary vector field V^{λ}, we know that both $\nabla_{\mu}V^{\lambda}$ and $\widehat{\nabla}_{\mu}V^{\lambda}$ are tensors, so their difference must also be. This difference is simply

$$\nabla_{\mu}V^{\lambda} - \widehat{\nabla}_{\mu}V^{\lambda} = \partial_{\mu}V^{\lambda} + \Gamma^{\lambda}_{\mu\nu}V^{\nu} - \partial_{\mu}V^{\lambda} - \widehat{\Gamma}^{\lambda}_{\mu\nu}V^{\nu} \tag{3.35}$$

$$= S^{\lambda}_{\mu\nu}V^{\nu} \quad . \tag{3.36}$$

Since V^{λ} was arbitrary, and the left hand side is a tensor, $S^{\lambda}_{\mu\nu}$ must be a tensor. As a trivial consequence, we learn that any set of connection coefficients can be expressed as some fiducial connection plus a tensorial correction,

$$\Gamma^{\lambda}_{\mu\nu} = \widehat{\Gamma}^{\lambda}_{\mu\nu} + S^{\lambda}_{\mu\nu} \quad . \tag{3.37}$$

"Next notice that, given a connection specified by $\Gamma^{\lambda}_{\mu\nu}$, we can immediately form another connection simply by permuting the lower indices. That is, the set of coefficients $\Gamma^{\lambda}_{\nu\mu}$ will also transform according to

$$\Gamma^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\nu}}{\partial x^{\nu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\Gamma^{\lambda}_{\mu\nu}\frac{\partial x^{\mu}}{\partial x^{\mu}}\frac{\partial x^{\nu}}{\partial x^{\nu}}\frac{\partial^2 x^{\lambda'}}{\partial x^{\mu}\partial x^{\nu}} \quad . \tag{3.38}$$

[*Since the partial derivatives appearing in the last term of equation (3.38) commute*], they determine a distinct connection. There is thus a tensor we can associate with any given connection, known as the **torsion tensor**, defined by

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} = 2\Gamma^{\lambda}_{[\mu\nu]} \quad . \tag{3.39}$$

It is clear that the torsion is antisymmetric in its lower indices, and a connection that is symmetric in its lower indices is known as 'torsion-free.'"

Noting what Carroll has stated above regarding the difference of two connections being a tensor, we can set $A_{\mu}=0$ in the Kaluza–Klein metric, not calculate the bulk geometry, and simply take the difference of the connections $\Gamma^{C\,\pm}_{AB}$ in $\Sigma^{\pm}$ as the perturbation $h_{\mu\nu}$ in $\mathcal{H}$. Even when $\Sigma^{\pm}$ are empty 5D space, the fact that they have topologies O(1,4) and O(2,3) means that the connections will be different on either side of $\mathcal{H}$.

III.5 What is a Multiplex?

Implication (3.30) solves problem two about all the objects being linearly dependent on ψ but, by going the route with Ψ, we introduce a new problem: problem three. How can the 4D object ψ on the left be equal to the 5D object Ψ on the right?[1] This cannot be

[1] The objects ψ and Ψ were introduced in reference [10]

true in general, and we don't want to introduce any unnecessary complexity, so we should reformulate the rigid equality in implication (3.30) as

$$\partial_0^3\big|\psi(x^\mu);\hat{\pi}\big\rangle := \partial_5^3\big|\Psi(\chi^A);\hat{\pi}\big\rangle \quad . \tag{3.40}$$

Equation (3.40) uses the := symbol because we want to show a new kind of relationship, possibly between a topological object and an algebraic one, or between a geometric object and a group theoretical one, or some such juxtaposition, such as connecting the ontological basis to the ontological group, or perhaps vice versa. Since it is not clear, at this point, how the type of object on the RHS of equation (3.40) compares with the type on the LHS, we will focus on the symbol := in the center and, going forward, we will say "multiplex (3.40)" instead of "equation (3.40)" when this symbol appears (although we will not rely upon it heavily in this book.)

An equation has an LHS and an RHS, but a multiplex will have an LHS/RHS pair, as in multiplex (3.40), and also an imaginary hand side (IHS) which consists of all the objects needed to make the multiplectic relationship true. We can select any two elements of a multiplex as an RHS and an LHS which are logically related through an independent IHS. If there are N objects in a multiplex, then we can always write an "is defined according to" statement between any two objects leaving the remainder of $N-2$ objects as an IHS. Those familiar with group theory will see many obvious parallels, but we will not discuss the likeness here other than to say that the foundation of group theory is that every group has the identity operator as one of its elements so $\{\hat{1},\hat{\pi},\hat{2},\hat{\Phi},\hat{\imath}\}$ is a good set to explore for group structure but the ontological basis $\{\hat{\pi},\hat{2},\hat{\Phi},\hat{\imath}\}$ is not.

We will also use the word multiplex to refer to an IHS from which no LHS/RHS pair has been selected; the most general multiplex contains all the information about all possible := pairings. If we want to make some arbitrary example that shows how all the pieces of a multiplex work, we can consider a multiplex

$$Z_M \quad \equiv \quad \{Z_1, Z_2, Z_3, Z_4, Z_5\} \quad . \tag{3.41}$$

When we choose to consider, perhaps in an effort to understand an equation, two elements of a multiplex taken as an LHS and an RHS, then there is a corresponding IHS that contains the full multiplex minus the two elements that are being used to define a := relationship. If we take an LHS Z_1 along with an RHS Z_2 then that defines an IHS $Z_{\mathrm{M}:\{Z_1,Z_2\}}$. Together they look like

$$Z_1 := Z_2 \quad , \qquad \text{with} \qquad Z_{\mathrm{M}:\{Z_1,Z_2\}} = \{Z_3, Z_4, Z_5\} \quad . \tag{3.42}$$

$Z_{\mathrm{M}:\{Z_1,Z_2\}}$ is the set of objects that makes $Z_1 := Z_2$ true. The IHS contains everything needed to demonstrate how a LHS is defined by an RHS. Then we also have

$$Z_5 := Z_3 \quad , \qquad \text{with} \qquad Z_{\mathrm{M}:\{Z_5,Z_3\}} = \{Z_1, Z_2, Z_4\} \quad , \tag{3.43}$$

and so on.

Unlike relationships given with =, those defined with := are not self-contained. The full meaning is contained on the LHS, the RHS, *and* on the IHS, but the IHS does not need to be specified to understand the most important part of the := relationship. The LHS/RHS multiplex relationship is statement that a sufficient IHS exists, and there is an implication (or an expectation) that some IHS will connect the LHS and RHS in a fairly direct, or "irreducible," manner, and not be some mathematical Rube Goldberg of the type that can be made complicated enough to relate any two things.

III.6 Problems Three and Four

Problem three manifests in multiplex (3.40) when the LHS is a 4-vector and the RHS is a 5-vector [10], and the two cannot satisfy the = relationship. Here, we will rederive Ψ to show that there is yet another undiscussed arrangement for resolving the linear dependence issue.

After inserting Ψ into equation (3.5) it becomes

$$8\pi f^3\big|\psi;\hat{\pi}\big\rangle = i\big|\Psi;\hat{\Phi}\big\rangle + \big|\Psi;\hat{i}\big\rangle \quad , \tag{3.44}$$

and this solves problems one and two. Problem two is solved when the starting point already exhibits single orthogonality between ψ and Ψ, and problem one is solved when there is an obvious symmetry that will allow the full frequency spectrum as Ψ always becomes whatever is needed to satisfy equation (3.44) for a given f. Recalling that we introduced Ψ in reference [10] as the complete state of the universe, and noting that the complete state is unobservable, there is no constraint on how complicated we can make its wavefunction. Still, as mentioned above, there is a lingering issue with the Ψ workaround: when we introduced the 5D Ψ vector, we also defined it to contain ψ as a 4D subspace. This is a severe constraint! Ψ is almost completely determined by ψ so, in truth, we have just barely solved problem two. To add more freedom between ψ and whatever we put on the RHS of equation (3.44), and to resolve the dimensionality discrepancy of problem three, consider a third representation

$$8\pi f^3\big|\psi;\hat{\pi}\big\rangle = i\big|\phi;\hat{\Phi}\big\rangle + \big|\phi;\hat{i}\big\rangle \quad . \tag{3.45}$$

To get ϕ we take

$$\big|\Psi\big\rangle = 0 \tag{3.46}$$

$$= 1 + (-1) \tag{3.47}$$

$$= 1 - \left(\frac{1}{4\pi}\right)\hat{\pi} + \left(\frac{\varphi}{4}\right)\hat{\Phi} - \left(\frac{1}{8}\right)\hat{2} + \left(\frac{i}{4}\right)\hat{i} \tag{3.48}$$

$$= 1 - \psi \tag{3.49}$$

$$= \phi - \psi \tag{3.50}$$

with

$$\psi \quad \equiv \quad \big|\psi\big\rangle = \begin{pmatrix} -\dfrac{1}{4\pi} \\ \dfrac{\varphi}{4} \\ -\dfrac{1}{8} \\ \dfrac{i}{4} \end{pmatrix} \;, \qquad \text{and} \qquad \phi \quad \equiv \quad \big\langle\phi\big| = \begin{pmatrix} \dfrac{1}{4\pi} \\ -\dfrac{\varphi}{4} \\ \dfrac{1}{8} \\ -\dfrac{i}{4} \end{pmatrix}^T \;. \tag{3.51}$$

Obviously the constant state vectors presented here (and in reference [10]) will lead to more problems because the wavefunction should vary over space and time but ψ and ϕ do not, and certainly $\Psi = 0$ does not. When this structure was introduced in reference [10], we did not explicitly state a method by which we could introduce the coordinate dependence into the wavefunctions. The implication of $\Psi = 0$ would have to be that $\Psi \neq 0$ in some perturbative limit, or some other limit where it is equal to zero only on a certain level of $\aleph$. Note well that when $\Psi = 0$, we can use the algebraic structure that we have built with Ψ to host perturbations in the form of $h_{\mu\nu}$, $h_{\alpha\beta}$, and h_{AB}, and then move to the computational frame where the relevant qubit with coordinate dependence is encoded in the perturbation. That computational frame is likely what we have previously described as G-space [9], and perhaps the difference between $\hat{M}^3$ and $\tilde{M}^4$ is that $\hat{M}^3$ takes a qubit in a position or momentum representation and outputs a qubit in the other representation, but $\tilde{M}^4$ has an output in the same representation as its input.

We described the mode between the state $\{\Psi, \psi, \phi\}$ and the qubit $\{h_{\mu\nu}, h_{\alpha\beta}, h_{AB}\}$ as exhibiting Yangian symmetry in reference [9]. Reference [9] is among the most rigorous mathematical analyses undertaken so far in this research program.[1] When we add the coordinate dependence that will make our wavefunctions ψ and ϕ functions instead of constants, whatever change we make in ψ must be perfectly balanced in ϕ if equation (3.46) is to hold. However, it is not required that the $\hat{\pi}$ component of ψ is perfectly offset by the $\hat{\pi}$ component of ϕ. Both $\psi\hat{\pi}$ and $\phi\hat{\pi}$ can vary independently when one uses the other components to keep everything balanced. Finally, note that we can solve problem three simply by replacing 5D Ψ with some 4D vector ϕ that is otherwise not constrained. This means we only have

[1] From this writer's perspective, the main result of the work unit whose output was "Ontological Physics" [9] was that further inquiry in that direction should be carried out with computers. This writer considered it prudent to continue with the survey of that which is better analyzed without computers, which is sometimes called philosophy, because certainly there are thousands or millions of other people who are already well trained in computerization.

to introduce a second 4D wavefunction, and it doesn't need to have anything to do with equations (3.46-3.51).

In quantum theory, the ordinary application of any resolution of the identity is to say that every object is multiplied by one, and now we have the option to say, also, that every object has zero added to it which allows us to insert two identities as $\hat{0}=\hat{1}-\hat{1}$, and which also may be written as $\hat{0}=\hat{1}-\hat{\Phi}^0$. Furthermore, where Feynman has discussed the role of addition for the action and the role of multiplication for wavefunction, and with these new "ontological" resolutions of both one and zero, we can make use of either channel anywhere by adding zero or multiplying by one. We expect that this will be an important feature of infinite complexity but for now it will suffice to say that there are so many options in this regard (regarding ϕ) that we can classify problem three as solved. Regardless of ϕ's complete technical specification, it is 4D so problem three goes away.

With three problems out of the way, we come to problem four about how the linear dependence issue lingers inside the Einstein tensor when

$$\big|\phi;\hat{\Phi}\big\rangle \quad \mapsto \quad G_{\mu\nu} \quad \equiv \quad R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu} \quad . \tag{3.52}$$

ϕ resolves the issues discussed earlier, but the drawback is that they do not solve the problem of the linear dependence of the objects inside $G_{\mu\nu}$. As mentioned a few times already, we could use $\hat{2}$ so that the four ontological vectors correspond to the four terms of Einstein's equation written without the brevity of the Einstein tensor

$$8\pi T_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+g_{\mu\nu}\Lambda \quad , \tag{3.53}$$

but we have avoided this for two reasons. First, the MCM algebra generates three terms naturally, as in equation (3.44), and we would have to unnaturally pick one of the three terms to split in half with $\hat{2}$ before assigning one of the halves to $\hat{2}$. Second, it is nice when we can interpret general relativity as a manifestation of the principle

$$\widehat{LQC}\big|\text{bounce}\big\rangle := \big|t_+\big\rangle+\big|t_-\big\rangle \quad , \tag{3.54}$$

which first appeared in reference [7], and there is something very interesting we can say about this. The original intention with equation (3.54) had been to show that the operation of the LQC operator on the bounce state was not just to decompose it into the past and future, but rather into the past, present, and future via

$$\widehat{LQC}\big|\text{bounce}\big\rangle := \big|t_+\big\rangle+\big|t_\star\big\rangle+\big|t_-\big\rangle \quad . \tag{3.55}$$

In the course of the development of the idea, it became clear that the bounce state should be the present denoted with $|t_\star\rangle$ and that is how we arrived at equation (3.54) in the original paper: reference [7]. However, now that we have added $\Sigma^{\varnothing}$ with the $\varnothing$ coordinates, we are absolutely able to preserve the original interpretation by writing

$$\hat{M}^3\big|\text{bounce}\big\rangle := \big|t_+\big\rangle + \big|t_\star\big\rangle + \big|t_-\big\rangle \quad , \tag{3.56}$$

with an implication that

$$\big|t_\star\big\rangle := \Sigma^{\varnothing} \quad . \tag{3.57}$$

We refer the reader to reference [7] for a lot of the "irrelevant" details regarding the $\widehat{LQC}$ formalism.

The concept illustrated by equation (3.56) has four terms, just like would be needed to write Einstein's equation in long form with four terms corresponding to the four ontological basis vectors. The only remaining obstacle to using $\hat{2}$ from the outset is that the MCM algebra generates three terms and we don't know how to best add a fourth one. Luckily we can sidestep this issue because dS and AdS belong to a small set of spacetimes that are called **maximally symmetric**. The definition of a maximally symmetric space is one whose Ricci tensor is linearly dependent on the metric.

$$R_{\mu\nu} \propto g_{\mu\nu} \qquad \Longrightarrow \qquad R_{\mu\nu} := g_{\mu\nu} \quad . \tag{3.58}$$

Since we are only requiring the possibility of constructing a Lorentz frame at the three slices $\{\aleph, \mathcal{H}, \Omega\}$ of the cosmological unit cell, but not the bulk hypercosmos between slices, we have miraculously generated a constraint that means we don't need to change anything else. Problem four is not a problem at all because we have already, based on completely independent considerations, chosen $\{\aleph, \mathcal{H}, \Omega\}$ to be exactly those unique spaces where the metric and Ricci tensors are *never* independent from each other. In maximally symmetric spacetime, all of the tensors in Einstein's equations are linearly dependent on each other.

If the Ricci tensor and the metric are linearly dependent on each other then the stress-energy tensor must also be linearly dependent with them. This means problem two was never a problem to begin with because we had already constructed the MCM to be the special case where complete linear dependence on both sides of Einstein's equation is expected. Furthermore, as a statement of the non-problematic nature of any of the "problems," note how

$$\partial_0^3\big|\psi;\hat{\pi}\big\rangle = \partial_5^3\big|\psi;\hat{\pi}\big\rangle \qquad \longrightarrow \qquad \partial_0^3\big|\psi;\hat{\pi}\big\rangle = \partial_5^3\big|\psi';\hat{\pi}\big\rangle \quad , \tag{3.59}$$

also solves problems one and three. ψ and ψ' are both 4D and ψ' will vary to accommodate any frequency ω. Simply adding the tick mark to ψ is sufficient to completely put aside all four of the problems cited in section III.1.

III.7 Maximally Symmetric Spacetime

Noting that flat expanding space in not curved, the Friedman–Lemaître–Robertson–Walker (FLRW) metric of flat expanding space is

$$ds^2 = -(dx^0)^2 + a^2(t)[(dx^1)^2 + (dx^2)^2 + (dx^3)^2] \quad . \tag{3.60}$$

Consider what Misner, Thorne, and Wheeler write about this metric in reference [27].

> "Turn now to the 3-geometry $\gamma_{ij}dx^i dx^j$ for the arbitrary initial hypersurface $\mathcal{S}_I$. This 3-geometry must be homogeneous and isotropic. A close scrutiny of its three-dimensional Riemann curvature must yield no 'handles' to distinguish one point on $\mathcal{S}_I$ from any other, or distinguish one direction at a given point from any other. 'No handles' means that [*the 3D Riemann tensor* R^l_{ijk}] must be constructed algebraically from pure numbers and from the only 'handle-free' tensors that exist: the 3-metric γ_{ij} and the three-dimensional Levi-Civita tensor ϵ_{ijk}. (All other tensors pick out preferred directions or locations.) One possible expression for R^l_{ijk} is
>
> $$R_{ijkl} = K\big(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}\big); \quad K = \text{"curvature parameter"} = \text{constant} \quad . \tag{3.61}$$
>
> Trial and error soon convince one that this is the *only* expression that both has the correct symmetries for a curvature tensor and can be constructed solely from constants, γ_{ij} and ϵ_{ijk}. Hence, this must be the three curvature of $\mathcal{S}_I$. (One says that any manifold with a curvature tensor of this form is a manifold of '*constant curvature*.')
>
> "As one might expect, the metric for $\mathcal{S}_I$ is completely determined, up to coordinate transformations, by the form [*equation (3.61)*,] of its curvature tensor. [*sic*] With an appropriate choice of coordinates, the metric reads [*sic*],[1]
>
> $$d\sigma^2 = \gamma_{ij}dx^i dx^j = K^{-1}[d\xi^2 + \sin^2\xi(d\theta^2 + \sin^2\theta\ d\phi^2)] \quad \text{if} \quad K > 0 \quad , \tag{3.62}$$
>
> $$d\sigma^2 = \gamma_{ij}dx^i dx^j = d\xi^2 + \xi^2(d\theta^2 + \sin^2\theta\ d\phi^2) \quad \text{if} \quad K = 0 \quad , \tag{3.63}$$
>
> $$d\sigma^2 = \gamma_{ij}dx^i dx^j = (-K)^{-1}[d\xi^2 + \sinh^2\xi(d\theta^2 + \sin^2\theta\ d\phi^2)] \quad \text{if} \quad K < 0 \quad . \tag{3.64}$$
>
> Absorb[2] the $K^{-1/2}$ or $(-K)^{-1/2}$ into the expansion factor $a(t)$ [*sic*], and define the

[1] In the original text of this excerpt [27], the letter χ was used where we write ξ.

[2] Here, "absorb" refers to an area of physics where the mathematical rigor is less than superb with respect to what absorption is. We have previously chosen, in reference [12], to "suppress one power of π so $\partial_t|\psi\rangle = \Phi m|\psi\rangle$" and here the authors refers to the exact same thing as absorption.

function

$$\Sigma \equiv \sin\xi, \quad \text{if} \quad k \equiv K/|K| = +1 \quad (\text{“positive spatial curvature”}) \ , \tag{3.65}$$

$$\Sigma \equiv \xi, \quad \text{if} \quad k \equiv K = 0 \quad (\text{“zero spatial curvature”}) \ , \tag{3.66}$$

$$\Sigma \equiv \sinh\xi, \quad \text{if} \quad k \equiv K/|K| = -1 \quad (\text{“negative spatial curvature”}) \ . \tag{3.67}$$

Thus write the full spacetime geometry in the form

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \quad , \tag{3.68}$$

$$\gamma_{ij}\, dx^i\, dx^j = d\xi^2 + \Sigma^2(d\theta^2 + \sin^2\theta\ d\phi^2), \tag{3.69}$$

and the three-curvatures of the homogeneous hypersurfaces[1] in the form

$$^{(3)}R_{ijkl} = [k/a^2(t)][\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}] \quad . \tag{3.70}$$

The curvature parameter K, after this renormalization, is evidently

$$K = k/a^2(t) \quad . \tag{3.71}$$

“Why is the word ‘renormalization’ appropriate? Previously $a(t)$ was a scale factor describing expansion of linear dimensions relative to the linear dimensions as they stood at some arbitrarily chosen epoch; but the choice of that fiducial epoch was a matter of indifference. Now $a(t)$ has lost that arbitrariness. It has been normalized so that its value here and now gives the curvature of a spacelike hypersurface of homogeneity here and now. Previously the factor $a(t)$ was conceived as dimensionless. Now it has dimensions of a length. This length is called the ‘radius of the model universe’ when the curvature is positive. Even when the

[1]These solutions in particular, in addition to Ashtekar and Singh's question about emergent time in reference [45], are what motivated the original object $\{\aleph, \mathcal{H}, \Omega\}$. In reference [45], Ashtekar and Singh asked, “Can we extract, from the arguments of the wavefunction, one variable which can serve as emergent time with respect to which the other arguments ‘evolve?”’ It was the preexisting knowledge of this solution that made this writer immediately recognize the Gel'fand triple as the correct algebraic dual to the $K \in \{\pm1, 0\}$ geometries in the framework where Ashtekar and Singh's “emergent time” is the superposition of the positive and negative time modes. Brian Kennedy had assigned Chris Isham's book *Lectures on Quantum Theory* as part of this writer's first graduate course in quantum mechanics and, within that excellent book, this writer found the above mentioned reference to Gel'fand's object.

Kennedy criticized the students in this writer's first semester graduate course for doing so poorly on an exam. This writer was absent for the criticism because Kennedy was handing back the test and reviewing students' errors, but this writer was confident in having aced the test. However, Kennedy gave this writer 5/20 on one of three 20 point problems and left a note asking why the problem wasn't solved via the method he suggested in class. This writer went to Kennedy's office hours the following week and showed him that if the three parameters left in this writer's solution were set to unity then it was correct. Kennedy immediate agreed and upped the score to 15/20 which made put this writer well above the grade Kennedy had cited as a threshold in his criticism to the class. This writer was really bitter about that because if there has ever been a 19/20 partial credit answer in the history of physics, then this writer's solution was it. The extra four points would have given this writer the highest grade in the class. This writer would have almost got 100% on the heavily weighted test almost ensuring an A course grade after the final. However, Kennedy docked four points because he had made an error in grading by not recognizing this writer's alternative methodology. Kennedy compounded his error by leaving a note about not doing it his way, and did not even realize that this writer had aced the test during post-test lecture during the week proceeding the office hours in question. This writer was embittered by those four points and, after that, was not interested in trying to get an A or being particularly studious in Kennedy's second semester quantum mechanics course. However, Kennedy deserves thanks for adding a second textbook to the syllabus for the breadth of his students' exposure to the fundamental issues. Isham's book has been very helpful in this research program, and even inspirational in its direct straightforwardness.

curvature is negative one sometimes speaks of $a(t)$ as a 'radius.' Only for zero curvature does the normalization of $a(t)$ still retain its former arbitrariness. Thus, for zero curvature, consider two choices for $a(t)$, one of them $a(t)$ and the other $\bar{a}(t) = 2a(t)$. Then with $\bar{\xi} = \frac{1}{2}\xi$, one can write proper distances in the three dimensions of interest with perfect indifference in either of two ways:

$$\begin{pmatrix}\text{proper distance}\\ \text{in the direction}\\ \text{of increasing } \xi\end{pmatrix} = a(t)\,d\xi = \bar{a}(t)\,d\bar{\xi} \quad , \tag{3.72}$$

$$\begin{pmatrix}\text{proper distance}\\ \text{in the direction}\\ \text{of increasing } \theta\end{pmatrix} = a(t)\xi\,d\theta = \bar{a}(t)\bar{\xi}\,d\theta \quad , \tag{3.73}$$

$$\begin{pmatrix}\text{proper distance}\\ \text{in the direction}\\ \text{of increasing } \phi\end{pmatrix} = a(t)\xi\sin\theta\,d\theta = \bar{a}(t)\bar{\xi}\sin\theta\,d\phi \quad . \tag{3.74}$$

"No such freedom of choice is possible when the model universe is curved, because then the ξ's in the last two lines are replaced by a function, $\sin\xi$ or $\sinh\xi$, that is not linear in its argument.

"Despite the feasibility in principle of determining the absolute value of the 'radius' $a(t)$ of a curved spacetime, in practice [*1973's*] accuracy falls short of what is required to do so. Therefore it is appropriate in many contexts to continue to regard $a(t)$ as a factor of relative expansion, the absolute value of which one tries to keep from entering into any equation exactly because it is difficult to determine."

The first thing we will point out is that Misner, the first author of reference [27], is the "M" in ADM, whose result [46] we have rejected in reference [3] on the basis of modern CMB data that shows there is a heavenly multipole moment that we can very much "grasp" as a "handle" in the manifold. We are not using "no handles." The hats on all four of the ontological numbers are handles and we have formulated χ^5 as another sort of handle. In the above excerpt, the authors state that the implication of no-handles is that the Riemann tensor "must be constructed algebraically from pure numbers and from the only 'handle-free' tensors that exist: the 3-metric γ_{ij} and the three-dimensional Levi-Civita tensor ϵ_{ijk}." Since the MCM does include handles, we will need to verify whether or not the MCM allows additional pieces to contribute to the Riemann curvature tensor. We have not given much attention to the 3D metric γ_{ij} since the main avenue of complexification is expected along the high-dimensional channels unique to $\Sigma^{\pm}$. Of all the ways to deform the flat Minkowski space, there is a subset of simpler deformations that only change the geometry of 3-space. Among that subset, there is a simpler subset of scalar deformations that curve Minkowski space by some constant factor everywhere in the universe. The spaces $\aleph$ and Ω have been assigned as de Sitter space and anti-de Sitter space exactly because they of this last variety.

Minkowski space, de Sitter space, and anti-de Sitter space are all maximally symmetric. $\{\aleph, \mathcal{H}, \Omega\}$ is *especially* maximally symmetric.

It is important to note that while $\mathcal{H}$ has a simple metric

$$ds^2_{\pm} = -(dx^0)^2 + \sum_{i=1}^{3}(dx^i)^2 \quad , \tag{3.75}$$

$\aleph$ and Ω have different metrics that are non-trivial despite their respective conditions of maximal symmetry. Carroll develops those metrics as follows [17].

> "The maximally symmetric spacetime with positive curvature ($\kappa > 0$) is called **de Sitter space**. Consider a five-dimensional Minkowski space with metric $ds_5^2 = -du^2 + dx^2 + dy^2 + dz^2 + dw^2$, and embed a hyperboloid given by
>
> $$-u^2 + x^2 + y^2 + z^2 + w^2 = \alpha^2 \quad . \tag{3.76}$$
>
> Now introduce coordinates $\{t, \chi, \theta, \phi\}$ on the hyperboloid via
>
> $$u = \alpha \sinh(t/\alpha) \tag{3.77}$$
>
> $$w = \alpha \cosh(t/\alpha) \cos\chi \tag{3.78}$$
>
> $$x = \alpha \cosh(t/\alpha) \sin\chi \cos\theta \tag{3.79}$$
>
> $$y = \alpha \cosh(t/\alpha) \sin\chi \sin\theta \cos\phi \tag{3.80}$$
>
> $$z = \alpha \cosh(t/\alpha) \sin\chi \sin\theta \sin\phi \quad . \tag{3.81}$$
>
> The metric on the hyperboloid is then
>
> $$ds^2 = -dt^2 + \alpha^2 \cosh^2(t\alpha) \left[d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \quad . \tag{3.82}$$
>
> We recognize the expression in round brackets as the metric on a two-sphere, $d\Omega_2^2$, and the expression in square brackets as the metric on a three-sphere, $d\Omega_3^2$. Thus, de Sitter space describes a spatial three-sphere that initially shrinks, reaching a minimum size at $t = 0$, and then re-expands.[1] Of course this particular description is inherited from a certain coordinate system; we will see that there are equally valid alternative descriptions.
>
> "These coordinates cover the entire manifold. You can generally check this by, for example, following the behavior of geodesics near the edges of the coordinate system; if the coordinates were incomplete, geodesics would appear to terminate in finite affine parameter. The topology of de Sitter is thus $\mathbb{R} \times \mathbb{S}^3$. This makes it very

[1] Shrinking, reaching a minimum, and then reexpanding is also known as "bouncing."

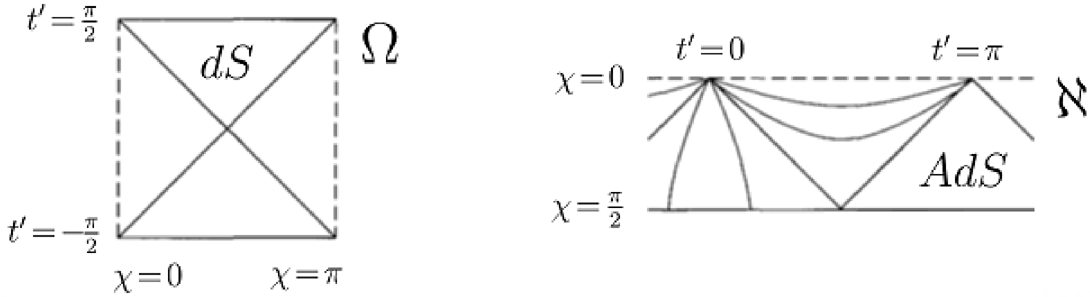


Figure 29: These conformal diagrams are most appreciatively reproduced from Carroll's textbook [17]: *Spacetime and Geometry: An Introduction to General Relativity.*

simple to derive the conformal diagram, since the important step in constructing conformal diagrams is to write the metric in a form which is conformally related to the Einstein static universe (a spacetime with topology $\mathbb{R} \times \mathbb{S}^3$, describing a spatial three-sphere with constant radius through time). Consider the coordinate transformation from t to t' via

$$\cosh(t/\alpha) = \frac{1}{\cos(t')} \quad . \tag{3.83}$$

[*Equation (3.82)*] now becomes

$$ds^2 = \frac{\alpha^2}{\cos^2(t')} d\bar{s}^2 \quad , \tag{3.84}$$

where $d\bar{s}^2$ represents the metric of the Einstein static universe,

$$d\bar{s}^2 = -(dt')^2 + d\chi^2 + \sin^2\chi d\Omega_2^2 \quad . \tag{3.85}$$

The range of the new time coordinate is

$$-\pi/2 < t' < \pi/2 \quad . \tag{3.86}$$

"The conformal diagram of de Sitter space will simply be a representation of the patch of the Einstein static universe to which de Sitter is conformally related. It looks like a square, as shown [*on the left side of figure 29*]. A spacelike slice of constant t' represents a three-sphere; the dashed lines at the left and right edges are the north and south poles of this sphere. The diagonal lines represent null rays; a photon released at past infinity will get precisely to the antipodal point on the sphere at future infinity. Keep in mind that the spacetime 'ends' to the past and future only through the magic of conformal transformations; the actual de Sitter space extends indefinitely into the future and past. Note also that two points can have future (or past) light cones that are completely disconnected; this reflects the fact that the spherical spatial sections are expanding so rapidly that light from one point can never come into contact with light from the other.

"A similar hyperboloid construction reveals the $\kappa < 0$ spacetime of maximal symmetry, known as **anti-de Sitter** space. Begin with a fictitious five-dimensional manifold with metric $ds_5^2 = -du^2 - dv^2 + dx^2 + dy^2 + dz^2$, and embed a hyperboloid given by

$$-u^2 - v^2 + x^2 + y^2 + z^2 = -\alpha^2 \quad . \tag{3.87}$$

Note all the minus signs. Then we can induce coordinates $\{t', \rho, \theta, \phi\}$ on the hyperboloid via

$$u = \alpha \sin(t') \cosh(\rho) \tag{3.88}$$

$$v = \alpha \cos(t') \cosh(\rho) \tag{3.89}$$

$$x = \alpha \sinh(\rho) \cos\theta \tag{3.90}$$

$$y = \alpha \sinh(\rho) \sin\theta \cos\phi \tag{3.91}$$

$$z = \alpha \sinh(\rho) \sin\theta \sin\phi \quad , \tag{3.92}$$

yielding a metric on this hyperboloid of the form

$$ds^2 = \alpha^2 \left(-\cosh^2(\rho) dt'^2 + d\rho^2 + \sinh^2(\rho) d\Omega_2^2 \right) \quad . \tag{3.93}$$

These coordinates have a strange feature, namely that t' is periodic.[1] From [*equation (3.88)*], t' and $t' + 2\pi$ represent the same place on the hyperboloid. Since $\partial_{t'}$ is everywhere timelike, a curve with constant $\{\rho, \theta, \phi\}$ as t' increases will be a closed timelike curve. However, this is not an intrinsic property of the spacetime, merely an artifact of how we have derived the metric from a particular embedding. We are welcome to consider the 'covering space' of this manifold, the spacetime with the metric given by (3.93) in which we allow t' to range from $-\infty$ to ∞. There are no closed timelike curves in this space, which we will take to be the definition of anti-de Sitter space.

"To derive the conformal diagram, perform a coordinate transformation analogous to that used for de Sitter, but now on the radial coordinate:

$$\cosh(\rho) = \frac{1}{\cos\chi} \quad , \tag{3.94}$$

so that

$$ds^2 = \frac{\alpha^2}{\cos^2\chi} d\bar{s}^2, \tag{3.95}$$

where $d\bar{s}^2$ represents the metric on the Einstein static universe (3.85). Unlike in de Sitter, the radial coordinate now appears in the conformal factor. In addition,

[1] The MCM proposal to modify spacetime by wrapping time around a cylinder, a.k.a. giving x^0 a circular topology, has the same effect of making time periodic.

for anti-de Sitter, the t' coordinate goes from minus infinity to plus infinity, while the range of the radial coordinate is

$$0 \leq \chi < \frac{\pi}{2} \quad . \tag{3.96}$$

Thus, anti-de Sitter space is conformally related to half of the Einstein static universe. The conformal diagram is shown [*on the right side of figure 29*], which illustrates a few representative timelike and spacelike geodesics passing through the point $t' = 0$, $\chi = 0$. Since χ only goes to $\pi/2$ rather than all the way to π, a spacelike slice of this spacetime has the topology of the interior of a hemisphere $\mathbb{S}^3$; that is, it is topologically $\mathbb{R}^3$ (and the entire spacetime therefore has the topology $\mathbb{R}^4$). Note that we have drawn the diagram in polar coordinates, such that a point on the left side represents a point at the spatial origin, while [*the*] one on the right side represents a two-sphere at spatial infinity. Another popular representation is to draw the spacetime in cross-section, so that the spatial origin lies in the middle and the right and left sides together comprise spatial infinity.[1]

"An interesting feature of anti-de Sitter is that infinity takes the form of a timelike hypersurface, defined by $\chi = \pi/2$. Because infinity is timelike, the space is not globally hyperbolic, we do not have a well-posed initial value problem in terms of information specified on a spacelike slice, since information can always 'flow in from infinity.' Another interesting feature is that the exponential map is not onto the entire spacetime; geodesics, such as those drawn [*on the right side of figure 29*], which leave from a specified point do not cover the whole manifold. The future-pointing timelike geodesics, as indicated, can initially move radially outward from $t' = 0$, $\chi = 0$, but eventually refocus to the point $t' = \pi, \chi = 0$ and will then move radially outward once again.[2]

"As an aside it is irresistible to point out that the timelike nature of infinity enables a remarkable feature of string theory, the 'AdS/CFT correspondence.' Here, AdS is of course the anti-de Sitter space we have been discussing, while CFT stands for conformally-invariant field theory defined on the boundary (which is, for an n-dimensional AdS, an $(n-1)$-dimensional spacetime in its own right). The AdS/CFT correspondence suggests that, in a certain limit, there is an equivalence between quantum gravity (or a supersymmetric version thereof) on an AdS background and a conformally-invariant non-gravitational field theory defined on the boundary. Since we know a lot about non-gravitational quantum field theory that we don't know about quantum gravity, this correspondence (if true, which seems likely but remains unproven[3]) reveals a great deal about what can happen in quantum gravity."

Carroll's example of embedded hyperboloids demonstrates exactly what we have done to embed the chronological dS and AdS metrics onto the chirological slices of $\Sigma^{\pm}$. We have included these long excerpts mainly to demonstrate the general relevance of the MCM:

[1] Note how Carroll's description of the options for drawing the conformal diagram mirror our own options for drawing the MCM unit cell centered on $\mathcal{H}$ or $\varnothing$

[2] This is ordinarily described as bouncing.

[3] See reference [3] for a proof.

almost every type of thing we have imposed as an “unmotivated” constraint was already an inherent constraint on the objects from which the MCM is assembled. Furthermore, Carroll shows how the conformal diagrams of dS and AdS are constructed from co-$\hat{\pi}$s or pairs of halves of co-$\hat{\pi}$s, as in figure 29. The important takeaway from figure 29 is that where dS is constructed naturally on two complete co-$\hat{\pi}$s, AdS is only constructed from one and a half co-$\hat{\pi}$s. Through the presence of half co-$\hat{\pi}$s we can see that it will be easy to modularize the coordinate systems described by Carroll for immediate application to the construction of complete $\hat{\pi}$-sites in the MCM. Any advanced calculation using these coordinates would be impractical to carry out by hand, as is the analytical style of this writer and others, as in figure 30. On the up side, the interested party should be excited to know that tensor algebra solver software exists and is described in reference [48]. Williams’ paper can likely be trusted as the source of record for many common formulae that appear in various states of correctness and incorrectness throughout the literature because he has derived them with software.[1]

III.8 Toward Geodesics

Geodesics are another topic better handled on computers. Wikipedia says, “The full Kaluza equations under the cylinder condition are quite complex, and most English-language reviews as well as the English translations of [*foreign language reports*] contain some errors.” If we

[1]Regarding imperfect formulae, even software can have errors, and, in any case, a comprehensive survey of the literature is always in order. Regarding literature known to contain errata: just as a literate person can understand a sentence with a misspelled word in it, so can a numerate person often recognize what an equation demonstrates even when its formalism is not algorithmically impeccable. Therefore, when the critic is numerate, criticisms of the form, “This is wrong,” or, “This is not even wrong,” can sometimes be rebutted with, “Did I demonstrate something there?” If the answer is yes then the target of the criticism might close his rebuttal with, “*Q.E.D.*”

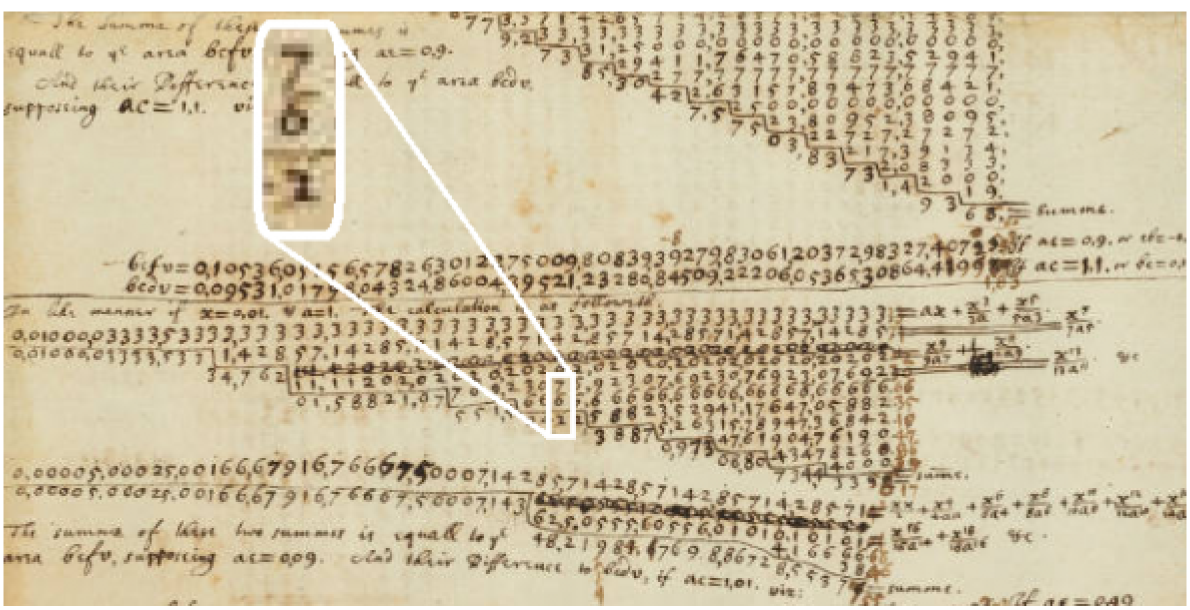


Figure 30: Calculation of the geodesics of the MCM will require a computational facility much greater than the one relied upon by this writer. That facility is demonstrated in this figure which shows a page excerpted from reference [47]

.

did undertake the effort to calculate by hand and then report the geodesics of the MCM, our report would almost certainly contain errors of this sort and thereby we would have added little to the literature in our publication of an erroneous or even error-riddled set of geodesics. Certainly this is an exercise best left to computer experts or until this writer becomes more expert with computers. We very much refer the reader to reference [48] wherein Williams has done a great service to mankind putting together a comprehensive and seemingly error-free review of common formulae related to the 5D Kaluza–Klein metric which is

$$\Sigma_{AB} = \begin{pmatrix} -1+A_1A_1\chi^5 & A_1A_2\chi^5 & A_1A_3\chi^5 & A_1A_4\chi^5 & A_1\chi^5 \\ A_2A_1\chi^5 & 1+A_2A_2\chi^5 & A_2A_3\chi^5 & A_2A_4\chi^5 & A_2\chi^5 \\ A_3A_1\chi^5 & A_3A_2\chi^5 & 1+A_3A_3\chi^5 & A_3A_4\chi^5 & A_3\chi^5 \\ A_4A_1\chi^5 & A_4A_2\chi^5 & A_4A_3\chi^5 & 1+A_4A_4\chi^5 & A_4\chi^5 \\ A_1\chi^5 & A_2\chi^5 & A_3\chi^5 & A_4\chi^5 & \chi^5 \end{pmatrix}$$

$$= \begin{pmatrix} g_{\alpha\beta}+A_\alpha A_\beta\chi^5 & A_\alpha\chi^5 \\ A_\beta\chi^5 & \chi^5 \end{pmatrix} \quad . \tag{3.97}$$

We will not complete the calculation here because geodesics are too complicated to do efficiently by hand. Note that the formulas in reference [48] use the cylinder condition throughout the 5D space. In the MCM, we are only forced to apply the cylinder condition on the slices $\aleph$ and Ω but not necessarily on each individual slice of $\Sigma^\pm$ where $\chi^5_\pm \not\in \{\Phi, 0, -1\}$. $\mathcal{H}$ and $\varnothing$ are further differentiated from $\aleph$ and Ω when we note that the cylinder condition is not a prerequisite for $\{\mathcal{H}, \varnothing\}$. $\mathcal{H}$ is a flat 4D brane embedded in a smoothly varying continuum of de Sitter branes and that continuum, including $\mathcal{H}$, is constrained to have a representation in general relativity where $x^4 = 0$ is the parameter of curvature. $\mathcal{H}$ can therefore be apart from a Kaluza–Klein theory with its inherent scalar field ϕ^2 induced through the cylinder constraint. $\varnothing$ is at infinity and its topology is defined as the topology of $\mathcal{H}$ because the only thing we might ever consider using $\varnothing$ for is to define a Lorentz frame. So $\varnothing$ is defined thusly and $\Sigma^\varnothing$ is constrained accordingly. The Lorentz approximation is that the lab frame is Minkowski space devoid of any perturbations due to the mass-energy of lab equipment, planets, moons, scientists, *etc.*, so, therefore, we can always define that frame in $\varnothing$. The MCM condition means that the origin of coordinates in $\varnothing$ would be the point of $\Sigma^\varnothing$ that stitches $\chi^5_\pm$ together in the piecewise definition of geodesics parameterized in χ^5. If we were going to undertake computerization, a first check on the robust stability of MCM would be to compare its geodesics in the cases where $A_\alpha = 0$ and $A_\alpha = 1$. Since it is only potential differences that contribute to observables, we should get the exact same geodesics for any constant potential A_α.

Everything in the geodesic equation comes from the derivatives of the metric. The non-tensorial Christoffel symbols (connection coefficients) are

$$\Gamma^C_{AB} = \tfrac{1}{2}\,\Sigma^{CD}\left(\partial_A\Sigma_{BD} + \partial_B\Sigma_{DA} - \partial_D\Sigma_{AB}\right) \quad , \tag{3.98}$$

and they appear in the geodesic equation as

$$\frac{d^2\chi^C}{d\lambda^2} + \Gamma^C_{AB}\frac{d\chi^A}{d\lambda}\frac{d\chi^B}{d\lambda} = 0 \quad . \tag{3.99}$$

The geodesic equation reflects an impressive amount of complexity already present in Einstein's theory from 100 years ago. Certainly, Einstein carried out more intense manual computations than we have in this research program. In fact, it is due to Einstein's efforts that we recognized the dimensionless coefficient of proportionality 8π as highly relevant in reference [12]. It must be noted that Einstein had no hope of using computers to make his calculations and that his contemporaries such as Schwarzschild, Nordström, and others were the first to make many of the most elementary calculations in general relativity. Likewise, any of this writer's contemporaries could be the first to make elementary calculations in the MCM but it seems science has taken a turn since the olden times.

To find the connection coefficients we need the inverse metric which is

$$\Sigma^{AB} = \begin{pmatrix} -1 & 0 & 0 & 0 & A_1 \\ 0 & 1 & 0 & 0 & -A_2 \\ 0 & 0 & 1 & 0 & -A_3 \\ 0 & 0 & 0 & 1 & -A_4 \\ A_1 & -A_2 & -A_3 & -A_4 & \left(\eta^{\alpha\beta}A_\alpha A_\beta + \dfrac{1}{\chi^5}\right) \end{pmatrix} \quad . \tag{3.100}$$

In reference [10], we pointed out that since $\chi^5 \equiv \chi^5_+ \otimes \chi^5_\varnothing \otimes \chi^5_-$ is a non-standard pseudo-dimension, we will have issues understanding $\hat{M}^3 \equiv \partial^3$ in terms of the chirological time. However, x^0 is not a pseudo-dimension so, in the chronological sense, $\hat{M}^3 \equiv \partial^3_t$ and this is where Einstein's 8π comes from. We had to define an alternate convention for chirological ∂^3_χ which appears in reference [10]. That convention is

$$\partial^3_5 \quad \equiv \quad \frac{\partial}{\partial\chi^5_-}\frac{\partial}{\partial\chi^5_\varnothing}\frac{\partial}{\partial\chi^5_+} \quad . \tag{3.101}$$

To evaluate these derivatives, we assume a certain set of gauge transformations

$$\psi' = e^{i\Lambda}\psi \qquad \longleftrightarrow \qquad A'_\mu = A_\mu + \frac{1}{e}\,\partial_\mu\Lambda \quad , \tag{3.102}$$

according to the prescription in reference [10]. In that reference, we showed only how the wavefunction transforms but here we will also consider the transformation of the potential because it drives the transformation of the metric and geodesics. To derive the fully relativistic gauge transformation, we will need to substitute the covariant derivative for the partial derivative that appears in equation (3.102) but we will proceed in the limit where the covariant derivative is equal to the partial derivative. Therefore, what follows in this section is only an estimate.

To assess the feasibility of deriving a non-trivial "ontological" potential that could be used to define a specific metric and therefore a specific set of geodesics, we will start with

$$\psi_0 = e^{-i\omega t} \quad , \qquad \text{and} \qquad A_\mu = 0 \quad . \tag{3.103}$$

We will make gauge transformations to accommodate the operator ∂_5^3 according to equation (3.101). The first derivative in ∂_5^3 is with respect to χ_+^5 so we make the gauge transformation with

$$\Lambda_1 = \Phi\chi_+^5 + \omega t \quad . \tag{3.104}$$

This gives the wavefunction

$$\psi_1 = e^{i\Lambda_1}\psi_0 = e^{i(-\omega t + \Phi\chi_+^5 + \omega t)} = e^{i\Phi\chi_+^5} \quad . \tag{3.105}$$

Gauge theory requires a corresponding transformation of the potential, as in equation (3.102). The transformations employed here do not affect the A_i components of the 4-potential. Those components, as they appear in the gauge transformed potential, will be proportional to the ∂_i derivatives of Λ_1 and it is obvious that those will all vanish. The new potential is

$$A_0^{\{1\}} = \frac{1}{e}\,\partial_0\big(\Phi\chi_+^5 + \omega t\big) = \frac{\omega}{e} \tag{3.106}$$

$$A_i^{\{1\}} = A_i = 0 \quad , \tag{3.107}$$

To apply $\partial_\varnothing$ we need to apply another gauge transformation

$$\psi_2 = e^{i(\Phi\chi_+^5 + \Lambda_2)} \quad , \qquad \text{with} \qquad \Lambda_2 = \pi\chi_\varnothing^5 - \Phi\chi_+^5 \quad , \tag{3.108}$$

which gives

$$\psi_2 = e^{i\pi\chi_\varnothing^5} \quad . \tag{3.109}$$

The derivative of Λ'' with respect to x^μ vanishes so there is no change in the potential and we see

$$A_0^{\{2\}} = A_0^{\{1\}} = \frac{\omega}{e} \tag{3.110}$$

$$A_i^{\{2\}} = A_i^{\{1\}} = A_i = 0 \quad . \tag{3.111}$$

We need to complete taking ∂_5^3 with one more gauge transformation for ∂_- which is

$$\psi_3 = e^{i(\pi\chi_\varnothing^5 + \Lambda_3)} \quad , \qquad \text{with} \qquad \Lambda_3 = -\Phi\chi_-^5 - \pi\chi_\varnothing^5 \quad , \tag{3.112}$$

and this gives

$$\psi''' = e^{-i\Phi\chi_-^5} \quad . \tag{3.113}$$

Again, the derivative of Λ_3 with respect to x^μ vanishes so

$$A_0^{\{3\}} = A_0^{\{2\}} = A_0^{\{1\}} = \frac{\omega}{e} \tag{3.114}$$

$$A_i^{\{3\}} = A_i^{\{2\}} = A_i^{\{1\}} = A_i = 0 \quad . \tag{3.115}$$

The final remaining step is to convert back to a wavefunction of the x^μ coordinates which we can do in a few separate ways. This is very interesting and surely contains a lot more nuance than will be discussed here. We have started with an ordinary wavefunction $e^{-i\omega t}$ and used gauge transformation operations to take derivatives with respect to the three chirological times in the $\hat{M}^3 \equiv \partial_5^3$ operator. The final gauge transformation will be applied to convert the wavefunction back into a form that does not depend on chiros.

These two simple options, a and b, for completing $\psi_0(x^\mu) \rightarrow \psi_4(x^\mu)$ are

$$\psi_4^{a\pm} = e^{\mp i\omega t} \quad , \qquad \text{or} \qquad \psi_4^{b\pm} = e^{\pm ikx} \quad , \tag{3.116}$$

and there are also non-simple forms where the phase has mixed dependence on both time and space with $\Delta \equiv kx - \omega t$. Even if we choose the simpler option where the initial and final states both depend on t but not x, we still have the option to choose the sign that changes the direction of the wavepacket's propagation. Figure 31 illustrates how this arrangement

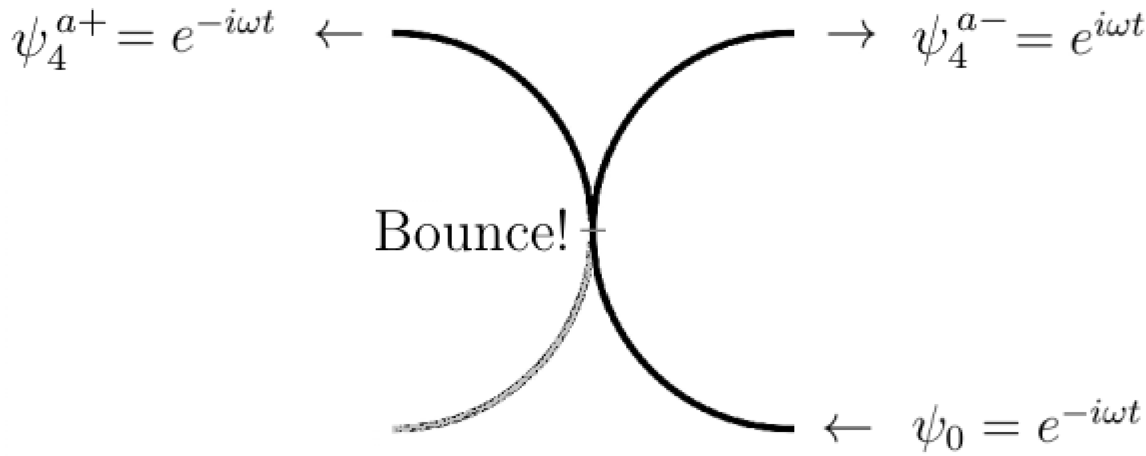


Figure 31: A plane wave is incident on the bounce state from the bottom right. The bounce is generally understood, by now, to lie at either the endpoints of the MCM unit cell or the central point corresponding to $p \in \Sigma^{\varnothing}$. The incident wave can be either transmitted or reflected by the bounce.

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is very natural to the MCM. Also note that ψ_4 seems logically related to the new process $\tilde{M}^4$ though it arose purely through consideration of $\hat{M}^3 \equiv \partial^3$. We must associate the two forms ψ_4^a and ψ_4^b with the idea that the topology on either side of $\mathcal{H}$ has different numbers of spacelike and timelike dimensions. A 5D plane wave incident on $\mathcal{H}$ from Σ^-, if transmission is possible, must have some component converted from a spacelike domain onto a timelike domain. Since timelike dimensions use dimensional transposing parameters such as c, we can expect that this change of domain significantly alters the character of the transmitted wave. In fact, it is not outlandish to think that the dimensional transposing parameter might even alter the energy of the wave as it passes through $\mathcal{H}$. However, here we remain focused on the gauge potential that arises due to the gauge transformation of the wavefunction presented above.

To achieve ψ_4^a or ψ_4^b will have to consider two Λ_4's

$$\Lambda_4^{a\pm} = \mp\omega t + \Phi\chi_-^5 \quad , \qquad \text{and} \qquad \Lambda_4^{b\pm} = \pm kx + \Phi\chi_-^5 \quad . \tag{3.117}$$

If we choose to arrive at ψ_4^a then the $A_i^{\{4a\}}$ components will all vanish and the final gauge transformed potential is

$$A_0^{\{4a\mp\}} = \frac{\omega}{e} + \frac{1}{e}\partial_0\big(\mp\omega t + \Phi\chi_-^5\big) = \frac{\omega}{e} \mp \frac{\omega}{e} \tag{3.118}$$

$$A_i^{\{4a\mp\}} = 0 \quad . \tag{3.119}$$

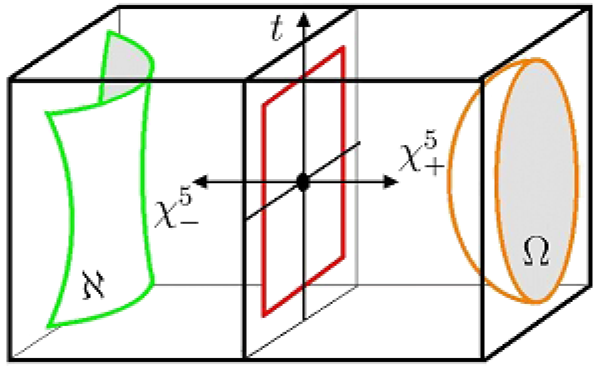

Figure 32: The original graphical iteration of the MCM unit cell is, perhaps, the most intuitive representation of its important geometric features.

These values for $A_\mu^{\{4\}}$ look very good. If we select the initial and final wavefunctions as $e^{-i\omega t}$ then the potential at the end is equal to the potential at the start: $A_\mu = 0$. However, at some point (not presently), we will need to study the frequency doubling in the induced potential

$$A_0^{\{4a+\}} = \frac{2\omega}{e} \quad , \tag{3.120}$$

associated with a reflection of the wavepacket, as in the upper right figure 31.

For Λ_4^b we have

$$A_0^{\{4b\pm\}} = \frac{\omega}{e} + \frac{1}{e}\partial_0\big(\pm kx + \Phi\chi_-^5\big) = \frac{\omega}{e} \tag{3.121}$$

$$A_i^{\{4b\pm\}} = 0 + \frac{1}{e}\,\partial_i\big(\pm kx - \Phi\chi_-^5\big) = \pm\frac{k}{e} \quad , \tag{3.122}$$

which is more complicated than $A_\alpha^{\{4a\}}$ because it involves both x and t. In this case, the final potential will have non-vanishing components $A_i^{\{4a\}}$.

Here, we will state a motivation for assigning two timelike dimensions to Σ^+ and only one timelike dimension to Σ^-. For $\hat{M}^1$, we want to evolve some initial state along the chirological time instead of the chronological time. This is, in general, how the MCM mode of evolution differs from old physics. If we evolved the qubit with chronological time, then we would be in perfect agreement with what the classical theory says to do, sans modifications. In order to construct a second timelike path for new physics, we should assign the O(2,3) topology to Σ^+. To do this, we simply redefine the scalar field that appears in the 5D metric as

$$\phi^2 \quad \equiv \quad \chi^5 \qquad \longrightarrow \qquad \phi^2 \quad \equiv \quad -\chi^5 \quad . \tag{3.123}$$

Now the signs in $\Sigma^{\pm}_{AB}$ will show that Σ^{+} has two timelike dimensions and Σ^{-} will have four spacelike dimensions.

Consider the role of the second timelike dimension in Σ^{+} after we formulate an initial condition at $\chi^5=0$. We know that the qubit will evolve in proper time according to the Schrödinger equation but we want to add the chirological mode of evolution along geodesics so that we may explain aspects of quantum weirdness with "interference effects" between the two timelike evolutionary channels. We evolve the initial qubit through $\chi^5_+\in(0,\Phi]$ in Σ^{+}, and note that the interval $(0,\Phi]$ is just long enough to hold one half of one co-$\hat{\pi}$, or one half of one $\hat{\pi}$ vector, just as is required to construct the conformal diagram of AdS shown in figure 29. This follows because

$$\frac{\pi}{2}\approx 1.57 \quad , \qquad\qquad \text{and} \qquad\qquad \Phi\approx 1.62 \quad , \tag{3.124}$$

and it also follows that, in the modularization process for AdS and dS on co-$\hat{\pi}$s, we can expect characteristic remainder terms on the order of $1.62-1.57=0.05<<1$ which might be used to make perfect geometry fuzzy. We will discuss the remainder $\Phi-\pi/2$ extensively in sections IV.1 and IV.5.

III.9 Dark Energy and Expanding Space

Here, we return to the MCM's first result: dark energy. Unification of the theories of gravitation and quantization is a vastly more primary problem than dark energy because it predates all of what we now call modern physics, or post-war physics, but dark energy is still very important. The MCM dark energy result [2, 7] was followed by a new numerical formula for the fine structure constant [22, 12], a derivation of Einstein's equation based on the numbers in that formula [12], a new geometric explanation for the AdS/CFT correspondence [3], an explanation for the structure of the fundamental particles in the standard model as facets of the AdS/CFT geometry [11], a prediction for two new spin-1 particles [3], and many other interesting results [9, 49, 13, 4, 5, 23, 10, 8]. The MCM mechanism for dark energy, called inverse radial spaghettification, can be thought of as ordinary spaghettification toward the past (to the left) in the cosmological unit cell where the radial coordinate can be intuitively extrapolated from the rectangular coordinates in the figures.

Using equation (3.123), consider the Kaluza–Klein metric in the limit of vanishing complexity. When $A_\alpha=0$, we have a pair of "ground state" metrics

$$\Sigma^{+\,O(2,3)}_{AB}=\begin{pmatrix}-1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&-\chi^5_+\end{pmatrix} \quad , \tag{3.125}$$

and

$$\Sigma_{AB}^{-\ O(1,4)} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\chi_{-}^{5} \end{pmatrix} \quad . \tag{3.126}$$

The form of each metric is identical and it is only the sign of $\chi_{\pm}^{5}$ that distinguishes the number of timelike dimensions. (χ_{-}^{5} is negative so $-\chi_{-}^{5} > 0$ and we are ignoring a likely complex phase between $\chi_{\pm}^{5}$.) The metrics $\Sigma_{AB}^{\pm}$ are essentially the same so we may follow the ordinary prescription to derive the line element for a generic Kaluza–Klein metric Σ_{AB}.

$$ds_{MCM}^{2} = \Sigma_{AB}\, d\chi^{A}\, d\chi^{B} = -\big(d\chi^{1}\big)^{2} + \big(d\chi^{2}\big)^{2} + \big(d\chi^{3}\big)^{2} + \big(d\chi^{4}\big)^{2} - \chi^{5}\big(d\chi^{5}\big)^{2} \quad . \tag{3.127}$$

On cosmological scales, $A_{\alpha}=0$ is good approximation for the nearly negligible intergalactic magnetic field so we should consider the implications of equation (3.127) as physical implications. At first glance, ds_{MCM}^{2} is very nearly the FLRW line element

$$ds_{FLRW}^{2} = -\big(dx^{0}\big)^{2} + a^{2}\big(x^{0}\big)\left[\big(dx^{1}\big)^{2} + \big(dx^{2}\big)^{2} + \big(dx^{3}\big)^{2}\right] \quad . \tag{3.128}$$

of flat expanding space. The FLRW metric is flat expanding rather than curved expanding because the scale factor $a^{2}(x^{0})$ only depends on x^{0} but not the spatial coordinates x^{i}. Note how the expansion factor in the FLRW metric is a squared function of a timelike dimension exactly like the expansion factor in equation (3.127). Using equation (3.123) to substitute the scalar field $\phi^{2}(\chi^{5})\equiv-\chi^{5}$ back into ds_{MCM}^{2} we see the two line elements are indeed similar with $a^{2}(x^{0})\to\phi^{2}(\chi^{5})$ in

$$ds_{MCM}^{2} = -\big(d\chi^{1}\big)^{2} + \big(d\chi^{2}\big)^{2} + \big(d\chi^{3}\big)^{2} + \big(d\chi^{4}\big)^{2} + \phi^{2}\big(\chi^{5}\big)\left[\big(d\chi^{5}\big)^{2}\right] \quad . \tag{3.129}$$

In equation (3.128), the scale factor stretches all three spatial dimensions equally but, in equation (3.129), the scale factor only stretches the fifth dimension. In Σ^{+}, where χ_{+}^{5} is a positive valued timelike dimension, the metric shows contracting time. On the other side of $\mathcal{H}$, where χ_{-}^{5} is negative and spacelike, the metric shows expanding space but not in all four spacelike dimensions.[1] **Expanding space** in the past is exactly what astrophysicists observe, and contracting time in the future is more or less the mechanism by which we have proposed to generate **dark energy** [2, 7]. Furthermore, the expansion of space by a parameter in $[-1,0)$ is unlikely to be exactly counterbalanced by a contraction of time proportional to a parameter in $(0,\Phi]$, and that imbalance can lead to a pressure gradient which causes the arrow of time to point preferentially in one direction.

[1] This might allow for the propagation of tachyons, as in reference [5].

The FLRW metric describes flat space because the expansion parameter is a function of a timelike variable. In Σ^-, the fifth dimension is spacelike so not only do we get expanding space there, we get expanding curved space exactly like what will be required to embed slices that contain 4D dS and AdS. Our example of expanding space only shows expanding space in one dimension so there is a natural interpretation related to increasing levels of $\aleph$ along χ^5. There is another natural interpretation would be to let the expansion of three spatial dimensions, which are properly astrophysical, be taken as a reciprocal effect derived from a modularized dependence on the expansion of the fourth spacelike dimension in Σ^-.

III.10 Advanced and Retarded Potentials

When approaching $\mathcal{H}$ from the directions of $\chi^5_\pm$, the discrepancy between the O(1,4) and O(2,3) topologies makes it impossible to model the boundary as a simple 1D transmission problem for the quantum mechanical probability current to pass through $\mathcal{H}$ (or $\varnothing$.) In the future Ω, it should be a productive exercise to model the MCM in 1D (motion along χ^5 only) to derive a hypercomplex potential function V that plugs into the Hamiltonian of the Schrödinger equation as

$$\hat{\mathbb{H}} = \frac{\hat{p}^2}{2m} + \hat{V} \quad . \tag{3.130}$$

With a well defined $\hat{V}$, we would solve for the transmission and reflection coefficients for the probability current in the hypercosmos. We mention that in this section because we may be able to identify those coefficients, T and R, with the advanced and retarded potentials. If T is the amplitude of the signal in the forward time direction from $\mathcal{H}_1$, then it reaches the next moment $\mathcal{H}_2$ as a retarded signal from the past. However, the signal that is reflected from $\mathcal{H}_1$ with amplitude R will move in reverse time, meaning that it will be incident on an earlier $\mathcal{H}$-brane (or an earlier $\varnothing$-brane) from the future, as is expected of the advanced potential.

Consider the place of the advanced potential as given by reference [50].

