Derivation method of numerous dynamics in the Special Theory of Relativity

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Abstract:

The article presents our innovative method of deriving dynamics in the Special Theory of Relativity. This method enables to derive infinitely dynamics in relativistic mechanics. We have shown five examples of these derivations. In this way, we have shown that the dynamics known today as the dynamics of Special Theory of Relativity is only one of infinitely possible. There is also no reason to treat this relativistic dynamics as exceptional, either for experimental or theoretical reasons. Therefore, determination of which possible dynamics of relativistic mechanics is a correct model of reality remains an open problem of physics.

1. Introduction

Kinematics deals with the movement of bodies without taking their physical characteristics into account. The basic concepts of kinematics are: time, location, transformation, speed and acceleration.

Dynamics deals with the movement of material bodies under the action of forces. The basic concepts of dynamics are: mass of inertia, force, momentum and kinetic energy.

Kinematics and dynamics are resulting in mechanics. In the article we deal with relativistic mechanics, i.e. the Special Theory of Relativity, which unlike classical mechanics, also applies to high-speed.

Currently, only one dynamics of the Special Theory of Relativity is known. In the article we presented the author's method of deriving numerous dynamics for this theory. Relativistic dynamics is derived based on the relativistic kinematics and one additional assumption, which allows the concept of mass, momentum and kinetic energy to be introduced into the theory.

2. Kinematic assumptions of the Special Theory of Relativity

The kinematics of the Special Theory of Relativity is based on the following assumptions:

I. **All inertial systems are equivalent**.
This assumption means that there is no such a physical phenomenon, which distinguishes the inertial system. In a particular case, it means that there is no such phenomenon for which the absolute rest is needed to explain. Mathematically, it results from this assumption that time transformation and position coordinates between any two inertial systems has an identical form, depending only on the relative velocity of these inertial systems.

II. **Velocity of light *c* in vacuum is the same in every direction and in each inertial system.**

III. **Transformation of time and position coordinates between the inertial systems is linear**.

These assumptions are often written in other equivalent forms.

Based on mentioned assumptions, it is possible to derive Lorentz transformation on which the Special Theory of Relativity is based. There are many different derivation ways of this transformation. Two derivations are presented in monograph [3].

Markings adopted in Figure 1. will be convenient for our needs. Inertial systems move along their x-axis. The velocity $v_{2/1}$ is a velocity of U_2 system measured by the observer from U_1 system. The velocity $v_{1/2}$ is a velocity of U_1 system measured by the observer from U_2 system. In the Special Theory of Relativity occurs that $v_{2/1} = -v_{1/2}$.

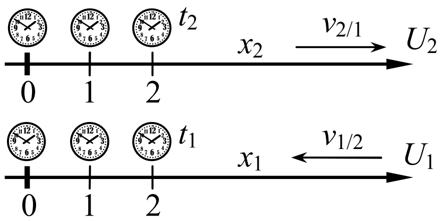

Fig. 1. Relative movement of inertial systems U_1 and U_2 ($v_{2/1} = -v_{1/2}$).

Lorentz transformation from U_2 to U_1 system has a form of:

$$t_1 = \frac{1}{\sqrt{1-(v_{2/1}/c)^2}}\left(t_2 + \frac{v_{2/1}}{c^2}x_2\right) \tag{1}$$

$$x_1 = \frac{1}{\sqrt{1-(v_{2/1}/c)^2}}(v_{2/1}t_2 + x_2) \tag{2}$$

$$y_1 = y_2, \qquad z_1 = z_2 \tag{3}$$

Lorentz transformation from U_1 to U_2 system has a form of:

$$t_2 = \frac{1}{\sqrt{1-(v_{1/2}/c)^2}}\left(t_1 + \frac{v_{1/2}}{c^2}x_1\right) \tag{4}$$

$$x_2 = \frac{1}{\sqrt{1-(v_{1/2}/c)^2}}(v_{1/2}t_1 + x_1) \tag{5}$$

$$y_2 = y_1, \qquad z_2 = z_1 \tag{6}$$

Transformation (1)-(3) and (4)-(6) includes complete information on the relativistic kinematics.

3. Selected properties of relativistic kinematics

In order to derive dynamics we will need two formulas from kinematics, i.e. (20) and (23) from kinematics. We will derive them out of transformation (1)-(3).

3.1. Transformation of velocity

Determine the differentials from transformation (1)-(3)

$$dt_1 = \frac{1}{\sqrt{1-(v_{2/1}/c)^2}}(dt_2 + \frac{v_{2/1}}{c^2}dx_2) \tag{7}$$

$$dx_1 = \frac{1}{\sqrt{1-(v_{2/1}/c)^2}}(v_{2/1}dt_2 + dx_2) \tag{8}$$

$$dy_1 = dy_2, \qquad dz_1 = dz_2 \tag{9}$$

From the inertial system U_1 and U_2, the moving body U_3 is observed. In U_1 system, it has a velocity of $v_{3/1}$, while in U_2 system has a velocity of $v_{3/2}$. The components of these velocities were presented in Figure 2.

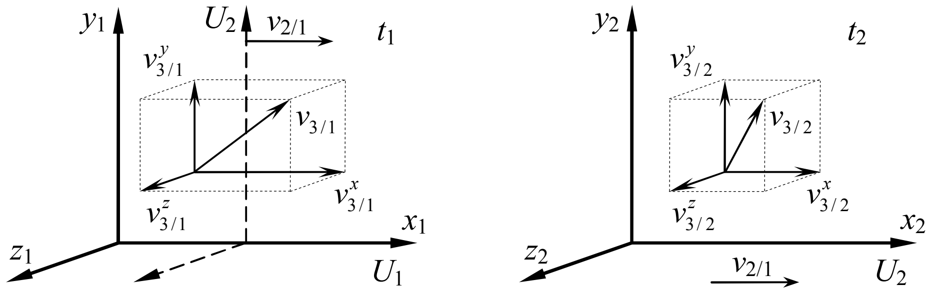

Fig. 2. Movement of the body from two inertial systems U_1 and U_2.

The body velocity U_3 in inertial system U_2 has the following components

$$v^x_{3/2} = \frac{dx_2}{dt_2}, \qquad v^y_{3/2} = \frac{dy_2}{dt_2}, \qquad v^z_{3/2} = \frac{dz_2}{dt_2} \tag{10}$$

The body velocity U_3 in inertial system U_1 has the following components

$$v^x_{3/1} = \frac{dx_1}{dt_1}, \qquad v^y_{3/1} = \frac{dy_1}{dt_1}, \qquad v^z_{3/1} = \frac{dz_1}{dt_1} \tag{11}$$

When to equations (11) we put differentials (7)-(9) then we will receive

$$\begin{cases} v^x_{3/1} = \dfrac{\frac{1}{\sqrt{1-(v_{2/1}/c)^2}}(v_{2/1}dt_2 + dx_2)}{\frac{1}{\sqrt{1-(v_{2/1}/c)^2}}(dt_2 + \frac{v_{2/1}}{c^2}dx_2)} \\ v^y_{3/1} = \dfrac{dy_2}{\frac{1}{\sqrt{1-(v_{2/1}/c)^2}}(dt_2 + \frac{v_{2/1}}{c^2}dx_2)} \\ v^z_{3/1} = \dfrac{dz_2}{\frac{1}{\sqrt{1-(v_{2/1}/c)^2}}(dt_2 + \frac{v_{2/1}}{c^2}dx_2)} \end{cases} \tag{12}$$

i.e.

$$\begin{cases} v^x_{3/1} = \dfrac{v_{2/1} + dx_2/dt_2}{1 + \dfrac{v_{2/1}}{c^2}(dx_2/dt_2)} \\ v^y_{3/1} = \sqrt{1-(v_{2/1}/c)^2}\,\dfrac{dy_2/dt_2}{1 + \dfrac{v_{2/1}}{c^2}(dx_2/dt_2)} \\ v^z_{3/1} = \sqrt{1-(v_{2/1}/c)^2}\,\dfrac{dz_2/dt_2}{1 + \dfrac{v_{2/1}}{c^2}(dx_2/dt_2)} \end{cases} \tag{13}$$

