Conjecture that any square of a prime p^2 can be written as p+q+(nq-n±1) where q and nq-n±1 primes

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Abstract. In this paper I make the following conjecture: Any square of a prime p^2, where p > 3, can be written as p + q + (n*q – n + 1) or as p + q + (n*q - n – 1), where q and n*q – n + 1 respectively n*q - n – 1 are primes and n positive integer. Examples: 11^2 = 121 = 11 + 37 + (2*37 – 1), where 37 and 2*37 – 1 = 73 are primes; 13^2 = 169 = 13 + 53 + (2*53 – 3), where 53 and 2*53 – 3 = 103 are primes. An equivalent formulation of the conjecture is that for any prime p, p > 3, there exist n positive integer such that one of the numbers q = (p^2 – p + n – 1)/(n + 1) or q = p^2 – p + n + 1)/(n + 1) is prime satisfying also the condition that p^2 – p – q is prime.

Conjecture:

Any square of a prime p^2, where p > 3, can be written as p + q + (n*q – n + 1) or as p + q + (n*q - n – 1), where q and n*q – n + 1 respectively n*q - n – 1 are primes and n positive integer.

Note that p^2 – p – (6*k – 1) is divisible by 3 for p of the form 6*h – 1 so in this case q can only be of the form 6*h + 1 while both p^2 – p – (6*k ± 1) are not divisible by 3 for p of the form 6*h + 1.

Verifying the conjecture:
(up to p = 41)

: p^2 = 5^2 = 25 = 5 + 7 + (2*7 – 1) and [7, 13] primes;

: p^2 = 7^2 = 49 = 7 + 5 + (9*5 – 8) and [5, 37] primes; also p^2 = 7^2 = 49 = 7 + 11 + (3*11 – 2) and [11, 31] primes;

: p^2 = 11^2 = 121 = 11 + 37 + (2*37 – 1) and [37, 73] primes;

: p^2 = 13^2 = 169 = 13 + 53 + (2*53 – 3) and [53, 103] primes;

: p^2 = 17^2 = 289 = 17 + 31 + (8*31 – 7) and [31, 241] primes;

: p^2 = 19^2 = 361 = 19 + 11 + (31*11 – 30) and [11, 331] primes;

: p^2 = 23^2 = 529 = 23 + 7 + (83*7 – 82) and [7, 499] primes; also p^2 = 23^2 = 529 = 23 + 9 + (27*19 – 26) and [23, 487] primes; also p^2 = 23^2 = 529 = 23 + 43 + (11*43 – 10) and [43, 463] primes; also p^2 = 23^2 = 529 = 23 + 73 + (6*73 – 5) and [73, 433] primes; also p^2 = 23^2 = 529 = 23 + 127 + (3*127 – 2) and [127, 379] primes;

: p^2 = 29^2 = 841 = 29 + 271 + (2*271 – 1) and [271, 541] primes;

: p^2 = 31^2 = 961 = 31 + 11 + (92*11 – 93) and [11, 919] primes; also p^2 = 31^2 = 961 = 31 + 311 + (2*311 – 3) and [311, 619] primes;

: p^2 = 37^2 = 1369 = 37 + 223 + (5*223 - 6) and [223, 1109] primes; also p^2 = 37^2 = 1369 = 37 + 11 + (132*11 - 131) and [223, 1109] primes;

: p^2 = 41^2 = 1681 = 41 + 19 + (90*19 – 89) and [19, 1621] primes; also p^2 = 41^2 = 1681 = 41 + 43 + (38*43 – 37) and [43, 1597] primes; also p^2 = 41^2 = 1681 = 41 + 43 + (38*43 – 37); also p^2 = 41^2 = 1681 = 41 + 547 + (2*547 – 1).

Note that p^2 = 23^2 can be written the way mentioned for five different pairs of primes [q, n*q – n + 1]; also p^2 = 41^2 can be written the way mentioned for four different pairs of primes [q, n*q – n + 1]

Note that, from the primes above, 13, 31 and 37 can be written as p + q + (n*q – n – 1) and 5, 7, 11, 17, 19, 23, 29, 41 can be written as p + q + (n*q – n + 1) and 37 can be written in both ways.