> "Electromagnetic radiation will here be considered as an example for wave phenomena in general.[1] It may be described in terms of the four-potential A^μ, which in the Lorenz gauge obeys the wave equation
>
> $$-\partial^\nu\partial_\nu A^\mu(\mathbf{r},t) = 4\pi j^\mu(\mathbf{r},t) \quad , \qquad \text{with} \qquad \partial^\nu\partial_\nu = -\partial_t^2 + \Delta \quad , \tag{3.131}$$
>
> with $c = 1$, where the notations[2] $\partial_\mu := \partial/\partial x^\mu$ and $\partial^\mu := g^{\mu\nu}\partial_\nu$ are used together with Einstein's [*summation convention*]. When an appropriate boundary condition is imposed, one may write A^μ as a functional of the sources j^μ. For two well known boundary conditions one obtains the *retarded* and the *advanced* potentials,

[1] Although the Schrödinger equation is the heat equation rather than the intuitive wave equation, it does certainly fall under the topic of wave phenomena in general.

[2] Δ is the Laplacian operator: $\Delta \equiv \nabla^2 \equiv \nabla \cdot \nabla$.

$$A^{\mu}_{\rm ret} = \int \frac{j^{\mu}(\mathbf{r}, t - |\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' \tag{3.132}$$

$$A^{\mu}_{\rm adv} = \int \frac{j^{\mu}(\mathbf{r}, t + |\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' \quad . \tag{3.133}$$

These two functionals of $j^{\mu}(\mathbf{r}, t)$ are related to one another by a reversal of retardation time $|\mathbf{r}-\mathbf{r}'|$ [*sic*]. Their linear combinations are solutions of [*equation (3.131).*]"

The origin of $A^{\mu}_{\rm adv}$ and $A^{\mu}_{\rm ret}$ is a tedious subject in classical electromagnetism and will be beyond the scope of this book. The point of this book is (mostly) to go into greater detail regarding the general relevance of that which has been published already. However, we do want to emphasize that when we impose the MCM condition that sets $t = 0$, the second argument in j^{μ} is completely natural to the MCM coordinates. For j^{μ} evaluated at the advanced time, we can expect

$$|\mathbf{r} - \mathbf{r}'| = \Phi \quad , \tag{3.134}$$

and for j^{μ} evaluated at the retarded time we expect

$$|\mathbf{r} - \mathbf{r}'| = -\varphi \quad . \tag{3.135}$$

Consider the following from reference [51].

"In 1909 Walter Ritz and Albert Einstein (former classmates at the University of Zurich) debated the question of whether there is a fundamental temporal asymmetry in electrodynamics, and if so, whether Maxwell's equations (as they stand) can justify this asymmetry. As mentioned above, the potential field equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -4\pi\rho \quad , \tag{3.136}$$

is equally well solved with either of two functions

$$\phi_1 = \int \frac{\rho(x, y, z, t - r/c)}{r}\, dx\, dy\, dz \qquad \phi_2 = \int \frac{\rho(x, y, z, t + r/c)}{r}\, dx\, dy\, dz \quad , \tag{3.137}$$

where ϕ_1 is called the retarded potential and ϕ_2 the advanced potential. Ritz believed the exclusion of the advanced potentials represents a physically significant restriction on the set of possible phenomena, and yet it could not be justified in the context of Maxwell's equations. From this he concluded that Maxwell's equations were fundamentally flawed, and could not serve as the basis for a valid theory of electrodynamics. Ironically, Einstein too did not believe in Maxwell's equations, at least not when it came to the micro-structure of electromagnetic radiation, as he had written in his 1905 paper on what later came to be called photons. However, Ritz's concern was not related to quantum effects (which he rejected along with special relativity), it was purely classical, and in the classical context Einstein was not troubled by the exclusion of the advanced potentials. He countered Ritz's argument by pointing out (in his 1909 paper "On the Present State of the Radiation Problem") that the range of solutions to the field equations is not reduced by restricting ourselves to the retarded potentials, because all the same overall force-interactions can be represented equally well in terms of advanced or retarded potentials (or some combinations of both). He wrote

> 'If ϕ_1 and ϕ_2 are [*retarded and advanced*] solutions of the [*potential field*] equation, then $\phi_3 = a_1\phi_1 + a_2\phi_2$ is also a solution if $a_1 + a_2 = 1$. But it is not true that the solution ϕ_3 is a more general solution than ϕ_1 and that one specializes the theory by putting $a_1 = 1, a_2 = 0$. Putting $\phi = \phi_1$ amounts to calculating the electromagnetic effect at the point x, y, z from those motions and configurations of the electric quantities that took place prior to the instant t. Putting $\phi = \phi_2$ we are determining the above electromagnetic effects from the motions that take place after the instant t. In the first case the electric field is calculated from the totality of the processes producing it, and in the second case from the totality of the processes absorbing it. If the whole process occurs in a (finite) space bounded on all sides, then it can be represented in the form $\phi = \phi_1$ as well as in the form $\phi = \phi_2$. If we consider a field that is emitted from the finite into the infinite, we can naturally use only the form $\phi = \phi_1$, precisely because the totality of the absorbing processes is not taken into consideration. But here we are dealing with a misleading paradox of the infinite. Both kinds of representations can always be used, regardless of how distant the absorbing bodies are imagined to be. Thus one cannot conclude that the solution $\phi = \phi_1$ is more special than the solution $\phi = a_1\phi_1 + a_2\phi_2$ where $a_1 + a_2 = 1$.'

"Ritz objected to this, pointing out that there is a real observable asymmetry in the propagation of electromagnetic waves, because such waves invariably originate in small regions and expand into larger regions as time increases, whereas we never observe the opposite happening. Einstein replied that a spherical wave-shell converging on a point is possible in principle, it is just extremely improbable that a widely separate set of boundary conditions would be sufficiently coordinated to produce a coherent in-going wave. Essentially the problem is pushed back to one of **asymmetric boundary conditions** [*emphasis added*]."

A field emitted from the finite into the infinite is exactly the problem of how to begin to propagate fields in $\mathcal{H}_1$ through the unit cell to the higher level of $\aleph$ on $\mathcal{H}_2$. Recall the transfinite origin of the concept of a level of $\aleph$ [9]. We will rely on a general understanding of the coefficients

$$a_1 + a_2 = 1 \qquad \longleftrightarrow \qquad \Phi + \varphi = 1 \quad , \tag{3.138}$$

while noting that complexified solutions of the form

$$\phi = a_1\phi_1 + a_2\phi_2 + a_3\phi_3 + a_4\phi_4 \quad , \qquad \text{with} \qquad a_1 + a_2 + a_3 + a_4 = 1 \quad , \tag{3.139}$$

are independently interesting on account of

$$\hat{1} = \frac{1}{4\pi}\,\hat{\pi} - \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{i} \quad . \tag{3.140}$$

I was there when he set the heavens in place, when he marked out the horizon on the face of the deep, when he established the clouds above and fixed securely the fountains of the deep, when he gave the sea its boundary so the waters would not overstep his command, and when he marked out the foundations of the earth.
– Proverbs 8:27-29

IV Computation and Analysis in Quantum Cosmology

In the first section of this chapter, we review the details set forth in the earliest MCM-related publications [2, 7]. Section two presents the details of a specific set of closed timelike curves known as Tipler sinusoids as an example of the kinds of field solutions that will have to be encoded into the energy function of the hypercosmological lattice.[1] Section three discusses quantum mechanics and improves the motivations for what we have called the MCM hypothesis. Section four reviews Penrose's seminal paper on conformal infinity [52] and discusses MCM applications of the conformal principles. In section five, we discuss the analytical features of covering spaces with the intention to emphasize what it means to break U(1) symmetry and how the strong force can be incorporated into the MCM. We will directly show the $\mathrm{U}(1)\times\mathrm{SU}(2)$ property of electroweakness and discuss the extension to SU(3). Section six treats the double slit experiment in the context of the cosmological lattice. Section seven is dedicated to analysis with a concentration on numerical analysis. In section eight, we present a toy model of the mass parameters of the universe $\{\Omega_{\mathrm{Matter}},\Omega_{\mathrm{DM}},\Omega_{\mathrm{DE}}\}$ that is in perfectly within the parameter space allowed by ΛCDM.

With the conclusion of this book, we can reasonably say that the survey of fundamentals in this research program has progressed to about 1920. Certainly there is a lot of work left to be done. If this book was longer, we would take the extra pages to show how non-relativistic quantum theory should be generalized from $\mathbb{C}$ to $^\star\mathbb{C}$, and once that is complete we could begin to develop an MCM energy function to use for $\hat{V}$ in some 1D MCM application of the Schrödinger equation. From there, we would progress to a more thorough study of the Dirac equation. That would lay the foundation to move on to the work of Schwinger, and we would hope to show that the radiative corrections he found are natural to the MCM. After that, we should study the work of Feynman and take great care to look for complexity hidden in or passed over by his rules for writing integrals based on diagrammatic representations of QFT interactions. After that we would revisit the Landau–Yang theorem to (hopefully) show that

[1] The word "encoded" here refers to the nebulous justification for writing the Einstein–Hilbert action (or similar) which one does because it gives the geodesics of spacetime as the equations of motion.

spin-1 to two photons is allowed when the mathematical analysis is carried out with $^\star\mathbb{C}$ in the dynamical hypercosmos instead of with $\mathbb{C}$ on an uncoupled spacetime background. After that, the results of Lee and Yang regarding parity violation would need to be converted into the MCM language so that we might address the technical aspects of the Sakharov conditions for the matter/anti-matter imbalance. From there, we could move on to selected results in modern QFT such as the electroweak theory which posits that every spacetime point has its own SU(2) subspace. Then we would hopefully show that the SU(3) symmetry of QCD arises from a projection of SU(2) into a higher (or lower) level of $\aleph$.

IV.1 The Modified Cosmological Model

An early idea in the MCM was to solve the problem of the divergent energy of the vacuum predicted by quantum field theory. In reference [7], we proposed to divide infinite energy by zero volume to obtain a finite energy density, and the MCM unit cell embodies that concept wholeheartedly, as in figures 33 and 34. Those figures show where we can obtain $V=0$ for the finite energy density and also the $(\Phi\pi)^3+2\pi$ needed for

$$\alpha^{-1}_{MCM}=\big(\Phi\pi\big)^3+2\pi \quad . \tag{4.1}$$

The empirical application of $\alpha^{-1}_{QED}\approx137$ is mainly in the splitting of energy levels of electrons trapped in spherically radial atomic potential wells. When the MCM matches chronos to leptons and chiros to quarks, the pieces x^0 and $\{\chi^5_+,\chi^5_\varnothing,\chi^5_-\}$ give, generally, the electron and three quarks in the exactly solvable hydrogen atom. If we then associate a transit of the MCM unit cell with the transition of an atom in one state to an atom in another then there is a picture of the electron changing from one spherical harmonic state to another across $\mathcal{H}$ or $\varnothing$. We want to exploit the inherent non-unitarity of the ontological basis $\{\hat{\imath},\hat{\Phi},\hat{2},\hat{\pi}\}$ to generate the fine structure constant as a property of the ontological topology. Therefore, if we consider the MCM exactly as it defined, namely a state $|\psi;n\rangle$ in $\mathcal{H}_1$ and $|\psi;m\rangle$ in $\mathcal{H}_2$, such that, per figure 33, the normalized magnitude of $\hat{\Phi}$ is 2π and the normalized magnitude of $\hat{\pi}$ is $(\Phi\pi)^3$, then we will come to a direct motivation for not only the number 137 itself, but also its main context in physics: the splitting of hydrogenic energy levels.

The MCM borrows a concept from solid state physics to consider the universe as one quantum of spacetime bouncing around in a lattice, or possibly transiting a lattice in a superconducting analogue phase. We label the lattice “the hypercosmos” but, in actuality, the hypercosmos needs be nothing more than a mathematical potential. In the ordinary case of dynamics on a lattice, there is a clear geometric picture of the overall periodicity such BCC, FCC, HCP, and other complete geometric representations but there is as yet no complete geometric representation of the MCM unit cell, at least not in the familiar sense of geometry on a single level of $\aleph$. We have not been able to fully draw the diagram connecting adjacent $\mathcal{H}$-branes because there is a non-geometric psychological process included in the periodicity. Even then, we have been able to deduce a lot of its properties through various philosophical analyses. However, one idea will be to join $\Sigma^\pm$ beyond the boundaries of the unit cell where $\chi^5_\pm=\pm\infty$. In that case, we can remove the topological incongruity of dS and AdS by joining $\Sigma^\pm$ across a singular manifold of infinite curvature. Another idea is to

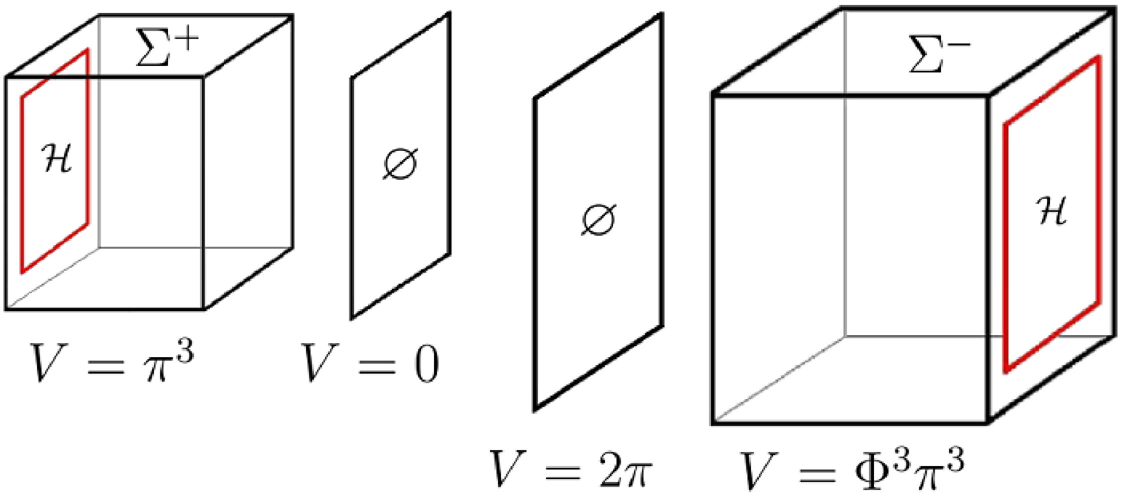

Figure 33: This figure shows the relative volumes of the major elements of the unit cell. Notably $\varnothing$ must have $A = \pi^2$ on the left and something like $A = \Phi^2\pi^2$ on the right. Therefore, during $\hat{M}^3$, $\varnothing$ can be uniformly rarefied by Φ or it can be rarefied only along one direction (probably the timelike direction) by Φ^2. However, we expect that the unit cell centered on $\mathcal{H}$ will have the same area on both sides, as in figure 34.

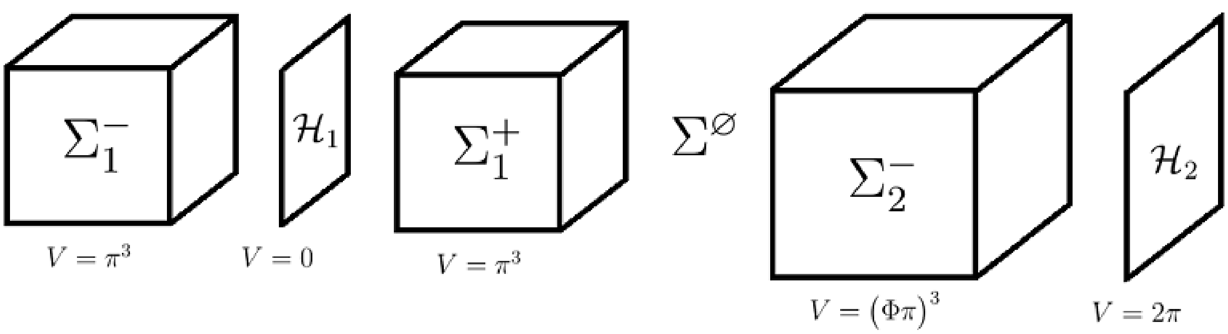

Figure 34: This figure shows another intuitive concept for the ultimate origin of α_{MCM}. This figure demonstrates the non-unitary component of forward chirological evolution and also the fundamental difference between $\mathcal{H}$- and $\varnothing$-branes. $\mathcal{H}$ connects to $\Sigma^\pm$ where the 5D spaces do not contain their $\chi^5 = 0$ boundary but $\Sigma^\pm$ both contain their boundary where they are joined across $\varnothing$. $\mathcal{H}$-branes get inserted into an empty slot but $\varnothing$-branes are something extra which must be added.

connect Ω and $\aleph$ as elements of a symplex and we will develop that concept throughout this chapter.

The MCM and TOIC were developed to describe the process $\hat{M}^3$ by which an observer can make a prediction for something to happen, then wait for an event which can confirm or deny the prediction, and then wait a while longer for the retarded signal from that event to reach him in the present where he can compare the reality to his predicted expectation. We say that the calculation of the dynamics across one full unit cell would constitute an impossible calculation and, by that, we mean that no solution exists within an analytical framework that exists only on a single level of $\aleph$. These situations of analytical intractability are common in physics for excited and perturbed states but they are exceptional in the case of a ground state or vacuum solution like the MCM unit cell with $A_\mu = 0$ and no matter-energy in 4- or 5D. One such exceptional case is the energy of the QFT vacuum. When we do $\hat{M}^3$ with that calculation, it says our theory is wrong. The usual prescription for dealing with this common problem of computational impossibility (in the absence of expeditious fudge factors) is to simply substitute the numerical solution for the analytical one and, here, the reader must note that numerical solutions *are* analytical solutions. They are simply a different sort designed to minimize the error associated with the numerical solution and it is even possible that the error term associated with the numerical solution can be reduced to zero so that it is exactly the analytical solution. In that case, what would seem to be an analytically intractable problem will have a solution and everything that was "impossible" will have to be reclassified as "impossible until now."

Unintegrable integrals of transcendental functions (such as trigonometric functions and the associated complex exponential) are a common example in physics where analytical solutions do not exist. Nearly everything in quantum theory other, than the harmonic oscillator potential, is of this mathematically intractable variety. In relativity, there are certainly very few solutions to the geodesic equation that can be written down without numerical methods of approximation. Therefore, if we develop a new numerical algorithm that can improve the approximation of such unintegrable solutions then that will be a good advance. Since we have little hope for computing $\hat{M}^3$ directly, we have, in this regard, introduced $\tilde{M}^4$ which we define precisely as

$$\tilde{M}^4 \quad \equiv \quad \hat{\Upsilon} \ , \qquad \text{where} \qquad \hat{\Upsilon} \quad \equiv \quad \hat{U} + \hat{M}^3 \ . \tag{4.2}$$

The operator $\hat{\Upsilon}$ was added to the MCM in 2011. In reference [22], we proposed to generate α_{MCM}^{-1} with this operator and in section II.5 we described a new method to construct the MCM structure constant α_{MCM}^{-1}: 2π radians of freedom in a U(1) theory comes from two zenith coordinates $\theta \in [0, \pi]$ covering the Riemann sphere's topology $\mathbb{S}^2$ and $(\Phi\pi)^3$ comes from the composition of an O(3) theory with $\hat{\Phi}$ during transit across the MCM unit cell such that $\hat{\Phi} : \pi^3 \to \Phi^3\pi^3$.

The intention at the start of this section is to clarify the geometric framework in which we find the all-important fine structure constant but not to derive it specifically. We will use Feynman's non-inclusion of the endpoints of the time axis as evidence of freedom to map the interval x^0 onto $\mathbb{S}^1$ with a covering space representation and then we will return to covering spaces in section IV.5. Feynman makes some comments in reference [6] (excerpted in chapter one) about how additive terms in the action show up as multiplicative terms in the wavefunction. He is able to show how one representation makes the other simpler and we propose to make the entire theory simpler with the operator $\hat{\Upsilon} \equiv \tilde{M}^4$ that acts via addition and multiplication simultaneously.[1] Ordinary methods of non-gravitational quantum field theory in curved spacetime have no coupling between the field theory and the fabric of spacetime but, here, we can supplement those methods by guiding spacetime with $\hat{M}^3$ during unitary field evolution with $\hat{U}$. This method can be accomplished with $\int d\hat{\gamma}$ but we will not rely on it in this section which aims, mostly, to review references [2] and [7]. There is still work remaining to clarify the MCM processes but the conceptual framework is clear and exceedingly well motivated, and we will make some exquisitely nice remarks on $\hat{\gamma}$ in section IV.3.

As a theory of everything, the lack of coupling between spacetime and fields on spacetime is only one of many problems solved by the MCM. In addition to the energy density of the vacuum, there are many examples in QFT where action integrals explode and we propose to eventually rewrite all of them (most of them) in non-exploding form with the $^\star\mathbb{C}$ formalism of the TOIC.[2] Recall that if Feynman did not impose an arbitrarily short duration of time for his calculation [6] then his integrals would explode, and that we have called Feynman's operation (or similar) by the name $\hat{U}$. Using the theory of infinite complexity, we can ensure everything that would make a solution explode in $\hat{U}$ is simultaneously imploded by the the effect of $\hat{M}^3$. In general, we can send divergences to points at "conformal infinity" and then switch in and out of covering space representations to bring those terms in or push them out as required. This directly represents a source of information and a sink of information. In the TOIC, we will encode these sources and sinks on unpaired elements that exist as a remainder after taking the inner product of a series with an even infinite number of terms and a series with an odd infinite number of terms. Using the $\int d\hat{\gamma}$ notation, we will have a manifold plus a scalar as the output of the new hypercomplex inner product that relies on δ'_{jk}, as in section II.2.

Before the MCM, the closest physicists had come to discovering a mechanism of unification between gravity and electromagnetism was in the work of Kaluza. Around 1920, Kaluza showed that a 4D general relativity with matter-energy and a 4D gauge theory (conformal field theory) with electric charges can be encoded in a 5D space when there is no 5D matter-energy. This work was produced almost immediately following Einstein's formulation of gravitation but it was nearly a century later that we proposed [3] to use two such 5D theories to define a boundary condition in 10D likely relevant to string theory. Kaluza's theory, called Kaluza–Klein theory today, doesn't work by itself because it predicts that the electromagnetic field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \tag{4.3}$$

[1]Interestingly, the Riemann zeta function [8] has independent representations as a sum over integers $\zeta(s) = \sum_{n=0}^{\infty} \frac{1}{n^s}$ and as a product over prime numbers $\zeta(s) = \prod_p \frac{1}{1-p^{-s}}$.

[2]Recently Arkani-Hamed *et al.* published a paper titled *Scattering Amplitudes For All Masses and Spins* [53] which is likely a good stab in this direction but his refusal to acknowledge this writer's contributions to his research program is at times irksome and always frustrating.

should always vanish. In the MCM, we introduce *two* 5D spaces with which we might sidestep this problem. As we have shown very many times, four electromagnetic potential 4-vectors A_μ arise in the MCM as

$$\Sigma^+_{AB} = \begin{pmatrix} \Sigma^+_{\mu\nu} & A^+_\mu \\ A^\nu_+ & \chi^5 \end{pmatrix} \qquad \text{and} \qquad \Sigma^-_{AB} = \begin{pmatrix} \Sigma^-_{\mu\nu} & A^-_\mu \\ A^\nu_- & \chi^5 \end{pmatrix} \quad . \tag{4.4}$$

Two of these four 4-vectors $\{A^+_\mu, A^\nu_+, A^-_\mu, A^\nu_-\}$ are constrained to satisfy $F_{\mu\nu}=0$ but the other two can be used to define a non-vanishing $F_{\mu\nu}$ in $\mathcal{H}$. The general idea is to have two pairs of vectors on two levels of $\aleph$ when equation (4.3) is only a constraint on one such level but we have not yet written the field strength tensor in $\mathcal{H}$ in the way that does this while maintaining the physical interpretation of everything else in the theory. What we have undertaken is to let there be two 5-spaces which generate a 10D stringlike boundary condition where the two spaces are separated by a topological obstruction containing the four spacetime dimensions of $\mathcal{H}$: x^μ. $\mathcal{H}$ is the 4D boundary where 5D Σ^-_1 is connected to 5D Σ^+_1 and there is some subtlety associated with $\varnothing$ connecting Σ^+_1 to Σ^-_2 since the level of $\aleph$ increases there. We might consider either obstruction, $\mathcal{H}$ or $\varnothing$ (or both), as a Goldstone boson of the broken symmetry arising from the projection of U(1) $\theta\in(0,2\pi]$ onto a helical interval $\theta\in[-\infty,\infty]$ (a covering space) but, instead, we are considering the obstruction as Minkowski space. However, this is the general idea of MCM quantum cosmology: the whole universe is like one quantum of spacetime [2, 12].

As of the writing of this chapter in late 2017, more than eight years have passed since reference [2] was submitted to arXiv. Over the following 3,000 or so days, we have refined arguments, made some of them quantitative (recall that diagrams are already quantitative topology to begin with), and still mostly conformed to the original theory which is the main topic of this section. Here, we will go through reference [2], almost sentence by sentence, and then we will also revisit the follow on attempt [7] submitted to arXiv two years later in 2011. It was in reference [7] that we coined the phrase "modified cosmological model."

Reference [2] begins as follows.

> "Consider the spacetime diagram in a region surrounding the big bang where $ct=0$ corresponds to the apex of a quantum geometric 'bounce' rather than a divergent singularity. At $ct=0$, allow the superposition bounce state to decay to two time arrow eigenstates. By convention we say our universe is the eigenstate moving forward in time along the positive ct axis. The other decay product is another universe experiencing forward flowing time in the direction of our negative ct axis."

To expand on this, we note that the canonical spin algebra of operators on $|\!\uparrow\rangle$ and $|\!\downarrow\rangle$ can be used to describe the MCM quantum cosmology based on $|t_\pm\rangle$. Consider, in analogy, that a quantum mechanical state can be in a superposition of spin up and spin down so that there is no preferred orientation for the net angular momentum in the $\hat{z}$-direction. Likewise, we can simulate a big bang with no arrow of time as a superposition of two time arrow eigenstates. $\mathcal{H}'$ is spanned by the possible states of a system in a given moment with no quantum mechanical time and this constraint of timelessness on the information in the wavefunction is given by

$$\big|t_\star\big\rangle = \frac{1}{\sqrt{2}}\big|t_+\big\rangle + \frac{1}{\sqrt{2}}\big|t_-\big\rangle \quad . \tag{4.5}$$

This will also be a good model for describing the big bang, or the big bounce, or the big crunch, or the singular topological obstruction that exists at the location of an MCM observer. With these time eigenstates, we defined a superposition that gives no net arrow of time in the present moment, which is singular. It cannot support an arrow which must have a tip somewhere away from its anchor point. An absent arrow of time is a requirement for an accurate representation of the big bang at the beginning of the universe before time existed, or for a timeless Hilbert space of states at some given moment $\mathcal{H}'(t)$. With spin up and spin down, there is no way to measure a state of zero net angular momentum for one quantum; a measurement will yield either spin up or spin down. However, the observer cannot make a measurement that would collapse $|t_\star\rangle$ into one of $|t_\pm\rangle$ so the state of timelessness is persistent and we should associate this with the timelessness of the present moment.

A common yarn in the teaching of quantum mechanics is to explain to students that the orientation of the axis for the $\hat{S}_z$ operator is not relative to an objective background but, rather, is only relative to the way the observer defines the eigenfunctions that describe the possible outcomes of his measurements of angular momentum along an arbitrary axis in 3-space. Therefore, without repeating the entire history of quantum mechanics, when the observer is invariably forced to observe time flowing in one direction as the present moment keeps changing position in spacetime, that will set the analogue of $\hat{S}_z$, call it $\hat{T}$, along the axis of time. $\hat{T}$ is the time arrow operator. We are using two universes U and $\bar{U}$ so the fermion algebra may already contain the $\hat{T}$ operator if we say that the two universes' simultaneous wavefunction transforms as a spinor. In that case, when $\hat{S}_z$ acts on spinors to give the eigenvalues $\pm 1/2$, $\hat{T}$ acting on universes will give $\pm$ so we would have $\hat{T} = \text{sign}(\hat{S}_z)$.

The potential energy landscape of the MCM is a periodic repetition of that shown in figure 35. In one moment, the observer is drawn toward the future and again in the next moment, and in the moment after that, *etc.* Time always marches on. This energy condition is such that the farthest objects in the observable universe, *i.e.*: the objects most distant on the observer's past light cone, should appear to be accelerating away from observers on Earth. This is exactly what is observed and called dark energy. *Voilà*! A Nobel Prize, *s'il vous plaît*. When we extend the periodic potential beyond one unit cell, we have to introduce the transfinite component because the depth of the energy well has to get to infinity before the next unit cell can start. This must be associated with the changing level of $\aleph$ that precedes the restoration of finiteness after each application of $\hat{M}^3$ (or $\hat{M}^4$.) It is nice to see that forward flowing time and the accelerating recession of cosmological objects at high redshift $z >> 1$ are two aspects of the same energy condition. In section III.9, we showed how the metric predicts dark energy and expanding space, and now, with the inherent chirological directionality of increasing levels of $\aleph$, we have shown that forward flowing time is an additional implication of the MCM.

Reference [2] continues as follows.

> "To allow dynamical interaction between the universes to take place via a familiar mechanism wrap the ct axis around a cylinder. The big bang occurs at $\phi = 0$

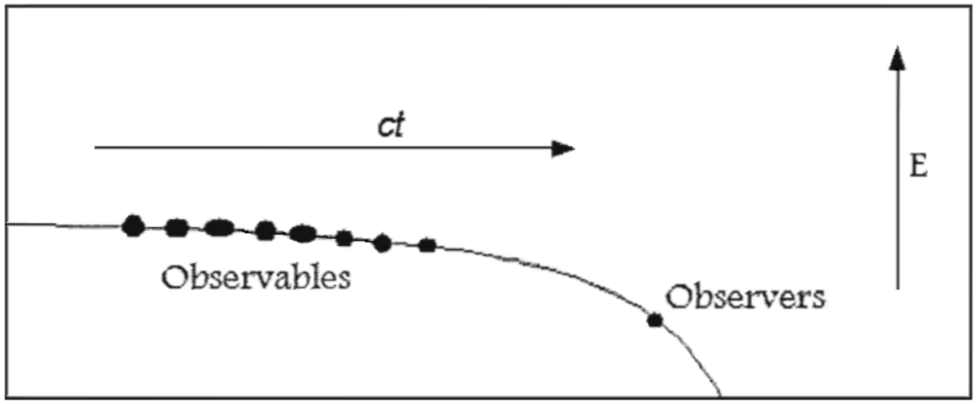

Figure 35: This figure first appeared in a 2009 manuscript [2] rejected for publication by arXiv but later published on viXra. This figure ignores the local energy wells of the observer and observables to show that the mass-energy of the lattice site on the higher level of ℵ dominates on cosmological scales. The energy curve defined by the two masses in figure 36 contributes as a infinitesimal when taken in superposition with the energy curve in this figure because it is the energy well of some $m > \infty$ on a higher level of ℵ than the universe that contains the labeled observers and observables.

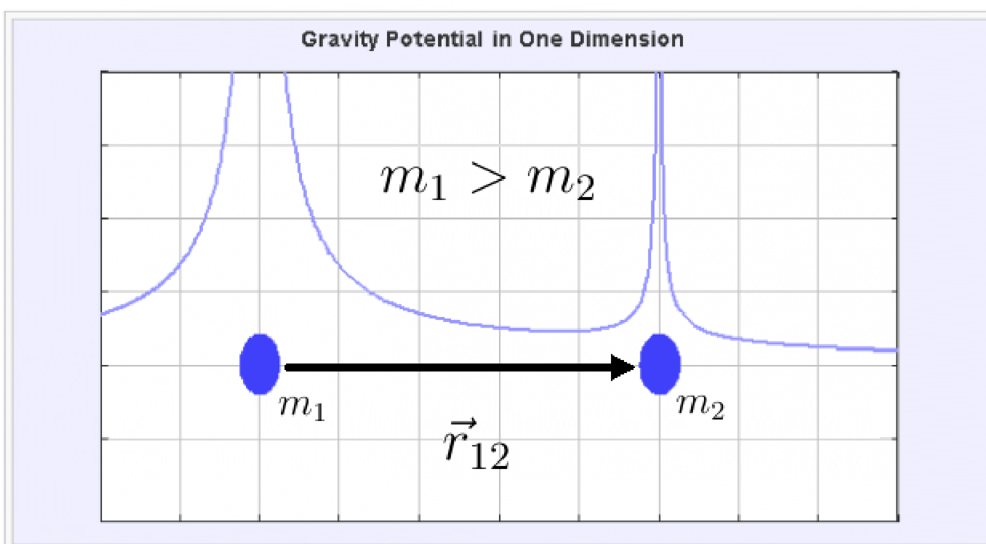

Figure 36: In this figure of Newtonian gravity adapted from Wikipedia, the energy curve shows the local landscape due to m_1 and m_2 which fairly well correspond to the observers and observables in figure 35. Figure 35 does not include these energy wells because it demonstrates the cosmological energy well of an entire universe on a higher level of ℵ.

and the big crunch for each universe occurs at $\phi = \pi$."

When we say "allow dynamical interaction between the universes to take place via a familiar mechanism" we refer to the mechanism by which ordinary masses gravitate when they can be connected by a curve in a manifold, as in figure 36. The curve we refer to is $\vec{r}_{12}$, not the energy curve. When the connecting curves such as $\vec{r}_{12}$ are geodesics, they show the path of an unaccelerated point mass in spacetime. When m_1 and m_2 deform spacetime, the geodesics will be such the masses move directly toward each other (assuming there is no angular momentum.) A point mass is a poorly defined object because there are no physical point masses and, in any case, the geodesic equation unphysically ignores the gravitational backreaction but these methods of approximation, together with the exponential maps that approximate the relationship between a manifold and its tangent space, do altogether form a framework for $\hat{M}^3$ that will say the theory is correct. It may even be that the two disparate theories of modern physics can be connected through their methods of approximation more easily than they might be connected in the exactly solvable analytical sector. On the cosmological scales inherent to general relativity, we can ignore, say, the backreaction of a hydrogen ion against the gravitational background of a galaxy but the nature of these approximations should be an important feature at the small scales expected of quantum gravity. Also note that, in the MCM, the small scale of quantum gravity is actually a small number on a higher level of $\aleph$ so it is even larger than the cosmological scale.

The curve along which two masses gravitate can be Euclidean (Newtonian) $\vec{r}_{ij}$ or it can be some other path in curved space that needs to be parameterized with an affine parameter like λ in $x^\mu(\lambda)$, or a quasi-affine parameter like χ^5. In general, all of these curves are conformally equivalent to a straight line with no curvature (or torsion), such as $\vec{r}_{12}$ in figure 36. Therefore, there is a conformal invariance between any curved chronological trajectory through spacetime, generally parameterized as $x^\mu(\lambda)$, and an axiomatically flat path across the MCM unit cell parameterized with χ^5 such that

$$x'(\chi^5) \quad \equiv \quad \chi^\alpha_+(\chi^5_+)\,\hat{\Phi}^j \quad \cup \quad p \in \Sigma^\varnothing \quad \cup \quad \chi^\alpha_-(\chi^5_-)\,\hat{\Phi}^{j+1} \quad . \tag{4.6}$$

This representation shows the general structure under which we bisect π with a point and then have two disconnected domains $\theta \in (0, \pi/2)$ such that 0 is the location of the observer and $\pi/2$ is at conformal infinity.[1] If we envision the origin with two abstract directions $\chi^5_\pm$ attached then we might say that we could assemble 2π dimensionless radians of freedom from two such arrangements of bisected π: one centered on the origin of $\mathcal{H}$ and another centered on the origin of $\varnothing$. We should consider that the factor 2π in α^{-1}_{MCM} inherits one π from the zenith coordinate of one chart on $\mathbb{S}^2$ and another from the other zenith coordinate in the second chart. When we bisect each π, we have four objects like $\pi/2$ on which we will shortly propose to encode bispinor structure. In general, we can associate one chart with the unit cell centered on $\mathcal{H}$ and the other with the unit cell centered on $\varnothing$. When considering two different charts on $\mathbb{S}^2$, we are not constrained to take disconnected charts at opposite poles. We might take two charts at the same pole where one chart is on $\mathbb{S}^2$ and the other chart is

[1]Conformal infinity is derived in most cases from $\tan(\pm\pi/2) = \pm\infty$.

over the hypercomplexly infinitesimal annulus around the origin of coordinates of the first chart, as in reference [8]. Then these two charts cover an abstract $\mathbb{S}^2$ because the chart on the small annulus is self-similar to the standard coordinate chart on $\mathbb{S}^2$ in every way.

One further thing that we will call attention to in the excerpt above is this: we wrote, at that time, that both universes have a big crunch at $\phi=\pi$ but we might go a bit further now. An easy way to model a topological defect might be to have one universe reach future timelike infinity at $\phi=\pi/2$ and the other at $\phi=\Phi$. To that end consider the operations in figure 37. On the right, where we apply $\hat{2}^{-1}$ to go back to the union of two co-$\hat{\pi}$s, there is a small remaining element. On these small elements, we can define localized sections of finite width that can house scaled coordinate space representations of quantum state vectors. The idea will be to use $\int d\hat{\gamma}$ to construct a new general relativity which can then serve as a dynamical spacetime background included in the hypercomplex inner product $\langle\psi;\hat{\Phi}^{j+1}|\psi;\hat{\Phi}^{j}\rangle$ whose remainder is written onto a segment like $\Phi-\pi/2$. Using $\hat{2}^{-1}$, we may reform U(1) and then use $\hat{\pi}$ to define the remaining interval as a compactified U(1) dimension. Note that we are using the operator $\hat{\pi}$ to construct a circle[1] and that the $\hat{\pi}$ MCM object is generally associated with the U(1) electromagnetic theory. U(1) is the symmetry of a circle, hence $\hat{\pi}$. If we use $\hat{\Phi}$ on both co-$\hat{\pi}$s separately, as in figure 38, then we can construct the system with which we have argued against the Riemann hypothesis [8], as in figure 39. If we operated on the circle with $\hat{\Phi}$ then all four intervals would be affected through the distributive property of multiplication. Therefore, the operation with $\hat{\Phi}$ in figure 38, which does not satisfy the distributive property, looks like a bispinor product. In future work, therefore, we should consider the bispinor representation of the geometric basis of the manifold (its twistor space representation) such that the eigenvalues of the metric are the $\{+-+-\}$ components of the ontological resolution of the identity and not the $\{-+++\}$ components of the ontological basis. The properties of the ontological basis are manifest in the O(3,1) Lorentz symmetry of spacetime fields that can be represented as objects depending on the bispinor representation of the manifold's cotangent basis, also called its 1-forms. dx is a one-form and the volume element of spacetime $d^4x\equiv dx^\mu$ can be $d\hat{\gamma}$ in twistor space... or these objects can be combined in some such similar scheme. A good idea for further development will be to take the twistor representation of the cotangent basis.

We have obtained, in figure 38, a hidden SU(2) symmetry exactly as required for electroweak theory. In the process of doing so, each time we add a small U(1) symmetry, it replicates the general idea of a compactified fifth dimension in Kaluza–Klein theory. It is precisely the U(1) symmetry of the fifth dimension that distinguishes specific Kaluza–Klein theories from general Kaluza theory. Considering the center of figure 38, if we say that the two free segments are co-$\hat{\pi}$s on $\hat{\Phi}^{j+1}$ and $\hat{\Phi}^{j-1}$ respectively then the repeating unit of the lattice in this representation is equal to the $\hat{\Phi}^{j}$ ring with a little $\hat{\Phi}^{j\pm1}$ sphere attached. Although the standalone piece has one higher segment and one lower, we can consider the repeating unit as one ring with one little sphere on one side. A 2-sphere and a circle are constructed from three circles so this representation is evocative of the π^3 term in α_{MCM}.

We can obtain the line from the circle by taking the limit of the radius increasing to infinity and this is how we can ensure the identical topological flatness of χ^5 across the unit cell: always use the level of $\aleph$ so that $R=\infty$ defines a straight line. Then the trajectory across the unit cell requires a scaling $\infty\,\hat{\Phi}^{j}\to R\hat{\Phi}^{j+1}$ as $\mathcal{H}_1\to\varnothing$ increments the level of $\aleph$ by

[1] The number π is sometimes referred to as "the circle number."

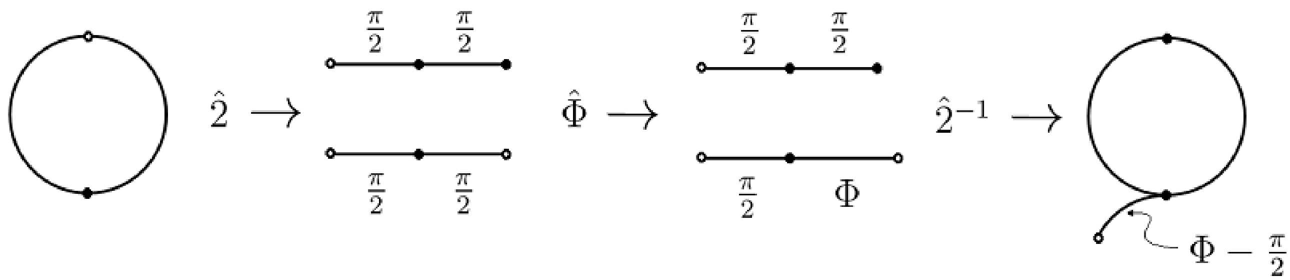

Figure 37: This figure shows how we can use the numbers in the ontological basis as operators to create complex structures.

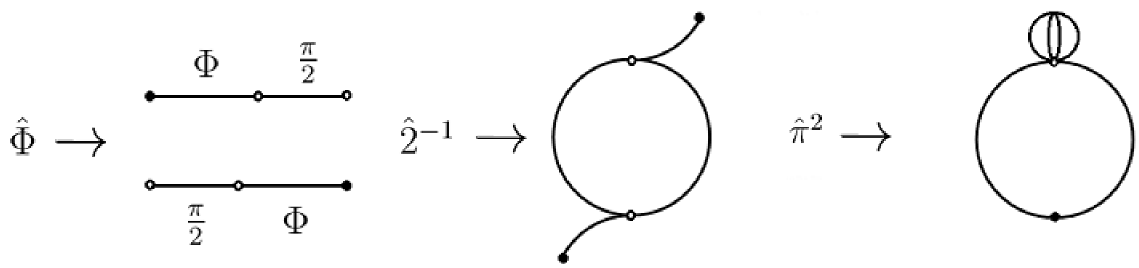

Figure 38: A further manipulation beyond those shown in this figure would be to change the level of $\aleph$ in the rightmost figure so that the radius of the large circle becomes infinite. If the operation is applied judiciously then we could obtain a small sphere attached to flat space as is required for the topology of electroweak theory.

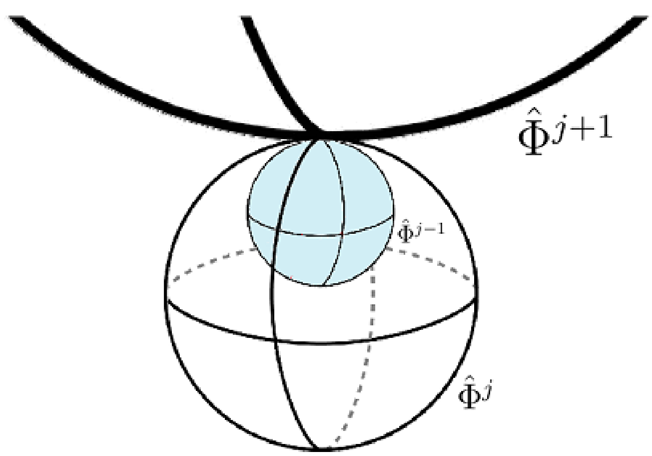

Figure 39: When repeated many times, the operations in figure 38 can generate the nested fractal structure developed, in reference [8], to argue against the Riemann hypothesis. The three spheres in this figure are on different levels of $\aleph$ which means that the radius of each larger sphere is infinite with respect to the radii of the spheres nested within.

one, and then $\varnothing \to \mathcal{H}_2$ increments it by another during $R\hat{\Phi}^{j+1} \to 0\,\hat{\Phi}^{j+2}$. Then $\mathcal{H}$ and $\varnothing$ are out of phase as even and odd levels of $\aleph$. This is an important feature that we will return to later in this section.

Consider a fundamental utility of the ontological basis $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ in a simplified yet descriptive manner. The MCM presupposes a cylindrical symmetry so we have π and 2 *a priori* in the relationship $C = 2\pi R$ between the cylinder's radius and its circumference. Use $\hat{2}$ to split the circumference into two intervals that will be the time axes of U and $\bar{U}$. Splitting two intervals at a point necessarily leaves one interval with a missing endpoint. Invoking the physical constraint that the observer in U or $\bar{U}$ is between his past and future light cones, we move a null point from the end of each interval to its center. This is shown in figure 38 after acting with $\hat{\Phi}$ but, presently, we let the null points go to the center after acting with $\hat{2}$ because we have not yet used $\hat{\Phi}$. Thus, we have separated the past and future of each universe. We may derive from $\theta \in (0, 2\pi]$ four intervals $\{\vartheta_+, \vartheta_-, \bar{\vartheta}_+, \bar{\vartheta}_-\}$ all ranging from 0 to $\pi/2$, two of which will reach "the big crunch" at conformal future timelike infinity where $\vartheta_+ = \bar{\vartheta}_+ = \pi/2$ and the other two will have "the big bang" in their history at $\vartheta_- = \bar{\vartheta}_- = \pi/2$. $\hat{\Phi}$ will act on one interval from ϑ and one from $\bar{\vartheta}$ but we have not yet used $\hat{\Phi}$. In the ϑ representation, there is no $\vartheta < 0$ but there is negative parameter when the parameter on each co-$\hat{\pi}$ is $\theta \in (-\pi/2, \pi/2)$. Therefore, in Gaussian terminology two of these ϑ are inverse in θ and two are direct, and that is a good motivator for the spinor product with $\hat{\Phi}$. When $\hat{\Phi}$ operates, it only stretches the direct (or inverse) intervals leading to future timelike infinity in U and $\bar{U}$. Here we have described θ and ϑ as the domain of chronological time x^0 and, in general, we can associate $\vartheta_+ > 0$ with ϑ that increases from $\mathcal{H}$ through the future light cone or across Σ^+ and then $\vartheta_- > 0$ continues to increase through the past light cone, or across Σ^-, on the way back to $\mathcal{H}$.

Moving on with the review of reference [2], it goes on to state that, "Interaction through the point $\phi = \pi$ is plausible since we are not considering any kind of conformal infinity that would be required if our bangs and crunches were singular." This passage refers to the non-singular bounce of LQC but in the present formulation we do consider conformal infinity; that which we do not consider is divergent infinity. The interaction in question is the gravitational attraction of U and $\bar{U}$ through the bounce point which results in the effect known as dark energy. It is "plausible" because we have added the chirological component to the system such that the dead end at an ordinary singularity is replaced with the door to a higher level of $\aleph$ (as in figure 10.) This is obviously something that has been revised.

Instead of singularities or bounces at past and future timelike infinity, we have a new concept of changing levels of transfinite infinitude ($\aleph$). The assumption of a non-singular big crunch at the end of the universe was inherited from the bouncing framework but we have subsequently generalized the MCM away from LQC so we also need to discuss how big bangs or crunches relate to the present formulation in $^\star\mathbb{C}$. Currently, we take the singular moment as the moment of the present $\mathcal{H}$ but, in the historical development, the singular moment was the big bang. Along two co-$\hat{\pi}$s (in U and $\bar{U}$) we say that $\{\vartheta_-, \bar{\vartheta}_-\} \in (0, \pi/2)$ is associated with the past and $\pi/2$ is the distance along $\{\vartheta_+, \bar{\vartheta}_+\}$ from the present to future timelike infinity, which replaces the earlier definition "big crunch." We say that $\hat{2}$ splits the U(1) symmetry into a pair of co-$\hat{\pi}$s and then we use $\hat{\Phi}$ to change lengths. The exact change will be a property of the exact algebra but, for the purposes of qualitative discussion, we presume that $\hat{\Phi}$ changes the dimensionless length between the present and future infinity

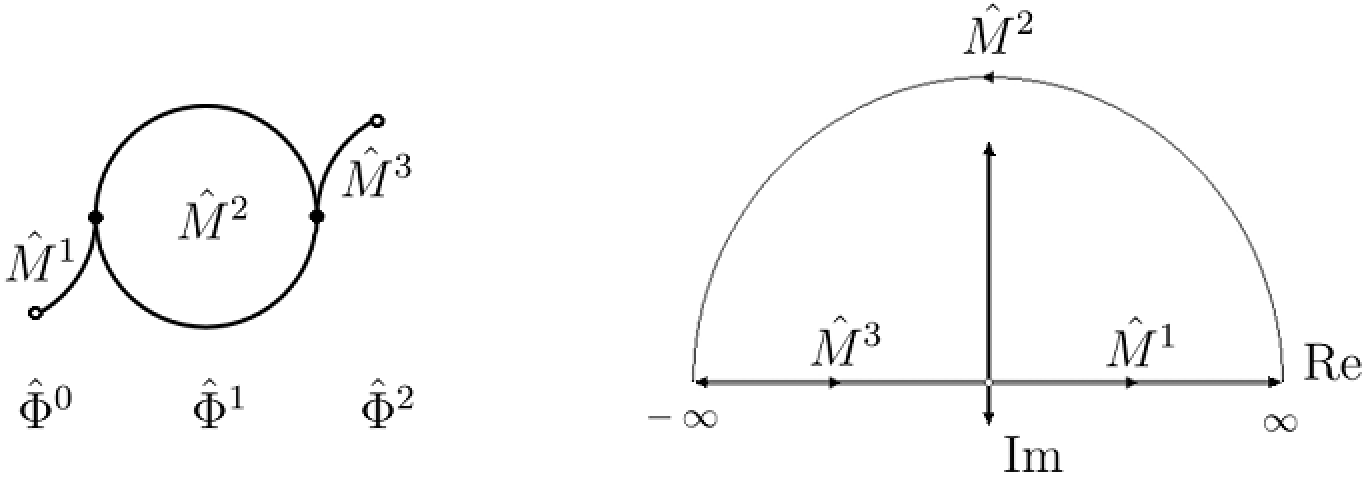

Figure 40: The trajectory of maximum action has an intermediate path of integration that could lie in the upper or lower complex half-plane.

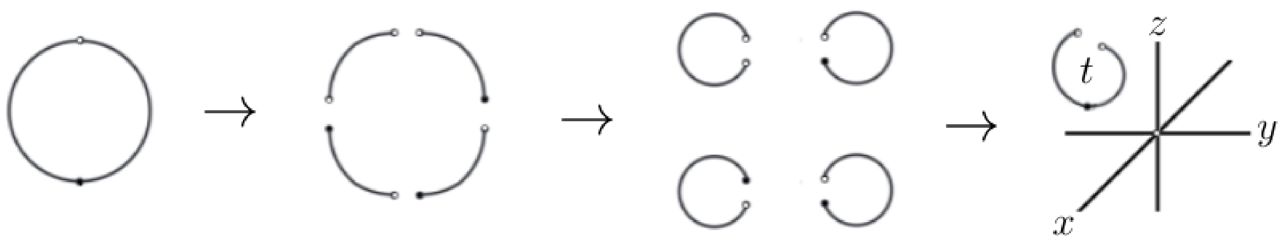

Figure 41: When the interval $\theta \in (-\pi, \pi)$ is decomposed into four intervals of length $\pi/2$, one of the intervals is unique because it has two null endpoints.

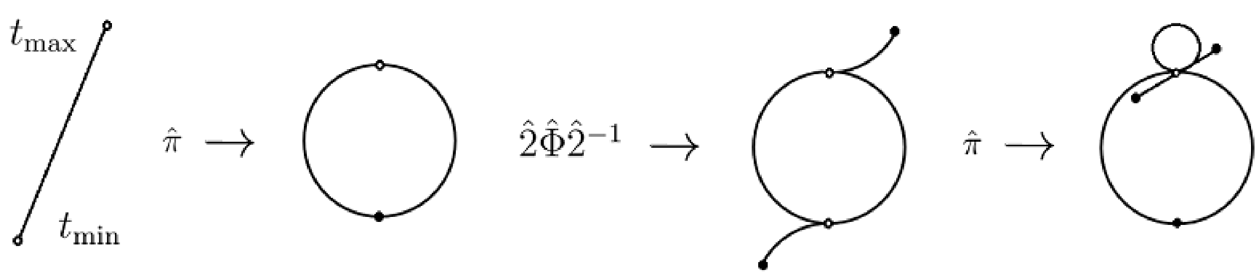

Figure 42: When we take the operator $\hat{2}\hat{\Phi}\hat{2}^{-1}$ such that $\hat{\Phi}^j \to \hat{\Phi}^{j+1}$, there is an intuitive association that $\hat{2}$ takes a qubit out of $\mathcal{H}_1$ at which point $\hat{\Phi}$ operates before we use $\hat{2}^{-1}$ to put the qubit back into $\mathcal{H}$ at $\mathcal{H}_2$.

from $\pi/2$ to Φ. After we make this change and then recombine all four elements, two futures and two pasts, into a circle, we will have a small extra interval of length $|\pi/2-\Phi|\approx 0.05$, as in figure 37. We can further use $\hat{\pi}$ to define the small remainder as a circle to make a small loop attached to the original circle. These little loops will have relative length $|\pi/2-\Phi|$ on the same level of $\aleph$ as the big circle but length 2π on the level of $\aleph$ directly associated with the small element, as in figure 40. Figure 37 shows that $\hat{\Phi}$ has operated on only one of the co-$\hat{\pi}$s but if we use it on both, as in figure 38, then it is very easy to see that the topology will support the path of integration shown on the right in figure 40. The convention in figure 40 shows that $\hat{M}^3$ increases the level of $\aleph$ by two which was the original convention before we showed a more compact mechanism [13] that increases the level of $\aleph$ in unit increments. The compact mechanism required that $\hat{\Phi}$ specify a point in the bulk of Σ^+ specifically to avoid increasing the level of $\aleph$ by two during each $\hat{M}^3$, and now we have good evidence suggesting that it should, in fact, always increase by two. Perhaps even levels of $\aleph$ are on shell and odd levels are off shell, or there exists some such condition that we will not clarify presently where one level describes a hidden sector attached to every observable sector. In various MCM references, we have made the argument that the observer connects two of $\{\aleph,\mathcal{H},\Omega\}$ at each step of $\hat{M}^3$ so we are in good order say that the observer spans two levels of $\aleph$ and that, therefore, an increase of two levels is needed if $\hat{M}^3$ sends the observer completely into the future.

Figure 41 shows the general principle of topology change in the MCM [5]. The figure shows that one of the four four quadrants on $\theta\in(-\pi,\pi)$ is different than the other three. Three of them have one missing endpoint and the fourth has two missing endpoints. This gives the O(3,1) character of the system but we do not want this property when we set the past and future light cones of U and $\bar{U}$ on four ϑ. Therefore, perhaps the odd and even levels of $\aleph$ shall refer to a temporal description, as with U and $\bar{U}$, and a spatial description, as in figure 41. In reference [7], we wrote the following.

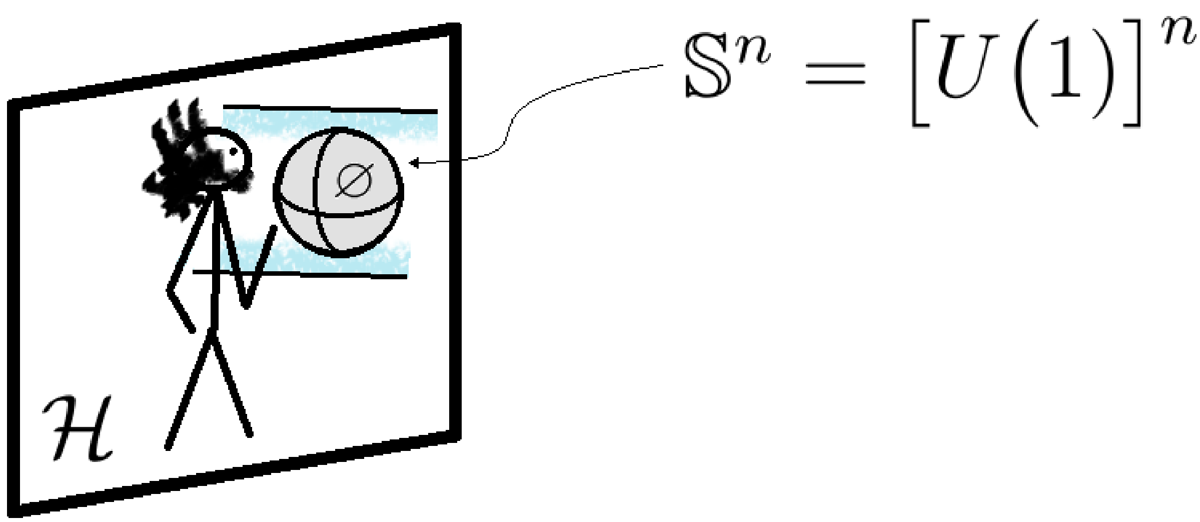

Figure 43: This figure calls attention to the psychological nature of physics. This figure calls further attention to the application of the sphere theorem to the framework depicted in figures 37-42.

> "We have assigned a spatial 3-sphere $[x^i]$ to each dimension of the temporal sphere $[\{t_+, t_\star, t_-\}]$. Space serves as the radial coordinate of the temporal ball just as time serves as the radial coordinate in the 3+1 dimensional space of $[g]$eneral $[r]$elativity. [*sic*] By alternating temporal and spatial spheres, diameters are mapped to circumferences and it is clear that the MCM is a fractal matrix theory of infinite complexity. The embedding and re-embedding is the physical manifestation of T-duality."

Note well, $\hat{\pi}$ is exactly the map from a diameter to circumference, as in the ancient formula

$$C = \pi D \qquad \Longrightarrow \qquad \hat{\pi} \;:\; D \;\mapsto\; C \;. \tag{4.7}$$

This can be written as

$$C = 2\pi R \;, \qquad \text{so} \qquad \hat{2}\hat{\pi} \;:\; R \;\mapsto\; \pi R \;\mapsto\; C \;. \tag{4.8}$$

Perhaps on even (odd) levels of $\aleph$, we will construct temporal spheres like ϑ^μ and on odd (even) levels we will construct spatial spheres like x^μ. We also mention T-duality in this excerpt. T-duality is the equivalence of theories under the change $r \to 1/r$ and is, therefore, exactly the inversion operation on the Riemann sphere. We invert it once to get from $\mathcal{H}$ to $\varnothing$ and then we uninvert it such that it goes to $\mathcal{H}$ on the higher level of $\aleph$. In general, we have two dualities in the MCM: T-duality between x and $1/x$, an also circle duality between x and e^{ix}. Clearly, there is a lot of material to finalize and by now the reader should understand that the purpose of this book is to discuss the material but not to finalize it.

Above we have taken π *a priori* as part of the MCM but we can do better, as in figure 42. We can begin with the Feynman theory wherein the the endpoints of the time axis are not included and then operate on that axis with $\hat{\pi}$ to get a circle *ab initio*. Then we apply $\hat{2}\hat{\Phi}\hat{2}^{-1}$ to increase the level of $\aleph$ and, if we finish with $\hat{\pi}$ as opposed to the $\hat{\pi}^2$ shown in figure 38, we will end up with a small embedded cylinder. If we say that the $\hat{\pi}$ to the left of figure 42 operates on the $\hat{\Phi}^j$ level of $\aleph$ then the second $\hat{\pi}$ operates on the small interval at the right of figure 42 because the large circular interval has been sent to $\hat{\Phi}^{j+1}$ with $\hat{2}\hat{\Phi}\hat{2}^{-1}$.

Considering figures 38, 39, and 42, note the critical importance of $\hat{\pi}^2$ for constructing the small embedded sphere. Furthermore, the reader is invited to recall the prominent role of π^2 in our derivation of the Einstein equation. In reference [4], we described a requirement to "cast a factor of π^2 into the information current." Starting with the MCM hypothesis

$$\omega^3 \big|\psi;\hat{\pi}\big\rangle = i\pi\Phi^2 \big|\psi;\hat{\pi}\big\rangle \;\;, \tag{4.9}$$

we can easily derive

$$8\pi^3 f^3 \big|\psi;\hat{\pi}\big\rangle = i\pi^2\big|\psi;\hat{\Phi}\big\rangle + \pi^2\big|\psi;\hat{i}\big\rangle \;\;, \tag{4.10}$$

which has an extra factor of π^2 in it. Therefore, using tentative notation, we can write

$$8\pi f^3\big|\psi;\hat{\pi}\big\rangle \ \cdot \ \pi^2 = \Big(i|\psi;\hat{\Phi}\big\rangle + |\psi;\hat{\imath}\big\rangle\Big) \ \cdot \ \pi^2 \quad . \tag{4.11}$$

At this point, we can replace some prosaic "casting" with $\pi^2 \equiv \hat{\pi}^2$ such that we use $\hat{\pi}^2$ to construct the topology of the embedded 2-sphere.

In whichever manner we twist the lines and circles, we need to be able to define the correspondence between the test particles that travel on them and the field representation in the bulk that would specify the path of an off shell virtual particle. These paths cannot be fully described with θ or ϑ but we have proposed to use $\hat{\Upsilon}$ to act with addition and multiplication simultaneously. Keeping in mind that all of the above operations were multiplicative in nature, and without going too far off on a tangent, this will be a good place to mention a possible future definition for $\hat{\Upsilon}$ like

$$\hat{\Upsilon} \quad \equiv \quad \hat{\imath} + \hat{\pi}\hat{2}\hat{\Phi} \quad . \tag{4.12}$$

We can use $\hat{\imath}$ to add the bulk space beyond the scaffolding to what we have done with the scaffolding itself in $\hat{M}^3$. We have associated $\hat{U} \equiv \hat{\imath}$ with ordinary quantum theory and i is featured prominently in the Schrödinger and Dirac equations

$$i\hbar\frac{\partial}{\partial t}\psi := \hat{H}\psi \quad , \qquad \text{and} \qquad i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu} = mc\psi \quad . \tag{4.13}$$

Among the most notable features of the motions derived from these equations is that the quantum wavefunction will often penetrate the classically forbidden regions of the energy landscape.[1] If we consider the lines of the the figures in this section as strings then we can send classical vibrations along them but those vibrations will never leave the lines that represent the strings. In the quantum regime, the off-string forbidden region becomes allowed. Therefore, if desired, we can encode the Dirac and Schrödinger equations on $\hat{\imath}$, and that leaves the other three operators for the topological deformations which create the little SU(2) symmetry at every point in spacetime. In general, we say that the lines are associated with real axes since, in the present convention at least, they have real length and we can add to that real domain the entire bulk space of fields with imaginary components by, perhaps, using $\hat{\Upsilon}$ as defined in equation (4.12). Earlier, we proposed to add matrix-valued *or other* multiplectic coefficients to the derivatives in $\hat{M}^3 \equiv \partial^3$ and the hatted objects in equation (4.12) fit the bill.

To continue the review of reference [2], consider the statement, "We are dealing with quantum geometric 'tunnelling' where everything remains pleasantly continuous." We inherited this smoothness condition from LQC but, at the time of the appearance of reference [2] in 2009, we had not yet made the connection to the Poincaré conjecture. That appeared

[1] They will always penetrate the forbidden region when the potential is physical, meaning that the energy differences are all finite.

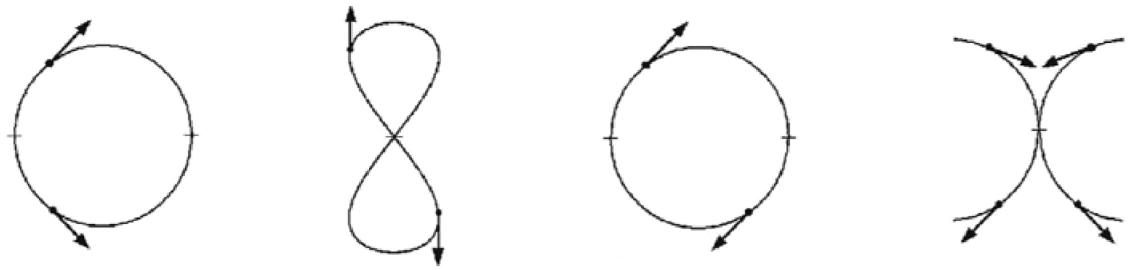

Figure 44: The MCM proposes to wrap the ct axis of the Minkowski diagram around a cylinder.

about two years later in reference [7]. Originally, the motivator of a singularity-free universe was "the repulsive force of quantum geometry" and, though not stated explicitly in 2009, the lack of any singularities implies that a trivial manifold has to be diffeomorphic to the real universe, and that, therefore, in qualitative terms of loose language, all of the the topological deformations in figure 44 are allowed.

We have migrated away from LQC as a motivator so now we can motivate the diffeomorphism in another way. First, remove from all of spacetime the interiors of any event horizons that contain topological singularities. We can leave other kinds of singularities in the fields defined in the universe[1] but, to construct a trivial 3-ball, there can be no gravitational singularities where the curvature of the manifold becomes infinite. We have removed the interiors of all the black holes so there are no places where the manifold's continuum breaks down but, now, we have punched holes in it, meaning that the topology is still not a trivial 3-ball. More changes are required. Therefore, increase the distances $\vec{r}_{jk}$ between the centroids of all the event horizons such that $\vec{r}_{jk}\hat{\Phi}^1 \rightarrow \vec{r}_{jk}\hat{\Phi}^2$. We have not increased the effective radius of the black holes so their sizes are on the order of an infinitesimal with respect to the new scale of the universe on the higher level of $\aleph$. Even these pointlike black holes prevent us from assembling a trivial 3-ball so we need to make a trick. We know that we can use the Banach–Tarski result to add points so we can also go in the reverse direction and remove all these points leaving a smooth manifold which is a topological 3-ball. Even then, there remain questions about the derivatives at these points which would break the diffeomorphism down to simple homeomorphism. This is insufficient for the purposes of the MCM; we need full diffeomorphism.