On the basis of (10) we obtain the desired velocity transformation from U_2 to U_1 system

$$\begin{cases} v^x_{3/1} = \dfrac{v^x_{3/2} + v_{2/1}}{1 + \dfrac{v^x_{3/2}v_{2/1}}{c^2}} \\ v^y_{3/1} = \sqrt{1-(v_{2/1}/c)^2}\,\dfrac{v^y_{3/2}}{1 + \dfrac{v^x_{3/2}v_{2/1}}{c^2}} \\ v^z_{3/1} = \sqrt{1-(v_{2/1}/c)^2}\,\dfrac{v^z_{3/2}}{1 + \dfrac{v^x_{3/2}v_{2/1}}{c^2}} \end{cases} \tag{14}$$

In special case, when U_3 body moves parallel to x-axis then occurs

$$v^x_{3/1} = v_{3/1}, \qquad v^x_{3/2} = v_{3/2}, \qquad v^y_{3/1} = v^y_{3/2} = 0, \qquad v^z_{3/1} = v^z_{3/2} = 0 \tag{15}$$

Then velocity transformation (14) takes the form of formula to sum-up parallel velocities

$$v_{3/1} = \frac{v_{3/2} + v_{2/1}}{1 + \dfrac{v_{3/2}v_{2/1}}{c^2}} \tag{16}$$

3.2. Change of velocity seen from different inertial systems

The body is inert in U_3 system and performs a momentary acceleration to U_3 system. The body movement is observed from U_1 and U_2 systems. The velocities of inertial systems are parallel to each other. We adopt markings shown in Figure 3.

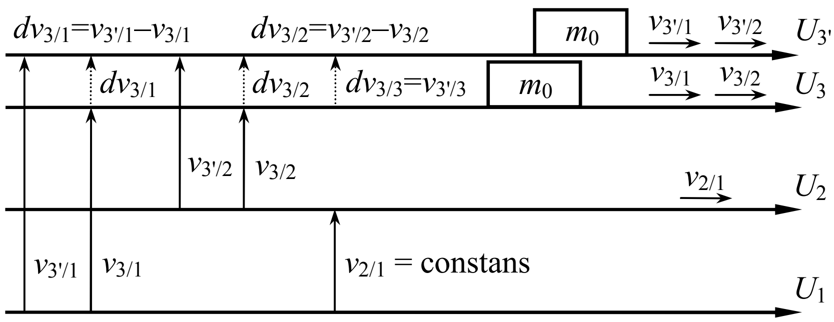

Fig. 3. Increases in the velocity seen in inertial systems U_1 and U_2.

We will determine the differentials from formula (16)

$$dv_{3/1} = \frac{d\dfrac{v_{3/2}+v_{2/1}}{1+(v_{3/2}v_{2/1})/c^2}}{dv_{3/2}}dv_{3/2} = \frac{1+\dfrac{v_{3/2}v_{2/1}}{c^2}-(v_{3/2}+v_{2/1})\dfrac{v_{2/1}}{c^2}}{\left(1+\dfrac{v_{3/2}v_{2/1}}{c^2}\right)^2}dv_{3/2} \tag{17}$$

$$dv_{3/1} = \frac{1-\dfrac{v_{2/1}^2}{c^2}}{\left(1+\dfrac{v_{3/2}v_{2/1}}{c^2}\right)^2}dv_{3/2} \tag{18}$$

If U_3 system is U_2 system then it is necessary to replace index 3 with 2. We will receive

$$dv_{3/1} = dv_{2/1}, \quad v_{3/2} = v_{2/2} = 0, \quad dv_{3/2} = dv_{2/2} \tag{19}$$

On this basis, the formula (18) takes a form of

$$dv_{2/2} = \frac{dv_{2/1}}{1-(v_{2/1}/c)^2} \tag{20}$$

Relation (20) is related to the change of body velocity seen in the inertial system U_2, in which the body is located ($dv_{2/2}$), and the change of velocity seen from another inertial system U_1 ($dv_{2/1}$).

3.3. Time dilatation

If motionless body is in U_2 system, then for its coordinates occurs

$$\frac{dx_2}{dt_2} = 0 \tag{21}$$

Based on time transformation (7) we receive

$$\frac{dt_1}{dt_2} = \frac{1}{\sqrt{1-(v_{2/1}/c)^2}}\left(1+\frac{v_{2/1}}{c^2}\frac{dx_2}{dt_2}\right) \overset{\frac{dx_2}{dt_2}=0}{\Rightarrow} \frac{dt_1}{dt_2} = \frac{1}{\sqrt{1-(v_{2/1}/c)^2}} \tag{22}$$

On this basis we receive the formula for time dilatation of motionless body with regard to U_2 system

$$\frac{dx_2}{dt_2} = 0 \quad \Rightarrow \quad dt_2 = \sqrt{1-(v_{2/1}/c)^2} \cdot dt_1 \tag{23}$$

4. Dynamics in the Special Theory of Relativity

All dissertations will be conducted only for one-dimensional model, i.e. all analyzed vector values will be parallel to x-axis. Each derived dynamic can easily be generalized into three-dimensional cases.

In order to derive dynamics in the Special Theory of Relativity, it is necessary to adopt an additional assumption, which allows the concept of mass, momentum and kinetic energy to be introduced into the theory. Depending on the assumption, different dynamics of bodies are received.

The mass of inertia body resting in inertial frame of reference is determined by m_0 (rest mass). The mass of inertia body mass U_2, as seen from U_1 system, is determined by $m_{2/1}$ (relativistic mass). It is worth to note that the relativistic mass in this case is an inertia mass that occurs in the Newton's second law, rather than mass occurring in the formula for momentum, as assumed in the Special Theory of Relativity. In this way, we have adopted a different definition of relativistic mass, than adopted in the Special Theory of Relativity. Such a definition of the relativistic mass is more convenient in deriving dynamics.

For force, momentum, and kinetic energy, definitions identical as in classical mechanics apply.

The body of m_0 mass of inertia is in U_2 system. It is affected by force $F_{2/2}$ that causes acceleration of $dv_{2/2}/dt_2$. Therefore, for the observer from U_2 system, the Newton's second law takes a form of

$$F_{2/2} = m_0 \cdot a_{2/2} = m_0 \frac{dv_{2/2}}{dt_2} \tag{24}$$

For the observer from U_1 system, mass of inertia of the same body is $m_{2/1}$. For this observer, the force $F_{2/1}$ acts on the body, causing acceleration of $dv_{2/1}/dt_1$. Therefore, for the observer from U_1 the Newton's second law takes the form of

$$F_{2/1} = m_{2/1} \cdot a_{2/1} = m_{2/1} \frac{dv_{2/1}}{dt_1} \tag{25}$$

For the observer from U_2 system, the change of this body momentum can be recorded in the following forms

$$dp_{2/2} = F_{2/2} \cdot dt_2 = m_0 \cdot a_{2/2} \cdot dt_2 = m_0 \frac{dv_{2/2}}{dt_2} dt_2 = m_0 \cdot dv_{2/2} \tag{26}$$

For the observer from U_1 system, the change of this body momentum can be recorded in the following forms

$$dp_{2/1} = F_{2/1} \cdot dt_1 = m_{2/1} \cdot a_{2/1} \cdot dt_1 = m_{2/1} \frac{dv_{2/1}}{dt_1} dt_1 = m_{2/1} \cdot dv_{2/1} \tag{27}$$

where:

- $dp_{2/2}$ is a change of body momentum with rest mass m_0 in the inertial system U_2, measured by the observer from the same inertial system U_2,
- $dp_{2/1}$ is a change of body momentum in the inertial system U_2, measured by the observer from the same inertial system U_1.

Kinetic energy of the body is equal of the work into its acceleration. For the observer from U_1 system, the change of kinetic energy of this body is as follows

$$dE_{2/1} = F_{2/1} \cdot dx_{2/1} = m_{2/1} \cdot a_{2/1} \cdot dx_{2/1} = m_{2/1} \frac{dv_{2/1}}{dt_1} dx_{2/1} = m_{2/1} \frac{dx_{2/1}}{dt_1} dv_{2/1} = m_{2/1} \cdot v_{2/1} \cdot dv_{2/1} \tag{28}$$

where:

- $dE_{2/1}$ is a change of kinetic energy of the body in inertial system U_2, measured by the observer from the inertial system U_1.

4.1. STR dynamics with constant force (STR/*F*)

In this section, a model of dynamics of bodies based on the assumption that the force accelerating of the body (parallel to *x*-axis) is the same for an observer from every inertial system will be derived (hence indication *F*).

4.1.1. The relativistic mass in STR/*F*

In the model STR/*F* we assume, that

$$F_{2/1}^{F} = F_{2/2} \tag{29}$$

Having introduced (24) and (25), we obtain

$$m_{2/1}^{F} \frac{dv_{2/1}}{dt_1} = m_0 \frac{dv_{2/2}}{dt_2} \tag{30}$$

On the base (20) and (23), we have

$$m_{2/1}^{F} \frac{dv_{2/1}}{dt_1} = m_0 \frac{dv_{2/1}}{1-(v_{2/1}/c)^2} \cdot \frac{1}{\sqrt{1-(v_{2/1}/c)^2} \cdot dt_1} \tag{31}$$

Hence, we obtain a formula for relativistic mass of the body that is located in the system U_2 and is seen from the system U_1, when assumption (29) is satisfied, as below

$$m_{2/1}^{F} = m_0 \left[\frac{1}{1-(v_{2/1}/c)^2} \right]^{3/2} \tag{32}$$

4.1.2. The momentum in STR/*F*

The body of rest mass m_0 is associated with the system U_2. To determine the momentum of the body relative to the system U_1 we substitute (32) to (27)

$$dp_{2/1}^{F} = m_{2/1}^{F} \cdot dv_{2/1} = m_0 \left[\frac{1}{1-(v_{2/1}/c)^2} \right]^{3/2} dv_{2/1} = m_0 c^3 \frac{1}{(c^2 - v_{2/1}^2)^{3/2}} dv_{2/1} \tag{33}$$

The body momentum is a sum of increases in its momentum, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$p_{2/1}^{F} = m_0 c^3 \int_0^{v_{2/1}} \frac{1}{(c^2 - v_{2/1}^2)^{3/2}} dv_{2/1} \tag{34}$$

From the work [1] (formula 72, p. 167) it is possible to read out, that

$$\int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 - x^2}}, \qquad a \neq 0 \tag{35}$$

After applying the integral (35) to (34) we receive the formula for the body momentum in U_2 system and measured by the observer from U_1 system in a form of

$$p_{2/1}^{F} = m_0 c^3 \frac{v_{2/1}}{c^2 \sqrt{c^2 - v_{2/1}^2}} = \frac{m_0}{\sqrt{1-(v_{2/1}/c)^2}} v_{2/1} \tag{36}$$

This formula is identical to the formula for momentum known from the Special Theory of Relativity, for the same reasons as in the case of momentum. This is because the dynamics known from the Special Theory of Relativity is derived from the assumption (29). It was adopted unconsciously, because it was considered as necessary. The awareness of this assumption allows to its change and derives other dynamics.

As already mentioned above, the definition of relativistic mass adopted by us is different from the definition adopted in the Special Theory of Relativity. In our case, the relativistic mass is the one, which occurs in the Newton's second law (25). In this particular case, it is expressed in terms of dependency (32). In the Special Theory of Relativity, the relativistic mass is the one, which occurs in the formula (36) per momentum.

4.1.3. The momentum in STR/*F* for small velocities

For small velocity $v_{2/1} << c$ momentum (36) comes down to the momentum from classical mechanics, because

$$v_{2/1} << c \quad \Rightarrow \quad p^F_{2/1} \approx m_0 v_{2/1} \tag{37}$$

4.1.4. The kinetic energy in STR/*F*

We will determine the formula for kinetic energy. To the formula (28), we introduce the dependence for the relativistic mass (32)

$$dE^F_{2/1} = m^F_{2/1} \cdot v_{2/1} \cdot dv_{2/1} = m_0 \left[\frac{1}{1-(v_{2/1}/c)^2} \right]^{3/2} v_{2/1} dv_{2/1} = m_0 c^3 \frac{v_{2/1}}{(c^2 - v^2_{2/1})^{3/2}} dv_{2/1} \tag{38}$$

The kinetic energy of body is a sum of increases in its kinetic energy, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$E^F_{2/1} = m_0 c^3 \int_0^{v_{2/1}} \frac{v_{2/1}}{(c^2 - v^2_{2/1})^{3/2}} dv_{2/1} \tag{39}$$

From the work [1] (formula 74, p. 167) it is possible to read out, that

$$\int \frac{x dx}{(a^2 - x^2)^{3/2}} = \frac{1}{\sqrt{a^2 - x^2}} \tag{40}$$

After applying the integral (40) to (39) we receive the formula for the kinetic energy of the body in U_2 system and measured by the observer from U_1 system in a form of

$$E^F_{2/1} = m_0 c^3 \frac{1}{\sqrt{c^2 - x^2}} \Bigg|_0^{v_{2/1}} = m_0 c^3 \left(\frac{1}{\sqrt{c^2 - v^2_{2/1}}} - \frac{1}{c} \right) = m_0 c^2 \frac{1}{\sqrt{1-(v_{2/1}/c)^2}} - m_0 c^2 \tag{41}$$

This formula is identical to the formula for kinetic energy known from the Special Theory of Relativity, for the same reasons as in the case of momentum (36).

4.1.5. The kinetic energy in STR/*F* for small velocities

Formula (41) can be written in the form

$$E^F_{2/1} = m_0 c^2 \frac{1 - \sqrt{1-(v_{2/1}/c)^2}}{\sqrt{1-(v_{2/1}/c)^2}} \cdot \frac{1 + \sqrt{1-(v_{2/1}/c)^2}}{1 + \sqrt{1-(v_{2/1}/c)^2}} \tag{42}$$

$$E^{F}_{2/1} = \frac{m_0 v^2_{2/1}}{2} \frac{2}{1 - \frac{v^2_{2/1}}{c^2} + \sqrt{1 - \frac{v^2_{2/1}}{c^2}}} \tag{43}$$

On this basis, for small values $v_{2/1} << c$ we receive

$$v_{2/1} << c \quad \Rightarrow \quad E^{F}_{2/1} \approx \frac{m_0 v^2_{2/1}}{2} \frac{2}{1+1} = \frac{m_0 v^2_{2/1}}{2} \tag{44}$$

4.1.6. The force in STR/*F*

Due to the assumption (29) value measurement of the same force by two different observers is identical.

4.2. STR dynamics with constant momentum change (STR/Δ*p*)

In this section, a model of dynamics of bodies based on the assumption that the change in momentum of the body (parallel to *x*-axis) is the same for an observer from every inertial system will be derived (hence indication Δ*p*).

These dynamics seem particularly interesting, because the conservation law of momentum is a fundamental law. Assumption that the change of body momentum is the same for every observer seems to be a natural extension of this law.

4.2.1. The relativistic mass in STR/Δ*p*

In the model STR/Δ*p* we assume, that

$$dp^{\Delta p}_{2/1} = dp_{2/2} \tag{45}$$

Having introduced (26) and (27), we obtain

$$m^{\Delta p}_{2/1} dv_{2/1} = m_0 dv_{2/2} \tag{46}$$

On the base (20), we have

$$m^{\Delta p}_{2/1} dv_{2/1} = m_0 \frac{dv_{2/1}}{1 - (v_{2/1}/c)^2} \tag{47}$$

Hence, we obtain a formula for relativistic mass of the body that is located in the system U_2 and is seen from the system U_1, when assumption (45) is satisfied, as below

$$m^{\Delta p}_{2/1} = m_0 \frac{1}{1 - (v_{2/1}/c)^2} \tag{48}$$

4.2.2. The momentum in STR/Δ*p*

The body of rest mass m_0 is associated with the system U_2. To determine the momentum of the body relative to the system U_1 we substitute (48) to (27)

$$dp^{\Delta p}_{2/1} = m^{\Delta p}_{2/1} \cdot dv_{2/1} = m_0 \frac{1}{1 - (v_{2/1}/c)^2} dv_{2/1} = m_0 c^2 \frac{1}{c^2 - v^2_{2/1}} dv_{2/1} \tag{49}$$