In an ordinary shrinking operation, where the sizes of black holes in the universe shrink instead of the distance between them increasing, the manifold is smooth up to a point (after shrinking the volume of the event horizon to a point) which is then removed "manually" to reduce the size of the object to zero. It is normal to remove these points so that the manifold is perfectly smooth as needed to construct a 3-ball but, even then, the fields and tangent fields will have kinks at the location of the point which was removed. This means that the model universe has only been made homeomorphic to the 3-ball but not diffeomorphic. When we increase the distance between the singularities instead of reducing their horizons, the picture is different. Since the mass of the singularity is $m_{BH} = m\hat{\Phi}^1$, it will only have an interaction distance of zero in the universe that exists on $\hat{\Phi}^2$. Then the tangent fields will not have kinks

[1] An example of the allowable singularities is the the infinite electrical potential predicted at the tip of sharp, charged conductor.

and we obtain diffeomorphism because masses like $m\!=\!0$ do not curve spacetime. Therefore, even when the real universe does contain singularities we can still say that it is diffeomorphic to the 3-ball: we simply have to encode the singularity qubits on a lower level of $\aleph$. When we want to consider singularities on our cosmological 3-ball, we only have to zoom in on the empty fabric of the manifold to consider a local region around the point where we place the singularity for consideration. We call the this local region the hypercomplexly infinitesimal neighborhood around the point to emphasize that this is different than an ordinary rescaling to zoom in on a local neighborhood. The hypercomplexly infinitesimal neighborhood around a point exists on a lower level of $\aleph$ than the initial framework of analysis. In this way, we can add singularities as small perturbations on a smooth manifold without disrupting the all-important topological invariance. The topological invariance of the MCM unit cell is the foundation of everything and must not falter.

Reference [2] progresses as follows.

> "The gravitational interaction [*between the time arrow eigenstates*] acts along a single axis so anisotropies in one universe's matter density do not appear as gradients in force on the other universe. This is in good agreement with the uniform [*effect*] of the controversial cosmological constant."

Indeed, it is in good agreement. The point made here about matter anisotropy is subtle but vital. The accelerating redshift z of deep space objects (dark energy) is observably uniform across the universe so, given a pair of real physical universes with clumpy features, we are constrained in our modeling of dark energy as an interaction between them such that these clumpy features do not lead to clumpy dark energy. We can suppress the anisotropic clumpiness in the interaction by transmitting it along a 1D manifold χ^5 that connects universe A with universe B, that is, it connects $\mathcal{H}_1$ to $\mathcal{H}_2$, instead of another idea where some 4D vector field grows off the mass distribution in universe A, reaches across the MCM unit cell, and then acts on universe B as an anisotropic effect that cannot be interpreted as dark energy. A 4D vector field connecting the two universe is unallowed both because it would transmit the information about clumpiness and it would mean that A and B were really just one universe with a large void in the middle. There would be no fifth dimension.

Moving forward with the review, consider the following further passage from reference [2].

> "Given an assumption of interaction through the big crunch, forward in time points 'downhill' toward a lower energy state [*as in figure 35.*] Observations have been made supporting this. Since we are further along in time than the astrophysical objects we observe, we can think of ourselves as deeper into a gravitational well than these [*light years*] distant objects. If this were so, we would be accelerating away from the observed object images. This is an alternative interpretation of data suggesting that the more distant an object lies, the more quickly it accelerates away from us. Acceleration is relative and in the [*MCM*] it is more intuitive to conclude that this data shows us to be accelerating away from the past, toward the future."

At this point, we are forced to concede that we should have followed the example of Einstein and published a few very short papers rather than putting three sections into one short paper[1] [2] because the groundbreaking, Nobel Prize worthy[2] result that appeared in the third paragraph of reference [2] seems to have been lost on readers by the time they finished reading the eighth and final paragraph. This all-important third paragraph is reviewed extensively in reference [54]. Higgs received a Nobel Prize for his brief observation in a two page paper [34] that a certain differential equation could be recast as a wave equation but reference [2] is not about differential equations. It is about topology. The language of topology is not differential equations so detractors err when they insist that Higgs' paper is categorically better than reference [2] on account of his research in differential equations being reported in the language of differential equations while our research in topology was not also reported in the language of differential equations. Topology does have its own jargon but that jargon is in support of the diagrammatic component; it is not the primary channel for communication of topological ideas. In any case, the point here is not to argue for a Nobel Prize but, rather, to argue that reference [2], with its groundbreaking and likely correct explanation for dark energy, was good enough to appear on arXiv alongside thousands of other papers that will be never be cited and possibly not even read.

The above excerpts paraphrase the first section of reference [2] and the second section begins stating, "The only thing relevant to the quantification of entropy is a system's macrostate as defined by a set of parameters." The macrostate of $\mathcal{H}$ is determined by the parameters $\{x^\mu, x^\mu_\pm, x^\mu_\varnothing, \chi^A_\pm, \chi^A_\varnothing\}$.[3] A key feature of the MCM is that we constrain these parameters in the boundary condition defined by the MCM unit cell; the phase space does not run rampant across all of parameter space. Therefore, in the MCM, we have already laid the groundwork for a new push toward a general relativistic statistical mechanics.[4] As of 2010 no such statistical mechanics had yet been devised and, as of this writing in 2017, no such mechanics are known to exist to this writer.

Another key feature that uncovers a nice tunnel to complexity can be seen in the MCM parameters $\{x^\mu, x^\mu_\pm, x^\mu_\varnothing, \chi^A_\pm, \chi^A_\varnothing\}$. The fifth chirological coordinates $\chi^5_\pm$ have a connection with a hypothetical fifth chronological coordinate $x^4_\pm$. It would be attached to $x^\mu_\pm$ as the de Sitter parameter of curvature for the embedded metric on the slice at each value of $\chi^5_\pm$ but there is no χ^A corresponding to x^μ in the way that we take for the $\pm$ and $\varnothing$ coordinate variants. It is quite common to describe the coordinates in dS or AdS as x^a when the the fifth "coordinate" takes the same value everywhere in the model universe whose curvature is equal to x^4 (and the lower case Latin index runs from zero to four.) Since $\Sigma^\pm$ do not contain their boundaries at $\chi^5=0$, and since $\mathcal{H}$ is a 4D analytical Minkowski space, there is no connection for χ^5 to a hypothetical x^4. Recall that if we adapted the x^a coordinates of de Sitter space to Minkowski space then the fifth coordinate would be zero and the metric would not be invertible leading to a host of related technical problems. None of these problems are intractable but, in the MCM, we say that $\mathcal{H}$ is purely 4D and none of those problems exist. However, we can use a 5D analogue metric for $\varnothing$ since we will never need to invert its metric. The only common requirement for an inverse metric is in the connection coefficients but we do not anticipate needing to know those for $\varnothing$ where the

[1] Here we do not mean to imply that the sections should have been rewritten. We only mean to say that the paragraphs in each section of reference [2] should have been divided among three papers rather than three sections of one paper.

[2] It is worthy of a Nobel Prize upon confirmation, as per usual. However, it is unlikely that this writer would bow his head to the King of Sweden to accept the medal or even don the clown costume preferred by the attendees of such ceremonies.

[3] There are four x^μ variables and four χ^A variables so, if desired, we could condense the notation so that the only relevant parameters are $x^\mu{}_\nu$ and $\chi^A{}_\mu$.

[4] See reference [55] for an introduction to general relativistic statistical mechanics.

only purpose is to smoothly sew together $\chi^5 \equiv \chi^5_+ \otimes \chi^5_\varnothing \otimes \chi^5_-$. Therefore, we can encode the chirological metric $\Sigma^\varnothing_{AB}$ at $\chi^5_\pm = \pm\infty$ with matrix valued coefficients Σ_{55} pertaining to an algebra altogether disconnected from everything described by the coordinates of the unit cell. This algebra would contain the operations that construct the cosmological lattice from any given qubit. As Dirac added matrix coefficients to the Klein–Gordon equation, we can add any kind of multiplectic structure here. Operations such as rotations and inversions, and all the mundane topological manipulations we have motivated for the MCM, are typically described in equation form with matrices. Rotation matrices, shear matrices, and the like can be encoded in the fifth diagonal position of a hypothetical metric Σ_{AB} which would define the chirological line element if $\mathcal{H}$ was actually embedded in a 5-space. We could embed it in 5-space, if we wanted to, but we have designed the MCM unit cell such that $\mathcal{H}$ is topologically a surface on the terminating edge of a semi-infinite 5-space: either Σ^+ or Σ^-. Rather than using an obscure corner of $\mathcal{H}$ in x^4, we will use an obscure corner of $\varnothing$ which was already obscure to begin with. Therefore, we motivate the concept of double orthogonality between the hypercosmological lattice and the dynamics inside a particular universe $\mathcal{H}$.

Reference [2] continues as follows.

> "[*C*]onsider the following topological manipulations [(*figure 44.*)] The big bang and crunch are identical and may be mapped into each other by twisting the *ct* circle into a figure eight. Twist it once further so that time forms a circle once again but now [*the forward time direction*] for each universe is in the clockwise direction. Finally center the dynamics on the bounce state so the death and rebirth of each universe is schematically clear.
>
> "In the final frame of [*figure 44*], the semicircle on the left represents our perception of the larger system just before the crunch. Our universe is nearing a state of maximum entropy and the reverse time universe is converging on its minimum entropy big bang. After the bounce we again find one universe at a maximum of entropy and another at a minimum. As there is no way to tell which is the forward or reverse time universe, this interaction is analogous to the rearrangement of identical particles — a process long known to be isentropic. To alleviate problems with human intuition in perceiving the flow of time, let us replace the spacetime diagram with the familiar Feynman diagram where a rigorous framework is already in place for dealing with interacting bodies moving in different directions through time [*figure 45*]. If we give ourselves fully over to the Feynman diagram, we should consider the reverse time universe to be the [*anti-particle*] of our universe. Then it has negative baryon number and the greater system at hand becomes baryon neutral in good agreement with predictions."

This excerpt contains two further insights that are good ideas on the order of the good idea about dark energy. Thermodynamics is (mostly) beyond the scope of this book but the other idea embodied by figure 45 is very much in scope. Most of the mathematical content of the early MCM papers was encoded in the diagrams but we used Feynman's formulation of the path integral as an analytical foundation. We did not reproduce in 2009 what Feynman had written 70 years earlier but Feynman's results did exist and did serve as the foundation

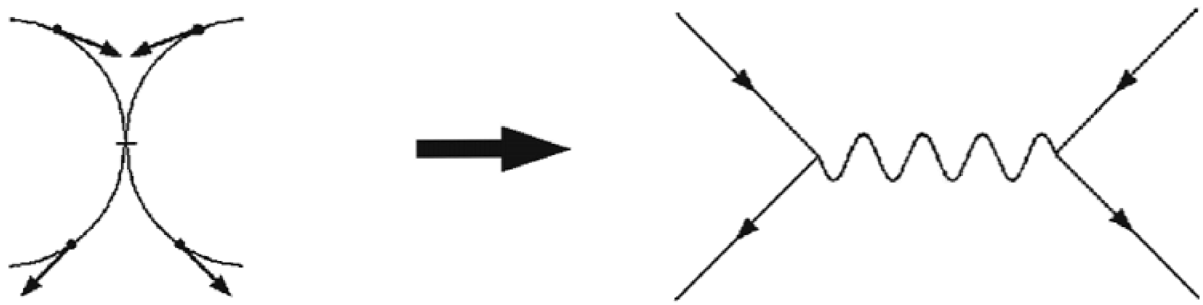

Figure 45: This figure show the isomorphism of interacting universes in the MCM with interacting fermions in QED.

of reference [2], and all the subsequent work in this research program. Detractors will surely claim that if we have not transcribed Feynman's words then they could not have possibly served as a rigorous analytical foundation for the work in this research program. However, reasonable people can not possibly accept detractors' faulty argument. The isomorphism between Feynman's idea and the MCM, as in figure 45, is *absitively posolutely* undeniable.

The final bit of this excerpt has to do with the matter/anti-matter imbalance that we claim to solve without addressing the Sakharov conditions.[1] It is possible that the MCM has subsequently and incidentally conformed to those requirements but it will be another topic for future inquiry to examine whether or not the theory predicts a matter/anti-matter imbalance in each individual universe. One likely motivator for this imbalance is that t_+ and $t_\star$ must both be associated with forward time so that there is a fundamental imbalance between $\{t_+, t_\star\}$ and $\{t_-\}$. This imbalance might further be connected to the fractional electric charges of the quarks: $+2/3$ and $-1/3$. Another motivator for an imbalance is that infinity is not symmetric about the origin. Recall that the ordinary concepts of plus and minus infinity do not carry over into the framework of hypercomplex analysis. We can have plus and minus infinity on $\hat{\Phi}^j$ but infinitely big $\hat{\Phi}^{j+1}$ lies at positive infinity and infinitely small $\hat{\Phi}^{j-1}$ lies at the origin. We have defined $\hat{\Phi}$ as a 1D object pointing though, or to, future timelike infinity so it cannot point to the entire ring at infinity that would be symmetric around the origin. That would require $\hat{\Phi}$ to also point to spacelike and null infinity but we have it not defined it to do that (yet.) Asymmetry about the origin induces a lack of symmetry in the related analytical framework and we might try to associate that with the non-vanishing baryon numbers of the two individual universes U and $\bar{U}$.

The above concludes what we will repeat from 2009's reference [2] and, now, we will continue the theme of this section with a few excerpts from reference [7]. Just as the theme of this section has been to rehash reference [2], the purpose in producing reference [7] in 2011 was to rehash it, and also to see if arXiv would accept a manuscript typeset in LaTeX with a lot of (a fair amount of at least) irrelevant prose added. They did not and had instituted an endorsement system since this writer's first attempt to publish with them in 2009. In 2011, this writer obtained an endorsement from Pablo Laguna and sent a LaTeX manuscript to arXiv again. Thereafter, arXiv both rejected the manuscript and revoked the endorsement according to some unpublished censorship criterion that many who publish on arXiv might

[1] The excerpt states that there is no imbalance but that is only when considering U and $\bar{U}$ together; within U there is still an imbalance.

believe does not exist.

Reference [7] begins with the following irrefutable statements that detractors have surely sought to refute: "A new paradigm is needed in physics and the MCM is such a construction," and, "Diagrams are used in physics to transmit information with a clarity not present in excessively quantified arguments." Not only was a new a paradigm needed around 2009-2011 but, despite the laborious, decades-spanning efforts of those to whom the MCM's detractors show good will, new paradigms have been few and far between. Most paradigmatic proposals can be ruled out in minutes and supersymmetry — the main unrefuted paradigmatic proposal, possibly even irrefutable, as in "not even wrong" — isn't so super.[1] Therefore, the scientific merit of the MCM is self-evident when it presents a novel new paradigm falsifiable in many ways, most notably in its prediction for new spin-1 fundamental bosons. Note that a paradigm is one thing and a formal mathematical theory polished by thousands of the world's best and brightest over a century of sustained collaborative effort is quite another. References [2] and [7] describe a paradigm, or what might be called a framework, or simply a new idea. All formalized theories are built on the new ideas that preceded them. Indeed, history shows that, between formulating an idea and polishing one, the formulation is the bottleneck which throttles progress in physics.

Misner, Thorne, and Wheeler write the following about diagrams in reference [27].

> "Pictures are no substitute for computation. Rather, they are useful for (a) suggesting geometric relationships that were previously unsuspected and that one verifies subsequently by computation; (b) interpreting newly learned geometric results."

The primary output of this MCM research program has been to suggest the geometric relationships that produce physics' three most important dimensionless constants $\{\alpha_{MCM}, 8\pi, 1/4\pi\}$. When detractors focus only on the computational component, which is not primary in this research program, they ignore that the main difficulty in physics in recent decades has been the lack of any new geometric relationships on which new computations could be based. After studying physics in college for almost a decade, this writer became familiar with the general framework of physical computation and sought to develop a new set of geometric relationships of the type that would be relevant. We do not disagree with Misner, Thorne, and Wheeler when they claim that pictures are no substitute for computation, but we do disagree with detractors who claim that pictures are worthless without computation.[2] Following Misner, Thorne, and Wheeler, we must conclude that the pictorial component should precede the computational component. Therefore, considering that physics is a collaborative effort with no one person doing the entire thing, it is most odd (and indeed personally vexing to this writer) that other physicists have not been publicly excited at the possibility of making new computations based on the MCM geometry. Indeed, in analogy, it is as if detractors believe that the machinist who mills a part for the LHC should not be paid or praised for his work because he did not also assemble the LHC himself, interview and hire

[1] In the paradigm of supersymmetry there is a rotation that will swap fermions with bosons and the MCM contains this symmetry. We simply associate fermions with chiros and bosons with chronos so that any rotation which swaps x^0 with χ^5 necessarily swaps bosons with fermions.

[2] We in no way imply that this research program is devoid of computation. We only allude to the type of very involved, tedious calculations that detractors look for when they scan over papers without reading them.

all the staff at CERN, run the machine and do the full data analysis by himself, and also organize the conferences in which he might disseminate his work.

During this writer's time in academia, 2003-2011, it became apparent to him that the area of critical need in physics was definitely not in the computation department. The critical need was in the conceptual department and, although this writer has supplied an exquisite concept to the physics community at large, none of the other physicists will admit to making the computations which must follow. This writer concedes that the computational component is usually considered the primary component in modern physics but detractors should concede that such is likely the main reason for physics having gone into a stall for so many decades. Computation is no substitute for creative imagination. While we may or may not have misquoted Einstein in reference [3], but it is widely accepted that Einstein said, "Imagination is more important than knowledge. For knowledge is limited, whereas imagination embraces the entire world, stimulating progress, giving birth to evolution." In this research program, we have sought to blend knowledge with imagination to derive a new paradigm without becoming bogged down in the irrelevant technical details that detractors are so in love with. Rather than work out the details they would like to see for themselves, detractors have chosen to block both progress and evolution seemingly out of spite for this writer's creativity and outstanding mastery of the foundations of physics.

To continue the review of reference [7], consider the following.

> "Perelman's [*and Hamilton's*] proof of the Poincaré conjecture can be applied to LQC in a way not possible with other cosmologies. The conjecture is this: every simply-connected closed three-manifold is homeomorphic to the three-sphere. Bojowald has shown that the divergent singularities of classical [*g*]eneral [*r*]elativity do not exist in Nature. Given this, the Poincaré conjecture can be applied to LQC as: every simply connected, closed three-manifold is diffeomorphic to the three-sphere."

In 2011, we presumed that singularities do not exist in Nature due to the principles of LQC but now we can completely ignore LQC and say that singularities don't make our theory explode due to the axioms of hypercomplex analysis in the theory of infinite complexity. With $^\star\mathbb{C}$ numbers, we can embed singularities in a non-singular 3-ball as perturbations on a lower level of $\aleph$. This is very good because singularities are generally believed to exist in the physical universe on the interior of real black holes. Analyses of other theories run into a dead end at infinity but hypercomplex analysis is transfinite so there is no dead end.

It is most interesting to note that we began with a solution to a Millenium Problem and then progressed to a solution of another one regarding the Riemann hypothesis [8] about five years later. Detractors must acknowledge the validity, and now proven superiority, of using geometry rather than differential/integral equations as the mathematical tool driving new concepts.

Continue to consider reference [7] with the following.

> "The conformal equivalence of the Minkowski diagram and the Penrose diagram [*figure 46*] is trivial. The universe defined by I and II in the Penrose diagram travels forward through time and this motion constitutes a component of its 4-momentum. If momentum is conserved, the big bang must have thrown an equal amount of

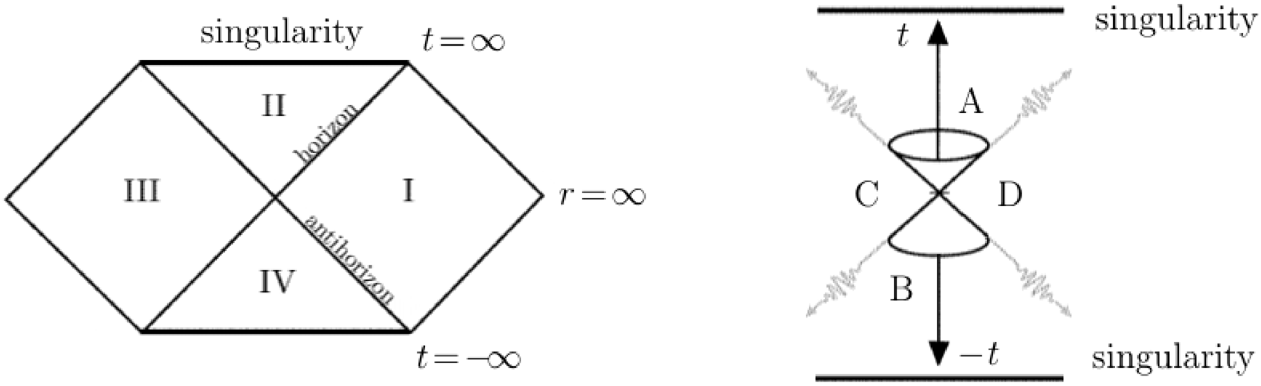

Figure 46: On the left is the Penrose diagram of a Schwarzschild black hole. On the right is a quasi-Minkowski diagram describing the big bang. It shows two future light cones opening in opposite directions away from the big bang. The black arrows on the right indicate the arrows of time in the universes U and $\bar{U}$.

> matter and energy along both time directions as in [*figure 46*]. This is not posed as an assumption but rather [*as*] an absolute fact of momentum-conserving Nature.
>
> "Regions III and IV of the Penrose picture are unphysical in the [*standard cosmological model*]. This fact stems from the big bang singularity forbidden in LQC. In the present divergence-free paradigm, all four regions are physical and coexist. Penrose's reverse time description of III and IV is accepted without question in the MCM; such a reverse time regime is needed to satisfy [*conservation of momentum*].
>
> "In [*figure 46*], region A is a conformal map of Penrose regions I and II. Region B represents Penrose III and IV. Spacelike regions C and D are orthogonal to the Penrose diagram and do not appear in it."

This excerpt contains one of the MCM's finest qualities: conservation of momentum. The MCM *does* universally respect conservation of momentum but ΛCDM cosmology can not honestly put that feather in its hat.

The comparisons between {I,II,III,IV} in the Penrose diagram of a Schwarzschild black hole (figure 46, left) and {A,B,C,D} in the Minkowski diagram (figure 46, right) were framed around the idea of the bounce being at the big bang. The minimal Schwarzschild model of a singularity in spacetime is a good model for the big bang singularity in an abstract analytical manifold that we can use as a proxy for the spacetime which did not exist at the special moment where everything in the universe was condensed to a point (or however small the apex of a quantum geometric bounce is taken to be.) We have subsequently migrated to the framework in which the singularity is associated to the present moment $\mathcal{H}$ and we have not much treated the other universe $\bar{U}$ which we call Penrose III and IV. Furthermore, the Penrose diagram is 4D so the full extension to the high dimensional MCM has a lot more complexity than the one-to-one correspondence of regions suggested in figure 46. For instance, one particular issue is that it makes sense for U and $\bar{U}$ to be next to each other at the big bang but it is unclear what that might mean at an arbitrary time during the late universe. One feature to note is that, since we have encoded the past of $\mathcal{H}$ on $\hat{\varphi}$ instead of $\hat{\Phi}^{j-1}$, there is room to say that U and some $\bar{U}$ are still adjacent at every moment and shortly we will make a neat connection in that regard.

One of the first things to note about Penrose diagrams, which Penrose himself called conformal diagrams, is that they *do* explicitly contain points at infinity which are not necessarily included in the Minkowski diagram. Therefore, the main venue for Minkowski diagrams is the description of calm spacetime around an inertial (Lorentz) frame and the Penrose diagram is better suited to describing black holes and cosmologies. The well known Penrose diagrams of the different black holes are not themselves so important for the MCM because they only describe the slice $\mathcal{H}$, and not the bulk hyperspacetime or the requisite non-commutative aspects of chirological time, but Penrose's ideas about conformal infinity are completely indispensable in the MCM. Conformal equivalence in spacetime is always reducible to a Penrose diagram of some sort (a conformal diagram) and the MCM is completely dependent on that equivalence. We say that two manifolds are conformally equivalent if the line element in one can be written in terms of the other as

$$ds^2 = \Gamma d\tilde{s}^2 \quad , \tag{4.14}$$

where Γ is some conformal scale factor. Penrose's idea of a map between physical spacetime and another manifold where the location of infinity lies at a location specified with a finite parameter is the foundation of just about everything we call hypercomplex analysis. Generally, we derive a map from infinite to finite with some change of coordinates like

$$\tau = \tan(\rho) \quad , \qquad \text{where} \qquad \tan\left(\pm\frac{\pi}{2}\right) = \pm\infty \quad . \tag{4.15}$$

This will put infinity at the endpoints of the four intervals $\vartheta \in (0, \pi/2)$ introduced above. Penrose's seminal insights into conformal infinity are reported in reference [52]. That paper was reproduced by Springer in 2010 with a well deserved label: Golden Oldie. We will study reference [52] more closely in section IV.4.

Before moving on with the review of reference [7], consider what Hamilton writes about Penrose diagrams in reference [56].

> "The Penrose diagram shows that the horizon is really two distinct entities, the Horizon, and the Antihorizon. The Horizon is sometimes called the true horizon. It's the horizon you actually fall through if you fall into a black hole. The Antihorizon might reasonably be called the illusory horizon. In a real black hole formed from the collapse of the core of a star, the illusory horizon is replaced by an exponentially redshifting image of the collapsing star.[*sic*]
>
> "However, the Schwarzschild geometry has a simple mathematical form, and that form can be extended analytically. The mathematical extension consists of a second copy of the Schwarzschild geometry, reversed in time, glued along the Antihorizon. The complete analytic extension of the Schwarzschild geometry contains not only a Universe and a Black Hole, but also a Parallel Universe and a White Hole. This is simply a mathematical construction, with no basis in reality. Still, it is cute that even the simplest kind of black hole, a Schwarzschild black hole,

harbors alien mathematical passageways."

Where Hamilton writes, "the illusory horizon is replaced by an exponentially redshifting image of the collapsing star," he fairly concisely sums up what Poplawski was writing about in his prolific "Is the Universe Inside a Black Hole?" publications. When the whole universe is inside a black hole, rather than the redshifted image of a star we will see the redshifted image of the whole early universe showing up as dark energy. In the MCM we describe dark energy as inverse radial spaghettification and this is the gist of it. Furthermore, it does not appear that Hawking, in his initial derivation of his eponymous radiative effect, has made allowances adequately for the "alien mathematical passageways."

During the time that a large portion of physicists were preoccupied with firewalls versus complementarity instead of the MCM, 2012 and the following years, it was postulated that one of the following three things must be false [57]: "(i) Hawking radiation is in a pure state, (ii) the information carried by the radiation is emitted from the region near the horizon, with low energy effective field theory valid beyond some microscopic distance from the horizon, and (iii) the infalling observer encounters nothing unusual at the horizon." It is mind-boggling to this writer that hundreds or thousands of physicists juggled the "rigorous mathematical implications" of (i-iii) without considering that none of their juggling was rigorous at all because Hawking's result on which they were building was not rigorous to begin with. To this writer's knowledge, no one, at any point in the well published debate, ever pointed out that the entire principle of firewalls or complementarity was founded on Hawking's non-rigorous result. As a last aside before continuing with reference [7], note the following from reference [5].

> "Consider Hawking radiation in the context of topological obstructions. Near the horizon there is a quantum fluctuation and one particle starts falling inward before it annihilates with its partner. Once its trajectory pierces the horizon, its phase space is spontaneously truncated so that no future trajectory ever leaves the interior of the event horizon. (In reference [13] we showed how a fractal embedding of event horizons in a charged, rotating black hole is a good descriptor for cosmological lattice translations.)
>
> "The radius of the black hole is proportional to the mass enclosed so it has respective radii r_{out} and r_{in} before and after the particle falls behind the horizon. When is the moment that the particle's phase space changes? It can't change until the particle passes the horizon, and when it does the black hole's radius has already changed to r_{in} meaning the particle is inside by more than a differential element of distance. Hence the moment we are examining can no longer be the moment the radius changed. This is an unresolved paradox.
>
> "To derive Hawking radiation, it is necessary to advance a trajectory from r_{far} to r_{close}. When the trajectory is very close, all the relevant information is exported to a parameter file. The information is injected inside the black hole and then someone starts the stopwatch running again. With one particle inside, the other escapes as Hawking radiation despite there being no physical trajectory through the horizon."

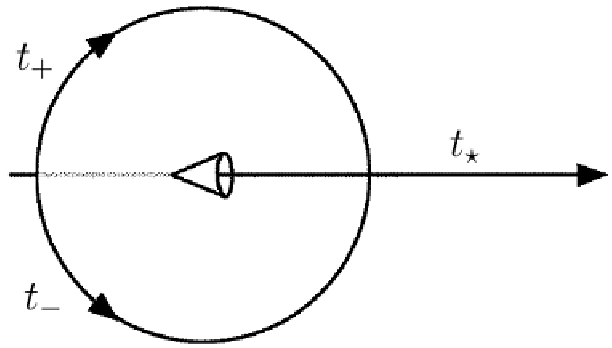

Figure 47: This figure shows the general superposition of positive and negative time that the observer must occupy when he is unable to make a measurement to determine if he is in $|t_+\rangle$ or $|t_-\rangle$.

Alien mathematical passageways indeed.

Returning to reference [7], we treat an experimental constraint on theorists' ability to guess random cosmologies as follows.

> "The WMAP data rules out the curvature of the universe postulated here. To avoid this contradiction let us assume WMAP observes a superposition of $|t_+\rangle$ and $|t_-\rangle$ so that $|t_\star\rangle = \alpha|t_+\rangle + \beta|t_-\rangle$ as in [*figure 47*]. Then WMAP samples two oppositely curved spaces which obey the superposition principle. The result is the observation of flat space. [O]bservers will never be able to say if they belong to $|t_+\rangle$ or $|t_-\rangle$. The wavefunction is diffuse and the postulation of $|t_\star\rangle$ is confirmed. When an observation cannot be made, both possibilities must coexist as a superposition of states."

NASA's WMAP website [58] reports the following about the curvature of the universe.

> "The WMAP spacecraft can measure the basic parameters of the Big Bang theory including the geometry of the universe. If the universe were flat, the brightest microwave background fluctuations (or "spots") would be about one degree across. If the universe were open, the spots would be less than one degree across. If the universe were closed, the brightest spots would be greater than one degree across.
>
> "Recent measurements (c. 2001) by a number of ground-based and balloon-based experiments, including MAT/TOCO, Boomerang, Maxima, and DASI, have shown that the brightest spots are about 1 degree across. Thus the universe was known to be flat to within about 15% accuracy prior to the WMAP results. WMAP has confirmed this result with very high accuracy and precision. We now know (as of 2013) that the universe is flat with only a 0.4% margin of error.[1] This suggests that the Universe is infinite in extent; however, since the Universe has a finite age, we can only observe a finite volume of the Universe. All we can truly conclude is that the Universe is much larger than the volume we can directly observe."

[1] A likely unrelated point of interest is that α_{MCM} disagrees with α_{QED} by about 0.4%.

One of the main differences between references [2] and [7] is the embedding of flat spacetime in the latter between two curved spaces: hyperbolic and spherical. During the preparation phase of reference [7], this writer had not yet been expelled from Georgia Tech; that would not happen until the month following the attempted publication of reference [7]: December 2011. While doing unrelated professional research at Georgia Tech regarding shock induced phase transitions in bulk metallic glasses, this writer had a weekly meeting with Seung-Soon Jang who was co-advising this writer's professional research with Naresh Thadhani. In these weekly meetings, we spoke not only about glasses but about scholarly things in general. On a particular day, likely in October 2011, this writer was explaining to Jang that the WMAP result which shows a flat universe prohibits the kind extra dimensions that now constitute $\Sigma^{\pm}$. When this writer told him, "Extra cosmological dimensions are ruled out," he responded, "No, I don't think so." This writer subsequently verified the veracity Jang's statement and, upon deep contemplation, the extra dimensions were added to the MCM. Jang's comment was at least as great an influence on the MCM as the lectures given by Ashtekar and Penrose and has previously gone unmentioned. This writer extends his gracious thanks to Seung-Soon Jang and acknowledges learning the Greek word *chiros* after inquiring about the names of some of the computing clusters in Jang's group. One was named Veritas and another was named Chiros, and there were a few other clusters.

The excerpt above finishes with a statement that the universe is much larger than what is observable so we are drawn toward a comparison with the topological system derived in reference [8] wherein we study the Riemann sphere. When considering the hypercomplexly infinitesimal discs around the polar singularity on $\mathbb{S}^2$, we were able to define them as flat by taking the radius of the curvature of the sphere as infinity. Therefore, a similar principle may be at play in the physical universe where our local region of flat spacetime smoothly transitions into non-vanishing curvature beyond the CMB.

NASA's WMAP website [58] reports the following about WMAP in general.

> "The Wilkinson Microwave Anisotropy Probe (WMAP) mission reveals conditions as they existed in the early universe by measuring the properties of the cosmic microwave background radiation over the full sky. This microwave radiation was released approximately 375,000 years after the birth of the universe. WMAP creates a picture [*figure 48*] of the microwave radiation using differences in temperature measured from opposite directions (anisotropy). The content of this image tells us much about the fundamental structure of the universe."

The proper age of the universe $t=375,000y$ corresponds to the time of baryogenesis. One second after the big bang, the universe was much too hot for baryons such as nucleons to form. Even at that early time, the age of the universe was already far beyond the inflationary epoch which itself took place after the universe had fully transitioned out of the Planck epoch, which had some non-deterministic origin in a big bang or bounce of some variety (according to ΛCDM.) Hundreds of thousands of years later, around $t=375,000y$, the universe cooled off enough for hydrogen atoms to form. At that time, the fluid of the universe became transparent to electromagnetic radiation. The cosmic microwave background is the farthest (oldest) thing we can see in the universe because, for $t<375,000y$, all the photons were getting knocked around in the pre-baryonic plasma. For this reason, the CMB is sometimes

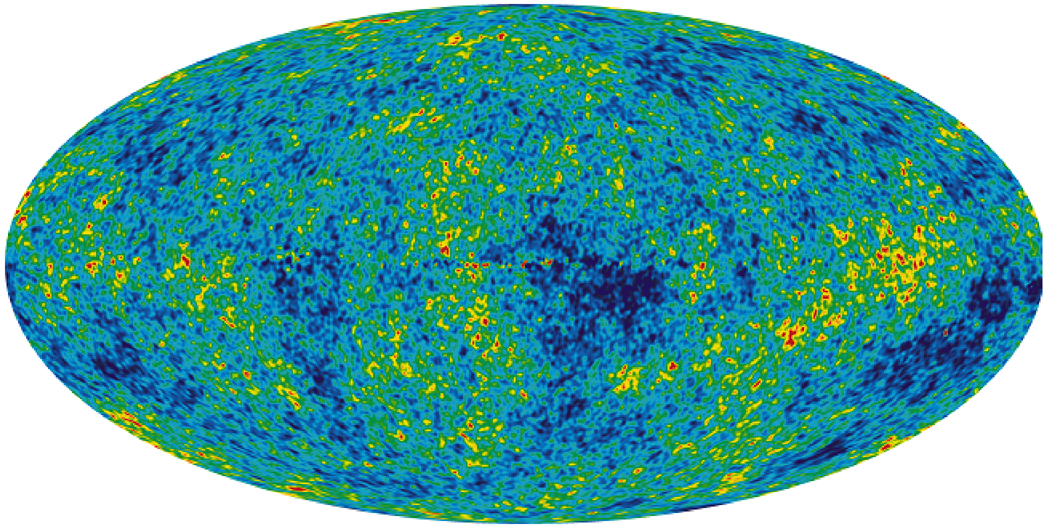

Figure 48: The results of the WMAP experiment show that the angular scale of temperature fluctuations in the cosmic microwave background is about 1°. This implies that the spacetime on the interior of the CMB has very little or no net curvature.

called the surface of last scattering. Frequent scattering means we cannot infer that a photon's source must lie along the path that the photon took to reach a detector on Earth. For photons created after $t\!=\!375,000y$, when the universe became clear, most of the photons in the universe that come to our telescopes get here along a straight path. When a telescope detects one of these photons, we have good reason to conclude that the photon was created in the region of sky at which a particular telescope is pointed at the time of the detection. When we detect CMB photons, there is no expectation that the photon was created in the region of sky that is the the object of the detector.

We see that, in the canonical ΛCDM framework, there are many nested layers of complexity between the present day and the big bang: last scattering, inflation, the Planck epoch, and a non-deterministic first moment of the universe after the singularity. Therefore, we have good reason to say that our own hand waving between a pointlike big bang and a non-pointlike bounce is likely to be a subset of the canonical hand waving in ΛCDM.[1] In ΛCDM, there is significant hand waving when we presume that the cosmological topology is such that the special relativistic light cone can be analytically continued all the way back to a singularity at $t\!=\!0$. This topology is valid for the present day and the early universe but it is not valid for the very early universe. Therefore, we have good reason to move the bounce to $\mathcal{H}$ and then set the big bang or the big crunch at $\varnothing$ where we have a fifth diagonal term in the metric $\Sigma_{55}^{\varnothing}$ which will modify the topology. If we want to use a single topology for the entire lifetime of the universe then our chosen lattice hypercosmos model means that "cosmological phonons" could come into the universe from the past or the future (the bang or the crunch) and this is in line with our repeated calls to include dependence on the advanced time in the field equations of $\Sigma^{\pm}$ and $\mathcal{H}$. The only place that we are modifying ΛCDM is in the early universe that we cannot see because the universe was at that time filled with plasma,

[1]This particular circumstance of hand waving was confined to the early MCM only. We do not presently wave hands in this fashion because there is no bounce in the modern iteration constructed about some present moment.

and plasma physics is the canonical example of why it is sometimes necessary to include the advanced potential in the field dynamics if one wishes to make a verifiable prediction.

NASA goes on in reference [58].

> "To address its key scientific questions, WMAP measures small variations in the temperature of the cosmic microwave background radiation. These variations are minute: one part of the sky has a temperature of 2.7251° Kelvin (degrees above absolute zero), while another part of the sky has a temperature of 2.7249° Kelvin. In 1992, NASA's Cosmic Background Explorer (COBE) satellite detected these tiny temperature differences on large angular scales. WMAP measures anisotropy with much finer detail and greater sensitivity than COBE did. These measurements reveal the size, matter content, age, geometry and fate of the universe. They also reveal the primordial structure that grew to form galaxies and will test ideas about the origins of these primordial structures."

The temperature of the CMB is important because we can compare it to the temperature of the universe at which we predict baryons were able form. If Doppler shift due to the expansion of the universe has cooled those pre-baryonic photons to their present temperature then that gives us a measure of how much the universe has expanded since baryogenesis. Figure 48 is shaded to show the fluctuations in the temperature of the CMB but the results of the experiment were that the CMB is surprisingly thermally flat. The even temperature of the CMB (isotropy) is very important for building the MCM lattice. The boundary condition of isotropy at the outer edges of the anisotropic physical universe is an unintuitive constraint. When we look at the intermediate stuff of the universe between Earth and the CMB, there are great fluctuations in what is observed. We see void, planets, stars, planetary nebulae, galaxies of various geometries, *etc.* The gradient from darkness to starlight in the night sky is strongly pronounced and, except in very deep space images, we do not need to shade for contrast. It is surprising that the CMB does not have these features because all of the anisotropy in the present day universe is supposed to be the deterministic continuation of the isotropic surface of last scattering at $t = 375,000y$.

Here, we point out one of the major problems with the standard cosmological model: the horizon problem. We say that the age of the universe is about 13.7Gy because the CMB is about 13.7Gcy away from Earth. If we observe the CMB from a position on the surface of the Earth for about 12 hours, the final position in the sky will be about 27.6Gcy from the initial position. This follows from the 24 hour Earth day. If one looks to the sky for 12 hours then the final view is in the opposite direction to the initial view and $27.6 \approx 13.7 \times 2$. It is paradoxical that both sides of the CMB, the side that is 13.7Gcy from Earth in one direction and the side that 13.7Gcy in the opposite direction, are in thermal equilibrium. According to the standard cosmological model, the universe is not yet old enough for any information to have propagated across 27.6Gcy as would be required for two points separated by π radians in the CMB to come to thermal equilibrium with each other. Among the many different phases of the universe between the present day and the big bang, the standard workaround for the horizon problem is included in the inflation phase. We simply say that the universe came to thermal equilibrium when it was very small and then inflation kicked in making the observed equilibrium conform to the expectation. However, inflation itself is contrived (inflation is just $f(x) = e^x$ for the size of the universe), there is at least as much evidence against it as there is for it, and, in any case, modern inflation is just the latest iteration of e^x after several earlier versions were ruled out by experiment.[1] In the MCM, we should therefore consider that the hypercosmos exists forever so that the thermal equilibrium of the CMB is not paradoxical. We can take an eternal universe or we might even take a finite duration, big bang/crunch universe where the CMB shows a state of equilibrium that reflects the equilibration of the cosmological lattice phonons at every time in the eternal existence of the hypercosmos. In that case, what we would call "the universe" exists inside the CMB but does not include it; the CMB would be a domain wall in an eternal cosmological lattice that houses universes which come and go as vacuum or other fluctuations. These solutions are purely speculative but are no more or less contrived than inflation itself.

Another problem, one not solved by inflation, is that when the cosmological horizon is 13.7Gcy away from Earth in every direction, there is only one geometry that will accommodate the arrangement. The CMB surface must be a sphere and the observer must be at its center. Therefore, in the paradigm where the universe is about 13.7Gcy old, the equidistance of the CMB implies that the Earth is in the center of the universe. On one hand, since the MCM universe describes what can be observed relative to an observer at $x^\mu = 0$, it is not so strange that cosmological phenomena should be centered on the position of the observer. Given the common understanding, however, that the observer is not the center of the universe, we might consider a diver in the ocean. The water around the diver is clear out to a certain distance where he observes a bluish and opaque wall. This is the analogue surface of last scattering in the water. The distance to this surface is approximately the mean free path of the photons in the water. Beyond this distance, the diver no longer has an expectation that photons come to him from the direction in which he observes them. It is the diffusion of the scattered photons from beyond the mean free path that cause the underwater optical horizon to look like a featureless blue wall. Regardless of where the diver swims in the ocean, he will always be in the center of this sphere of visibility so, in the cosmological setting, we may formulate some similar scenario where the observer is always in the center of a larger sphere of visibility in space. Whatever the scenario should be, we will not be drawn to the conclude that the age of the water is X years because the distance to the scattering surface is X light years. Therefore, we should likewise not do so for the age of the universe. The flat, empty intergalactic medium of the cosmological arena does not generally attenuate light like water does so, in analogy, we should say that, perhaps, 13.7cy is the distance at which metrical turbulence sets in [7].

What is metrical turbulence? To understand this feature, what we called hyperturbulence in reference [7], or to know if it is even a valid idea for an effect, would require a significant analysis which will not appear in this book. However, since the idea is a prominent and novel feature of the MCM we will discuss the underlying principle. To that end, note that the lines in figures like figure 47 are the proper time axes of different universes and that, on some level of $\aleph$, we have a description where t_+ is the time axis of de Sitter space, $t_\star$ Minkowski space, and t_- anti-de Sitter space. We have constructed the cosmos so that there is a representation where the parallels, meridians, and hypermeridians on the surface of $\mathbb{S}^3$ (figure 49) are associated with flat, hyperbolic, and spherical geometries [7]. Parallels are

[1] Reference [59] gives a good review of the modern state of cosmic inflation.

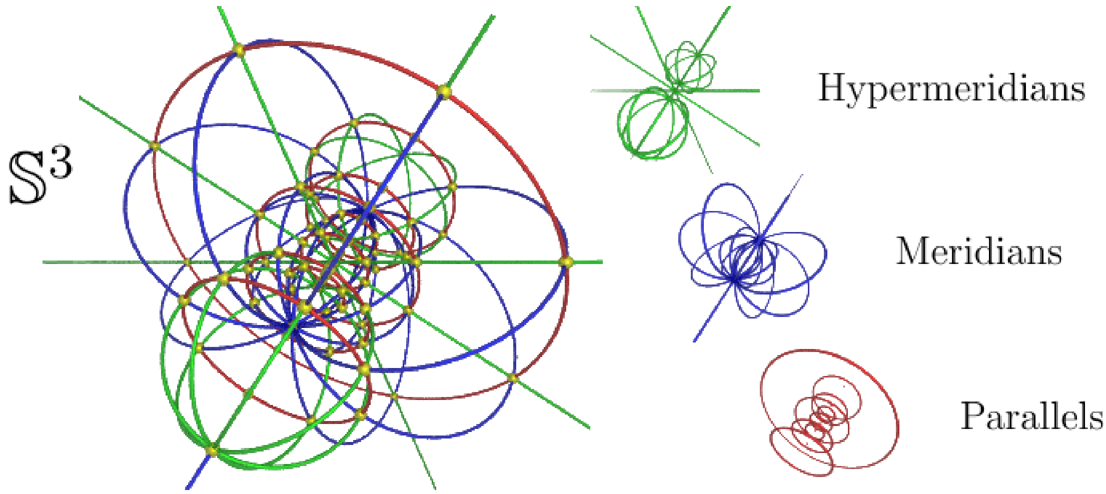

Figure 49: Each type of circle in this figure, by definition, implies a component π and, when we consider all three simultaneously, we are drawn toward π^3 which is an important ingredient for α^{-1}_{MCM}. This figure is mostly taken from Wikipedia.

likely flat and should foliate[1] Minkowski space but it is not obvious, between meridians and hypermeridians, which should foliate de Sitter or anti-de Sitter space. Note that all three types of lines — parallels, meridians, and hypermeridians — are circles or straight lines, and, by changing levels of $\aleph$, circles of radius R can be made flat with $R\to\infty$ and straight lines can be made circular with $\infty\to R$. Furthermore, one parallel, one meridian, and one hypermeridian can be used to construct the three element structure of a circle with a sphere attached that we have developed in this section, and depicted in figure 49.

The 3-sphere representation contrasts with the unit cell representation because $\{\aleph,\mathcal{H},\Omega\}$ all come together at the vertices seen in figure 21. The unit cell is designed to show the χ^5 direction but it is clear that x^0 also leads to $\aleph$ and Ω in its distant past and future respectively. These representation can be reconciled by considering that chiros must wind around chronos (chirally.) It will likely be a source of topological incongruity leading to a condition of non-conservation of information that chiros must wind around chronos chirally while also being identically flat, as in the MCM unit cell.

To develop a concept of hyperturbulence, consider that the metric associated with each of $\{t_+,t_\star,t_-\}$ extends into the bulk, away from the lines, through the foliation and that the foliations must therefore overlap at the points where parallels, meridians, and hypermeridians intersect at right orthogonal vertices. Even if we define the foliation on a lower level of $\aleph$ so that the foliation only extends into the hypercomplexly infinitesimal neighborhood around each line, these lines intersect at the vertices. Therefore, even in the hypercomplexly infinitesimal neighborhoods around each line, the "metrical fields" $\{g_{\mu\nu},\Sigma^{\pm}_{\alpha\beta}\}$ are bound to

[1] Just as $\mathbb{S}^2$ missing one point is equivalent to the plane, $\mathbb{S}^3$ missing one point is equivalent to 3-space. Therefore when we complete 3-space with a point from one of the lines $\{t_+,t_\star,t_-\}$ we will obtain $\mathbb{S}^3$. Therefore, the "foliation" refers to that which propagates along each axis using a single point from it to maintain at all times the $\mathbb{S}^3$ topology. It is through this mechanism of self-similarity, namely 3-spheres $\{x,y,z\}$ attached to the surface of the primary 3-sphere of $\{t_+,t_\star,t_-\}$, that we propose to generate an infinite amount of complexity. When we complete physical 3-space with a point of time along with the axis of that time, that gives a 3-ball whose surface is a 3-sphere entirely other than the 3-sphere of $\{t_+,t_\star.t_-\}$. If we were going to complete that sphere with another axis to create a 3-ball, that axis would likely be spacelike. Therefore, one imagines a fractal embedding, as in reference [7], "consisting of three 3-balls embedded on the surface of another 3-ball," and we can continue the embedding forever by taking any surface direction of any 3-sphere as the radial direction of an embedded 3-ball.

overlap because there is only one hypercomplexly infinitesimal neighborhood around the point of intersection.[1] Therefore, when the line element at these points of intersection has simultaneous contributions from topologically irreconcilable hyperbolic and spherical metrics we may speculate that the flat transparency of deep space breaks down to give a turbulent line element that defines an opaque region where light no longer propagates along straight lines. In that case, the 13.7Gcy distance to the CMB will not be a measure of the age of the universe but, rather, a measure of how much the metric of $t_\pm$ bleeds onto the metric of $t_\star$.

Without pursuing the concept of "overlapping metrical fields" directly we can consider a similar phenomenon: the dark energy interaction arising from the gravity of one universe overlapping on the other. We have required that this interaction is isotropic so, therefore, the anisotropic distributions of matter-energy in one universe cannot bleed over into the other universe. For this, to first order at least and only for the purposes of qualitative discussion, note that $\aleph := t_-$ and $\Omega := t_+$ bleeding onto $\mathcal{H} := t_\star$ is an altogether different phenomenon from $\mathcal{H}_2$ bleeding onto $\mathcal{H}_1$ or $\bar{U}$ bleeding onto U. The proposed effect of hyper-turbulence refers to flat space mingling with curved space but dark energy is an effect we attribute to an interaction between two flat universes. We have not totally clarified if dark energy acts across $\{U, \bar{U}\}$ or $\{\mathcal{H}_1, \mathcal{H}_2\}$, the latter being two adjacent instances of a single universe U, and we will do so now using some of the principles discussed earlier in this section.

We now say $\mathcal{H}_2$ is two levels of $\aleph$ higher[2] than $\mathcal{H}_1$ so we can set U and $\bar{U}$ on adjacent levels of $\aleph$. In this arrangement, if the first instance of U ($\mathcal{H}_1$) is on $\hat{\Phi}^j$ and the second instance of U ($\mathcal{H}_2$) is on $\hat{\Phi}^{j+2}$ then we clearly have space to put $\bar{U}$ on $\hat{\Phi}^{j+1}$. In the previous formulation, we were constrained to put the past of $\hat{\Phi}^{j+2}$ on $\hat{\Phi}^{j+1}$ but, now that we have encoded the record of the past on $\hat{\varphi}$, we have room to include $\bar{U}$ on $\hat{\Phi}^{j+1}$. When we consider the original argument that U and $\bar{U}$ should interact through the big bang, we can make the extension to the present convention to say that U and $\bar{U}$ have a dark energy interaction that is centered on each present moment. Furthermore, when we take the fundamental unit of the MCM lattice as a circle with a small sphere attached and consider that the circle is on a different level of $\aleph$ than the sphere, as in figure 50, we can say that the permanent pull toward the future is encoded into this system. This is the scenario where dark energy arises between U and $\bar{U}$, and shortly we will describe another arrangement where the interaction is between U and U in the future, and we will conclude that dark energy is not an interaction between U and $\bar{U}$ but between $\mathcal{H}_1$ and $\mathcal{H}_2$. Figure 50 shows how each unit element in the cosmological lattice can include the dark energy interaction given in reference [2] but it begs another question. If U and $\bar{U}$ are two universes then how can one be a circle while the other one is a sphere? A further question asks that if the observer is in $t_\star$ then how can he also be in t_+, as in figure 47? To answer both questions, we can say that U is always taken in the $t_\star$ representation and that $\bar{U}$ is taken in the $t_\pm$ representation. Therefore, we naturally find that the circle $\mathbb{S}^1$ should correspond to U with $t_\star$ on even levels of $\aleph$ and the sphere $\mathbb{S}^2$ should correspond to $\bar{U}$ with $t_\pm$ on odd levels. Here, the reader should note that since we have no way to the determine the absolute level of $\aleph$ it will only be a convention to say that

[1] If we say that $\{t_+, t_\star, t_-\}$ are on three different levels of $\aleph$ then they will not necessarily overlap. However, if $\mathcal{H}_1$ is on $\hat{\Phi}^j$ and $\mathcal{H}_2$ is on $\hat{\Phi}^{j+2}$ then there is only one level of $\aleph$ for $\aleph$ and Ω so they must overlap. Furthermore, it has been the convention in the MCM unit cell to say that $\{\aleph, \mathcal{H}, \Omega\}$ are all on the same level of $\aleph$ so we are justified to consider $\{t_+, t_\star, t_-\}$ not on three separate tiers of infinitude.

[2] Three levels of $\aleph$, one level and two higher ones, are a good amount to associate with $\hat{M}^3$ because they are sufficient to cover infinitesimal elements, finite elements, and infinite elements. In reference [8], we defined hypercomplexity as the limit of infinite complexity when it is restricted to only three simultaneous levels of $\aleph$.

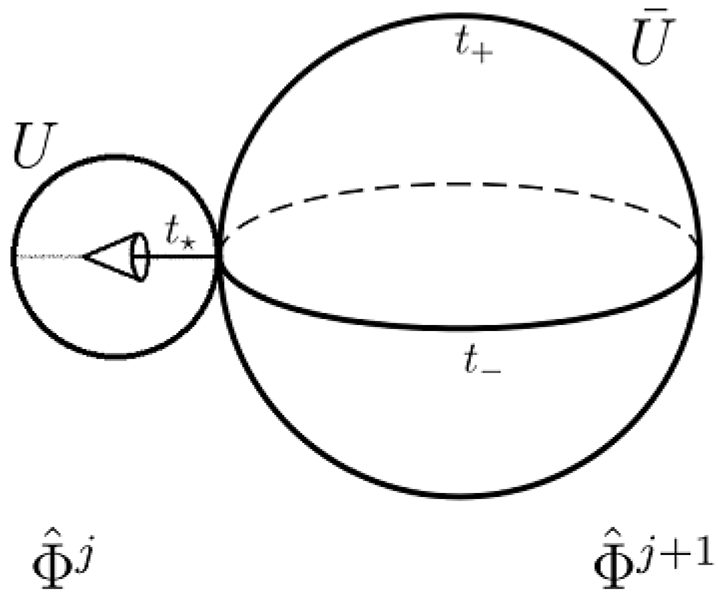

Figure 50: This figure shows a universe U which will experience dark energy due to the gravitational pull of $\bar{U}$ on a higher level of $\aleph$. If we convert to the $t_\pm$ representation in U then we could add another universe on $\hat{\Phi}^{j-1}$ which would experience dark energy in its $t_\star$ representation due to the pull of U.

one sector is odd and the other even.

When we have operated with the ontological basis to construct $\mathbb{S}^1 \times \mathbb{S}^2$, as in figures 37, 38, and 42, which must be the $\mathrm{U}(1) \times \mathrm{SU}(2)$ electroweak sector of the standard model, we initially derived a circle with an unpaired interval at each pole. When we assemble a lattice constructed from these objects, the interval from each end will be paired with the interval from the next object so that there are always two of these unpaired intervals between each circle. In analogy with solid state physics, the fundamental repeating unit of crystal structure sometimes shows half of an atom on each side and sometimes it shows an entire atom on one side, but not the other. Therefore, we have considered the repeating unit as a circle with a sphere but now we we will consider the fundamental object that reflects the spinor properties of $\hat{\Phi}$, as in figure 51. Furthermore, since there is a manifest duality between $t_\star$ and $t_\pm$ we will also consider the arrangement in figure 52 where $t_\star$ is associated with the outer regions of the repeating unit instead of the center. It will be hard to build the periodic lattice with this representation because we do not want two $t_\star$ intervals in the same place but figure 52 is the one that motivates dark energy as an interaction between $\mathcal{H}_1$ and $\mathcal{H}_2$ instead of between U and $\bar{U}$. Figure 52 is like figure 50 with part of a second unit cell attached so, if we consider it in that way, there will be no difficulty building the lattice in such a representation.

Figure 52 does not show that the interaction between the two U will be 1D but if we take U only on even levels of $\aleph$ by requiring that odd levels of $\aleph$ are purely imaginary with respect to the even levels, then we will arrive at figure 53 which does show a 1D transmission. Figure 53 is exactly like the integration path around what we have called the Cauchy C curve. We have associated $\hat{\imath}$ with the off shell region and, in the usual picture of on and off shell, there is only one shell but the utility of the lattice is such that there are multiple shells separated by off shell regions. We should associate every point that can be described as "on shell"

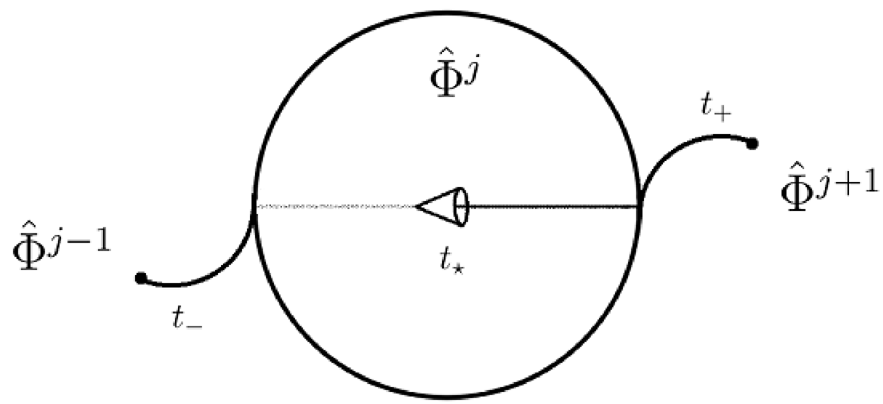

Figure 51: This figure contrasts figure 50 to show the repeating unit in the fundamental form without moving both outer intervals to one side.

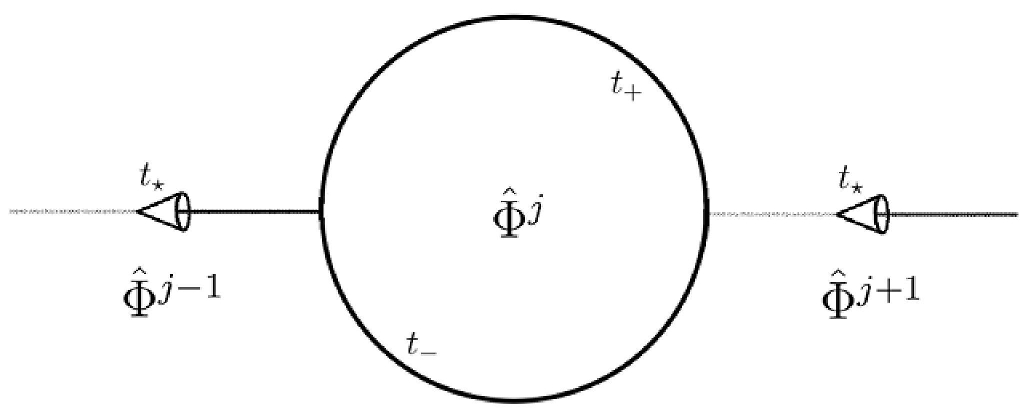

Figure 52: This figure shows a dual representation to figure 51 where $t_\pm$ are each associated with one co-$\hat{\pi}$.

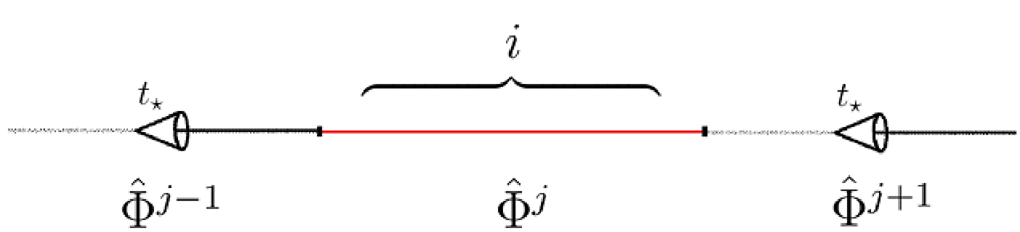

Figure 53: This figure demonstrates how we may transmit the dark energy interaction in only one dimension. The central region is imaginary but that does not strictly imply that it is associated with $\hat{i}$.

with the $\hat{\pi}$-sites. Furthermore, in reference [3], we showed that, for the unit cell centered on $\mathcal{H}$, the fields in Σ^- should be imaginary with respect to real fields in Σ^+. However, it does not matter which fields are real and which are imaginary between $\Sigma^\pm$; it only matters that there is a factor of i between them. If we put i into Σ^+ around $\mathcal{H}_1$ but put i into Σ^- around $\mathcal{H}_2$ then the entire intermediate region between them will be imaginary exactly as in figure 53. We know that there is more than one dimension in $\Sigma^\pm$ but, if we make everything in there imaginary, that will have the effect of sending the interaction through $\hat{i}$ alone, which is 1D. Clearly there is a lot of hand waving associated with this picture since we have not mentioned χ^5 at all, and the O(4,1) and O(3,2) topologies of $\Sigma^\pm$ imply that some of the dimensions will always be real regardless of where we put i, but, then again, we have not yet invoked the $\sqrt{i}$ channel. If we add imaginary phases like $\pm i^{3/2}$ and $\pm i^{1/2}$ then, possibly, we can arrive at figure 53. We could accommodate both O(4,1) and O(3,2) topologies purely in $\hat{i}$ by separating timelike and spacelike dimensions in $\Sigma^\pm$ with roots of i instead of i and 1 (which are roots of unity). Furthermore, if everything in $\Sigma^\pm$ is imaginary in this way then that will naturally enforce the $R_{AB}=0$ condition in the bulk of the MCM unit cell.

To construct a cosmological lattice we have to put another universe on the other side of the CMB and that universe would have the same anisotropic structure as ours. If the entire hypercosmos is filled with cells containing anisotropic universes smoothly connected by spacetime to the their CMBs then we would have to come up with some good idea for why the CMB lattice domain walls screen that information. In general, we can use the same mechanism from figure 53 which screens the anisotropy from contributing to dark energy. If we made a trivial lattice of ΛCDM cosmologies immediately adjacent to each other then we could immediately say that the model was unphysical because it would lead to anisotropies in the CMB on the scale of the anisotropies in the universes. In the MCM, to the contrary, we connect the adjacent universes by the path around infinity so they are not right next to each other, as in figure 40. Therefore, $\bar{U}$, the unseen parallel universe, must lie entirely within the path around infinity and that is described by figure 53. When we integrate around that path in the complex plane, we can take semicircle in either the upper or lower complex plane and, in figures 40 and 52, we have the choice to make use of either of $t_\pm$ when we want to connect $\mathcal{H}_1$ to $\mathcal{H}_2$ lying beyond $\bar{U}$. Intuitively, this should be t_+. Furthermore, all of the anisotropy from the universe on the lower level of $\aleph$ contributes even less than a differential element because the lower level is two levels of $\aleph$ lower.

Earlier in this section, we proposed to encode the Dirac and Schrödinger operators on $\hat{i}$ although $\hat{i}$ does not appear in equations (4.12). Here, we clarify that as

$$||\hat{i}|| = i \qquad \not\Rightarrow \qquad \hat{i} \quad \equiv \quad i \ , \tag{4.16}$$

so that

$$\hat{i} \quad \not\equiv \quad i \ . \tag{4.17}$$

This is obvious because, for purposes of self-similarity, we will need to define complex fields in the real sector $\{\hat{\Phi}, \hat{2}, \hat{\pi}\}$ of the ontological basis. Due to the tensor transformation law, those same fields in $\hat{\imath}$ will be shifted by $\pi/2$ when represented in $\mathbb{C}$. Therefore we can associate the i in figure 53 with the i in the Schrödinger and Dirac operators so that the off shell domain wall is a quantum sector that complements the classical sector $\mathcal{H}_j$.

One of the most interesting features of the CMB is how perfectly the spectrum of CMB photons aligns with the spectral distribution for blackbody radiation. It conforms to the blackbody law better than anything that can be created with classical machine parts on Earth. Therefore, we can ideate that the CMB conforms to the blackbody spectrum better than any classical device because it is a quantum object quantized to do so. For example, during the $n=2$ to $n=1$ atomic transition, we will always get a photon with energy hf_0 and we can say that the macroscopic CMB object on $\hat{\Phi}^j$ is a a quantized object on $\hat{\Phi}^{j+1}$ quantized such that the energy distribution is the blackbody spectrum. In a certain isotope, ignoring magnetic effects, recoil effects, *etc.*, the $n=2$ to $n=1$ transition can only happen if it emits a photon with $E=hf_0$ and we can theorize that an effect from a higher level of $\aleph$ constrains the spectrum of energy in a distribution of photons instead of the energy of a single photon.

The origin of Planck's constant h in physics is directly in the law for the distribution of blackbody photons. Before Planck inserted his constant into the distribution there was a problem called the ultra-violent catastrophe where the distribution would agree with observations at higher energies but diverged at lower energies to the ultra-violet side of visible light in the total spectrum of electromagnetic radiation. Wikipedia says the following about Planck's law for blackbody radiation.

> "Planck derived the correct form for the intensity spectral distribution function by making some strange (for the time) assumptions. In particular, Planck assumed that electromagnetic radiation can only be emitted or absorbed in discrete packets, called quanta, of energy:
>
> $$E_{\text{quanta}} = h\nu = h\frac{c}{\lambda} \quad , \tag{4.18}$$
>
> where h is Planck's constant. Planck's assumptions led to the correct form of the spectral distribution functions:
>
> $$B(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/(\lambda k_B T)}-1} \quad . \tag{4.19}$$
>
> Albert Einstein solved the problem [*of discretized energy packets*] by postulating that Planck's quanta were real physical particles—what we now call photons, not just a mathematical fiction. He modified statistical mechanics in the style of Boltzmann to an ensemble of photons. Einstein's photon had an energy proportional to its frequency and also explained an unpublished law of Stokes and the photoelectric effect."