The body momentum is a sum of increases in its momentum, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$p_{2/1}^{\Delta p} = m_0 c^2 \int_0^{v_{2/1}} \frac{1}{c^2 - v_{2/1}^2} dv_{2/1} \tag{50}$$

From the work [1] (formula 52, p. 160) it is possible to read out, that

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right|, \qquad a \neq 0 \tag{51}$$

After applying the integral (51) to (50) we receive the formula for the body momentum in U_2 system and measured by the observer from U_1 system in a form of

$$p_{2/1}^{\Delta p} = m_0 c^2 \frac{1}{2c} \ln\left|\frac{c+x}{c-x}\right|\Bigg|_0^{v_{2/1}} = \frac{m_0 c}{2} \ln\left(\frac{c+v_{2/1}}{c-v_{2/1}}\right) \tag{52}$$

4.2.3. The momentum in STR/Δ*p* for small velocities

Formula (52) can be written in the form

$$p_{2/1}^{\Delta p} = \frac{m_0 v_{2/1}}{2} \frac{c}{v_{2/1}} \ln\left(\frac{c+v_{2/1}}{c-v_{2/1}}\right) = \frac{m_0 v_{2/1}}{2} \ln\left(\frac{(1+v_{2/1}/c)^{c/v_{2/1}}}{(1-v_{2/1}/c)^{c/v_{2/1}}}\right) \tag{53}$$

$$p_{2/1}^{\Delta p} = \frac{m_0 v_{2/1}}{2} \ln\left(\frac{\left(1+\frac{1}{c/v_{2/1}}\right)^{c/v_{2/1}}}{\left(1-\frac{1}{c/v_{2/1}}\right)^{c/v_{2/1}}}\right) \tag{54}$$

On this basis, for small values $v_{2/1} << c$ we receive

$$v_{2/1} << c \quad \Rightarrow \quad p_{2/1}^{\Delta p} \approx \frac{m_0 v_{2/1}}{2} \ln\left(\frac{e}{1/e}\right) = \frac{m_0 v_{2/1}}{2} \ln(e^2) = m_0 v_{2/1} \tag{55}$$

4.2.4. The kinetic energy in STR/Δ*p*

We will determine the formula for kinetic energy. To the formula (28), we introduce the dependence for the relativistic mass (48)

$$dE_{2/1}^{\Delta p} = m_{2/1}^{\Delta p} \cdot v_{2/1} \cdot dv_{2/1} = m_0 \frac{1}{1-(v_{2/1}/c)^2} v_{2/1} dv_{2/1} = m_0 c^2 \frac{v_{2/1}}{c^2 - v_{2/1}^2} dv_{2/1} \tag{56}$$

The kinetic energy of body is a sum of increases in its kinetic energy, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$E_{2/1}^{\Delta p} = m_0 c^2 \int_0^{v_{2/1}} \frac{v_{2/1}}{c^2 - v_{2/1}^2} dv_{2/1} \tag{57}$$

From the work [1] (formula 56, p. 160) it is possible to read out, that

$$\int \frac{x}{a^2 - x^2}\,dx = -\frac{1}{2}\ln\left|a^2 - x^2\right| \tag{58}$$

After applying the integral (58) to (57) we receive the formula for the kinetic energy of the body in U_2 system and measured by the observer from U_1 system in a form of

$$E_{2/1}^{\Delta p} = -m_0c^2\frac{1}{2}\ln\left|c^2 - x^2\right|\Bigg|_0^{v_{2/1}} = -\frac{m_0c^2}{2}\ln(c^2 - v_{2/1}^2) + \frac{m_0c^2}{2}\ln(c^2) \tag{59}$$

$$E_{2/1}^{\Delta p} = \frac{m_0c^2}{2}\ln\frac{c^2}{c^2 - v_{2/1}^2} = \frac{m_0c^2}{2}\ln\frac{1}{1-(v_{2/1}/c)^2} \tag{60}$$

4.2.5. The kinetic energy in STR/Δ*p* for small velocities

Formula (60) can be written in the form

$$E_{2/1}^{\Delta p} = \frac{m_0v_{2/1}^2}{2}\frac{c^2}{v_{2/1}^2}\ln\frac{1}{1-(v_{2/1}/c)^2} = \frac{m_0v_{2/1}^2}{2}\ln\frac{1}{[1-(v_{2/1}/c)^2]^{(c/v_{2/1})^2}} \tag{61}$$

$$E_{2/1}^{\Delta p} = \frac{m_0v_{2/1}^2}{2}\ln\frac{1}{\left[1-\dfrac{1}{(c/v_{2/1})^2}\right]^{(c/v_{2/1})^2}} \tag{62}$$

On this basis, for small values $v_{2/1} << c$ we receive

$$v_{2/1} << c \quad \Rightarrow \quad E_{2/1}^{\Delta p} \approx \frac{m_0v_{2/1}^2}{2}\ln\frac{1}{1/e} = \frac{m_0v_{2/1}^2}{2} \tag{63}$$

4.2.6. The force in STR/Δ*p*

Body with rest mass m_0 is related to U_2 system. It is affected by force that causes acceleration. For the observer from this system, the acceleration force has in accordance with (24) the following value

$$F_{2/2} = m_0\frac{dv_{2/2}}{dt_2} \tag{64}$$

For the observer from U_1 system, acceleration force has in accordance with (25) the following value

$$F_{2/1}^{\Delta p} = m_{2/1}^{\Delta p}\frac{dv_{2/1}}{dt_1} \tag{65}$$

If we will divide parties' equation (65) by (64), then on the basis of (20) and (23) we will receive

$$\frac{F_{2/1}^{\Delta p}}{F_{2/2}} = \frac{m_{2/1}^{\Delta p}}{m_0}\cdot\frac{dt_2}{dt_1}\cdot\frac{dv_{2/1}}{dv_{2/2}} = \frac{m_{2/1}^{\Delta p}}{m_0}(1-(v_{2/1}/c)^2)^{3/2} \tag{66}$$

On the basis of (48) we obtain a relation between measurements of the same force by two different observers

$$F_{2/1}^{\Delta p} = \sqrt{1-(v_{2/1}/c)^2} \cdot F_{2/2} \tag{67}$$

The highest value of force is measured by the observer from the inertial system in which the body is located.

4.3. STR dynamics with constant mass (STR/*m*)

In this section, a model of dynamics of bodies based on the assumption that body weight is the same for an observer from each inertial reference system will be derived (hence indication *m*).

4.3.1. The relativistic mass in STR/*m*

In the model STR/*m* we assume, that

$$m_{2/1}^{m} = m_0 \tag{68}$$

Therefore, for the observer from inertial system U_1, the body mass in U_2 system is the same as the rest mass.

4.3.2. The momentum in STR/*m*

The body of rest mass m_0 is associated with the system U_2. To determine the momentum of the body relative to the system U_1 we substitute (68) to (27)

$$dp_{2/1}^{m} = m_{2/1}^{m} \cdot dv_{2/1} = m_0 dv_{2/1} \tag{69}$$

The body momentum is a sum of increases in its momentum, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$p_{2/1}^{m} = m_0 \int_0^{v_{2/1}} dv_{2/1} = m_0 v_{2/1} \tag{70}$$

In this relativistic dynamics the momentum is expressed with the same equation as in classical mechanics.

4.3.3. The kinetic energy in STR/*m*

We will determine the formula for kinetic energy. To the formula (28), we introduce the dependence for the relativistic mass (68)

$$dE_{2/1}^{m} = m_{2/1}^{m} \cdot v_{2/1} \cdot dv_{2/1} = m_0 v_{2/1} dv_{2/1} \tag{71}$$

The kinetic energy of body is a sum of increases in its kinetic energy, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$E_{2/1}^{m} = m_0 \int_0^{v_{2/1}} v_{2/1} dv_{2/1} = \frac{m_0 v_{2/1}^2}{2} \tag{72}$$

In this relativistic dynamics the kinetic energy is expressed with the same equation as in classical mechanics.

4.3.4. The force in STR/*m*

Body with rest mass m_0 is related to U_2 system. It is affected by force that causes acceleration. For the observer from this system, the acceleration force has in accordance with (24) the following value

$$F_{2/2} = m_0 \frac{dv_{2/2}}{dt_2} \tag{73}$$

For the observer from U_1 system, acceleration force has in accordance with (25) the following value

$$F_{2/1}^m = m_{2/1}^m \frac{dv_{2/1}}{dt_1} = m_0 \frac{dv_{2/1}}{dt_1} \tag{74}$$

If we will divide parties' equation (74) by (73), then on the basis of (20) and (23) we will receive

$$\frac{F_{2/1}^m}{F_{2/2}} = \frac{dt_2}{dt_1} \cdot \frac{dv_{2/1}}{dv_{2/2}} = (1-(v_{2/1}/c)^2)^{3/2} \tag{75}$$

i.e.

$$F_{2/1}^m = (1-(v_{2/1}/c)^2)^{3/2} \cdot F_{2/2} \tag{76}$$

The highest value of force is measured by the observer from the inertial system in which the body is located.

4.3.5. Discussion on the STR/*m* dynamics

Obtaining a relativistic dynamics, in which there is no relativistic mass, and equations for kinetic energy and momentum are identical as in classical mechanics can be surprising, because in relativistic mechanics it is believed that the accelerated body can achieve maximum speed c. However, this dynamics is formally correct.

If the body velocity $v_{2/1}$ reaches c value, then according to (76)

$$F_{2/1}^m = (1-1^-)^{3/2} \cdot F_{2/2} \approx 0 \tag{77}$$

In the inertial system U_2, in which the body is located, can be affected by acceleration force $F_{2/2}$ of any, but finite value. However, from a perspective of the inertial system U_1, towards which the body has c velocity, the same force is zero. This means that from a perspective of U_1 system, it is not possible to perform work on the body, which will increase its kinetic energy indefinitely. From the relation (72) it results that the kinetic energy, that a body with mass m_0 and velocity c has, a value has

$$E_{\max}^m = \frac{m_0 c^2}{2} \tag{78}$$

4.4. STR dynamics with constant force to its operation time (STR/*F*/Δ*t*)

In this section, a model of dynamics of bodies based on the assumption that the force that accelerates of the body (parallel to *x*-axis) divided by the time of operation of this force is the same for an observer from every inertial system will be derived (hence indication *F*/Δ*t*).

4.4.1. The relativistic mass in STR/*F*/Δ*t*

In the model STR/*F*/Δ*t* we assume, that

$$\frac{F_{2/1}^{F/\Delta t}}{dt_1}=\frac{F_{2/2}}{dt_2} \tag{79}$$

Having introduced (24) and (25), we obtain

$$m_{2/1}^{F/\Delta t}\frac{dv_{2/1}}{dt_1}\frac{1}{dt_1}=m_0\frac{dv_{2/2}}{dt_2}\frac{1}{dt_2} \tag{80}$$

On the base (20) and (23), we have

$$m_{2/1}^{F/\Delta t}\frac{dv_{2/1}}{dt_1^2}=m_0\frac{\dfrac{dv_{2/1}}{1-(v_{2/1}/c)^2}}{(1-(v_{2/1}/c)^2)dt_1^2} \tag{81}$$

Hence, we obtain a formula for relativistic mass of the body that is located in the system U_2 and is seen from the system U_1, when assumption (79) is satisfied, as below

$$m_{2/1}^{F/\Delta t}=m_0\left[\frac{1}{1-(v_{2/1}/c)^2}\right]^2 \tag{82}$$

4.4.2. The momentum in STR/*F*/Δ*t*

The body of rest mass m_0 is associated with the system U_2. To determine the momentum of the body relative to the system U_1 we substitute (82) to (27)

$$dp_{2/1}^{F/\Delta t}=m_{2/1}^{F/\Delta t}\cdot dv_{2/1}=m_0\left[\frac{1}{1-(v_{2/1}/c)^2}\right]^2 dv_{2/1}=m_0c^4\frac{1}{(c^2-v_{2/1}^2)^2}dv_{2/1} \tag{83}$$

The body momentum is a sum of increases in its momentum, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$p_{2/1}^{F/\Delta t}=m_0c^4\int_0^{v_{2/1}}\frac{1}{(c^2-v_{2/1}^2)^2}dv_{2/1} \tag{84}$$

From the work [1] (formula 54, p. 160) it is possible to read out, that

$$\int\frac{dx}{(a^2-x^2)^2}=\frac{x}{2a^2(a^2-x^2)}+\frac{1}{4a^3}\ln\left|\frac{a+x}{a-x}\right|,\quad a\neq 0 \tag{85}$$

After applying the integral (85) to (84) we receive the formula for the body momentum in U_2 system and measured by the observer from U_1 system in a form of

$$p_{2/1}^{F/\Delta t} = m_0 c^4 \left[\frac{x}{2c^2(c^2 - x^2)} + \frac{1}{4c^3} \ln \frac{(c+x)}{(c-x)} \right]_0^{v_{2/1}} = m_0 c \left[\frac{c v_{2/1}}{2(c^2 - v_{2/1}^2)} + \frac{1}{4} \ln \frac{(c + v_{2/1})}{(c - v_{2/1})} \right] \tag{86}$$

$$p_{2/1}^{F/\Delta t} = m_0 v_{2/1} \frac{1}{2} \left[\frac{1}{1 - (v_{2/1}/c)^2} + \ln \left(\frac{c + v_{2/1}}{c - v_{2/1}} \right)^{\frac{c}{2v_{2/1}}} \right] \tag{87}$$

4.4.3. The momentum in STR/*F*/Δ*t* for small velocities

Formula (87) can be written in the form

$$p_{2/1}^{F/\Delta t} = m_0 v_{2/1} \left[\frac{1}{2(1 - (v_{2/1}/c)^2)} + \frac{1}{4} \ln \left(\frac{(1 + v_{2/1}/c)^{c/v_{2/1}}}{(1 - v_{2/1}/c)^{c/v_{2/1}}} \right) \right] \tag{88}$$

$$p_{2/1}^{F/\Delta t} = m_0 v_{2/1} \left[\frac{1}{2(1 - (v_{2/1}/c)^2)} + \frac{1}{4} \ln \left(\frac{\left(1 + \frac{1}{c/v_{2/1}}\right)^{c/v_{2/1}}}{\left(1 - \frac{1}{c/v_{2/1}}\right)^{c/v_{2/1}}} \right) \right] \tag{89}$$

On this basis, for small values $v_{2/1} << c$ we receive

$$v_{2/1} << c \quad \Rightarrow \quad p_{2/1}^{F/\Delta t} \approx m_0 v_{2/1} \left[\frac{1}{2} + \frac{1}{4} \ln \left(\frac{e}{1/e} \right) \right] = m_0 v_{2/1} \left[\frac{1}{2} + \frac{1}{4} \ln(e^2) \right] = m_0 v_{2/1} \tag{90}$$

4.4.4. The kinetic energy in STR/*F*/Δ*t*

We will determine the formula for kinetic energy. To the formula (28), we introduce the dependence for the relativistic mass (82)

$$dE_{2/1}^{F/\Delta t} = m_{2/1}^{F/\Delta t} \cdot v_{2/1} \cdot dv_{2/1} = m_0 \left[\frac{1}{1 - (v_{2/1}/c)^2} \right]^2 v_{2/1} dv_{2/1} = m_0 c^4 \frac{v_{2/1}}{(c^2 - v_{2/1}^2)^2} dv_{2/1} \tag{91}$$

The kinetic energy of body is a sum of increases in its kinetic energy, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$E_{2/1}^{F/\Delta t} = m_0 c^4 \int_0^{v_{2/1}} \frac{v_{2/1}}{(c^2 - v_{2/1}^2)^2} dv_{2/1} \tag{92}$$

From the work [1] (formula 58, p. 160) it is possible to read out, that

$$\int \frac{x dx}{(a^2 - x^2)^2} = \frac{1}{2(a^2 - x^2)} \tag{93}$$

After applying the integral (93) do (92) we receive the formula for the kinetic energy of the body in U_2 system and measured by the observer from U_1 system in a form of

$$E_{2/1}^{F/\Delta t} = m_0 c^4 \frac{1}{2(c^2 - x^2)}\bigg|_0^{v_{2/1}} = \frac{m_0 c^4}{2}\frac{1}{(c^2 - v_{2/1}^2)} - \frac{m_0 c^4}{2}\frac{1}{c^2} \tag{94}$$

$$E_{2/1}^{F/\Delta t} = \frac{m_0 c^2}{2}\frac{1}{1-(v_{2/1}/c)^2} - \frac{m_0 c^2}{2} = \frac{m_0 v_{2/1}^2}{2}\frac{1}{1-(v_{2/1}/c)^2} \tag{95}$$

The formula for kinetic energy (95) was derived from the work [2], due to the fact that the author adopted a different assumption than the one on which the dynamics known from the Special Theory of Relativity was based.