Taking only passing notice of the relationship between the highly abnormal factor λ^5 and the dimensionality of $\Sigma^{\pm}$, we need to associate quantized spin angular momentum with Dirac's constant

$$\hbar \quad \equiv \quad \frac{h}{2\pi} \quad . \tag{4.20}$$

The general idea in the MCM has been to use the modular topology operators to twist, or *spin*, the time axes of different universes joined on the topological singularity $\varnothing$ where the level of $\aleph$ increases. If we want to make rigorous the connection between the MCM twisting and quantum mechanical spin, we should compute the angular momentum vectors of the different universes and see if there is an implication of integer angular momenta, which can be described classically, being split in half on the i surface that facilitates the twisting.

In the unit cell centered on $\varnothing$, we propose to join Σ_1^+ to Σ_2^- where $\chi_+^5 = \infty$ and $\chi_-^5 = -\infty$ such that the topology of de Sitter space with $x_+^4 = \infty$ is not incompatible the topology of anti-de Sitter space with $x_-^4 = -\infty$. If we could not rely on "the bounce point" $\varnothing$ at $\chi_\pm^5 = \pm\infty$ then we could never join dS topology with AdS topology, and we have associated the i surface of the quantum cosmological sector with the bounce point. We have the freedom to include two bounce points in the model, one at the origin and one at infinity, because we have two charts on the Riemann sphere which is inherent to the topology of quantum states: quantum amplitudes are always $\mathbb{C}$ numbers. Both charts on the Riemann sphere have a coordinate singularity at $\phi = \pi$ relative to the origin $\phi = 0$ of the chart in question.[1] The MCM wraps the time axis around a cylinder so, when the hypersurface of the present propagates forward in time, there is manifestly an associated angular momentum. Therefore, it is likely that if we examine the material presented in this section, as it relates to the law of conservation of angular momentum, then we might show that half-integer spin momenta are implied. We will return to this important point in section IV.3.

Before moving on, we make a point regarding the significant hand waving, or, more technically, the lack of specificity, that we have relied on to make the preceding qualitative statements in this section. There is no issue taking different representations to describe different effects. One might be inclined to say, "All effects have to be demonstrated in all representations," but that is false. We can operate with any and all of $\{\hat{i}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ on any line or circle, on any level of $\aleph$, to generate any required representation in the required orientation with respect other representations. The lattice site addresses that we assign to different representations will make them unique with respect to other representations that we use to describe other effects.

IV.2 Tipler Sinusoids

Here, we give an example of the kinds of analytical field solutions that will need to be converted to machine language before a meaningful prediction can be extracted for an experimental test of the accuracy of some application in cosmology. The purpose of this section is not to analyze Tipler's result in the MCM but, rather, to discuss the complexity of its features as they relate to the general principles of the MCM.

Consider what Tipler wrote in reference [60], a paper containing in its abstract the interesting phrase, "would act as a time machine."

> "Since the work of Hawking and Penrose, it has become accepted that classical general relativity predicts some sort of pathological behavior. However, the exact nature of the pathology is under intense debate [*sic*] because solutions to the field equations can be found which exhibit virtually any type of bizarre behavior. It is

[1] We can associate this pair of singularities with the MCM condition that the observer is always at the origin. When the observer can never measure something at his own location, we introduce a new boundary condition $\psi(0) = 0$ to supplement the usual condition $\psi(\infty) = 0$.

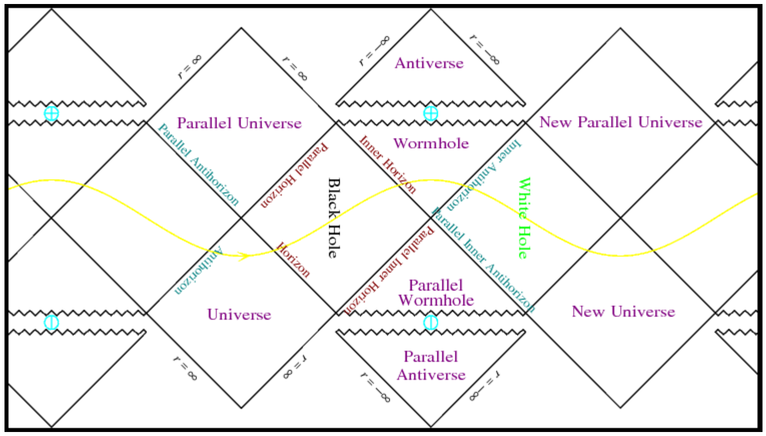

Figure 54: This figure shows the conformal diagram of Reissner-Nordström spacetime. Hypercomplex coordinates on chronos and chiros are well suited to defining a continuous parameter through the periodic boundaries. Where we have previously made reference to the Kerr–Newman metric of a charged rotating black hole, the Reissner–Nordström metric describes a a charged black hole with no net angular momentum.

thus of utmost importance to know what types of pathologies might be expected to occur in actual physical situations. One of these pathologies is causality violation, and [*it can be argued*] that if we make the assumptions concerning the behavior of matter and manifold usual in general relativity, then it should be possible in principle to set up an experiment in which this particular pathology could be observed.

"Because general relativity is a local theory with no *a priori* restrictions on the global topology, causality violation can be introduced into solutions quite easily by injudicious choices of topology;[1] for example, we could assume that the timelike coordinate in the metric is periodic,[2] or we could make wormhole modifications in Reissner–Nordström space [(*figure 54.*)] In both of these cases the causality violation takes the form of closed timelike lines (CTL) which are not homotopic to zero, and these need cause no worries since they can be removed by reinterpreting the metric in a covering space (following Carter, CTL removable by such means will be called trivial–others will be called nontrivial)."

Regarding trivial CTL, which we will refer to as CTC (closed timelike curves), Tipler writes the following about Carter's result.

"Carter's causality theorem can be stated as follows: A necessary and sufficient condition for nontrivial causality violation in a connected, time-oriented spacetime with a timewise orthogonally transitive Abelian isometry group is the nonexistence of a covariant vector in the Lie algebra such that the corresponding differential form in the surface of transitivity is everywhere well behaved and everywhere timelike.

[1]Tipler's comment is evocative of an axiom stated in reference [8], "If the topology required by the new definitions is not reducible to a [*sufficient form*] then the definitions will amount to what Laithewaite has called 'the multiplication of bananas by umbrellas' meaning that the definitions are contrived."

[2]This is what we have done in the MCM [2], obviously.

> If the above criterion is satisfied, then there exist both future- and past-directed timelike lines between any two points of the spacetime."

Although the Wikipedia page for Lie algebras is straightforward enough, we prefer to focus on simpler aspects of Tipler's result. Indeed, beginning with Tipler's reference to Hawking and Penrose, then his reference to Carter's theorem, and his ultimate reliance on the results of van Stockum, the expertise in general relativity far exceeds the general relevance of the modified cosmological model. There are, however, a number of features in reference [60] that are generally relevant and amenable to immediate analysis, qualitative as it may be, with the tools that we have developed and reviewed in this book.

CTC which are not homotopic to zero cannot be removed by smooth variations. "Homotopic to zero" means "can be removed by smooth variation" so we see the obvious distinction between trivial and non-trivial CTC. Tipler says that pathological, non-trivial CTC cannot be removed via smooth deformation but that they can be removed "by reinterpreting the metric in a covering space." As we will discuss in section IV.5, the covering space relies on the modularization of the topology in a way that is more complex than what is required to build a simple topological homeomorphism. If a given CTC is homotopic to zero then we can remove it with an ordinary coordinate transformation that doesn't know about modularized topology. Non-trivial CTC can only be removed by changing the topology and, for that, our intent is to expand circular U(1) into a helix. We have associated $\{\hat{2},\hat{\pi},\hat{\imath}\}$ with the circle through $e^{ix}=e^{ix+2\pi i}$ and we add $\hat{\Phi}$ at the point of discontinuity in the domain of the parameter of the broken U(1) symmetry $\theta\in(0,2\pi)$ to get rid of the periodic boundary condition by creating a helix whose central axis is parallel to $\hat{\Phi}$. This will transform the finite circular interval into a infinite linear interval wrapped around a cylinder. This infinite interval $\theta\in(-\infty,\infty)$ is the covering space of the circular interval $\theta\in(0,2\pi)$. Since we are starting with U(1) and then changing it, we are also starting with the Euler formula and then changing it. We will say the Euler formula is true in the space spanned by $\{\hat{2},\hat{\pi},\hat{\imath}\}$ but not true in $\{\hat{\imath},\hat{\Phi},\hat{2},\hat{\pi}\}$ which is a space akin to U(1) with a winding number attached.

All of the fields used by Tipler, originally derived by van Stockum, are built almost entirely from functions related through the Euler formula. Regarding a rapidly rotating, infinite massive cylinder, Tipler writes, "van Stockum has developed a procedure which generates an exterior solution for all $aR>0$," where a is the angular velocity and R is the radius of the cylinder. That solution is

$$0<aR<\frac{1}{2}: \quad \begin{cases} H &= e^{-a^2R^2}\big(r/R\big)^{-2a^2R^2} \\[2ex] L &= \dfrac{Rr\sinh(3\epsilon+\theta)}{2\sinh(2\epsilon)\cosh(\epsilon)} \\[2ex] M &= \dfrac{r\sinh(\epsilon+\theta)}{\sinh(2\epsilon)} \\[2ex] F &= \dfrac{r\sinh(\epsilon-\theta)}{R\sinh(\epsilon)} \end{cases} ,\quad \text{with} \quad \begin{cases} \theta &= \big(1-4a^2R^2\big)^{1/2}\ln(r/R) \\ \epsilon &= \tanh^{-1}(1-4a^2R^2)^{1/2} \end{cases} \tag{4.21}$$

$$aR = \frac{1}{2}: \quad \begin{cases} H &= e^{-1/4}\big(r/R\big)^{-1/2} \\ L &= \frac{1}{4}\big[3 + \ln(r/R)\big] \\ M &= \frac{1}{2}\big[1 + \ln(r/R)\big] \\ F &= \big(r/R\big)\big[1 - \ln(r/R)\big] \end{cases} \tag{4.22}$$

$$aR > \frac{1}{2}: \quad \begin{cases} H &= e^{-a^2R^2}\big(r/R\big)^{-2a^2R^2} \\ L &= \dfrac{Rr\sin(3\beta+\gamma)}{2\sin(2\beta)\cos(\beta)} \\ M &= \dfrac{r\sin(\beta+\gamma)}{\sin(2\beta)} \\ F &= \dfrac{r\sin(\beta-\gamma)}{R\sin(\beta)} \end{cases} \quad , \quad \text{with} \quad \begin{cases} \gamma &= \big(4a^2R^2-1\big)^{1/2}\ln(r/R) \\ \beta &= \tan^{-1}(4a^2R^2-1)^{1/2} \end{cases} \quad . \tag{4.23}$$

We find hyperbolic sines and cosines in the central region $0 < aR < 1/2$, ordinary sines and cosines in the exterior region $aR > 1/2$, and logarithms where they are joined. A first check on physicality might be to examine the piecewise continuousness or discontinuousness of the three fields' tangent and cotangent fields at the point where they are sewn together. On the other hand, a first mathematical analysis using the tools of the TOIC, which is also a physical analysis, will be to expand all the fields out into infinite series such as those used to demonstrate the validity of the Euler formula. In that case, we would look for pathological rarefication of information where certain series expansions are sampled more or less heavily than others that appear equally weighted in van Stockum's representation.

The first thing to note in the hypercomplex analysis of van Stockum's solutions (not carried out here) will be that $\ln(x)$ is not an object in the Euler formula like $\sin(x)$, $\cos(x)$, and e^{ix}. All of the hyperbolic functions can be represented with exponentials, e^x rather than e^{ix}, but $\ln(x)$ is not going to enable any useful symmetry reductions. Already we can see that the MCM/TOIC analysis of the solution will not be directly easy because $\ln(x)$ is a wrench in the sprocket. Equations (4.21-4.23) contrast with the metrical solutions in equations (3.65-3.67) where the hyperbolic and spherical solutions are joined at the flat interface with a simple linear argument. At the boundary where the interior field is sewn to the exterior field with the $aR = 1/2$ solution, we have the logarithm which is somehow different than the other functions. Given the structure of the MCM, it is likely that we can move this logarithmic analytical pathology to $\Sigma^{\varnothing}$.

The structure of the solutions for the Tipler sinusoid here and for the FLRW metric in section III.7 both have a similar structure which should be familiar to any student of physics: the solutions in two disconnected regions are sewn together with a third solution

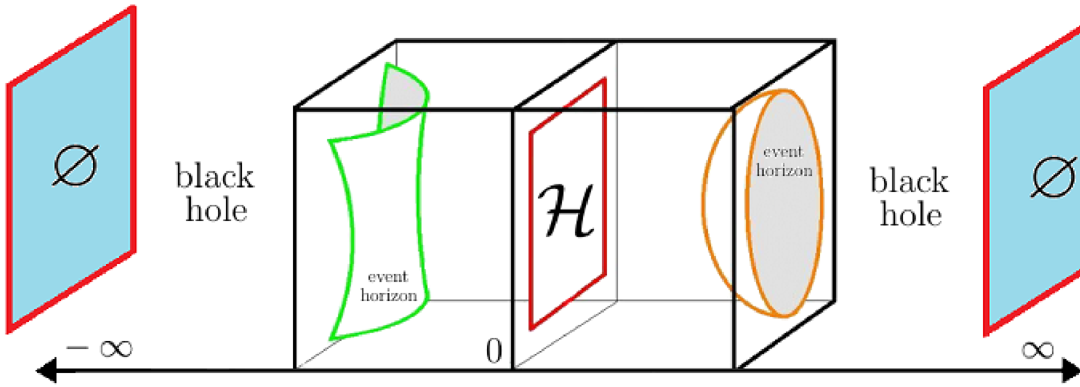

Figure 55: When we join across levels of $\aleph$ with singular topology $\varnothing$ we can say that the region denned by $\aleph$ and Ω is surrounded by black holes (pictured) or equivalently $\mathcal{H}$ can be the singularity denned within the $\{\aleph, \Omega\}$ event horizons. In the latter case we directly obtain the notion of a universe inside a black hole.

defined on the surface that separates them. This structure, which is very common in physics, lends itself naturally to $\chi^5 \equiv \chi^5_+ \otimes \chi^5_\varnothing \otimes \chi^5_-$. It also lends itself naturally to coordinate transformations such that we should be able to join regions on either of $\mathcal{H}$ or $\varnothing$. Note the phase change between the hyperbolic trigonometry functions in the interior regions and the ordinary trigonometry functions in the exterior region: the hyperbolic variants depend on e^x so, across the surface $aR = 1/2$, we obtain a change $e^x \to e^{ix}$ that mirrors $i\chi^5_- \to \chi^5_+$ across $\mathcal{H}$, as derived in reference [3]. There will be some nuance joining on a topological singularity that will not support a coordinate space representation when $\varnothing$ is located at $\chi^5_\pm = \pm\infty$; there, we might use the twistor representation or similar. Before we would undertake such an effort we must first show why that would useful but our intent here is only to show the similarity between classically ubiquitous piecewise field solutions sewn together at a point and MCM fields in $\Sigma^\pm$ sewn together at one of two points: $\mathcal{H}$ or $\varnothing$.

One task that is already well motivated is to show causal trajectories across the MCM unit cell and Tipler explains that the pathological behavior containing non-trivial CTC can be removed through the covering space. Therefore, we might consider that the MCM unit cell shows the covering space representation where identically flat χ^5 has no pathologies at all. Tipler writes [60], "The region of causality violation is confined within an event horizon," so if we do put an infinite rotating cylinder (such as spacetime wrapped around a cylinder) into an MCM cosmos then causality violating CTC can fill the hypercosmos in the x^0 direction but would be constrained never to close across the unit cell in the χ^5 direction. This follows because we have taken χ^5 as the non-physical covering space of x^0 which is the object of closure in any CTC. Another possibility is that these closed CTC are hidden behind the event horizons in figure 55 so that no CTC ever intersect with $\mathcal{H}$. Perhaps hyperspacetime ends at $\{\aleph, \Omega\}$ and the rest of $\Sigma^\pm$ for large $|\chi^5_\pm|$ are not constrained by $R_{AB} = 0$. This would allow us to put a source behind one horizon (white hole) and a sink (black hole) behind the other which send a gravitational displacement current (or similar) through hyperspacetime whose quantum aspects are called the information current. We can take the system shown in figure 55 where the black hole behind $\aleph$ becomes a white hole that will be our transmitter; information *does* generally come to an observer along his past light cone. Furthermore, causality violation is precisely the anomalous interpretation assigned to the advanced potential which we independently expect to violate causality everywhere in the MCM unit cell. Therefore, perhaps we should consider that there is no constraint on CTC that would prevent them from intersecting $\mathcal{H}$ meaning that the central region in figure 55 is the interior of a black hole rather than the exterior of two black holes. If we include the advanced potential in the field equations, then it is almost a foregone conclusion that there will be CTC in $\mathcal{H}$.

The traditional idea of a white and black hole pair, as in the Penrose diagram of a Schwarzschild black hole, would send matter through hyperspacetime in violation of $R_{AB}=0$. Therefore, in the limit where we ignore any non-causal loop holes in the Kaluza–Klein constraint $R_{AB}=0$, we want the source and sink to be a transmitter and a receiver for gravitational radiation. Note that there is no dipolar gravitational radiation and that when we use the sphere theorem to invert the interior and exterior solutions for modular continuation[1] we will be using a topology that we have described, in reference [8], as having four singularities: a pair of singularities in each of a pair of charts. If we can associate each pair of charts with Σ^+ and Σ^- respectively then there will be four singularities in each union of Σ^+ and Σ^- and we should generally associate these with $\{\vartheta_+,\bar{\vartheta}_+,\vartheta_-,\bar{\vartheta}_-\}$. Each such union contains one MCM unit cell which is the region of hyperspacetime between $\aleph$ and Ω where we want the gravitational waves to propagate. Therefore, with four singularities, or four poles, we have exactly the ingredients needed to make quadrupolar gravitational radiation. Perhaps this line of reasoning might even uncover novel modularized field solutions describing a gravitational autodynamo such as the geon described by Wheeler [61].

The next steps following onto Tipler's work, should we decide to do an depth MCM analysis, are mostly clear. The primary goal of that research would be to examine how CTC initially come into existence as aR starts at zero and then increases, and also to see if we can forge a connection between the conditions simultaneous with the cylinder in the present and the character of the information that comes into the present along the CTC. A few other things that should be reanalyzed include Tipler's proof of Carter's theorem. We need to back away from Euler's formula and decompose all the objects into infinite sums. An additional point of contention that could be revealed under further analysis is a reliance on a line element ds^2 defined according to an angular coordinate that is double valued at the point $\{0,2\pi\}$. Tipler takes the polar coordinate of his infinite cylinder as $\varphi\in[0,2\pi]$ where the MCM would suggest $\varphi\in[0,2\pi)$ or $\varphi\in(0,2\pi]$. We find, therefore, two points where we might add hypercomplexity: the center of the phase space at $aR=0$ and possibly one point where we break the U(1) symmetry with $f(0)\neq f(2\pi)$. We might associate the latter with a non-commutative frame dragging effect.

All of the above gives a good assessment of what MCM cosmology will look like in practice but the aspect of reference [60] that has the most bearing on the MCM specifically is found in one of the footnotes.

[1]In proffering an idea of "modular continuation" we refer to a spherical system where an interior solution is joined to an exterior solution by a solution on the surface of a sphere with radius R. Due to the conformal invariance of the theory, we should be able to apply the sphere theorem so that the interior and exterior solutions are permuted, and then apply a coordinate transformation such that the interior solution in the exterior region can be joined to another exterior solution at a larger radius $R'>R$. Here, we rely on a system, as in reference [8], of concentric shells so that any exterior region is also an interior region with respect to some larger radius. "Modular continuation" then refers to joining interior and exterior regions across surfaces at different radii.

> "For $a^2 + e^2 > m^2$ [*where a is the angular velocity, e is the charge, and m is the mass*] there are no event horizons and so causality violation is global, but it is not clear that a star with such high values of angular momentum and/or charge would collapse sufficiently far to uncover the region where $g_{\varphi\varphi}$ changes sign [*sic*]. Penrose has argued [*sic*] that a naked Kerr singularity would be a good model for a rapidly rotating star which has collapsed into a disk. CTL would be expected when $e \neq 0$, but one might contend that these occur so close to the singularity (and hence in regions where we expect general relativity to break down anyway) that they are without physical significance. van Stockum's work shows, however, that CTL are not necessarily associated with extreme curvature in physically significant situations."

Noting that Tipler uses $g_{\varphi\varphi}$ to describe what we have called Σ_{55}, the changing sign of Σ_{55} is the most critical aspect of the MCM unit cell. On one side of $\mathcal{H}$, the fifth diagonal position is negative, and on the other side it is positive. Any system containing this changing sign must have direct relevance to the MCM and the alternation of $\Sigma^{\pm}$ as nested spherical shells (when we take radial χ^5 rather than Cartesian.) Tipler also writes that the CTC for a charged black hole, such as the Kerr–Newman black hole, would be too close to the singularity for them to have any physical significance but we have developed the concept of the hypercomplexly infinitesimal neighborhood around a singularity expressly for the purpose of assigning physical significance to such regions. Finally, while much of Tipler's discussion of CTC has focused on regions of extreme curvature, he states that CTC are not exclusively associated with regions of extreme curvature and, therefore, we may expect such pathologies in the region of mild curvature around $\mathcal{H}$ within the ground state MCM unit cell.

IV.3 MCM Quantum Mechanics

In discussing quantum cosmology, we conform to the following distinction between classical mechanics and quantum mechanics: classical phenomena have equations of motion that minimize the action and quantum phenomena come from solutions that are maxima of the action. In section I.3, it was demonstrated that any maximum of the action must follow a trajectory in phase space that goes around the Cauchy C curve at infinity, and that quantum phenomena are so strange because a topological obstruction along that path leaves an analytical remainder. The "Dirac bracket" inner product of a bra with a ket is modified such that dx is augmented by $d\hat{\gamma}$. The remainder of the MCM inner product is associated with what is called Dirac orthogonality: a property specific to wavefunctions represented in the eigenbasis of a continuous spectrum of eigenstates. The position space representation is one such example. The non-orthogonality of position eigenstates gives the Dirac orthogonality condition

$$\langle x_1 | x_2 \rangle = \delta(x_1 - x_2) \quad , \qquad \text{where} \qquad |\psi\rangle = \int_{-\infty}^{\infty} \psi(x) |x\rangle \, dx \quad , \tag{4.24}$$

which is completely different than genuine orthogonality in the case of discrete eigenvectors

$$\big\langle\psi_n\big|\psi_m\big\rangle=\delta_{mn}\quad,\qquad\text{where}\qquad\big|\psi\big\rangle=\sum_{n=1}^{\infty}\alpha_n\big|\psi_n\big\rangle\quad.\tag{4.25}$$

Continuous position eigenstates, analytical Dirac delta functions $\delta(x-x')$, live in an infinite dimensional non-Hilbert space called Ω', as in $\{\aleph',\mathcal{H}',\Omega'\}$. Therefore, we can identify the sector of relevance for the "analytical remainder" that will be operated on with $d\hat{\gamma}$: when the inner product uses finite dimensional representations use dx, and when there is an infinite dimensional basis, directly related to "levels of $\aleph$" through $\psi\!:=\!\delta$, then use $d\hat{\gamma}$.

Consider the Dirac formalism for the representation of ψ in a discrete basis ψ_n or in the continuous basis of position eigenstates $|x\rangle$:

$$\big\langle\psi_n\big|\psi\big\rangle=\alpha_n\quad,\qquad\text{but}\qquad\big\langle x\big|\psi\big\rangle=\delta(x-x')\quad.\tag{4.26}$$

Note the integration variable in $\delta(x-x')$. Why isn't there an integration variable with $\langle\psi_n|$? This is an important question because all of quantum field theory comes equations (4.26). The answer has to with the idea that the observer can never test a question like, "Is the particle at position x?," but, "Is the particle in discrete state n?," is very often testable. Among other things, the observer does not have any devices that can measure mathematically specific points in space but he does have devices that can measure analytically singular properties like electronic transition energy. Therefore, depending on the sector of the quantum theory that is being probed, finite or infinite dimensionality in the vector space, we have two different probability functionals that need to be written into $\hat{M}^3$. $\hat{M}^3$ cannot be periodic if sometimes we compute $P[\psi_n]\!\in\!\mathbb{R}$ to get a number for comparison with experiment but sometimes we compute $P'[\psi(x)]dx\notin\mathbb{R}$ and then obtain a real-valued probability with $P\!=\!\int P'[\psi(x)]dx$. The probability in the finite dimensional sector gives

$$P[\psi_n]=|\alpha_n|^2\quad,\qquad\text{so}\qquad P[\psi_n]\in\mathbb{R}\quad.\tag{4.27}$$

If we simply replace ψ_n with $\psi(x)$ then we get the nonsense answer

$$P[\psi(x)]=|\delta(x-x')|^2\qquad\Longrightarrow\qquad P[\psi(x)]\notin\mathbb{R}\quad.\tag{4.28}$$

This is nonsense because we can't do $\hat{M}^3$ with it. Predictions can only be checked with $\mathbb{R}$ numbers but we still need to apply more operations to equation (4.28) before we can reach the final requisite step $P[\psi]\!\in\!\mathbb{R}$. Equation (4.28) does not satisfy this constraint. However, the Dirac delta function does make some qualitative sense as a position space wavefunction; if the delta is within the limits of the dx integration in $P[\psi(x);a,b]$ then the integral will return unity and otherwise it will return zero.

This section closely examines $P[\psi]$ and so we will restate its relevance. At the end of the day in quantum theory, whatever the application, the numbers that will be compared to experiment come out at as

$$P[\psi] \in \mathbb{R} \quad , \qquad \text{with} \qquad 0 \leq P[\psi] \leq 1 \quad . \tag{4.29}$$

When evaluating whether or not a theory's prediction is correct, $P[\psi]$ is the object of interest. It is the main philosophical object at the heart of $\hat{M}^3$ wherein an observer does science when he computes an expectation *and then also checks it.* In ordinary quantum theory, we have the two different interpretations for $P[\psi]$ discussed above but we want to define a universal process for $\hat{M}^3$ that has only one interpretation and only one algorithm. Before a prediction can be checked, the prediction must return a real number. Therefore we may write

$$\hat{M}^3 \; : \; P[\psi_n] = |\alpha_n|^2 \quad , \qquad \text{and} \qquad \hat{M}^3 \; : \; P[\psi(x);a,b] = \int_a^b |\psi(x')|^2 dx' \quad . \tag{4.30}$$

In the discrete eigenbasis, there is no integration variable and $P[\psi] \in \mathbb{R}$ is obtained directly. In the continuous eigenbasis, we introduce an integration variable with the concept of Dirac orthogonality and then eliminate that variable with an integral operation. Most notably, it is the analytical structure of $P[\psi]$ that first suggested the maximum action path in chapter one, and we might consider that the conjuration and elimination of an integration variable is something that happens along the Cauchy C curve around infinity where the level of $\aleph$ changes. The probability functional is the backbone of everything about $\hat{M}^3$ and we need to define an operator or functional that will always return a real number. For the probability interpretation to hold, the normalization of $P[\psi]$ is such that

$$P[\psi_n] = \sum_{n=1}^{N} |\alpha_n|^2 = 1 \quad , \qquad \text{and} \qquad P[\psi(x);-\infty,\infty] = \int_{-\infty}^{\infty} |\psi(x')|^2 \, dx' = 1 \quad . \tag{4.31}$$

On the left, the continuum of available eigenstates introduces a new variable into the P functional and then it gets eliminated in the integral over dx'. We will not go off, too much, on a tangent here but it must be noted how this is very much like the concept of an information source and sink. When we introduce a new variable and then integrate it away, there is always a lot of wiggle room in measure theory that we can use to add arbitrary complexity if desired. It is within this wiggle room that we have made the change $dx \to d\hat{\gamma}$.

For a finite, discrete set of eigenfunctions we have

$$P[\psi_n] = |\alpha_n|^2 \quad , \qquad \text{where} \qquad \psi = \sum_{n=1}^{N} \alpha_n \psi_n \quad , \tag{4.32}$$

but the case is altogether different for the infinite dimensional case of continuous spectra of eigenfunctions. Then the interpretation for $P[\psi]$ changes so that $P[\psi]dx$ is the probability of finding the particle between x and $x+dx$ (at time t.) Therefore, we have a good concept for integrating over the hypercomplex neighborhood between x and $x+dx$.[1] For the infinite dimensional case, we have

$$P[\psi(x);a,b]=\int_a^b \psi^*(x)\psi(x)\,dx \ , \qquad \text{where} \qquad \psi(x')=\big\langle x\big|\psi\big\rangle=\int_{-\infty}^{\infty}\delta(x-x')\psi(x)\,dx \ . \tag{4.33}$$

In long form, the formula to obtain a real probability from $\psi(x)=\langle x|\psi\rangle$ is

$$P[\psi(x);a,b]=\int_a^b \left[\int \delta(x_1-x')\psi(x_1)\,dx_1\right]\left[\int \delta(x_2-x')\psi^*(x_2)\,dx_2\right]dx' \ . \tag{4.34}$$

This can be rewritten in MCM the as

$$P[\psi(x);a,b]=\int_a^b \left[\int \delta(\chi_+-x)\psi(\chi_+)\,d\chi_+\right]\left[\int \delta(\chi_--x)\psi^*(\chi_-)\,d\chi_-\right]dx \ . \tag{4.35}$$

To obtain $P[\psi]\in\mathbb{R}$ on an arbitrary level of $\aleph$, we can add $\chi_\varnothing$ such that

$$P[\psi(x);-\infty,\infty;\hat{\Phi}^j]=\int_{\hat{0}}^{\hat{\Phi}^j}\left(\int_{-\infty}^{\infty}P'[\psi(x)]\,dx\right)d\chi_\varnothing=\hat{\Phi}^j \ . \tag{4.36}$$

The integration over $d\chi_\varnothing$ happens only in the hatted channel so it is easy to understand why $\chi^5_\varnothing$ should have no width in $\chi^5\equiv\chi^5_+\otimes\chi^5_\varnothing\otimes\chi^5_-$. That integration increases the level of $\aleph$ and exists purely in the hatted channel

$$\int_{\hat{0}}^{\hat{\Phi}^j} dx_\varnothing=\hat{\Phi}^j \ . \tag{4.37}$$

[1] Often it is noted that there is a problem calling Dirac delta functions "functions" and all of this semanticism can almost certainly be assuaged with regular functions on different levels of $\aleph$.

Everything about limits of $\Sigma^{\pm}$ converging on $\mathcal{H}$ or $\varnothing$ should be derivable from equation (4.35). However, we will only examine the fundamentals of quantum mechanics in this section. The fundamental that we presently consider is that $P[\psi]$ is algorithmically different for $\psi(x)$ and ψ_n, and we need something that is algorithmically the same for $\hat{M}^3$.

Note how the two functionals

$$P[\psi_n] = |\alpha_n|^2 \quad , \tag{4.38}$$

and

$$P[\psi(x);a,b] = \int_a^b \int_0^\infty \int_{-\infty}^0 \delta(\chi_+ - x)\delta(\chi_- - x)\psi^*(\chi_-)\psi(\chi_+)\,d\chi_-\,d\chi_+\,dx \quad , \tag{4.39}$$

for obtaining a real-valued probability are somewhat similar in structure to

$$\hat{\Upsilon} = \hat{U} + \hat{M}^3 \quad , \tag{4.40}$$

with

$$\hat{U} := P[\psi_n] \quad , \qquad \text{and} \qquad \hat{M}^3 := P[\psi(x);a,b] \quad . \tag{4.41}$$

One gets the impression that the derivation of the probability $0 \leq P[\psi] \leq 1$ can be universalized with a projection operator into the $\hat{U}$ or $\hat{M}^3$ component of $\hat{\Upsilon}$. For $P[\psi_n]$, the eigenfunctions are exactly orthogonal and we expect no remainder, and that can be associated with $\hat{U}$. Then there is a natural extension to rules for dx and $d\hat{\gamma}$ when the latter operates on the remainder of the $\hat{M}^3$ component. The dx^3 in $P[\psi(x);a,b]$ is very much like $\hat{M}^3$, and $\hat{M}^3$ is the operator that makes the connection to the gravitational sector. In fact, when we make the simplistic conversion $|\alpha_n|^2 \to |\delta(x-x')|^2$, as in equation (4.28), it is likely not irrelevant that $|\delta(x)|^2 \sim dx^2$ because the metric $g_{\mu\nu}$ goes like dx^2. All of the equations given so far in this section assume the Euclidean metric and we will continue in that convention having taken note that $|\delta(x-x')|^2 \sim dx^2$ gives a good indicator for a fruitful avenue of complexification.

The intention is to use the ontological basis as the basis for general relativity in a manifold constructed from the analytical remainder when the MCM inner product is between two series represented with unequally infinite numbers of terms. We will also want to use the ontological basis as a discrete eigenbasis in quantum theory. Here, we will consider various constructions of the MCM inner product of objects on various levels of $\aleph$. A first inner product that we could take is

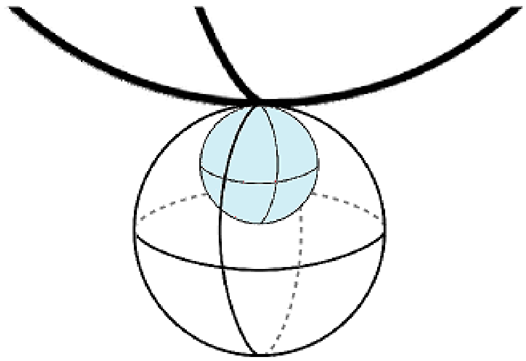

Figure 56: The $\hat{\pi}^2$ operator adds topological hypercomplexity to the Riemann sphere.

$$P[\psi] = \big\langle \psi;\hat{\pi};\hat{\Phi}^2 \big| \psi;\hat{\pi};\hat{\Phi}^1 \big\rangle \quad . \tag{4.42}$$

We already have

$$\big\langle \psi \big| \psi \big\rangle = 1 \quad , \qquad \text{so} \qquad \big\langle \psi;\hat{\pi} \big| \psi;\hat{\pi} \big\rangle = \hat{\pi}\hat{\pi} := \pi^2 \quad , \tag{4.43}$$

and we can assume that π^2 gets used to create the topology described in figure 56. $\hat{\pi}$ is a real number so $\hat{\pi}^* = \hat{\pi}$. We have not completely decided if the level of $\aleph$ should increase by one or two in $\hat{M}^3$. In the convention where $\hat{M}^3$ increases the level of $\aleph$ by one, we have

$$\big\langle \psi;\hat{\pi};\hat{\Phi}^{j+1} \big| \psi;\hat{\pi};\hat{\Phi}^j \big\rangle = \hat{\pi}\hat{\Phi}^{2j+1}\hat{\pi} \quad . \tag{4.44}$$

For the probability interpretation to be valid, we need to obtain $\hat{\pi}\hat{\Phi}^{2j+1}\hat{\pi} \to 1$ so that equation (4.44) says that a state on $\hat{\Phi}^j$ will always transition to $\hat{\Phi}^{j+1}$.

We have proposed in this research program to label the $\hat{\pi}$-sites with integers and that $\hat{M}^3 : \hat{\pi}_1 \to \hat{\pi}_2$. Therefore, if we want to make rigorous the statement "use $\hat{\pi}^2$ to create $\mathbb{S}^2$," we can define a projection operator into the $\hat{\pi}_2$-site. Therefore, consider $\hat{\Upsilon}$ such that it produces the qubit at $\mathcal{H}_2$ from the input qubit on $\mathcal{H}_1$ *and* also another part that we will suppress with

$$\hat{\mathcal{P}}_{\pi_2} = \frac{1}{\pi^2}\,\hat{\pi}_2 \quad . \tag{4.45}$$

The projection operator $\hat{\mathcal{P}}_{\pi_2}$ is normalized for the probability interpretation and we might associate the remainder with that which is suppressed by the projection. The dynamical geometry which complements ψ via the MCM mechanism of quantum gravity [4] must be related to that which does not survive $\hat{\mathcal{P}}_{\pi_2}$. It has been the convention in the MCM to use the definition that $\hat{\pi}_j \equiv \hat{\pi}^j$ meaning that

$$\hat{\pi}_1 \cdot \hat{\pi}_1 = \pi^2 \quad , \qquad \text{and} \qquad \hat{\pi}_1\hat{\pi}_1 = \hat{\pi}^2 \quad , \tag{4.46}$$

The reader should take careful note that, for the probability interpretation, and without modifying any of the normalization built into ψ already, we need to remove π^2 from equation (4.44) and then also remove Φ^{2j+1}. Regarding π^2, we have already defined $\hat{M}^3$ to be such that $\hat{\pi}_1 \mapsto \hat{\pi}_2$ so the intuitive projection operator $\hat{\pi}^2$ to select $\mathcal{H}_2$ is the one that enforces the normalized probability.

We have shown the projection operator with the dot product in equation (4.11), but the projection operator in quantum mechanics is usually written as a dyadic so we should examine that formalism as well. The first thing to clarify in that direction is that $\{x^\mu_-, x^\mu, x^\mu_+, x^\mu_\varnothing\}$ is continuous in $\{\aleph, \mathcal{H}, \Omega, \varnothing\}$ but the basis vectors $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ themselves are discrete. The purpose of the ontological basis is to give four discrete position space representations to ψ so, in a sense, they are discrete but position eigenvectors are continuous. Consider the properties of discrete states

$$\big|\psi\big\rangle = \sum_{\alpha=1}^{4} \big|\hat{e}_\alpha\big\rangle\big\langle\hat{e}_\alpha\big|\psi\big\rangle \quad , \qquad \mathbb{I} = \sum_{\alpha=1}^{4} \big|\hat{e}_\alpha\big\rangle\big\langle\hat{e}_\alpha\big| \quad , \qquad \text{where} \qquad \hat{e}_\alpha \in \{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\} \quad . \tag{4.47}$$

The four discrete ontological basis vectors need to be orthogonal. We can demonstrate this with the general property of independent basis vectors $\hat{e}_1 \cdot \hat{e}_2 = 0$ such that

$$\big\langle\hat{e}_1\big|\hat{e}_2\big\rangle = \int \hat{e}_1^* \cdot \hat{e}_2 \, dx = \int 0 \, dx = 0 \quad . \tag{4.48}$$

Quantum mechanical projection operators have the property that $\hat{\mathcal{P}}^2 = \hat{\mathcal{P}}$ and we should show that as well. For example, if $|\hat{e}_j\rangle = \hat{e}_j$ we have

$$\hat{\mathcal{P}}_i^2 = \big|\hat{\imath}\big\rangle\big\langle\hat{\imath}\big|\hat{\imath}\big\rangle\big\langle\hat{\imath}\big| = \big|\hat{\imath}\big\rangle\, \hat{\imath} \cdot \hat{\imath}\, \big\langle\hat{\imath}\big| = -\hat{\mathcal{P}}_i \quad . \tag{4.49}$$

This raises an important question regarding $\hat{\imath}$. Should we take $\hat{\imath}^* \equiv -\hat{\imath}$ or should we take $\hat{\imath}^* \equiv \hat{\imath}$ because, as a basis vector, $\hat{\imath}$ is outside the realm of complex conjugation? $\hat{\pi}$ offers a simpler case when we may unambiguously write

$$\hat{\mathcal{P}}_\pi^2 = \big|\hat{\pi}\big\rangle\big\langle\hat{\pi}\big|\hat{\pi}\big\rangle\big\langle\hat{\pi}\big| = \big|\hat{\pi}\big\rangle\, \hat{\pi} \cdot \hat{\pi}\, \big\langle\hat{\pi}\big| = \pi^2\hat{\mathcal{P}}_\pi \quad . \tag{4.50}$$

Evidently

$$\big|\hat{e}_\alpha\big\rangle = \frac{1}{|\hat{e}_\alpha|}\,\hat{e}_\alpha \quad . \tag{4.51}$$

Note that equation (4.51) is what gives the term $1/\pi^2$ that appeared in equation (4.45). Then we obtain

$$\hat{\mathcal{P}}_\pi^2 = \big|\hat{\pi}\big\rangle\big\langle\hat{\pi}\big|\hat{\pi}\big\rangle\big\langle\hat{\pi}\big| = \big|\hat{\pi}\big\rangle\left(\frac{1}{\pi}\right)\hat{\pi}^*\cdot\left(\frac{1}{\pi}\right)\hat{\pi}\big\langle\hat{\pi}\big| = \big|\hat{\pi}\big\rangle\left(\frac{1}{\pi^2}\right)\big(\pi^2\big)\big\langle\hat{\pi}\big| = \hat{\mathcal{P}}_\pi \quad . \tag{4.52}$$

Using $|\hat{\imath}\rangle = -i\,\hat{\imath}$ and $\langle\hat{\imath}| = i\,\hat{\imath}$, we can rewrite equation (4.49) as

$$\hat{\mathcal{P}}_i^2 = \big|\hat{\imath}\big\rangle\big\langle\hat{\imath}\big|\hat{\imath}\big\rangle\big\langle\hat{\imath}\big| = \big|\hat{\imath}\big\rangle(i)\,\hat{\imath}\cdot(-i)\,\hat{\imath}\,\big\langle\hat{\imath}\big| = \big|\hat{\imath}\big\rangle\,\hat{\imath}^*\cdot\hat{\imath}\,\big\langle\hat{\imath}\big| \quad . \tag{4.53}$$

Therefore, we need to take

$$\big\langle\hat{\imath}\big|\hat{\imath}\big\rangle = \hat{\imath}^*\cdot\hat{\imath} = -\hat{\imath}\cdot\hat{\imath} = 1 \quad , \tag{4.54}$$

if we are to recover $\hat{\mathcal{P}}_i^2 = \hat{\mathcal{P}}_i$. However, it must be noted we still have the option to set $\hat{\imath}\cdot\hat{\imath} = 1$ as an added layer of complexity when the $\hat{\mathcal{P}}_i$ operator has some pathology likely associated with the $\sqrt{i}$ channel.

Take special note of the butterfly operator $|\hat{e}_j\rangle\langle\hat{e}_j|$ as a projection operator. In equation (4.45), we introduced the projection operator into the $\hat{\pi}_2$-site and we can rewrite it as

$$\hat{\mathcal{P}}_{\pi_2} = \big|\hat{\pi}\big\rangle\big\langle\hat{\pi}\big| = \left(\frac{1}{\pi}\hat{\pi}\right)^*\left(\frac{1}{\pi}\right)\hat{\pi} = \frac{1}{\pi^2}\hat{\pi}_2 \quad . \tag{4.55}$$

Interestingly, the pairwise butterfly operator is a dyad. It is only possible to write the operator in this form for the projector into $\hat{\pi}_2$. We could not write the dyadic with three ontological basis vectors for $\hat{\pi}_3$. Consider the other dyadic representation

$$\big|\hat{\pi}\big\rangle\big\langle\hat{\pi}\big| \quad \equiv \quad \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi^2 \end{pmatrix} \quad . \tag{4.56}$$

Therefore, we have two dyadic products: a unitary one $|\hat{e}_j\rangle\langle\hat{e}_j|$ and a non-unitary one $\hat{e}_j\hat{e}_j$. Before we show what $\langle\hat{e}_j|\psi\rangle$ is, let us demonstrate the property of $\mathbb{I}$ in equations (4.47) by writing

$$\sum_{\alpha=1}^{4}\big|\hat{e}_\alpha\big\rangle\big\langle\hat{e}_\alpha\big| = \begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix} \ . \tag{4.57}$$

Indeed, these four matrices do sum to the identity matrix $\mathbb{I}$.

When the spectrum of eigenvalues is continuous, as it is for position, then the completeness relation changes as

$$\mathbb{I}_4 = \sum_{\alpha=1}^{4}\big|\hat{e}_\alpha\big\rangle\big\langle\hat{e}_\alpha\big| \quad \longrightarrow \quad \mathbb{I}_\infty = \int \big|x\big\rangle\big\langle x\big|\,dx \ \ , \tag{4.58}$$

and the orthonormality condition

$$\big\langle\hat{e}_\alpha\big|\hat{e}_\beta\big\rangle = \left(\frac{1}{|\hat{e}_\alpha|}\hat{e}_\alpha\right)^{*}\cdot\frac{1}{|\hat{e}_\alpha|}\hat{e}_\beta = \delta_{\alpha\beta} \ \ , \qquad \text{becomes} \qquad \big\langle x'\big|x\big\rangle = \delta(x-x') \ \ , \tag{4.59}$$

when

$$\big\langle\hat{e}_1\big|\psi\big\rangle = \psi(x) = \delta(x-x') \tag{4.60}$$

$$\big\langle\hat{e}_2\big|\psi\big\rangle = \psi(x_+) = \delta(x_+-x') \tag{4.61}$$

$$\big\langle\hat{e}_3\big|\psi\big\rangle = \psi(x_-) = \delta(x_--x') \tag{4.62}$$

$$\big\langle\hat{e}_4\big|\psi\big\rangle = \psi(x_\varnothing) = \delta(x_\varnothing-x') \ \ . \tag{4.63}$$

Discrete eigenstates are orthogonal but continuous states are not orthogonal because $\delta(x-x')$ is never equal to zero. The Dirac delta is identically never equal to zero, which is totally different than the Kronecker delta $\delta_{\alpha\beta}$ which is identically zero for $\alpha=\beta$. However, $\delta(x-x')$ is kind of like zero so the concept of "Dirac orthogonality" is introduced. Furthermore, the hypercomplexly infinitesimal neighborhood has been introduced specifically for the analysis of objects that are very close zero but still non-vanishing.

We have been able to define the projection operators as per normal but there is some unresolved complexity that lingers. Writing out the full form of ψ we find

$$\big|\psi\big\rangle=\sum_{\alpha=1}^{4}\big|\hat{e}_\alpha\big\rangle\big\langle\hat{e}_\alpha\big|\psi\big\rangle=\big\langle\hat{i}\big|\psi\big\rangle\big|\hat{i}\big\rangle+\big\langle\hat{\Phi}\big|\psi\big\rangle\big|\hat{\Phi}\big\rangle+\big\langle\hat{2}\big|\psi\big\rangle\big|\hat{2}\big\rangle+\big\langle\hat{\pi}\big|\psi\big\rangle\big|\hat{\pi}\big\rangle \tag{4.64}$$

$$=c_i\,\big|\hat{i}\big\rangle+c_\Phi\,\big|\hat{\Phi}\big\rangle+c_2\,\big|\hat{2}\big\rangle+c_\pi\,\big|\hat{\pi}\big\rangle \quad . \tag{4.65}$$

This formula is properly normalized when all of the coefficients are equal to one half but it seems wrong that $\langle\hat{e}_\mu|\psi\rangle=\langle\hat{e}_\nu|\psi\rangle$ for any μ or ν. If we use the ontological coefficients

$$\big|\psi\big\rangle=\frac{-i}{4}\,\big|\hat{i}\big\rangle+\frac{-\varphi}{4}\,\big|\hat{\Phi}\big\rangle+\frac{1}{8}\,\big|\hat{2}\big\rangle+\frac{1}{4\pi}\,\big|\hat{\pi}\big\rangle \quad , \tag{4.66}$$

then the unitarity is broken with

$$\sum_{\alpha=1}^{4}|c_\alpha|^2=\left(\frac{1}{4\pi}\right)^2+\left(\frac{-\varphi}{4}\right)^2+\left(\frac{1}{8}\right)^2+\left(\frac{-i}{4}\right)^2\neq 1 \quad . \tag{4.67}$$

In fact, we have not yet carefully considered what the objects in equations (4.60-4.63) are supposed to be. Certainly, if we write that the c_α are all equal to $1/2$ then it makes no sense to write

$$\big\langle\hat{e}_\alpha\big|\psi\big\rangle=\int_{-\infty}^{\infty}\hat{e}_\alpha\psi(x_\alpha)\,dx \quad , \tag{4.68}$$

because it will not generate any complexity. The likely issue is that even though we have written the basis discretely, the corresponding objects are defined as, for example

$$\big|\psi;\hat{\pi}\big\rangle=\psi(x)=\big\langle x\big|\psi\big\rangle \quad . \tag{4.69}$$

This is a position space representation with continuous eigenfunctions. When we have a discrete orthonormal basis $|\hat{e}_\alpha\rangle$, what we actually have is four sets of continuous eigenfunctions. Therefore let there be a "psi-langle" and a "psi-rangle" such that, for example

$$\big\{\hat{\pi}\big|\psi\big\rangle=\big|\psi;\hat{\pi}\big\rangle \tag{4.70}$$

Then the operation specified by

$$\langle \psi;\hat{\pi}\big|\psi;\hat{\pi}\rangle = \int \psi^{*}(x)\psi(x)\,dx \quad , \tag{4.71}$$

will be the integral that is required to obtain $P[\psi(x)] \in \mathbb{R}$ from $P'[\psi(x)]dx$. This is the interpretation for equation (4.69) where it says a ket is equal to a bra and a ket. In this notation, we require one more integration operation and that must be the one which ensures $P[\psi(x);a,b]\in\mathbb{R}$ If we write

$$\big|\psi\rangle = \big\{\hat{i}\big|\psi\rangle\big|\hat{i}\big\} + \big\{\hat{\Phi}\big|\psi\rangle\big|\hat{\Phi}\big\} + \big\{\hat{2}\big|\psi\rangle\big|\hat{2}\big\} + \big\{\hat{\pi}\big|\psi\rangle\big|\hat{\pi}\big\} \tag{4.72}$$

$$= \psi(x_-)\big|\hat{i}\big\} + \psi(x_+)\big|\hat{\Phi}\big\} + \psi(x_\varnothing)\big|\hat{2}\big\} + \psi(x)\big|\hat{\pi}\big\} \quad , \tag{4.73}$$

then we can let the purpose of the new notation be to disrupt the normalization of the probability with the non-unitary properties of the ontological basis such that $\sum|c_\alpha|^2 \neq 1$. However, it is not the intention to reformulate all of quantum mechanics in this book, and breaking unitarity certainly requires a reformulation of the entire theory.

We do have a few more interesting *and rigorous* results to show in this section so we will return to $\langle\psi;\hat{e}_\mu;\hat{\Phi}^\Delta|\psi;\hat{e}_\lambda;\hat{\Phi}\rangle$. If we use the projection operator $\hat{\pi}^2$ then we may obtain from equation (4.44)

$$\big\langle\psi;\hat{\pi};\hat{\Phi}^{j+1}\big|\psi;\hat{\pi};\hat{\Phi}^{j}\big\rangle\cdot\hat{\pi}^2 = \hat{\Phi}^{2j+1} \quad . \tag{4.74}$$

For the probability interpretation, we need $\Phi^{2j+1}\to 1$. Solving $2j+1=0$ for $j=1/2$ gives a nonsensical answer because levels of $\aleph$ are discrete and it doesn't make any sense to have a half of one. Therefore the levels of $\aleph$ in $\hat{M}^3$ must go to $\hat{\Phi}^{j+2}$ instead of the $\hat{\Phi}^{j+1}$ shown above. In the convention where $\hat{M}^3:\hat{\Phi}^j\to\hat{\Phi}^{j+2}$, we can write

$$\big\langle\psi;\hat{\pi};\hat{\Phi}^{j+2}\big|\psi;\hat{\pi};\hat{\Phi}^{j}\big\rangle = \hat{\pi}\hat{\Phi}^{2j+2}\hat{\pi}\cdot\hat{\pi}^2 = \hat{\Phi}^{2j+2} \quad , \tag{4.75}$$

which requires us to solve $2j+2=0$ to get the correct normalization with $j=-1$. Written simply, we have

$$\big\langle\psi;\hat{\pi};\hat{\Phi}^{1}\big|\psi;\hat{\pi};\hat{\Phi}^{-1}\big\rangle = \hat{\pi}\hat{1}\hat{\pi} \quad . \tag{4.76}$$

This equation makes it look like $\hat{M}^3$ should increase the level by two. However, it involves $\hat{\Phi}^{-1}$ which is something that we want to avoid. We can't increase by $j\to j+1$ in equation (4.76) because that will mess up the probability with $\Phi^1\neq 1$. There is another way that we can get the correct normalization with $\hat{M}^3$ that increments the level of $\aleph$ by one instead of two. We might encode the initial quantum state on the same level of $\aleph$ as $\mathcal{H}$, that is $\hat{\Phi}^0 = \hat{1}$, with $\hat{\varphi}$. Then we can write

$$\big\langle \psi; \hat{\pi}; \hat{\Phi}^1 \big| \psi; \hat{\pi}; \hat{\varphi} \big\rangle = \hat{\pi} 1 \hat{\pi} \quad . \tag{4.77}$$

This formulation no longer requires that the level changes by two. In contrast to equation (4.76), we might associate $\hat{\Phi}^{-1}$ with $\bar{U}$ so that $t_\pm$ are associated with $\hat{\Phi}^{\pm 1}$, exactly as in figure 52 and many other descriptions of the same scenario. Equation (4.77) relies on the magnitude of φ being proportional to Φ^{-1} even though $\hat{\varphi}$ itself exists on the same level of $\aleph$ as $\mathcal{H}$ on $\hat{\Phi}^0$

Equation (4.77) looks exactly like a quaternion rotation. A quaternion rotation when the angle is π will have the effect of a 2π rotation through the complex plane. If we rotate $\hat{1}$ by $\pi/2$ radians through $\mathbb{C}$ then it becomes $\hat{i}$, if we rotate it by π then it becomes $-\hat{1}$ *etc.* Then

$$e^{-\mathbf{u}\pi} \hat{1} e^{\mathbf{u}\pi} = \hat{1} \quad , \tag{4.78}$$

is the unique quaternion rotation that will give the correct probability interpretation of the MCM inner product. Quaternion rotation by any angle other than $n\pi$ will not preserve the probability interpretation. Quaternion rotations operate from the left and right. With that in mind, we might use the psi-langle and psi-rangle together in a "psi-gangle"

$$\big\{ \hat{e}_\alpha \big| \; \big| \psi \big\rangle \; \big| \hat{e}_\beta \big\} = \big| e^{-\mathbf{u}\theta_\alpha} \psi e^{\mathbf{u}\theta_\beta} \big\rangle \quad , \tag{4.79}$$

as a way around the issue in equation (4.69) where, in essence, a ket was set equal to a bra-ket. However, we leave these details to a specific treatment to appear elsewhere.

Figure 57 shows how we can account for the two different types of rotation. The single operator rotation causes a rotation only, but the two operator rotation causes the rotation and also changes the anchor point of the object in question. In this case, we can see how $\vec{V} e^{\mathbf{u}\theta}$ forms an intermediate object that we should associate with the Cauchy C curve. If we organize the operation as in figure 58 then we can see easily how the object that connects the two anchor points $\mathcal{O}$ and $\mathcal{O}'$ is in a lateral direction to the shared direction of the other two objects $\vec{V}$ and $e^{-\mathbf{u}\theta} \vec{V} e^{\mathbf{u}\theta}$. In that case, we should associate this intermediate path with the off shell region i that separates adjacent instances of $t_\star$. Furthermore, the quaternions have the property $\mathbf{ijk} = -1$ whereas the Lorentz transformation, rigorously a rotation by an angle in $\mathbb{C}$, uses the regular imaginary number $i^2 = -1$. $\hat{\mathcal{R}}$ is a Lorentz transformation when the imaginary part of the complex angle vanishes. There is motivation for a third step in $\hat{M}^3$ when we we consider that there is only one on shell region in spacetime but, in hyperspacetime, we can rotate objects into the imaginary off shell region, apply another operation completely within the off shell region, and then rotate back onto the on shell region at the next $\hat{\pi}$-site. Therefore, if we want to make the connection in a certain picture

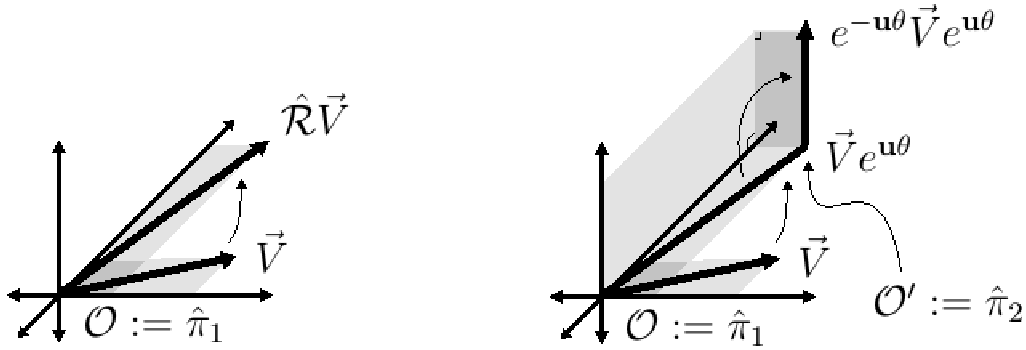

Figure 57: We can let quaternion rotation (quaternion analogue rotation) differ from ordinary rotation when the quaternion operation changes the anchor point of the object while the single operator rotation does not do so.

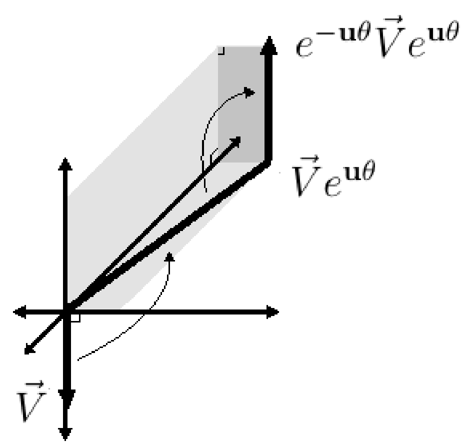

Figure 58: This figure seeks to replicate the mechanism shown in figure 53. Here the initial and final directions are both along $\hat{z}$.

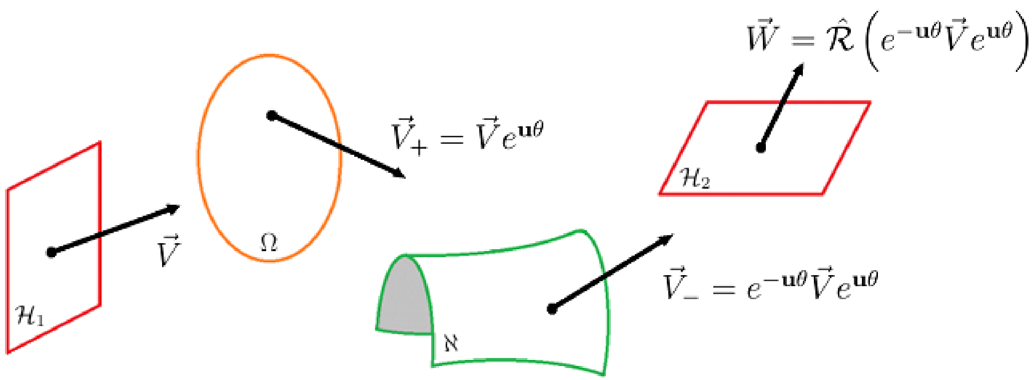

Figure 59: This figure demonstrates how a quaternion rotation and an ordinary rotation together fill the requirement for $\vec{V} \mapsto \vec{W}$ which was developed in chapter one. Notably, $\vec{W}$ is of the form $\hat{M}^3\vec{V}$ and a direction for future inquiry will be to make the change $\mathbf{u}\theta \to \mathbf{\Phi}$ when the rotation angle becomes $2\Phi \approx \pi$.

$$\hat{\pi} := e^{\mathbf{u}\pi} \quad , \tag{4.80}$$

we may let the objects in the ontological basis go like

$$\hat{\pi} \quad \mapsto \quad e^{\boldsymbol{\pi}} \quad , \tag{4.81}$$

In equation (4.81), we have used the notation from reference [5] that $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ have operator properties in a quaternion picture $\{\boldsymbol{i}, \boldsymbol{\Phi}, \boldsymbol{2}, \boldsymbol{\pi}\}$. We might speculate that the place of the exponential map in converting the basis vectors into quaternion operators will be such that the object appears in different incarnations on different levels of $\aleph$, possibly identifiable as the odd and even levels. For instance, we might say that $\hat{\pi}$ on even levels of $\aleph$ is like e^{π} on odd levels. Where we have supposed in reference [7] to build a fractal matrix theory of infinite complexity via the alternation of diameters and circumferences on spatial and temporal spheres, the alternation between $\hat{\pi}$ and e^{π} on successive levels of $\aleph$ is, in essence, is the same principle. $\hat{\pi}$ is like a diameter and e^{π} is like a circumference.

Here, a question remains about how we can switch between $\hat{\pi}$ and $e^{\boldsymbol{\pi}}$. The first thing to note is that the quaternion in the exponent plays the same role as i in an ordinary rotation operator.[1] Noting that the change of parameterization $x \leftrightarrow e^{ix}$ is associated with the change of topology $\mathbb{R} \leftrightarrow \mathbb{S}^1$, and noting that we have proposed to use the changing level of $\aleph$ to switch between circular and straight paths with $R \leftrightarrow \infty$, we can suppose that $\hat{\pi}$ on the flat level of $\aleph$ becomes $e^{\boldsymbol{\pi}}$ on the circular level of $\aleph$. There were a lot of unanswered questions regarding MCM quaternions in reference [5], and we did not answer them in section II.7, but we will say a little more about map (4.81) here. We will not go into a lot of detail about quaternions but it suffices to say that almost everything in quantum theory has an isomorphism with the quaternions. Since electromagnetism can also be formulated with quaternions instead of vectors, and quantum theory can be formulated with quaternions instead of Pauli matrices, it may be wise to revert to the quaternion formulation of physics when teaching the material to graduate students.

Relationships like that in map (4.81) are known as exponential maps. In general relativity, the exponential map is a map from the a manifold's tangent space back to the manifold itself. Therefore, the exponential map is well-suited as a map like those in figure 59 where the mapping is from one space to another. Defining an exponential map between the ontological basis and quaternions is very natural for a number of reasons, many of which appear in reference [5]. Another reason is that the Pauli matrices, together with the identity, are like the quaternions. Quaternion rotation requires two operators, one on the left and one on the right, and we are proposing to derive quaternion structure from the inner product which will always have two operators: one in the bra and one in the ket. While not directly related to quaternions, ancillary support is found when the dyadic representation of pairs of ontological

[1] In reference [5], when we first proposed to use the ontological basis as quaternions, we noted an issue with $\hat{\imath} \mapsto e^{\mathbf{i}}$ because it induces an imaginary coefficient that does not exist in Hamilton's quaternions $\mathbb{H}$. Therefore, we should examine the case when $\mathbf{i}$ is different than the other quaternions because it is like the ordinary rotation operator $e^{i\Delta}$ through another exponential map $\mathbf{i} \mapsto e^{i}$.

basis vectors contains 16 distinct matrices like the algebra of physical space for the Dirac equation.

With an appropriately chosen $\hat{\Phi}^j$, we can enforce the probability interpretation of the inner product with

$$\hat{\pi}^* \hat{1} \hat{\pi} \quad \mapsto e^{-\pi} \hat{1} e^{\pi} = \hat{1} \quad , \tag{4.82}$$

and figure 59 shows how we can use this structure inside the MCM unit cell. We can define the transport of $\vec{V}$ as

$$e^{-\mathbf{u}\theta} \vec{V} e^{\mathbf{u}\theta} = e^{-\mathbf{u}\theta} \vec{V}_+ = \vec{V}_- \quad . \tag{4.83}$$

How can we get to $\vec{W}$ from $\vec{V}_-$? Recall that the ontological basis differs from the quaternions in $\hat{i}$ because the rigorous definition of the quaternions $\mathbb{H}$ requires that all four of them have real coefficients. Also note that, among the ontological basis, $\hat{i}$ is the only one that has unitary properties. Therefore, we should consider a unitary rotation such that

$$\vec{W} = \vec{V}_- e^{i\delta} \quad . \tag{4.84}$$

When $\hat{M}^3$ is constituted with two quaternion rotation operators and an ordinary rotation, we find a structure similar to the gauge theory developed in reference [10]. In that reference, we showed that the chirological derivative $\hat{M}^3 = \partial_+ \partial_\varnothing \partial_-$ introduced constants $\pm\Phi$ for $\partial_\pm$ and π for $\partial_\varnothing$. The oppositely signed Φ are easily attributable to the two quaternion operators, and π may be attributed to the ordinary rotation, which is somehow connected with $\partial_\varnothing$ though $\varnothing$ makes no appearance in figure 59. The quaternion operator on the right rotates $\vec{V}$ and then the left quaternion operator changes the anchor point and rotates it once again. Let the final step be a non-rotating phase change that implements some duality between AdS in $\aleph$ and the CFT of ψ in $\mathcal{H}$. In reference [3], we described AdS/CFT correspondence through a general treatment of bulk/boundary correspondence but the final operation that sends $\vec{V}_-$ in $\aleph$ to $\vec{W}$ in $\mathcal{H}_2$ is a natural home for the technical criteria that formally define the AdS/CFT correspondence. This is notable because we require the correspondence between AdS and the CFT, but not between dS and the CFT. There is no dS/CFT correspondence and, in the series of operations proposed here, we remove any suggestion that the bulk/boundary correspondence developed between $\mathcal{H}$ and $\Sigma^\pm$ in reference [3] would require some new dS/CFT correspondence beyond the AdS/CFT correspondence that has been well studied in the 21$^{\text{st}}$ century.

Where can we get the extra term $e^{i\delta}$ required for equation (4.84)? We can use the unitary evolution operator in the MCM inner product so that

$$\langle \psi; \hat{\pi}; \hat{\Phi}^1 | \hat{U} | \psi; \hat{\pi}; \hat{\varphi} \rangle = \hat{\pi} e^{i\hat{H}t} \hat{\pi} \quad . \tag{4.85}$$

This brings us to an important point. We have been considering the topological remainder in the MCM inner product of ψ with ψ but we have motivated the existence of the remainder only in the case of orthogonal states. There is some nuance related to how we will use $|\hat{e}_\alpha\}$ but, in general, ψ is not orthogonal to itself. Therefore, it will be helpful to consider the property of the unitary evolution operator

$$\big\langle\psi\big|\hat{U}\big|\psi\big\rangle = \big\langle\vartheta\big|\psi\big\rangle \quad , \tag{4.86}$$

where ϑ and ψ are orthogonal states. For instance, in the position space representation the unitary evolution operator will take a state $|x_1\rangle$ and return a state $|x_2\rangle$ that is Dirac orthogonal to $|x_1\rangle$. Then

$$\big\langle\vartheta;\hat{\pi};\hat{\Phi}^{j+2}\big|\hat{U}\big|\psi;\hat{\pi};\hat{\Phi}^{j}\big\rangle := \big\langle\vartheta\big|\psi\big\rangle = \delta(x-x') \quad , \tag{4.87}$$

which has the backbone of Dirac orthonormalism that will be associated with the remainder term. Note well, not every MCM inner product will produce a remainder. We have not produced the remainder in any of the above but we are progressing toward that feature.

When assigning quaternion properties to the ontological basis we needed to send them into the exponent of Euler's number with $\hat{\pi}\mapsto e^{\pi}$ and, in general, we have an exponential map $x\mapsto e^{ix}$ as the relationship between linear and circular intervals. To restate the connection to levels of $\aleph$, when the radius of a circle is finite it is a circle, but when the radius becomes infinite the circle becomes a line. In developing quantum theory, it is shown to students that the Hamiltonian operator is a Hermitian matrix and that e to the power of a Hermitian matrix is always a unitary operator. How does the matrix get into the exponent and what does it mean for Euler's number to be raised to the power of a matrix? This is another example of an exponential map. For square matrices $\mathbb{M}$ we have

$$e^{\mathbb{M}} \quad \equiv \quad \sum_{j=0}^{\infty}\frac{\mathbb{M}^j}{j!} = \mathbb{I} + \mathbb{M} + \frac{1}{2}\,\mathbb{M}^2 + \frac{1}{6}\,\mathbb{M}^3 + \cdots \quad , \tag{4.88}$$

and we will examine the exponential map again in section IV.6.

Referring again to figure 59, take note that dS spacetime has no boundary but AdS spacetime does have one. When we increase the level of $\aleph$ we necessarily introduce a boundary term. Therefore, when we say that the MCM inner product has a remainder due to an extra term in the series expansion of the wavefunction, we can let the dual vector on the higher level of $\aleph$ in the MCM inner product always be defined such that it includes more terms than the same series on the lower level of $\aleph$. This is an alternative possible source of extra terms besides the argument for odd and even infinities made in reference [9]. Since the level of $\aleph$ is expected to increase by two in the MCM inner product, we should assume that there will be at least two more terms in the series expansions on the higher level of $\aleph$ without choosing one source for those terms or another. This is simply another proposal. The most important feature is that the MCM inner product does or can produce a remainder that can be analytically connected to an imbalance in the countability of two countably infinite series.

One of this writer's first conversations with David Finkelstein occurred in 2011 during the weeks following the distribution of reference [7] on the Georgia Tech campus. At that time, David made some comments that may be paraphrased as, "Oh, you decided to go to the big end instead of the small end." He was referring to our introduction of what has evolved into the Cauchy C curve and he was comparing it to traditional attempts to fundamentally understand the universe by probing the realm of quanta. Most physicists working on quantum gravity attempt to develop quantum gravity by trying to show a self-consistent framework for the graviton as the quantum of the Newtonian gravitation field. In the MCM, we begin with Einstein gravity so that there is no field to quantize, and let the scale of the entire gravitational manifold representing the universe in one moment be on the order of an infinitesimal with respect to the scale of the manifold representing the universe in the next moment. By "going to the big end" we consider the entire universe as one quantum on the higher level of $\aleph$, and integration is necessarily the operation that goes toward the big end. When we consider two simultaneous charts on $\mathbb{S}^2$, one of them is on a higher level of $\aleph$ so that is another example of going toward the big end. Furthermore, when we say those two levels of $\aleph$ are $\hat{\Phi}^0$ and $\hat{\Phi}^1$, to the exclusion of $\hat{\Phi}^{-1}$ (associated with t_-), then we once again uncover a fundamental asymmetry of the MCM. If it was symmetric then $\hat{\Phi}^0$ wouldn't preferentially pair with either of $t_\pm$ but it does preferentially pair with t_+ and, using very broad strokes, we can probably use this to explain why the baryon number of the universe does not vanish but, rather, is manifestly positive.

The convention to define the MCM inner product with $\hat{\Phi}$ and $\hat{\varphi}$, as in

$$\left\langle \psi;\hat{\pi};\hat{\Phi}^{j+1}\middle|\psi;\hat{\pi};\hat{\varphi}\right\rangle = \hat{\pi}^{-1}\hat{1}\hat{\pi} \quad , \tag{4.89}$$

is interesting because, with $\hat{\varphi}$ being on $\hat{\Phi}^j$, the numbers of the discretized levels of $\aleph$ are $j+1$ and j respectively. As we will show at the end of this section, the integer values j and $j+1$ form the backbone of the most complex eigenvalues of the angular momentum operators in quantum mechanics. If we associated an angular momentum with each of the $t_\pm$ universes as they travel in different directions around the circle then the two momenta should combine in superposition to give no net angular momentum but there is an altogether different scenario with $t_\star$. In the representation where $t_\star$ goes around a circle rather than along a straight line, as per the MCM initiative to wrap the the x^0 axis around a cylinder, there is evidently an associated angular momentum $\vec{L}$. As $\vec{L}$ is conserved from one moment to the next ($\mathcal{H}_1$ to $\mathcal{H}_2$), it must be decomposed onto $t_\pm$ for the integration path around infinity, as in figure 52. In the decomposition, we expect that $\vec{L}/2$ will go into t_+ and $\vec{L}/2$ will go into t_-. $t_\pm$ run clockwise and anti-clockwise between $\mathcal{H}_1$ and $\mathcal{H}_2$ so the rotation that contributes to the momentum in t_- must be reversed with respect the rotation in t_+. This is required to generate $\vec{L}/2$ in t_- which adds, rather than cancels, with the angular momentum of t_+. Therefore, we can associate $\vec{L}/2$ with half integer spin, and the two opposite types of rotation should be called "spin up" and "spin down."

Above we have shown how the amplitude for forward evolution needs to normalized to unity. The formalism also needs to show that the amplitude for staying in the present is forbidden with

$$\left\langle \psi;\hat{\pi};\hat{\Phi}^0\middle|\psi;\hat{\pi};\hat{\varphi}\right\rangle = 0 \quad . \tag{4.90}$$

This formula says that the probability for a qubit on $\hat{\Phi}^0$ to stay on $\hat{\Phi}^0$ is zero. However, the formula has simply reduced j by one with respect to equation (4.89) so we can immediately write

$$\left\langle \psi;\hat{\pi};\hat{\Phi}^0\middle|\psi;\hat{\pi};\hat{\varphi}\right\rangle = \hat{\pi}^{-1}\hat{\varphi}\hat{\pi} \quad . \tag{4.91}$$

This does not equal zero so we need to take a closer look. Here, we might uncouple $\hat{\varphi}$ from $\mathcal{H}$ in some fashion such that

$$\hat{\varphi} \quad \mapsto \quad \hat{\Phi}^{-1} \quad . \tag{4.92}$$

With this map, we say that everything on $\hat{\Phi}^{-1}$ is an infinitesimal. When the probability is proportional to $\hat{\varphi}$, as in equation (4.91), it is subfinite and can be associated with zero as required for equation (4.90).

What about the probability for a state to start on $\hat{\Phi}^j$ and then end up on a level that is much higher? If we consider

$$\left\langle \psi;\hat{\pi};\hat{\Phi}^j\middle|\psi;\hat{\pi};\hat{\varphi}\right\rangle = \hat{\pi}^{-1}\hat{\Phi}^{j-1}\hat{\pi} \quad , \tag{4.93}$$

with $j\geq 2$ we do not get zero. If we use the equivalent of map (4.92), trivially

$$\hat{\Phi}^j \quad \mapsto \quad \hat{\Phi}^j \quad , \tag{4.94}$$

then equation (4.93) gives a number on a higher level $\aleph$ which is infinite, and therefore a problem. To build the workaround in this case, note that we have not specified the initial level of $\aleph$ so we may simply say that $\hat{\varphi}$ was encoded on the level of $\hat{\Phi}^{j-2}$. From this we recover $P[\psi] = 1$ with a finite (but arbitrary) normalization. Absolute level of $\aleph$ does not factor into consideration so it is feasible that we can normalize such that the level of $\aleph$ on the right of the modified Dirac bracket is always $j-2$. This will hard code into the bracket that it always increases the level of $\aleph$ by two. Therefore, we do not obtain $\langle\psi;\hat{\pi};\hat{\Phi}^{\{j\geq 2\}}|\psi;\hat{\pi};\hat{\varphi}\rangle = 0$ but instead exclude it from consideration. When the future is on $\hat{\Phi}^j$, and the qubit is on $\hat{\Phi}^{j-2}$, then the baseline level of $\aleph$ is evidently $j-1$. The map that will identify a transition amplitude into $\hat{\Phi}^j$, as in equation (4.93), with the amplitude into $\hat{\Phi}^1$, as in equation (4.89), is

$$\Phi^{1-j} \quad : \quad \hat{\Phi}^j \quad \mapsto \quad \hat{\Phi}^1 \quad . \tag{4.95}$$

If we add this to equation (4.93), we get

$$\big\langle \psi;\hat{\pi};\hat{\Phi}^j\big|\Phi^{1-j}\big|\psi;\hat{\pi};\hat{\varphi}\big\rangle = \hat{\pi}^{-1}\left(\frac{\hat{\Phi}^{j-1}}{\Phi^{j-1}}\right)\hat{\pi} = \hat{\pi}^{-1}\hat{1}\hat{\pi} \quad , \tag{4.96}$$

This shows that the probability for the qubit to end up two levels of $\aleph$ higher than where it started is 100%. The standard use of a normalization coefficient in quantum theory is precisely so that the inner product is normalized to unity. This shows that the normalization constant Φ^{1-j} in equation (4.96) is well motivated. Therefore, with the exception of the rule $\hat{\varphi}\mapsto\hat{\Phi}^{-1}$, we have only used the ordinary methods of quantum mechanics to study these amplitudes. The eccentric step $\hat{\varphi}\mapsto\hat{\Phi}^{-1}$ was introduced for amplitudes when j was not large enough. When studying the case of j too large, we redefined the inner product so that it is always across two levels of $\aleph$, and this definition means that we do not have to rely on the map $\hat{\varphi}\mapsto\hat{\Phi}^{-1}$. The operation which returns unity is always from $j-2$ to j.

The above stated rules will work, but the notation with two hatted specifiers $|\psi;\hat{e}_\mu;\hat{e}_\nu\rangle$ is not something we have previously considered. Also, it is unsatisfying that, when j is too big, we use a normalization but in the case of $\hat{\varphi}\mapsto\hat{\Phi}^{-1}$ we say that the number exists on a different tier of infinitude. The normalization procedure should work the same in either case so we have imposed an arbitrary rule to do one thing one case and another in the other case. Therefore. we will propose another inner product where we do not require the normalization to unity. Consider

$$\big\langle \psi;\hat{\Phi}^2\big|\psi;\hat{\pi}\big\rangle = \pi\Phi^2 \quad . \tag{4.97}$$

This is the amplitude for the $\hat{\pi}$ component to end up two levels of $\aleph$ higher than where it started. It returns the all-important critical value [4] at the foundation of all of the MCM's quantitative results:

$$\partial^3\psi := i\pi\Phi^2\psi \quad , \tag{4.98}$$

where the factor of i is relegated to the $\sqrt{i}$ channel in equation (4.97). If we put the unitary evolution operator in there as

$$\big\langle \psi;\hat{\Phi}^2\big|\hat{U}\big|\psi;\hat{\pi}\big\rangle = \big\langle \vartheta;\hat{\Phi}^2\big|\psi;\hat{\pi}\big\rangle = \pi\Phi^2\delta(x-x') \quad , \tag{4.99}$$

then we have defined an excellent MCM remainder, but if $\langle\vartheta|\psi\rangle=\delta(x-x')$ then $\langle\vartheta;\hat{e}_\lambda|\psi;\hat{e}_\mu\rangle$ is supposed to give a number in $\mathbb{R}$. In this section, we will replicate the derivation of general relativity using equation (4.97) but the reader should note that, by propagating $\delta(x-x')$ through the algebra, we can describe increasingly complex connections between quantum states ψ and the geometric conditions in a gravitational manifold.

The result that $\hat{\mathcal{P}}_{\pi_2}=\hat{\mathcal{P}}_{\pi_1}$ is very important because it shows that there is fundamentally only one $\hat{\pi}$-site. When we construct the projection operators for a hypothetical $\hat{\pi}_j$-site, they are all the same and this is indicative of fractal structure. Linear algebra requires that basis vectors appear by themselves and not in products so it was ambiguous when we wrote the bra-ket with both $\hat{\pi}$s and $\hat{\Phi}$s in it as in

$$\left\langle\psi;\hat{\pi};\hat{\Phi}^1|\psi;\hat{\pi};\hat{\varphi}\right\rangle=\hat{\pi}\hat{1}\hat{\pi}\quad. \tag{4.100}$$

The second $\hat{\pi}$-site is actually $\hat{\pi}_2\equiv\hat{\pi}^2$ so a fully specified version of equation (4.100) is

$$\left\langle\psi;\hat{\pi}_2;\hat{\Phi}^1|\psi;\hat{\pi}_1;\hat{\varphi}\right\rangle=\hat{\pi}_2\hat{1}\hat{\pi}_1=\hat{\pi}\cdot\hat{\pi}\cdot\hat{\pi}\quad. \tag{4.101}$$

There are redundant $\hat{\pi}$s in there. This notational deficiency is likely attributable to the psi-langle formalism where the product of a psi-langle with a ket is another ket. This means we could take the output ket with another $\{\hat{\pi}|$ which would be associated with the third instance of $\hat{\pi}$ in equation (4.101). If we write the probability for a state to transition into a level of $\aleph$ that is two levels higher than where it started as

$$P[\psi(x)]_{\Phi^2}=\left\langle\psi;\hat{\Phi}^2\middle|\psi;\hat{\pi}\right\rangle\quad, \tag{4.102}$$

as in equation (4.97), then this will have just one $\hat{\pi}$ in it. Therefore, the condensed psi-langle/ket $|\psi;\hat{e}_\lambda\rangle$ notation is favored. This is excellent because we have treated objects like $|\psi,\hat{e}_\lambda\rangle$ throughout this research program but objects like $|\psi;\hat{e}_\lambda;\hat{e}_\rho\rangle$ are of a different sort altogether. In equation (4.97), we have reduced to the intuitive form $\langle\psi;\hat{e}_\mu|\psi;\hat{e}_\nu\rangle$ and it looks as if we don't need to write $\hat{\pi}$ redundantly.

$$\left\langle\psi;\hat{\Phi}^2\middle|\psi;\hat{\pi}\right\rangle=\hat{\Phi}^2\cdot\hat{\pi}\quad. \tag{4.103}$$

We have the native definitions in linear algebra $\langle\hat{e}_\alpha|\hat{e}_\beta\rangle=\delta_{\alpha\beta}$ but equation (4.103) is a non-linear relationship wherein we cannot enforce the rules of linear algebra. The most famous non-linear application in quantum mechanics is the harmonic oscillator

$$V_{\text{harmonic}}(x)=\frac{1}{2}m\omega x^2\quad. \tag{4.104}$$

This potential along with the hydrogen atom mentioned earlier are the most important exactly solvable problems in quantum mechanics.[1] The harmonic oscillator is non-linear in x but the hydrogenic potential

$$V_{\text{hydrogenic}}(x) = -\frac{1}{4\pi}\frac{e^2}{\epsilon_0}\frac{1}{x} \quad , \tag{4.105}$$

is linear on the other side of a Riemann sphere somewhere when $1/x \to x$. The harmonic oscillator differs significantly from the hydrogen atom when its quantized energy levels increase as integers. The hydrogenic energy levels

$$E_n^{\text{hydrogenic}} = \frac{m_e}{2}\left(\frac{1}{4\pi}\frac{e^2}{\epsilon_0 \hbar}\right)^2 \frac{1}{n^2} \quad , \tag{4.106}$$

kind of increase as levels of $\aleph$ because n is an integer, but the quantum energy levels in the harmonic oscillator

$$E_n^{\text{harmonic}} = \hbar\omega\left(\frac{1}{2}+n\right) \quad , \tag{4.107}$$

do increase exactly as integers. The inverse linear r^{-1} atomic potential has energies that go like n^{-2} and the quadratic harmonic potential has energies that go like n. There is likely some deep reciprocity between these two famously exactly solvable problems of hydrogen and harmony when

$$\left\{x^{-1};n^{-2}\right\} \quad \longleftrightarrow \quad \left\{x^2;n^1\right\} \quad , \tag{4.108}$$

but instead we will remain focused on equation (4.103).

What we have done in equation (4.103) is to let there always be a remainder, not just in the case of Dirac orthogonal vectors. We say equation (4.103) shows a remainder because of the non-linear pathology. We have treated these objects mostly in the Dirac formalism but consider the case when we write out the inner product in the form that highlights the potential for an unpaired term. In long form, we have

$$\left\langle \psi;\hat{\Phi}^2\middle|\psi;\hat{\pi}\right\rangle = \sum_{j=0}^{\infty}\sum_{k=1}^{\infty} c_k^* c_j \delta'_{jk}\int \psi_k^*\psi_j\,dx = c_0\pi\Phi^2\int\psi_0\,d\hat{\gamma} \quad . \tag{4.109}$$

[1] In quantum field theory there are exact solutions for fourth order interactions and those are essentially the square of the harmonic oscillator problem. The third order interaction is an unsolved problem well suited to analysis in $^\star\mathbb{C}$.

At this point, it is very easy to associate the unpaired term ψ_0 with the (questionable) delta function in equation (4.99). Furthermore, equation (4.99) shows the remainder only associated with the Dirac orthogonal states which is what we want. It is natural to write

$$\big\langle \psi;\hat{\Phi}^2\big|\hat{U}\big|\psi;\hat{\pi}\big\rangle = \big\langle \vartheta;\hat{\Phi}^2\big|\psi;\hat{\pi}\big\rangle = c_0\pi\Phi^2\int \delta(x-x')\,d\hat{\gamma} \quad . \tag{4.110}$$

Depending on what $d\hat{\gamma}$ does, and what we use for x and x', this representation offers a lot of potential for complexification.

Consider a reformulation of the MCM hypothesis as

$$\big\langle \psi\big|\hat{M}^3\big|\psi\big\rangle \quad \equiv \quad \big\langle \vartheta;\hat{\Phi}^2\big|\psi;\hat{\pi}\big\rangle \quad . \tag{4.111}$$

The interpretation is that the returned value of $\hat{M}^3$ is equivalent to the amplitude for the level of $\aleph$ of the $\hat{\pi}$-site to increase by two as the particle moves from one location to another. In that case, using the remainder notation and setting $c_0 = 1$, we have

$$\big\langle \psi\big|\hat{\partial}_t^3\big|\psi\big\rangle = \omega^3\psi_0 \quad , \qquad\qquad \text{and} \qquad\qquad \big\langle \vartheta;\hat{\Phi}^2\big|\psi;\hat{\pi}\big\rangle = \pi\Phi^2\vartheta_0 \quad , \tag{4.112}$$

from which we can infer directly that

$$\omega^3\psi_0 = \pi\Phi^2\vartheta_0 \quad . \tag{4.113}$$

Then we may write, as per usual,

$$8\pi^3 f^3\big(\psi_0\,\hat{\pi}\big) = \pi\Phi^2\big(\vartheta_0\,\hat{\pi}\big) \tag{4.114}$$

$$= \pi\Phi\big(\vartheta_0\,\hat{\pi}\big) + \pi\big(\vartheta_0\,\hat{\pi}\big) \tag{4.115}$$

$$= \pi^2\big(\vartheta_0\,\hat{\Phi}\big) - i\pi^2\big(\vartheta_0\,\hat{\imath}\big) \quad . \tag{4.116}$$

We must get rid of π^2 in some fashion: either we construct $\mathbb{S}^2$, we use the projection operator $\pi^{-2}\,\hat{\pi}^2$, we use the quaternion identity $\hat{\pi}^{-1}\beta\hat{\pi} = \beta$, or do some such operation that we recover

$$8\pi f^3\big(\psi_0\,\hat{\pi}\big) = \big(\vartheta_0\,\hat{\Phi}\big) - i\big(\vartheta_0\,\hat{\imath}\big) \tag{4.117}$$

$$= \frac{\Phi}{4}\big(\vartheta_0\,\hat{2}\big) + \frac{1}{2}\big(\vartheta_0\,\hat{\Phi}\big) - i\big(\vartheta_0\,\hat{i}\big) \quad . \tag{4.118}$$

This replicates the earlier matching of the ontological basis to the elements of Einstein's equation

$$f^3\big(\psi_0\,\hat{\pi}\big) \quad \mapsto \quad T_{\mu\nu} \tag{4.119}$$

$$\frac{\Phi}{4}\big(\vartheta_0\,\hat{2}\big) \quad \mapsto \quad R_{\mu\nu} \tag{4.120}$$

$$\big(\vartheta_0\,\hat{\Phi}\big) \quad \mapsto \quad Rg_{\mu\nu} \tag{4.121}$$

$$i\big(\vartheta_0\,\hat{i}\big) \quad \mapsto \quad g_{\mu\nu}\Lambda \quad . \tag{4.122}$$

Here, we must repeat, again, that the matching in these maps is arbitrary and that we expect future investigations will show unique relationships between the ontological pieces of the remainder and the tensors in general relativity. Specifically, the manner in which we have chosen to derive the $\hat{2}$ sector solely through the $\hat{\Phi}$ sector is very arbitrary. However, regardless of the form of the matching, here we have achieved exactly what we proposed to do with $\int d\hat{\gamma}$. The important principle demonstrated is that there does exist some set of matchings between the objects in Einstein's equations and the objects in equation (4.118). Furthermore, $T_{\mu\nu}$ is the stress-energy tensor which we have set proportional to the frequency cubed and there exists an independent result in quantum theory wherein the energy density of the vacuum depends on the cube of the frequency.

Moving on with quantum mechanics, in reference [62] Gatland writes the following about why half-integer spin states must have multiplectic coefficients. While we have used the concept of even and odd levels of $\aleph$ separating on shell regions from an off shell bulk, Gatland makes a novel argument regarding the odd- or evenness of the analytical wavefunctions themselves.

> "One problem in presenting the theory of angular momentum in the context of quantum mechanics is explaining the absence of orbital momentum states that are half-integer (in units of $\hbar$). The angular momentum are initially defined in terms of position and momentum operators and the commutation relations for the components are obtained. The remainder of the algebraic analysis only uses the commutation relations for the angular momentum components so that some of the information contained in the position-momentum commutation relations is lost. As a result both integer and half-integer momentum states appear. [*sic*] In the usual development, the raising and lowering operators from a state with z-component angular momentum number m to the neighboring m states (expressed in terms of the polar angle θ and the azimuthal angle φ) are

$$L_{\pm} = -i\hbar e^{\pm i\varphi}\big(\pm i\partial/\partial\theta - \cot\theta\partial/\partial\varphi\big) \quad , \tag{4.123}$$

and the z-component angular momentum operator is

$$L_z = i\hbar\partial/\partial\varphi \quad . \tag{4.124}$$

If $F_{l,m}(\theta,\varphi)$ is an eigenfunction of L^2 and L_z with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$, respectively, the according to [equation 4.124],

$$F_{l,m}(\theta,\varphi) = f_{l,m}(\theta)e^{im\varphi} \quad . \tag{4.125}$$

We combine [*equations (4.123) and (4.125)*] and find that the neighboring states (after normalization) are given by

$$\hbar\big(\pm d/d\theta - m\cot\theta\big)f_{l,m} = \rho_{l,\pm m}f_{l,m\pm1} \quad , \tag{4.126}$$

where

$$\rho_{l,m} = \Big[\big(l(l+1) - m(m+1)\big)\Big]^{1/2}\hbar \quad . \tag{4.127}$$

Raising the top state or lowering the bottom state gives zero for the right-hand side of [*equation (4.126)*]. From the resulting differential equations we find

$$f_{l,\pm l} = N_{\pm}\sin^l\theta \quad . \tag{4.128}$$

The normalization constants N_+ and N_- in [*equation (4.128)*] have the same magnitude but not necessarily the same phase. All of the states for a given l may be obtained by starting from the bottom state, [*equation (4.128)*] with $m = -l$, and using [*equations (4.126-4.127)*] to move up in m."

Quantum mechanical eigenfunctions are purely a study of differential equations and, here, we will say a little about how analytical wavefunctions might be modified, if required, to describe the MCM, and also what l and m might describe in the MCM system. l is called the azimuthal quantum number and we proposed to modify the azimuthal angle in the MCM by extending the periodic domain $\theta\in(0,2\pi]$ onto the infinite helical interval $\theta\in(-\infty,\infty)$ in some covering space representation. Therefore, we might insert an additional layer of decoupling by removing from the functions $\{F_{l,m}, f_{l,m}, \rho_{l,m}, e^{im\varphi}\}$ the topological U(1) periodicity. This complexification may make it impossible to separate the functions into products of simpler functions but this kind of separated analysis is not required for modern computing facilities which can return solutions to arbitrarily complex sets of differential equations without ever having to represent them analytically.

In quantum theory, we move up in m which is associated with the zenith angle on the z-direction in the $\hat{S}_z$ operator. Furthermore, the zenith angle in the MCM must be identified with a set of two such angles through which we derive bispinor structure on $\{\vartheta_+, \bar{\vartheta}_+, \vartheta_-, \bar{\vartheta}_-\}$. We might use the concept of double orthogonality to join the changing level of $\aleph$ to the magnetic quantum number m such that the raising and lowering operator on m is associated with a raising (or lowering) operator for the level of $\aleph$. However, not everything must be connected and m and $\hat{\Phi}^j$ could be completely separate.

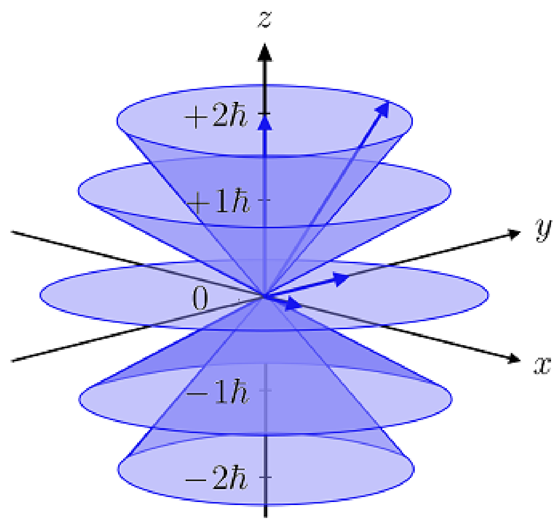

Figure 60: This figure shows the possible angular momentum configuration for $l = 2$. For any l-state, there are $2l + 1$ possible values of m giving the $2 \times 2 + 1 = 5$ possible values shown in this figure, which is taken from Wikipedia.

Gatland's main idea in reference [62] is that it is impossible to have half-integer values of the orbital angular momentum which could be explained with classical mechanics. This is shown in figure 60 and the reader should understand that if half-integer values were allowed then the flat sections of z would be separated by units of

$$\frac{\hbar}{2} = \frac{1}{4\pi} h \quad , \tag{4.129}$$

as opposed to the integer separation shown in figure 60. As it is, there are no such sets of eigenfunctions that have this property so we need to describe our experiments in this limit with eigenspinors. Gatland writes the following about the symmetry of that representation.

> "We now consider the symmetries of the $f_{l,m}$ functions under the transformation $\theta \to \pi - \theta$. Under such a sign transformation $\sin\theta$ is even (no sign change), $\cos\theta$ is odd (changes sign), and $\cot\theta$ is odd. Also, if $g(\theta)$ is even (odd), then $dg/d\theta$ is odd (even). It follows from [*equation (4.126)*] that if $f_{l,m}$ is an even function, then $f_{l,m\pm1}$ is an odd function, and vice versa. But, according to [*equation (4.128)*], the bottom state is even, so the states are alternatively even and odd.
>
> "However, according to [*equation (4.128)*], the top state is also even so there must be an odd number of m states. This requirement in turn implies that l be an integer. The integer angular momentum states are allowed by the symmetry analysis, but the half-integer angular momentum states are forbidden. Half-integer angular momentum states cannot be represented in terms of three-dimensional space functions."

Thus spinors are introduced and we get the complicated $\hat{J} = \hat{L} + \hat{S} + ...$ operators. All of these operators are used to describe the angular momentum of an electron in a hydrogen atom. All systems with many electrons and/or nucleons are solved by methods of approximation based on the beautiful solution for hydrogen. Note well that, in the MCM particle model (section II.8), quarks are like chiros and leptons are like chronos so the electron and three quarks in a hydrogen atom are exactly like $x^0 \cup \{\chi^5_+, \chi^5_\varnothing, \chi^5_-\}$.

Note that the transformation $\theta \to \pi - \theta$ is exactly the operation that will represent decoupling two co-$\hat{\pi}$s generating a U(1) symmetry and then reassembling them such that $\mathcal{H}$ and $\varnothing$ are swapped between the two representations of the MCM unit cell. When we take two coordinates $\theta \in (-\pi/2, \pi/2)$ that are initially the same, and then we send one of them as $\theta \to \pi - \theta$, that will have the effect of reversing the direction of increasing coordinate along that θ. This is exactly what will be required for constructing the various iterations of $t_\pm$. Furthermore, if we change the level of $\aleph$ so that the parameter on each co-$\hat{\pi}$ goes like $\theta \to e^{i\theta}$ then

$$e^{i\theta} \quad \longrightarrow \quad e^{i(\pi-\theta)} = e^{i\pi}e^{-i\theta} = -e^{-i\theta} \quad . \tag{4.130}$$

In this instance, we have before and after objects that essentially transform as $x \to 1/x$ with

$$e^{i\theta} \quad \longrightarrow \quad -\frac{1}{e^{i\theta}} \quad , \tag{4.131}$$

which is the inversion map on $\mathbb{S}^2$

$$\zeta = \frac{1}{\xi} \quad , \tag{4.132}$$

up to a sign. We can get that extra sign anywhere, most directly by considering an initial change $\theta \to \theta - \pi$ instead of $\theta \to \pi - \theta$. In equation (4.131), the ζ and ξ functions ($e^{i\theta}$ and $e^{-i\theta}$) increase in opposite directions around the complex unit circle exactly as we expect t_+ and t_- will. The inversion shown in equation (4.132) is between two coordinate charts on the surface and if we consider another inversion that exchanges the sphere's interior region with its exterior region then the extra minus sign will be obtained as a normal vector pointing from the surface toward the interior or the exterior.

IV.4 Conformalism and Infinity

Penrose gives a definition for conformal infinity in spacetime which begins as follows in reference [52].

> "Let $\tilde{\mathscr{M}}$ denote physical space-time with metric $d\tilde{s}$. The idea is to construct another 'unphysical' manifold $\mathscr{M}$ with a boundary $\mathscr{I}$ and metric ds, such that $\tilde{\mathscr{M}}$ is conformal to the interior of $\mathscr{M}$ with $ds = \Omega d\tilde{s}$, and so that the 'infinity' of $\tilde{\mathscr{M}}$ is represented by the 'finite' hypersurface $\mathscr{I}$ (see [*figure 61*].) This last property

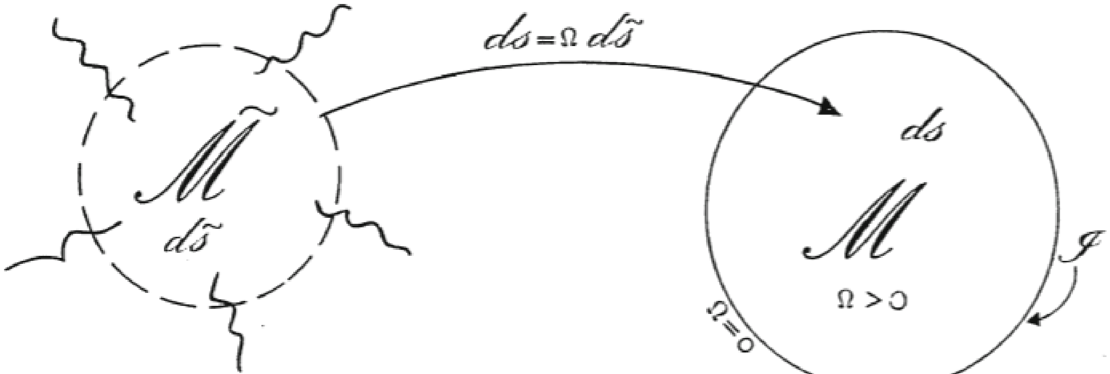

Figure 61: This figure originally appeared in reference [52] where the original caption read, "The infinite physical space-time $\tilde{\mathscr{M}}$ is mapped into an unphysical 'finite' conformally equivalent manifold $\mathscr{M}$, with boundary $\mathscr{I}$ corresponding to the 'infinity' of $\tilde{\mathscr{M}}$." The squiggly lines leaving $\tilde{\mathscr{M}}$ fairly well represent CMB photons coming from the the horizon of the observable universe and the included boundary of $\mathscr{M}$ is like a domain wall in the hypercosmos.

is expressed by the condition that $\Omega = 0$ on $\mathscr{I}$, that is to say, the metric at $\mathscr{I}$ is stretched by an infinite factor in the passage from $\mathscr{M}$ to $\tilde{\mathscr{M}}$ so $\mathscr{I}$ gets mapped to infinity. *Asymptotic* properties of $\tilde{\mathscr{M}}$ and the fields in $\tilde{\mathscr{M}}$ can now be investigated by studying $\mathscr{I}$, and the *local* behavior of the fields at $\mathscr{I}$–provided that all the relevant concepts can be put into conformally invariant form."

Here, will refer to the conformal factor as Ω_{conf} for disambiguation with de Sitter space Ω. When Penrose describes stretching by an infinite factor, he necessarily refers to the changing level of $\aleph$. The hypercosmos is such that the information on the $\hat{\pi}$-site corresponding to $\mathcal{H}$ propagates *beyond* the boundary at conformal infinity to another $\hat{\pi}$-site on a higher level of $\aleph$. The open physical universe that is feasibly amenable to measurement is $\mathcal{H} \equiv \mathscr{M}$. Ω is another unbounded space; its spherical topology means that it does not have a boundary. In $\hat{M}^3$, the boundary is not obtained until projection into the hyperbolic topology of $\aleph$. Therefore, let $\hat{\Phi}$ anchored to $\mathcal{H}$ point to Ω and $\hat{\Phi}$ anchored to Ω point to $\aleph$ such that the level of $\aleph$ increases by two in each application of $\hat{M}^3$. When the metric is stretched by a conformal factor equal to infinity on $\mathscr{I}$, that is a requirement for the metric to be dynamical at the $\hat{\pi}_2$-site. Without getting stretched by at least ∞, the metric would be $g_{\mu\nu} = \text{diag}(0,0,0,0)$ in $\mathcal{H}_2$. Where Penrose completes the physical manifold with a point at infinity, we want to complete it with at least $\mathbb{S}^0$: one point for countable infinity and one for uncountable infinity. Furthermore, we might wish to take analytical continuations of $\mathscr{M}$ into the region beyond $\mathscr{I}$ without venturing into the universe $\mathcal{H}_2$ on the other side. Then we should complete the physical manifold with a linear segment whose endpoints are countable and uncountable infinity. The purpose of $\hat{\Phi}$ is then, first, to point to countable infinity at $\mathscr{I}$, and then from countable to uncountable infinity such that there is a rotation completely denned within the region beyond conformal infinity but before the next $\hat{\pi}$-site. Therefore, when Penrose suggests examining the behavior of fields at $\mathscr{I}$ we might suggest to examine the behavior of the analytical continuation of fields into the region inside conformal infinity. This region should be associated with the cosmological domain walls that separate adjacent universes in the hypercosmos.

In reference [27], Misner, Thorne, and Wheeler write the following.

> "When performing calculations in asymptotically flat spacetime, one often must examine the asymptotic forms of the fields (e.g., the metric, or the curvature tensor, or the electromagnetic field) 'at infinity.' For example, the mass and angular momentum of an isolated system are determined by the asymptotic form of the metric [*sic*]. It is rarely sufficient to examine asymptotic forms near 'spatial infinity.' For example, if one wishes to learn how much mass was carried away by gravitational and electromagnetic waves during a supernova explosion, one must examine the asymptotic form of the metric not just at 'spatial infinity,' but at 'future null infinity."'

What does it mean to be conformal? A map is conformal if it preserves the angles at which lines intersect. If we draw an orthogonal basis in a manifold and use a conformal map to send it to another manifold $\tilde{\mathscr{M}}\mapsto\mathscr{M}$, or $\mathcal{H}\mapsto\Omega\mapsto\aleph\mapsto\mathcal{H}$, or even $\mathcal{H}\hat{\Phi}^j\mapsto\mathcal{H}\hat{\Phi}^{j+2}$ which could be written $\mathcal{H}\mapsto\tilde{\mathcal{H}}$, then the basis will still be orthogonal.[1] An important application of conformal invariance is to preserve the right angle of intersection of the two null intervals $x^1=\pm cx^0$ in a 2D Minkowski diagram of $\mathcal{H}\equiv\mathscr{M}$. Conformal invariance is familiar from special relativity where the conformal deformation of the null interval is always elsewhere from the observer due to the rigidity of the assumed Lorentz frame. In figure 62, we see the two lines intersect at right angles and that is always the topology of the Lorentz frame. Without regard for the motion of an observer, photons will always recede from and arrive at the position of the observer at the speed of light. This requires, at all times, that the light cone centered on the observer is describable with a conformal field theory. This is a property of relativity. The light cone must open at the 90° angle of intersection of two lines whose slopes are $c=\pm1$. In figure 63, we see that all the lines still intersect at right angles but are not everywhere orthogonal. Conformal maps preserve the orthogonality that is the general structure of physical modes that we use to make physical descriptions. The "general structure" referred to is that $\sin(x)$ and $\cos(x)$ are out of phase by $\pi/2$ in the context that the harmonic oscillator potential is approximately the only exactly solvable problem in physics. We can use the Euler formula to add complexity with an imaginary component via the exponential map $\theta\mapsto e^{i\theta}$. This is how we go from the lines in the diagrams to the bulk space. In the O(3,1) topology of $\mathbb{S}^3$ with a phase shifted radial degree of freedom (a 3-ball) we have three real dimensions on the surface of the sphere and then we go into the bulk space with a fourth imaginary dimension (or this can be real and the sphere can be imaginary, as in section II.4.) Without considering a full 3-ball, we can consider the simplified MCM system in figure 64 which shows that the MCM worldlines are sinusoidal. Then we can associate an on shell condition with the sine waves $\sin(x)$ and $\cos(x)=\sin(x-\pi/2)$ and we should associate the exponential plane waves e^{ix} with the bulk region off of the sinusoidal lines. It is precisely the introduction of operators whose commutators are proportional to i that allows one to probe the off shell region in QFT.

[1] The two options for writing $\hat{M}^3$ here, $\mathcal{H}\mapsto\Omega\mapsto\aleph\mapsto\mathcal{H}$ and $\mathcal{H}\hat{\Phi}^j\mapsto\mathcal{H}\hat{\Phi}^k$, reflect what we called the 4D $\hat{\Phi}$ and the 5D $\hat{\Phi}$ in section I.2 (figures 3 and 4.)

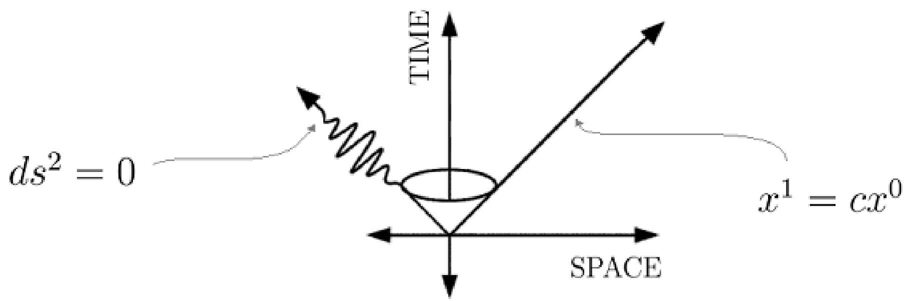

Figure 62: The null interval $ds^2 = 0$ is conformally invariant and that makes conformal field theory important for physics. Length contraction contracts the horizontal axis which is a conformal Lorentz transformation that would make the angle of the light cone acute. However, the Lorentz approximation that defines an inertial frame says the angle is always $\pi/2$ right angle radians. Therefore, the meaning of the word "relativity" is that the slope of the light cone's edges at a point away from the observer depends on an observer's *relative* motion.

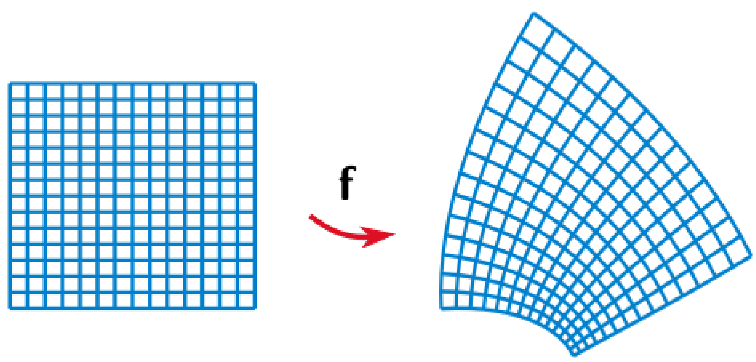

Figure 63: **f** is a conformal map because it preserves the angles of intersection of all lines. However, the distance between the lines, and therefore the angles between them away from their points of intersection, are not preserved in a conformal transformation. This figure is taken from Wikipedia.

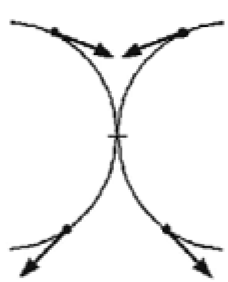

Figure 64: The structure of time wrapped around a cylinder in the MCM is generally described by two sine waves out of phase by π radians.

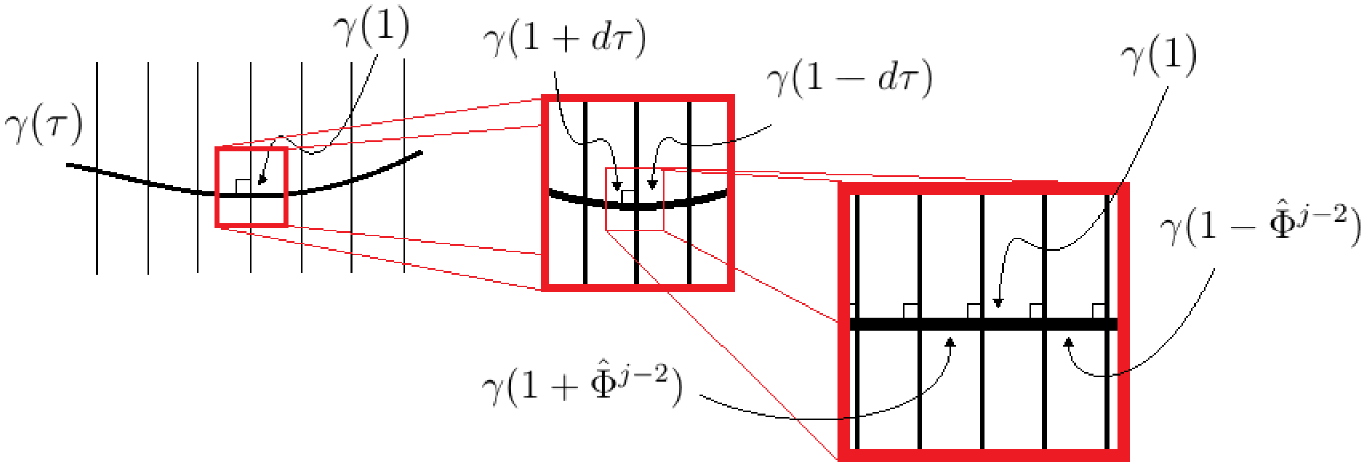

Figure 65: This figure demonstrates that conformal transformations preserve the angles of intersection of lines but only at the point of intersection, not in a local neighborhood of the intersection... unless we consider the hypercomplexly infinitesimal neighborhood.

A good way to understand a conformal map from one manifold to another, the general sense in which we use the word "conformal" in the MCM, is that the conformal manifold can be constructed by smooth deformations of the original manifold. In conformal transformations, the angles of intersections are always preserved but the angle between lines is not preserved everywhere. In figure 63, all the lines intersect at 90 degrees ($\pi/2 \approx \Phi$ radians.) In the conformal manifold produced by $\mathbf{f}$, the angle between the lines is not everywhere $\pi/2$ as it is in the pre-image of $\mathbf{f}$. The concept of local preservation of angles does not mean that there is a finite area around the point of intersection where the angle of intersection has been preserved. In figure 63, one might assume incorrectly that there is a small region in the corner of each box where the lines are still perpendicular. In fact, the lines are only perpendicular at the point of intersection, as in the center of figure 65. Regarding that figure, consider a non-straight line γ parameterized in the deformed manifold with τ. Also consider another set of parallel lines z (a direction) in the conformal manifold such that one of these lines is perpendicular to γ at $\tau = 1$. The fact that the lines are only perpendicular at a point can be extended to the idea in general relativity that a coordinate basis $\hat{e}^\mu$ can only be defined at a point; it is extended throughout an "inertial frame" only in the Lorentz approximation. When $\hat{z}$ is perpendicular to $\gamma(\tau)$ at $\tau = 1$, we must say that, even at the local point $\gamma(1 + d\tau)$, the parallel lines are no longer perpendicular to $\gamma(\tau)$ because it is identically non-straight. This is shown in the center of figure 65. However, in the MCM, we have the freedom to consider another local region around the point of intersection which we call the hypercomplexly infinitesimal neighborhood of any point $\gamma(\tau)$. At that level of zoom, there are points $\gamma(\tau_j \hat{\Phi}^j + \tau_{j-2} \hat{\Phi}^{j-2})$ where the z-direction is perpendicular to γ across a local region described by the parameter τ_{j-2}, as in figure 65 (right.) For this, we use the definitions that

$$\tau \pm d\tau \quad \equiv \quad \tau_j \, \hat{\Phi}^j \pm \hat{\Phi}^{j-1} \ , \qquad \text{and} \qquad \hat{\Phi}^{j-1} \quad \equiv \quad \infty \hat{\Phi}^{j-2} \ . \tag{4.133}$$

Due to the axioms of hypercomplexity, this small region defined with τ_{j-2} on $\hat{\Phi}^{j-2}$ can extend all the way out to any finite value until

$$\gamma(\Phi^j + \infty\hat{\Phi}^{j-2}) \quad \equiv \quad \gamma(\Phi^j + \Phi^{j-1}) \ \ , \tag{4.134}$$

where we use the notation that

$$\gamma(\Phi^j + \Phi^{j-1}) \quad \equiv \quad \gamma(1 + d\tau) \ \ . \tag{4.135}$$

Therefore, infinity on $\hat{\Phi}^j$ is unity on $\hat{\Phi}^{j+1}$. Infinity on $\hat{\Phi}^j$ is like an infinitesimal on $\hat{\Phi}^{j+2}$. In the conformal manifold, we may consider the neighborhood where $\tau_{j-2} \in (-\infty, \infty)$ to be such that the angles between γ and the other lines are everywhere as they were in the original flat manifold at $\gamma(1)$. We see that the parameter τ_{j-2} is allowed to vary across any finite range in the hypercomplexly infinitesimal neighborhood around $\gamma(1)$ because $\tau_{j-1}d\tau$ will always be infinitesimal for finite τ_{j-1}. See reference [8] for a more detailed exposition of the notation in equations (4.133-4.135).

The hypercomplexly infinitesimal neighborhood will be very important if we are to describe the entire universe as a single quantum of spacetime with language that usually describes atoms in a lattice. This notation allows us to resolve a continuum inside the quantum which is another example of that to which Finkelstein referred to as "going to the big end." Building a lattice with the requisite non-commutative properties, likely one that implies that the reciprocal lattice of the cosmological reciprocal lattice is not the original cosmological lattice itself, will be a significant technical feat. However, we have made use of the orthogonal intersection of parallels, meridians, and hypermeridians on $\mathbb{S}^3$ to set the relative orientations of the flat, spherical, and hyperbolic spaces of the MCM unit cell so we have a lot of guidance for what the lattice might look like once we make the inevitable conversion from the rectangular coordinates that are easy to draw to the spherical coordinates which are not so easy to represent without specialized software. Conformal invariance guarantees that parallels, meridians, and hypermeridians will always intersect at the same angle of perfect orthogonality and, therefore, the relevant features of the MCM unit cell will be preserved even if we represent them with two cubes $\Sigma^{\pm}$. Notably, we will never be able to cover the rectangular representation of the unit cell with a conformal deformation of the 3-spherical representation because a conformal transformation will never be able to squeeze all the way into the corners of the MCM unit cell as it appears in this book.

In reference [52], Penrose gives the definitions of conformal invariance of field equations.

> "The utility of [*conformal infinity*] rests on the fact that the zero rest-mass free-field equations for each spin value are conformally invariant if interpreted suitably. For example, for spin zero, if the wave equation is written as
>
> $$\left\{ \nabla_\mu \nabla^\mu + \frac{R}{6} \right\} \phi = 0 \tag{4.136}$$

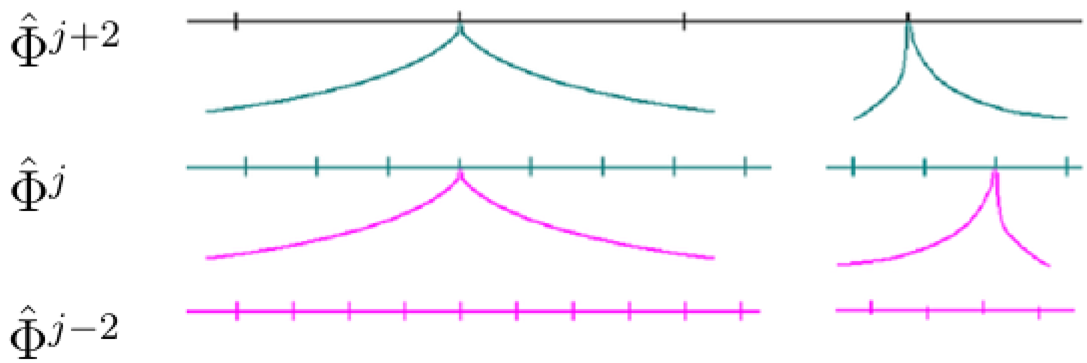

Figure 66: This figure shows tiers of infinitude as $\{\hat{\Phi}^{j+2}, \hat{\Phi}^{j}, \hat{\Phi}^{j-2}\}$. When this figure appeared in chapter one, there was in intuitive idea that this should show three adjacent levels of $\aleph$ but clearly it does not. This shows the entire infinite expanse of the central level being like a point in the upper level so it necessarily skips the intermediate level. A finite element on $\hat{\Phi}^{j+1}$ is like a point on $\hat{\Phi}^{j+2}$ or the entire real line on $\hat{\Phi}^{j}$.

where R is the scalar curvature and ∇_μ denotes the covariant derivative—both according to the metric $g_{\mu\nu}$ of $\mathscr{M}$, then

$$\left\{\tilde{\nabla}_\mu \tilde{\nabla}^\mu + \frac{\tilde{R}}{6}\right\} \tilde{\phi} = 0 \tag{4.137}$$

where $\tilde{\nabla}_\mu$, $\tilde{R}$ refer to the metric $\tilde{g}_{\mu\nu} = \Omega^{-2} g_{\mu\nu}$ of $\tilde{\mathscr{M}}$ and where

$$\tilde{\phi} = \Omega\phi \quad . \tag{4.138}$$

For spin 1 we have Maxwell's free-field equations

$$\nabla^\mu F_{\mu\nu} = 0 \quad , \qquad \nabla_{[\lambda} F_{\mu\nu]} = 0 \tag{4.139}$$

with $F_{\mu\nu} = F_{[\mu\nu]}$. The tilde version of this holds if we put

$$\tilde{F}_{\mu\nu} = F_{\mu\nu} \quad . \tag{4.140}$$

For spin 2, we can use a tensor $K_{\mu\nu\rho\sigma}$ with the symmetries

$$K_{\mu\nu\rho\sigma} = K_{[\rho\sigma][\mu\nu]} \quad , \qquad K_{\mu[\nu\rho\sigma]} = 0 \quad , \qquad K^\mu{}_{\nu\rho\sigma} = 0 \tag{4.141}$$

satisfying

$$\nabla_{[\gamma} K_{\mu\nu]\rho\sigma} = 0 \tag{4.142}$$

or equivalently $\nabla^\mu K_{\mu\nu\rho\sigma} = 0$. Conformal invariance is achieved if

$$\tilde{K}_{\mu\nu\rho\sigma} = \Omega^{-1} K_{\mu\nu\rho\sigma} \quad .\text{”} \tag{4.143}$$

If the conformal factor Ω_{conf} is a constant then equation (4.138) is exactly what we have shown in section II.2 regarding the "size" of the coordinates at one lattice site compared to another. Stretching and shrinking are absolutely conformal transformations. Tensor transformations are conformal; the coefficients of proportionality in the tensor transformation law can be labeled Ω_{conf}. Also note that equations (4.136-4.137) are the wave equation so ϕ cannot be a quantum mechanical wavefunction. Wavefunctions satisfy the heat equation. (The Schrödinger equation is a heat equation and that is why the probability density diffuses like heat.) The zero mass Klein–Gordon equation in flat space is

$$\nabla_\mu \nabla^\mu \phi = 0 \quad , \tag{4.144}$$

and it generalizes to curved space as

$$\left(\nabla_\mu \nabla^\mu + \zeta R\right) \phi = 0 \quad , \tag{4.145}$$

where the Ricci scalar R is a mass analogue term. The equation is conformally invariant when $\zeta = 1/6$, meaning that equations (4.136-4.137) would have different values ζ and $\tilde{\zeta}$ for any place in parameter space other than $\zeta = 1/6$.

Penrose gives the definition $ds = \Omega_{\text{conf}}\, d\tilde{s}$ which implies $ds^2 = \Omega^2_{\text{conf}}\, d\tilde{s}^2$ and that is mirrored in Penrose's later definition $\tilde{g}_{\mu\nu} = \Omega^{-2}_{\text{conf}}\, g_{\mu\nu}$. If we reduce equations (4.136-4.137) to 1D so that ϕ describes, say, the displacement of a vibrating string then it is very easy to understand equation (4.138). It says that, after mapping the string to a conformal string in a conformal manifold, the amplitude of the vibration on the string is scaled by the linear conformal factor. In the case of a string, ϕ describes the transverse displacement so it is a distance. Obviously it follows that the metric will be scaled by Ω^2_{conf} because it depends on the distance as $dx^\mu dx^\nu$. The displacement field of a string is a good example of a spin-0 field. The string has no inherent angular momentum and there is no such thing as a polarized classical string vibrating in the plane.

We have included Penrose's comments about spin fields because we can make a nice connection to the spin-1 field specifically but, first, we will say a little more about spin and spinors to complement a few concepts developed earlier in this book. The simplest representation of non-classical spin is that for spin-1/2. The Pauli spin matrices are 2D so, to connect with Hilbert space, we need to make the change $\mathcal{H}' \to \mathcal{H}' \otimes \mathbb{C}^2$. A $\mathbb{C}^2$ number is a 1×2 column array and those connect to 2D matrices via the rules of matrix algebra. For spin-1, the spin matrices are 3D and we have a corresponding state space $\mathcal{H}' \to \mathcal{H}' \otimes \mathbb{C}^3$. Spin-3/2 matrices are 4D and there is a corresponding state space, spin-2 matrices are 5D, *etc.* In the MCM, we have proposed to construct all these spaces with the product of $\mathcal{H}$ and the neighboring dimensions in the unit cell. For instance, to construct the space of spin-1/2 wavefunctions [11], we have modified the state space as

$$\mathcal{H}' \quad \to \quad \mathcal{H}' \otimes \chi^5_+ \otimes \chi^5_- \quad . \tag{4.146}$$

$\chi^5_\pm$ are separated by $\chi^5_\varnothing$ so we can assign one $\mathbb{C}$ number to each of $\Sigma^\pm$ to generate $\mathbb{C}^2$. Then "spin is weird" because $\Sigma^\pm$ are disconnected on the same level of $\aleph$ but Σ^+ is connected to Σ^- on the higher level by the Cauchy C curve. The full MCM unit cell includes the surfaces $\{\aleph, \mathcal{H}, \Omega\}$ whose coordinates are $\{x^\mu_-, x^\mu, x^\mu_+\}$ and we can construct the $\mathbb{C}^3$ needed for spin-1 matrix algebra with

$$\mathcal{H}' \quad \to \quad \mathcal{H}' \otimes x^0_+ \otimes x^0 \otimes x^0_- \quad . \tag{4.147}$$

Here, we generate $\mathbb{C}^3$ with three functions that take a point from each time axis and return a $\mathbb{C}$ number. Therefore spin-1 can be described with three numbers in three disconnected spaces $\{\aleph, \mathcal{H}, \Omega\}$.

The abstract vector space $\mathcal{H}'$ is modified for spinors by adding an n-tuple from $\mathbb{C}$ and this must have a corresponding modification in the position space representation. We have shown that half integer spin states cannot be accommodated with simple functions of position space. To get the corresponding object for the position space representation of $\psi \in \mathcal{H}' \otimes \mathbb{C}^2$, we would need to add a function of χ^5_+ that returns a complex number and another function of χ^5_- that does the same. For the spin-1 spinor, we would take three functions of the three expansion dimensions shown in equation (4.147). There are no fundamental particles with spin-3/2 so we can ignore that case and progress to the spin-2 graviton which is sometimes thought to exist. Gravitons have never been detected and no one has ever developed a consistent mathematical framework for them. In the MCM, the space for spin-2 would have to be constructed as

$$\mathcal{H}' \quad \to \quad \mathcal{H}' \otimes t^{\{j+1\}}_- \otimes t^{\{j\}}_+ \otimes t^{\{j\}}_\star \otimes t^{\{j\}}_- \otimes t^{\{j-1\}}_+ \quad . \tag{4.148}$$

This construction spans three levels of $\aleph$ and cannot exist within a single unit cell so the MCM prediction, originally, was that there are no spin-2 gravitons. This aligns well with the MCM's alternative approach to quantum gravity: we have made the quantum sector classical instead of seeking to quantize the classical sector. For the case of half-integer spin, it is provably impossible to make the quantum sector classical but we have developed a workaround with piecewise geometry. The graviton is expected to be the quantum of the gravitational field but there is no gravitational field in general relativity and that further corroborates the MCM prediction that there are no spin-2 gravitons. How could there be a quantized gravitational field when there is no classical gravitational field to begin with? However, there may well be physical spin-2 gravitons in Nature and, now that we have introduced $\hat{2}$ and $\varnothing$, we might reformulate definition (4.148) as

$$\mathcal{H}' \quad \to \quad \mathcal{H}' \otimes t^{\{j\}}_{\varnothing+} \otimes t^{\{j\}}_+ \otimes t^{\{j\}}_\star \otimes t^{\{j\}}_- \otimes t^{\{j\}}_{\varnothing-} \quad . \tag{4.149}$$

which would exist on a single level of $\aleph$, as in figure 67.

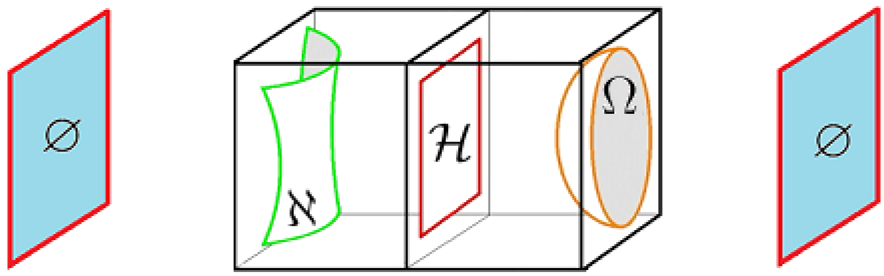

Figure 67: We have previously cited a problem with the physical interpretation for the extension of the MCM spin scheme to spin-2. The scheme in this figure, which includes $\varnothing$, assuages the interpretive problem so spin-2 fundamental MCM particles should not be ruled out on interpretive grounds alone.

We should further note that the electromagnetic field strength tensor $F_{\mu\nu}$ is conformally invariant between $\mathscr{M}$ and $\tilde{\mathscr{M}}$ without any reliance on Ω_{conf}. The relativistically invariant Lagrangian for electromagnetism is

$$\mathcal{L}_{EM} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad . \tag{4.150}$$

We have placed the $\hat{\pi}$-site in the cosmological lattice such that it corresponds to $\mathcal{H}$ which is the classical domain of electromagnetism and the coefficient of $\hat{\pi}$ in

$$\hat{1} = \frac{1}{4\pi}\,\hat{\pi} - \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{\imath} \quad , \tag{4.151}$$

is the dimensionless electromagnetic coupling constant $1/4\pi$. Therefore, this gives us a lot of guidance about how to arrange spin fields in the lattice constructed from $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ but that will be beyond the scope of this section. It suffices to say that we have taken $\hat{\pi} \in \mathcal{H}$ as the object of "old physics" and the MCM unit cell centered on $\mathcal{H}$ has a prominent feature $\{\aleph, \mathcal{H}, \Omega\}$ which will support the $\mathbb{C}^3$ numbers needed for the spin-1 matrix algebra of the electromagnetic force carrier. The masslessness constraint on the photon will likely provide further guidance in a full analysis of all physical spin fields whose conformal properties are described in reference [52]. An alternative formulation of $\mathcal{L}_{EM}$ in the ontological formalism is

$$\hat{\mathcal{L}}_{EM} = \frac{1}{4\pi} F_{\mu\nu} F^{\mu\nu}\,\hat{\pi} \quad , \tag{4.152}$$

which shows there is likely some deep ontological connection between the inherent conformal invariance of $F_{\mu\nu}$ and the energy function $\mathcal{L}_{EM}$. Furthermore, if we take only the $\hat{\pi}$-site as the sector of absolute conformal invariance then we might associate the 1/4 pre-factor of $\mathcal{L}_{EM}$ to

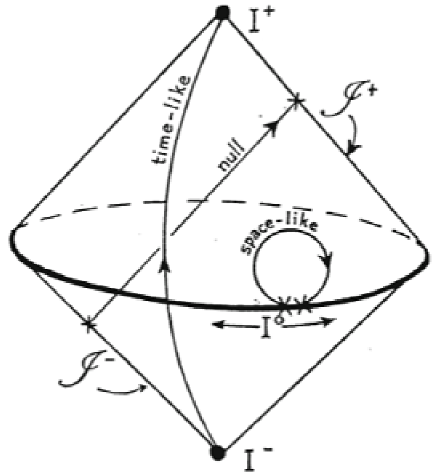

Figure 68: This figure is taken from reference [52]. The finite universe $\tilde{\mathscr{M}}$ is conformal to the region between the two elements of $\mathscr{M}$ labeled $\mathscr{I}^+$ and $\mathscr{I}^-$.

one fourth of the total energy, where energy is apparently conserved because we only consider one fourth of $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$. Unitarity is associated with probability but it is also associated with conservation of energy. Therefore, when we extend the theory into the non-unitary sector, perhaps the $\hat{\pi}$ component still conserves energy. It is possibly mere coincidence but still worth noting that when we take the ontological resolution of identity with the formula

$$\hat{e}_\alpha = |\hat{e}_\alpha|\big|\hat{e}_\alpha\big\rangle \ , \qquad \text{then} \qquad \hat{1} = \sum_{\alpha=1}^{4} \frac{1}{4}\big|\hat{e}_\alpha\big\rangle \ , \tag{4.153}$$

has the same coefficient as the electromagnetic energy function $\mathcal{L}_{EM}$. In this case, the projection operator $\hat{\mathcal{P}}_\pi$ would select the sector in which energy is conserved.

Regarding conformal infinity itself, and specifically figure 68, reference [52] goes on as follows.

> "The meaning of I^-, $\mathscr{I}^-$, I^0, $\mathscr{I}^+$, I^+, can be seen by considering the behaviour of curves in $\mathscr{M}$ corresponding to the straight lines in $\tilde{\mathscr{M}}$ [*which is the finite manifold not including points at infinity.*] The image of a time-like straight line originates at I^- and terminates at I^+; the image of a space-like straight line originates and terminates at I^0; the image of a null straight line originates at a point of $\mathscr{I}^-$ and terminates at a point of $\mathscr{I}^+$. Thus, at I^- represents past infinity; I^0 represents spatial infinity; I^+ represents future *null* infinity. We expect, therefore, that zero rest-mass fields are to be significant at $\mathscr{I}^-$ and $\mathscr{I}^+$, but that fields of finite rest-mass are important at I^- and I^+ and not at $\mathscr{I}^-$ and $\mathscr{I}^+$."