4.4.5. The kinetic energy in STR/*F*/Δ*t* for small velocities

For small velocity $v_{2/1} << c$ kinetic energy (95) comes down to the kinetic energy from classical mechanics, because

$$v_{2/1} << c \quad \Rightarrow \quad E_{2/1}^{F/\Delta t} \approx \frac{m_0 v_{2/1}^2}{2} \cdot \frac{1}{1} = \frac{m_0 v_{2/1}^2}{2} \tag{96}$$

4.4.6. The force in STR/*F*/Δ*t*

Body with rest mass m_0 is related to U_2 system. It is affected by force that causes acceleration. For the observer from this system, the acceleration force has in accordance with (24) the following value

$$F_{2/2} = m_0 \frac{dv_{2/2}}{dt_2} \tag{97}$$

For the observer from U_1 system, acceleration force has in accordance with (25) the following value

$$F_{2/1}^{F/\Delta t} = m_{2/1}^{F/\Delta t} \frac{dv_{2/1}}{dt_1} \tag{98}$$

If we will divide parties' equation (98) by (97), then on the basis of (20) and (23) we will receive

$$\frac{F_{2/1}^{F/\Delta t}}{F_{2/2}} = \frac{m_{2/1}^{F/\Delta t}}{m_0} \cdot \frac{dt_2}{dt_1} \cdot \frac{dv_{2/1}}{dv_{2/2}} = \frac{m_{2/1}^{F/\Delta t}}{m_0}(1-(v_{2/1}/c)^2)^{3/2} \tag{99}$$

On the basis of (82) we obtain a relation between measurements of the same force by two different observers

$$F_{2/1}^{F/\Delta t} = \frac{1}{\sqrt{1-(v_{2/1}/c)^2}} \cdot F_{2/2} \tag{100}$$

The lowest value of force is measured by the observer from the inertial system in which the body is located.

4.5. STR dynamics with constant mass to elapse of observer's time (STR/*m*/Δ*t*)

In this subchapter a model of body dynamics will be derived based on the assumption that the body mass divided by the elapse of time in observer system is the same for the observer from each inertial frame of reference (hence indication *m*/Δ*t*).

4.5.1. The relativistic mass in STR/*m*/Δ*t*

In the model STR/*m*/Δ*t* we assume, that

$$\frac{m_{2/1}^{m/\Delta t}}{dt_1} = \frac{m_0}{dt_2} \tag{101}$$

On the base (23), we have

$$\frac{m_{2/1}^{m/\Delta t}}{dt_1} = \frac{m_0}{\sqrt{1-(v_{2/1}/c)^2}\cdot dt_1} \tag{102}$$

Hence, we obtain a formula for relativistic mass of the body, that is located in the system U_2 and is seen from the system U_1, when assumption (101) is satisfied, as below

$$m_{2/1}^{m/\Delta t} = m_0 \frac{1}{\sqrt{1-(v_{2/1}/c)^2}} \tag{103}$$

4.5.2. The momentum in STR/*m*/Δ*t*

The body of rest mass m_0 is associated with the system U_2. To determine the momentum of the body relative to the system U_1 we substitute (103) to (27)

$$dp_{2/1}^{m/\Delta t} = m_{2/1}^{m/\Delta t}\cdot dv_{2/1} = m_0 \frac{1}{\sqrt{1-(v_{2/1}/c)^2}} dv_{2/1} = m_0 c \frac{1}{\sqrt{c^2 - v_{2/1}^2}} dv_{2/1} \tag{104}$$

The body momentum is a sum of increases in its momentum, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$p_{2/1}^{m/\Delta t} = m_0 c^2 \int_0^{v_{2/1}} \frac{1}{\sqrt{c^2 - v_{2/1}^2}} dv_{2/1} \tag{105}$$

From the work [1] (formula 71, p. 167) it is possible to read out, that

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a}, \qquad a > 0 \tag{106}$$

After applying the integral (106) to (105) we receive the formula for the body momentum in U_2 system and measured by the observer from U_1 system in a form of

$$p_{2/1}^{m/\Delta t} = m_0 c \cdot \arcsin\frac{v_{2/1}}{c}\Bigg|_0^{v_{2/1}} = m_0 c\cdot \arcsin\frac{v_{2/1}}{c} \tag{107}$$

4.5.3. The momentum in STR/*m*/Δ*t* for small velocities

Formula (107) can be written in the form

$$p_{2/1}^{m/\Delta t} = m_0 v_{2/1} \frac{\arcsin \frac{v_{2/1}}{c}}{\frac{v_{2/1}}{c}} \tag{108}$$

On this basis, for small values $v_{2/1} << c$ we receive

$$v_{2/1} << c \quad \Rightarrow \quad p_{2/1}^{m/\Delta t} \approx m_0 v_{2/1} \tag{109}$$

4.5.4. The kinetic energy in STR/*m*/Δ*t*

We will determine the formula for kinetic energy. To the formula (28), we introduce the dependence for the relativistic mass (103)

$$dE_{2/1}^{m/\Delta t} = m_{2/1}^{m/\Delta t} \cdot v_{2/1} \cdot dv_{2/1} = m_0 \frac{1}{\sqrt{1-(v_{2/1}/c)^2}} v_{2/1} dv_{2/1} = m_0 c \frac{v_{2/1}}{\sqrt{c^2 - v_{2/1}^2}} dv_{2/1} \tag{110}$$

The kinetic energy of body is a sum of increases in its kinetic energy, when the body is accelerated from the inertial system U_1 (the body has velocity 0) to the inertial system U_2 (the body has velocity $v_{2/1}$), i.e.

$$E_{2/1}^{m/\Delta t} = m_0 c \int_0^{v_{2/1}} \frac{v_{2/1}}{\sqrt{c^2 - v_{2/1}^2}} dv_{2/1} \tag{111}$$

From the work [1] (formula 73, p. 167) it is possible to read out, that

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} \tag{112}$$

After applying the integral (112) do (111) we receive the formula for the kinetic energy of the body in U_2 system and measured by the observer from U_1 system in a form of

$$E_{2/1}^{m/\Delta t} = -m_0 c \sqrt{c^2 - v_{2/1}^2}\Big|_0^{v_{2/1}} = -m_0 c \sqrt{c^2 - v_{2/1}^2} + m_0 c \sqrt{c^2} \tag{113}$$

$$E_{2/1}^{m/\Delta t} = m_0 c^2 - m_0 c \sqrt{c^2 - v_{2/1}^2} = m_0 c^2 (1 - \sqrt{1-(v_{2/1}/c)^2}\,) \tag{114}$$

4.5.5. The kinetic energy in STR/*m*/Δ*t* for small velocities

Formula (114) can be written in the form

$$E_{2/1}^{m/\Delta t} = \frac{m_0 v_{2/1}^2}{2} \cdot \frac{2c^2}{v_{2/1}^2} \cdot \frac{(1 - \sqrt{1-(v_{2/1}/c)^2}\,)(1 + \sqrt{1-(v_{2/1}/c)^2}\,)}{1 + \sqrt{1-(v_{2/1}/c)^2}} \tag{115}$$

$$E_{2/1}^{m/\Delta t} = \frac{m_0 v_{2/1}^2}{2} \cdot \frac{2c^2}{v_{2/1}^2} \cdot \frac{1 - (1 - (v_{2/1}/c)^2)}{1 + \sqrt{1-(v_{2/1}/c)^2}} = \frac{m_0 v_{2/1}^2}{2} \frac{2}{1 + \sqrt{1-(v_{2/1}/c)^2}} \tag{116}$$