For the purposes of the MCM, we need arrange $\{\mathrm{I}^-, \mathscr{I}^-, \mathrm{I}^0, \mathscr{I}^+, \mathrm{I}^+\}$ in the unit cell but the full conformal analysis goes far beyond the general relevance of the MCM, and the rectangular representation is not well suited to conformalism at infinity. We have put future

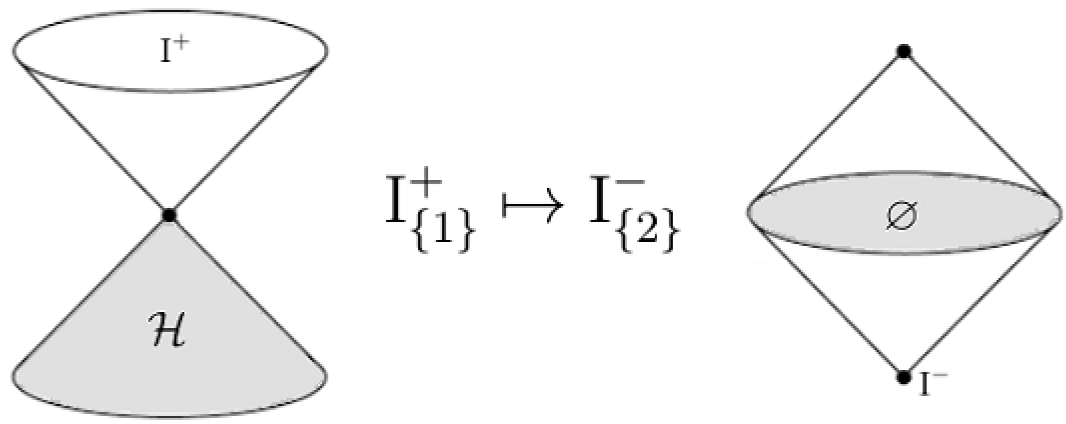

Figure 69: To devise an analytic continuation beyond infinity we must map future timelike infinity in the physical manifold to past timelike infinity in the conformal manifold. Here the 2π radians around I^+ are condensed to a point so in some sense the continuation onto the higher level of $\aleph$ is like the inverse of the Hopf fibration. To construct the Hopf fibration, each point in $\mathbb{S}^2$ "fibrates" a circle of 2π radians and all the circles together form $\mathbb{S}^3$ which the topology of all of space.

and past timelike infinity on Ω and $\aleph$ respectively so we should at least assume that I^+ is a point in Ω and I^- is a point in $\aleph$. Zero rest-mass fields are relevant because dark energy is observed through the photon field and figure 68 looks exactly like the topological piece we will need to continue the topology of the hypercosmos beyond the future timelike infinity of a given universe, as in figure 69. In $\mathcal{H}\equiv\tilde{\mathscr{M}}$, future timelike infinity I^+ is, in the low dimensional representation, a disc of infinite radius that closes the future light cone at $x^0=\infty$. The map from I^+ to I^- requires that the level of $\aleph$ change by two so that the infinite radius, first, becomes finite with one increment of $\aleph$, and then becomes infinitesimal for smooth integration with the representation centered on $\varnothing$. When the circle of 2π radians on $\hat{\Phi}^1$ becomes a point on $\hat{\Phi}^2$, that is like a contraction of the fibers in the Hopf fibration such that $\mathbb{S}^2$ is recovered from $\mathbb{S}^3$.

Consider the 4D Lorentz frame represented in Cartesian spacetime coordinates and assume that the topology of the Lorentz frame is sufficient for a model of the physical cosmos $\tilde{\mathscr{M}}$. Further consider the geometry of the light cones on the left in figure 69. Within the MCM, we have encoded the past on $\hat{\varphi}$ so that the interior of the past light cone is actually $\bar{U}$. This is an example of topological modularization because, in the direct, non-modular topology, the past light cone is filled with U. We are able to make this change because the timelike region inside the past light cone, be it U or $\bar{U}$, will never be intermingled with the region outside the cone. Special relativity makes a stark distinction between the timelike region inside the light cone and the spacelike region outside. When the spacetime interval associated with a motion is

$$\Delta s^2=-\big(c\Delta t\big)^2+\Delta x^2+\Delta y^2+\Delta z^2 \quad , \tag{4.154}$$

the following definitions tell where x^μ_{final} lies in relation to the light cone structure whose apex was at x^μ_{initial}. We have

$$\Delta s^2 > 0 \qquad \Longrightarrow \qquad \text{Spacelike separation} \tag{4.155}$$

$$\Delta s^2 = 0 \qquad \Longrightarrow \qquad \text{Lightlike separation} \tag{4.156}$$

$$\Delta s^2 < 0 \qquad \Longrightarrow \qquad \text{Timelike separation} \quad . \tag{4.157}$$

Here, we briefly consider the global synergies of the above concepts of conformalism in the MCM. The spacelike interval is separated from the timelike interval by the null interval which is conformally invariant. The field strength tensor $F_{\mu\nu}$ does not need to be rescaled in the conformal manifold because the force carrier of electromagnetism, the photon, only moves between spacetime points separated by $\Delta s^2 = 0$. Conformal invariance of the gauge theory means that, no matter how we scale[1] the objects in the theory, the overall picture of physics will not break. We can apply conformal transformations to the objects in any region of the theory and we know that the null interval will always be a topological obstruction between the spacelike and timelike regions. This allows us to say, "$\bar{U}$ is inside the past light cone." If some events have a given separation in $\tilde{\mathscr{M}}$, as in equations (4.155-4.157), then they will have that same separation in $\mathscr{M}$. Furthermore, we have associated the timeless spacetime slice of a position representation $\mathcal{H}$ with the past light cone where $ds^2 = 0$ already defines the surface of conformal invariance. Even furthermore, we have chosen $\mathcal{H}$ as a topological obstruction in the MCM unit cell, and the null interval is already a topological obstruction between the spacelike and timelike regions. Still further, the coefficient in the non-unitary case of $\hat{\pi}$ is the dimensionless electromagnetic coupling constant $1/4\pi$, the topology of the electromagnetic sector of the standard model is U(1), U(1) is the topology of a circle, and π is sometimes known as the circle number. Thus $\hat{\pi}$ is very well suited to the role it plays in the MCM.

Moving on, consider a pair of cones pointing in the direction of space, to the side of the past and future light cones in the Minkowski diagram. The cones have circular openings and we cannot fill all of space with four conical solids. Even if we go from the Minkowski diagram to the 3D version with two spacelike dimensions, and consider six conical solids, two with a shared axis in the direction perpendicular to the plane of the page, we will not fill all of space. The same in 4D, we can never fill all of spacetime with a finite number of conical solids. In 4D, there is no analogue of the inside of the light cone that could be considered a complementary conical element such that the two topologies could be joined on the null region to form O(3,1). We must take the conical interior region with a toroidal exterior region. Therefore, the topology of spacelike infinity is different than the topology of timelike infinity, and that topological incongruity is *not* built into the topology of the Lorentz frame. The difference between spacelike and timelike infinity in spacetime is most obvious in the notation O(3,1). Timelike infinity only has one dimension but spacelike infinity has three.

[1]Regarding gauge theory, "scale" is a better translation than "gauge" of the original German word "eich." The word was popular in Germany around the time gauge theory was being discovered because they were using different gauges of train rail to build different kinds of transportation networks. Scale is a better word to describe how the steel I-beam can be made bigger to support a heavier train. "Scale theory" is what people should say instead of "gauge theory."

Furthermore, Penrose had relied upon null infinity in his framework for conformalism so we might later consider that the path around the Cauchy C curve in 4D spacetime (or 5D) could be much more complex than the simple semicircle we have used to describe it thus far. We have considered that all of spacetime cannot be filled with 4D right hyperconical sections and we have also considered that a conformal transformation can never stretch a spherical or conical manifold (radial manifold) all the way into the corners of a rectilinear Cartesian manifold. There is no conformalism between a circle and a square because the square's corners intersect at $\pi/2$ and any conformal transformation would preserve that angle. A similar example would be the Fourier series approximation to a step function; there is no number of waves considered in superposition that will generate a true step function. We can get very close, but it will always be an approximation.

It is known that the spacelike region created by removing a pair of timelike hypercones from 4D spacetime can never be equivalent to a rectangular manifold but in reference [46] wherein Arnowitt, Deser, and Misner prove that the total energy of the universe is a positive definite quantity, the authors rely on a rectangular boundary at spacelike infinity. To discuss reference [46], we should discuss its relevance to the modified cosmological model, and also to David Finkelstein. During the short period of time between the attempted November 2011 publication of reference [7] and this writer's December 2011 expulsion from Georgia Tech, reference [7] was submitted for peer review. Among the very many journals that received MCM manuscripts in 2011 and 2012, something approaching proper peer review was only granted at one of them. All the others said that the work was "inappropriate" or issued no response at all. After speaking with David Finkelstein, who did in person extend this writer's research a warm regard, this writer was complaining to Andrew Zangwill that the normal publication process was not working as expected for the modified cosmological model. Zangwill assured this writer that if reference [7] was submitted to IJTPD, it would absolutely be granted ordinary review by a subject matter expert. This writer did submit the manuscript to IJTPD and it was accepted for review. Soon afterward, this writer received the reviewer's comments which may be paraphrased as, "The author knows nothing, and certainly nothing about the ADM mass-energy." The reviewer's criticism of the MCM was focused on the idea to have two universes with oppositely signed p^0 in their 4-momenta traveling in different directions through time. The energy of the universe with $p^0 < 0$ would, by construction, be negative and that violates the theorem which appears in reference [46]. This writer responded to the reviewer's criticism by pointing out some fine nuance related to the definition of the surface at spacelike infinity but received no response and the manuscript was soon removed from the online submission system. Shortly after, this writer noticed on Finkelstein's Georgia Tech faculty web page that he had been the editor of IJTPD for decades so we presume that Finkelstein was the reviewer. This writer takes his experience with IJTPD as representative of peer review in general. In person, Finkelstein agreed that the manuscript had some good points and there was a dialogue consisting of criticisms and responses. However, when Finkelstein donned his anonymous reviewer mask, the manuscript had no redeeming value whatsoever because of the "total ignorance" regarding, among other things, the not highly relevant work of Arnowitt, Deser, and Misner. When it was demonstrated in rebuttal that the issue raised by Finkelstein (presumably) was not problematic at all, IJTPD simply chose to end the communication and ignore this writer's valid point. The manuscript was removed from consideration for publication at IJTPD very soon thereafter.

One issue raised here with the ADM theorem is that the volume element bounded by the surface whose differential element is

$$dS_i = \frac{1}{2}\,\epsilon_{ijk}\,dx^j dx^k \quad , \tag{4.158}$$

cannot possibly be the entire spacelike region because the number of topological corners disagrees with the conical representation. However, the small contribution of the small sliver of extra volume that would solve the problem would have no effect on the theorem's validity so it is irrelevant to Finkelstein's criticisms, or possibly the criticisms of Finkelstein's former coworker. In response to the ADM positive definiteness theorem [46], we have proposed in reference [3] to add complexity to the surface element at spacelike infinity by defining

$$d\hat{S}_i = \begin{pmatrix} 0 & \frac{1}{2}\epsilon_{ijk}dx^j dx^k \\ -\frac{1}{2}\epsilon_{ijk}dx^j dx^k & 0 \end{pmatrix} \quad , \tag{4.159}$$

but there are very many other proofs of the positive definiteness theorem for the total energy of the universe in the modern literature. Without focusing on any specific detail, we may refute all of them in one swoop if we introduce a convention such that binding energy is positive in the universe where time flows in reverse, end of story. However, it should be emphasized that reversing the normal vector on the surface at spacelike infinity such that

$$dS_i = -\frac{1}{2}\,\epsilon_{ijk}\,dx^j dx^k \quad , \tag{4.160}$$

is allowed and almost exactly equivalent to reversing time so that t increases in the direction toward $-\infty$. By right-handed convention, we choose, under ordinary circumstances, the unit normal in equation (4.158) but in the MCM we take the convention that gives positive energy in U and negative energy in $\bar{U}$. The unit normal vector is defined according to handedness of the axes so, when we swap $+t$ for $-t$, we are well motivated to select the differential element of area in equation (4.160). However, if we simply say that binding energy is positive inside the universe that moves along t_- then we need not consider the unit normal vector on the surface at spacelike infinity at all.

Here, we begin to get to one of the most important points made in this book: deterministic evolution is not, in general, a conformal transformation. In quantum mechanics, there is an idea to generate spin from very many infinitesimal rotations and here we will show that evolution cannot be generated from very many small conformal transformations. In the 1940s, Dirac and Feynman (and very many others) each noted the shortcomings of their own theories relating to the extension of Maxwell's theory into the quantum realm. Neither Dirac nor Feynman mentioned that, even among all the infinite loops of quantum field theory, no one has a mechanism through which to break or reconnect field lines. This shows that the problem has been considered historically so impossible that prominent physicists were not even talking it about it any more almost a century after Maxwell formulated electromagnetism. Maxwell saw that light was a propagating electromagnetic wave but his much lauded theory did not describe how such waves can detach from sources, as in figure 70. No one had solved the problem by the time of Dirac and Feynman and it remains officially unsolved. Nevertheless, Maxwell's theory is wildly successful despite this major interpretive deficiency. Again, we do not criticize the deficiency, but rather we want to emphasize that first ideas appear, and later they are polished, and both stages of that process should be recorded in the literature. This process of postulation and refinement was normal in the days of Maxwell, Dirac, and Feynman, but it is treated abnormally in the present day. The breaking of field lines looks exactly like figure 71. Demonstrating the broad application of the principle, we can see a similar feature to figure 70 in simulations of quark-gluon plasma, as in figure 72, and in accelerations of non-Newtonian fluids, as in figure 73. All of the complexity in simulations of the generalized dynamical process can be described via the bifurcation of one sheet into two or the joining of two sheets into one which is exactly what happens to the MCM topology every time the trajectory along the χ^5 direction is computed.

Figures 72 and 73 show many points in a manifold where field lines "bloop" together or bloop apart. These figures are demonstrative of all physical systems because the physical vacuum fluctuates in this manner at all times. In reference [10], we suggested that $\hat{\Phi}$ might point to exceptional points that we will refer to here as the apex of blooping. However, the physical fields do not treat these points as exceptional points; they are completely mundane and any point in a field can at any time become the apex for a given blooping apart or blooping together. These reconnections and disconnections can happen anywhere and, in reference [10], we showed that new frequencies can appear at any point along the real line during the frequency doubling cascade into chaos. The applications are different — the blooping of field lines or the blooping of a new frequency mode into or out of existence — but the topology of a transition through a saddle point is the same unexpected behavior when the expectation is that the dynamical processes would slide away from the critical blooping point exponentially quickly.

In the MCM language, we can define a field theory completely within the infinite extent of the hypercomplexly infinitesimal neighborhood around the point of unstable equilibrium at the "apex" of a dynamical saddle. We refer to the bloop as a saddle point because as a system oscillates its field lines should be squeezed together or rarefied but then, according to determinism, return to their original arrangement instead of crossing and breaking, or breaking and joining. A saddle point describes an unstable equilibrium and the "x-point" in figure 70 is exactly a saddle point. We expect that physical trajectories will come arbitrarily close to it and then veer away from it in the downhill saddle direction. However, in the case of blooping, we have irrefutable evidence that trajectories do reach these points and there is not a well established concept in physics for what happens when they get there. It is easy to envision the topological bloops in figures 72 and 73 as two spheres coming together or separating at a point, and the non-trivial parameter χ^5 is already organized so that the three cases in figure 71 are the unique properties of $\{\aleph,\mathcal{H},\Omega\}$. The central part of figure 71 is a singular geometry and must be associated with $\chi^5=0$ or $\chi^5=\pm\infty$ while hyperboloids of one sheet or two are non-singular and can be associated with a range of values for $\chi^5_\pm$. Hyperboloids of one sheet have a negative hyperboloid parameter and hyperboloids of two sheets have a positive one.

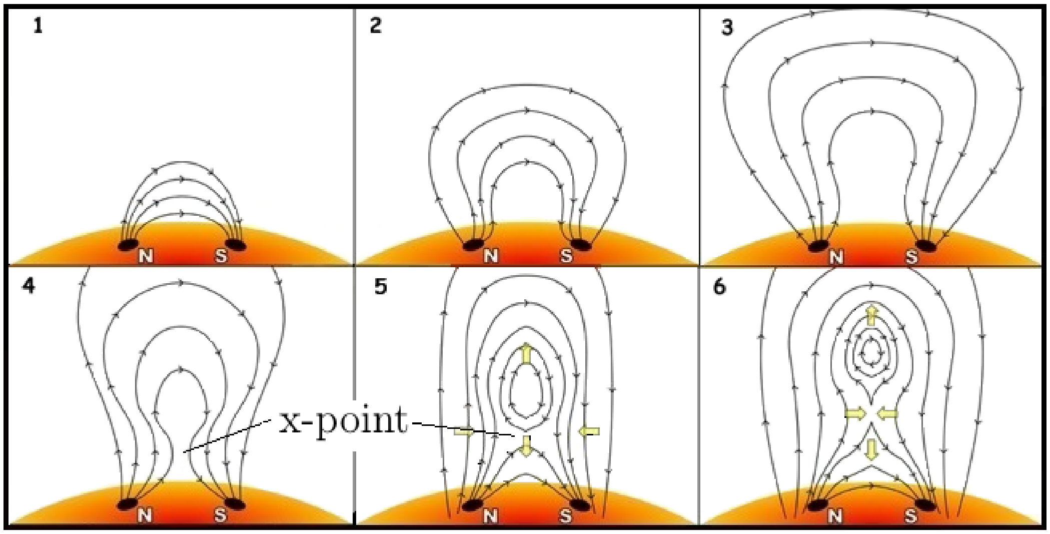

Figure 70: The angle of intersection of the field lines between panes four and five, the so called x-point, is not preserved throughout the field evolution so the evolution operator cannot be constructed from conformal transformation operators. This figure is adapted from one found on the website of the Department of Astronomy of the University of Wisconsin.

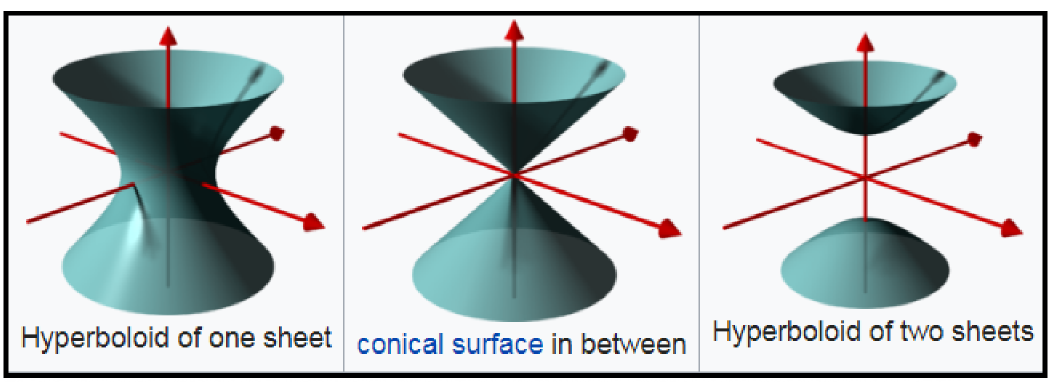

Figure 71: The hyperboloid associated with negative curvature is cohesive but the hyperboloid of positive curvature has two discrete topological elements. This mechanism will be useful in attempts to describe the breaking and reconnection of classical electromagnetic field lines, and also the splitting of $t_\star$ on even levels of $\aleph$ into $t_\pm$ on odd levels. In the center, we see the light cone structure that is already present in $\mathcal{H}$ even without the MCM. It it is unsurprising that the light cone structure will be replicated on $\mathcal{H}$ via the $\chi^5 = 0$ curvature parameter. This figure is taken from Wikipedia.

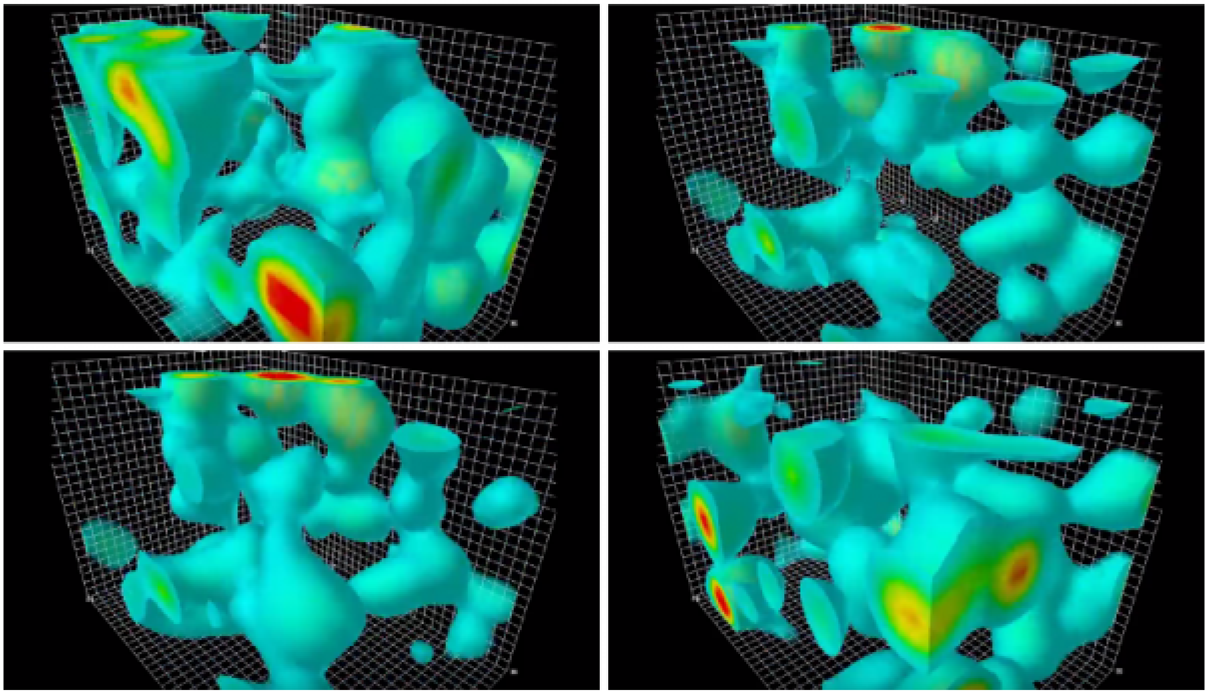

Figure 72: Simulations of quark-gluon plasma show smooth deformations of hyperboloidal perturbations.

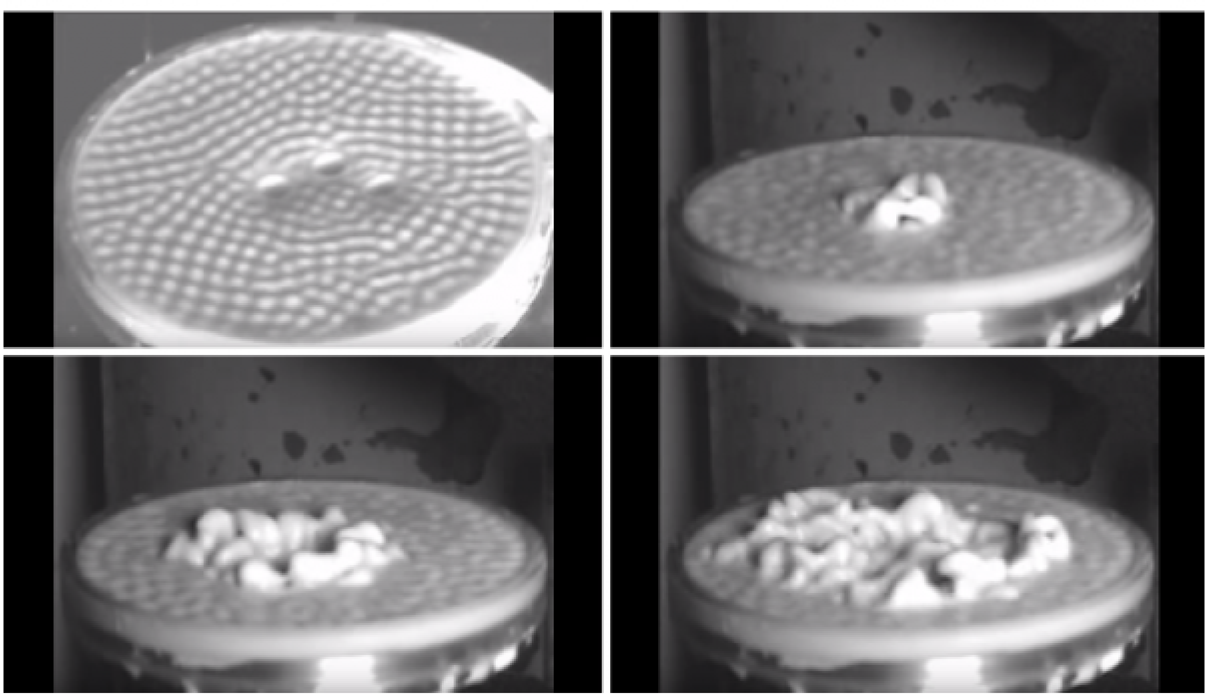

Figure 73: The bloop effect can be seen in experiments such as the one in this picture where experimenters from the University of Texas at Austin applied extraordinary mechanical accelerations to non-Newtonian fluids. The fluid in this figure is corn starch.

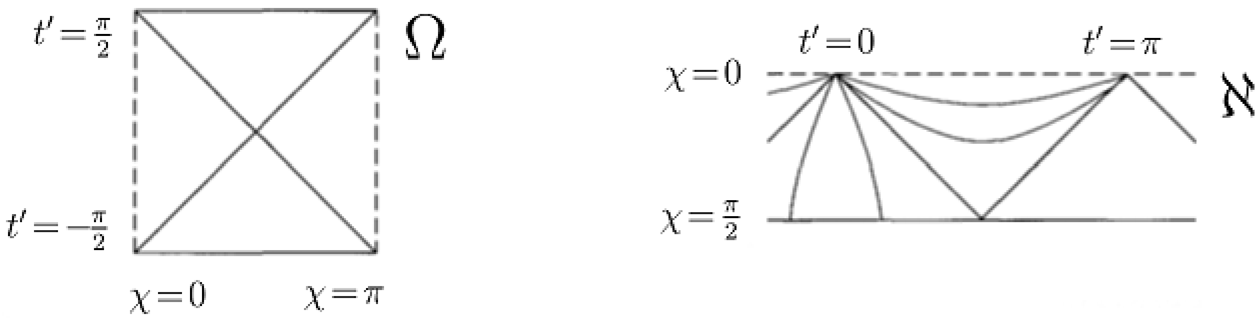

Figure 74: This figure contains conformal diagrams of de Sitter space Ω and anti-de Sitter space $\aleph$.

Figure 70 encapsulates the argument we will make for why evolution is non-conformal. At the apex of the bloop, the field lines intersect, and conformal transformations preserve the angle of intersection, but the lines only intersect at that one special apex point. Therefore, if we have an evolution operator whose effect is to take one state and return another state that could have been obtained by a conformal transformation of the initial state, we should conclude that there is some problem with the evolution operator. In the MCM, we have smooth conformal variance across Σ^+ and Σ^- individually but there is no smooth variation that will connect Σ^+ to Σ^-. Even when we smoothly deform the manifold to a de Sitter singularity of infinite curvature at $\chi_+^5 = \infty$, if we continued to conformally transform the Σ^+ manifold in any way, we would never obtain the AdS manifold that we need on the other side of Σ^+ at $\chi_-^5 = -\infty$. A conformal transformation of de Sitter space will never result in an anti-de Sitter space but this is required for the operation that propagates information across the unit cell. We need to add a non-conformal piece where the level of $\aleph$ changes. Furthermore, we have proposed to use $\hat{\Phi}$ as the object that points the apex of blooping and we already have it pointing from $\mathcal{H}$ to $\varnothing$, which we have now placed at the apex of the bloop. To get the change of topology from dS to AdS, we need a mechanism that will change the sign of the fifth position of the metric.

In section IV.2, we pointed out that Tipler has considered just such a changing sign near a rapidly rotating infinite cylinder but, in section II.1, we excerpted Carroll [17] who wrote that putting the metric into canonical form with conformal transformations can never change the signature of the metric. Any conformal transformation is allowed to deform the metric as required, but if none of them can ever change the metric signature then evolution can never be a conformal transformation because it requires a change like $\{-++++ +\} \to \{-++++-\}$. Therefore, even away from the context of the MCM, and based solely on the requirement that a physical theory of determinism must support blooping, any evolution operation whose output can be obtained via conformal transformation of its input must have some problem with it. It is true that, even in the MCM, often times we will obtain a system in $\mathcal{H}_2$ that superficially looks like a conformal transformation of the system in $\mathcal{H}_1$ but that superficial appearance ignores that $\mathcal{H}_2$ is on a different level of $\aleph$ and that the level of $\aleph$ cannot be changed with a conformal transformation. $\hat{\Phi}^j \to \hat{\Phi}^{j+2}$ is a discrete operation. Therefore, when detractors declare that the intermediate MCM steps at $\{\Omega, \varnothing, \aleph\}$ are superfluous, the wise physicist may rebut the declaration by showing that dS$\to$AdS enforces the non-conformal constraint of the evolution operator which can be glossed over in something like $\mathcal{H} \mapsto \mathcal{H}$.

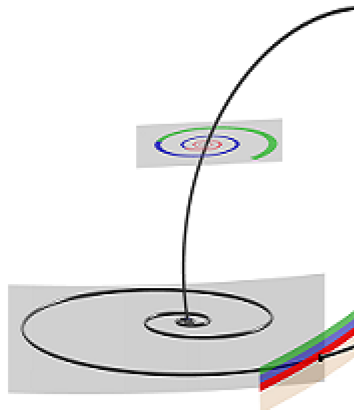

Figure 75: The Shilnikov bifurcation is strangely evocative of the logo for John Titor. The potential for CTC in the real universe [60, 12] is such that there may be a connection between Titor's alleged time travel technology and the MCM research program into the fundamental nature of time itself.

To say a little more about conformal infinity at the apex point $\varnothing$, note that the descriptions of de-Sitter space and anti-de Sitter space that we excerpted from reference [17], in section III.7 (figure 74), reflect the conformal coordinates used by Penrose in reference [52]. A hyperboloid of one sheet is defined with a negative hyperboloid parameter so it is like $\aleph$, and a hyperboloid of two sheets has a positive hyperboloid parameter but it is not quite like Ω. Ω is constructed from only one surface, not two as in figure 71. Furthermore, in figure 74, we see that Ω is constructed on a complete co-π but $\aleph$ is constructed on only one half of a co-$\hat{\pi}$. Therefore there is some manifest complementarity when $\aleph$ is a complete mathematical hyperboloid constructed on half of a co-$\hat{\pi}$ and Ω is only half of a mathematical hyperboloid but is constructed on a complete co-$\hat{\pi}$. We can likely make a connection here to the splitting of $t_\star$ on even levels of $\aleph$ into t_+ and t_- on the odd levels.

Above, we claimed that the problem of field line breaking was no longer being considered by prominent physicists but the problem has been worked on by very many people. Another example of blooping is to consider is the dripping of water droplets from a leaky faucet, as in reference [63]. If we study the water molecules, obviously there is no blooping but, in the fluid equations that contain the field line description, there is clearly a small bloop each time a droplet forms whose surface is disconnected from the small reservoir of water under the faucet. This particular blooping is described in reference [63] as an Andronov–Shilnikov[1]

[1] As an undergraduate this writer took an ODE class and a dynamical systems class with Andrey Shilnikov at Georgia State University. This writer shamefully burned Shilnikov with the student review form for the the dynamical systems course. This writer had gone into college with an idea assembled from a childhood introduction to mathematics and a slew of psychedelic experiences as a young adult. The idea began to crystallize during three semesters of calculus and other advanced math courese. This writer signed up for Shilnikov's dynamical systems course hoping that it would contain the missing link to this writer's idea but it did not. This writer was not, at that time, aware that the missing link was as yet undiscovered. This writer had a good understanding of what would be covered in the other math classes he would take as an undergraduate and none of them were relevant to that missing link. When Shilnikov's dynamical systems course concluded, it became apparent that no one was going to explain the extra piece of mathematics to this writer. In frustration and disappointment, and ignorance that the missing piece had never been formulated by anyone, and under the completely wrong assumption that no one reads or cares about student review forms, this writer cathartically burned Shilnikov with the student review although Shilnikov was an excellent professor, exceptionally erudite, and very friendly. This writer was frustrated to have to wait until graduate school to take a course that would explain the extra mathematical component but this writer did not encounter the material in graduate school. Then, again due in no small part to personal frustration, this writer set out to figure out the missing piece on his own. This book is a work unit of that investigation. The understanding of dynamical systems imbued to this writer by Shilnikov was absolutely essential to that effort; without a foundation in those concepts, we would not have even had the words to begin to describe the idea. This writer extends his gratitude, warm regards, and an apology to Andrey Shilnikov.

saddle-node bifurcation. Figure 75 depicts the distinct modalities of bloop approach followed by bloop egress in the Shilnikov bifurcation [64]. Just as there is a question in the MCM about how Σ^+ can connect to Σ^-, there is an open question in the theory of dynamical systems regarding how the planar spiraling sector can be connected to the perpendicular component at the saddle point. The extra pieces attached to the broad outer sweep of figure 75 indicate that there is an extra piece needed to connect one sector to the other. We have shown the relevance of the saddle, but what is the node in the saddle-node bifurcation? We want $\hat{\Phi}$ to point to spacetime points where bloops happen and we can put a 2-form on the end of $\hat{\Phi}$. We will discuss the feature more thoroughly in sections IV.5 and IV.6 but we mention it here because of the general relevance to blooping. We can interpret a 2-form as a multiplex containing two different normal vectors so we should say that one is in the plane of spiraling and the other is in the perpendicular direction such that the 2-form is the "node."

IV.5 Covering Spaces

When we propose in the MCM to wrap x^0 around a cylinder we are invoking the concept of a covering space. We often say that a circle is the target space of the real line as a covering space. We know that we can obtain a parameter on a circle from a parameter on a line by making the change of variables $x^0 \to e^{ix^0}$. This *must* define a circle because we may write

$$e^{ix^0} = \cos(x^0) + i\sin(x^0) \quad . \tag{4.161}$$

If we make the further change of variables

$$y = \cos(x^0) \quad , \qquad \text{and} \qquad z = \sin(x^0) \quad , \tag{4.162}$$

then we may use the identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to write the equation of the unit circle centered on the origin

$$y^2 + z^2 = 1 \quad . \tag{4.163}$$

Therefore, if we use the linear algebraic formalism

$$\vec{r} = y^2\hat{1} + z^2\hat{\imath} \quad , \qquad \text{where} \qquad |\vec{r}| = 1 \quad , \tag{4.164}$$

and then move to the plane spanned by some non-unitary basis like $\{\hat{\Phi}, \hat{2}, \hat{\pi}\}$, we will deform the circle into an ellipse such that $|\vec{r}| \neq 1$.

In section IV.2, we showed that pathological closed timelike curves can be removed from a manifold via the covering space representation. If we consider a circle as representative of any CTC then it is obvious how the pathological behavior is removed in the helical

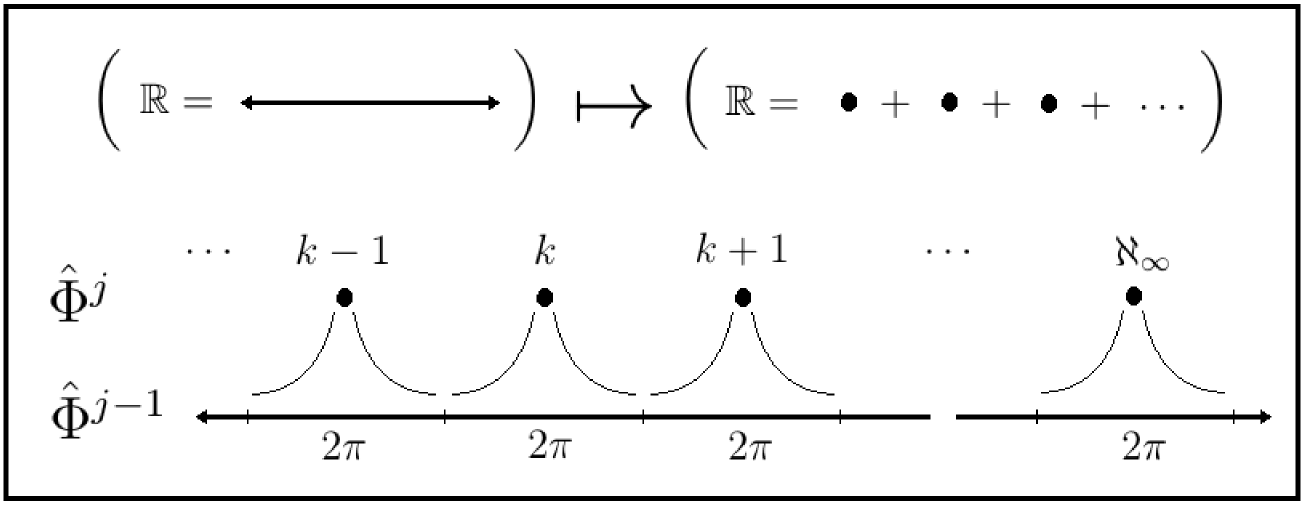

Figure 76: It is known $\mathbb{R}$ is composed of uncountably infinitely many points. If we change levels of $\aleph$ such that points of vanishing width becomes an interval of finite width then there will be $\aleph_\infty$ such intervals. This figure is similar nature to the construction of the Hopf fibration which results when each of the $\aleph_\infty$ points in $\mathbb{S}^2$ is replaced with a circle of finite circumference 2π radians.

covering space of the circle: the circle is closed but the helix is not. Open timelike curves are not pathological in any way; every mundane physical process in general relativity is described with open timelike curves. In the MCM, however, since we are moving in the opposite direction from the covering space onto the target space (mapping the ct axis onto a circle) we expect that there will be pathological CTC. If we are to include the effects inherent to classical electromagnetism that depend on the advanced time, a time greater than the observer's proper time since the beginning of the universe, then we will be required to incorporate some spacetime geometry that will allow information from the future to contribute to determinism in the present. Closed timelike curves are the penultimate example of this geometry, pathological as they may be.

The covering space will have a lot of application to the extension of conformal infinity from a point to a segment that can accommodate countable infinity $\aleph_0$ and uncountable infinity $\aleph_\infty$. Another place in the MCM where the covering space has a direct application is in the breaking of the U(1) symmetry of quantum mechanics by imposing the framework of hypercomplex analysis in which the Euler formula is not always exactly correct. To consider the domain of quantum mechanical eigenfunctions when the Euler formula does not hold we will necessarily consider the covering space of U(1) which is most generally the unbounded real line $\mathbb{R}$. If we want to re-impose all the symmetries of U(1) in the covering space then we can simply make the domain structure on $\mathbb{R}$ periodic in 2π dimensionless units of length. One might superficially assume that both representations, the circle or a line with a periodic domain structure, are equivalent but there are some global differences.

If there are uncountably many points along $\mathbb{R}$ and we expand each one into an interval of 2π by reducing the level of $\aleph$, as in figure 76, then there will be uncountably many periodic cells of width 2π. We have not changed the structure of $\mathbb{R}$ at all. The $\mathbb{R}$ topology is identical on every level of $\aleph$. Therefore, if there are uncountably many such intervals on $\hat{\Phi}^{j-1}$ there are uncountably many of them on any level of $\aleph$. Numbering the uncountably infinite points

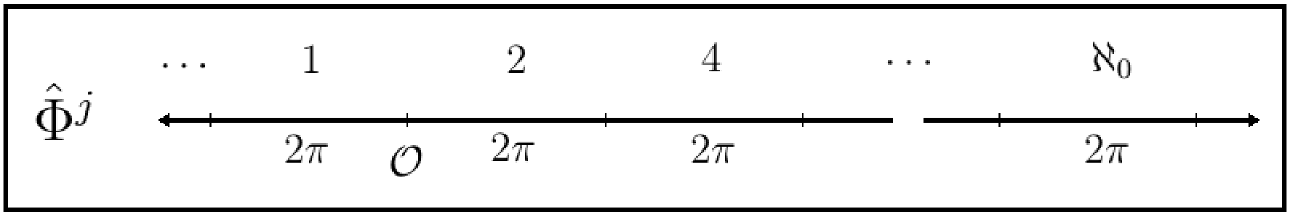

Figure 77: This figure demonstrates another method for segmenting $\mathbb{R}$ with intervals of finite width 2π. Because we have included the origin, we are able to define a first interval, a second one, and so on such that there are only countably infinitely many such intervals in $\mathbb{R}$. This disparity between possible periodic domain structures on $\mathbb{R}$ must be hidden in the region beyond conformal infinity. In the MCM, we extend Penrose's concept of a non-physical point at infinity [52] to an entire non-physical space beyond infinity.

in figure 76 with k instead of $1, 2, 3...$ demonstrates that there is no first point that we could begin to count and so, hence, there are uncountably many of them. It is understood that there are uncountably many points in $\mathbb{R}$ and if we reduce the level of $\aleph$ by one $j \to j-1$ then points of zero width become intervals of finite width. However, if we did not change the level of $\aleph$ and simply declared, "Let the domain on $\mathbb{R}$ be periodic in 2π," then we could make an argument that there are only countably many such intervals. In figure 77, we include the origin and note that, in general, the analysis of $\mathbb{R}$ begins with a cut. That cut in $\mathbb{R}$ can be any number so we will call it the origin $\mathcal{O}$. Relative to that point, one can very much begin to count the intervals of 2π. There is one to the left of $\mathcal{O}$, a second one to the right of $\mathcal{O}$, a third one to left of the first one, *etc*, as in figure 77. This is relevant for the MCM because the MCM condition places the observer exactly at $\mathcal{O}$. Therefore, if we stay on the same level of $\aleph$, there will be countably many intervals of 2π but, if we change the level of $\aleph$, we can obtain uncountably many such intervals. From this we see that countable infinity can be covered with uncountably infinity when the target and covering spaces are both $\mathbb{R}$. Indeed, there is a natural association between chiros and $\mathbb{R}$ with countably many intervals, and between chronos and $\mathbb{R}$ with uncountably many.

Consider the covering spaces in figure 78. On the left is a trivial covering space but on the right we show a non-trivial covering space. Each element $\pi/2$ in the circle is followed by another element $\Phi - \pi/2$. This covering space reflects a simple application of the operations shown in section IV.1. The N^{th} interval of $\pi/2$ (which can be rescaled to 2π as needed) begins at the position $N\Phi$ in the non-trivial covering space. In figure 78, the covering space has small gaps between elements that are adjacent in the target space and we can twist these extra elements into small circles to recover the trivial projective covering space, as in figure 79. However, the simultaneous existence of the non-trivial covering space with the trivial one implies that the interval $\Phi - \pi/2$ becomes a symplectic point in the trivial covering space rather than vanishing completely. In this way, it is easy to associate the trivial and non-trivial covering spaces with different levels of $\aleph$; the point on one level is a finite interval on the other.

To make the connection between the trivial and non-trivial covering spaces, we need to assign a 2-form everywhere a sphere appears on the helix. To move in that direction, first consider that, when treating the ADM positive definiteness theorem, we introduced a

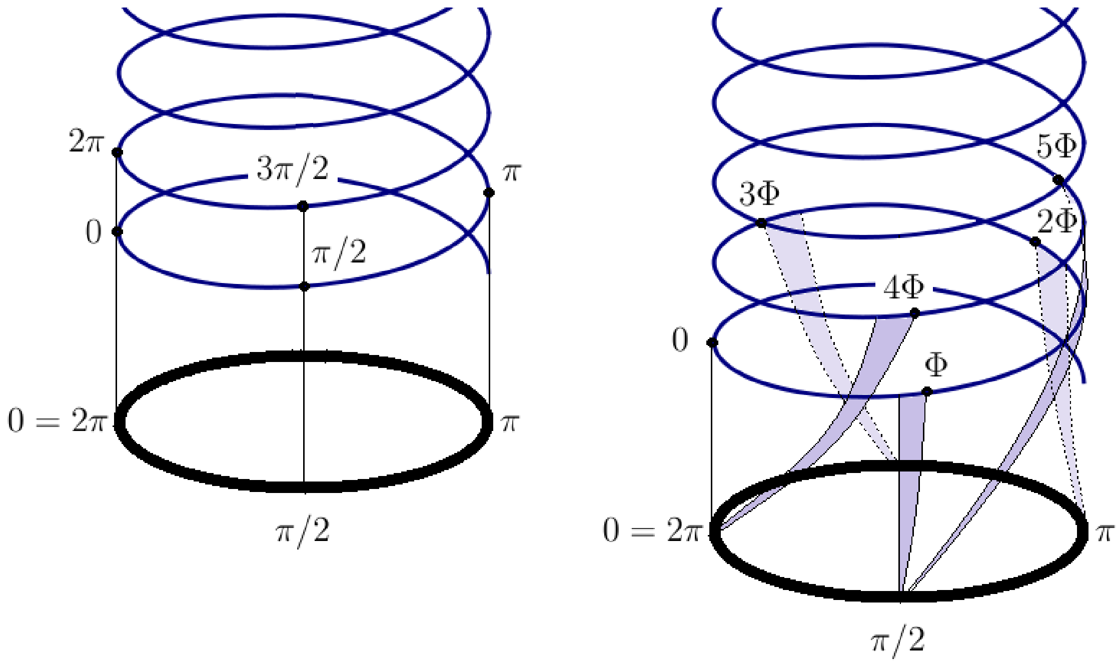

Figure 78: Here, we can see an intuitive placement for the idea to encode half of a co-$\hat{\pi}$ ($\pi/2 \approx 1.57$) on each $\hat{\Phi}$ (because $\Phi \approx 1.62$.) Each purple region has an element $\Delta = \Phi - \pi/2$ that can be associated with $\Sigma^{\varnothing}$ in the base space. We will establish complex behavior when we make a representation where the $\Sigma^{\varnothing}$ element has non-zero width. The affine parameter along the helix that contains the covering space is a natural object for such a definition.

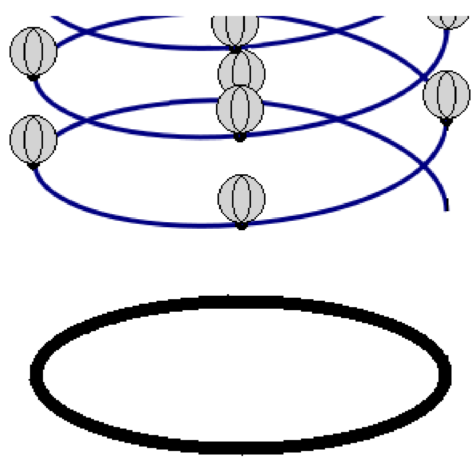

Figure 79: By compactifying the extra elements $\Phi - \pi/2$, it is possible to recover the trivially projective covering space. This figure is evocative of the mechanism through which points in spacetime become the Riemann sphere in twistor space.

symplectic 2-form

$$d\hat{S}_i = \begin{pmatrix} 0 & \frac{1}{2}\epsilon_{ijk}\, dx^j dx^k \\ -\frac{1}{2}\epsilon_{ijk}\, dx^j dx^k & 0 \end{pmatrix} \quad , \tag{4.165}$$

on the surface at spacelike infinity. The physical interpretation for this surface element is that the normal vector on the surface points in the left- or right-handed direction (to the interior or exterior of all of space) and it is a general property of 2-forms that we can assign multiple directions to a point. In the polar coordinates $\{r, \theta, z\}$ natural to the helix, the unit vector that points in the direction of increase along the helix is

$$\hat{e}_{\text{helix}} = c_\theta \hat{\theta} + c_z \hat{z} \quad . \tag{4.166}$$

When we add a generalized "2-form"

$$\hat{e} = \begin{pmatrix} 0 & \hat{e}_{\text{helix}} \\ \hat{e}_{\text{sphere}} & 0 \end{pmatrix} \quad , \tag{4.167}$$

at the points where the spheres are attached then a trajectory approaching each such point can continue on the helix or be diverted into the sphere. Indeed, noting that the spinor is comprised of two unit vectors

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \equiv \quad \hat{e}_1 \quad , \qquad \text{and} \qquad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \equiv \quad \hat{e}_2 \quad , \tag{4.168}$$

we see that the spinor definition of the multiplex that defines the path at the symplectic point might be superior to the matrix definition in equation (4.167). In either case, the multiplectic form is such that the continuation through the symplectic point is a smooth extension of the trajectory in either exit direction. Figure 80 shows the scheme by which we introduce local bifurcative symplex at the points where the spheres are attached. The U(1) symmetry only needs to be preserved along the $\hat{e}_{\text{helix}}$ direction. Furthermore, when contracting the interval $\Phi - \pi/2$ into the small sphere so that the trivial covering space is recovered, we can do that contraction at the $N\Phi$ points or the $N\pi/2$ points in a way that will produce a variety of projections into the target space.

The inversion operation on the Riemann sphere changes the anchor point of every vector in its attached vector field. If a vector is anchored at $\phi = \phi_0$ and $\theta = \theta_0$ relative to an origin of coordinates at one of the sphere's poles then, after the inversion operation, they will be located at $\phi' = \phi_0$ and $\theta' = \theta_0$ relative to the primed origin of coordinates at the other pole. We have mostly treated this problem as a discrete operation but if we use a continuous inversion operation, such as that in figure 81, then we see a direct application

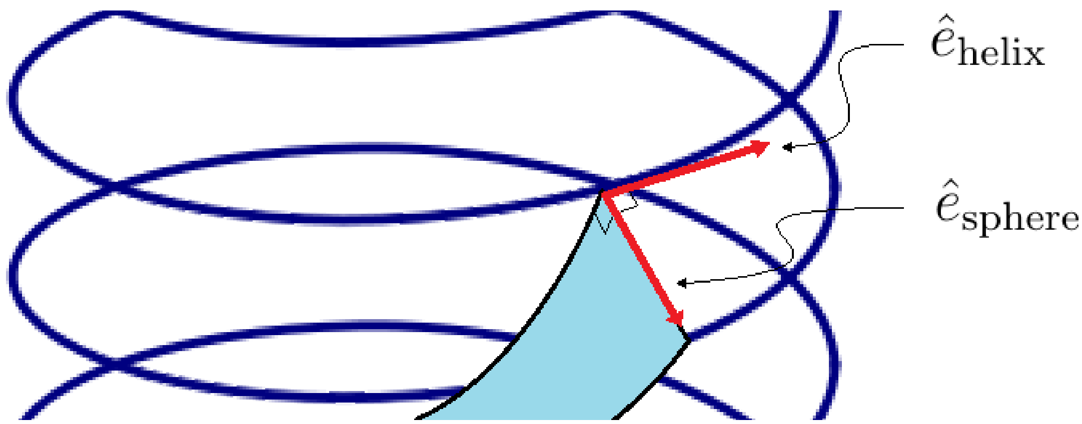

Figure 80: The small intervals $\Phi - \pi/2$ that are the basis of the small spheres inherent to the non-trivial covering space can be encoded on the trivial covering space with a 2-form at the periodic points. We can measure parameter along the helix such that these points lie at $\theta = N\pi/2$ or $\theta = N\Phi$.

of the symplectic form in equation (4.165). The origin of the two non-zero numbers in the symplectic matrix $d\hat{S}_i$ comes from the dot product of the differential area vector with the normal vector

$$d\hat{S} := d\vec{A} \cdot \pm\hat{n} \quad , \tag{4.169}$$

where $\pm\hat{n}$ point in the left or right handed directions, as in figure 82. Therefore, if we use the sphere theorem to invert the Riemann sphere with a continuous deformation, as in figure 81, then the inward facing normal vector will be swapped with the outward facing one and vice versa.

In figure 79, we have many small spheres whose interiors and exteriors can be permuted with the sphere theorem and, in general, we can use this to create a lot of non-commutative or fractal complexity in the cosmological lattice. Then using equation (4.167), we can either preserve or break the U(1) symmetry. $\hat{e}_{\text{helix}}$ can point in the direction of preserved symmetry and $\hat{e}_{\text{sphere}}$ can point in the direction of broken symmetry. Furthermore, we should note the fractal quality when each circle in the small sphere has its own helical covering space with yet smaller embedded spheres at other bifurcative points where we can impose or not impose the U(1) constraint. This mechanism should have direct bearing on the construction of the cosmological lattice. By using a system of nested covering spaces, we can implement the topological component of the information current, as in figure 83. This mechanism will transport information throughout the hypercosmos and across various levels of $\aleph$. This type of information exchange is more natural to matrix operators but we have shown the topological component here in keeping with the theme of this research program.

Earlier in this book, we noted that the path across the unit cell is qualitatively similar in many regards to a threading of the Hopf fibration. The Hopf fibration is itself defined when each point of $\mathbb{S}^2$ has a corresponding circle in $\mathbb{S}^3$ such that if every point in $\mathbb{S}^2$ is replaced with a circle then we obtain $\mathbb{S}^3$. The utility of fibrations in mathematics is to parameterize one topological space in terms of another and the direct utility in the MCM will be parameterize

Figure 81: The sphere theorem offers another method to invert the Riemann sphere beyond the ordinary discrete inversion map $\zeta = 1/\xi$. In addition to swapping the interior and exterior regions, this inversion operation also turns the sphere upside down which will implement the changing anchor points in the associated vector field. The normal vector pointing outward from the north pole of the sphere in the first frame becomes an inward pointing normal vector anchored to the south pole in the last frame. The sphere theorem demonstrates an important application in which we must consider the differential element of surface area as $dS_i = \pm\epsilon_{ijk}dx^j dx^k$ instead of simply $dS_i = \epsilon_{ijk}dx^j dx^k$. This figure is taken from reference [65]

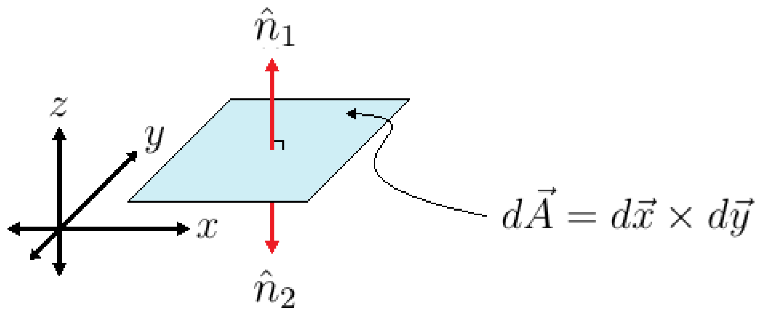

Figure 82: This figure demonstrates a symplectic 2-form. Noting that the 1/2 factors in equation (4.165) come from the double counting in the Einstein notation, we see that, when $\hat{n}$ is a unit vector $|\hat{n}| = 1$, the possible values for $dS = d\vec{A} \cdot \hat{n}$ are $\pm dxdy$ depending on which normal vector is used.

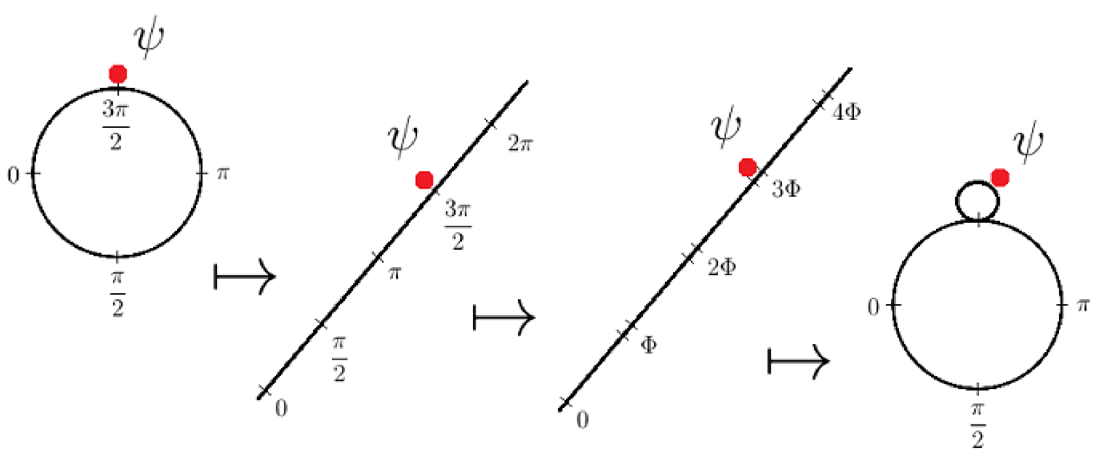

Figure 83: This figure demonstrates the information current by changing the domain of ψ from the big circle to the little circle.

everything on $\hat{\Phi}^{j+1}$ in terms of objects on $\hat{\Phi}^j$. In the MCM, we say that the relationship between the circle of zero radius (a point) and a circle of finite radius is the changing level of $\aleph$ so, indeed, it is likely that the Hopf fibration appears in every $\hat{M}^3$ where the level of $\aleph$ increases. Furthermore, the Hopf fibration is important in twistor theory and we have proposed to use the twistor representation as the representation for $\varnothing$ where no position space exists because of the infinite curvature of the embedded metrics on the slices of $\Sigma^\pm$ at $\chi^5_\pm = \pm\infty$. Even furthermore, the example at the beginning of this section about assigning an interval of dimensionless length 2π to every point in $\mathbb{R}$ is almost exactly what Hopf has done assigning a circle in $\mathbb{S}^3$ to every point in $\mathbb{S}^2$. We will not be sidetracked presently with these matters but we have a laid the foundation for a lot of future inquiry. We might undertake this inquiry at some point in the future Ω or some detractor might choose to desist from his program of detractions and unproductive research in order make these likely fruitful inquiries on his own.

In reference [8], when we were considering the polar coordinate singularity on $\mathbb{S}^2$ we defined the changing level of $\aleph$ such that the location of the initial point was the center of the circle when the point was represented on the other level of $\aleph$. However, if we use the idea that each point in $\mathbb{S}^2$ comes from a circle in $\mathbb{S}^3$ then we will probably be constrained to say that the irreducibly singular location of the point in $\mathbb{S}^2$ is not in the center of the circle in $\mathbb{S}^3$ but rather that it is a point in the circle. The Hopf fibration refers to circles as fibers attached to, not centered on, the points of $\mathbb{S}^2$. If the point is on the large circle, and not at its center, then we have likely changed the anchor point of some object. Furthermore, a point is a circle on a lower level of $\aleph$ (objects become smaller on higher levels) so there is some nuance when one intuitively associates the higher dimensionality of $\mathbb{S}^3$ with the higher level of $\aleph$ because it is obtained by reducing the level of $\aleph$ of a set of points to obtain a set of circles.

The formal definition of a covering space has to do with the neighborhood around a point. When we want to consider the point at conformal infinity in the Penrose scheme of conformalism, we cannot use a covering space because there is only a neighborhood to consider on one side (because $\mathscr{I}$ is the boundary of $\mathscr{M}$.) However, we have only considered the general properties of covering spaces. We have not yet added hypercomplexity and, when we do, the hypercomplexly infinitesimal neighborhood around the point at conformal infinity is amenable to canonical covering. Furthermore, when we extent Penrose's point at infinity to a segment between $\aleph_0$ and $\aleph_\infty$ we will be able to consider the neighborhood around $\aleph_0$ as per usual.

Consider the endpoints of the zenith angle $\phi \in (-\pi, \pi)$, as in figure 84. There is a hypercomplexly infinitesimal neighborhood around the null point so we can make the transition from ordinary complexity to hypercomplexity by including or not including $\hat{\Phi}$. Figure 84 shows $\hat{\Phi}$ on the same level of $\aleph$ as $\hat{\pi}$. If we call that level j and then consider $\hat{\pi} + \hat{\Phi}^{j+1}$, we will recover the circle constructed with $\hat{\pi}$ alone because $|\hat{\Phi}| = \Phi$ is a finite quantity that becomes an infinitesimal quantity on a higher level of $\aleph$. Therefore, we should consider that the circle does always include $\hat{\Phi}$. When we require that $\hat{\Phi}$ has different boundary conditions at its endpoints, that provides a natural mechanism through which to break the U(1) symmetry. Furthermore, we can use $\hat{\Phi}$ to point in a direction that deforms the circle into a helical interval as needed to construct the covering space considered in this section. That is to say: we can use $\hat{\Phi}$ to create $\hat{z}$. Figures 85 and 86 are not directly related to the covering space

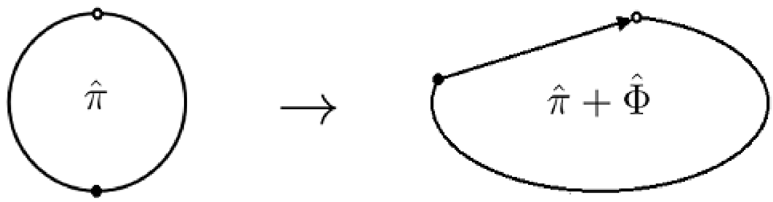

Figure 84: We may enforce the broken U(1) symmetry by requiring unequal boundary conditions at either end of $\hat{\Phi}$. The circle can be recovered from $\hat{\pi}+\hat{\Phi}$ by considering $\hat{\Phi}$ on a lower level of $\aleph$ such that it is an object inside the hypercomplexly infinitesimal neighborhood around the null point of the non-included boundary when $\theta \in (0, 2\pi]$.

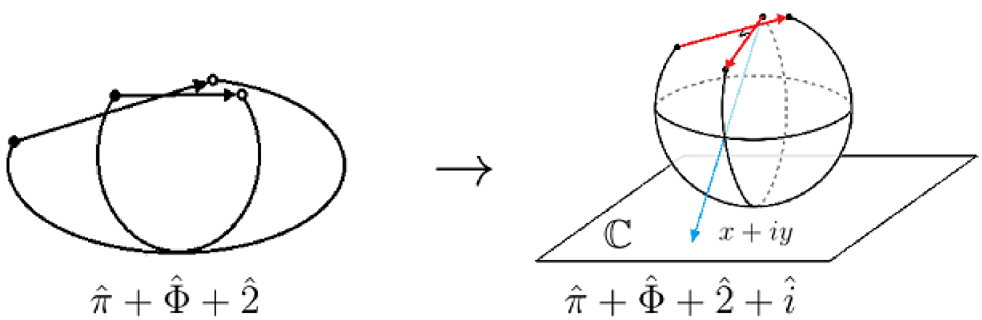

Figure 85: This figure shows an application of the ontological basis toward building the requisite MCM topology. On the left, note the role of $\hat{2}$ when $\mathbb{S}^1$'s 2π radians become 4π steradians on $\mathbb{S}^2$.

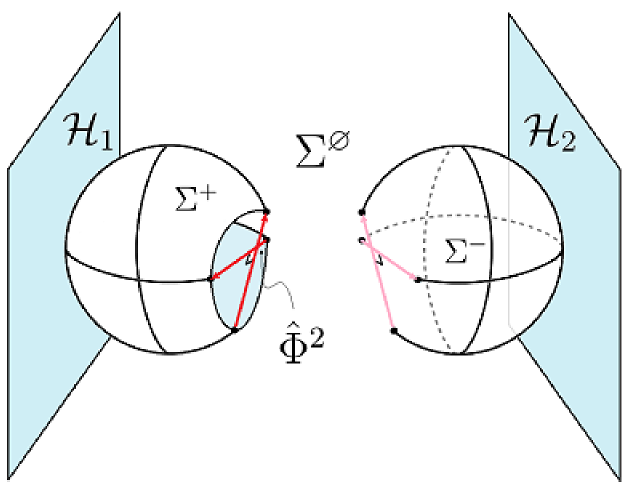

Figure 86: In this book, we have focused on attaching the small sphere to the circle to achieve SU(2)⊗U(1) but, for the complete standard model, we need to achieve SU(3)⊗SU(2)⊗U(1) which is the plane with a 2-sphere attached on one side and a 3-sphere on the other. This figure does not directly motivate the group theoretical structure of the standard model but it does demonstrate an intuitive arrangement for the $j \to j+2$ change in the level of $\aleph$ under $\hat{M}^3$. If both $\mathcal{H}_1$ and $\mathcal{H}_2$ have a sphere with two $\hat{\Phi}$ objects on the right side, then, on the left side of $\mathcal{H}_2$, we could, for instance, have a sphere missing the $\hat{\Phi}^2$ which it receives from $\mathcal{H}_1$ during propagation across the unit cell with the inversion operation on the Riemann sphere. This figure shows a double inversion where the first inversion moves the sphere's origin to $p \in \Sigma^{\varnothing}$ and the second moves it to a point in $\mathcal{H}_2$.

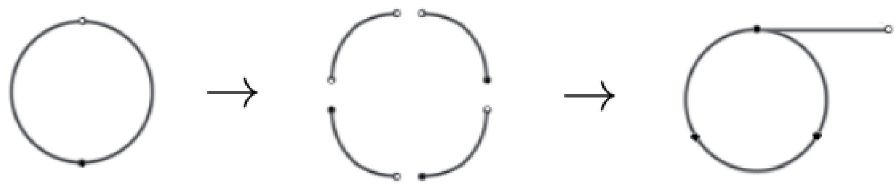

Figure 87: This figure demonstrates a topological application for the unpaired terms discussed in this section. When an inner product in the arithmatic sector leaves an unpaired element, it should act as a connector in the topological sector.

but they show a notable extension of the scheme $\hat{\pi}\to\hat{\pi}+\hat{\Phi}$ to the full ontological basis.

The idea to have an unpaired element in the MCM inner product comes from the feature of quantum mechanics that the eigenvectors of a continuous basis of eigenfunctions are not orthogonal. In general, we can associate this unpaired element with an included endpoint getting shuffled to a place where a non-included endpoint is required. We have some guidance for building the cosmological lattice when we can take any number of intervals with a non-included endpoint, place all of those non-included endpoints at one shared lattice site, and then pin them all together with an included endpoint from another interval. Indeed, simply taking a polar great circle of the Riemann sphere $\theta\in(-\pi,\pi)$, if we want to construct a circle without a null point via this method of pinning then we naturally obtain a $\hat{\pi}+\hat{\Phi}$ helical unit, as in figure 87. Incidentally, this gives a good demonstration of what it means for $\hat{\Phi}$ to live inside the hypercomplexly infinitesimal neighborhood around the null point in $\theta\in(-\pi,\pi)$. If the circle is on the j level of $\aleph$ then so is the $\hat{\Phi}$ on the right side of this figure but there is another $\hat{\Phi}^{j-1}$ attached to the circle on the left. It exists inside the missing north polar point. Figure 87 shows one construction but it should be clear that there are any number of constructions that can be assembled from an unlimited number of finite intervals with various conditions of missing or included endpoints.

In figures 84-86, we have used the ontological basis to make the extension from real analysis on $\hat{\pi}$ to hypercomplex analysis on $\{\hat{\imath},\hat{\Phi},\hat{2},\hat{\pi}\}$. Often we add the null point to the U(1) topology citing the null point on the Riemann sphere but we may equally well cite the isomorphism of $\theta\in(-\pi,\pi)$ with $\mathbb{R}:x\in(-\infty,\infty)$. Let $\hat{2}$ and $\hat{\Phi}$ both be associated with the transfinite component so that simple complex analysis takes place in $\mathbb{C}$ spanned by $\hat{\pi}$ and $\hat{\imath}$. One of the most important formulae in the analysis of $\mathbb{C}$ is the Cauchy integral formula. Wikipedia says the following.

> "Cauchy's formula shows that, in complex analysis, 'differentiation is equivalent to integration.' Complex differentiation, like integration, behaves well under uniform limits – a result denied in real analysis."

The statement that integration is the same as differentiation in complex analysis enables us to use derivative and integral operations to "change the level of $\aleph$." Wavefunctions are typically infinitely differentiable so the Cauchy differentiation formula (which is beyond the scope of this book in which we do not even fully define the Cauchy C curve) applies. Cauchy's differentiation formula is a concise formula for the n^{th} derivative of a holomorphic function such as the wavefunction. We do not have formulae like this in real analysis.

However, we might expect to see new complexity when the "Cauchy C curve" of relevance is not the boundary of an ordinary disc but is, instead, along a helix composed of many discs sewn together at symplectic points. One expects that, in general, the various Cauchy formulas describing integrations around the boundary of a disc in $\mathbb{C}$ can be adapted to integrations across a window function of width 2π radians on the helical covering space. If we disrupt the symmetry of the U(1) topology of a disc's boundary with a series of embedded symplectic points on the helix then we expect the relationship between spaces and tangent spaces will become more complex than that which is represented so concisely with the Cauchy differentiation formula in the space spanned by only $\hat{\pi}$ and $\hat{\imath}$. Indeed, the hidden degrees of freedom of the non-trivial covering space developed in this section might disrupt the regularity of the Cauchy differentiation formula across different levels of $\aleph$ such that decoherence is observed when boundary conditions that match on a value and a first derivative only exactly match on the value on another level $\aleph$, but are still constrained to almost match on the first derivative. While complex analysis is typically the domain of quantum theory, we should note the boundary condition on the metric in the Lorentz approximation is such that its first derivatives all vanish. Therefore replacement of the Cauchy disc with a window function on the MCM helix might lead to quantum decoherence and fuzziness in spacetime. In fact, we can avoid having to modify the well known boundary conditions of the exactly solvable problems in physics by only introducing this decoherence on the odd levels of $\aleph$ between the even levels associated with $\mathcal{H}_1$ and $\mathcal{H}_2$. It will be a fruitful exercise to consider in the future the case when the two unit vectors attached to the embedded symplectic points are considered in the non-unitary sector as two crossed $\hat{\Phi}$ vectors, one pointing into the trivial covering space and one pointing in a orthogonal direction in the non-trivial covering space.

IV.6 The Double Slit Experiment

One of the primary results of reference [9] was a logical explanation for the odd wave/particle duality observed in the double slit experiment. The formalism developed in reference [9] led notably a reanalysis of Bell's inequality in reference [49] which shows that local hidden variables such as the chirological MCM coordinates are always allowed to exist even when Bell's theorem "proves" that they are not. The main result in reference [9] was to begin to develop a transfinite definition of the exponential function. As a physical motivator for that extension of e^{ix} onto every level of $\aleph$, we developed the double slit experiment in the context of the cosmological lattice. As an illustration of the prominent features of the cosmological lattice we will show in this section how the structure removes the "weirdness" from this famous example of unintuitive quantum phenomena.

The first known double slit experiment was carried out by Young in 1801. He used photons and, in 1927, Davisson and Germer did the double slit experiment with electrons. The general experimental set up is shown in figure 88. Since that time, the wave/particle mystery effect has been confirmed for various quanta incident on two slits. It is one of the most important results in quantum theory and its empirical constraints on theory greatly influenced the development of the MCM. Even after solving dark energy [2], deriving the fine structure constant and Einstein's equation [12], as well as reproducing the particle structure of the standard model [11], finding an intuitive explanation in the MCM for the double slit experiment stands out as a feat of great importance. Wikipedia says the following about

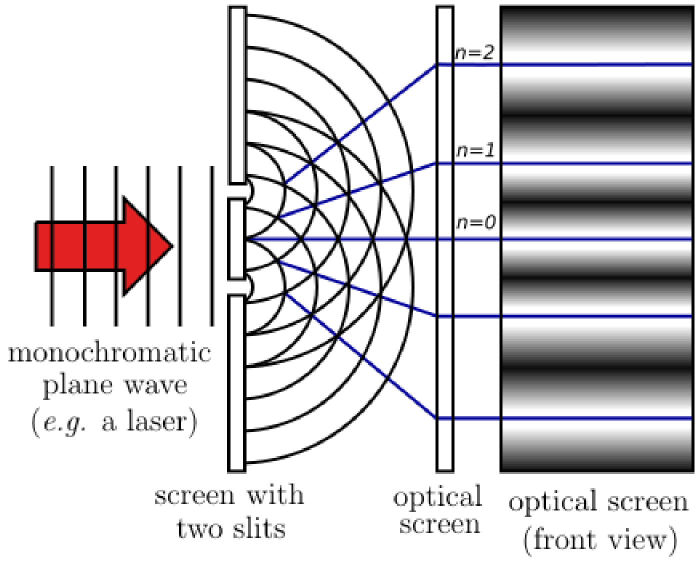

Figure 88: Wave interference requires that the flux incident on the diffraction screen pass through both slits equally. This figure is taken from Wikipedia and it ignores the decreasing amplitudes of the maxima away from the beamline.

this hallmark of quantum weirdness.

> "In the basic version of this experiment, a coherent light source, such as a laser beam, illuminates a plate pierced by two parallel slits, and the light passing through the slits is observed on a screen behind the plate. The wave nature of light causes the light waves passing through the two slits to interfere, producing bright and dark bands on the screen – a result that would not be expected if light consisted of classical particles. However, the light is always found to be absorbed at the screen at discrete points, as individual particles (not waves), the interference pattern appearing via the varying density of these particle hits on the screen. Furthermore, versions of the experiment that include detectors at the slits find that each detected photon passes through one slit (as would a classical particle), and not through both slits (as would a wave). However, such experiments demonstrate that particles do not form the interference pattern if one detects which slit they pass through. These results demonstrate the principle of waveparticle duality.
>
> "Other atomic-scale entities, such as electrons, are found to exhibit the same behavior when fired towards a double slit. Additionally, the detection of individual discrete impacts is observed to be inherently probabilistic, which is inexplicable using classical mechanics. [*sic*]
>
> "The double-slit experiment (and its variations) has become a classic thought experiment for its clarity in expressing the central puzzles of quantum mechanics. Because it demonstrates the fundamental limitation of the ability of the observer to predict experimental results, Richard Feynman called it 'a phenomenon which is impossible [*sic*] to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery of quantum mechanics.'"

The critical behavior that leads to this famous weirdness is whether or not the observer checks which slit the particles go through. If the observer does not make this observation then the particles show up on the optical screen according to a wavelike interference pattern of probability density, but if the observation is made then the particles show up on the screen as if each slit were an independent source of particles. In pre-MCM physics this was hard to accommodate in a theoretical description of the process because a psychological event such as an observation is just another point in spacetime the same as any other. The behavior that is an observation should have no effect on the system producing the observed result. For some strange reason, it does have a pronounced effect and, for a long time, this was considered an inexplicable phenomenon (or explicable only with an unsatisfying explanation.) In the MCM, we expand spacetime into hyperspacetime such that the observer's observation is not just another event in spacetime; these events are $\hat{\pi}$-sites. Observations at successive $\hat{\pi}$-sites define the periodic boundary of the MCM unit cell. This allows us to make a formal distinction between generalized spacetime events (points in spacetime) and psychological events that collapse wavefunctions.

Even if Feynman says there is only one mystery, the issues in the double slit experiment are many. When one particle at a time is sent through the slits, each particle will register on the screen individually indicating that it has not been split apart by the diffraction grating. This particulate behavior can be confirmed by placing a detector at the location of the slits which will confirm that a single quantum always goes through one slit or the other, but never both. The mysterious component of the experiment arises when many individual quanta are sent through the grating and no observations are made at the slits. If the impact of each particle is marked on the screen then the total pattern that forms is that of wave interference which is incompatible with the particle having gone through one slit or the other. Wave interference implies that the particle goes through both slits, as in figure 88. Another example of wave/particle duality is when a gamma ray beam of weakening intensity shows up on a Geiger counter as increasingly infrequent clicks rather than as a decreasing amplitude of the loudness of the clicks. In seeking to develop an MCM motivation for the double slit result, one in which the result is intuitive rather than non-intuitive, we reject that the idea that the particle chooses to go through slit or the other. Since the electron is inanimate and cannot make a choice, it must always go through both slits. The only options, intuitively, for the electron's path are for it to go through one or both slits and we must choose the option for both because it is a superset of the other possibility.[1] Rather than claiming that an electron will choose on its own to go through one slit or the other, we will say that the observer chooses (randomly) to observe one mode or another. This problem of choosing one slit or the other is the same the problem of why individual particles show up on the screen at one location and not another. This is a probabilistic problem that we do not treat in this book.

To motivate the double slit interpretation with $\hat{\pi}$-sites, we will need to consider that the probability amplitude is complex-valued and, therefore, must be associated with the off shell region of reality between $\hat{\pi}$-sites. Here, we use the terms on and off shell more broadly than only referring to the mass shell in phase space: on shell means at a $\hat{\pi}$-site ($\mathcal{H}$) and off

[1]Here we ignore some fine nuance related to the Cauchy C curve around infinity. It is possible that the particle doesn't go through either slit and instead traverses the whole universe before showing up on the screen but we will not treat that possibility in this section. That possibility induces a requisite superluminal motion but we might assuage that problem by traversing the universe on the lower level of $\aleph$ while measuring distance according to the scale of the higher level.

shell means in the bulk of the MCM unit cell. In the region where no measurement is made between the plate and the screen, we have no particle, only amplitude. Then we must say that the collapse of the wavefunction on the optical screen is a topological bloop of a sort where the particle comes into pointlike existence disconnected from the diffuse wavefunction. We give the same wavefunction to every particle that goes through the slits but when we observe a particle, the wavefunction collapses to show that all of the probability is focused at the single point where it is observed. The two slit wave interference pattern is always a sinusoid pinched inside of an exponential envelope function and the goal in physics, ultimately, is to explain how the particle can show up at random places on the screen while the behavior of very many particles will always conform to the wave pattern. Furthermore, if we start measuring which slit the photons go through then we will not observe wave interference at all regardless of the large number of particles that we send into the apparatus. The brief treatment in reference [9] was focused on this latter mystery about detection at the intermediate position where the slits are located between the source and the optical screen. That detection will destroy the interference pattern. The mystery about why large numbers of individual particles always assemble the wavelike pattern is a question in stochasticism which lies far beyond anything considered to date in the MCM.

To go into a little more depth here, note that we have the same wave behavior in front of the slits as we do behind it. If particles were aimed at the center of the two slits then they would hit the material that divides the two slits and not show up at the screen at all. The probability density for where the particles will hit the screen with the slits in it is centered on this point but, for some stochastic reason, sometimes the particle does not hit the material in the center and instead goes through one slit or the other. Why the particle should go through one slit or the other is the same question of stochasticism regarding why it hits the optical screen in one part of the wave pattern and not the other. However, the question, "Why here and not there?," as it regards the optical screen is more interesting because the probability density has maxima *and* minima meaning that there are certain places on the screen where particles *never* arrive. On the first screen with the slits in it, there is just one blob of probability density centered between the slits but, on the second plate, when there is no measurement at the first, there are multiple blobs that decrease in amplitude away from the axis of the beam.

To get a wave interference pattern on the final screen, we need to take two waves in superposition. We will call these ψ_a and ψ_b corresponding to wavefronts of probability amplitude emanating from each slit which we will label a and b. Particles will preferentially show up where these waves are in phase and they will rarely show up where the waves are out of phase (and they will never show up where their phases differ by exactly π radians.) Separating the initial beam into two components a and b, we may write the wave and particle modes as

$$\text{Waves} \quad \longrightarrow \quad \begin{cases} \big|\psi_a(t_{\text{source}});\,\hat{\pi}_1\big\rangle \quad \to \quad \big|\psi_a(t_{\text{slits}});\hat{\pi}_1\big\rangle \quad \to \quad \big|\psi_a(t_{\text{screen}});\hat{\pi}_2\big\rangle \\ \\ \big|\psi_b(t_{\text{source}});\hat{\pi}_1\big\rangle \quad \to \quad \big|\psi_b(t_{\text{slits}});\hat{\pi}_1\big\rangle \quad \to \quad \big|\psi_b(t_{\text{screen}});\hat{\pi}_2\big\rangle \end{cases} \tag{4.170}$$

$$\text{Particles} \quad \longrightarrow \quad \begin{cases} \big|\psi_a(t_{\text{source}});\hat{\pi}_1\big\rangle \quad \to \quad \big|\psi_a(t_{\text{slits}});\hat{\pi}_2\big\rangle \quad \to \quad \big|\psi_a(t_{\text{screen}});\hat{\pi}_3\big\rangle \\ \\ \big|\psi_b(t_{\text{source}});\hat{\pi}_1\big\rangle \quad \to \quad \big|\psi_b(t_{\text{slits}});\hat{\pi}_1\big\rangle \quad \to \quad \big|\psi_b(t_{\text{screen}});\hat{\pi}_2\big\rangle \end{cases} . \quad (4.171)$$

At time t_{source}, the particle starts moving toward the screen. The observer must have some way to know if his device is emitting electrons and when he confirms that an electron has been emitted that marks his first measurement at $\hat{\pi}_1$. If the observer does not check to see which slit the electron went through then he has not yet reached chirological $\hat{\pi}_2$ at the chronological time t_{slits}. The observer wishes to observe either wavelike or particulate behavior on the screen so he will definitely make a measurement at chronological t_{screen}. This event will be at the $\hat{\pi}_2$ site when he does not check which slit the electron went through, and it will be $\hat{\pi}_3$ when he does. The formulation in equations (4.170-4.171) resolves the pathological behavior wherein checking which slit a particle goes through destroys the wave interference pattern on the optical screen. When the particles are detected at the slits, the density of particles on the optical screen becomes two blobs corresponding to two particle sources: slit a and slit b. We can derive this behavior from equations (4.170-4.171) when two waves on $\hat{\pi}_2$ will interfere but two waves distinctly on $\hat{\pi}_2$ and $\hat{\pi}_3$ will not. In the future, this mechanism will likely be adapted to the $\hat{\Phi}^j$ formalism instead of $\hat{\pi}_j$ but here we simply reproduce that which originally appeared in reference [9].

The case of waves in equation (4.170) is straightforward enough. If we send the particles though one at a time, they will always show up as particle impacts on the screen that only form an interference pattern when recorded and considered all together. The probability density on the screen, in this case, shows the interference of two waves with the same ontological identifier. The case of particles in equation (4.171) differs from the case of waves because the probability density on the screen is just the superposition of the waves from slit a and slit b with no interference effects. Here we rely on the concept that interference is only a property of like waves. If we fire neutrons through one slit and water molecules through the other then it will not matter if we observe the slits or not, there will be no interference. When a single beam is incident on the slits then obviously the waves going through each slit are alike and will interfere, but if we use the levels of $\aleph$ on the different $\hat{\pi}$-sites to make the waves unalike then interference is forbidden even for like particles. The observation at the slits collapses the wavefunction only on the slit in which the particle is detected. The wavefunction coming to the screen through the other slit originated on $\hat{\pi}_1$ but now the wave emanating from the slit through which the particle was observed to pass comes from $\hat{\pi}_2$ which is on a different level of $\aleph$.[1]

Reference [9] begins with some definitions that make a $\hat{\pi}_1$ state different than a $\hat{\pi}_2$ state. The structure of the relevant wavefunction in the ordinary notation is

$$\Psi_{\text{scr}} = \Psi_a + \Psi_b \tag{4.172}$$

$$\Psi_a = \Psi_0 e^{i\omega t} \tag{4.173}$$

[1]The phrase "level of $\aleph$" was first coined in reference [9].

$$\Psi_b = \Psi_0 e^{i\omega t+\delta} \tag{4.174}$$

$$P'_{\rm scr}[\psi] = \big\langle \Psi_{\rm scr} \big| \Psi_{\rm scr} \big\rangle \quad , \tag{4.175}$$

where $\Psi_{\rm scr}$ is the wavefunction on the screen. As above, Ψ_a and Ψ_b are the contributions to $\Psi_{\rm scr}$ from each slit, and the probability for finding the particle on the screen between z_1 and z_2 is

$$P[\psi(z)] = \int_{z_1}^{z_2} \Psi^*_{\rm scr} \Psi_{\rm scr} \, dz \quad . \tag{4.176}$$

Regardless of any quantum weirdness, dissimilar waves will not create interference patterns so we must have the same ω in Ψ_a and Ψ_b. To modify the existing framework, we will use lower case ψ. The structure of the MCM wavefunction in equations (4.170-4.171) is

$$\psi_{\rm scr} = \psi_a + \psi_b \tag{4.177}$$

$$\psi_a = \psi_0 e^{i\omega_n \chi^5} \tag{4.178}$$

$$\psi_b = \psi_0 e^{i\omega_m \chi^5 + \delta} \tag{4.179}$$

$$P_{\rm scr}[\psi] = \big\langle \psi_{\rm scr}; \hat{\pi}_m \big| \psi_{\rm scr}; \hat{\pi}_n \big\rangle \quad . \tag{4.180}$$

If we take $m = n$ then this formulation exactly replicates the ordinary formulation in equations (4.172-4.175) (up to the final integration over dz which is included in the MCM notation $\langle \psi; \hat{\pi} | \psi; \hat{\pi} \rangle$.)

If the MCM wavefunction starts and ends on a $\hat{\pi}$-site then it must go through some intermediate sites as well and here we will detail that process. The wavefunction is

$$\psi(x,t) = \psi(x) e^{i\omega t} \quad , \tag{4.181}$$

and for simplicity we will let

$$\psi(x) = \psi_0 = 1 \quad . \tag{4.182}$$

When the wavefunction evolves in the chronological channel, t begins to increase from $t_{\text{source}} \equiv t_i$ until the trajectory terminates at time $t_{\text{screen}} \equiv t_f$. To examine the chirological channel $\mathcal{H} \mapsto \Omega \mapsto \aleph \mapsto \mathcal{H}$, we need to use some of the new operators developed in section IV.3. We will introduce time dependence as

$$\psi(t) = \big\{ \hat{\pi} \big| \psi(t) \big\rangle = \big| \psi(t); \hat{\pi} \big\rangle \quad . \tag{4.183}$$

To get the wavefunction out of $\mathcal{H}$, we need to get in the Σ^+ representation. That looks like

$$\psi_+(\chi^5_+) = \big\{ \hat{\Phi} \big| \psi(t); \hat{\pi} \big\rangle \quad , \tag{4.184}$$

but what is the analytical form of $\psi_+(\chi^5_+)$? χ^5 is dimensionless so, if we trivially make the conversion $t \to \chi^5_+$, it will mess up the units in the exponent. Euler's number is typically raised to the power of a dimensionless number in physics; that is why the familiar term $\Delta = kx - \omega t$ comes out in dimensionless radians. Furthermore, the information about t_i is expected to be relevant at t_f when the chronological and chirological paths intersect again. We say "again" because these paths necessarily intersect at t_i when make the conversion in equation (4.184). For consideration of the chirological path, we will take the definition

$$\psi_+(\chi^5_+) = \big\{ \hat{\Phi} \big| \psi(t_i); \hat{\pi} \big\rangle = -\varphi\pi \big| \psi(\chi^5_+); \hat{\Phi} \big\rangle \tag{4.185}$$

$$= -\varphi\pi \, e^{\chi^5_+(i\omega t_i)} \quad , \tag{4.186}$$

where the $-\varphi\pi$ comes from $\hat{\pi} = -\varphi\pi\hat{\Phi}$, and the exponent is fully dimensionless. Now that we have taken the wavefunction out of $\mathcal{H}$, we need to send it to Ω. According to figure 1, the length of χ^5_+ across Σ^+ is Φ. Therefore the wavefunction on Ω is

$$\psi_+(\Phi) = -\varphi\pi \, e^{(i\omega t_i)\Phi} \quad , \tag{4.187}$$

and we have achieved $\mathcal{H} \mapsto \Omega$. For $\Omega \mapsto \aleph$, we need the $\Sigma^{\varnothing}$ representation. When obtaining the $\Sigma^{\varnothing}$ representation, we will preserve $\chi^5_+ = \Phi$ as we have preserved $t = t_i$.

$$\psi_{\varnothing}(\chi^5_{\varnothing}) = \big\{ \hat{2} \big| \psi_+(\Phi); \hat{\Phi} \big\rangle = -\varphi\pi \left(\frac{\Phi}{2} \right) \big| \psi(\chi^5_{\varnothing}); \hat{2} \big\rangle \tag{4.188}$$

$$= \frac{\pi}{2} \, e^{\chi^5_{\varnothing}(i\omega t_i)\Phi} \quad , \tag{4.189}$$

$\chi^5_\varnothing$ has no width in figure 1 but we here we will use another length of Φ, as in figure 89, which gives

$$\psi_\varnothing(\Phi) = \frac{\pi}{2}\, e^{(i\omega t_i)\Phi^2} \quad . \tag{4.190}$$

This has the interesting coefficient $\pi/2$ and it lies exactly between $\hat{\pi}_1$ and $\hat{\pi}_2$ This number, $\pi/2$, is usually associated with orthogonality and we want $\varnothing$ to be topologically orthogonal to $\mathcal{H}$. It is through this concept that we introduce double orthogonality between adjacent $\hat{\pi}$-sites. With equation (4.190), we have achieved $\mathcal{H}\mapsto\Omega\mapsto\aleph$ and we need the Σ^- representation of the wavefunction for $\aleph\mapsto\mathcal{H}$. Then

$$\psi_-(\chi^5_-) = \big\{\hat{i}\big|\psi_\varnothing(\Phi);\hat{\Phi}\big\rangle = \frac{\pi}{2}\big(-2i\big)\big|\psi(\chi^5_-);\hat{i}\big\rangle \tag{4.191}$$

$$= -i\pi\, e^{\chi^5_-(i\omega t_i)\Phi^2} \quad , \tag{4.192}$$

Sticking to the convention from figure 1 that the length of χ^5_- across Σ^- is unity, we can advance

$$\psi_-(1) = -i\pi\, e^{(i\omega t_i)\Phi^2} \quad . \tag{4.193}$$

We have evolved the wavefunction across $\chi^5\equiv\chi^5_+\otimes\chi^5_\varnothing\otimes\chi^5_-$ that span the unit cell and now we need to put it back into the position space representation on $\hat{\pi}_2$. We write

$$\psi(t_f) = \big\{\hat{\pi}_2\big|\psi_-(1);\hat{i}\big\rangle = \big(-i\pi\big)\left(\frac{i}{\pi}\right)\big|\psi(t_f);\hat{\pi}_2\big\rangle \tag{4.194}$$

$$= e^{(i\omega t_f)\Phi^2} \quad , \tag{4.195}$$

where the projection back into $\mathcal{H}$ replaces, for some reason, t_i with t_f. This reason is likely a boundary condition that says the chirological and chronological wavefunctions have to be equal at $\hat{\pi}$-sites. Information about t_i is preserved, if required, through $\Delta t{=}t_f - t_i$.

At this point, we use the ω_n notation from equations (4.178-4.179). In reference [9], ω_n is defined [9] such that $\omega_n = \omega\Phi^n$. This reflects $\hat{M}^3$ that increases the level of $\aleph$ by one. Here, we have included the $\varnothing$ component which agrees with the level increasing by two so we should introduce the definition

$$\omega_n = \Phi^{2n}\omega \quad , \tag{4.196}$$

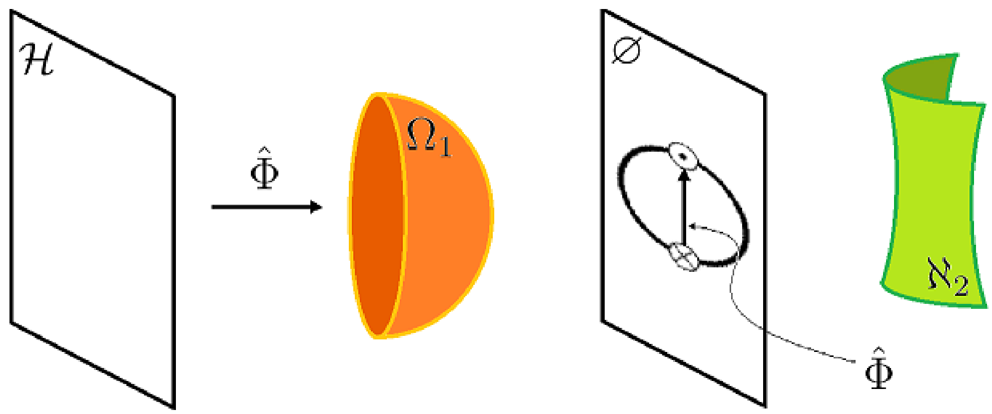

Figure 89: This figure shows where we can obtain two copies of $\hat{\Phi}$ during $\hat{M}^3$ such that the level of $\aleph$ is increased by two across each unit cell. The perpendicular directions of the $\hat{\Phi}$ in this figure are evocative of the rotation operation depicted in figure 58.

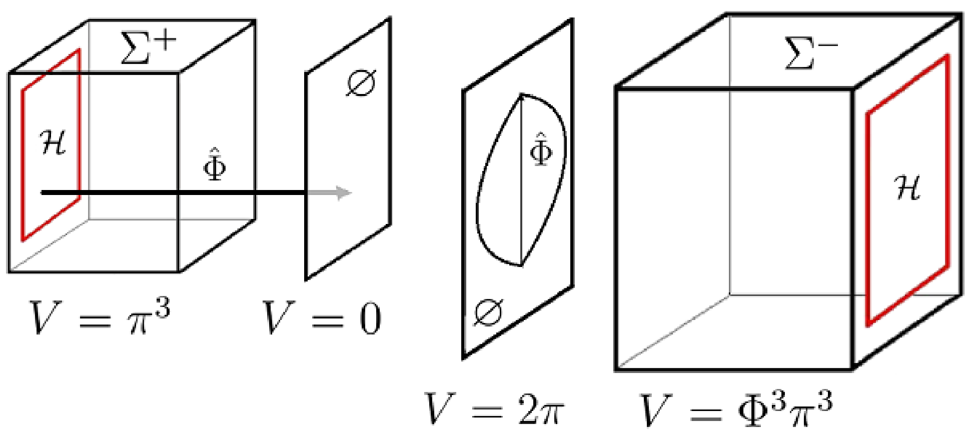

Figure 90: This figure shows where we can obtain $\alpha_{MCM} = 2\pi + (\Phi\pi)^3$ via the manner of rarefaction demonstrated by figures 33 and 34. A rotation of the second $\hat{\Phi}$ about the first $\hat{\Phi}$ axis will directly implement the torsion so this is a very natural representation. It is evocative of figure 91.

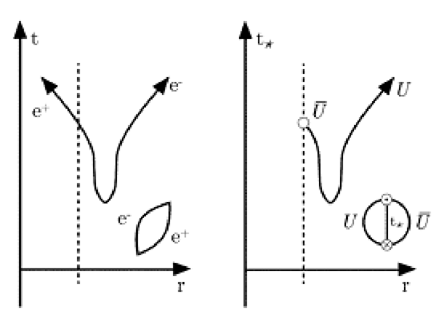

Figure 91: This figure from reference [7] (excerpted in part from reference [17]) demonstrates the general principle of $\hat{\Phi}$ pointing to, and lying within, a worldsheet.

so that

$$\psi(t_{\text{screen}}) = \big\{\hat{\pi}_2\big|\psi_-(1);\hat{i}\big\rangle = e^{i\omega_1 \Delta t} \quad , \tag{4.197}$$

is obtained. We have used the operators $\{\hat{e}_\lambda|$ here but we could just as well use the $\hat{\mathcal{P}}$ formalism so that $|\hat{e}_\lambda\}\{\hat{e}_\lambda|$ appears instead. Then, where $\hat{\pi}_2$ is used in equation (4.197), we could examine the property $\hat{\mathcal{P}}^2 = \hat{\mathcal{P}}$ demonstrated in section IV.3 as it relates to the psi-rangle/psi-langle non-unitary butterfly operator. Indeed, when the projection operator into the $\hat{\pi}_1$-site is the same as the projection operator into the $\hat{\pi}_2$ site we can preserve the information about which is which by associating $\hat{\pi}_1$ with t_i and, as in equation (4.197), $\hat{\pi}_2$ with t_f.

Equation (4.197) is exactly what the wavefunction should look like at t_f on $\hat{\pi}_2$ when we write the initial frequency as ω_0. There is a normalization that must be introduced to conserve energy under the change of frequency, but we have achieved the mechanism through which $\hat{\pi}_n$ waves will not form interference patterns with $\hat{\pi}_m$ waves when $m \neq n$. If we repeated the process to derive

$$\big\{\hat{\pi}_3\big|\psi_-(1);\hat{i}\big\rangle = e^{i\omega_2 t_f} \quad , \tag{4.198}$$

then it is clear that the two waves will not form an interference pattern because their frequencies are different. Furthermore, when we say that there needs to be some normalization of the frequency to conserve energy, we should consider the MCM solution for dark energy [2, 7, 54]. Acceleration into the future indicates a changing passage of time so we can implement the energy normalization and set a constraint on dark energy when

$$\omega_1 = [\omega_0]\frac{[\text{rad}]}{[\sec(\hat{\pi}_1)]}[\Phi^2] \qquad \longrightarrow \qquad \omega_1 = [\omega_0]\frac{[\text{rad}]}{[\sec(\hat{\pi}_2)]}[\Phi^2] \quad , \tag{4.199}$$

where

$$[\sec(\hat{\pi}_1)] = [\sec(\hat{\pi}_2)][\Phi^2] \qquad \Longrightarrow \qquad \omega_0 \quad \longrightarrow \quad \omega_0 \quad . \tag{4.200}$$

IV.7 Fundamentals of Hypercomplex Analysis

Real physical systems are usually too complicated to compute by hand so we use computers to compute expectation values for comparison with experiment. Many a graduate student has been frustrated by seemingly impossible homework problems only to have another student or professor later say, “Here one must use the small angle approximation,” or, “Here one must take the approximation as a Taylor series in arbitrarily few terms to get an answer that is good enough for full credit.” Even then, homework problems are often trivial compared to real life applications at the edge of what can be done. For instance, one such problem might be to consider the electric field above a finite charged conductor with irregular, asymmetric geometry, and then derive the equations of motion of a charged particle in its vicinity moving with some velocity $\dot{\vec{r}}$. The geometry is irregular so it will be impossible derive the analytic form of the electric field (due to the irregularly distributed charge.) Whatever the field's equations are, the field is a vector field. At each point in space, a vector is anchored that points in the direction of the electric force that will be exerted on a positive electric charge located there. Even if we are somehow able to write the field equations, likely with finite element numerical methods, it still remains to compute the second order differential equation $\vec{F}=q(\vec{E}+\dot{\vec{r}}\times\vec{B})=m\ddot{\vec{r}}$ (and even when we compute it, we have ignored the Abraham–Lorentz force.) At that point, and beyond the approximations that were involved in deriving $\vec{E}$ to begin with, we would resort to more numerical methods to approximate the motion of the particle in question. Such methods, likely Runge–Kutta methods, ask where the particle is and what the field is at that point, and, although the motion is continuous, we will say that the particle moves some discrete step in the direction of the field vector. We set the step as a very small increment and then approximate the total motion as the sum of very many small steps. Among the downsides is that each discrete step necessarily ignores all the field vectors between teach step's initial and the final positions. If the step is small enough, however, and then numerical methods conforming to this prescription are generally an excellent approximation.

Electromagnetic homework problems live in flat space so the the small step of discretized approximate motion always stays inside the manifold that represents all of space. If we were computing gravitational equations of motion in curved space instead of Euclidean electric motion then the tangent vectors would point outside of spacetime and this represents another source of error that needs to be minimized. One tool for solving this problem is known as the exponential map and, after a few more fundamental concepts, the exponential map will serve as our segue into numerical analysis for the MCM at the end of this section. The exponential map is a map from a manifold's tangent space back to the manifold itself. In general, it must be noted that, when we develop the theory, we are like an artist scribbling an impression, refining it, developing it, and then crafting it (hopefully) into a masterpiece, and then we put it through the wood chipper get a computer to accept it is input.

One common place of approximation in quantum theory is to calculate the amplitude of small oscillations around some valley in the energy landscape. Higgs does so in his seminal paper [34]. Therefore, consider the minimum of the harmonic oscillator potential $V=kx^2/2$ in the conformal coordinates $x\in(-\pi/2,\pi/2)$. The potential spans all of space $x'\in(-\infty,\infty)$ and the conformal relationship is $x'=\tan(x)$. If we want to consider the region close to $x=0$, as per the MCM methodology, then we should also consider the hypercomplexly infinitesimal neighborhood around that point. The coordinates of that neighborhood are some other conformal zenith $y\in(-\pi/2,\pi/2)$ on another level of $\aleph$ such that we assemble two co-$\hat{\pi}$s with the domain of the potential and the domain of the hypercomplexly infinitesimal neighborhood around $x=0$, as per the prescription developed in reference [8]. These two coordinates x and y should be related through the hypercomplex inversion map $x=1/y$ from which it follows that the small angle approximation in x fails completely in y. However, where the ordinary relationship for hyperreal numbers ${}^*\mathbb{R}$ is $\infty=1/\epsilon$, which we would write as $\hat{\Phi}^1=1/\hat{\Phi}^{-1}$, we have introduced the intermediate level $\hat{\Phi}^0$ in ${}^\star\mathbb{C}$ where $\hat{\Phi}^{j+2}=\aleph_0\hat{\Phi}^{j+1}$ and

$\hat{\Phi}^{j+1}=\aleph_0\hat{\Phi}^j$ so there is room to let the small angle approximation fail without completely destroying physics in the hypercomplexly infinitesimal neighborhood around the minimum. Indeed, it is better, here, to write the inversion map with the notation $x\hat{\Phi}^j \mapsto 1/y\hat{\Phi}^{j-1}$ than as an equation $x = 1/y$ which does not properly account for hypercomplexity. The quantum harmonic oscillator is usually a student's first introduction to zero point energy and, in the MCM, we might associate that energy with the internal degrees of freedom of the hypercomplexly infinitesimal neighborhood around the energy minimum. Often one asks the question, "Is it possible to tap the zero point energy?" Perhaps an improved analysis of the small oscillations about the lowest $n=1$ energy eigenstate will yield new insight on this important question.

Another application of complexity in numerical analysis relates to the representation of a physical continuum with a discrete set of grid points. Consider two computational grid representations of de Sitter space and anti-de Sitter space respectively. Every set of grid points contains its first and last point so where does the topological nuance go when AdS contains its boundary at infinity but dS does not? Where does this topological degree of freedom to either include or not include a boundary at infinity go when we simply compute hyperbolic and spherical field theories on topologically equivalent grids of discrete points in spacetime? To see the implication of the topological equivalence of all sets of grids points, consider a 1D simulation on 100 evenly spaced grid points x_j on the unit circle. We can enforce U(1) symmetry on the interval $\theta\in[0,2\pi]$ by maintaining a boundary condition $x_1 = x_{100}$. When the angular coordinate ranges from $[0,2\pi)$ we can put the first grid point at $\theta=0$ but we cannot put another grid point at $\theta=2\pi$ because that point is not in the domain of θ. The last grid point will appear at $\theta=99\epsilon=2\pi-\epsilon$. θ does not contain its boundary at 2π so we are forced to define our point lattice on the closed interval $\theta\in[0,2\pi-\epsilon]$. Here, we have completely lost the feature of the system in question that it does not contain its boundary. Where did the interval $\theta\in(99\epsilon,2\pi)$ go? Our simulation leaves a small gap that is usually considered negligible but should not be considered so in the analysis of $^\star\mathbb{C}$. In figure 83, we have demonstrated how qubits stored in the small gap at the end of the grid can be shuffled into the primary grid domain with MCM operations so in hypercomplex numerical analysis we should account in the grid for the openness or closedness of the underlying analytical topology.

Consider a simple application of the topological discrepancy between closed or open spacetimes and sets of grid points which are always closed. Spherical coordinates can never be deformed to cover a Cartesian manifold because the angles of intersection of the rectilinear coordinates at the corners of the considered region are not preserved in the polar coordinate representation. However, we can increase the conformal parameter Ω_{conf} to such a degree that all of the deviation between a radially charted manifold and the Cartesian manifold is contained in the missing interval $\theta\in(99\epsilon,2\pi)$. Figure 92 shows the principle through which it is possible to increase Ω_{conf} until the apex point is squeezed arbitrarily tightly into the corner. If the deviation from rectangularism is negligible everywhere in the grid except for the last box beyond x_{100} and below y_1 then we are on firm ground to say that the rectangular grid representation is a very good approximation for the curve $x^\mu(\theta)$.

Here, we suggest that, by including an extra abstract grid point for the apex point, all the tools of conformal transformation will become exactly available in the realm of numerical analysis. All of the information that would be truncated beyond the last grid point can be

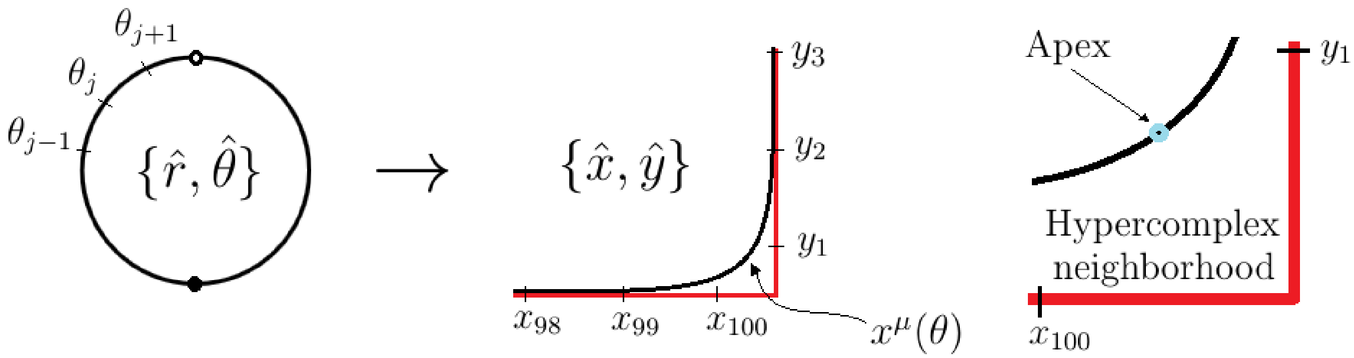

Figure 92: In the analytical realm, there is no conformal transformation between the circle and the square. In the realm of numerical approximation, we can use a large conformal scaling factor to squeeze the discrepancy into the region hidden beyond the final grid point. It will be possible to modify numerical computation algorithms such that they include one more abstract grid point which will correspond to $\hat{\Phi}$ pointing from, in this case, (x_{100}, y_1) to the non-grid apex point which contains information stored on another level of $\aleph$.

encoded on the apex point with arbitrary multiplectic structure. We should define the last grid point, the new hypercomplex grid point, such that $\hat{\Phi}$ points to the location of the apex of the curve from the final grid point. The apex point is the point that would eventually end up in the corner if such a conformal rescaling were possible. On that point, we can encode all the information about the real curvature of $x^\mu(\theta)$ that is lost by approximation as a set of rectangularized grid points. In finite analysis, the contributions of the small features that are ignored between conformal manifolds and rectangular grids can, in most cases, rightly be taken as insignificant. When Arnowitt, Deser, and Misner have used the Cartesian coordinates for the surface at spacelike infinity [46], they have ignored corners of the type shown in figure 92. They have rightly done so in this context because that last element is insignificant as a contribution to the total energy of the universe. However, there is room in hypercomplex analysis to define the opposite case where the corner cannot be ignored. For instance, we might desire to put an infinite amount of matter-energy in the corner to model the MCM dark energy interaction between a universe and another universe on a higher level of $\aleph$. In transfinite analysis, we can encode hypercomplex qubits on these missing elements such that they are insignificant on one level of $\aleph$ but might completely dominate on another level of $\aleph$. Note the global consistency of the most general case where a grid of Cartesian points is taken between two apex points: the apex point near x_1 is like $\hat{\Phi}^j$ pointing from a lower level of $\aleph$ to the set of grid points x_k on the j^{th} level of $\aleph$, and the other apex point is like $\hat{\Phi}^{j+1}$ pointing to one or more grid points on a higher level of $\aleph$ from x_{100}. Therefore, we see how the topological remainder that cannot be moved onto a computer without some special accommodations can have a lot of relevance regarding the outcome achieved by a given method of approximation. The MCM application of the apex principle arises at least when we want to join evolution in boundaryless Ω with continued evolution in bounded $\aleph$.[1] This joining will likely be accomplished through the a singularity at $\chi^5_\pm = \pm\infty$ but it is impossible to represent a singularity with set of grid points... unless we

[1]The 3-space in AdS is strictly bounded but the time axis is unbounded so $\aleph$ does not have one cohesive boundary $\partial\aleph$ with regards to chronological time. However, the change of topology O(3,2) $\rightarrow$ O(4,1) can likely be used to define $\partial\aleph$ relative to chirological time.

consider the hypercomplexly infinitesimal neighborhood around the singularity. Therefore, there is reason to consider the relevance of the abstract grid point to representations of the singular point $p \in \Sigma^{\varnothing}$.

Consider an application in which flat space is defined as the superposition of spherical and hyperbolic spaces. Hyperbolic space includes its boundary so a grid of points does have the correct topology. To define the superposition we might simply put another set of grid points corresponding to de Sitter space between the Anti-de Sitter grid points. As long as the first and last grid points corresponding to the combined flat space are AdS grid points, we do not invoke any kind of topological discrepancy and it is very natural to say that $\hat{\Phi}$ connects the first AdS grid point with the first dS grid point, and likewise for the last points. For example, if the AdS grid points are $\{0, 1, 2, ...100\}$ then we might take the dS grid points as $\{1/2, 3/2, ...199/2\}$. Then the relationship between the density of grid points in Minkowski space $\{0, 1/2, 1, 3/2, ...199/2, 100\}$ and the contributing dS and AdS spaces is the same as the density of terms in the Euler formula. The Euler formula is proven by showing that half of the terms in the series expansion of e^{ix} are like the sine series and the other half are like the cosine series.

Consider some trajectory coming into the $[0, 2\pi]$ or $(0, 2\pi]$ region (the 1D grid representation of $\mathcal{H}$) at the end of a trajectory in the bulk which reaches its asymptotic behavior by entering the planar grid region. When we have let $\hat{\Phi}^j$ come into the set of grid points from a lower level of $\aleph$, we now consider the trajectory along the connection between adjacent levels. The trajectory begins on a lower level of $\aleph$ and then enters the region represented with grid points via the $\hat{\Phi}$ object connecting some conformal apex to the grid itself. We say the behavior is asymptotic because the bulk spaces $\Sigma^{\pm}$ do not include their boundary at $\mathcal{H}$ and, presumably, the grid representation in question is that of $\mathcal{H}$. As an example, consider a simulation of the lifetime of the universe on a set of grid points along the x^0 axis. In very many cases, the first grid point will lie after the Planck and inflationary epochs. Therefore, we could consider the inflationary period as corresponding to the curve in the region smaller than x_1. The leftward apex point in this scenario would represent the big bang. When the first grid point x_1 is already later than inflation, we will completely crop the inflationary period from the domain of the simulation and, therefore, also anything that might have happened before the big bang. Once reaching the asymptote representing the post-inflationary phase of the universe at the first grid point, the topology of the system is truncated such that there is never again any component of the manifold in the direction perpendicular to the line defined by the 1D grid points x_j. We say "never again" only in reference to the domain spanned by the grid; when we come the end of the grid and consider a second apex point, likely the big crunch, then once again we can consider a perpendicular direction. To restate this concept for clarity, the vectors that points from one non-apex grid point to another are always strictly in the x-direction but the vector that points from the first and last grid points to the apex points can have a component in a perpendicular direction. We can generate a 3-space in this way when the first apex point deviates in the y-direction but the final apex point deviates in the z-direction. In this example, the inflation field never contributes to what we compute for any grid point $x_j \in [x_1, x_{100}]$. We might compute the mass density of the universe at each grid point, or perhaps its optical opacity. Just as we can ignore the effects of inflation before the first grid point, we can add new effects beyond the last grid point. For now, consider only monotonic entry into the grid at the first grid point: inflation begins after the apex point but then smoothly tapers off before time $t{=}x_1$. Therefore, the topology has some small $d\theta$ where the real curve has to leave the computational manifold (the set of grid points.) Regardless of how tightly we squeeze "the circle" into "the corner," there will always be some deviation between the line defined by the grid points and the physical manifold but, for numerical purposes, we only require that the deviation is small compared to the spacing of the grid points. In the analytical sector, the deviation of the physical and computational manifolds can be represented with $d\theta$. The vector pointing from one grid point to the next along the time axis is $\hat{x}^0$ but to account for the curved area outside the boundary of the simulation, at some level we will need to add $d\theta$ so that we can recover the apex point that does not lie along the axis. This $d\theta$ refers the appearance of components in the $\hat{y}$- and $\hat{z}$-directions; it is not θ as in $x^\mu(\theta)$.

Note how capital Greek letters Θ and Φ show a line with its endpoints either within or beyond the interior of a circle. To discuss the features of the distinction between closed and open intervals, we can consider that Θ does include the line's endpoints but Φ does not. When we consider a straight line that either does or does not include its endpoints, all of the tangent vectors to that line are collinear with the line. From a geometric perspective, this is a self-evident property of straight lines: they are their own tangent spaces. When the topology of the space represented by the grid is such the boundary is not included in the space, then we need to include the apex point. The apex point can never be squeezed all the way down onto the axis defined by the grid so there exists some $d\theta$. Depending on how we define the convention, this $d\theta$ will introduce a dy or a dz, and that fully defines a second independent tangent vector. Therefore, the tangent space to a grid without apex points is 1D but the tangent space to the more realistic grid that does include the apex point(s) is at least 2D. Therefore, the inclusion of the apex point directly motivates a new realm of complexity through the expansion of the tangent space.

In numerical analysis, the forward approximation for the first derivative is

$$\dot{f}(x) \approx \frac{f(x_{j+1}) - f(x_j)}{|x_{j+1} - x_j|} \quad . \tag{4.201}$$

Our purpose here is to compute the "tangent vectors" to the set of grid points to get an idea of the topology of the grid points. This will be useful for comparing the grid topology to the topology of the non-discretized progenitor manifold where analytically exact solutions can exist. To examine the grid itself, we set $f(x_j) = \vec{x}_j$, and the tangent vector at every grid point is

$$\dot{\vec{x}} \approx \frac{\vec{x}_{j+1} - \vec{x}_j}{|x_{j+1} - x_j|} = \frac{\epsilon\,\hat{x}}{\epsilon} = \hat{x} \quad . \tag{4.202}$$

How do we take the forward derivative approximation at the last grid point? This is impossible and usually a little hand waving is introduced such that we take the backward approximation

$$\dot{f}(x) \approx \frac{f(x_j) - f(x_{j-1})}{|x_j - x_{j-1}|} \quad . \tag{4.203}$$

However, when we add another grid point for the apex point we can still use the forward formula at x_{100}. Now the tangent vector at that last point will not be collinear with the other vectors and this means that the tangent space to the computational domain has two basis vectors and must sweep out a plane. Using the last grid point and the apex point, the forward derivative formula is

$$\dot{\vec{x}} \approx \frac{\vec{x}_{\rm apex} - \vec{x}_{100}}{|x_{\rm apex} - x_{100}|} = \frac{\big[\big(100+\varepsilon_x\big)\epsilon\,\hat{x} + \varepsilon_y\,\hat{y}\big] - 100\epsilon\,\hat{x}}{\beta} \quad , \tag{4.204}$$

from which it immediately follows that

$$\dot{\vec{x}} \quad \propto \quad \hat{x} + \hat{y} \ \ . \tag{4.205}$$

We have demonstrated the concept for the grid with $f(x) = \vec{x}$ but a problem is that we generally do not have a value for $f(x_{\rm apex})$. If we simply ignore the final endpoint $x_{\rm apex}$ then, rather than an open interval, we have modeled a closed interval with one less grid point. To make the distinction between the closed and open intervals in the numerical language, we can say that the tangent vector at the endpoint has a component that is perpendicular to the tangent space of the closed interval, as in equation (4.205). Then the tangent space increases from a line to a plane or from the plane to a space, *etc.* The tangent space of the grid space that includes its boundary has co-dimension $1N$ or $2N$ with the tangent space of the grid space that does not include its boundary. Here N is the number of grid dimensions, $N = 1$ in the example we have discussed, and we say $1N$ or $2N$ because the extra direction at the apex point can be the same at each end of the grid or those directions can be perpendicular to $\hat{x}$ and also mutually perpendicular as $\hat{y}$ and $\hat{z}$.

Penrose suggests [52] that completing the physical manifold with a point at conformal infinity is useful because we can consider the behavior of fields at conformal infinity $\mathscr{I}$. Therefore, when we complete the physical manifold with a conformal point, we necessarily induce the extra point $x_{\rm apex}$ and we should consider that the apex points at conformal infinity are what will distinguish a hypercomplex grid from an ordinary one. By building rotations around conformal infinity, we can rotate between, perhaps, a set of grid points representing $\mathcal{H}$ and another representing $\Sigma^\pm$ such that the analytical intractability of the disconnection of $\mathcal{H}$ from $\Sigma^\pm$ is not an issue at all in the discretized domain of computer language. Even when $\mathcal{H}$ and $\Sigma^\pm$ are separate, the last grid point of $\mathcal{H}$ is next to the first grid point in Σ^+, and this facilitates continuation in the manner of any adjacent grid points. The mathematical operation required to twist a tangent vector out of the plane is rotation (or possibly torsion which we not treat here) and we need, in this example, to get the tangent vector at the last $\mathcal{H}$ grid point to point to the first Σ^+ grid point or an intermediate apex point. This will be important if we do not have information about the apex point *a priori* but instead want to determine it from the information generated on the grid; we might devise a rotation operator $\hat{\mathcal{R}}$ that takes the last tangent vector and makes it point to the apex point which, in this example, holds a multiplex containing all the grid points of Σ^+. Without regard for the density of the grid points, it will only be the final computer language tangent vector between $\mathcal{H}$'s last grid point and the apex point that expands the tangent space, as in equation (4.205). The apex point can be squeezed arbitrarily tightly into the corner so the angle of deviation assigned to that rotation out of the plane should be $d\theta$, which we already know is appended to the generator of rotations when writing a 3D rotation operator $\hat{\mathcal{R}}_z := \hat{L}_z$. Therefore, we might expect that the apex point is associated with the Cauchy C curve and quantum phenomena will be obtained by going "through the big end" of classical simulations when we join sets of grid points on an apex point.

Let us examine how we can derive this second basis vector $\hat{y}$ for the tangent space to x_j without knowing about the apex point before hand. The formula for an infinitesimal rotation operator is

$$\hat{\mathcal{R}}_z(d\theta) = \hat{1} - i\,d\theta\hat{L}_z + O(d\theta\,d\theta') \quad . \tag{4.206}$$

For the purposes of hypercomplexity, we want to examine the feasibility for constructing this operator using nothing but the ontological basis $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$. Equation (4.206) can be replaced with the sum of the identity with a linear (trivially conformal) part and a symplectic part. The $d\theta\,d\theta' \cong dx^\mu dx^\nu$ term motivates us to replace the error term with $\hat{\Phi}$. We will say $\hat{\Phi}$ should have a symplectic form on it so that it can point to the surface at spacelike infinity where the normal vector points either back inside the universe or outside of the universe altogether. Earlier in this section, we proposed to use $\hat{\Phi}$ to point in the direction of the apex point so that it could be defined without including $\hat{y}$ or $\hat{z}$, and now we want to directly replace the error term in the definition of $\hat{\mathcal{R}}$ with $\hat{\Phi}$. The notation $O(d\theta\,d\theta')$ means that the first two terms in equation (4.206) are correct up to terms of order $d\theta^2$ and we propose to build an ontological rotation

$$\hat{\mathcal{R}}_z(d\theta) = \hat{1} - i\,d\theta\hat{L}_z + \hat{\Phi}(dx^\mu \wedge dx^\nu) \quad , \tag{4.207}$$

that does not have an error term. Equation (4.207) puts each of the three terms on three different levels of $\aleph$ in the normalization that $\hat{\Phi} = \hat{\Phi}^{j+1}$, $\hat{1} = \hat{\Phi}^j$, and $d\theta := \hat{\Phi}^{j-1}$. This is a favorable structure in hypercomplexity because it can specify rotations as they relate to one lower and one higher level of $\aleph$. The symplectic form that we have assigned to $\hat{\Phi}$ offers a lot of promise for replacing the $O(d\theta\,d\theta')$ term. We need a term like $d\theta^2$ and we have it in

$$\hat{\Phi} := dx^\mu \wedge dx^\nu \quad \equiv \quad \begin{pmatrix} 0 & dx^\mu\,dx^\nu \\ -dx^\mu\,dx^\nu & 0 \end{pmatrix} \quad . \tag{4.208}$$

This multiplectic definition is presented to define $\hat{\Phi}$ such that it has a symplectic form at its tip. The symplectic form attached to the tip of $\hat{\Phi}$ will be useful for moving beyond spacelike infinity, for steering through non-trivial covering spaces or for steering through the cosmological lattice in general, and also for pointing to bloop points where field lines break as in section IV.4, or where new frequencies appear, as in reference [10]. Indeed, we have shown that the symplectic points embedded on the helix in section IV.5 appear at the points $N\Phi$. We can construct $\mathbb{R}$, which is the covering space in question, with the embedded symplectic points by taking a stack of $\hat{\Phi}$ where each $\hat{\Phi}$ has a symplectic form at its tip, as in equation (4.208). Furthermore, when $dx\wedge dy\neq dy\wedge dx$, we see some small hint of why the quaternion rotation uses two operators instead of just one. Perhaps two operators are needed to define the orientation in a way that is not required for a non-symplectic 2-form like $d\theta d\theta'=d\theta' d\theta$.

The normal analysis of rotations discards the $\mathrm{O}(d\theta\, d\theta')$ term and only considers the linear approximation

$$\hat{\mathcal{R}}_z(d\theta) = \hat{1} - i\, d\theta \hat{L}_z \quad , \tag{4.209}$$

which we could write with $\hat{\Upsilon}=\hat{\imath}+\hat{2}\hat{\pi}\hat{\Phi}$ such that

$$\hat{\mathcal{R}}_z(d\theta) = \hat{\Upsilon}\cdot -\hat{\imath} = 1 - \hat{2}\hat{\pi}\hat{\Phi}\cdot\hat{\imath} \quad . \tag{4.210}$$

This gives the general form of $\hat{\mathcal{R}}_z(d\theta)$ but we must make some accommodation for the extra two hats after $\hat{\imath}$ is dotted into the second polynomial term $\hat{2}\hat{\pi}\hat{\Phi}$. For that, we can use the extra two hatted objects to construct a dyadic. The dyadic is a projection operator so we may evaluate $\hat{2}\hat{\pi}\hat{\Phi}\cdot\hat{\imath}$ as dot product of $\hat{\imath}$ with a certain projection of one of $\hat{2}$, $\hat{\pi}$, or $\hat{\Phi}$. Another issue with equation (4.210) is that this is only a representation of the linear approximation of $\hat{\mathcal{R}}_z(d\theta)$. We want to obtain an exact expression of the form of equation (4.207). To that end, consider a different definition

$$\hat{\mathcal{R}}_z(d\theta) = \hat{1} + \hat{\Upsilon}^\mu \quad , \tag{4.211}$$

where we introduce

$$\hat{\Upsilon}^0 = \hat{\imath} + \hat{\Phi}\hat{2}\hat{\pi} \tag{4.212}$$

$$\hat{\Upsilon}^1 = \hat{\Phi} + \hat{2}\hat{\pi}\hat{\imath} \tag{4.213}$$

$$\hat{\Upsilon}^2 = \hat{2} + \hat{\pi}\hat{\imath}\hat{\Phi} \tag{4.214}$$

$$\hat{\Upsilon}^3 = \hat{\pi} + \hat{\imath}\hat{\Phi}\hat{2} \quad . \tag{4.215}$$

Clearly $\hat{\Upsilon}^1$ is the operator we are looking for. It allows us to write

$$\hat{\mathcal{R}}_z(d\theta) = \hat{1} + \hat{2}\hat{\pi}\hat{i} + \hat{\Phi} \quad , \tag{4.216}$$

which is of the exact form of equation (4.207). Here, we simply need to solve a matching problem on

$$\hat{2}\hat{\pi}\hat{i} \quad \longleftrightarrow \quad -i\, d\theta \hat{L}_z \quad . \tag{4.217}$$

Before examining the matching, consider that $\hat{\mathcal{R}}$ needs to be able to rotate about any spatial axis if there is any hope of devising a working rotation operator from the outlandish formulation in equation (4.211). Multiple rotation axes are easily accommodated with

$$\hat{\mathcal{R}}_x(d\theta) = \hat{1} - \left(\hat{2}\hat{\pi}\right)\hat{i} + \hat{\Phi} \tag{4.218}$$

$$\hat{\mathcal{R}}_y(d\theta) = \hat{1} - \left(\hat{i}\hat{2}\right)\hat{\pi} + \hat{\Phi} \tag{4.219}$$

$$\hat{\mathcal{R}}_z(d\theta) = \hat{1} - \left(\hat{\pi}\hat{i}\right)\hat{2} + \hat{\Phi} \quad . \tag{4.220}$$

Since each dyadic is such that $\hat{e}_\mu\hat{e}_\nu \neq \hat{e}_\nu\hat{e}_\mu$, equations (4.218-4.220) give two possibilities for each rotation. We could say that this justifies an extension to the quaternion rotations where two operators would uniquely determine a unique operation or we could say that the $\hat{e}_\alpha\hat{e}_\beta \neq \hat{e}_\beta\hat{e}_\alpha$ freedom is associated with clockwise or counterclockwise rotations. We could even let the operator be such that one dyad gives the instruction to rotate and the other gives the instruction to swap the tip with the anchor point and then do the rotation. Before we decide any of that, we need to determine the matching in relationship (4.217).

We have previously written the $\hat{L}_z$ operator in polar coordinates but, to accommodate the symmetries required for a rotation about an arbitrary spatial axis, we should work in Cartesian coordinates. When we write

$$\hat{L}_z = -i\frac{\partial}{\partial\phi} \quad , \tag{4.221}$$

in the $\{z, \phi, \theta\}$ coordinates, there are no ϕ or θ axes that we might rotate about because the $\hat{\phi}$ and $\hat{\theta}$ unit vectors point in different directions at different points in space. Instead, we will consider

$$\hat{L}_z = -i\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) \quad . \tag{4.222}$$

$\hat{L}_z$ looks like the total differential of some generic $f(x, y)$

$$df = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy \quad . \tag{4.223}$$

Indeed, the dyadic, being composed of only two pieces, is generally amenable to representation as $f(x, y)$. The relationship between equation (4.222) and equation (4.223), up to the function f itself, is mostly that the dx and dy have been integrated over

$$\hat{L}_z \overset{?}{=} -i\frac{\partial}{\partial y}\int dx + i\frac{\partial}{\partial x}\int dy \quad . \tag{4.224}$$

Therefore, we might consider an exotic qubit $\hat{d}f$ such that $\hat{L} := \hat{d}$ and $\hat{d}f$ integrates over dx and dy. Perhaps there is a natural association with the weird quantum operator for the observable quantization of angular momentum $\hat{L}_z$ in that it represents a total differential that has been half integrated over. Equation (4.224) is then of the same form as the remainder of the MCM inner product. When $|\vartheta\rangle$ and $|\psi\rangle$ are Dirac orthogonal states

$$\langle\vartheta|\psi\rangle = \sum_{j=0}^{\infty}\sum_{k=1}^{\infty} c_k^* c_j \delta'_{jk}\int \psi_k^{-\,*}\psi_j^+\,dx = \big(\mathsf{null}\big)c_0\delta'_{jk}\int\big(\mathsf{null}\big)\psi_0^+\,dx \quad , \tag{4.225}$$

represents something half integrated over. We only mention this as an aside because the partially integrated form of df in equation (4.224) is in similar in quality to equation (4.225).

Now, we continue with the analysis of $\hat{\mathcal{R}}_z$. With the Cartesian $\hat{L}_z$ from equation (4.222), we may obtain from equation (4.206)

$$\hat{\mathcal{R}}_z(d\theta) = 1 - \,d\theta\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + O(d\theta\,d\theta') \quad . \tag{4.226}$$

One of the interesting features of $\hat{\mathcal{R}}_z(d\theta)$ is that the quantum mechanical angular momentum operator $\hat{L}_z$ is the generator of classical rotations. From equation (4.222), we can write

$$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x \quad , \tag{4.227}$$

where the idea in quantum theory is that $\hat{p}_i \equiv \partial_i$. We assign special significance to z when we say that there can be only a simultaneous eigenbasis in quantum theory for the total angular momentum and the projection onto one of the three spatial axes, call it z. Therefore, we have judiciously arranged equation (4.220) such that $\hat{2}$ is outside of the dyadic in $\hat{\mathcal{R}}_z(d\theta)$. The quantum operator consists of two terms with like structure, as in equation (4.227), and $\hat{2}$ is the operator that splits one term into two. This is what it means to choose $\hat{2}$ judiciously: if we look at $\hat{\imath}$ or $\hat{\pi}$ outside the dyad in equations (4.218-4.219) then we would not expect an inherent polynomial of two like terms. We could choose to split any term into two terms if desired, but that structure is only innate to $\hat{2}$.

Beginning to develop the requisite connection $-id\theta\hat{L}_z \leftrightarrow (\hat{\pi}\hat{i})\hat{2}$, we write

$$-d\theta\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right) \quad\longleftrightarrow\quad (\hat{\pi}\hat{i})\hat{2} \ \ . \tag{4.228}$$

Where does the differential angle $d\theta$ come from? To include it we should revise the ansatz in equation (4.211) such that

$$R_z(d\theta) = \hat{1} + \delta\hat{\Upsilon}^1 \ \ , \tag{4.229}$$

where δ indicates the small variation. As an illumination of relevant the principles, consider that we have added an ontological resolution of 0 to the ontological resolution of the identity such that

$$\hat{1} = \frac{1}{4\pi}\,\hat{\pi} - \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{i} \ \ , \qquad \text{and} \qquad \hat{0} = \hat{1} + \big(-\hat{1}\big) \ \ . \tag{4.230}$$

Therefore, when we consider the infinitesimal rotation $R_z(d\theta)$, we might take a variation about either of zero or one. We can examine $R_z(d\theta)$ as the variation around one with $R_z(0) = \hat{1}$ or we can examine it as the variation around zero when $R_z(0) = \hat{1} + \hat{0}$. In general, this principle illustrates the connection between multiplication and addition that is likely amenable to group theoretical applications. $\hat{0}$ introduces additive complexity and $\hat{1}$ introduces multiplicative complexity. When we attempt to recover relationship (4.228), we can write the linear approximation to $\hat{\mathcal{R}}$ as

$$R_z(d\theta) = \delta\hat{\Upsilon}^0_z = \delta\left[\hat{i} + \big(\hat{\Phi}\hat{\pi}\big)\hat{2}\right] \ \ , \tag{4.231}$$

and we can write the total operator as

$$R_z(d\theta) = \hat{1} + \delta\hat{\Upsilon}^1_z = \hat{1} + \delta\left[\hat{\Phi} + \big(\hat{i}\hat{\pi}\big)\hat{2}\right] \ \ . \tag{4.232}$$

Equation (4.231) is our tentative representation of the first order approximation to $\hat{\mathcal{R}}$ and equation (4.232) is the tentative representation for the exact form of $\hat{\mathcal{R}}$. Equations (4.231-4.232) show the convention that rotation about the z axis is selected by taking $\hat{2}$ outside of the dyadic. Since we are not concerned with an ontological representation of the linear approximation to $\hat{\mathcal{R}}$, we will only work with equation (4.232).