On this basis, for small values $v_{2/1} << c$ we receive

$$v_{2/1} << c \quad \Rightarrow \quad E_{2/1}^{m/\Delta t} \approx \frac{m_0 v_{2/1}^2}{2} \cdot \frac{2}{2} = \frac{m_0 v_{2/1}^2}{2} \tag{117}$$

4.5.6. The force in STR/*m*/Δ*t*

Body with rest mass m_0 is related to U_2 system. It is affected by force that causes acceleration. For the observer from this system, the acceleration force has in accordance with (24) the following value

$$F_{2/2} = m_0 \frac{dv_{2/2}}{dt_2} \tag{118}$$

For the observer from U_1 system, acceleration force has in accordance with (25) the following value

$$F_{2/1}^{m/\Delta t} = m_{2/1}^{m/\Delta t} \frac{dv_{2/1}}{dt_1} \tag{119}$$

If we will divide parties' equation (119) by (118), then on the basis of (20) and (23) we will receive

$$\frac{F_{2/1}^{m/\Delta t}}{F_{2/2}} = \frac{m_{2/1}^{m/\Delta t}}{m_0} \cdot \frac{dt_2}{dt_1} \cdot \frac{dv_{2/1}}{dv_{2/2}} = \frac{m_{2/1}^{m/\Delta t}}{m_0} (1-(v_{2/1}/c)^2)^{3/2} \tag{120}$$

On the basis of (103) we obtain a relation between measurements of the same force by two different observers

$$F_{2/1}^{m/\Delta t} = (1-(v_{2/1}/c)^2) \cdot F_{2/2} \tag{121}$$

The highest value of force is measured by the observer from the inertial system in which the body is located.

5. The general form of dynamics

In presented examples, assumptions have been adopted which can be written in forms (30), (46), (68), (80) and (101). On this basis, it can be seen that the assumption for relativistic dynamics is as follows

$$m_{2/1}^{\{a,b\}} \frac{dv_{2/1}^a}{dt_1^b} = m_0 \frac{dv_{2/2}^a}{dt_2^b}, \qquad a,b \in R \tag{122}$$

On the basis of (20) and (23) we receive

$$m_{2/1}^{\{a,b\}} \frac{dv_{2/1}^a}{dt_1^b} = m_0 \frac{\dfrac{dv_{2/1}^a}{(1-(v_{2/1}/c)^2)^a}}{(1-(v_{2/1}/c)^2)^{b/2} \cdot dt_1^b} \tag{123}$$

We are adopt markings

$$\{x\} \equiv \{a,b\} \qquad \wedge \qquad x = a + \frac{b}{2} \in R \tag{124}$$

Now on the basis of (123) the relativistic mass of inertia of body in U_2 system, seen from U_1 system, when an assumption is fulfilled (122), is expressed in dynamics $\{x\}$ by the following formula

$$m_{2/1}^{\{x\}} = m_0 \left[\frac{1}{1-(v_{2/1}/c)^2} \right]^x \tag{125}$$

Each such relativistic mass defines a different relativistic dynamics.

According to presented examples, based on formulas (27) and (125), the momentum in dynamics $\{x\}$ is expressed by the following formula

$$p_{2/1}^{\{x\}} = \int_0^{v_{2/1}} dp_{2/1}^{\{x\}} = \int_0^{v_{2/1}} m_{2/1}^{\{x\}} \cdot dv_{2/1} = m_0 \int_0^{v_{2/1}} \left[\frac{1}{1-(v_{2/1}/c)^2} \right]^x dv_{2/1} \tag{126}$$

$$p_{2/1}^{\{x\}} = m_0 c^{2x} \int_0^{v_{2/1}} \frac{1}{(c^2 - v_{2/1}^2)^x} dv_{2/1} \tag{127}$$

According to presented examples, based on formulas (28) and (125), the kinetic energy in dynamics $\{x\}$ is expressed by the following formula

$$E_{2/1}^{\{x\}} = \int_0^{v_{2/1}} dE_{2/1}^{\{x\}} = \int_0^{v_{2/1}} m_{2/1}^{\{x\}} \cdot v_{2/1} \cdot dv_{2/1} = m_0 \int_0^{v_{2/1}} \left[\frac{1}{1-(v_{2/1}/c)^2} \right]^x v_{2/1} dv_{2/1} \tag{128}$$

$$E_{2/1}^{\{x\}} = m_0 c^{2x} \int_0^{v_{2/1}} \frac{v_{2/1}}{(c^2 - v_{2/1}^2)^x} dv_{2/1} \tag{129}$$

According to presented examples, based on formulas (24), (25) and (20), (23), the relation between forces in dynamics $\{x\}$ is expressed by the following formula

$$\frac{F_{2/1}^{\{x\}}}{F_{2/2}} = \frac{m_{2/1}^{\{x\}} \dfrac{dv_{2/1}}{dt_1}}{m_0 \dfrac{dv_{2/2}}{dt_2}} = \frac{m_{2/1}^{\{x\}} \dfrac{dv_{2/1}}{dt_1}}{m_0 \dfrac{dv_{2/1}}{1-(v_{2/1}/c)^2} \cdot \dfrac{1}{\sqrt{1-(v_{2/1}/c)^2} \cdot dt_1}} = \frac{m_{2/1}^{\{x\}}}{m_0} (1-(v_{2/1}/c)^2)^{3/2} \tag{130}$$

On the basis of (125) we receive

$$\frac{F_{2/1}^{\{x\}}}{F_{2/2}} = \left[\frac{1}{1-(v_{2/1}/c)^2} \right]^x (1-(v_{2/1}/c)^2)^{3/2} = \left[\frac{1}{1-(v_{2/1}/c)^2} \right]^{x-\frac{3}{2}} \tag{131}$$

6. Summary of dynamics

Summary derived formulas for momentum and kinetic energy:

Dynamics **x = 0**

$$p_{2/1}^{m} = m_0 v_{2/1} \tag{132}$$

$$E_{2/1}^{m} = \frac{m_0 v_{2/1}^2}{2} \tag{133}$$

Dynamics **x = 1/2**

$$p_{2/1}^{m/\Delta t} = m_0 c \cdot \arcsin\frac{v_{2/1}}{c} = m_0 v_{2/1} \frac{\arcsin(v_{2/1}/c)}{v_{2/1}/c} \tag{134}$$

$$E_{2/1}^{m/\Delta t} = m_0 c^2 (1 - \sqrt{1-(v_{2/1}/c)^2}) = \frac{m_0 v_{2/1}^2}{2} \frac{2}{1+\sqrt{1-(v_{2/1}/c)^2}} \tag{135}$$

Dynamics **x = 1**

$$p_{2/1}^{\Delta p} = \frac{m_0 c}{2} \ln\left(\frac{c+v_{2/1}}{c-v_{2/1}}\right) = m_0 v_{2/1} \ln\left(\frac{c+v_{2/1}}{c-v_{2/1}}\right)^{\frac{c}{2v_{2/1}}} \tag{136}$$

$$E_{2/1}^{\Delta p} = \frac{m_0 c^2}{2} \ln\frac{1}{1-(v_{2/1}/c)^2} = \frac{m_0 v_{2/1}^2}{2} \ln\frac{1}{[1-(v_{2/1}/c)^2]^{(c/v_{2/1})^2}} \tag{137}$$

Dynamics **x = 3/2**
(currently recognized STR dynamics)

$$p_{2/1}^{F} = m_0 v_{2/1} \frac{1}{\sqrt{1-(v_{2/1}/c)^2}} \tag{138}$$

$$E_{2/1}^{F} = m_0 c^2 \frac{1}{\sqrt{1-(v_{2/1}/c)^2}} - m_0 c^2 = \frac{m_0 v_{2/1}^2}{2} \frac{2}{\sqrt{1-\frac{v_{2/1}^2}{c^2}}\left(1+\sqrt{1-\frac{v_{2/1}^2}{c^2}}\right)} \tag{139}$$

Dynamics **x = 2**

$$p_{2/1}^{F/\Delta t} = m_0 v_{2/1} \frac{1}{2}\left[\frac{1}{1-(v_{2/1}/c)^2} + \ln\left(\frac{c+v_{2/1}}{c-v_{2/1}}\right)^{\frac{c}{2v_{2/1}}}\right] \tag{140}$$

$$E_{2/1}^{F/\Delta t} = \frac{m_0 v_{2/1}^2}{2} \frac{1}{1-(v_{2/1}/c)^2} \tag{141}$$

In Figure 4 were compared momentums from derived relativistic dynamics.