The formula for the variation is $\delta J = J(u_0 + \delta u) - J(u_0)$, and we should not take the variation of the basis vectors but only the variation of the dyadic. We can represent equation (4.232) in the convention where $\hat{e}_1 = \hat{\pi}$, $\hat{e}_2 = \hat{\Phi}$, $\hat{e}_3 = \hat{2}$, and $\hat{e}_4 = \hat{i}$ as

$$R_z(d\theta) = \hat{1} + \delta \begin{pmatrix} 0 \\ 1 \\ \hat{i}\hat{\pi} \\ 0 \end{pmatrix} \quad , \tag{4.233}$$

so the J in the variation formula is $J(i,\pi) = \hat{i}\hat{\pi}$. Then

$$\delta\big(\hat{i}\hat{\pi}\big) = \big(\hat{i} + \delta i\big)\big(\hat{\pi} + \delta\pi\big) - \hat{i}\hat{\pi} = \hat{i}\,\delta\pi + \hat{\pi}\,\delta i + \delta i\,\delta\pi \quad . \tag{4.234}$$

Here, we ignore terms of order δ^2 so we have gone off on quite a tangent pursuing the ontological representation of the rotation. However, in section IV.5, we showed that $\mathbb{C}$ is simply constructed from $\hat{i}$ and $\hat{\pi}$ so, in some vague sense, we have taken the variation of the complex plane $\delta(\hat{i}\hat{\pi})$ wherein all quantum phases have the form of rotation operators. Here, we update equation (4.233) as

$$R_z(d\theta) = \hat{1} + \Big[\delta\big(\hat{i}\hat{\pi}\big)\hat{2} + \hat{\Phi}\Big] = \hat{1} + \big(\hat{i}\,\delta\pi + \hat{\pi}\,\delta i\big)\hat{2} + \hat{\Phi} \quad , \tag{4.235}$$

and this is what we are trying to put in the form of equation (4.207), which contains $-id\theta\hat{L}_z$. If we use the mapping

$$\hat{i} \quad \mapsto \quad -1 \tag{4.236}$$

$$\delta\hat{\pi} \quad \mapsto \quad d\theta \tag{4.237}$$

$$\hat{2} \quad \mapsto \quad \hat{L}_z \quad , \tag{4.238}$$

then we almost get the right form with

$$R_z(d\theta) = \hat{1} - i\,d\theta\hat{L}_z + \hat{\Phi} + \big(\hat{\pi}\,\delta i\big) \quad , \tag{4.239}$$

but we have the extra piece $\hat{\pi}\,\delta i$. This could be another hint that only quaternion rotations can be built from $\{\hat{i}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$; we might use two operators such that the extra piece gets canceled.

Before coming to that, it is not clear why $\hat{2}$ should be $\hat{L}_z$ or that we are even well motivated in the maps (4.236-4.238) to begin with. When we defined a similar looking set of maps for Einstein's equation, we were motivated by the coefficient of proportionality 8π but we have not demonstrated a corresponding piece of evidence in this case. Therefore, consider the relationship between the angular momentum operator and Pauli spin matrix commutation relations

$$[\hat{L}_x, \hat{L}_y] = i\varepsilon_{abc}\,\hat{L}_z \quad , \qquad \text{and} \qquad [\sigma_a, \sigma_b] = 2i\varepsilon_{abc}\,\sigma_c \quad . \tag{4.240}$$

The Pauli matrices are the half-integer angular momentum operators and we are not too far from firm ground when want to associate the dyadic $(\hat{e}_\alpha \hat{e}_\beta)$ with the Pauli matrices. The Pauli matrices are the simplest matrices that have the requisite properties for a physical spin algebra but they are by no means the only set of matrices that can be used. Even when 2 appears in the commutator for the Pauli matrices, the mechanism is not so clear as the appearance of 8π in the MCM derivation of Einstein's equation but it does define some fundamental linkage between $\hat{2}$ and the quantum mechanical angular momentum operator $\hat{L}_z$. Perhaps the difference between $\hat{L}_z$ and $\hat{2}$ is the former constructs $\hat{\mathcal{R}}_z(d\theta)$ for an arbitrary rotation around the z axis but the latter is only used to construct $\hat{\mathcal{R}}_2(\Phi)$ which points exclusively to the apex point. Here, we have raised very many more questions tangentially, especially regarding the representation of Pauli matrices with ontological dyads, but we will remain in this section focused on $\hat{\Upsilon}$ before progressing into numerical analysis for hypercomplexity.

Regarding $\hat{\Upsilon}$, why should the operator

$$\hat{\Upsilon} = \hat{e}_\mu + \hat{e}_\nu \hat{e}_\rho \hat{e}_\sigma \quad , \tag{4.241}$$

be more important than

$$\hat{\Upsilon} = \hat{e}_\mu \hat{e}_\nu + \hat{e}_\rho \hat{e}_\sigma \quad , \qquad \text{or} \qquad \hat{\Upsilon} = \hat{e}_\mu \hat{e}_\nu \hat{e}_\rho \hat{e}_\sigma \quad ? \tag{4.242}$$

On this last question, we might propose that the form of equation (4.241) relates to even levels of $\aleph$ and equations (4.242) relate to odd levels. Another representation of $\hat{\Upsilon}$ is

$$\hat{\Upsilon} = \hat{e}_\mu + \hat{e}_\nu + \hat{e}_\rho + \hat{e}_\sigma \quad , \tag{4.243}$$

and we have already shown that this one is very important. This is the linear algebra version of $\hat{\Upsilon}$ and we might say that equation (4.241) is the Lorentz invariant version of $\hat{\Upsilon}$, or even that $\hat{\Upsilon} = \hat{e}_\mu \hat{e}_\nu + \hat{e}_\rho \hat{e}_\sigma$ is the twistor version. The intention here is only to call attention to these objects and not necessarily to define them. As the fundamental objects of the theory, we will definitely need to come up with answers for what these objects are and so, in prudence, we cite them here.

To sum up the above excursion into complexity consider the nicest MCM result regarding Einstein's operator

$$\widehat{8\pi} \quad : \quad T_{\mu\nu} \quad \mapsto \quad R_{\mu\nu} + g_{\mu\nu}\Lambda \quad . \tag{4.244}$$

In our analysis of the fundamentals, it is notable that while π shows up linearly and at third order in $\alpha^{-1}_{MCM} = 2\pi + (\Phi\pi)^3$, 2 only shows up linearly and Φ only shows up in its cube. It is further notable that 2 *does* show up at third order in the theory when we write

$$\widehat{2^3\pi} \quad : \quad T_{\mu\nu} \quad \mapsto \quad R_{\mu\nu} + g_{\mu\nu}\Lambda \quad , \tag{4.245}$$

and Φ shows up at first order in $\hat{\Phi}$. Therefore, when detractors say, "Prove it," we can write

$$\frac{2^3\pi^3}{\pi^2} = 2^3\pi = 8\pi \quad , \tag{4.246}$$

which is the general relativity operator. This operator is derived with the map $\widehat{2\pi} : f \mapsto \omega$ that is usually associated with the Fourier transform which we have not yet even begun to examine. However, 2π is known to be a spurious constant of the normalization of the Fourier transform with the inverse Fourier transform between the frequency domain and the angular frequency domain. After writing equation (4.246) for detractors, we can cite an extensive body of work [2, 7, 12, 11, 3, 9, 13, 49, 4, 5, 23, 10, 8] which defines a specific enough set of requirements for a new mathematical mechanism which will describe, by principle of analytic rigor, a new sector in physics. Thus it is demonstrated; detractors have no ground upon which to stand.

The angular frequency is also referred to as the wavenumber domain but when angular frequency and wavenumber are taken as the same, it disregards conformalism. The wavenumber domain uses a Cartesian chart but the angular momentum domain uses a radial coordinate chart. Therefore if one domain completed with its surface at conformal infinity is deformed into the other with its surface at infinity, then the corners in the Cartesian chart will be pinched into symplectic points in the polar coordinates. This is not allowed in conformal transformations so this distinction of the Fourier transform needs to be rigorously accounted for. Likely, the angular frequency domain can be associated with $\hat{\pi}$, and wavenumber with $\hat{\Phi}$.

As another aside about the complexity of the ontological objects, consider the place of the affine connection in general relativity. Relativity deforms the relationships between spaces and their tangent spaces such that the partial derivative does not transform as a tensor, and we have to introduce multiplectic "connection coefficients" $\Gamma^{\nu}_{\mu\lambda}$ to derive a "covariant" derivative

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda \quad . \tag{4.247}$$

If we write $\hat{\Upsilon}$ as

$$\hat{\Upsilon}^\lambda{}_{\mu\nu\sigma} = \hat{e}_\lambda + \hat{e}_\mu \hat{e}_\nu \hat{e}_\sigma \quad , \tag{4.248}$$

then we get a lot more complexity than the $\hat{\Upsilon}^\mu$ in equations (4.212-4.215) because any combination of $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ is allowed but we have only considered $\hat{\Upsilon}^\mu$ such that each object appears exactly once. To update the notation, we may write

$$\hat{\Upsilon}^1 \quad \equiv \quad \hat{\Upsilon}^\Phi{}_{ijk} = \hat{\Phi} + \hat{e}_i \hat{e}_j \hat{e}_k \quad . \tag{4.249}$$

Even here, there is more complexity because repeats are allowed in $\{i, j, k\}$ but none appeared in equation (4.213). We should represent the second term in equation (4.249) with one index of one type and two of the type that will contribute to the dyadic. As an ansatz, let

$$\hat{\Upsilon}^\lambda{}_{\mu\nu\sigma} = \hat{e}_\lambda + \Gamma^\mu_{\nu\sigma} \quad , \qquad \text{where} \qquad \Gamma^\mu_{\nu\sigma} = \big(\hat{e}_\nu \hat{e}_\sigma\big)\hat{e}_\mu \quad . \tag{4.250}$$

In equation (4.240), we showed how the commutators of the Pauli matrices and angular momenta are likely relatable through $\hat{2}$, and can arise from taking the two objects in parenthesis as a dyadic tensor matrix coefficient for $\hat{2}$. Specifically we have examined

$$\Gamma^2_{i\pi} = \big(\hat{\imath}\hat{\pi}\big)\,\hat{2} \quad . \tag{4.251}$$

We have analyzed a possible connection between the 3D rotation operator and the ontological basis and shown that the new connection has the same form as what is already called the connection $\Gamma^\lambda_{\mu\nu}$.

What about $\hat{M}^3$? Can it be represented in this way? It is not infeasible that some maps exist between $\{\partial_+, \partial_\varnothing, \partial_-\}$ and some configuration of $\{\hat{e}_\mu, \hat{e}_\nu, \hat{e}_\lambda\}$. Therefore, consider the most general operator

$$\big\langle \psi \big| \hat{M}^3 \big| \psi \big\rangle \quad \longrightarrow \quad \big\langle \psi \big| \hat{e}_\mu \hat{e}_\nu \hat{e}_\sigma \big| \psi \big\rangle \quad . \tag{4.252}$$

We can define a quaternion exponential map and an ordinary exponential map as

$$\hat{e}_\mu \hat{e}_\nu \hat{e}_\lambda \quad \mapsto \quad e^{-\mathbf{u}_\mu} M[\hat{e}_\nu] e^{\mathbf{u}_\lambda} \quad , \tag{4.253}$$

where $M[\hat{e}_\nu]$ is not a quaternion rotation operator or a 3D rotation operator, and

$$\hat{e}_\mu\hat{e}_\nu\hat{e}_\lambda \quad \mapsto \quad M^+[\hat{e}_\nu]e^{i\hat{H}t}M^-[\hat{e}_\lambda] \quad . \tag{4.254}$$

where $M^\pm[\hat{e}_\rho]$ are likewise not quaternion rotations or 3D rotations. This might be a demonstration of what it means to make an impossible calculation. If there are two exponential maps required, that can not be considered simultaneously, then how can we make sense of it? We will be compelled to include sufficient complexity in $\hat{M}[\hat{e}_\lambda]$ such that some unpaired qubits from $\hat{M}^3$ get paired with partners from $\hat{U}$ as per $\hat{\Upsilon} \equiv \hat{U} + \hat{M}^3$.

Equation (4.254) is the matrix exponential map with some extra pieces. In quantum theory, the Hamiltonian operator $\hat{H}$ is a square matrix and we use the definition

$$e^{\hat{H}} \quad \equiv \quad \sum_{j=0}^{\infty} \frac{\hat{H}^j}{j!} = \mathbb{I} + \hat{H} + \frac{1}{2}\,\hat{H}^2 + \frac{1}{6}\,\hat{H}^3 + \cdots \quad . \tag{4.255}$$

The scalar exponential map is the one that alternates lines with circles via $x \mapsto e^{ix}$ such that

$$e^x \quad \equiv \quad \sum_{j=0}^{\infty} \frac{x^j}{j!} = 1 + x + \frac{1}{2}\,x^2 + \frac{1}{6}\,x^3 + \cdots \quad , \tag{4.256}$$

but what is the exponential map for $e^{\vec{V}}$? $\hat{H}$ is a square matrix and any product of N square matrices will have the same dimensionality as $\hat{H}$. Likewise with e^x, a scalar to any power is still a scalar. When we write

$$e^{\vec{V}} \quad \equiv \quad \sum_{j=0}^{\infty} \frac{\vec{V}^j}{j!} = \hat{1} + \vec{V} + \frac{1}{2}\,\vec{V}\cdot\vec{V} + \frac{1}{6}\,\vec{V}\cdot\vec{V}\cdot\vec{V} + \cdots \quad , \tag{4.257}$$

there are a few problems. $\hat{1}$ doesn't make sense in that context, $\vec{V}$ is a vector but $\vec{V}\cdot\vec{V}$ is a scalar, and $\vec{V}\cdot\vec{V}\cdot\vec{V}$ is poorly defined. Nevertheless, the exponential map for vectors is an important object in general relativity where it takes the form

$$\exp_p(\vec{V}) \quad : \quad T_p\Sigma \quad \mapsto \quad \Sigma \tag{4.258}$$

as a map from a manifold's tangent space at a point $T_p\Sigma$ back to the manifold Σ itself. What is called the exponential map in general relativity is critically important in any simulation of general relativity and, naively, it is the map shown in equation (4.257) although one never uses that representation because of the problem in the notation.

In the realm of rigor, even hydrogen atoms curve flat space and the tangent vectors to the geodesics in the manifold containing the hydrogen will point outside of the manifold. This is important because the geodesics of non-symmetric natural geometries cannot be written in closed form and instead we approximate the path with the tangent vector. However, if the tangent vector points outside of the manifold then there is a problem using it to define the path which must lie exclusively within the manifold. One of the simplest (and best) methods for determining the path, often called a method of evolution, is the Gaussian method, also called Euler's method. Often one asks, "If particle A is at location B then where will it be later?" The Gaussian approximation answers this question by taking the tangent vector to a point and then defining the next point in the path by taking some small step Δt in the direction of the tangent vector. If Δt is very small then, often times, this is a very good approximation for the real path which never leaves the manifold. For example, the time derivative of position is velocity measured in meters per second; the velocity vector is the tangent vector to the path at a point. If we multiply meters per second by some small amount of seconds then we will obtain a distance. In Euler's method, we can approximate the path by moving a small distance in the direction of the velocity vector after taking its product with some small time step Δt. We say this gives the next point in the path. Then we take the tangent vector to that point, multiply it by the standard time step, obtain the next point, *etc*. The problem is that this cannot possibly be the real path because every tangent vector points to a place outside of spacetime. This is useful when we take a very small time step because the outcome is very much like the physical expectation but, when we want to account for effects like symplectic points or the hypercomplex neighborhood around a point then the discrete nature of Δt makes it highly unlikely that the discretized approximate path will intersect these mathematically specific points even if the continuous trajectory would pass through them.

The exponential map is developed, for instance, in reference [17]. The main useful property of the exponential map is that a tangent vector k^μ to a manifold at point p defines a unique geodesic $x^\mu(\lambda)$ running through p as

$$x^\mu(\lambda) = \lambda k^\mu \quad , \tag{4.259}$$

when $\lambda = 0$ at p. Therefore, as a matter of practicality, the exponential map is Euler's method

$$x_{j+1} = x_j + \Delta t \, \dot{x}_j \quad , \tag{4.260}$$

which is very easy to understand. Equation (4.260) is equation (4.259) when $x_j = 0$, $\dot{x}_j = k^\mu$, and $\Delta t = \lambda$.

Leading into our discussion of Runge–Kutta methods, note the general utility of this classical method of numerical approximation. Let there be some field

$$\dot{x} = \sigma\big(y - z\big) \tag{4.261}$$

$$\dot{y} = -xz + \alpha x - y \tag{4.262}$$

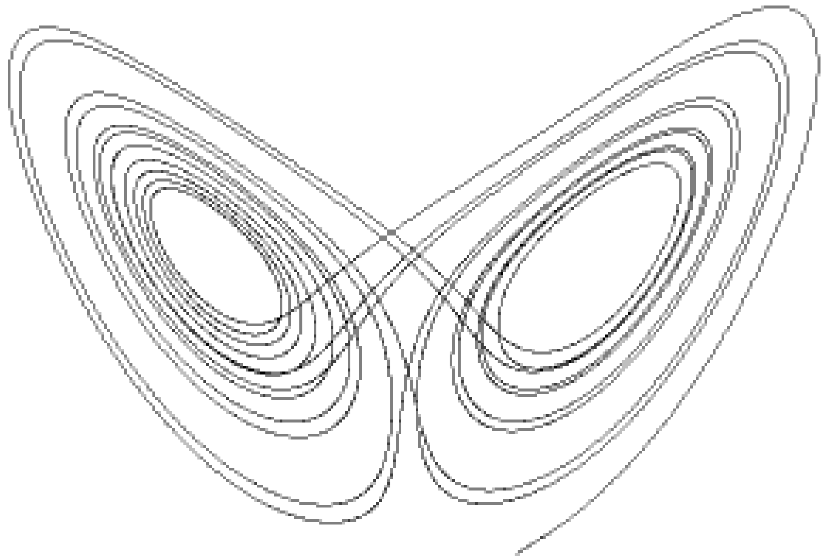

Figure 93: This figure shows the Lorenz attractor.

$$\dot{z} = xy - \beta z \quad , \tag{4.263}$$

where $\{\sigma, \alpha, \beta\}$ are some constants. Altogether equations (4.261 -4.263) define a vector at every point in space. One asks, "If we examine point p in the field where does it go?" If $p = (x_1, y_1, z_1) = (1, 2, 3)$ then, after Δt, we say it goes to

$$x_2 = 1 + \Delta t\, \dot{x}(p) \tag{4.264}$$

$$y_2 = 2 + \Delta t\, \dot{y}(p) \tag{4.265}$$

$$z_2 = 3 + \Delta t\, \dot{z}(p) \quad . \tag{4.266}$$

If we repeat this process very many times, and connect each p_j to each p_{j+1}, then we will obtain something that looks very much like figure 93. This is called the Lorenz attractor after Lorenz who developed equations (4.261-4.263) to describe chaotic weather patterns. This example shows the importance of the step size Δt; if we chose $\{\sigma, \alpha, \beta\}$ on the order of 10^0 and Δt on the order of 10^6 then the first step would move the point far beyond the attractive basin and the interesting behavior would not be observed. Only when the step is appropriately sized do we observe the interesting behavior. If the step was too large then all of the complexity would be glossed over so, obviously, when simulating systems with very fine features, a very small step size is required. Therefore, when we do computations in general relativity near, for instance, a black hole whose event horizon is a mathematically perfect surface, it is likely that any finite step size will represent an insufficient resolution. In the context of Hawking radiation, we have raised an issue regarding the increment of an event horizon's radius associated with one electron falling into the black hole and it would require an outrageously small Δt to resolve this in the numerical realm. Here, we have demonstrated the general difference between the exactly solvable problems that are few and far between in physics, and the other problems that must be solved by approximation. The approximated solution always reflects the physical input *and* artifacts related to the choice of approximation. It is even possible to conflate effects derived from a certain approximation with actual physical effects but, in the MCM, we have only considered that which can be written exactly and are, therefore, free from any potential approximation biases. Indeed, there are many potential sources of bias in any numerical solution and the step size is only one of them. Consider what Burden and Faires write in reference [66].

> "We began with a discussion of the most elementary numerical technique, Euler's method. The procedure is not sufficiently accurate to be of use in applications, but it illustrates the general behavior of the more powerful techniques, without the accompanying algebraic difficulties. The Taylor methods [*are*] generalizations of Euler's method. They [*are*] accurate yet cumbersome because of the need to determine extensive partial derivatives of the defining function of the differential equation.[1] The Runge–Kutta formulas simplified the Taylor methods, while not significantly increasing the error."

Euler's method, or Gauss', in equation (4.260) computes x_{j+1} using only data from x_j but Runge–Kutta methods use data from a few other points and are more accurate. Consider the Runge–Kutta fourth order algorithm as excerpted from reference [66].

To approximate the solution of the initial value problem

$$y' = f(t, y), \quad a \leq t \leq b, \quad y(a) = \alpha,$$

at $(N+1)$ equally spaced numbers in the interval $[a, b]$.

INPUT endpoints a, b; integer N; initial condition α.
OUTPUT approximation w to y at the $(N+1)$ values of t.

[1] Equation (2.47), in section II.1, is an excellent example of this cumbersome task.

1. Set $\varepsilon = (b-a)/N$;
   $t = a$
   $w = \alpha$
   OUTPUT(t, w).

2. For $j = 1, 2, ...$ do steps $3-5$.

   3. Set $K_1 = \varepsilon f(t, w)$;
      $K_2 = \varepsilon f(t + h/2, w + K_1/2)$;
      $K_3 = \varepsilon f(t + h/2, w + K_2/2)$;
      $K_4 = \varepsilon f(t + h, w + K_3)$.

   4. Set $w = w + (K_1 + 2K_2 + 2K_3 + K_4)/6$; Compute w_j
      $t = a + j\varepsilon$. Compute t_j

   5. OUTPUT(t, w)

6. STOP.

The statement of the initial value problem is the same that we have considered for Euler's method. As in equations (4.261-4.263), we have a tangent vector specified, here it is called y'. For the Lorenz attractor, we defined a first point $p = (1, 2, 3)$ and here we have some generalized $y(a) = \alpha$. a is the start time and this Runge–Kutta algorithm specifies how long the simulation will run where, above, we simply considered many time steps. This method differs from the Gaussian method when it computes four different values for the next point and then takes a weighted average of them as the returned value. Notably, the K_1 contribution to the weighted average is exactly the Gaussian approximation. However, even when the error associated with this method is less than in the previous method, the discrete stepping is still going to ignore anything related to hypercomplexly infinitesimal neighborhoods. Toward that problem, consider what Burden and Faires write in reference [66] regarding adaptive step methods.

> "The appropriate use of varying step size [*produces*] computationally efficient integral approximating methods. In itself, this might not be sufficient to favor these methods due to the **increased complication** [*emphasis added*] of applying them. However, they have another feature that makes them worthwhile. They incorporate in the step size procedure an estimation of the truncation error that does not require the approximation of the higher derivatives of the function. These methods are called *adaptive* because they adapt the number and position of the nodes used in the approximation to ensure that the truncation error is kept within a specified

bound."

The geodesic equation and equations like those that define the Lorenz attractor are differential equations that need to be integrated before we can use them to plot trajectories like those in figure 93. For this reason, the corresponding Runge–Kutta methods are called integral approximating methods. If we could integrate the equations of motion exactly then there would be no need but, in practice, when one derives the equations of motion for a real physical system, they are not exactly solvable. If we want to simulate a black hole of finite mass existing in some large region of the universe then an adaptive step method will allow us to simulate that which we have referred to as embedding the singularity in the manifold on a lower level of ℵ. One adaptive method is the Runge–Kutta–Fehlberg method. Consider the Runge–Kutta–Fehlberg algorithm, also excerpted from reference [66].

To approximate the solution of the initial value problem

$$y' = f(t, y), \quad a \leq t \leq b, \quad y(a) = \alpha,$$

with local truncation error within a given tolerance.

INPUT endpoints a, b; initial condition α; tolerance TOL; maximum step size ε_{max}; minimum step size ε_{min}.
OUTPUT t, w, ε where w approximates $y(t)$ and the step size ε was used, or a message that the minimum step size was exceeded.

1. Set $t = a$;
   $w = \alpha$;
   $\varepsilon = \varepsilon_{max}$;
   $FLAG = 1$;
   OUTPUT(t, w).

2. While $(FLAG = 1)$ do steps $3 - 11$.

3. Set $K_1 = \varepsilon f(t, w)$;
$K_2 = \varepsilon f(t + \frac{1}{4}h, w + \frac{1}{4}K_1)$;
$K_3 = \varepsilon f(t + \frac{3}{8}h, w + \frac{3}{32}K_1 + \frac{9}{32}K_2)$;
$K_4 = \varepsilon f(t + \frac{12}{13}h, w + \frac{1932}{2197}K_1 - \frac{7200}{2197}K_2 + \frac{1932}{2197}K_3)$;
$K_5 = \varepsilon f(t + h, w + \frac{439}{216}K_1 - 8K_2 + \frac{3680}{513}K_3 - \frac{845}{4104}K_4)$;
$K_6 = \varepsilon f(t + \frac{1}{2}h, w - \frac{8}{27}K_1 + 2K_2 - \frac{3544}{2565}K_3 + \frac{1859}{4104}K_4 - \frac{11}{40}K_5)$.

4. Set $R = \frac{1}{h}\left|\frac{1}{360}K_1 - \frac{128}{4275}K_3 + \frac{2197}{75240}K_4 - \frac{1}{50}K_5 - \frac{2}{55}K_6\right|$;

5. If $R \leq TOL$ then do steps 6 and 7.

6. Set $t = t + \varepsilon_{max}$; Approximation accepted.
$w = w + \frac{25}{216}K_1 + \frac{1408}{2565}K_3 + \frac{2197}{4104}K_4 + \frac{1}{5}K_5$.

7. OUTPUT(t, w, ε).

8. Set $\delta = 0.84(TOL/R)^{1/4}$.

9. If $\delta \leq 0.1$ then set $\varepsilon = 0.1\varepsilon$
else if $\delta \geq 4$ then set $\varepsilon = 4\varepsilon$
else set $\varepsilon = \delta\varepsilon$. Calulate new h.

10. If $\varepsilon > (TOL)\varepsilon_{max}$ then set $\varepsilon = \varepsilon_{max}$.

11. If $t \geq b$ then set $FLAG = 0$
else if $t + \varepsilon > b$ then set $\varepsilon = b - t$
else if $\varepsilon < \varepsilon_{min}$ then
set $FLAG = 0$;
OUTPUT('min ε exceeded').

12. STOP.

One shortcoming of this algorithm will be that the adaptive step, no matter how small we allow it to go, will always be finite and, therefore, will always gloss over infinitesimal features such as symplectic points (which become symplectic Riemann spheres in twistor space.) However, where this algorithm can give an error message in step 11 that the step has gotten too small, we can replace that with an instruction to move the simulation to another level of $\aleph$ where, ostensibly, the symplectic point becomes a finite region or the distances describing a $\hat{\Phi}^j$ black hole embedded in a $\hat{\Phi}^{j+1}$ universe become finite. This would have a lot of direct application in machine language where numbers are defined not as elements of $\mathbb{R}$ but, rather, as floats, doubles, and the like where there is a limit imposed *a priori* on the relative scale of any two numbers in the simulation.

Why have we included these algorithms in full? To answer that question, we must first say a little about why we have not included any MCM energy function $\mathcal{L}_{MCM}$ or H_{MCM}.[1] It is traditionally thought in physics that the energy function is where the magic happens but consider the full scope of the magic in physics. First one (hopefully) draws a system diagram and then one labels the system's degrees of freedom such that all the energy channels (modes) are enumerated. Then one solves the Euler-Lagrange equations which involve functionals of the energy function or one applies the action principle which involves the same. By some canonical method, one obtains the equations of motion and detractors are wrong to cite the absence of a retranscription of the canon in MCM publications as a defect in the MCM itself. After obtaining the equations of motion one asks, "If the system is like this now, how will it be later?" One quickly realizes that the equations of motion offer no answer to this important question! The equations of motion are written as not so intuitive differential equations. To determine the motion, one must integrate the equations of motion to solve for the behavior of the degrees of freedom. If one is very lucky, these can be solved exactly but, often in physics, one is not that lucky. At that point, one resorts to methods like the Runge–Kutta methods presented in this section, and there are other popular Runge–Kutta methods in physics. For instance, the Runge–Kutta–Nordström algorithm will conserve energy along the approximated trajectory while Runge–Kutta–Fehlberg will not. Even with Runge–Kutta–Nordström, the approximate trajectory will not conserve in all cases angular momentum so, if one wishes to see that, one has to use an even more powerful algorithm. If one wants to simulate particles randomly arriving as discrete packets of mass, energy, and linear and angular momentum on the the final screen in the double slit experiment then one likely relies upon what are called Monte Carlo methods. An entire landscape of quadrature methods will give an approximation to any integral $P[\psi(x)]$ even when it is analytically intractable, and,in general, analytical intractability in physics is treated with numerical analysis.

To the point of why we have included the Runge–Kutta algorithms, we argue that the algorithmic computational component is really where the magic happens in physics. When every analytical method has already been adapted to some algorithm in numerical analysis, we view the extant algorithms as a better place to scan for hidden features related to $P[\psi(x)]$ than the analytical representations themselves. The MCM has been so successful because we have discovered many hidden features in the analytical sector but these results are related

[1] Part of the reason we have never treated an MCM energy function is that, even if we did invest the time and energy to derive one, we do not have a powerful computing facility in which to integrate the resultant equations of motion. Without those integrated equations, it would be very difficult (impossible) to determine if the energy function in question was correct or if it contained an error. Indeed, one expects that the first such function would contain errors and, without an efficient computational facility in which to identify those errors and iteratively correct them, the initial exercise to develop an energy function at all with would have constituted a suboptimal allocation of efforts and attention.

only to the fundamentals which contribute to $P[\psi(x)]$. When dealing with $P[\psi(x)]$ directly, the numerical sector is a superior fishing grounds. In general, the numerical representation allows one to define and enumerate the modes of a system with the same simplicity as a diagram, and with the added benefit of actually being able to produce a real-valued expectation value with the push of a button that is tantamount to, if not identically, $P[\psi(x)]$. Writing an energy function in the analytical sector has no simplicity, especially in general relativity, and is only indirectly connected to the empirical sector. For instance, the algorithmic representation of the process for integrating the equations of motion derived from the Einstein–Hilbert action will give a much clearer picture of the overall issues than would be found by simply writing

$$\mathcal{S}_{\text{EH}} = \frac{1}{2}\int R\sqrt{-\det(g_{\mu\nu})}\,d^4x \quad . \tag{4.267}$$

Given the approach we have taken in the MCM, we can examine equation (4.267) and identify a likely $\hat{2}$, the Ricci scalar R is like a delta function choosing a specific slice of χ^5, but what is the integral over the square root of the negative determinant of the metric supposed to be in the diagrammatic component? The minus sign will cancel the minus sign from the O(3,1) structure of a diagonal metric to ensure a real root, but what is the rest of it?

Indeed, where we have presented $\hat{M}^3$ in the analytical channel as ∂^3, that does not fully encompass the procedure through which an observer computes a prediction for an event, waits for the event to happen, and then compares the real to event to his prediction. To fully encompass all of that, $\hat{M}^3$ will need to be written algorithmically in language similar to what we have presented for the Runge–Kutta fourth order and Runge–Kutta–Fehlberg algorithms. The analytical sector will present a lot of challenges when we want to extend the full action formalism of the standard model of particle physics toward the maximum action path across $\hat{\Phi}^j \to \hat{\Phi}^{j+2}$. Whatever that analytical formalism is, it will have the structure of two sets of grid points in the numerical sector. In practice, $P[\psi(x)]$ will be computed in the numerical sector and never in the analytical sector. The precise energy function of the MCM universe is irrelevant because the MCM modifies the way arbitrary energy functions are crunched in numerical integrators.

Within the Lorenz attractor, defined by differential equations (4.261-4.263), are three qubits, all of the general form $\hat{2}(\delta i\,\hat{\pi} - \delta\pi\,\hat{i})$ where the constants are like $\hat{e}_\mu$ and the variables are like δe_μ. Lorenz' $\dot{x}$ and $\dot{z}$ are exactly of this form and his $\dot{y}$ is of this form with an additive term $-y$ which means that y is self-referentially damped by its own magnitude. Even the Riemann tensor can be written as $R_{ijkl} = K(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk})$. Indeed, the partially integrated total differential $\hat{d}f$ was of this same form and the connection between d and $\hat{d}$ would be an eccentric feat in the analytical sector.

In the MCM grid, which is a lattice, we have defined $\hat{\Phi}$ pointing from the planar grid topology to some apex point where there is another $\hat{\Phi}$ pointing from the apex point into some topologically distinct set of grid points. Whereas a grid of position space and a grid of momentum space are orthogonal, the two grids that we can derive by sampling the two disks in the Lorenz attractor can be considered double orthogonal. Whatever the operation for smooth evolution across levels of $\aleph$ in the analytical sector, if we project the two Lorenz disks, meaning all the $\vec{x}_j$ that were plotted in figure 93, onto two planes, and then further project the projections onto the x and z axes, we will obtain the two requisite orthogonal sets of grid points. Indeed, we can assign a spinor to each $\vec{x}_j$ that specifies if it belongs to the left disc or the right disk. Furthermore, we can select one of the transitional $\vec{x}_j$ that appear to belong to neither disk as an apex point of the sort referred to a bounce point in reference [2]. Indeed, with a judicious choice of initial condition we can set the parameters $\{\sigma, \alpha, \beta\}$ and take the grid spacing such that there is only one point in the non-disk transition region. Indeed this limited and tailored run of the simulation should be associated with one application of $\hat{M}^3$. The trajectory starts in one disk where it spins for a while until it hits the apex point and ends up in the other disk which we will say is on a higher level of $\aleph$. Therefore, since $P[\psi(x)]$ is, according the experimental standards of detractors and others, an object defined in numerical analysis, it is irrelevant what the analytical representation of the mechanism is. Therefore, the analytical representation is irrelevant.

IV.8 Two Stringy Universes in the Information Current

One of the best and mathematically most advanced features of the MCM is that it is a natural framework for 10D string theory when we define 10D geometric boundary conditions with five dimensions in each of $\Sigma^\pm$ $(5+5=10.)$ There are very many ways we can define the ten boundary conditions needed to solve a 10D system in the MCM. We can use χ_+^A and χ_-^A to define five boundary conditions in each, or we could define five boundary conditions on a string $\phi(\chi^5)$ in only one of $\Sigma^\pm$ and then define another five conditions for $\dot{\phi}(\chi^5)$ in either of $\Sigma^\pm$. Between the $\chi_\pm^A$ coordinates and the $x_\pm^\mu$ coordinates, we have nine dimensions in each of $\Sigma^\pm$ that we can evolve according to x^0 such that the MCM has two 10D strings as the fundamental unit whereas contemporary string theory uses only one 10D string as the fundamental unit. There are very many possible arrangements. The MCM condition sets $\phi(0)=0$ and that never changes because we compute everything else relative to that feature. It is the defining property of the MCM that $\phi(0)=0$ can never change so we also find $\dot{\phi}(0)=0$, $\ddot{\phi}(0)=0$, $\dddot{\phi}(0)=0$, *etc.* When two strings terminate on two sides of $\mathcal{H}$, a likely D-brane, and they both share the $\phi(0)=0$ condition, then they can be smoothly joined across the brane. They need not match on $\varnothing$, if placed at $\chi_\pm^5=\pm\infty$, because the infinite curvature dominates.

The Riemann hypothesis is a very important in string theory because the Riemann ζ function is a common object in its analytical representation. Where we have examined the Riemann ζ function in reference [8], we have strongly argued that the Riemann sphere must have two null points instead of just one. The first frame of figure 94 has one null point because it shows a polar great circle of the Riemann sphere $\theta\in(-\pi,\pi)$. Therefore, we should consider that the circle is composed of two strings that are topologically disconnected at the south polar point but unified there with a shared boundary condition. In fact, the main result of reference [8] was that a circle with one null point is not a sufficient topology on which to host hypercomplexity. There must be at least two null points. Therefore, when we have proposed to generate $\hat{\pi}+\hat{\Phi}$ by adding $\hat{\Phi}$ at the null point, here we should have two $\hat{\Phi}$s. Since the two $\hat{\Phi}$ are concomitant pointing through each other, we can say that we have generated $\hat{\pi}+\hat{\Phi}^2$. Two $\hat{\Phi}$s stuck together at an apex point are like "Rubik's Scissors" that can support a modularized fractal topology constructed from base units of $\hat{\pi}$ and $\hat{\Phi}$. $\hat{\pi}$ is an inherent curved piece and $\hat{\Phi}$ is an inherent straight piece which can be defined as the $R\to\infty$ limit of a semicircular piece of radius R. Indeed, this fairly well aligns with "levels of $\aleph$" and the

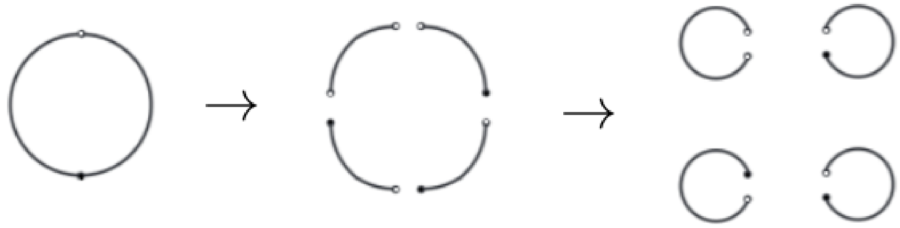

Figure 94: We can use inversion operations to change the direction of increasing parameter along a given element and we can attach qubits to missing endpoints such that they will be shuffled around among different elements in a self-similar scheme of covering spaces.

non-linear operation $\hat{\Phi}^2 \cdot \hat{\pi}$ at the heart of MCM general relativity. Furthermore, the duality between one 10D string defined with $\chi^A_\pm$, two 10D strings $\{x^0, x^\mu_+, \chi^5_+\}$, and $\{x^0, x^\mu_-, \chi^5_-\}$ is a likely host for bispinor structure. The only dimensionful parameter in string theory is the length of the string and we propose to construct a representation where the two strings in the MCM concept of string theory are the two co-$\hat{\pi}$s which have appeared throughout this book, each of length π, or perhaps a $\hat{\pi}$ and a $\hat{\Phi}$, or even two co-$\hat{\Phi}$s.

We will induce an information current in the model when we consider the inclusion or non-inclusion of the endpoints of the strings such that qubits can be shuffled into or out of the circle formed by the two co-$\hat{\pi}$s and two co-$\hat{\Phi}$s through a dynamical series of nested covering space representations. By including the endpoints or not including them, we will alternate bounded and unbounded topologies in the manner of $\tilde{\mathscr{M}}$ and $\mathscr{M}$, or Ω and $\aleph$. The two MCM manifolds $\{\aleph, \Omega\}$ have boundedness or unboundedness built into the spatial slices of their de Sitter definitions and we can take $\tilde{\mathscr{M}} \equiv \mathcal{H}$ with $\mathscr{M} \equiv \varnothing$. These are manifolds that do or do not include a point at conformal infinity, and $p \in \Sigma^\varnothing$ is such a point. Figure 94 shows the endpoint structure when the co-$\hat{\pi}$s are further divided about their own origins and this is representative of the MCM fractal geometry; either co-$\hat{\pi}$ can form a circle with half the circumference of the two co-$\hat{\pi}$s together, and then the topology argument regarding a second null point implies that a single co-$\hat{\pi}$ should have two $\hat{\Phi}$s as it did in the larger circle. When we consider four elements, as in figure 94, we have considered two layers of complexity: decomposition of a circle into co-$\hat{\pi}$s and then a second nested layer of complexity. This decomposition is under the general sense that two layers of complexity demonstrate an attachment to one higher level of $\aleph$ and one lower.

We want to show how to pass two wavefunctions through the machinery that we have presented in this book as pertains to a single wavefunction. Therefore, we will consider the forward passthrough of one wavefunction through a series of matrix operations, and then we will scan for the existence of a parallel reverse passthrough channel. This operation will be accomplished with notation related to the tensor indices $\alpha, \beta \in \{1,2,3,4\}$ or $\dot{\alpha}, \dot{\beta} \in \{2,3,4,5\}$. In section IV.6, we showed the propagation of the wavefunction across each step of $\mathcal{H} \mapsto \Omega \mapsto \aleph \mapsto \mathcal{H}$ and here we consider the matrix representation of the metric at each step of $\hat{M}^3$. For the step $\mathcal{H}_1 \mapsto \Omega$ we wrote equation (4.184) which was

$$\psi_+(\chi^5_+) = \big\{ \hat{\Phi} \big| \psi(t); \hat{\pi} \big\rangle \quad . \tag{4.268}$$

Here we will develop the multiplectic component of the scheme that followed equation (4.184), which should be representable with matrices even when tensors are more familiar to some. The MCM quanta of the quantum gravity theory are inherently dynamical in the geometry so we also want to propagate the metric across the unit cell with the wavefunction. We need to embed the 4D metric in the upper left corner of a 5D matrix as

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mapsto \quad \begin{pmatrix} \boxed{\text{-1}} & \boxed{0} & \boxed{0} & \boxed{0} & 0 \\ \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} & 0 \\ \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} & 0 \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & \chi^5_+ \end{pmatrix} \quad . \tag{4.269}$$

On the left, the O(3,1) topology exists with the complex O(1) in the upper left corner but, in the 5D matrix, we can encode O(3,1) using either corner. χ^5_+ is positive so for the specific 5D matrix in map (4.269) the Lorentz qubit can only exist in the upper left corner.[1] Also note the dependence on χ^5_+; it varies across Σ^+ because Σ^+_{55} is not a constant like g_{00} and, when we move the complex O(1) part to χ^5_-, the variation within Σ^- is a conformal deformation of the dimensional transposing parameter between the timelike and spacelike regions. The next step of $\hat{M}^3\ \Omega\mapsto\aleph$ is where the level of $\aleph$ increases and we will say that the 4D metric moves to the other corner. This looks like

$$\begin{pmatrix} \boxed{\text{-1}} & \boxed{0} & \boxed{0} & \boxed{0} & 0 \\ \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} & 0 \\ \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} & 0 \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & \chi^5_+ \end{pmatrix} \quad \mapsto \quad \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{0} \\ 0 & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} \\ 0 & \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} \\ 0 & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{\chi^5_-} \end{pmatrix} \quad . \tag{4.270}$$

Here we can use the idea that χ^5_- is a negative number to preserve the O(3,1) topology through this inversion operation. Now, the Lorentz qubit exists in the lower right hand corner and we must associate this inversion with the two-step rotation that rotates a vector, swaps its anchor point with its tip, and then does another rotation. The last step $\aleph\mapsto\mathcal{H}_2$ looks like

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{0} \\ 0 & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} \\ 0 & \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} \\ 0 & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{-\chi^5_-} \end{pmatrix} \quad \mapsto \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad . \tag{4.271}$$

[1] In the convention where $\Sigma^\pm_{55} = -\chi^5_\pm$ this will be reversed, obviously.

To achieve this step, we have seemingly normalized χ^5_- to unity but upon closer inspection, as in figure 1, one sees that one is the length of χ^5_- that spans Σ^-. The propagation across the unit cell involves χ^5_+ going to Φ, then we swap $\hat{\Phi}$'s anchor point and rotate it again to accumulate Φ in $\varnothing$ (along a symplectic direction inherent to the symplectic form at the tip of the first $\hat{\Phi}$) to give a second higher level of $\aleph$ when we anchor $\hat{\Phi}$ at the point $\chi^5_+ = \Phi$. We measure χ^5_- from zero on the higher level of $\aleph$ toward its limit $\chi^5_- = 1$ at $\mathcal{H}$. All of the tensor notation will have to be rewritten to get the timelike part of the O(3,1) topology back into the zero tensor index, or else everything else will have to be modified, but we have demonstrated the broad strokes. Furthermore, it is obvious that these matrices can hold $\bar{U}$ which starts in the lower right corner and then moves in the opposite direction across the matrices with respect to the direction demonstrated here for a universe U.

If we compute the above matrix operations then how can we use the $\bar{U}$ channel? We can say that the output of $\bar{U}$ that exists at the beginning of the forward operation is the a source of information, and that the initial $\bar{U}$ at the end of the forward operation is a sink. When the source is like Φ and the sink it is like φ, we have a derived what should be a working thermodynamics. Where does the extra information go when the source and sink are imbalanced? It must go into the entropy. We are presently considering non-entropic systems but this is highly evocative of what we have referred to as Penrose's entropy dilemma in reference [2].

Regarding two universes, we will say a little more about the change of topology between O(1,4) and O(2,3) that arises from

$$\chi^5 \quad \equiv \quad \chi^5_+ \otimes \chi^5_\varnothing \otimes \chi^5_- \quad , \qquad \text{with} \qquad \chi^5 \in [\varphi, \Phi] \quad . \tag{4.272}$$

$\Sigma^\pm_{55} \equiv \chi^5_\pm$ so the change of sign of χ^5 at $\Sigma^\varnothing$ implies a change in the total number of timelike and spacelike dimensions in Σ^+ and Σ^-. O(4,1) and O(3,2) can both hold an O(3,1) manifold so there is no problem to pass ordinary spacetime through the unit cell. Consider the symplectic point: the two basis vectors that span the space of possible exit directions from the symplectic points can guide timelike trajectories onto timelike trajectories, and spacelike trajectories onto their own smooth continuations. Likely, null trajectories are conformally invariant because they never pass through symplectic points. Indeed, the union of the rings at infinity for each spatial dimension in the strictly real (relative to some phase convention) O(3) part of the spacetime topology is like three $\hat{\pi}$s, or π^3 as in α_{MCM}, or even the three mixing angles for quarks in the Cabibbo–Kobayashi–Maskawa matrix. It is very natural, therefore, to assume that the wavefunction of U and $\bar{U}$ together is an anti-symmetric spinor wavefunction even if the classical limit of a wavefunction for U alone would suggest a symmetric wavefunction. As one last bit about this change of topology O(4,1) $\to$ O(3,2), consider the case when $\Sigma^\pm$ are joined on a singularity at $\chi^5_\pm = \pm - \infty$. Somehow, smooth evolution toward $+\infty$ is replaced with smooth evolution away from $-\infty$. This is exactly the kind of thing we can accomplish with symplectic points that contain unit vectors pointing in different directions. If there is one symplectic point on Ω, and one on $\aleph$, then we can join them through local symplex instead of the non-local singularity proposed for $\chi^5_\pm = \pm\infty$.

To finish this book, we will propose a new formulation of the mass parameters of the universe $\{\Omega_{\text{Matter}}, \Omega_{\text{Dark Matter}}, \Omega_{\text{Dark Energy}}\}$. All of the fundamental matter particles that make

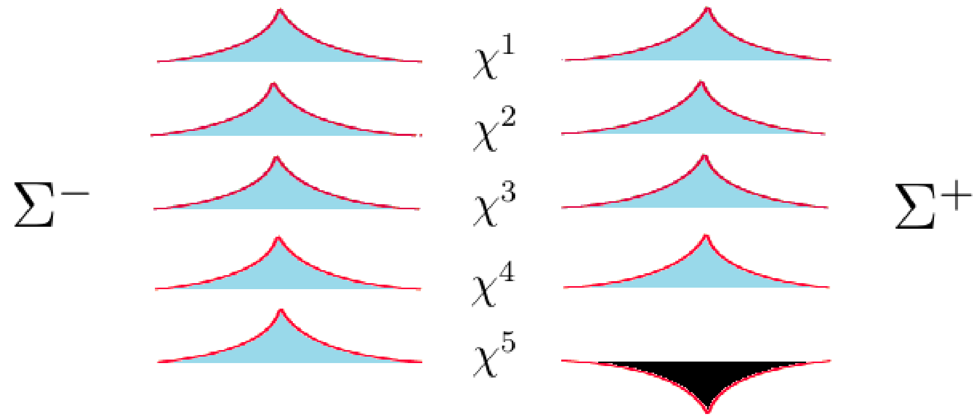

Figure 95: χ^5 has an antisymmetric wavefunction because there is a topological obstruction at $\chi^5=0$ that invokes a twisted path around infinity.

up the stuff in the universe have antisymmetric wavefunctions so we will associate the classical massiveness of Ω_{Matter} with some solitonic element, likely a left/right mover in string theory, associated with the shaded region χ^5_+ in figure 95. Therefore, consider some moment of $\Sigma^\pm$ such that, when the total moment of $\chi^A_\pm$ is one, the moment of χ^5_+, the anti-symmetric piece easily identified in figure 95, is one tenth. We will examine only the moment of the χ^5_+ antisymmetric piece because it is the generator of antisymmetry. When the matrix representation presented in this section is such that U evolves forward in time while $\bar{U}$ evolves simultaneously in reverse time, we will have to define spinor valued coefficients for each matrix position. This will allow simultaneous passthrough channels for U and $\bar{U}$ when the spinor has one forward passthrough channel $|\uparrow\rangle\equiv|t_+\rangle$ and one reverse passthrough channel $|\downarrow\rangle\equiv|t_-\rangle$: a source and sink of information. Indeed, the fundamental MCM principle is that $|t_\star\rangle$ is a superposition of the $|t_\pm\rangle$ eigenstates [7]. Let the spinor on Σ^+_{55} be such that half of the moment goes to U and half goes to $\bar{U}$ so that the moment associated with U is half of one tenth. Noting that ΛCDM sets $\Omega_{\text{Matter}}\approx 0.05$, we can model the universe as

$$\Omega^U_{\text{Matter}}=0.05 \quad , \qquad \text{and} \qquad \Omega^{\bar{U}}_{\text{Matter}}=0.05 \quad , \tag{4.273}$$

where the anomalous contribution to the total mass-energy is divided among

$$\Omega_{\text{Dark Matter}}=0.25 \quad , \qquad \text{and} \qquad \Omega_{\text{Dark Energy}}=0.65\approx|\varphi| \quad . \tag{4.274}$$

This has a nice ontological feel to it and it sits square in the middle of the parameter space favored by ΛCDM. These parameters describe the distribution of mass-energy in the universe through

$$\Omega^U_{\text{Matter}}+\Omega^{\bar{U}}_{\text{Matter}}+\Omega_{\text{Dark Matter}}+\Omega_{\text{Dark Energy}}=1 \quad . \tag{4.275}$$

Regarding dark matter, we might replace the infamously unsatisfying particulate explanation with an interaction of U and $\bar{U}$ through a series of cosmological symplectic points such that the associated moment is that of one channel in

$$\hat{1} = \frac{1}{4}\left|\hat{\pi}\right\rangle + \frac{1}{4}\left|\hat{\Phi}\right\rangle + \frac{1}{4}\left|\hat{2}\right\rangle + \frac{1}{4}\left|\hat{i}\right\rangle \quad . \tag{4.276}$$

We have already proposed that dark energy should be an interaction of U and $\bar{U}$ through the ends of the unit cell and "dark matter" can be a similar interaction along a different avenue. Here, dark matter is described as an interaction through the symplectic points and dark energy is described as an interaction through the apex point, as in reference [2].

A knife at detractors' throat.
Why didn't you read
that which I wrote?
Geometry will be the mathematical tool.
Death to you,
you malicious fool.

V Death to Detractors

Appendices

Appendix A

Synopsis of Historical Development

Here we list the abstracts of the papers that most directly document the historical development of the modified cosmological and theory of infinite complexity. We comment on some of them to point out key details with the benefit of hindsight.

A.1 Modified Spacetime Adresses Dark Energy, Penrose's Entropy Dilemma, Baryon Asymmetry, Inflation, and Matter Anisotropy

"A model of modified spacetime is discussed. Implications for causality regarding modern anomalies and paradoxes are made. Topics include a dark energy candidate without induced gravitational screening. The dynamics of the repulsive force of quantum geometry allow the validity of the second law continuously through a universe's death and rebirth. The baryon asymmetry is explained without addressing the Sakharov conditions. Inflation and anisotropies in an FLRW universe are also attributed to quantum bounce phenomena. No attempt at quantification is made."

Presented without comment.

A.2 Dark Energy in M-Theory

"Dark Energy is yet to be predicted by any model that stands out in its simplicity as an obvious choice for unified investigative effort. It is widely accepted that a new paradigm is needed to unify the standard cosmological model (SCM) and the minimal standard model [*of fundamental particles*] (MSM). The purpose of this article is to construct a modified cosmological model (MCM) that predicts dark energy and contains this unity. Following the program of Penrose, geometry rather than differential equations will be the mathematical tool. Analytical methods from loop quantum cosmology (LQC) are examined in the context of the Poincare conjecture. The longstanding problem of an external time with which to evolve quantum gravity is resolved. The [*dark energy*] and WMAP data are reexamined in this framework. No exotic particles or changes to General Relativity are introduced. The MCM predicts dark energy even in its Newtonian limit while preserving all observational results. In its General Relativistic limit, the MCM describes dark energy as an inverse radial spaghettification process. Observable predictions for the MCM are offered. AdS/CFT correspondence is discussed. The MCM is the 10 dimensional union of de Sitter and anti-de Sitter space and has M-theoretical application to the five string theories which lack a unifying conceptual component. This component unifies gravitation and electromagnetism."

Note the specific goal of this research program: to use geometry rather differential equations. Part of the purpose of the present book is to show that the geometric picture illustrated by sufficiently logical diagrams is equivalent to, *or better than*, the clunky differential equation formulation of certain big-picture issues in physics. This paper introduces many of the most fundamental pieces of the MCM such as the golden ratio Φ and non-flat spacetime.

The procrastination between 2009's reference [2] and this paper [7] ended in response to the superluminal neutrino non-event. Even after two years, this writer had not heard back regarding his manuscript [2] and sought to prepare another one for possible entrance into the superluminal neutrino fray. It is this writer's opinion that the arc of the story of his life was greatly influenced by his simultaneous decision in 2011 to attend an Occupy Protest™.

A.3 Tempus Edax Rerum

"A non-unitary quantum theory describing the evolution of quantum state tensors is presented. Einstein's equations and the fine structure constant are derived. The problem of precession in classical mechanics gives an example."

This paper contains a derivation of Einstein's equation from the framework in which we have derived α_{MCM}. On the day that detractors finally relent from their persistent detractions, they will be hard pressed to say how they might have ever been so miserly as to detract from the irrefutable mathematical results in this paper. To detract from the interpretation or application of the result is to miss the main point of this paper: Einstein's equation is encoded into the MCM geometry. There is no margin for detractions regarding the MCM's early dark energy result, and that result was circumstantially confirmed with the derivation of α_{MCM}. When we have also demonstrated Einstein's equation as further confirmation of all previous results, and yet detractors detract, there exists a serious problem in academia. The result in this paper is magnificent in the most spectacular way unlike 100% of the unverified theories that are praised 100 times a week at conferences around the world. Even still, detractors say, "It is not verified," as if verification is needed to confirm that $8\pi^3/\pi^2 = 8\pi$, or that $(\Phi\pi)^3 + 2\pi \approx 137$, or that a downhill energy condition on the time axis will cause an effect exactly like dark energy. It makes no sense. Furthermore, this paper derives a characteristic length scale for the new effects which offers experimentalists a direct window into the principles. Despite this, it appears there has been no work done toward forging an experimental inquiry regarding this length scale as it might relate to the anti-gravity effects of classical mechanical precession. As of the time of this writing in late 2017, more than five years have passed without this valuable result being publicly recognized. There is a conspiracy afoot.

A.4 Geometric Cosmology

> "The modified cosmological model (MCM) is explored in the context of general relativity. A flaw in the ADM positive-definiteness theorem is identified. We present an exposition of the relationship between Einstein's equations and the precessing classical oscillator. Kaluza theory is applied to the MCM and we find a logical motivation for the cylinder condition which leads to a simple mechanism for AdS/CFT."

Here, we are content to explore the most general instance of bulk-boundary correspondence and then assume it is a reformulation of the well-known AdS/CFT correspondence. Finding the fine structure constant or the coefficient of Einstein's equation in an invented framework might not have been a sign, but finding both of them in the same framework made it a sure thing that this was a correct physical formulation. A further confirmation appeared in this paper when the previously unexplained cylinder condition in Kaluza-Klein theories naturally describes a periodic boundary condition on a succession of fractally nested worldsheets.

A.5 Quantum Structure

> "The logical structure of the standard model is isomorphic to the geometric structure of the modified cosmological model (MCM). We introduce a new particle representation scheme and show that it is invariant under CPT. In this representation spin arises as an ordinary physical process. The final character of the Higgs boson is predicted. Wavefunction collapse, the symmetry (anti-symmetry) of the wavefunction and some recent experimental results are discussed."

Another huge confirmation of the appropriateness of the framework appeared in this paper when the structure of the standard model of particle physics was shown to be the exact structure of the MCM. This is where the theory graduated from even the most obtuse, codgerly accusations of not-even-wrongness by deriving an experimental prediction akin to Higgs' prediction for a scalar boson – less that six months after discovering the connection to Einstein's equation. Particularly amazing was the accounting for the eight flavors of gluon.

A.6 Ontological Physics

> "Ambiguity in physics makes many useful calculations impossible. Here we reexamine physics' foundation in mathematics and discover a new mode of calculation. The double slit experiment is correctly described by the new mode. We show that spacetime emerges from a set of hidden boundary terms. We propose solutions to problems including the limited spectrum of CMB fluctuations and the anomalous flux of ultra-high energy cosmic rays. A fascinating connection between biology and the new structure should have far reaching implications for the understanding and meaning of life."

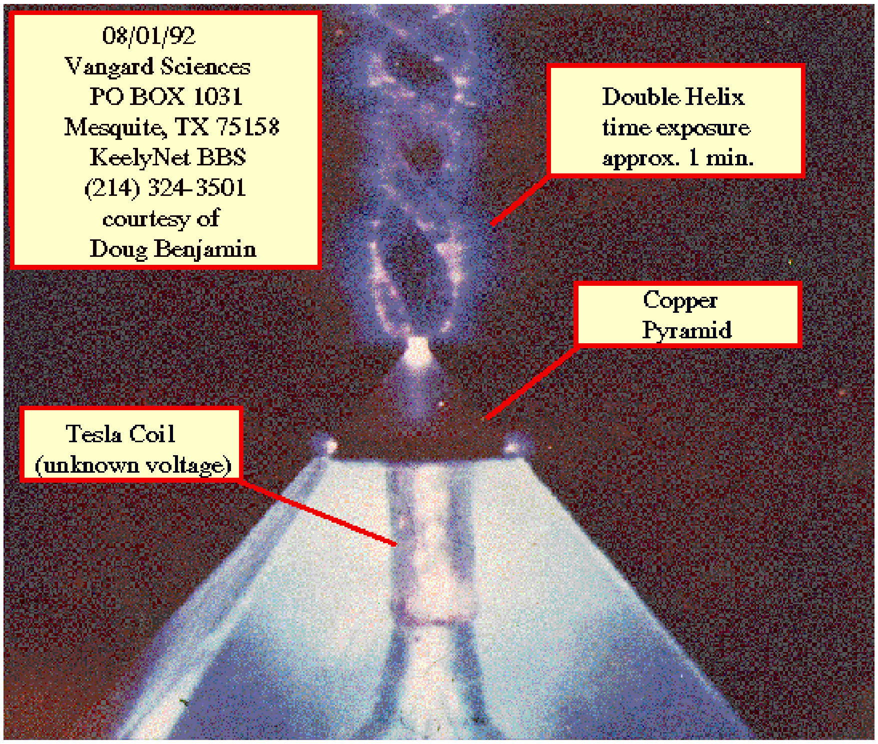

Figure A.1: The effect shown in this figure, if it is real, certainly should be added to the modern physics lab in the undergraduate curriculum.

Here, we developed the argument that has motivated our subsequent reliance on hyperreal numbers. In this book, we aim to continue the formalization of definitions required for the development of "ontological" systems of differential equations of hypercomplex-valued quantities and what they mean for the implied divergences when integrating over singularities. A singularity is a common object even in classical electromagnetism. An example is the potential at the vertex of a pointed, charged object. A real object will not have a mathematically perfect vertex, but when we solve the field equations for the mathematical boundary condition that does have a mathematical vertex, we observe a singularity in a field of real numbers. This paper included figure A.1 which purports to show that unusual behaviors should be assigned to sharp points where cusps lead to singularities.

Among the main results of this paper we to demonstrate that the new interpretation for "levels of $\aleph$" leads to a direct resolution of the classical interpretive paradox in the double slit experiment. This is an important thing that $\hat{M}^3$ does even beyond the myriad previous utilities of $\hat{M}^3$. We have not followed up on this direction very much because we have been taking a fine tooth comb through classical field theory before attempting to wade into quantum field theory, but significant attention is given in this book. Furthermore, the concept of levels of $\aleph$ has driven all subsequent work on the theory of infinite complexity. The new interpretation for the double slit experiment that appeared in this paper is notable because it refers a bifurcation of the particle's worldline in the same way that bifurcating number lines were later treated in reference [5]. This method is expected to have an application to one of the main paradoxes in classical field theory: the breaking and reconnection of field lines.

In this paper from 2013, we point out that $\hat{M}^3$ moves Dirac vectors from $\mathcal{H}_1$ to $\mathcal{H}_2$, though we work with the simpler, non-multiplectic scalar wavefunction in this book. Further, where we have titled the first section of this book An Abstract Psychological Dimension, the primary philosophical predicate of the methodology demonstrated in this paper was that "chiros tracks the observer's attention." We emphasized that the original idea for using Feynman diagrams in our first paper to use the word "modified" [2] was absolutely correct, and the maximum action path derived here in chapter one is certainly the requisite modified version of the action principle identified in 2013. Furthermore, this paper is where we first emphasized that "Dirac orthonormal" functions are a subset of "not orthonormal" functions for analytical purposes. We later went on to develop this showing that Bell's inequality, under a certain very reasonable assumption, actually gives the limiting case when local hidden variables are always allowed [49]. It was in this paper that we first discussed the DNA-like structure in the MCM and this will be a priority subject during the advanced computerized phase of this research program. Another highlight of this paper was to suggest a novel boundary condition for astrophysical searches for evidence of cosmic strings.

A.7 Kerr-Newman, Jung, and the Modified Cosmological Model

> "Where physical theory normally seeks to describe an objective natural world, the modified cosmological model (MCM) seeks to describe an observer's interaction with that world. Qualitative similarities between the psychological observer, the MCM, and the Kerr-Newman black hole are presented. We describe some minimal modifications to previously proposed processes in the MCM. Inflation, large-scale CMB fluctuations and the free energy device are discussed."

This paper was mostly a discussion of esoteric, or *metaphysical*, properties relating the observer and the environment, and the pesky measurement dependence of the quantum mechanical wavefunction. In this paper, we showed that $\hat{M}^3$ can be taken such that it increments the level of $\aleph$ by one or two in each application.

A.8 Infinitely Complex Topology Changes with Quaternions and Torsion

> "We develop some ideas that can be used to show relationships between quantum state tensors and gravitational metric tensors. After firmly grasping the math by α and Einstein's equation, this is another attempt to shake it and see what goes and what stays. We introduce slightly more rigorous definitions for some familiar objects and find an unexpected connection between the chirological phase Φ^n and the quaternions $\mathbf{q}\in\mathbb{H}$. Torsion, the only field in string theory not already present in the theory of infinite complexity, is integrated. We propose a solution to the Ehrenfest paradox and a way to prove the twin primes conjecture. The theory's

apparent connections to negative frequency resonant radiation and time reversal symmetry violation are briefly treated."

The main results of this paper were to explain that topology change should be modeled as a boundary condition on Hilbert space, and that there exists a likely exponential map between the ontological basis $\{\hat{\imath}, \hat{\Phi}, \hat{2}, \hat{\pi}\}$ and the quaternions $\mathbb{H}$, or some modified quaternion $\{\mathbf{i}, \mathbf{\Phi}, \mathbf{2}, \pi\}$ called $\mathbb{H}'$. The dimensionless constant of the MCM $1/4\pi$, which is also the dimensionless coupling constant of electromagnetism, was first reported in this paper.

A.9 Quantum Gravity

"This paper uses a small set of mathematical principles to describe a very wide swath of physics. These principles define a new theory of quantum gravity called the theory of infinite complexity. The main result is that Einstein's equation for general relativity can be derived from unrelated, mathematically novel quantum phenomena. That the theory takes no free parameters should be considered strong evidence in favor of a real connection between physics and mathematics."

This paper condensed previous work to derive a logically minimal mathematical representation that has been the basis of subsequent work, and is the origin of the notation convention used in the present paper.

A.10 On the Reimann Zeta Function

"We discuss the Riemann zeta function, the topology of its domain, and make an argument against the Riemann hypothesis. While making the argument in the classical formalism, we discuss the material as it relates to the theory of infinite complexity (TOIC). We extend Riemann's own (planar) analytic continuation $\mathbb{R} \to \mathbb{C}$ into (bulk) hypercomplexity with $\mathbb{C} \to {}^\star\mathbb{C}$. We propose a solution to the Banach–Tarski paradox."

The main result of this paper was an argument against the Riemann hypothesis. In the course of making that argument, we showed that the topology of the Riemann sphere is not sufficient for the purposes of hypercomplexity and we proposed to add a second null point to $\mathbb{S}^2$. When we have proposed in this book to attach Φ to the null point, a second null point indicates a second $\hat{\Phi}$ of the sort required to derive the MCM's critical value $i\pi\Phi^2$.

A.11 The Truth about Evolution

"The purpose of this report is to debunk Darwin's theory of evolution and any variant theory that relies on the natural rate of mutation to explain the origin of new genes. We construct a model of DNA and show that the minimum rate of mutation needed to produce humans within the geological age of the Earth is too high. It is much higher than any realistic model of random mutations. The calculation presented here should end the evolution debate, at least in its Darwinian limit. Other problems with evolution are discussed."

This paper shows that the mainstream scientific observer is most likely fundamentally wrong about his or her place in the universe and should be open to alternative interpretations in general, and particularly so regarding systems of extreme complexity.

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