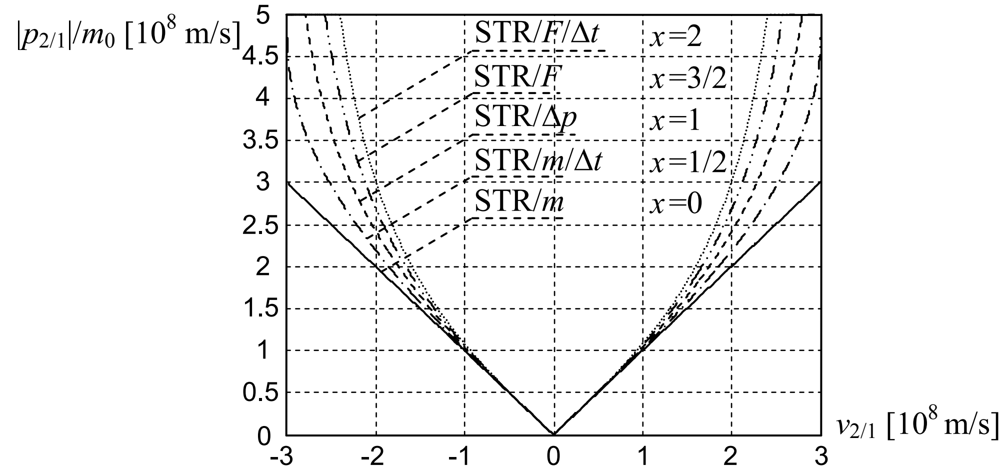

Fig. 4. Module of the momentum in dynamics:
STR/*m* (x=0), STR/*m*/Δ*t* (x=1/2), STR/Δ*p* (x=1), STR/*F* (x=3/2) and STR/*F*/Δ*t* (x=2).

In Figure 5 were compared kinetic energies from derived relativistic dynamics.

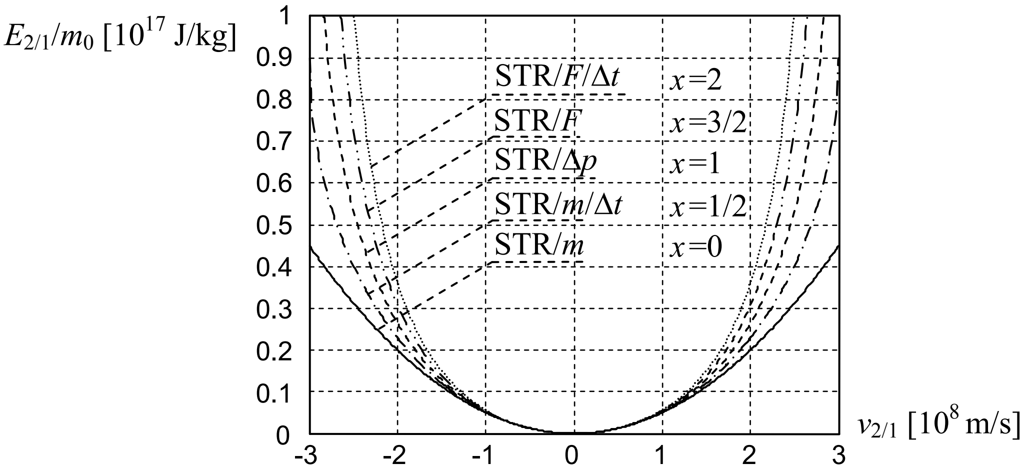

Fig. 5. Kinetic energies in dynamics:
STR/*m* (x=0), STR/*m*/Δ*t* (x=1/2), STR/Δ*p* (x=1), STR/*F* (x=3/2) and STR/*F*/Δ*t* (x=2).

In Figure 6 were compared relation between measurements of the same.

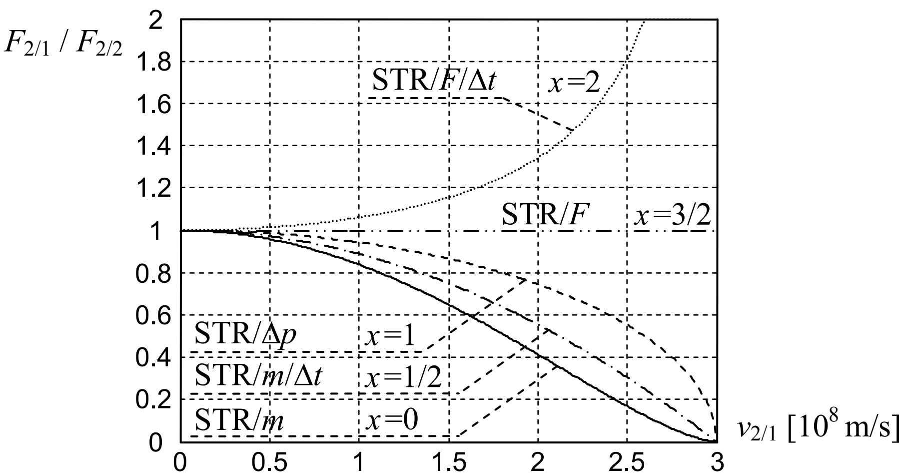

Fig. 6. Relation between measurements of the same force by two different observers in dynamics:
STR/*m* (x=0), STR/*m*/Δ*t* (x=1/2), STR/Δ*p* (x=1), STR/*F* (x=3/2) and STR/*F*/Δ*t* (x=2).

7. Even more general form of dynamics

Relation (125) to the relativistic mass can be even more generalized. In the general case, it is possible to assume that the relativistic mass is expressed by the following formula

$$m_{2/1}^{\{f\}} = m_0 \cdot f(v_{2/1}) \tag{142}$$

Where $f(v_{2/1})$ is any continuous function with the following properties

$$f(v_{2/1}) \geq 0 \tag{143}$$

$$f(0) = 1 \tag{144}$$

$$f(v_{2/1}) = f(-v_{2/1}) \tag{145}$$

Each function $f(v_{2/1})$ defines a different dynamics of the Special Theory of Relativity.

8. Final conclusions

The article presents our author's method of deriving dynamics in the Special Theory of Relativity. Five examples of such deriving were shown.

Derivation of dynamics is based on two formulas applicable in the kinematics of STR, i.e. (20) and (23). In order to derive the dynamics of STR, it is necessary to adopt an additional assumption in kinematics, which allows the concept of mass, kinetic energy and momentum to be introduced into the theory.

The dynamics of STR/*F* is nowadays recognized as the dynamics of the Special Relativity Theory. It is based on the assumption that each force parallel to *x*-axis has the same value for the observer from each inertial frame of reference. However, other dynamics are possible in accordance with the kinematics of the Special Theory of Relativity. In order to derive them, it is necessary to base on a different assumption.

Decision which from all possible dynamics of the Special Theory of Relativity is a correct model of real processes, should be one of the most important tasks of future physics. A calorimeter can be useful for verification of different dynamics. This device can measure the amount of heat released when stopping particles to high speed. On this basis, it is possible to determine graphs of the kinetic energy of accelerated particles as a function of their velocity, analogous to those presented in Figure 5. On this basis, it is possible to indicate the dynamics in which the kinetic energy of particles is compatible with experiments.

The fact that as a part of the Special Theory of Relativity, numerous dynamics can be derived greatly undermines the authenticity of the formula $E = mc^2$. According to our research, on the basis of relativistic mechanics, it is impossible to derive a formula expressing the internal energy of matter [4]. All derivations of this formula are wrong. The relation between mass and energy ($E = mc^2$) can be introduced into the STR as an independent assumption, but it does not result from Lorentz transformation, nor from the assumption (29) on which the dynamics of STR is based. But then there is a need to experimentally show what exactly is the form of such a dependency (e.g. why not $E = mc^2/2$) and experimentally investigate whether sometimes the form of such a dependency does not depend on the type of matter that this formula regards.

The presented method of dynamism derivation can also be used in other theory of body kinematics. In the monograph [3] we have used it to derive four dynamics in the Special Theory of Ether.

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